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Abstract
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Keywords
Nondestructive Evaluation

Disciplines
Materials Science and Engineering
MICROWAVE EDDY-CURRENT TECHNIQUES
FOR QUANTITATIVE NDE

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ABSTRACT

The objectives of this work are to develop an electromagnetic scattering model that can be used to predict the scattering from a crack in a conducting surface and to evaluate the feasibility of using this model in conjunction with microwave-measurement techniques to determine the dimensions of such a crack. Such a theory has been developed, and its derivation is presented. Theory and experiment are compared for rectangular slots measured at 100 GHz, and the agreement is found to be good. The necessary measurement protocol for determining the dimensions of a crack is discussed, and an example of inverting the measured data to determine the dimensions of a rectangular slot is presented.

INTRODUCTION

In the low-frequency eddy-current testing of metals, currents are caused to flow in a test specimen by placing it in the magnetic field of an induction coil. The flow of currents is affected by the electrical properties and shape of the test specimen, and by the presence of discontinuities and defects. In turn, these currents react on the exciting coil and affect its impedance. Thus, the presence of a defect is determined by monitoring the test coil impedance.

Such eddy-current tests are typically conducted at frequencies of less than 1 MHz where induction fields predominate and the electromagnetic wavelength is greater than 300 m. However, in quantitative NDE, where it is desired to obtain the defect dimensions from an analysis of the measured data, the use of such low frequencies does not provide the degree of sensitivity to changes in defect dimensions that is necessary for obtaining an accurate determination of these dimensions from an inversion of the eddy-current data. The problem of obtaining sufficient accuracy becomes more difficult as the flaws of interest become smaller.

This problem would be alleviated if higher frequencies were to be used in eddy-current inspection. Thus, the work reported here addresses the possibility of conducting eddy-current measurements in the microwave-frequency regime (1 GHz to 100 GHz). Previous work using frequencies in the range 10 GHz to 30 GHz has shown that good sensitivity to small cracks can be obtained, and that there is a clear correlation between crack depth and the detected signal.

In using microwave frequencies, the radiation fields associated with the sensors become an important consideration, and the physics involved is best described in terms of fields and waves. For example, a defect should be thought of as producing a change in the scattering of electromagnetic waves from the metal surface. It should also be noted that, since the use of microwave frequencies causes the currents induced in the test object to flow essentially on the surface (i.e., the skin depth is typically less than 1 μm at 100 GHz), microwave eddy-current techniques are limited in metals to surface inspection, e.g., to detection and characterization of surface-breaking cracks.

In order to invert the measured microwave eddy-current data to obtain crack dimensions, it is necessary to have a theoretical model that relates the electromagnetic scattering from a crack to the crack dimensions. The requisite theory should be variational so that approximate solutions for irregular crack geometries can be obtained. In addition, the theory should not be restricted to any particular frequency range so it can be used to clarify any distinctions between conventional (low-frequency) and microwave (high-frequency) eddy-current techniques. Finally, such a model would be useful for establishing an optimum measurement protocol.

A suitable general theory has been developed, and its derivation is outlined below. Then, as an example, the theory is applied to cross-polarized backscattering of a plane wave from a rectangular slot in an aluminum plate, and the measurement protocol necessary to determine the slot dimensions is discussed. The results of this theoretical example are compared with experimental results obtained at 100 GHz and are found to be in good agreement. Finally, graphical inversion of the theoretical electromagnetic scattering from the slot is performed to illustrate the process of obtaining the slot dimensions from the measured data.

A THEORETICAL MODEL FOR ELECTROMAGNETIC SCATTERING FROM SURFACE-BREAKING CRACKS IN METALS

A general electromagnetic scattering-measurement system is shown schematically in Fig. 1, which illustrates the general bistatic case where the transmitter and receiver are separated. The probes are arbitrary, but it is assumed that a single electromagnetic mode propagates at some point in the transmission line(s) [waveguide(s)] that connect the probe(s) to the transmitter and receiver. It is also assumed that the metal shields and test body exhibit finite conductivity.

