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Optimal pricing strategies for a cluster of goods: Own- and cross-price effects with correlated tastes

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Willingness to pay, reservation price, cluster of goods, multi-variate probit analysis, revenue maximizing strategy, unobserved

Disciplines

Behavioral Economics | Econometrics

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Abstract

Contingent valuation methods are used to identify observed and unobserved preferences of goods and services. We apply these methods to compute willingness to pay (WTP) for a product conditional on having purchased another offered product. We provide a derivation for own-price and compensated cross-price elasticities whose results suggest a pricing strategy considering all offered goods simultaneously. Therefore, we solve the social planner's problem maximizing a weighted function of producer's revenues and consumer's utility for the set of optimal prices. We show an application to collegiate sports, but these methods can be extended in a straightforward fashion to other goods.

JEL Codes: C30, C35, C40, D12, D40, D60, Z20

Key words: Willingness to pay, reservation price, cluster of goods, multi-variate probit analysis, revenue maximizing strategy, unobserved taste, compensated cross price elasticities, bundling, sports tickets

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1. INTRODUCTION

Pricing goods and services is a challenging task. Whether it is performed by a producer interested in maximizing profits or revenues, or by a social planner interested in maximizing the welfare of producers and consumers, the information about the market, product prices, purchasing decisions by customers, their characteristics, as well as attributes of the products, are required to properly conduct a price analysis. Contingent Valuation Methods (CVM) are appropriate in situations when the required market information is not readily available or when transaction information does not include consumers or producers who might enter the market at alternative prices. We will develop a strategy that may provide useful information for establishing pricing policy and marketing strategies, regardless of whether the decision maker is a producer or a social planner.

More precisely, in this study we use a CVM survey to extract willingness to pay for goods and services in the context of joint purchasing decisions. Our context are goods that fit naturally in clusters such that the demand for any one good cannot be treated as independent because each item in the group has unobserved attributes which are shared by all other goods in the group, and prompt consumers to reveal their taste for the group due to these common attributes. Multivariate probit analysis is shown to be helpful in capturing these unobserved taste and its impacts on the complementary or substitutability of the goods.

Besides employing standard willingness to pay estimation methods, we extend the analysis by adapting Greene's (2003) conditional marginal effects to estimate reservation prices (or willingness to pay) conditional on having decided to purchase another offered product. This allows the identification of unobserved taste for the offered goods which informs marketing and pricing strategies. To our knowledge, none of the previous studies exploited this capacity of conditional probabilities.

Additionally, after estimating standard own-price elasticities that can be easily computed directly from demand equations, we propose a method to estimate compensated cross-price elasticities of substitution among goods by using the correlation among the errors in the demand equations. In the context of these goods, ignoring this correlation may induce inaccurate estimation of the cross-price effects. These compensated elasticities may be useful in cases when i) only pure substitution effects are of interest, ii) when decomposition of uncompensated elasticity between income and price effects is relevant, iii) or in cases when the uncompensated effect is the objective but an approximation can be achieved provided income effect is sufficiently low because, for example, the product's share in income is low due to its relatively low price.

Finally, the existence of cross-price effects in demand prompts setting a pricing strategy which considers all offered goods simultaneously instead of establishing each price independently from the rest of the products. In our approach, we let the social planner to solve for the optimal pricing strategy which maximizes both producer's revenues and consumer's joint latent utility, conditional on the weight assigned by the planner to each agent.

We illustrate our methods using a dataset from an artificial market survey in which each participant is offered successive randomly priced tickets for alternative varsity sports at a Midwestern University. These methods can be extended in a straightforward fashion to other private or public goods and services.

The rest of the paper is organized as follows. Section 2 contains a literature review relevant for our study and section 3 presents the methodology. Section 4 discusses the data used in the application, section 5 presents results of the application, and section 6 concludes.

2. LITERATURE REVIEW

An important input for establishing prices and pricing policy is the assessment of willingness to pay by consumers for products of interest, and there are numerous methods employed to elicit willingness to pay. The most direct is using information from actual purchase data. Transaction information does not suffer from hypothetical bias, and additionally, precise information from price changes of similar products can be exploited. However, observed variation in prices is limited, and so, many potential consumers are not observed making purchases at prevailing prices although they would enter at lower prices. For some markets, transaction prices do not exist as with public or environmental goods and services. Experimental price information such as that obtained through CVM are often used in place of transactions data when researchers require a more complete characterization of the full range of consumer demands.[1,2]

The CVM also allows computation of own- and cross-price effects when market data is not available. Examples include price responses for environmental services and public goods (Hökby and Söderqvist 2003; Briscoe et al. 1990; Cummings et al. 1994; and Thomas and Syme 1988), products grown in South Carolina (Carpio and Isengildina-Massa 2009), wild bear bile in China (Hepburn & Macdonald, 2011), and six types of orange juice products (Shi et al. 2014).

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[1] By varying hypothetical prices, even outside the range of existing market prices, the researcher can observe how many more consumers enter the market as prices fall. If the deviations from true purchase decisions are random, the averaged responses will still provide accurate predictions of market demands. Among its drawbacks, it requires large samples to capture market responses across a large range of prices, and it is possible that respondents are not truthful about their answers.

[2] For a more detailed review of Contingent Valuation approach and its limitations we derive the reader to Diamond and Hausman (1994), Carson et al. (2001), Lusk and Schoreder (2004) and Venkatachalam (2003).

Other studies used CVM in the context of categorical dependent variable estimation methods. Ma and Seetharaman (2004) estimated the choice of a set of household items using multivariate logit models,[3] and Mataria et al. (2007) used survival analysis to estimate the own price effect of certain health care improvements. Maynard et al. (2004), by means of an experimental store in which individual choose between 5 types of meat products, computed demand elasticities.[4]

In the studies listed above, demand equations contain own- and cross- prices as explanatory variables, and so cross-price effects can be computed from the marginal effects of prices on demand. To our knowledge, no other study have restored to the use of the cross correlation across demand equations to compute cross-price effects.

The multivariate probit model, first used by Ashford and Swoden (1970), at first suffered from limited use due to computing power limitations but its utilization has increased with the improvements in computing capacity. Prominent applications include Young et al. (2009) who analyzed claims in a portfolio of insurance policies, Contoyannis and Jones (2004) who studied health production functions and lifestyle equations, and Chib and Greenberg (1998) who evaluated tradeoffs between commuting and buying a car. The application closest to ours is Chib et al. (2002) who used a 12th-variate probit model to study the effects of prices on household purchasing decisions, but employing household's market transactions and a Bayesian approach. However, although Chib *et al* (2002) report price effects, they do not compute elasticities of substitution, and do not compute willingness to pay.

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[3] They state that this model allows the estimation of the cross-correlation coefficients separately from the joint purchase outcomes. Additionally, they state that the cross-correlation estimates are smaller in magnitude but equal in sign to those of multivariate probit.

[4] In this two-stages approach, first a probit model describes the participation decision (i.e. a non-zero consumption decision) and then a Poisson regression truncated at zero identifies the consumption decision.

In the sports literature, CVMs are mainly restricted to capturing aggregated willingness to pay to assess whether a sport generated sufficient positive externalities to justify government construction of new sports venues (Johnson and Whitehead, 2000), attracting a professional hockey team (Johnson, Groothuis, and Whitehead, 2001), retaining professional football and basketball teams (Johnson, Mondello, and Whitehead, 2007); or supporting amateur sports and recreation programs (Johnson et al. 2007).

3. METHODOLOGY

Our context will be purchase decisions regarding sporting event tickets, but ours is a special case of purchase decisions regarding items in a product group where the items are not perfect substitutes for one another. An individual *i* is offered sequential options to purchase *J* products at exogenously given prices, denoted as P_i^1, P_i^2, …, P_i^J. The purchase choices are given by

$$\begin{aligned} B_i^{1*} &= \beta_0^1 + \beta_1^1 P_i^1 + \boldsymbol{Z}_i'\boldsymbol{\delta}^1 - \eta_i^1 \\ B_i^{2*} &= \beta_0^2 + \beta_1^2 P_i^2 + \boldsymbol{Z}_i'\boldsymbol{\delta}^2 - \eta_i^2 \\ &\vdots \\ B_i^{J*} &= \beta_0^J + \beta_1^J P_i^J + \boldsymbol{Z}_i'\boldsymbol{\delta}^J - \eta_i^J \end{aligned} \tag{1}$$

where B_i^{j*} , $j = 1, 2, \ldots, J$ represents the latent utility associated with purchasing product *j* at the offered price, P_i^j, holding constant a vector of individual characteristics Z_i and unobservable factors represented by η_i^j. In practice, we do not observe B_i^{j*}, but we do observe the choice of whether or not the individual buys each product. For $j = 1, 2, \ldots, J$, indexing each option, let the binary choices be represented by

$$B_i^j = \begin{cases} 1 & if\ B_i^{j*} > 0 \\ 0 & if\ B_i^{j*} \leq 0 \end{cases} \tag{2}$$

which implies that individual *i* purchases item *j* if it yields positive utility.

