"Portfolio Balance And Exchange Rate Stability"

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Abstract
Much of the ongoing debate concerning the relative advantages of a fixed vs. a flexible exchange rate regime has centered around the stabilizing or destabilizing effects of speculation and the magnitudes of the elasticities of demand for foreign goods and services (1). Using a "small-country" model which implicitly ignored portfolio balance effects, Mundell (1960) added another dimension to the controversy by demonstrating that the stability properties of either type of exchange rate system depend upon the degree of capital mobility. In particular, Mundell shows that when capital is perfectly mobile internationally, a fixed exchange rate ensures a direct approach towards equilibrium while a flexible rate can produce a cyclical approach. In contrast, if capital is immobile, a flexible exchange rate ensures a direct approach towards equilibrium while a fixed rate makes a cyclical approach likely.

Disciplines
Banking and Finance Law | International Business | Portfolio and Security Analysis | Taxation-Transnational

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"PORTFOLIO BALANCE AND EXCHANGE RATE STABILITY"

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LIST OF SYMBOLS

Money Market

\[ M^D = \text{nominal U.S. demand for cash balances} \]
\[ M = \text{nominal money holdings of the U.S. private sector} \]
\[ N = \text{nominal U.S. money holdings generated via the balance of payments} \]
\[ R = \text{world stock of the reserve asset} \]
\[ R^c = \text{U.S. central bank holdings of the reserve asset} \]
\[ \dot{R}^c = \text{change in } R^c, \text{ at time } t. \]

Bond Market

\[ B^D = \text{nominal U.S. demand for bonds} \]
\[ B^p = \text{nominal bond holdings of the U.S. private sector} \]
\[ B = \text{world stock of bonds} \]
\[ B^c = \text{U.S. central bank holdings of bonds} \]
\[ B = \text{stock of bonds available to be held in the private sector} \]

Goods Market

\[ Y = \text{current real U.S. income} \]
\[ Y^p = \text{U.S. permanent income} \]
\[ E = \text{total U.S. expenditures} \]
\[ X = \text{U.S. exports} \]
\[ W = \text{nominal U.S. private sector wealth} \]
\[ W = \text{real desired U.S. wealth} \]

Prices, Rates, and Miscellaneous

\[ P = \text{U.S. price level} \]
\[ k = \text{dollar price of pounds} \]
\[ \sigma = \text{dollar price of the reserve asset} \]
\[ r = \text{rate of return on bonds} \]
\[ \eta = \text{price elasticity of the U.S. demand for imports} \]
\[ \alpha \text{ and } \beta = \text{constants} \]

NOTE: A primed variable refers to the U.K. counterpart of that variable and a starred variable refers to the desired level of that variable.
PORTFOLIO BALANCE AND EXCHANGE RATE STABILITY*  
by Walter Enders**

Much of the ongoing debate concerning the relative advantages of a fixed vs. a flexible exchange rate regime has centered around the stabilizing or destabilizing effects of speculation and the magnitudes of the elasticities of demand for foreign goods and services (1). Using a "small-country" model which implicitly ignored portfolio balance effects, Mundell (1960) added another dimension to the controversy by demonstrating that the stability properties of either type of exchange rate system depend upon the degree of capital mobility. In particular, Mundell shows that when capital is perfectly mobile internationally, a fixed exchange rate ensures a direct approach towards equilibrium while a flexible rate can produce a cyclical approach. In contrast, if capital is immobile, a flexible exchange rate ensures a direct approach towards equilibrium while a fixed rate makes a cyclical approach likely.

Much of the current concern in macroeconomics has centered around portfolio balance effects (2) and recently this concern has spread into the area of international finance, wherein a growing number of stock-flow models of open economies have appeared (3).

The aim of this paper is to develop a large-country model, incorporating stock-flow effects, which can be used to analyze the stability properties of fixed and flexible exchange rate systems. It will be shown that asset supply changes play an important role in determining stability and that alternative exchange rate regimes have different effects on asset supplies. Mathieson (1973a) developed a portfolio balance model for a small country and found that with a
fixed exchange rate, either a cyclical or a direct approach towards equilibrium is possible; whereas, with a flexible exchange rate, the approach is direct. This conclusion held regardless of the degree of capital mobility. The properties of a small-country model are such that if capital is mobile, the small country cannot control the domestic supply of bonds; and if the exchange rate is fixed, the small country cannot control the nominal money supply. Large countries, however, can produce noticeable changes in the world stock of bonds or money regardless of the degree of capital mobility or fixity of the exchange rate. Since stability depends upon asset supply changes, the stability properties of large and small country models will differ (4), as will the stability properties of fixed and flexible exchange rate regimes.

The Model

The model postulates two countries (say the U.S. and the U.K.) which are large enough relative to each other so that one country's policy actions can potentially produce significant effects in the second country. In each of the two countries, only two assets are held, money and bonds. Residents of a country are assumed to hold only that country's money, whereas foreign denominated bonds can be held by domestics (5). Since the aim of this paper is to discuss the stability properties of fixed vs. flexible exchange rate regimes--over and above those produced by expectational effects--it seems appropriate to assume that asset holders have static exchange rate, interest rate and price expectations. The assumption of static expectations plus the assumption of perfect capital mobility imply that domestic and foreign bonds are perfect substitutes in an individual's portfolio. Thus, no significant loss of generality is incurred if it is further assumed that only one country--say the U.S.--issues bonds. In accord with the Mundell (1960) and Mathieson (1973a) models, the Keynesian assumption of fixed commodity prices and variable income levels is made.
For the U.S., the private sector's demands for nominal cash balances and nominal bond holdings are given by equations 1 and 2. These demands are functions of the current level of U.S. income ($Y$), the real (equal to the nominal) rate of return on bonds, and are homogeneous of degree one in terms of nominal U.S. private sector wealth.