The starting point of the theory is the Lorentz reciprocity theorem, which involves an integral

References are listed at the end of the paper.
over the closed surface, \( S \), equal to \( S_a + S_b + S_g + S_e + S_m \). If there are no sources enclosed within the volume defined by \( S \), the theorem takes the following form:

\[
\iiint_S (\hat{E} \times \hat{H}' - \hat{E}' \times \hat{H}) \cdot \hat{n} \, dS = 0 \tag{1}
\]

where \( \hat{n} \) is a unit vector that points outward from the enclosed volume. The quantities \( \hat{E} \) and \( \hat{H} \) are the electric and magnetic fields, respectively, that exist on the surface \( S \). The unprimed and primed fields are defined by the following conditions:

- Unprimed field—no crack is present, "a" is a transmitter, "b" is a receiver.
- Primed field—a crack is present, "a" is a transmitter, "b" is a receiver.

In addition, if gyromagnetic media exist within the closed volume, all dielectric and magnetic fields within the volume must be reversed in the two cases.\(^7\)

The evaluation of the surface integral in Eq. (1) requires a knowledge of the fields on the various parts of the surface \( S \). First, on all the metal surfaces the tangential electric field can be related to the surface magnetic field by means of a surface impedance, \( Z_s \):

\[
\hat{E} = -Z_s \hat{n} \times (\hat{n} \times \hat{A}) \tag{2}
\]

For the plane waves, \( Z_s \) is related to the skin depth, \( \delta_s \), by the well-known formula

\[
Z_s = \frac{j\omega \mu}{\sigma \delta_s} \tag{3}
\]

where \( \sigma \) is the conductivity of the metal. Second, on the surface at infinity one has the following radiation condition:

\[
\hat{E} = j\omega n \times (\hat{n} \times \hat{A}) \tag{4a}
\]

\[
\hat{H} = -j\frac{\omega}{\eta_0} (\hat{n} \times \hat{A}) \tag{4b}
\]

where \( \omega \) is the radian frequency, \( \eta_0 \) is the intrinsic impedance of free space, and \( \hat{A} \) is the vector potential. Third, since single modes are assumed to propagate in the transmission lines, it can be shown that the integrals over the transmission-line cross sections reduce to the following form:

\[
\iiint_S (\hat{E} \times \hat{H}' - \hat{E}' \times \hat{H}) \cdot \hat{n} \, dS = -4\pi P_a \tag{5a}
\]

\[
\iiint_S (\hat{E} \times \hat{H}' - \hat{E}' \times \hat{H}) \cdot \hat{n} \, dS = 4\pi P_b' \tag{5b}
\]

By using Eqs. (2), (4), (5), and (6), one can convert Eq. (1) into the following form:

\[
4\pi (P_b' - P_a) = -Z_s \iiint_S (\hat{E}' \cdot \hat{n}) \, dS + \iiint_S (\hat{H}' \cdot \hat{n}) \, dS \tag{7}
\]

Equation (7) expresses the difference between the scattering coefficients measured with probes "a" and "b" in terms of fields that exist in the crack mouth when it is either open or covered by a conductor having surface impedance \( Z_s \). Hence, to relate this theoretical result to an experimental measurement, the measurement system must be capable of measuring this difference in scattering coefficients.

In conventional eddy-current systems, one usually measures the change in impedance, \( Z' - Z \), of a probe as it passes over a crack, rather than the change in scattering coefficient. In the monostatic case, the distinction between "a" and "b" disappears, and one finds from transmission-line theory that the change in scattering coefficient and the change in impedance are related by

\[
4\pi (P_b' - P_a) = 11' (Z' - Z) \tag{8}
\]

where \( I \) and \( I' \) are the total currents flowing in the transmission line without and with a crack present, respectively. However, the scattering coefficients will be retained in the present development because they are more fundamental to a wave analysis.