Most studies treat these purchase decisions as independent. This is unlikely to be the case in reality, as almost any purchase has substitute or complement goods. In our context, attendance takes time, and so, spending time on one event leaves less time available for others, making competing events natural substitutes. On the other hand, brand or product line loyalty may create complementarity among goods sharing the brand so that buying one option would be positively correlated with purchasing decisions for other products in the cluster.[5] To accommodate that possibility, we assume that the unobserved factors in equation (1) are jointly distributed standard normal with correlation coefficients $\rho_{jk} \lesseqqgtr 0$, for $j \neq k$

$$\begin{bmatrix} \eta_i^1 \\ \eta_i^2 \\ \vdots \\ \eta_i^J \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} 1 & \cdots & \rho_{1J} \\ \vdots & \ddots & \vdots \\ \rho_{1J} & \cdots & 1 \end{bmatrix}\right) \tag{3}$$

By calculating the joint probabilities that Pr[B_i^1=1, B_i^2=1, …, B_i^J=1], Pr[B_i^1=1, B_i^2=1, …, B_i^J=0], and so on up to Pr[B_i^1=0, B_i^2=0, …, B_i^J=0], we find the following log-likelihood function for the model:

$$L\left(B_i^1, B_i^2, \ldots, B_i^J \mid P_i^1, P_i^2, \ldots, P_i^J, \boldsymbol{Z}_i, \boldsymbol{\beta_0}, \boldsymbol{\beta_1}, \boldsymbol{\delta}, \boldsymbol{\rho}\right) =$$
$$\sum_{i=1}^{n} \log\{\Phi_J\, [q_i^1\left(\beta_0^1 + \beta_1^1 P_i^1 + \boldsymbol{Z}_i^{'}\boldsymbol{\delta}^1\right), \ldots, q_i^J\left(\beta_0^J + \beta_1^J P_i^J + \boldsymbol{Z}_i^{'}\boldsymbol{\delta}^J\right), Q \cdot \boldsymbol{\rho}]\} \tag{3}$$

where $\boldsymbol{\beta_0} = [\beta_0^1, \ldots, \beta_0^J]$, $\boldsymbol{\beta_1} = [\beta_1^1, \ldots, \beta_1^J]$, $\boldsymbol{\delta} = [\delta^1, \ldots, \delta^J]$, Φ_J is the *J*-variate standard normal cumulative distribution function; and for all *j*, q_i^j is conveniently defined as $q_i^j = 2B_i^j - 1$. The

[5] For example, in sport events, loyalty to a team, often crosses sports because of an individual's residence, circle of friends, or school affiliation.

term $(Q \cdot \boldsymbol{\rho})$ is the $J \times J$ variance-covariance matrix but defined in terms of correlation coefficients. It equals the dot-product between the $J \times J$ symmetric matrix of correlation coefficients between equations $(\boldsymbol{\rho})$ and a $J \times J$ symmetric matrix (Q) where each entry $Q_{jk} = Q_{kj} = q_j q_k \rho_{jk}$ for $j \neq k$, and $Q_{jk} = 1$ for $j = k$.

Estimation of unconditional willingness to pay or reservation prices

The parameter estimates from (3) can be used to estimate willingness to pay for each product. This can be done following the contingent valuation approach by Cameron (1988) and Train (2003 pp. 168).

We measure willingness to pay by the reservation price: the price at which individual i is indifferent between purchasing and not purchasing an option that is offered. We can derive an estimate of the reservation price for each of the J purchase options as the value of P_i^j such that $\Pr\left[B_i^j = 1\right] = 0.50$. This probability of purchasing represents the demand for good j by the individual i. Algebraically, the reservation prices p_i^j implicitly solve the equation for each j:

$$\Pr\left[B_i^j = 1\right] = \Phi\left(\hat{\beta}_0^j + \hat{\beta}_1^j p_i^j + \boldsymbol{Z}_i' \widehat{\boldsymbol{\delta}}^j\right) = 0.5 \tag{4}$$

where Φ is the univariate standard normal cumulative distribution function and "^" indicates the estimated value of the parameter. Given that the standard normal probability density function is symmetric and centered on zero, the implicit equations for each p_i^j simplify to

$$\begin{aligned} &\hat{\beta}_0^j + \hat{\beta}_1^j p_i^j + \boldsymbol{Z}_i' \widehat{\boldsymbol{\delta}}^j = 0 \\ &p_i^j = -\frac{1}{\hat{\beta}_1^j}\left(\hat{\beta}_0^j + \boldsymbol{Z}_i' \widehat{\boldsymbol{\delta}}^j\right) \end{aligned} \tag{5}$$

The natural measure of the reservation price is the one that sets the probability of purchase equal to 0.5 as in equation (4). However, in principle, we could evaluate (4) at any common probability of ticket purchase. Because the probability of purchasing is monotonically decreasing in price, the rank order of individual preferences for each option is invariant to the choice of probability. Furthermore, as shown by Cameron (1988), a contingent valuation approach will yield numerically identical estimates of the willingness to pay as we derive at the probability of purchase equal to 0.5.[6]

Estimation of unobserved correlation between products

The elements of matrix $\boldsymbol{\rho}$, ρ_{jk} for $j \neq k$, measure the unobserved correlation in the error terms of the demand for products *j* and *k*. A positive estimate implies that unmeasured attributes that increase the likelihood of purchasing product *j* also increase the likelihood of purchasing product *k*. A positive and large correlation for any pair implies prospects for cross-marketing strategies between the two products.

Estimation of conditional reservation prices

Important conclusions can be derived from computing the probability of purchasing product *j* conditional on having purchased product *k*. For example, we can document the implications of common taste that individuals have for any pair of products (*j*,*k*) by comparing the reservation price that solves the conditional probability of purchasing product *j* conditioned on purchasing product *k*, with the reservation price that solves the unconditional probability of purchasing *j*. If the unobserved correlation between *j* and *k* is positive so the two goods have a source of complementarity in their unobserved tastes, independent of the regressors, then we would expect a greater probability of purchasing product *j* conditional on buying *k* than we would estimate using the unconditional probability.

Dropping the subscript *i* that indexes individuals in the sample, we are interested in computing the cross-reservation price for product *j* which arises from setting to 0.5 the expected

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[6] Rosas and Orazem (2014) provide a proof of the equivalence between this approach and the one proposed by Cameron (1988).

value of purchasing product j ($B^j = 1$) conditional on purchasing product k ($B^k = 1$), and given the observed decision each individual made regarding the remaining options ($B^{-j} = b^{-j}$). The conditional probabilities computations following Christofides et al. (1997), Greene (2003 pp. 713), and Greene (2009 pp.76) is:

$$\begin{aligned} 0.5 &= E\left[B^j = 1 | B^k = 1, B^{-j} = b^{-j}\right] \\ &= \Pr\left[B^j = 1 \middle| B^k = 1, B^{-j} = b^{-j}\right] \\ &= \frac{\Pr\left[B^j = 1, B^k = 1, B^{-j} = b^{-j}\right]}{\Pr[B^k = 1, B^{-j} = b^{-j}]} \end{aligned} \tag{6}$$

with $b^{-j} = \{0,1\}$ depending on the individual's decision, and where the last equality results simply from applying the definition of conditional probability as the ratio between the joint probability of the J products, to the joint probability of the conditioning J-1 random variables.

Applying equation (3), the cross-reservation price is the value of p^{jk} that solves the following highly nonlinear expression:

$$0.5 = \frac{\Phi_{\mathrm{J}}\left[\left(\beta_0^j + \beta_1^j p^{jk} + \boldsymbol{Z}'\boldsymbol{\delta}^j\right), \left(\beta_0^k + \beta_1^k P_i^k + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), q^{-j}\left(\beta_0^{-j} + \beta_1^{-j} P^{-j} + \boldsymbol{Z}'\boldsymbol{\delta}^{-j}\right), Q \cdot \boldsymbol{\rho}\right]}{\Phi_{\mathrm{J}-1}\left[\left(\beta_0^k + \beta_1^k P_i^k + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), q^{-j}\left(\beta_0^{-j} + \beta_1^{-j} P^{-j} + \boldsymbol{Z}'\boldsymbol{\delta}^{-j}\right), Q \cdot \boldsymbol{\rho}\right]} \tag{7}$$

The numerator represents an individual's joint decision to purchase all J products, arising from the value that takes the unobserved latent utility from deciding to buy the product at the offered price. The interpretation of the denominator is similar except that it is the joint decision for J-1 products. In general, as we have J purchase decisions, this comparison can be made either conditioning on purchasing one product k on the observed decision regarding the remaining (J-1) products, or any combination of purchasing/not purchasing the remaining (J-1) products. That is, without loss of generality, the same derivation is valid for analyzing any bundle of the J products.

Importantly, interesting conclusions can be obtained by comparing the reservation price p^j which solves (7) with the unconditional reservation price which solves (5), because this difference quantifies the effect of the unobserved common taste on willingness to pay between any pair of sports *j* and *k,* for $j \neq k$.

Estimation of own- and cross-price effects

We can also calculate own-price and cross-price elasticities using the maximum likelihood estimates. The probability of purchasing product *j* represents the estimated demand of individual *i* for that product. Therefore, the own-price demand elasticities of *j* is:

$$\varepsilon_i^{jj} = \frac{1}{n}\sum_{i=1}^{n} \frac{\partial P\left[B_i^j = 1\right]}{\partial P_i^j} \cdot \frac{P_i^j}{P\left[B_i^j = 1\right]} \tag{8}$$

Where *n* is the number of individuals in the sample.