1) $M^D = L(Y, r, W)$

2) $B^D = B^D(Y, r, W)$

and:

3) $W = B^P + M$

Where: $M^D =$ nominal demand for cash balances
$Y =$ current real U.S. income
$r =$ rate of return on bonds
$W =$ U.S. private sector wealth
$B^D =$ nominal demand for bonds
$B^P =$ nominal bond holdings of the U.S. private sector
$M =$ nominal money holdings of the U.S. private sector

At a moment in time, in which wealth is fixed, the balance sheet constraint imposes certain sign restrictions on the asset demand functions. In particular, as long as wealth is fixed, the sum of the asset demands must always be equal to the given stock of wealth since it is impossible to allocate more assets than the existing stock. The above condition will be met if the sum of the effects of changes in the interest rate and changes in the level of income both sum to zero across the portfolio, while the effect of a change in wealth sums to unity across the portfolio, i.e., $\frac{\partial M^D}{\partial Y} + \frac{\partial B^D}{\partial Y} = \frac{\partial M^D}{\partial r} + \frac{\partial B^D}{\partial r} = 0$; and $\frac{\partial M^D}{\partial W} + \frac{\partial B^D}{\partial W} = 1$. By assumption: $0 < \frac{\partial M^D}{\partial Y} < 1$; $\frac{\partial M^D}{\partial r} < 0$; and $0 < \frac{\partial M^D}{\partial W} < 1$.

Similarly the U.K. demands for money and bonds can be represented by:

4) $M^{D'} = L'(Y', r, W')$

Where: Primed symbols represent the U.K. counterpart of the U.S. variable.

5) $B^{D'} = B^{D'}(Y', r, W')$

and:

6) $W' = B^{P'} + M'$
In this two country world money has no backing, but the rules of the game are such that there is a reserve asset in which international payments are made. When a resident of one country receives the reserve asset, the central bank immediately exchanges the reserve asset for the domestic currency. Thus, one component of a country's money supply is the cumulated sum--either positive or negative--of the central bank's accumulations of the reserve asset, each times the then prevailing currency price of the reserve asset, i.e.,

\[ N = \int_0^T R^c(t) \sigma(t) dt \]

Where: \( N \) = nominal money holdings generated via U.S. central bank reserve accumulation

\[ N' = \int_0^T R'^c(t) \sigma'(t) dt \]

\( R^c \) = U.S. central bank reserves
\( R'^c \) = change in \( R^c \), at time \( t \)
\( \sigma \) = dollar price of the reserve asset

Allowing "\( k \)" to represent the dollar price of pounds, at any moment in time, \( k(t) = \frac{\sigma(t)}{\sigma'(t)} \). Furthermore, it should be clear that a change in the current currency price of the reserve asset does not alter the private sector's money holdings, rather a change in the central bank's net worth is involved.

In such a world, the total stock of the reserve asset is given by:

\[ R = R^c + R'^c \]

Where: \( R \) = world stock of the reserve asset.

If the exchange rate is initially set equal to unity and the stock of the reserve asset is fixed, then: \( dN = -dN' \).

The second component of a country's money supply is equal to the cumulated sum of bonds purchased by the central bank, for an open market operation will directly change the money supply and governments are assumed to sell bonds to the central bank only to finance a discrepancy between their expenditures and tax revenues. The money supply in each country can be represented by:

7) \( M = B^c + N \)

8) \( M' = B'^c + N' \)

Where: \( B^c \) = cumulated sum of U.S. central bank purchases of bonds. These bond holdings are assumed to be exogenous.
Equilibrium in the money markets requires that:

9) $B^C + N = L(Y, r, W)$

10) $B' + N' = L'(Y', r, W')$

It will be convenient to assume that the U.S. Government issues a fixed price bond which is denominated in terms of dollars (7). Since U.K. asset holders demand a certain pound value of bonds, bond market equilibrium requires:

11) $B^P = B^D(Y, r, W)$

12) $B - B^P = kB^D(Y', r, W')$

The above equations relate to demands for stocks at a point in time and convey no information concerning asset accumulation. It seems reasonable to assume, however, that savers base their saving decision on the discrepancy between desired and actual wealth, and that saving is proportional to the difference between the two. Since the desired or target level of nominal wealth is solely a function of permanent income and the current (equal to the expected) interest rate, as well as being homogeneous of degree one in terms of the domestic price level, saving can be represented by:

13) $\frac{dW}{dt} = \alpha \left[ PW(Y, r, w) - W \right]$ \hspace{1cm} Where: $P = \text{U.S. price level}$

14) $\frac{dW'}{dt} = \beta \left[ P'W'(Y', r, W') - W' \right]$ \hspace{1cm} Where: $W = \text{desired real wealth}$ $Y = \text{permanent income}$ $\alpha$ and $\beta$ are positive constants

$\frac{dW}{dY} > 0; \frac{dW}{dr} > 0$

Full stock equilibrium requires that the current level of wealth be equal to actual wealth and that permanent and actual income be equal. Once these conditions are substituted into equations 1 and 2, it is possible to obtain the desired level of U.S. money and bond holdings, i.e.,