Equation (7) is a linear integral equation that relates the unknown reflection coefficient, \( r' \), to the unknown tangential magnetic field, \( \hat{H}' \), and magnetic current, \( \hat{M}' \), in the crack mouth. The quantity \( \hat{H} \) is the magnetic field that exists on the surface of the metal test object in the absence of a crack. One way of solving this equation is to use the method of moments.\(^6\) Such a solution possesses the variational characteristics\(^7\) that are desired.
In the moment-method solution, one expands $\mathbf{M}'$ in a set of basis functions, $\mathbf{M}_n$:

$$
\mathbf{M}' = \sum_{n=1}^{N} V_n \mathbf{M}_n.
$$

(9)

The only conditions on the $\mathbf{M}_n$ are that they be linearly independent, and that their superposition approximate $\mathbf{M}'$ "reasonably well" (herein lies the "art" in the method of moments). One also needs to invoke the condition that the tangential magnetic field be continuous across the crack mouth; i.e.,

$$
\mathbf{H}_t^r = \mathbf{H}_t^i + \mathbf{H}_t^r (\mathbf{M}') = \mathbf{H}_t^c (-\mathbf{M}')
$$

(10)

where $\mathbf{H}_t^i$ is the incident magnetic field, $\mathbf{H}_t^r (\mathbf{M}')$ is the induced magnetic field just outside the crack mouth, and $\mathbf{H}_t^c (-\mathbf{M}')$ is the induced magnetic field just inside the crack mouth. $\mathbf{H}_t^c$ is a function of $\mathbf{M}'$ rather than of $\mathbf{M}'$ because of the equivalence principle. Also, because the $\mathbf{H}_t$ operators are linear, Eq. (9) can be substituted into Eq. (10) to give the result

$$
\sum_{n=1}^{N} V_n \mathbf{H}_t^r (\mathbf{M}_n) = \mathbf{H}_t^c (-\mathbf{M}')
$$

(11)

Next, one chooses a set of testing functions, $\mathbf{W}_m$, that are similar (but not necessarily equal) to the $\mathbf{M}_n$. By taking the dot product of each $\mathbf{W}_m$ with Eq. (11) and integrating that product over the crack mouth, one obtains the following result:

$$
\sum_{n=1}^{N} V_n \mathbf{H}_t^r (\mathbf{M}_n) + \sum_{n=1}^{N} V_n \mathbf{H}_t^c (\mathbf{M}_n) = -\mathbf{H}_t^c
$$

(12)

Thus, Eq. (10) has been converted into a set of scalar inhomogeneous linear equations, which thus can be solved for the unknown coefficients, $V_n$.

The result of eliminating the $V_n$ in Eq. (9) and substituting the result into Eq. (7) is best expressed in matrix form:

$$
\mathbf{F}_{LF CO} \mathbf{C}_c \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14a)

$$
\mathbf{C}_c \mathbf{Y}_n = \mathbf{F}_{LF CO} \mathbf{C}_c \mathbf{C}_c \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14b)

$$
\mathbf{y}_n = \mathbf{F}_{LF CO} \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14c)

$$
\mathbf{Y}_n = \mathbf{Y}_n \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14d)

$$
\mathbf{Y}_n = \mathbf{Y}_n \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14e)

$$
\mathbf{y}_n = \mathbf{Y}_n \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14f)

$$
\mathbf{y}_n = \mathbf{Y}_n \mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14g)

Here $\mathbf{a}_w$ and $\mathbf{a}_w$ are unit vectors pointing along $\mathbf{M}$ and $\mathbf{M}$, respectively.

Equation (13) applies to all electromagnetic eddy-current measurements of cracks. At this stage, no restrictions have been made on the operating frequency or the distance between the probe(s) and the sample. The first term in the equation--i.e., the one involving $Z_s$--can be interpreted as the change in scattering caused by removing a small volume (the crack) of metal that has finite conductivity. The second term can be interpreted as the change in scattering caused by energy being stored in the crack and being reradiated. The coefficient $F_{LF CO}$ contains the effects of changing the distance between the probe and the sample surface (lift off), and of the crack orientation. At low frequencies, one finds that the finite-conductivity term dominates; at high frequencies, where the crack becomes resonant, the effects of energy storage become predominant. Thus, the theory provides a clear distinction between conventional and microwave eddy-current techniques.