For the cross-price elasticities between two products, consider the simultaneous joint decision of purchasing two product (*j* and *k*) at the offered prices P^j andP^k, respectively, and the observed decision regarding the remaining product (-*j*). In terms of probabilities we have:

$$\begin{aligned} &\Pr\left[B^j = 1, B^k = 1, B^{-j} = b^{-j}\right] \\ &= \Phi_{\mathrm{J}}\left[\left(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j\right), \left(\beta_0^k + \beta_1^k P^k + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), q^{-j}\left(\beta_0^{-j} + \beta_1^{-j} P^{-j} + \boldsymbol{Z}'\boldsymbol{\delta}^{-j}\right), Q.\boldsymbol{\rho}\right] \\ &= C \end{aligned} \tag{9}$$

Which consists of the joint probability (demand) derived from the fact that, individually, the value of the unobserved latent utilities from consuming each good at the offered prices (P^j, P^k, P^{-j}) is higher than that of not buying in the cases the individual decided to purchase, and is lower in the cases the individual decides not to purchase. The value of C is a scalar between 0 and 1, and is the probability associated with this individual's decision given his or her particular characteristics. In order to find the cross-price elasticity of buying product *j* to changes in the price of k, let us define $P^{k*} = P^k(1 + 0.01)$.[7] By substituting P^k by P^{k*} in (9) and leaving everything else unchanged, we can evaluate the induced change in P^j which leaves unchanged the probability (C) of purchasing this combination of products. We denote this solution by P^{j*}, as the price of j which solves the following non-linear implicit function:

$$\Pr\left[B^j = 1, B^k = 1, B^{-j} = b^{-j}\right]$$

$$= \Phi_J\left[\left(\beta_0^j + \beta_1^j P^{j*} + \boldsymbol{Z}'\boldsymbol{\delta}^j\right), \left(\beta_0^k + \beta_1^k P^{k*} + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), q^{-j}\left(\beta_0^{-j} + \beta_1^{-j} P^{-j} + \boldsymbol{Z}'\boldsymbol{\delta}^{-j}\right), Q.\boldsymbol{\rho}\right] \quad (10)$$

$$= C$$

Finally, in order to obtain the cross-price elasticity between j and k, i.e. the change in demand of j due to a 1% change in price of k, we employ equation (8) but where the change in price P^j in the denominator (which is exactly equal to $[P^{j*}/P^j - 1]$) is that induced by the 1% change in P^k.

A restatement of equation (9) for the case of two goods implies $\Pr\left[B^j = 1, B^k = 1\right] = C$, and equation (10) can be expressed in the following way: $\Phi_2\left[\left(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j\right), \left(\beta_0^k + \beta_1^k P^k + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), Q.\boldsymbol{\rho}\right] = C$. Applying total differentiation on both sides of the previous equation we obtain:

$$\mathrm{d}\left(\Phi_2\left[\left(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j\right), \left(\beta_0^k + \beta_1^k P^k + \boldsymbol{Z}_i'\boldsymbol{\delta}^k\right), Q.\boldsymbol{\rho}\right]\right) = 0$$

Following Greene (2003) and remembering that P^j and P^k only appear in the demand equations for goods j and k respectively, we can compute the total differential as:

$$\left[\phi\left(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j\right)\beta_1^j\right] dP^j + \left[\phi\left(\beta_0^k + \beta_1^k P^k + \boldsymbol{Z}'\boldsymbol{\delta}^k\right)\beta_1^k\right] dP^k = 0 \quad (11)$$

where ϕ is the univariate standard normal density. Rearranging terms in (11) we can derive the marginal rate of substitution between j and k for the probability of purchase C as:

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[7] We arbitrary set the increment to 1%, but any other one could be defined.

$$MRS_1 = -\frac{dP^j}{dP^k} = \frac{\phi[(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j)]\beta_1^j}{\phi[(\beta_0^k + \beta_1^k P^k + \boldsymbol{Z}'\boldsymbol{\delta}^k)]\beta_1^k} \tag{12}$$

Equation (12) allows us to have, for a given pair of prices P^j and P^k, the point on the indifference curve with utility level set consistent with joint probability *C.* This is illustrated in Panel A of figure 1.[8] Additionally, these same prices are consistent with a level of marginal probability of buying products *j* and *k* as illustrated by $P(B^j = 1;\ P^j)$ and $P(B^k = 1;\ P^k)$ in panels B and C of figure 1, respectively. Applying the exercise previously described involves a movement along the indifference curve toward a new point with another marginal rate of substitution (MRS_2) that give us another marginal probability of buying product *j* and *k* associated with prices P^{j*} and P^{k*} that are illustrated by $P(B^j = 1;\ P^{j*})$ and $P(B^k = 1;\ P^{k*})$. Using this, we can compute the change in the probability of buying product *k* when we exogenously increase the relative price of product *j,* holding utility fixed at the level consistent with at joint probability level C. .

As will become clear in the estimation subsection, this procedure is applied when the randomization in the experimental design yields the outcome that each demand is uncorrelated with the prices of the other products. Therefore, this approach is exploiting the estimated unobserved correlation between equations. This approach could be useful in cases when pure substitution (compensated) effects are informative for practitioners or decision makers or to decompose uncompensated cross-price elasticities into income and substitution effects.

Optimal pricing strategy

In this application, the existence of cross-price effects determines that an optimal pricing strategy cannot be set individually by product, but rather, simultaneously for all products. We

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[8] Figure 1 assumes that the two goods are substitutes, but the same exercise can be conducted for complementary goods.

present an approach that is suitable for our particular case where cross-prices influence demand through the correlation in the errors.

We set this up as a social planner's problem with a utility function (W) depending on the consumers' utility (C), the producer's revenues (R),[9] and weights assigned to each, such that w_C between 0 and 1 is the one assigned to consumers. The social planner's problem becomes:

$$
\begin{aligned}
&\max_{P^1,\ldots,P^J} \{W\} = \max_{P^1,\ldots,P^J} f(C, R; w_C) \\
&C = \Phi_J\left[(\beta_0^1 + \beta_1^1 P^1 + \boldsymbol{Z}'\boldsymbol{\delta}^1), \cdots, \left(\beta_0^J + \beta_1^J P^J + \boldsymbol{Z}'\boldsymbol{\delta}^J\right), \boldsymbol{\rho}\right] \\
&R = \sum_{j=1}^{J} P^j \Pr\left(B^j = b^j\right) = \sum_{j=1}^{J} P^j \Phi\left[\left(\beta_0^j + \beta_1^j P^j + \boldsymbol{Z}'\boldsymbol{\delta}^j\right)\right]
\end{aligned}
\tag{13}
$$

Revenues are increasing in prices but utility (probability of purchase) is decreasing in prices, therefore there exists a tradeoff between these two objective functions. While higher prices reduce consumer's latent utility, they increase revenues through a direct effect. Nevertheless, they decrease revenues indirectly by reducing the probability of purchasing. As a result of this and the weighting preferences of the planner, together with the fact that these functions are well-behaved (a multivariate normal cdf's and the univariate normal cdf's), an optimal set of prices can be found.

If the producer is able to set prices regardless of individuals' welfare ($w_C \rightarrow 0$) our problem reduces to the planner solving for the price vector which maximize revenues. On the other hand, if the planner places the greatest weight on consumers ($w_C \rightarrow 1$), in our case with no budget constraint, maximization of the joint-purchase decision will imply $C \rightarrow 1$, which is only possible if prices become negative.

4. DATA

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[9] Revenue maximization is useful in case of goods and services characterized by a cost structure dominated by fixed costs, as in our case with sports events.

Our application will be taken from the sale of intercollegiate sports tickets. The National Collegiate Athletic Association (NCAA) regulates college and university sports in the United States. These sports are big business. In 2014, Division I-A Universities' spent $7.8 billion on men's and women's sports, generating revenues of $8.2 billion (USED, 2016). However, only a few sports actually generate sufficient revenue to cover their costs. The profitable sports include men's basketball which generated revenues of $1 billion against total expenses of $703 million, leaving a surplus of $298 million; and football with revenues of $3.7 billion against expenses of $2.1 billion, a surplus of $1.6 billion. Sports that fail to break even include women's basketball, volleyball, wrestling, gymnastics, hockey and soccer with accumulated deficits of $375 million. Most universities struggle to earn enough profit in football and men's basketball to cover their losses from the other sports.

Revenue generation depends on attendance. Men´s basketball and football have turnouts of 28 and 44 million spectators in 2014 respectively, while women´s basketball attracted only 8 million attendees during that year (NCAA, 2016).[10] However, fans willing to attend women's basketball may be revealing themselves to be atypically committed supporters who would be willing to attend other sports as well. If so, colleges and universities could augment revenues and/or increase attendance at less profitable sports by incorporating underlying complementary preferences for various sports into their ticket pricing strategies.

In this study, we compute the willingness to pay for 8 intercollegiate sports using data from a market experiment carried out at Iowa State University (ISU) in 2006. We demonstrate that willingness to pay for a given sports can be measured using decisions to accept or reject randomly priced men's and women's sports tickets in a simulated market. The parameters allow us to predict the probability of ticket purchase at any given price, and so we can estimate the

[10] We do not have access to systematized data on attendance for the rest of the mentioned sports.

reservation prices (or willingness to pay) at which each individual is indifferent between buying and not buying a ticket for each of the eight sports. These responses also allow us to compute own- and cross-price elasticities using a novel application of multivariate probit analysis which we present in the next section.