15) $M^{D*} = PL^{*}(Y, r)$

16) $B^{D*} = PB^{D*}(Y, r)$

By assumption (8): $\frac{dM^{D*}}{dY} > 0; \frac{dB^{D*}}{dY} > 0$

$\frac{dM^{D*}}{dr} < 0; \frac{dB^{D*}}{dr} > 0$
Similarly, for the U.K.:

17) \( M'D^* = P'L^*(Y', r) \)

18) \( B'D^* = \pi BiD^*(Y', r) \)

The price level which asset holders use to deflate nominal magnitudes is

... a function of the domestic price of the U.S. and the U.K. good. Since commodity

prices are assumed to be fixed, both the U.S. and U.K. price levels are solely

a function of the exchange rate (dollar price of pounds), i.e.,

19) \( P = P(k) \)

Where: \( 0 < \frac{\partial P}{\partial k} < 1; -1 < \frac{\partial P'}{\partial k} < 0 \)

20) \( P' = P'(k) \)

Future income streams are not known with certainty and asset holders alter

their conception of permanent income if current and permanent income are not equal. The simplest representation of this behavior is to allow the change in permanent income to be proportional to the difference between current and permanent income, i.e.,

21) \( \frac{dy}{dt} = \alpha' [y - Y'] \)

22) \( \frac{dy'}{dt} = \beta' [y' - Y'] \)

With saving behavior specified, the consumption or expenditure function becomes a redundant equation, i.e.,

\[
E = Y - \frac{dW}{dt} \quad \text{Where:} \quad E = \text{U.S. expenditures on the U.S. and on the U.K. good} = \text{income minus saving}
\]

\[
E' = Y' - \frac{dW'}{dt}
\]

Total consumption expenditures, by definition, sum to the demand for the domestic good plus the demand for the foreign good (imports). If the division of a fixed amount of expenditures between the domestic and the foreign good depends solely upon the exchange rate, the demand for imports is positively
related to total expenditures and negatively related to the price of foreign exchange.

The balance of payments condition states that the change in the U.S. money stock due to the balance of payments is equal to the difference between the dollar value of U.S. exports and imports plus the change in the U.K. holdings of U.S. bonds, i.e.,

\[
23) \frac{dN}{dt} = X(Y' - \frac{dW}{dt}, k) - kX'(Y - \frac{dW}{dt}, k) + \frac{dB}{dt} - \frac{dB^P}{dt}
\]

Where: \( X \) = U.S. exports \( X' \) = U.K. exports

and: \( 0 < \frac{dX}{dE} < 1; \frac{dX}{dk} > 0; 0 < \frac{dX'}{dE} < 1; \frac{dX'}{dk} < 0. \)

**Stock Equilibrium**

Full stock equilibrium requires that the current level of income equal permanent income, desired wealth equal actual wealth, and that asset stocks remain unchanged. As shown in the Mathematical Appendix, once these conditions are substituted into equations 1-23, the system reduces to five equations and six unknowns. Once it is known whether the exchange rate is fixed or flexible, it becomes possible to obtain a solution to the system since a flexible exchange rate means that changes in the holdings of international reserves are exogenous while a fixed exchange rate system fixes the value of "k."

**Fixed Exchange Rates**

The discussion of comparative steady states can be facilitated by examination of figure 1.
The curve labeled BB shows—for a given stock of bonds, interest rate and exchange rate—the locus of income levels which produce equilibrium in the world bond market (9). An increase in the private sector's holdings of bonds, a decrease in the interest rate or a decrease in the dollar price of pounds will shift this curve upwards as higher income levels will be needed to equate the world supply and demand for bonds. The curve labeled MM shows—for a given stock of money, interest rate and exchange rate—the locus of income levels which produce equilibrium in the world money market. An increase in the private sector's holdings of money or an increase in the interest rate will produce an upward shift in the MM curve. The effect of a change in the exchange rate on the MM curve is not clear, since a change in the exchange rate will have opposite effects on U.S. and U.K. price levels and, hence, nominal demands for money. The MM curve may have a greater or lesser slope than the BB curve. The ray kk shows—for a given exchange rate—the locus of income levels which maintain balance of payments equilibrium. A decrease in the dollar price of pounds shifts this curve upwards, if the Marshall-Lerner condition holds.

It will be useful to discuss the effects of a change in the money supply separately from a change in the stock of bonds. A change in the stock of bonds (money) without a corresponding change in the stock of money (bonds) can be thought of as occurring via a temporary increase in U.S. government expenditures which is financed by issuance of bonds (money). When government expenditures fall back to their original level, the private sector will be left with an enlarged stock of bonds (money) and an unchanged stock of money (bonds). This procedure allows one to view an open market operation as the difference between the effects of pure money issuance and pure bond issuance. Since the last section of this paper specifically deals with stability, the discussion concerning comparative steady states will assume that the system is stable.
Starting from a position of full equilibrium, such as point 1 in Figure 2, an increase in the U.S. money supply will shift the MM curve to $M_1 M'_1$. Any resulting increases in income levels will require a reduction in the interest rate such that the BB curve shifts upward and the $M_1 M'_1$ curve falls. Overall equilibrium can be restored at a point like 2 wherein income levels are increased and the interest rate reduced. With a higher level of U.K. income, and a lower interest rate, money market equilibrium requires a net inflow of money from the U.S. to the U.K. Thus, the U.S. experiences a temporary balance of payments deficit. The net direction of the short-term bond flow cannot be determined without specific knowledge of the relative sensitivities of the bond demands to incomes and the interest rate. The likelihood of a net inflow of bonds to the U.S. is directly related to the relative sensitivity of the U.S. vs. the U.K. bond demands to interest rates.