AN EXAMPLE

To illustrate how Eq. (13) can be evaluated in a specific case, consider the simple case of perpendicular plane-wave excitation of a rectangular slot cut in a perfectly-conducting plane. The geometry of such a slot is shown in Fig. 2. Assuming the receiver is cross-polarized to the transmitter, $\alpha = 0$ and Eq. (13) becomes

$$
\mathbf{F}_{LF CO} \mathbf{C}_c = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(13)

where the normalized coefficients and matrix elements are given by

$$
\mathbf{F}_{LF CO} = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14a)

$$
\mathbf{C}_c = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14b)

$$
\mathbf{y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14c)

$$
\mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14d)

$$
\mathbf{Y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14e)

$$
\mathbf{y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14f)

$$
\mathbf{y}_n = \frac{n_0}{4\pi} \int \frac{\sigma_0 (\mathbf{M}' \cdot \mathbf{H}) (\mathbf{a}_w \cdot \mathbf{H}_t^r)}{\mathbf{S}_c} d\mathbf{S}
$$

(14g)
FIGURE 2 RECTANGULAR SLOT GEOMETRY

\[
Y = Y_r + Y_c
\]

where the slot admittance is \( Y = Y_r + Y_c \) and it has been assumed that only one basis function is needed to approximate the fields in the slot mouth.

In this case, one can take advantage of the knowledge that exists concerning the solution for the fields in the aperture of a narrow-slot antenna. This solution should provide a good resonance-region approximation to the fields in the mouth of a slot that is deeper than it is wide. Hence, one chooses the basis function and test function to be

\[
\bar{M}_1 = \hat{a}_x \sin \left( \frac{k}{2} \left( x - \frac{a}{2} \right) \right) = \bar{W}_1
\]

where the slot admittance is \( Y = Y_r + Y_c \) and it has been assumed that only one basis function is needed to approximate the fields in the slot mouth.

Now, for a plane incident wave with magnetic field \( H_0 \), one has

\[
\hat{a}_x \cdot \bar{M}_1 = H_0 \cos \theta
\]

and

\[
\hat{a}_x \cdot \bar{M}_2 = 2H_0 \sin \theta
\]

where \( \theta \) is the angle between \( \bar{M}_2 \) and the x-axis and it has been assumed that there are no reflections from the probe. Thus, using Eqs. (16), (17), and (18) the coefficients of the normalized slot impedance \( (1/\eta_0 Y_s) \) in Eq. (15) become

\[
F_{LF} = \frac{\tau \bar{M}_2 \hat{a}_x}{\hat{a}_x A}
\]

The term resonance region refers to frequencies where the slot length is equal to or greater than one-half wavelength.

\[
F_{CO} = \sin 2\theta
\]

\[
C_1 = \sqrt{\frac{2}{\tan \theta}} \frac{4}{k \sqrt{ab}} \sin^2 \left( \frac{k a}{4} \right) \quad \text{and}
\]

\[
C'_1 = \frac{1}{\sqrt{2 \tan \theta}} \frac{4}{k \sqrt{ab}} \sin^2 \left( \frac{k a}{4} \right)
\]

Here, the quantity \( C_1C'_1 \) is a slot coupling factor that gives the frequency dependence of the coupling between the slot and the incident field. The normalized slot admittance, \( Y_0 Y_s \), is a slot parameter that is independent of the excitation. It is important to note that \( Y_s \) is the sum of a radiation admittance, \( Y_r \), that depends on the boundary conditions external to the slot, and a cavity admittance, \( Y_c \), that depends on the geometry inside the slot. Thus, if the geometry of the slot cavity changes, but not the geometry of the slot mouth, only the cavity admittance needs to be recalculated.