The theoretical model requires that each individual be given the opportunity to purchase or refuse a product. We also require sufficient variation in prices to identify the behavioral responses, $\hat{\beta}_1^j$ for $j = 1,2,\ldots,J$ in equation (1). Such data would not be commonly available in market transactions, and so we developed an artificial environment where appropriate data could be obtained. The universe of potential customers in this study is the undergraduate student population at Iowa State University.

In 2007, a random sample of 2000 students was invited by Email to participate in a web-based survey. Of these, 470 (23.5%) provided complete responses, and they represent the working sample for this study. The working sample reflected the attributes of the sample universe well, and so there does not appear to be any systematic relationship between observable student attributes and the likelihood of survey response. Table 1 describes the working sample.

The Iowa State student body is a particularly useful universe of potential subjects for this type of study. ISU students place a lot of importance on sports. In our sample, 43% of the participants in the study were fans of the college before attending ISU, 65% played intramural sports at the university and 68% played varsity sports in high school. Most continue to support ISU sports while students at the university. The ISU athletics budget follows the NCAA Division I-A pattern of profitable football and men’s basketball and negative returns to the rest of the sorts. While men´s basketball and football ended 2014 with surpluses of $4 million and $18 million respectively, the rest of the sports had an accumulated deficit of $15 million. Even women's basketball, which has consistently ranked among the top ten in attendance nationally, had losses of $3 million.

In addition to demographic information and questions related to participation and interest in sports, each respondent was asked whether they would purchase the next sports tickets at a stated price: a) a women's basketball ticket, b) a men's basketball ticket, c) a football ticket, d) a volleyball ticket, e) a wrestling ticket, f) a gymnastics ticket, g) a hockey ticket, and h) a soccer ticket.

Prices were generated from independent random draws from uniform price distributions. The men's basketball price was drawn from the uniform distribution U(7, 20) and rounded to the nearest whole number. Football prices were the nearest whole number drawn from the distribution U(13, 25). The rest of the sports prices were rounded to the nearest whole number from numbers drawn from the uniform distribution U(1, 10). The median value in each distribution was the current student ticket price.

Because each respondent received a randomly drawn price, there is no correlation with the control variables Z_i. As a result, we get virtually identical response parameters to the prices regardless of whether or not the Z_i's are included in the estimation. Nevertheless, it is useful to highlight some of the more interesting control variables used in the analysis. To control for overall interest in sports, we included information on whether the individual played varsity sports in high school and whether the individual participated in intramurals in school or in Iowa State. To control for demand for Iowa State sports more specifically, we ask whether the individual was an Iowa State fan before coming to college and whether they had parents or relatives graduate from Iowa State. We control for differences in tastes between men and women by including a gender dummy variable.

A useful check on the success of the price randomization is to plot the probability of a positive purchase response by random price offered. If respondents are following the law of demand, we should be able to trace out standard demand relationships from data plots. This is shown in figure 2. While the relationship is not perfectly monotonic, the fitted bivariate relationship between intent to purchase and price satisfies the law of demand: the higher the price, the lower the proportion of people willing to buy the ticket.

Additionally, figure 3 illustrates an example of the differences existing between the demand relationships for men's basketball between people who purchased another sport ticket (football) and people who did not. The fraction of people who stated they would buy a men's basketball ticket at the offered price is higher when conditioned on a positive response for purchasing a football ticket. This descriptive result shows the possibility of exploiting the unobserved common taste for pricing tickets, such as, creating bundling strategies. In general, and as our estimations will show, it is expected that willingness to purchase a given sport's ticket be different conditional on the decision the person made on other sports.

5. RESULTS

After presenting results for point estimates, we continue showing the results in the same order as structured in section 4.

A. Point estimates

The equations system (2) constitutes a seemingly unrelated 8-variate probit model. We estimate the system using simulated maximum likelihood.[11] A total of 400 draws from the likelihood function distribution are taken to compute the simulated likelihood in each iteration of the optimization algorithm. Table 2 reports the results.

[11] Simulated maximum likelihood is more accurate than conventional or exact maximum likelihood in estimating multivariate models, probits in particular, due to the difficulty in finding closed forms of the probabilities maximized as the number of equations increases (Cappellari and Jenkins, 2003).

For the above estimation, the Wald test shows that all estimated parameters in the regression equations jointly statistically significant. Also, the likelihood ratio test of the joint significance of the elements of the correlation matrix of the errors, ρ_{jk}, rejected the null hypothesis of zero correlations. Finally, the joint hypothesis that the individual characteristics used as control variables could be dropped from all the equations was soundly rejected with $\chi^2(48)$ =119.40. For that reason, we opted to include all control variables in all equations even though some of them are individually non-significant.

The own-price effects are all negative and significant. An increase on the price of the ticket reduces the probability of purchasing as the law of demand states. Men were more likely than women to buy all sports tickets except for men's basketball. Individuals who were Iowa State fans before coming to college were more likely to buy all sports tickets. Individuals who played sports in high school or college, whether organized sports or intramurals, were more likely to buy basketball and football tickets, but the relationship between sports participation and ticket demand is mixed for the other sports.

Another important issue related to the econometric design is that the order of the sports ticket price offered on the experimental design did not vary. Respondents were offered tickets in this order: women's basketball, men's basketball, football, volleyball, wrestling, gymnastics, hockey and soccer. This ordering could bias the estimation due to priming: the response to one price is influenced by the previously offered price. For example, if individuals received a low price offer and that causes them to buy the first sports ticket, then their decision on the second sports ticket may change compared to their response were they not have been offered a low price on the first sports ticket. To test whether this bias clouds our results, we conducted a re-estimation that includes the price of the sport that was previously offered. As shown in Appendix table A1, our own price coefficients are unaffected by the inclusion or exclusion of the other sports pries. Even in the few cases when it is significant, the own price coefficient hardly changed, suggesting that the randomized price strategy successfully avoided the potential priming problem.[12]

Our robustness tests confirmed that a) our model includes enough explanatory variables because adding other prices does not change the own-price coefficient; and b) the fact that the previously offered prices do not alter the own-price coefficient means that our price responses are interpretable as the outcome of simultaneous purchasing decisions rather than sequential purchasing decisions. These findings, and the fact that purchasing decision of one sport does not prevent from buying other sports, support the use of seemingly unrelated multivariate probit as the appropriate specification rather than conditional probits, multinomial probits, multinomial logits[13], and nested logits, all of whom assume sequential decisions.[14]

B. Correlation in unobserved demand for sport tickets

The cross-equation error correlations from equation (2) are reported in Table 3. All estimated correlations are positive, implying that the unobserved factors that prompt individuals to purchase tickets for one sport are positively correlated with the unobserved factors that prompt individuals to purchase tickets of all the other sports. Moreover, in all cases (except for the combination of football and soccer) estimations are statistically significant. The correlations suggest some interesting marketing strategies, such as bundling, particularly for sports that have difficulty attracting fans. For example, individuals who have an unusual strong interest in soccer

[12] We conducted a similar robustness check including all cross-prices, and similar results were obtained. Almost no price of sport *j* different from sport *i* was significant to explain decision of buying or not sport *i*.
[13] For a comparison between multivariate probit and multinomial logit see Young et al. (2009).
[14] Although we consider that the underlying latent utility is probabilistic and nonlinear, we also estimated an SUR linear probability model to test robustness to the functional form specification and the sensibility to the reparametrization of the variance and covariance matrix of the error terms embedded in the probit estimation. This model yields similar estimates of marginal effects and of unobserved correlation coefficients. Results are available upon request.

are also more likely to buy volleyball, gymnastics and hockey tickets. Because these sports have non-overlapping sections of their seasons, the time budget constraints do not force the sports to be substitutes.

C. Unconditional willingness to pay or reservation prices

Results for this section are derived by solving equation (5) for a reservation price for each individual in the sample for each of the 8 sports. A reservation price is defined as the price level that leaves the individual indifferent between purchasing or not purchasing a ticket for the corresponding sport. For each sport, Table 4 reports the mean, median and standard deviation of the reservation prices over the sample of 449 individuals. When comparing mean or median reservation price for each sport with the actual ticket price charged, we can identify the sports that charge less than students' average willingness to pay. Results show that this is the case for women´s and men´s basketball, football and hockey. They also show that wrestling prices are set at the same level as the reservation price.

While it is clear that a price increase may reduce demand by a quantity that make revenues to decrease, our setup let us compute the exact price that maximize revenues, because we identified the whole demand function for each sport, as shown in subsection 5.D.

D. Conditional reservation prices

Results in this section arise from numerically solving the nonlinear equation in (7) for a reservation price (p^{jk}) for each individual in the sample who decided to purchase both tickets, say for example, women's basketball (*j*) and men's basketball (*k*). The solution of equation (7) for a reservation price is performed by evaluating explanatory variables in vector $\boldsymbol{Z}$ at the observed value for each individual, and B^{-j} at the actual decision each individual made (which, in turn, implies computing the actual value of q^{-j} for each individual). The Bisection method is employed given that it consists of one equation in one unknown. Following this procedure, we compute conditional reservation prices for each pair of sports, which are then compared to the unconditional reservation prices estimated in the last section for the conditioned sport.

