5-1975

Heuristic Programs For Some Transshipment Plant Location Problems

George W. Ladd
Iowa State University

Follow this and additional works at: http://lib.dr.iastate.edu/econ_las_staffpapers
Part of the Behavioral Economics Commons, Databases and Information Systems Commons, Economic History Commons, Economic Theory Commons, and the Theory and Algorithms Commons

Recommended Citation
http://lib.dr.iastate.edu/econ_las_staffpapers/173

This Report is brought to you for free and open access by the Economics at Iowa State University Digital Repository. It has been accepted for inclusion in Economic Staff Paper Series by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Heuristic Programs For Some Transshipment Plant Location Problems

Abstract
The main objective of this paper is presentation of complete details on two procedures for solving transshipment plant location problems: procedures called "Simple Version of 0RA(1, t)" and "Simple Version of 0RA(2, t)" in Ladd and Lifferth [61. The second of these two procedures is a "heuristic" procedure or a "heuristic program." A number of heuristic procedures have already appeared in the literature for solving plant location problems: in papers by King and Logan Chern and Polopolus p3", Warrack and Fletcher [12] and Candler^ Snyder and Faught [2],

Disciplines
Behavioral Economics | Databases and Information Systems | Economic History | Economic Theory | Theory and Algorithms
HEURISTIC PROGRAMS FOR SOME
TRANSSHIPMENT PLANT LOCATION PROBLEMS

by

George W. Ladd

No. 10

May 1975
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heuristics</td>
<td>1</td>
</tr>
<tr>
<td>Notation</td>
<td>3</td>
</tr>
<tr>
<td>ORA(1, t)</td>
<td>4</td>
</tr>
<tr>
<td>ORA(2, t)</td>
<td>7</td>
</tr>
<tr>
<td>Stage 1</td>
<td>8</td>
</tr>
<tr>
<td>Stage 2</td>
<td>9</td>
</tr>
<tr>
<td>Stage 3</td>
<td>11</td>
</tr>
<tr>
<td>Stage 4</td>
<td>15</td>
</tr>
<tr>
<td>Stage 5</td>
<td>19</td>
</tr>
<tr>
<td>Use of existing facilities</td>
<td>20</td>
</tr>
<tr>
<td>Summary</td>
<td>21</td>
</tr>
<tr>
<td>Other heuristic procedures</td>
<td>22</td>
</tr>
<tr>
<td>Economies of size in transport</td>
<td>23</td>
</tr>
<tr>
<td>Predetermined demand at each final market</td>
<td>25</td>
</tr>
<tr>
<td>Nonlinear processing cost function</td>
<td>26</td>
</tr>
<tr>
<td>References</td>
<td>27</td>
</tr>
</tbody>
</table>
The main objective of this paper is presentation of complete details on two procedures for solving transshipment plant location problems: procedures called "Simple Version of ORA(1, t)" and "Simple Version of ORA(2, t)" in Ladd and Lifferth [61.

The second of these two procedures is a "heuristic" procedure or a "heuristic program." A number of heuristic procedures have already appeared in the literature for solving plant location problems: in papers by King and Logan [5], Chern and Polopolus [3], Warrack and Fletcher [12] and Candler, Snyder and Faught [2].

"The aim of heuristic is to study the methods and rules of discovery and invention" [8, p. 112]. "Modern heuristic endeavors to understand the process of solving problems, especially the \textit{mental operations typically useful in the process}" [Italics in original; 8, pp. 129-130]. "Heuristic reasoning is reasoning not regarded as final and strict but as provisional and plausible only, whose purpose is to discover the solution to the present problem" [8, p. 113]. Tonge defines heuristics as "... principles or devices that ... reduce search in problem-solving" [11].

Heuristics comes into common use in the solving of combinatorial problems (the problems studied in this paper are combinatorial problems) to reduce the amount of searching. The method Stollsteimer presented in his original papers [9, 10] for solving combinatorial problems could be labelled a "blind search procedure" because it involved studying every possible combination of every possible number of plants. But his procedure could be proved to provide the optimum solution. Conceptually, a blind search procedure
could be used for any combinatorial problem. But many combinatorial problems involve so many combinations that it is either impractical or impossible to investigate them all. In the problem that Warrack and Fletcher [12] studied, it was not possible to use the blind search procedure. Iowa State University computer programmers estimated that application of the Stollsteimer blind search procedure to Warrack and Fletcher's problem would require 10,000 hours of computer time on an IBM 360/50. Tonge [4, 11] used heuristic procedures to solve an assembly line balancing problem. A relatively small assembly line balancing problem like Tonge's [4, 11] would have required evaluating 9.3 \times 10^{157} different possible combinations, which would have taken about 3 \times 10^{114} years to complete with computers then available. If current computers were 10,000 times faster than computers available at the time of his study, it would now take only 3 \times 10^{111} years to evaluate all combinations. We developed a heuristic procedure for solving the second problem in this paper for two reasons: (a) The extremely large number of possible combinations to be investigated made a blind search procedure impractical. (b) It is not possible to determine a priori all the combinations that would have to be investigated in a blind search procedure.