To calculate the cavity admittance for a rectangular slot, one can expand \( \bar{R}_c \) in transverse-electric waveguide modes. Then Eq. (14e) becomes

\[
Y_c = -\frac{J}{\eta_0} \sum_{q \neq d} \frac{c_{th} \left( \frac{r}{r_q} \right)}{r_q \sin \left( \frac{ka}{2} \right)} \sin^2 \left( \frac{ka}{2} \right)
\]

where

\[
r_q^2 = (q \pi / a)^2 - k^2
\]

This result differs slightly from that obtained in Ref. 5 because of a difference in the definition of slot voltage.

The radiation admittance Eq. (14d) can be calculated by expanding the aperture fields in a plane-wave spectrum. Assuming \( kb << 1 \), one has

\[
\text{Re}(Y_r) = \frac{1}{\eta_0} \left\{ \text{Ci} (ka) \left[ \text{Ci} (ka) - 1/2 \text{Si} (2ka) \right] \cos ka \right. \\
\left. + \left[ \text{Si} (ka) - 1/2 \text{Ci} (2ka) \right] \sin ka \right\}
\]

and

\[
\text{Im}(Y_r) = \frac{1}{\eta_0} \left\{ \text{Si} (ka) \left[ \text{Si} (ka) - 1/2 \text{Ci} (2ka) \right] \cos ka \right. \\
\left. + \left[ \text{Ci} (ka) - 1/2 \text{Si} (2ka) \right] \sin ka \right\}
\]

where

\[
\text{Ci} (x) = \int_0^x \frac{1 - \cos u}{u} \, du \quad \text{and}
\]

\[
\text{Si} (x) = \int_0^x \frac{\sin u}{u} \, du
\]
to show explicitly how the liftoff factor [Eq. (19a)] depends on these parameters, it is necessary to relate $H_0$ and $PA$. For example, if the source antenna were equivalent to a magnetic dipole located at a large distance, $R$, from the slot, one would have

$$F_{LF} = \frac{-3}{16\pi} \frac{ab}{R^2} e^{-2kR}$$

(24)

Hence, if $k$ is large (wavelength is small), the dominant effect of changing $R$ will be to change the phase of the reflection coefficient. Thus, the locus of the reflection coefficient in the reflection-coefficient plane as $R$ is changed will be a nearly circular arc. This behavior can be used to discriminate between the signals produced by variations in liftoff and by a bona fide crack.

Having obtained Eqs. (19), (20), and (22), it is now possible to calculate the cross-polarized scattered power given by

$$P_{HV} = \left| T_{12} \right|^2 P_A$$

(25)

as a function of the frequency and slot dimensions. It is convenient, however, to normalize the scattered power first to suppress the dependence of the result on the characteristics of the probe and slot orientation, viz.,

$$P_{HV} \left( \frac{a}{d}, \frac{b}{d}, \frac{h_0^2}{a^2 b^2}, \frac{\omega}{c} \right) = \frac{64}{k^4 a^8 b^8} \sin^8 \left( \frac{ka}{4} \right) \frac{1}{\left| n_0 \right|^2}$$

(26)

The right-hand side of this equation is plotted (in dB) in Fig. 3 as a function of the product of frequency and slot length, with the ratios of slot width and depth to slot length as parameters. In this figure, $ka/d \geq 1$ defines the resonance region where electromagnetic energy can propagate into the slot with low attenuation. In this region, the scattering is seen to be a strong function of slot depth, which is a desirable characteristic from the standpoint of obtaining an accurate determination of depth from a scattering measurement. For frequencies below the resonance region, the fields inside the slot are evanescent, and so the sensitivity of the scattering to changes in slot depth decreases rapidly as the slot approaches one slot length in depth. Thus, eddy-current measurements for determining slot depth quantitatively are best conducted in the resonance region. It should be noted, however, that more than one slot depth can give the same value of scattered power in this frequency region, and so it may be necessary to conduct measurements at more than one frequency to resolve this ambiguity.

So far, this example has neglected the contribution of the surface-impedance term to the scattering. Indeed, in the cross-polarized case, $C_C = 0$ in Eq. (13), and the other term containing $Z_s$ is small for most metals. On the other hand, in the co-polarized case, $C_C = 1$, and $P_Q = Z_s^2 \cos^4 \theta$. At very low frequencies, $1/\sqrt{f} = 0$, and the quasi-static scattered power, $P_Q$, is determined entirely by the surface impedance.