Table 5 summarizes the results. Conditional reservation prices are higher than their unconditional counterparts for each combination of sports; this has an appealing economic interpretation, i.e., the unobserved common taste for the group of goods translates into willingness to pay, consistent with the complementarity concluded in subsection B. People who already have a ticket to one sport event have higher willingness to pay for the other sport. For example, individuals who purchased a men's basketball ticket are willing to pay an average of 41% more for a women's basketball ticket, compared to the unconditional estimated willingness to pay.[15]

In addition to these results, we can check how the variability of individual's choices shifts between the conditional and the unconditional case. We compare the coefficient of variation of unconditional reservation prices (standard deviation of P^j over median P^j) and the coefficient of variation of conditional reservation prices (standard deviation of P^{jk} over median P^{jk}).[16] In the majority of cases, the conditional reservation prices have more variability and are less concentrated than unconditional reservation prices, but there is no clear generalizable pattern. A similar exercise can be done for the change on the Skewness of conditional prices compared to unconditional prices. In almost every case, the difference is negative, meaning that the distribution of the reservation prices shifts upward when conditioned on other sports purchases.

E. Own-price and cross-price elasticities

[15] Tables A2 and A3 in the appendix show, respectively, the median P^{jk} and median P^j used to construct each cell of Table 5.

[16] Detailed results for each sports pair are not shown due to space, but are available from the authors upon request.

The Slutsky equations establish that the own price elasticity of demand for sport j is $\varepsilon_{jj} = \xi_{jj} - \theta_j \eta_j$, where ξ_{jj} is the compensated elasticity (i.e. the change in demand of j which leaves the individual in the same level of utility), θ_j is the budget share for the j^{th} sport, and η_j is the income elasticity of demand for the j^{th} sport. Similarly, the Slutsky equations for cross-price elasticities between any two sports j and k, $j \neq k$, establish that $\varepsilon_{jk} = \xi_{jk} - \theta_k \eta_j$, where ε_{jk} is the cross-price elasticity of demand due to a change in price of sport k, ξ_{jk} is the compensated cross-price elasticity (i.e. the change in demand of j which leaves the individual at the same level of utility after a change in price of k), θ_k is the budget share for the k^{th} sport, and η_j is the income elasticity of demand for the j^{th} sport.

The randomization of the experimental design implies that decisions for each sport are made independently from those for the other sports (which in this set up can be translated in the lack of income effects $\theta_j \cong 0$), yielding an approximate equality between the uncompensated demand elasticities and their compensated counterparts ($\varepsilon_{jk} \cong \xi_{jk}$ and $\varepsilon_{jj} \cong \xi_{jj}$). Having this in mind, we present in this section, results for own-price compensated demand elasticities for the eight sports and cross-price compensated elasticities for sport j after a change in sport k price using the methodology presented in section 4.

These are reported in table 6. To illustrate the interpretation, the first number shows that a 1% change in women's basketball price induces a median reduction of 0.46% in the probability of purchasing a women's basketball ticket. All the diagonal estimates are negative, and so price increases lower probability of sale. Only men's basketball is in the elastic range, and so prices could be raised to increase revenue for all other sports. The cross-price elasticities are all positive, and so all sport tickets are substitutes. We infer that while the unobserved correlation between sports (shown in part C) was estimated as positive (i.e. complementary goods) and can be interpreted as “pure taste for sports”, holding taste fixed, tickets are substitutes.

In other words, this complementarity is not large enough to drive a negative cross-price effect, but ignoring it would lead to an overestimation on average of the substitutability between sports. This overestimation would yield a pricing policy with at least some prices lower than what they would be if the unobserved correlation is taken into account, lowering revenue generation from the maximum possible. Therefore, our approach calls for the importance of setting pricing strategies in a multivariate framework when goods of interest can be considered members of the same cluster.

F. Optimal pricing strategy simultaneously determined for all sports

Results for this section arise from solving problem in (13) for a set of prices for each sport, assuming a linear utility for the planner. The problem is highly non-linear and of high dimension (8 dimensions), making it hard to find a closed-form solution. Therefore, we solve it numerically using the Davidon-Fletcher-Powell (DFP) Quasi-Newton optimization method.

A salient feature of these applications is the role played by the social planner who can decide how much to weight producer’s revenues versus consumer utility. In our application we could think of the social planner as University President or Board of Regents; the consumers being the public attending the events or the alumni monitoring sports success through the media; and the producer as the athletics department. Revenues are normalized by maximum revenues possible (the revenues attained if all the weight is assigned to the producer, that is w_C = 0) and therefore they range between 0 and 1, as it does joint utility because it is a probability.

One key aspect of this approach is that demographics of individuals (given by variable $\boldsymbol{Z}$ in problem (13)) condition the optimization results. If strategies for price discrimination are limited, as it is the case with sport tickets, we may describe individuals by a representative consumer.[17] First, suppose the representative consumer is the median individual[18] and the same weight is given to both the producer and consumers (w_C = 0.5). Table 7 shows the resulting optimal price strategy, which when compared to the actual prices, it implies to lower the prices of men's basketball, football and volleyball, and increase the rest. It turns out that actual prices yield lower revenues for the producer (and lower utility for the planner), but they do provide higher joint latent utility for the median consumer.

In order to show how the optimization works, figure 4 depicts the optimal solution in normalized revenues and joint utility scale (black diamond). As mentioned above, both joint utility and revenues range between 0 and 1. Points to the right and above the black lines, while Pareto superior, are not attainable because the maximum is obtained at the black diamond. However, points with either higher utility or higher revenues are possible but will not maximize planner's objective function; they will lie to the left and below the blue frontier. The blue frontier indicates all the utility and revenue pairs which leave the planner with a value function at a level equal to the optimum.[19] To its left, the planner's value function is lower. To illustrate this, we generate 2000 sets of 8 uncorrelated random prices and observe that the revenue and utility they yield lie in all cases in the mentioned region indicated by blue dots. Finally, this random set of prices also show that pairs of revenue and utility between the blue frontier and black lines are also not attainable due to the tradeoff between utility and revenue functions.

[17]As we already noticed, our setup allows pursuing a pricing strategy for each individual in the sample which is consistent with goods or services when a first-degree price discrimination is possible.
[18] A male, who played varsity sports in high school, did not play intramurals in high school but did in Iowa State, and neither has relatives graduated from Iowa State nor was a Cyclone fan before coming to school.
[19] The tradeoff between revenues and consumer's utility indicates that if consumers have zero joint utility, it requires the planner to have revenues higher than 1 (the maximum attainable) in order to have the same utility as that of the optimum.

If the planner gives more weight to the producer, for example if w_C goes from 0.5 to zero, it is expected that the pricing strategy yields higher revenues for the producer and lower joint utility for the median consumer. The extreme case, where all the weight is given to the producer ($w_C \rightarrow 0$) is indicated with a black asterisk in figure 5.[20] More cases are shown in the appendix.

The approach described above makes little use of the information available from the individual's demographics. A more "informed" strategy for both the planner and producer could be to rely on the knowledge of how these demographics determine optimal prices for each sport. For example, consider the specification of types of individuals which represent strata in the population. As each type yields a different pricing strategy, the planner may set the overall optimal strategy as a weighted average across strata.[21] Table 8 shows the optimal pricing strategy for each type (types are described at the bottom of the table). As expected, it is optimal to charge higher prices of women's basketball to women than to men, and higher prices of men's basketball tickets to men compared to women. It is also optimal to charge women higher prices for women's volleyball, soccer and gymnastics, but almost the same price as men for football, wrestling and hockey. Higher prices should also be charged to self-professed ISU fans, especially for basketball and football. Having played sports does not significantly affect optimal prices. Considering these differences, an optimal pricing strategy may be constructed as some weighted average of the different customer types or else strategic discounts from a common price could be targeted to the more price sensitive groups.

6. CONCLUSIONS

This analysis is an extension of Greene's (2003) conditional marginal effects framework to estimate reservation prices conditional on having decided to purchase another offered product.

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[20] It is likely that for most goods and services this is the only scenario of interest.

[21] Recall that price discrimination is not possible in this case, so only one price must be available for each sport at the tickets office.

This allows us to identify complementarities or substitutabilities among the offered goods which informs optimal marketing and pricing strategies. Additionally, besides estimating own-price elasticities in a standard fashion, we propose a method to estimate compensated cross-price elasticities of substitution among goods, exploiting the unobserved correlation among the demands of the goods modeled. The existence of cross-price effects in demand supports a pricing strategy which considers all offered goods simultaneously rather than separately. Our approach solves for the set of prices which maximize the utility of a social planner whose objective function depends on both producer's revenues and consumer's joint latent utility. Different weights can be assigned to each agent depending on the particular application, for example, all the weight collapsing to the producer or equal weight to both.

We illustrate the method using data from a contingent valuation survey administered at Iowa State University. Students were given the option to buy or reject 8 offered college sports tickets at randomly assigned prices. The correlations among the error terms from the equations predicting the buy/not buy decision are all positive and significant, proof that there are unobserved factors tied to tastes for sports which prompts attending sport events regardless of the discipline, i.e. inducing complementary between the goods.