![Figure 2](image_url)

An increase in the private sector's holdings of bonds will shift the BB curve upwards. Any increase in income levels requires that the interest rate rise such that the MM curve shifts upward and the now higher BB curve shifts downward. The higher U.K. level of income and the increased rate of return on bonds means that a temporary net inflow of bonds to the U.K. must occur. Higher income levels and the higher interest rate have offsetting effects on asset holders' demands for money. At net U.S. balance of payments surplus will occur if the U.S. demand for money rises; whereas, if the U.S. demand for money falls, a U.S. deficit will occur. The greater the relative sensitivity of the U.S.
demand for money to the interest rate (as compared to the U.K.), the more likely it is for the U.S. to experience a net balance of payments surplus.

Flexible Exchange Rates (10)

One of the more interesting results of the flexible exchange rate version of the model is that an increase in one country's money supply (again say that of the U.S.) may be expected to raise the level of U.S. income while causing U.K. income to fall. The reason for this result is that an increase in the U.S. money supply can be expected to reduce the rate of return on bonds. The lower interest rate acts to increase the U.K. demand for money; whereas, the U.K. money supply is fixed since flexible exchange rates prevent a balance of payments deficit or surplus from occurring. The resulting excess demand for money in the U.K. will produce a reduction in the U.K. level of income; and in conjunction with the lower rate of return on bonds, produce a flow of bonds from the U.K. to the U.S. U.S. income rises in order to restore equilibrium in the bond market. With a higher level of income in the U.S. and a lower level of income in the U.K., the dollar depreciates--if the Marshall-Lerner condition is met--in order that the value of U.S. exports remain equal to the value of U.K. exports.

The above discussion must be slightly modified since it ignored the effects of a change in the exchange rate on price levels and the U.K. demand for bonds. The increase in the dollar price of pounds can act to reduce the U.K. price level (necessarily reducing the nominal demand for money by U.K. residents) to such an extent that equilibrium in the U.K. money market need not be restored by a reduction in U.K. income. Further, since bonds are denominated in terms of dollars, while U.K. residents demand a pound value of bonds, an increase in the dollar price of pounds will increase the dollar value of the U.K. demand for bonds. This increase in the U.K. demand for bonds can offset those forces which
produce the flow of bonds from the U.S. to the U.K. Thus, the effects of exchange rate changes on price levels and the foreign demand for bonds can reverse all of the comparative state's results mentioned above.

In contrast to an increase in the U.S. money supply, an increase in the U.S. stock of bonds can be expected to increase the level of income in both the U.S. and the U.K. The increased stock of bonds results in a higher interest rate, and with unchanged money supplies, the level of income in each country rises. A higher U.K. income level and the increased rate of return on bonds means that bonds will flow from the U.S. to the U.K. resulting in a net increase in the stock of bonds in each country. It should be pointed out again that an exchange rate change can act to offset these results if the effects of a change in the exchange rate on price levels and bond demands are large.

**Stability**

As shown in the Mathematical Appendix, the dynamic model can be represented by seven equations and eight unknowns. Again, however, the conditions concerning the fixity of the exchange rate eliminate one unknown. Further, as should be clear, the solution to the steady state model is a particular solution to the dynamic model. Thus, the characteristic roots of the dynamic system bear directly on the stability properties of fixed as opposed to flexible exchange rates.

In order to discuss the stability properties of a system, it is necessary to postulate some disturbance to the system. In all cases examined, the discussion of stability will begin with an exogenous increase in the U.S. money supply, although the actual choice of the disturbance is irrelevant.

**Fixed Exchange Rates**

An increase in the U.S. money supply will create a discrepancy between
actual and desired holdings of cash balances, stimulating an increase in the U.S. demand for U.S. goods, U.K. goods and bonds. The increased U.S. demand for goods and bonds can normally be expected to produce an increase in the two income levels, a reduction in the interest rate and a deficit on both the U.S. trade and capital accounts. The U.S. balance of payments deficit, however, has the effect of increasing the U.K. money supply and hence stimulates the U.K. demand for U.S. and U.K. goods and bonds. The increase in these U.K. demands means that forces are set up which may tend to halt the flow of money from the U.S. to the U.K. It is possible, however, that the increased U.K. demand for goods and bonds is of such a magnitude that the flow of money from the U.S. to the U.K. tends to be reversed, rather than eliminated. Depending upon the magnitude of the forces tending to create this reverse flow, the system can approach equilibrium in either a direct or cyclical fashion, if it approaches equilibrium at all. As shown in the Mathematical Appendix, the possibility of a direct approach is increased:

a) the larger is \( \frac{\partial X'}{\partial E} \) relative to \( \frac{\partial X}{\partial E'} \), since this means that U.S. residents purchase relatively large amounts of the U.K. good,
b) the smaller is \( \frac{\partial L}{\partial W} \) relative to \( \frac{\partial L'}{\partial W'} \), since this means that U.K. residents are willing to hold relatively large amounts of money,
c) the smaller is \( \frac{\partial W}{\partial Y} \) relative to \( \frac{\partial W'}{\partial Y'} \), since this means that as income levels rise, U.K. residents will attempt to save relatively more than U.S. residents,
d) the smaller is \( \frac{\partial W'}{\partial E} \) relative to \( \frac{\partial W}{\partial E'} \), since this means that U.K. residents will dissave less as the interest rate falls.

Additionally, the system will tend towards stability if the term \( 1 - \frac{\partial X'}{\partial E} - \frac{\partial X}{\partial E'} \) is positive. This is analogous to strict Keynesian models in which the sum of the marginal propensities to import must be less than one for stability (11).