Of course, as the frequency is increased from zero, the energy stored in the slot also contributes to the scattering. For a deep slot $(d/a \approx 1)$, the Rayleigh scattering term, $P_R$, can be approximated by expanding Eq. (15) for small $ka$. The result is

$$P_R \left( \frac{a}{d}, \frac{b}{d}, \frac{h_0^2}{a^2 b^2}, \frac{\omega}{c} \cos^4 \theta \right) \approx \frac{\pi^3}{4} \frac{Z_s^2}{k^4 a^8 b^8}$$

(27)

These two normalized scattered powers are plotted as functions of frequency in Fig. 4. In making the computations it was assumed that $a = 2.5$ mm and that the material was aluminum with $Z_s = 3.26 \times 10^{-7} \sqrt{\omega (1 + j)}$ ohms, where $\omega$ is the frequency. This figure clearly shows the dominance of the surface-impedance term (quasi-static term) at low frequencies and the dominance of the energy-storage term (Rayleigh term) at high frequencies. The crossover occurs in this example at about 10 kHz. It is important to note that neither of these low-frequency approximations to the slot scattering contains any depth information. Hence, one concludes from this example that eddy-current measurements of crack depth are best conducted at frequencies where the wavelength is commensurate with the crack length.
This system uses an orthomode coupler to discriminate against co-polarized backscatter, and a homodyne detection system to provide in-phase (I) and quadrature (Q) output signals. The sensitivity of this system is currently about -75 dBm; this sensitivity is determined by the degree to which the transmitting and receiving portions of the system can be isolated in the absence of a crack by the orthomode coupler. The antenna used in the system is a lens-focused horn with a beamwidth at its focal point of about 3.5 mm at the operating frequency of 100 GHz.

An aluminum plate with six slots of different sizes electrodischarge-machined into its surface was prepared according to the layout shown in Fig. 6.* Slots 1, 2, and 3 have a cross section (a x b) of 2.54 mm x 0.25 mm; slots 4, 5, and 6 have a cross section of 1.27 mm x 0.25 mm. Thus, a/b = 10 for the first set of slots, and a/b = 5 for the second set of slots. Also, at 100 GHz, ka/\pi = 1.7 for the first set, and ka/\pi = 0.85 for the second set. Finally, slots 1 and 4 were specified to be 0.25 mm deep, slots 2 and 5, 0.5 mm deep, and slots 3 and 6, 1.0 mm deep.

The measured in-phase and quadrature voltages obtained at 98 GHz by translating the slots through the microwave beam are shown in Fig. 7. The plate was aligned perpendicularly to the microwave beam and was positioned so that a linear translation of the plate caused the centers of the slots to pass through the center of the beam, thereby maximizing the peak signal obtained from each slot. The slots were aligned with their lengths at an angle of about 60° to the electric polarization vector, thus ensuring that some of the incident energy would be coupled into the cross-polarized mode by each slot.

The in-phase and quadrature voltages were combined to form a polar display on a storage oscilloscope--common practice in low-frequency eddy-current work. The corresponding polar display for slots 1, 2, and 3 (the 2.5 mm-long slots) is shown in Fig. 7(c). This type of display clearly shows the differences in the amplitudes and phases of the scattered signals produced by the different-depth slots. In this case, the signal produced by slot 3 (1.0 mm deep) is very different from the signals produced by the other slots. However, all three signals are clearly distinguishable.

The approximate model (described previously) that assumes a sinusoidal distribution of electric field in the slot mouth can be used to calculate the theoretical slot response for the parameters used in the experiment. For a focused microwave beam, the change in excitation of the slot caused by moving the slot through the beam can be approximated by setting

\[
F_{LE} = e^{-j k x'^2 / R_0} \frac{2J_1(k x'/2)}{k x'^2/2}
\]  

where \( k \) is the wave number, \( x' \) is distance along the scanning direction measured from the center of the slot, \( R_0 \) is the distance between the microwave lens and the aluminum plate, and \( J_1 \) is the Bessel function of first kind and first order. Also, in this simple model, it is necessary to assume that the incident field is constant over the slot mouth for each position \( x' \).