When we compare the conditional to the unconditional reservation prices, we find that for almost all sports, people who buy any one ticket are more likely to pay more for other sports tickets. Because we show the demands are consistent with simultaneous decisions of whether to buy all 8 tickets, our results suggest that the sports should be marketed together rather than separately, for example opening the possibility of making product bundles. Estimation of cross-price elasticities show that, holding taste for sports constant, they are substitute goods. The mentioned complementarity is not large enough to drive a negative cross-price effect, but ignoring it would overestimate on average the substitutability between sports, leading to a pricing policy with at least some prices lower than what they would be otherwise with its consequences on revenue generation. As such, our approach calls for building pricing strategies in a multivariate framework, suggesting to avoid single equation estimations for goods categorized in clusters. Finally, the social planner's optimal pricing strategy yields that prices for some sports should be raised but others lowered.

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FIGURES

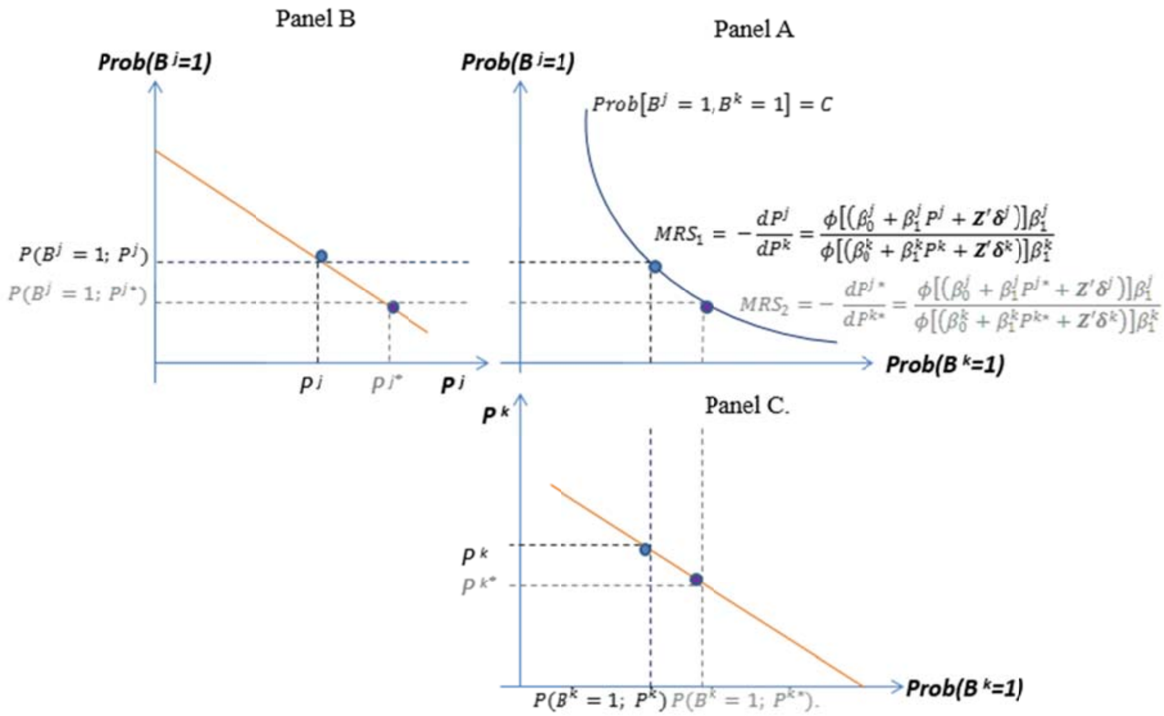

Figure 1. Derivation of cross-price elasticities between any pair of sports *j* and *k*

Note: Panel A. Indifference Curve between sports j and k. Panel B. Compensated demand Curve for Sport j. Panel C. Compensated inverse demand Curve for Sport k.

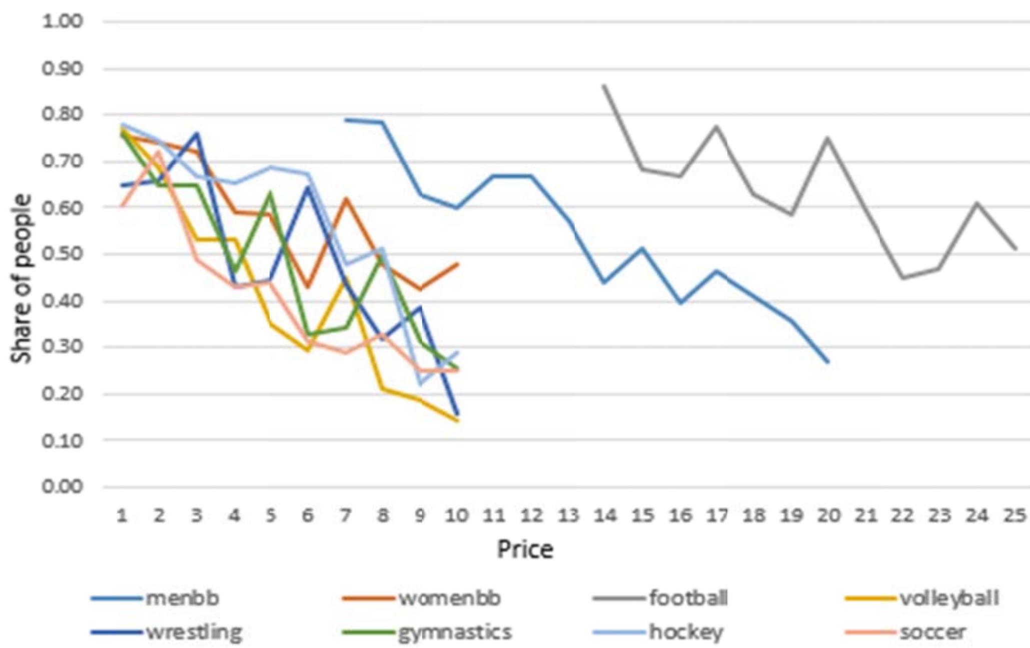

Figure 2. Panel A. Demand curves for each of 8 collegiate sports, measured by the proportion of people buying a sport at each price

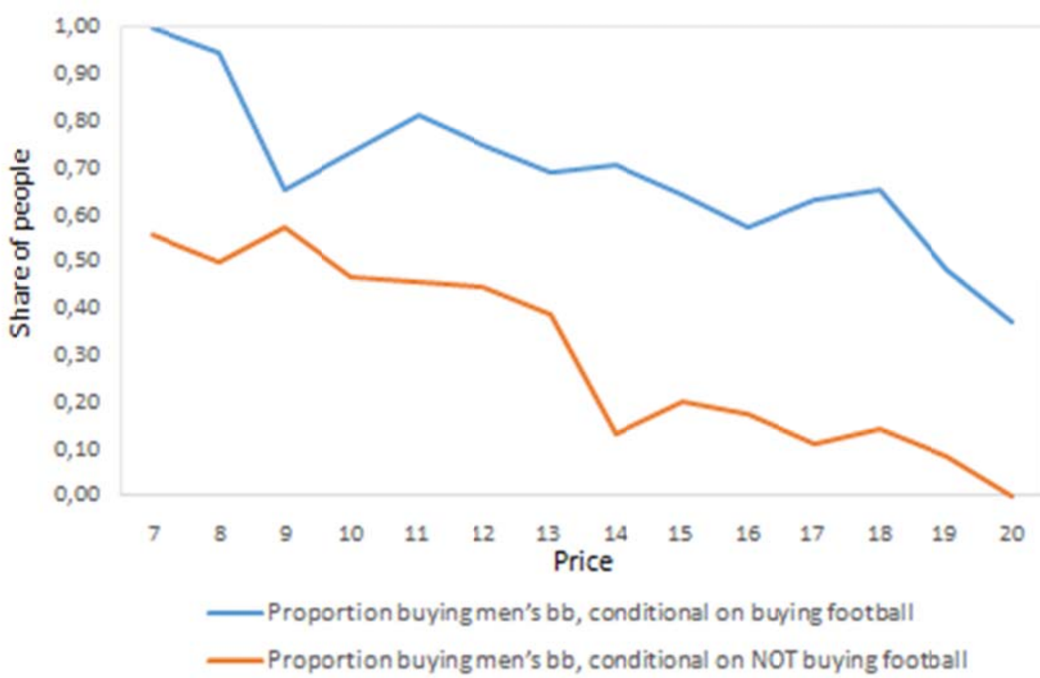

Figure 3. Panel B. Demand curves for men's basketball conditional on buying a football ticket, measured by the share of people that would buy a men´s basketball at each price.