Flexible Exchange Rates

The reason why a fixed exchange rate system can be unstable is, in effect,
that two assets are internationally mobile. An excess supply of one asset can create a disturbance such that additional quantities of that asset can enter into the country in question via the balance of payments. This situation is impossible in a system of flexible exchange rates since an exchange rate change prevents money from flowing across countries. This is not to say that a flexible exchange rate regime is necessarily stable, for exchange rate changes alter the real supplies of both money and bonds. Again, consider the effects of an increase in the U.S. money supply, which acts to increase the U.S. demands for the U.S. good, the U.K. good, and bonds. These increased demands, in accord with the steady state solution, act to reduce the interest rate, increase U.S. income and cause the exchange rate (dollar price of pounds) to rise. Equilibrium will be reached if, in addition, U.K. income falls and bonds flow to the U.S. The increase in the exchange rate, however, increases the U.S. price level, reduces the U.K. price level and imposes capital losses on U.K. holders of bonds which are denominated in the U.S. currency. The increase in the U.S. price level acts to decrease real U.S. wealth and stimulates U.S. saving, while real wealth in the U.K. can either rise or fall since the capital loss suffered by the U.K. bond holders can be offset by the decrease in the U.K. price level. Additionally, the exchange rate change means that U.K. residents have to save fewer pounds in order to accumulate bonds. Thus, U.S. saving can increase, whereas, U.K. saving can either increase or decrease. The greater the relative increase in U.K. saving, the greater the likelihood of a further increase in the exchange rate since U.S. exports will fall relative to U.K. exports. Furthermore, if U.K. wealth falls—due mainly to capital losses on bond holdings—U.K. asset holders desire to reshuffle their portfolios and acquire bonds for money (a fall in U.K. wealth will be spread out over a desired reduction in money and bond holdings). As U.K. residents attempt to sell money for bonds, upward pressure is put on the exchange rate. Thus, it is possible that a policy action
which produces an initial increase (decrease) in the exchange rate, can create pressure for further increases (decreases) in the exchange rate via the resulting effects which exchange rate changes have on real wealth.

Conclusion

The view taken in this paper has been that it is the market for stocks which are of paramount importance in the determination of the flows of financial assets. In particular, asset flows are a temporary phenomenon which will occur, if and only if, there is a disequilibrium between the stock demands and supplies of assets. These temporary flows of financial assets will act to produce adjustments in the variables determining stock demands (i.e., income levels, interest rate and exchange rate), and full equilibrium requires that the determinants of the stock demands for assets settle at a level at which desired and actual stocks are equal.

Whether the long-run equilibrium position is ever reached, however, depends upon whether an increase in the stock of an asset sets up forces which act to decrease the excess supply of that asset. Both fixed and flexible exchange rate systems can be stable or unstable, and if stable, the approach towards equilibrium can be cyclical or direct. Fixed exchange rates may produce instability by the fact that an excess supply of one asset in a country, may draw forth--via a balance of payments surplus or a deficit on the capital account--additional increases in the stock of that asset. A flexible exchange rate, while eliminating the possibility of monetary flows across countries, can be unstable since exchange rate changes alter the real wealth of asset holders. Thus, an initial excess supply of an asset can lead to an exchange rate change which stimulates a further excess supply of that asset.

It has also been demonstrated that the static properties of large-country
models are quite different from small-country models, such as the Mathieson (1973 a or b) models. Specifically, large countries are able to influence economic activity by money and bond issuance, even under conditions of capital mobility regardless of the fixity of the exchange rate. By focusing on the interaction effects between two economies, it was also demonstrated that the stability properties of large and small country models differ. Specifically, a small-country model cannot capture the effects of an exchange rate change on the real wealth of the private sector in the rest of the world. Yet, the change in foreign wealth due to a change in the exchange rate can produce instability in a flexible exchange rate system. Further, a small-country model cannot capture the effects of changes in the domestic asset stocks on the portfolios of foreign residents. It was found, however, that the interaction of domestic and foreign asset supplies can produce instability under a system of fixed exchange rates.
MATHEMATICAL APPENDIX

Comparative Statics

The steady state version of the model can be represented by:

\[
\begin{bmatrix}
\frac{\partial L^*}{\partial Y} & 0 & \frac{\partial L^*}{\partial r} & 0 & -1 & \frac{L^*}{\partial \theta} \\
\frac{\partial B^D}{\partial Y} & 0 & \frac{\partial B^D}{\partial r} & -1 & 0 & \frac{B^D}{\partial \theta} \\
0 & \frac{\partial L^*}{\partial Y} & 0 & 1 & \frac{L^*}{\partial \theta} \\
0 & \frac{\partial B^D}{\partial Y} & 1 & 0 & \frac{B^D(1 + \frac{\partial P}{\partial \theta})}{\partial \theta} \\
-\frac{\partial X^*}{\partial Y} & 0 & 0 & 0 & -(1 - \eta - \eta')
\end{bmatrix}
\begin{bmatrix}
dY \\
dY' \\
dr \\
dB^D \\
dX
\end{bmatrix}
= \begin{bmatrix}
dY \\
dY' \\
dr \\
dB^D \\
dX
\end{bmatrix}
\]

Note: Star (*) designates that desired wealth has been substituted into the point in time asset demand equation in order to obtain the desired stock of that asset.