Heuristic procedures incorporate "principles or devices that reduce search in problem solving." They are procedures that are "not regarded as final and strict but as provisional and plausible only."

A weakness of heuristic procedures for solving optimization problems is that these procedures cannot be guaranteed to provide an optimum solution. When one is faced with a combinatorial optimizing problem that is so big it is not possible or not feasible to use a
blind search procedure and evaluate all possible alternatives, he has
two choices. (a) He can either use a heuristic approach, being quite
certain that his approach will not yield an optimum solution. (b) He
can ignore the problem or refuse to work on it. Use of heuristic
procedures -- choice (a) -- is based on the assumption that "Good
answers, even though known not to be the best answers, are better than
no answers" and the assumption or hope that "I will develop a plausible
procedure that provides good answers."

NOTATION

Define the symbols

\[ S_i, L_j, M_h = \text{origin or source } i, \text{ plant site } j, \text{ final market } h; \]
\[ i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J; \quad h = 1, 2, \ldots, H. \]

\[ X(i\cdot) = \text{quantity of raw material shipped from } S_i, \text{ a known constant} \]

\[ X(ijh) = \text{quantity shipped from } S_i \text{ through } L_j \text{ to } M_h, \text{ in raw material equivalents} \]

\[ X(\cdot j\cdot) = \sum_{i=1}^{I} \sum_{h=1}^{H} X(ijh) = \text{quantity of raw material received} \]
\[ \text{and processed at } L_j \text{ and shipped to a final market.} \]

\[ X(\cdot \cdot h) = \sum_{i=1}^{I} \sum_{j=1}^{J} X(ijh) = \text{total quantity of product, measured} \]
\[ \text{in raw material equivalents, shipped to } M_h. \]

\[ X = \sum_{i=1}^{I} X(i\cdot) = \sum_{j=1}^{J} X(\cdot j\cdot) = \sum_{h=1}^{H} X(\cdot \cdot h) \]

\[ P(\cdot \cdot h) = \text{price at } M_h, \text{ in raw material equivalents, a known constant.} \]

\[ C(ij\cdot) = \text{per unit transport cost for transporting raw material} \]
\[ \text{from } S_i \text{ to } L_j, \text{ a known constant.} \]
\( C(\cdot \text{jh}) \) = per unit transport cost, in raw material equivalents, for transporting final product from \( L_j \) to \( M_h \); a known constant.

\( C(ijh) = C(\cdot \text{j}) + C(\cdot \text{h}) \)

\( TPC(j) = \alpha(j) + \beta(j) X(\cdot \text{j}) \) = total processing cost for plant at \( L_j \); \( \alpha(j) \) and \( \beta(j) \) known constants; \( \beta(j) \) is marginal processing cost at \( L_j \).

\( \lambda(km) = k\)-th set of \( m \) plant sites, \( m \leq J \). \( k = 1, 2, ..., J!/m!(J-m)! \)

The values of \( X, X(\cdot \text{i}), P(\cdot \text{h}), C(\cdot \text{j}), C(\cdot \text{h}), \alpha(j) \) and \( \beta_j \) are known constants. \( X(ijh), X(\cdot \text{j}) \) and \( X(\cdot \text{h}) \) are variables.

The values of \( X, X(\cdot \text{i}), P(\cdot \text{h}), C(\cdot \text{j}), C(\cdot \text{h}), \alpha(j) \) and \( \beta_j \) are known constants. \( X(ijh), X(\cdot \text{j}) \) and \( X(\cdot \text{h}) \) are variables.

\( ORA(1, t) \)

(\text{It is assumed that readers of this paper are familiar with Stollsteimer's original articles [9, 10].})

The problem to be solved is: Determine \( m, \lambda(km), \) and \( X(ijh) \) to maximize

\[
(1) \sum_{i} \sum_{h} \sum_{\lambda(km)} P(\cdot \text{h}) X(ijh) - C(ijh) - \beta(j) X(ijh) - \sum_{j} \alpha(j) j \in \lambda(km) h \in \lambda(km)
\]

\( \sum_{i} \sum_{h} P(\cdot \text{h}) X(ijh) \) is total value of product at final markets.

\( \sum_{i} \sum_{h} C(ijh) X(ijh) \) is total transport cost. \( \sum_{i} \sum_{h} \beta(j) X(ijh) \) is total variable processing cost (TVPC) in the \( m \) plants in \( \lambda(km) \); \( \sum_{j} \alpha(j) \) is total fixed processing cost (TFPC).