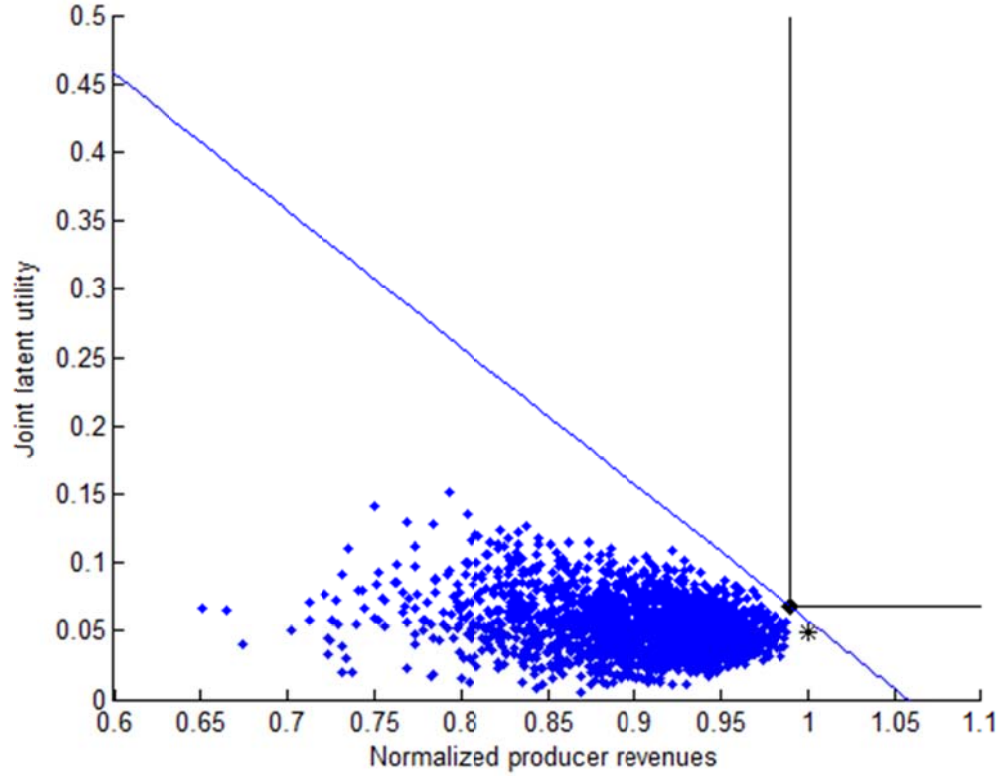

Figure 4. Social planner optimal strategy for w_C = 0.5 (black diamond), suboptimal strategies (blue dots) and indifference curve of the social planner between producer's revenue and consumers' utility (blue frontier). Numerical solutions of problem (13).

Note: If planner gives less weight to consumers, the optimal strategy implies higher revenues and lower joint utility (black asterisk, for $w_C \rightarrow 0$)

TABLES

Table 1. Summary statistics.

Variable	Mean	Standard deviation	Description
Bwbb	57.9%	49.4%	= 1 if buys women's basketball ticket
Bmbb	53.5%	50.0%	= 1 if buys men's basketball ticket
Bf	61.9%	48.6%	= 1 if buys football ticket
Bv	42.2%	49.4%	= 1 if buys volleyball ticket
Bwres	49.2%	50.0%	= 1 if buys wrestling ticket
Bgym	47.9%	50.0%	= 1 if buys gymnastics ticket
Bh	58.1%	49.4%	= 1 if buys hockey ticket
Bs	40.4%	49.1%	= 1 if buys soccer ticket
Pwbb	5.63	2.83	Random price of women's basketball ticket offered
Pmbb	13.65	3.99	Random price of men's basketball ticket offered
Pf	18.83	3.81	Random price of football ticket offered
Pv	5.46	2.86	Random price of voleyball ticket offered
Pwres	5.40	2.93	Random price of wrestling ticket offered
Pgym	5.71	2.89	Random price of gymnastics ticket offered
Ph	5.39	2.81	Random price of hockey ticket offered
Ps	5.56	2.90	Random price of soccer ticket offered
Gender	52.2%	50.0%	=1 if male
hi_sport	67.7%	46.8%	= 1 if played varsity sports in high school
hi_inter	38.3%	48.7%	= 1 if plays intramurals in high school
isu_inter	64.6%	47.9%	= 1 if plays intramurals in school
isu_family	40.8%	49.2%	= 1 if parents, grandparents or siblings attended ISU
isu_fan	42.9%	49.5%	= 1 if was a Cyclone fun before attending ISU
N	471		Number of observations

Note: ISU = Iowa State University

Table 2. Simulated maximum likelihood estimation of seemingly unrelated model in (2). Dependent variable: Binary choice of purchasing ticket of sport *j*, B^j, with *j* = 1,2,…,8. Standard errors in brackets.

	1	2	3	4	5	6	7	8
Dep. variable	womenbb	menbb	football	volleyball	wrestling	Gym	hockey	soccer
own price	-0.126***	-0.106***	-0.076***	-0.176***	-0.143***	-0.133***	-0.168***	-0.118***
	(0.022)	(0.017)	(0.018)	(0.024)	(0.024)	(0.023)	(0.023)	(0.022)
gender	-0.545***	0.084	-0.101	-0.294**	-0.144	-0.672***	-0.209	-0.268*
	(0.141)	(0.137)	(0.144)	(0.143)	(0.140)	(0.143)	(0.139)	(0.141)
hi_sport	0.183	0.037	-0.037	0.17	0.634***	0.034	-0.025	0.124
	(0.157)	(0.148)	(0.157)	(0.159)	(0.165)	(0.164)	(0.156)	(0.161)
hi_inter	0.052	0.049	0.224	-0.007	-0.229	-0.109	0.036	0.055
	(0.145)	(0.141)	(0.149)	(0.152)	(0.154)	(0.154)	(0.148)	(0.153)
isu_inter	0.239	0.315**	0.469***	-0.018	0.318**	-0.065	0.112	0.056
	(0.160)	(0.155)	(0.154)	(0.159)	(0.150)	(0.161)	(0.157)	(0.158)
isu_famil	0.004	-0.002	-0.068	-0.112	0.006	0.03	-0.187	-0.053
	(0.157)	(0.154)	(0.156)	(0.160)	(0.156)	(0.156)	(0.156)	(0.158)
isu_fan	0.490***	0.425***	0.444***	0.262*	0.335**	0.117	0.119	0.308**
	(0.152)	(0.156)	(0.153)	(0.157)	(0.158)	(0.156)	(0.155)	(0.154)
constant	0.653***	1.085***	1.304***	0.709***	0.1	1.046***	1.197***	0.277
	(0.189)	(0.278)	(0.383)	(0.195)	(0.220)	(0.212)	(0.214)	(0.197)
N = 449 – Simulation draws = 400 – LogLikelihood = -1,919 – Iterations = 33								

Table 3. Estimation of correlation coefficient parameters ρ_{jk}, with *j,k* = 1,2,…,8. Based on equation (3).

Dependent variable	women bb	men bb	footb all	volleyball	wrestling	gym	hockey	soccer
women bb	1.00							
men bb	0.51	1.00						
football	0.20	0.60	1.00					
volleyball	0.70	0.38	0.20	1.00				
wrestling	0.44	0.30	0.23	0.53	1.00			
gym	0.59	0.33	0.24	0.60	0.45	1.00		
hockey	0.42	0.28	0.27	0.52	0.42	0.55	1.00	
soccer	0.52	0.30	0.12	0.57	0.41	0.62	0.57	1.00

Note: All parameters are statistically significant at 5% except for ρ_{38} and ρ_{83}

Table 4. Estimated unconditional reservation prices of sport tickets, and comparison with actual ticket prices. Based on equation (5)

	women bb	**men bb**	**football**	**volleyball**	**wrestling**	**gym**	**hockey**	**soccer**
Actual price	5.00	13.50	19.00	5.00	5.00	5.00	5.00	5.00
				Estimated reservation price				
Median	7.08	14.46	23.33	4.07	5.17	3.95	6.72	3.42
Mean	7.00	14.67	23.40	4.12	5.01	5.28	6.76	3.31
Std. deviation	3.06	2.62	4.37	1.17	2.83	2.74	0.83	1.76
N	449	449	449	449	449	449	449	449

Table 5. Estimated percent increment of conditional reservation prices of sport tickets relative to unconditional reservation prices.

	k	1	2	3	4	5	6	7	8
j	Labels	womenbb	menbb	football	voleyball	wrestling	gymnastics	hockey	soccer
1	womenbb		40.5%	29.4%	48.4%	30.4%	40.9%	31.0%	41.5%
2	menbb	40.7%		37.0%	40.9%	29.7%	39.4%	33.9%	33.6%
3	football	21.8%	20.2%		19.8%	12.5%	20.8%	18.6%	16.4%
4	voleyball	85.4%	79.9%	74.2%		68.7%	78.6%	86.6%	95.1%
5	wrestling	54.4%	43.3%	40.6%	70.7%		67.4%	44.6%	57.4%
6	gymnastics	44.5%	52.0%	32.8%	48.7%	36.7%		46.5%	33.0%
7	hockey	38.6%	39.1%	32.5%	46.7%	23.7%	45.8%		44.6%
8	soccer	156.4%	160.8%	142.7%	169.8%	142.7%	172.7%	170.4%	

Note: Main diagonal elements have no significant meaning, therefore they are dropped. A useful hint to read this table and grasp the absolute values of conditional prices, increase the sport price in Table 3 by the percentage of interest corresponding to the row where the percentage is.

Percentage increment defined as $(P^{jk} - P^{j})/P^{j}$ where P^{jk} is the conditional price of j on k and P^{j} the unconditional price of j. Based on equation (7).