Where: \( \eta \) = price elasticity of the U.S. demand for the U.K. good. It will be assumed that the Marshall-Lerner condition is met, and by assumption:

\[
\frac{\partial L^*}{\partial r} < 0; \frac{\partial B^D}{\partial Y} > 0; \frac{\partial L^*}{\partial r} < 0; \frac{\partial B^D}{\partial Y'} > 0.
\]

Column six is deleted for a fixed exchange rate system while column five is deleted for flexible exchange rates.

a.) Fixed exchange rate system:

The determinant of the coefficient matrix (\( \Delta \)) is unambiguously positive, i.e.,

\[
\Delta = \left( \frac{\partial B^D}{\partial r} + \frac{\partial B^D}{\partial r} \right) \left[ \frac{\partial L^*}{\partial Y} + \frac{\partial X^*}{\partial Y'} \right] - \left[ \frac{\partial L^*}{\partial r} + \frac{\partial L^*}{\partial r} \right] \frac{\partial B^D}{\partial Y} = \frac{\partial X^*}{\partial Y} + \frac{\partial B^D}{\partial Y'} > 0
\]

Pure Monetary Policy (Note: For pure monetary policy, \( dB = 0 \ )):

\[
\frac{dY}{dB^c} = \frac{1}{\Delta} \frac{\partial X}{\partial Y} \left( \frac{\partial B^D}{\partial r} + \frac{\partial B^D}{\partial r} \right) > 0; \frac{dY'}{dB^c} = \frac{1}{\Delta} \frac{\partial X^*}{\partial Y} \left( \frac{\partial B^D}{\partial r} + \frac{\partial B^D}{\partial r} \right) > 0;
\]

\[
\frac{dr}{dB^c} = -\frac{1}{\Delta} \left[ \frac{\partial B^D}{\partial Y} \frac{\partial X}{\partial Y'} + \frac{\partial X^*}{\partial Y} \frac{\partial B^D}{\partial Y'} \right] < 0
\]
\[ \frac{dN}{dB^c} = \frac{1}{\Delta} \left[ \begin{array}{c} \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial r} - \frac{\partial X^*}{\partial Y} \left( \frac{\partial B^D*}{\partial r} + \frac{\partial B^D*}{\partial Y} \right) \end{array} \right] \Rightarrow -1 < \frac{dN}{dB^c} < 0; \]

\[ \frac{dB^P}{dB^c} = \frac{1}{\Delta} \left[ \begin{array}{c} \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial r} - \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial Y} \end{array} \right] \]

Pure Bond Issuance (Note: For pure bond issuance \( dB^c = 0 \)):

\[ \frac{dY}{dB} = -\frac{1}{\Delta} \frac{\partial X^*}{\partial r} \left( \frac{\partial L^*}{\partial Y} + \frac{\partial L^*}{\partial Y} \right) > 0; \]

\[ \frac{dy^*}{dB^c} = \frac{1}{\Delta} \left[ \frac{\partial L^*}{\partial Y} \frac{\partial X^*}{\partial Y'} + \frac{\partial L^*}{\partial Y} \frac{\partial X^*}{\partial Y'} \right] > 0; \]

\[ \frac{dk}{dB^c} = \frac{1}{\Delta} \left[ \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial r} - \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial Y} \right] > 0; \]

\[ \frac{dB^P}{dB^c} = \frac{1}{\Delta} \left[ \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial Y'} - \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial Y'} \right] > 0. \]

b.) Flexible Exchange Rate System:

The determinant of the coefficient matrix (\( \Delta' \)) has an ambiguous sign, i.e.,

\[ \Delta' = (1 - \eta - \eta') \left[ \begin{array}{c} \frac{\partial L^*}{\partial Y} \left( \frac{\partial B^D*}{\partial Y} + \frac{\partial B^D*}{\partial Y} \right) \end{array} \right] \]

Where: \( a_{16} \) (i = 1, ..., 4) = "i"th element in the 6th column of the coefficient matrix. If the effects of an exchange rate change on price levels and the U.K. demand for the U.S. bond are ignored [i.e., are of second order magnitude], then \( a_{16} \) (i = 1, ..., 4) can be set equal to zero and \( \Delta' \) is negative. The comparative statics results of this section will ignore these effects. The reader should keep in mind that the effects of changes in the exchange rate on price levels and asset demands can reverse any or all of the results which follow.

Pure Monetary Policy:

\[ \frac{dY}{dB^c} = -\frac{1}{\Delta'} \left( 1 - \eta - \eta' \right) \left[ \frac{\partial L^*}{\partial r} \frac{\partial B^D*}{\partial Y} + \frac{\partial L^*}{\partial r} \frac{\partial B^D*}{\partial Y} \right] > 0; \]

\[ \frac{dy^*}{dB^c} = \frac{1}{\Delta'} \left( 1 - \eta - \eta' \right) \left[ \frac{\partial L^*}{\partial Y} \frac{\partial B^D*}{\partial Y} \right] < 0; \]

\[ \frac{dk}{dB^c} = -\frac{1}{\Delta'} \left[ \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial r} - \frac{\partial B^D*}{\partial Y} \frac{\partial L^*}{\partial Y} \right] > 0; \]