* This test plate was prepared under the direction of Dr. O. Buck of the Rockwell International Science Center, Thousand Oaks, California.

** Figure 4 ** COMPARISON OF RAYLEIGH AND QUASI-STATIC SCATTERING FOR A DEEP SLOT

** An idealized measurement protocol **

Equation (13) provides the basis for defining an idealized eddy-current measurement protocol for determining crack dimensions. This protocol can be divided into four main steps:

1. Calibrate the system at each measurement frequency using a "standard crack" to determine the lift-off factor.
2. Detect the real crack while keeping the distance between the probe and the specimen the same as in the calibration.
3. Measure the crack in at least two different orientations to determine the orientation factor (assuming the crack length is much larger than the crack width).
4. Collect sufficient data to permit unambiguous inversion using the model to obtain the crack dimensions. (Ideally, a minimum data set would consist of amplitude and phase at two frequencies--the use of more frequencies may be required in the resonance region in order to resolve ambiguities).

Since the data will not be perfectly accurate and the crack geometry will not be known precisely, it is likely that statistical techniques, adaptive learning techniques, or both, will be required to obtain sufficient accuracy for crack dimensions determined from eddy-current measurements. In any case, Eq. (13) should provide a useful basis for designing experiments.

** Experiment **

The amplitude and phase of the cross-polarized backscattering from a series of rectangular slots in an aluminum plate were measured using the microwave system whose schematic diagram is shown in Fig. 5. This system uses an orthomode coupler to discriminate against co-polarized backscatter, and a homodyne detection system to provide in-phase (I) and quadrature (Q) output signals. The sensitivity of this
FIGURE 5  MICROWAVE SYSTEM FOR MEASURING CROSS-POLARIZED BACKSCATTER USING HOMODYNE DETECTION

FIGURE 6  LAYOUT OF SLOTTED ALUMINUM PLATE (slots are aligned in the x-direction)
The result of the calculation for slots 1, 2, and 3 is shown in Fig. 8. The absolute amplitude and phase is undetermined in this calculation; therefore, the theoretical plot has been normalized so that the peak response for slot 3 matches the experimental value for that slot. The experimental peak values of each slot response are indicated by Xs. A comparison of Figs. 7(c) and 8 shows remarkable agreement, considering the approximate nature of the model.

The experimental results obtained for slots 4, 5, and 6 are shown in Fig. 9. In that measurement, the gain was increased over that used for the larger slots and, as a result, liftoff effects became noticeable, as is evidenced by the high background or clutter in Figs. 9(a) and (b). However, as expected, the polar display [Fig. 9(c)] allows the slot signals to be clearly distinguished from the liftoff signal because the two types of signals are nearly orthogonal.

In this case, the length of the slots (1.27 mm) causes the operating frequency of 98 GHz to lie below the resonance region, with the result that changes in slot depth produce relatively little change in the phase of the scattered signal.

The corresponding theoretical response for these smaller slots is shown in Fig. 10. In this case, the theoretical plot was normalized to the experimental peak value for slot 6 after the clutter (liftoff signal) had been subtracted. Again, agreement between theory and experiment is fairly good.

Inversion

In view of the good agreement between theory and experiment, it appears worthwhile to examine the measurement-error sensitivity of an inversion process that is based on the simple model. Since all the slots were located at the same distance from the microwave lens and had the same orientation relative to the polarization of the incident wave, it was simplest to use one of the slots (slot 3) as a reference slot for calibrating the system via the model. Slot 2 was chosen as the unknown slot whose dimensions were being sought. Hence, the ratio of the measured complex signal for slot 2 to that for slot 3 was compared to the same ratio obtained from theory.