Table 6. Estimated elasticities for the eight sports evaluated at the median based on equation (8). Main diagonal are own-price elasticities and the rest are cross-price.

	k	1	2	3	4	5	6	7	8
j	Labels	womenbb	menbb	football	voleyball	wrestling	gymnastics	hockey	soccer
1	womenbb	-0.46	0.80	0.85	0.53	0.51	0.45	0.44	0.54
2	menbb	0.49	-1.10	0.68	0.68	0.54	0.46	0.45	0.46
3	football	0.33	0.95	-0.82	0.56	0.38	0.45	0.48	0.37
4	voleyball	0.28	1.06	0.87	-0.96	0.50	0.40	0.36	0.49
5	wrestling	0.32	0.74	0.66	0.39	-0.64	0.35	0.43	0.38
6	gymnastics	0.42	1.03	0.73	0.48	0.51	-0.61	0.38	0.53
7	hockey	0.43	0.83	0.78	0.37	0.45	0.42	-0.62	0.58
8	soccer	0.25	0.83	0.71	0.46	0.41	0.34	0.33	-0.66

Note: The median was chosen instead of the mean because of the skewness of the distribution of the elasticities.

Table 7. Comparison of optimal pricing strategy with actual prices and reservation prices. Optimal pricing is solution of problem (13) for w_C = 0.5.

	women bb	**men bb**	**football**	**volleyball**	**wrestling**	**gym**	**hockey**	**soccer**
				Actual prices and estimated reservation prices				
Actual price	5.00	13.50	19.00	5.00	5.00	5.00	5.00	5.00
Median WTP	7.08	14.46	23.33	4.07	5.17	3.95	6.72	3.42
				Optimal pricing strategy				
Mean	6.48	12.61	18.24	4.62	6.67	5.36	6.39	5.14

Table 8. Optimal pricing strategy by type of individuals. Solution of problem in (13) for different types and w_C = 0.5

Type	**women bb**	**men bb**	**football**	**volleyball**	**wrestling**	**gym**	**hockey**	**soccer**
I	8.27	15.07	22.88	4.51	6.61	4.49	5.63	5.81
II	10.62	14.08	23.44	5.09	6.63	7.14	6.08	6.55
III	7.26	14.60	21.37	4.04	5.10	5.40	5.88	5.16
IV	9.59	13.88	21.99	4.69	5.16	7.65	6.20	5.80
V	6.14	11.45	16.70	4.95	5.31	5.72	6.38	5.83
VI	7.32	10.30	16.73	5.09	4.17	7.13	6.27	5.37
VII	5.49	11.15	15.35	5.02	3.76	6.13	6.52	6.05
VIII	6.94	10.38	15.86	5.03	3.53	8.20	6.74	5.69

Note: Types I and II play sports and are ISU fans, male and female respectively. Types III and IV, do not play sports but are ISU fans, male and female respectively. Types V and VI play sports but are not ISU fans, also male and female respectively. Types VII and VII neither play sports nor are ISU fans, male and female respectively.

APPENDIX

Table A1. Simulated maximum likelihood estimation of seemingly unrelated model in (2) but including the price offered previously. Dependent variable: Binary choice of purchasing ticket of sport *j*, B^j, with *j* = 1,2,…,8.

	1	2	3	4	5	6	7	8
Dependent variable	womenbb	menbb	football	volleyball	wrestling	gym	hockey	soccer
own price	-0.114***	-0.098***	-0.078***	-0.179***	-0.138***	-0.132***	-0.166***	-0.118***
price previously offered	-	0.057**	0.024	-0.002	-0.019	0.029	-0.003	0.011
gender	-0.543***	0.109	-0.081	-0.294**	-0.137	-0.675***	-0.209	-0.267*
hi_sport	0.181	0.016	-0.035	0.164	0.632***	0.035	-0.030	0.123
hi_inter	0.057	0.067	0.226	-0.004	-0.227	-0.105	0.034	0.048
isu_inter	0.235	0.287*	0.461***	-0.017	0.314**	-0.067	0.112	0.058
isu_family	0.008	-0.004	-0.058	-0.114	0.008	0.018	-0.188	-0.052
isu_fan	0.492***	0.424***	0.456***	0.265	0.336**	0.127	0.118	0.309**
constant	0.587***	0.664**	0.984**	0.763**	0.178	0.884***	1.209***	0.220
N=449 - Simulation draws=400- Loglikelihood= 1,919 - Iterations= 34								

Table A2 Median of unconditional reservation prices of sport j for individuals who buy sport *j* and *k*. Based on equation (5).

	k	1	2	3	4	5	6	7	8
j	Labels	womenbb	menbb	football	voleyball	wrestling	gymnastics	hockey	soccer
1	womenbb		8.13	8.13	8.52	8.51	8.51	8.13	8.13
2	menbb	14.79		14.81	14.58	14.79	14.58	14.58	14.74
3	football	24.05	24.46		23.57	24.30	23.12	23.45	23.33
4	voleyball	4.67	4.67	4.36		4.67	4.67	4.32	4.36
5	wrestling	6.35	6.35	6.35	5.80		5.80	6.35	5.76
6	gymnastics	7.44	6.92	7.38	7.38	7.38		7.44	7.63
7	hockey	6.98	6.61	6.74	6.76	6.98	6.98		6.76
8	soccer	3.87	3.76	3.76	3.87	3.81	3.76	3.75	

Note: Main Diagonal elements are 0 because they have no meaning.

Table A3. Median of conditional reservation prices for individuals who buy the pair of sport *jk*. Based on equation (7).

	k	1	2	3	4	5	6	7	8
j	Labels	womenbb	menbb	football	voleyball	wrestling	gymnastics	hockey	soccer
1	womenbb	7.08	11.42	10.52	12.64	11.09	11.99	10.65	11.50
2	menbb	20.81	14.46	20.30	20.54	19.19	20.32	19.52	19.69
3	football	29.29	29.39	23.33	28.23	27.33	27.94	27.82	27.15
4	voleyball	8.66	8.40	7.59	4.07	7.88	8.34	8.06	8.50
5	wrestling	9.80	9.10	8.92	9.89	5.17	9.71	9.18	9.06
6	gymnastics	10.75	10.52	9.79	10.97	10.09	3.95	10.89	10.15
7	hockey	9.67	9.20	8.93	9.92	8.63	10.17	6.72	9.78
8	soccer	9.92	9.81	9.13	10.45	9.26	10.26	10.13	3.42

Note: Main diagonal is shows the median unconditional reservation price for all the sample (not only those who bought sports *jk* as it is the case with table A3).

Weighting strategies scenarios. If the planner gives more weight to the producer, it is expected that the pricing strategy yields higher revenues for the producer and lower joint utility for the median consumer. This pattern is shown in figure A.1 where blue dots represent optimal solutions as w_C decreases from 0.8 through 0.05; more precisely we set w_C = {0.80, 0.70, 0.65, 0.60, 0.55, 0.50, 0.45, 0.35, 0.25, 0.15, 0.05}. The black diamond is w_C = 0.5 and the black star is w_C = 0.05. As weight put into producer revenues approaches to one, higher revenues come at a cost for the planner, because require foregoing relatively more utility from consumers.

It is likely that for most goods and services, the situation where the weighting to consumers collapses to zero is the only scenario of interest, i.e., the case of the maximization of revenues alone.

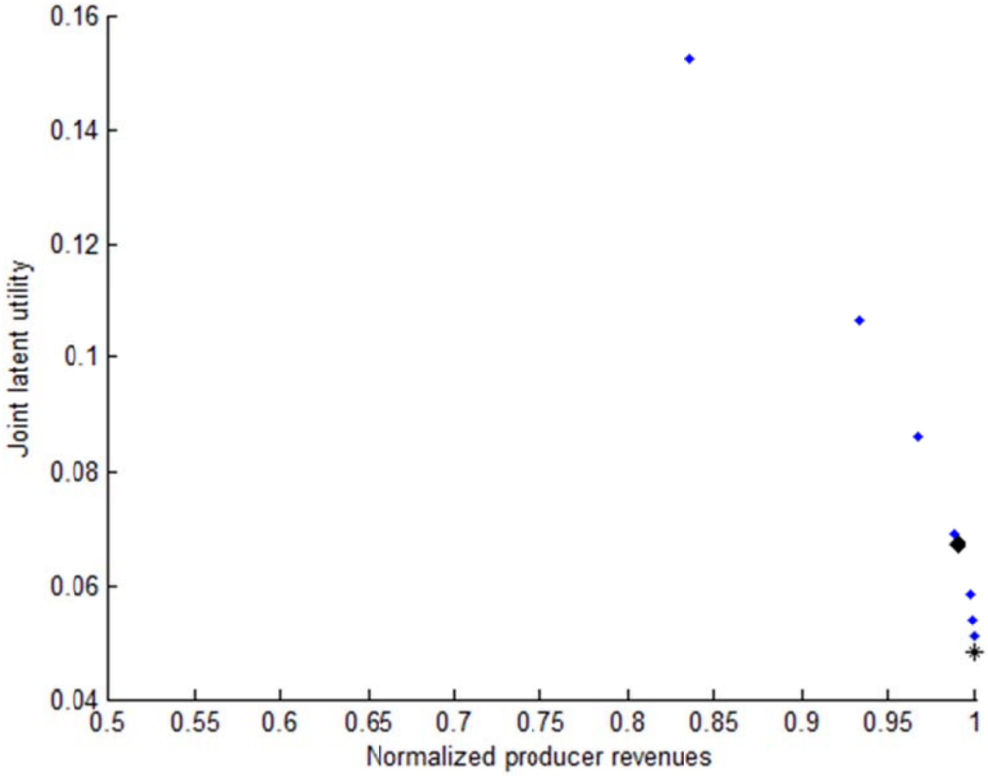

Figure A.1. Social planner optimal strategies for different weights to the consumers. Numerical solutions of problem (13).