\[ \frac{dB^P}{dB^c} = \frac{1}{\Delta'} \left( 1 - \eta - \eta' \right) \frac{\partial B^D*}{\partial Y} \left[ \frac{\partial L^*}{\partial Y} \frac{\partial B^D*}{\partial Y} - \frac{\partial L^*}{\partial Y} \frac{\partial B^D*}{\partial Y} \right] > 0. \]
Pure Bond Issuance:
\[
\frac{dy}{db} = \frac{1}{\Delta}(1 - \eta - \eta') \left( \begin{array}{c} \frac{dL^*}{dy} \\ \frac{dL^*}{dr} \end{array} \right)^* > 0; \\
\frac{dr}{db} = \frac{1}{\Delta}(1 - \eta - \eta') \left( \begin{array}{c} \frac{dL^*}{dy} \\ \frac{dL^*}{dr} \end{array} \right)^* > 0; \\
\frac{dk}{db} = \frac{1}{\Delta} \left( \begin{array}{c} \frac{dL^*}{dy} \\ \frac{dL^*}{dr} \end{array} \right)^* \left( \frac{dL^*}{dy} \frac{dB^*}{dr} - \frac{dL^*}{dy} \frac{dB^*}{dr} \right) > 0;
\]
\[
\frac{dL^*}{dy} \left( \frac{dL^*}{dy} \frac{dB^*}{dr} - \frac{dL^*}{dy} \frac{dB^*}{dr} \right)
\]

Dynamics

Allowing \( D \) to be the differential operator, the dynamic system's coefficient matrix post multiplied by the column vector of unknowns can be represented by
(Note that the initial exchange rate and prices have been set equal to unity and all non-linear functions have been approximated by use of a Taylor Expansion):
\[
\begin{bmatrix}
\frac{dL}{dy} & 0 & 0 & 0 & 0 & 0 & -\frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) & 0 \\
0 & \frac{dL^*}{dy} & 0 & 0 & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 \\
0 & 0 & \frac{dL^*}{dy} & 0 & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 \\
0 & 0 & 0 & \frac{dL^*}{dy} & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 \\
1 & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 & 0 & 0 \\
0 & 0 & 0 & \left( \frac{1}{\Delta} \left( \frac{dL^*}{dy} \right) \right) & 0 & -D & -D & -\frac{1}{\Delta^2} \left( \frac{dL^*}{dy} \right) \\
\frac{dL}{E} & \frac{dL}{E} & 0 & 0 & 0 & -\frac{1}{\Delta} \left( \frac{dL^*}{E} \right) - \frac{1}{\Delta^2} \left( \frac{dL^*}{E} \right) & -\frac{1}{\Delta} \left( \frac{dL^*}{E} \right) - \frac{1}{\Delta^2} \left( \frac{dL^*}{E} \right) & -\frac{1}{\Delta} \left( \frac{dL^*}{E} \right) - \frac{1}{\Delta^2} \left( \frac{dL^*}{E} \right) & -\frac{1}{\Delta} \left( \frac{dL^*}{E} \right) - \frac{1}{\Delta^2} \left( \frac{dL^*}{E} \right) \end{bmatrix}
\]

Column nine is deleted for a fixed exchange rate system while column eight is deleted for a flexible exchange rate system.

a.) Fixed exchange rate system:

The determinant of the coefficient matrix will take the form:
\[ D(1-D^3 + \lambda_2 D^2 + \lambda_3 D + \lambda_4) \]. Setting this equation equal to zero, the solution set for \( D \) will be the characteristic roots of the dynamic system. Using Descartes' rule of signs, no characteristic root will be positive if each \( \lambda_i(i = 1...4) \) is negative.
\[
\lambda_1 = \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} + \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \left( \frac{1}{\alpha^I \beta^I} \right) + \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha \beta^I} \right) \right] < 0
\]
\[
\lambda_2 = \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} + \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \left( \frac{1}{\alpha^I \beta^I} \right) + \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha \beta^I} \right) \right] + \left[ \frac{\partial \lambda}{\partial Y} \left( \frac{1}{\alpha^I \beta^I} \right) + \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha \beta^I} \right) \right] + \left[ \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha^I \beta^I} \right) + \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha \beta^I} \right) \right]
\]

This term is unambiguously negative if \( 1 - \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} \) is not negative.

\[
\lambda_3 = \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} + \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \left( \frac{1}{\alpha^I \beta^I} \right) + \frac{\partial \lambda}{\partial Y^I} \left( \frac{1}{\alpha \beta^I} \right) \right]
\]

\[
\lambda_3 \text{ is unambiguously negative if: a) } 1 - \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} \text{ is not negative, and if}
\[
b) \left[ \frac{\partial \lambda}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y^I} \right] \text{ is not positive.}
\]

\[
\lambda_4 = \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} + \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} + \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \right] - \left[ \frac{\partial \lambda}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y} + \frac{\partial \lambda^I}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} + \frac{\partial \lambda^I}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y^I} \right]
\]

\[
\lambda_4 \text{ is unambiguously negative if } \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y^I} \right] \left[ \frac{\partial \lambda}{\partial Y} \frac{\partial \lambda^I}{\partial Y^I} \frac{\partial \lambda^I}{\partial Y^I} \right]
\]

is not positive.

b.) Flexible exchange rate system:

The determinant of the coefficient matrix will take the form: \( D \left[ \lambda_1 \lambda_2^2 + \lambda_3 \lambda_4 \right] \)