Ideally, measurements at two frequencies that are far enough apart to produce measureable changes in scattering are needed in order to determine all three dimensions of a slot, as was mentioned in the section on measurement protocol. However, the existing experimental system did not permit significant changes in operating frequency to be made, therefore it was necessary to assume that one of the slot dimensions was known. The slot length, a, was chosen for this dimension, as it is the most likely to be known.

The amplitude and phase of the relative scattering from slot 2 at 99.9 GHz are shown in Fig. 11 as functions of slot depth, with slot width as a parameter. It is assumed that a = 2.5 mm. Also indicated in the figure are the estimated ranges for
FIGURE 8  THEORETICAL SLOT RESPONSE FOR SLOTS 1, 2, AND 3 (x indicates measured peak value)
Figure 9: Measured slot response for slots 4, 5, and 6.
FIGURE 10  THEORETICAL SLOT RESPONSE FOR SLOTS 4, 5, AND 6 (x indicates measured peak value)
the measured data. The measurement errors corresponding to these ranges are: amplitude ±1 dB; phase, ±4°. One can see from the figure that, if the slot width can be estimated to within ±20%, the measured data determines the slot depth to within ±11% (the cross-hatched area).

It is interesting to note that, in this case, the use of amplitude data alone would only increase the uncertainty in the depth determination by a small amount, namely to ±14%. The slot depth measured from a scanning electron micrograph of a rubber replica of slot 2 is \(d/a = 0.21\). This value lies approximately in the center of the cross-hatched slot-depth range shown in Fig. 11.

If the cross-sectional dimensions of a crack cannot be obtained by microscopic examination or by some other means, all three of the crack dimensions must be obtained from the eddy-current measurement alone. The accuracy of the required inversion solution will depend on how sensitive the scattering is to changes in each crack dimension. Calculations using the model developed here show that this sensitivity depends on the product of crack length and operating frequency, with maximum sensitivity obtained in specific portions of the resonance region. Thus, obtaining maximum accuracy in determining crack size will necessitate the use of a frequency that is appropriate to the size range being measured.

**SUMMARY**

A general theory for the electromagnetic scattering from a surface-breaking crack in a conducting material has been developed. The theory is valid for any frequency, and provides a basis for defining a measurement protocol for the purpose of determining crack dimensions from eddy-current measurements. The theory also shows that, at low frequencies, eddy-current measurements of cracks are dominated by effects of finite conductivity while, at high frequencies, the measured signals are determined mostly by energy storage in the crack.

Approximate numerical results have been obtained for the case of a rectangular slot. This example reveals all of the essential characteristics of the backscattering as a function of frequency and slot dimensions, and gives insight into what can be expected for the behavior of a signal scattered by a real crack. For example, when the slot length is greater than one-half wavelength, resonances can occur; these resonances make the determination of slot depth from measured scattering accurate but introduce ambiguities. For smaller slot lengths, there is a one-to-one relation between slot depth and scattered signal, but the amplitude of the scattered energy is smaller and slot depths that are greater than one slot length are not well resolved. The effects of liftoff and slot orientation are also elucidated in the example.
Experimental results obtained at 100 GHz using electrodischarge-machined slots in an aluminum plate were found to be in good agreement with theory. The smallest available slot, which was 1.27 mm long, 0.25 mm wide, and 0.25 mm deep, could be distinguished from clutter (lift off) by using phase-sensitive detection and a polar display. An example of using the measured data and the theoretical model to determine slot depth was given.

One can conclude that it should be possible to obtain an accurate determination of the dimensions of a surface-breaking crack from microwave scattering measurements. Although further improvements in the theory are possible, questions concerning the practical and economic realization of the technique should be addressed first.

ACKNOWLEDGEMENT

Technical discussions with Dr. B. A. Auld of Stanford University and Dr. R. B. Thompson of Rockwell International Science Center were most helpful in this work. Thanks are also due to Dr. A. C. Phillips of SRI for designing the 3-kHz portion of the phase-sensitive detection system used in the measurements.

This work was sponsored by the Center for Advanced NDE operated by the Rockwell International Science Center under Contract F33615-74-C-5180.

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