The effects of a change in the exchange rate on price levels and U.K. bond holdings can make the \( \lambda_i \) (\( i = 1 \ldots 4 \)) positive or negative. If these effects are ignored, however, all \( \lambda_i \) are negative if the Marshall-Lerner condition is met, i.e.,
\[
\lambda_1 = \left[ 1 - \eta - \eta' \right] \left[ \frac{\partial L}{\partial Y} \frac{\partial L'}{\partial Y'} \right] \left[ \frac{\partial \tilde{W}}{\partial r} \left( \frac{1}{\alpha' \beta'} \right) + \frac{\partial \tilde{W}'}{\partial r} \left( \frac{1}{\alpha' \beta'} \right) \right] < 0
\]

\[
\lambda_2 = \left[ 1 - \eta - \eta' \right] \left[ \left( \frac{1}{\alpha' \beta'} + \frac{1}{\alpha' \beta'} \right) \frac{\partial \tilde{W}}{\partial r} + \left( \frac{1}{\alpha' \beta'} + \frac{1}{\alpha' \beta'} \right) \frac{\partial \tilde{W}'}{\partial r} \right] \left[ \frac{\partial \tilde{W}}{\partial \tilde{Y}} - \frac{1}{\alpha' \beta'} \frac{\partial \tilde{W}}{\partial \tilde{Y}} \frac{\partial \tilde{Y}'}{\partial \tilde{Y}} - \frac{1}{\alpha' \beta'} \frac{\partial \tilde{W}'}{\partial \tilde{Y}} \frac{\partial \tilde{Y}'}{\partial \tilde{Y}} \right] < 0
\]

\[
\lambda_3 = \left[ 1 - \eta - \eta' \right] \left( \frac{\partial L}{\partial Y} \frac{\partial L'}{\partial Y'} \left( \frac{1}{\alpha \beta} + \frac{1}{\alpha \beta} \right) \frac{\partial \tilde{W}}{\partial r} + \left( \frac{1}{\alpha' \beta'} + \frac{1}{\alpha' \beta'} \right) \frac{\partial \tilde{W}'}{\partial r} \right] \frac{\partial L}{\partial \tilde{Y}} \left( \frac{\partial \tilde{L}}{\partial \tilde{Y}} \frac{\partial \tilde{W}}{\partial \tilde{Y}} + \frac{\partial \tilde{L}'}{\partial \tilde{Y}} \frac{\partial \tilde{W}}{\partial \tilde{Y}} + \frac{1}{\alpha' \beta'} \frac{\partial \tilde{W}}{\partial \tilde{Y}} \frac{\partial \tilde{Y}'}{\partial \tilde{Y}} \right) < 0
\]

\[
\lambda_4 = \left[ 1 - \eta - \eta' \right] \left( \frac{\partial L}{\partial Y} \left( \frac{\partial \tilde{L}}{\partial \tilde{Y}} \frac{\partial \tilde{W}}{\partial \tilde{Y}} + \frac{\partial \tilde{L}'}{\partial \tilde{Y}} \frac{\partial \tilde{W}'}{\partial \tilde{Y}} \right) - \frac{\partial \tilde{W}}{\partial \tilde{Y}} \frac{\partial \tilde{W}'}{\partial \tilde{Y}} \frac{\partial \tilde{Y}'}{\partial \tilde{Y}} \right) \frac{\partial L}{\partial \tilde{Y}} \left( \frac{\partial \tilde{L}}{\partial \tilde{Y}} \frac{\partial \tilde{W}}{\partial \tilde{Y}} - \frac{\partial \tilde{L}'}{\partial \tilde{Y}} \frac{\partial \tilde{W}'}{\partial \tilde{Y}} \right) < 0
\]

Thus, only the effects of exchange rate changes on real wealth can produce instability if the exchange rate is flexible.
This paper is based on portions of my dissertation, "A Two Country Portfolio Balance Model," (Columbia University, 1974) which was greatly aided by my advisors Ronald Findlay and Donald Mathieson. Further improvements for this paper were suggested by Harvey E. Lapan of Iowa State University. Any remaining errors are my own.

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1 For a discussion of the effects of speculation on exchange rate stability see Sohmen (1969) Chapter 3, and works cited therein. See Meade (1951) Chapter XXIV, Haberler (1949), or Machlup (1940), for seminal works on the role of elasticities in determining exchange rate stability.

2 For example, see: J. Tobin and W. Brainard (1968) or McKinnon (1969).

3 For example, see: Allen (1972), Mathieson (1973 a or b), or Dornbusch (1974).

4 Dornbusch (1974) has developed a two country model which only deals with flexible exchange rates under conditions where each country produces the same good. It is assumed here that countries produce differentiated products.

5 The model is easily adaptable to allow residents of each country to hold two monies.

6 The problems associated with incorporating interest payments on bonds into the analysis are quite burdensome relative to their actual importance. If each government is assumed to impose a lump sum tax, equal in magnitude to the interest paid to the private sector of that country, changes in interest payments will not alter income or appear in the balance of payments equation (equation 19). Obviously, it must be assumed that the amount of tax any individual pays is not commensurate with that individual's holdings of bonds.

7 The analysis would not be substantially altered if the U.S. Government issued consols or a variable priced bond with a fixed maturity date.

8 The problem of signing the effects of changes in income levels and interest rates on composition and size of a portfolio is a common problem in portfolio balance models.

9 The BB curve and MM curves are obtained by summation of U.S. and U.K. desired holdings of bonds and money. The ray kk is obtained by the steady state version of equation 23, i.e., \(X(Y', k) - kX'(Y, k) = 0\). Note that in obtaining the MM curve, a given rate of exchange is needed to add U.S. to U.K. desired money holdings. However, exchange rate changes do not alter the private sector's nominal money holdings.
A diagram similar to Figure 1 can be developed for flexible exchange rates. In the flexible exchange rate case, however, money supplies are independent. Thus, there are two money market equilibrium curves; a vertical U.S. curve and a horizontal U.K. curve.

See Meade (1951) Chapter XXVI.
Bibliography