This transshipment plant location problem can be converted into a problem that is essentially Case II in Stollsteimer's original paper [9, p. 638] and can be solved by a slight variation of the procedure that he suggested. \( P(\cdot \text{h}) - C(\cdot \text{h}) - C(\cdot \text{j}) - \beta(j) \) is
average revenue net of transport and variable processing cost (ARN) for product shipped from $S_i$, received and processed at $L_j$, and shipped to $M_h$. Denote the maximum ARN attainable at $S_i$ for product shipped through $L_j$ as

$$\pi(ij_{h_j}) = \max_h [P(h) - C(h_{ij}) - C(ij') - \beta(j)] = P(h_{i_j}) - C(ij') - \beta(j)$$

This maximum ARN is attained by shipping from $L_j$ to $M_{h_j}$. If some other source, say $S_u$, also ships to $L_j$,

$$\pi(uj_{u_j}) = \max_h [P(h) - C(h_{ij}) - C(uj') - \beta(j)] = P(h_{u_j}) - C(uj') - \beta(j)$$

Maximum ARN at $S_u$ for product shipped to $L_j$ is attained by shipping the final product to $M_{h_{u_j}}$. But $h_{u_j} = h_{i_j}$ for all $i$ and $u$. This is so because

$$P(h_{i_j}) - C(h_{i_j}) - C(ij') - \beta(j) = P(h_{u_j}) - C(uj') - C(ij') - \beta(j)$$

implies

$$P(h_{i_j}) - C(h_{i_j}) - C(uj') - \beta(j) = P(h_{u_j}) - C(uj') - C(ij') - \beta(j)$$

Thus, if maximum ARN attainable at $S_1$ for product shipped to $L_j$ is obtained by shipping the product made from $S_1$'s raw material to $M_{h_{i_j}}$, then maximum ARN attainable at any other source for product shipped to $L_j$ is obtained by transshipping the source's final product to $M_{h_{i_j}}$. We can, therefore, define $h_j = h_{i_j} = h_{u_j}$ for all $S_i$ and $S_u$, and write (2) as (3)

$$(3) \pi(ij_{h_j}) = \pi(ij_{h_{i_j}}) \text{ for all } S_i$$

Once $j$ is specified and (2) has been computed for $S_i = S_1$, $h_j$ is uniquely determined. Now specifying $L_j$ provides two bits of information:
(a) It identifies a plant site, as before. (b) It identifies the final market \( M_j \) to which all product received and processed at \( L_j \) will be shipped for maximum ARN.

To solve the problem, compute (2) for \( i = 1 \) and for all combinations of values of \( j \) and \( h \). These computations provide the values of ARN at \( S_i \) for raw material shipped through each \( L_j \) to each final market. These computations also determine the final market to be supplied by each \( L_j \), because \( h_{ij} = h_{uj} \) for all \( u \). For \( u > 1 \), compute

\[
(4) \quad \pi(u_{ij} h_j) = P(\cdot h_{ij}) - C(\cdot j h_{ij}) - C(\cdot u_{ij}) - \beta(j)
\]

for all \( j \). These computations of (2) and (4) provide an \( I \times J \) matrix \( [\pi(i_{ij} h_j)] \) of ARN. The problem now is Stollsteimer's original Case II, except that: (a) the objective is to maximize net revenue rather than to minimize cost and (b) \( \alpha(j) \) is a function of \( L_j \).

For a given \( \lambda(km) \), the maximum ARN attainable at \( S_i \) for shipping through some plant in \( \lambda(km) \) is given by

\[
(5) \quad \max_{j \in \lambda(km)} \pi(i_{ij} h_j)
\]

Obtaining (5) for each \( i \) is equivalent to defining an \( I \times m \) submatrix \( [\pi(i_{ij} h_j) | \lambda(km)] \) from \( [\pi(i_{ij} h_j)] \) and selecting the largest element in each row. Each row of \( [\pi(i_{ij} h_j) | \lambda(km)] \) contains values of ARN attained at one origin for shipping through each \( L_j \epsilon \lambda(km) \) to a final market.

For given \( \lambda(km) \), the maximum attainable net revenue ( = total revenue minus transport and variable and fixed processing costs = NR) is

\[
(6) \quad \overline{NR} | \lambda(km) = \sum_i \sum_j X(i; \cdot) \left[ \max_{h \epsilon \lambda(km)} \pi(i_{ij} h_j) \right] - \sum_{h \epsilon \lambda(km)} \alpha(j)
\]

The maximum attainable NR from \( m \) plants is

\[
(7) \quad \overline{NR} | m = \max_k \overline{NR} | \lambda(km)
\]

The set of plant sites and routings that maximizes (1) is the set that
provides

\[(8) \overline{NR} = \max_{m} \overline{NR} \mid m \]

ORA(2, t)

The preceding problem in effect ignores existing plants. It deals only with number, size and location of new plants. In ORA(2, t) some plants are in existence initially, but their total processing capacity is insufficient to handle all the available raw material. Additional processing capacity is needed. It can be obtained by increasing the size of (some or all) existing plants, or by building new plants at sites where no plants now exist, or by doing some of each of these.

Divide \( \lambda (\text{km}) \) into two subsets of plant sites:

\[ \lambda (\text{km}) = \{ \lambda (\text{km}_1), \lambda (\text{km}_2) \} \]

where

\( \lambda (\text{km}_1) \) = set of \( m_1 \) sites in \( \lambda (\text{km}) \) where plants now exist.

\( \lambda (\text{km}_2) \) = set of \( m_2 \) sites in \( \lambda (\text{km}) \) where plants do not now exist, but where new plants may be built.

\( m = m_1 + m_2 \) for all \( m \).

Also define

\( Z(\cdot j\cdot) \) = initial capacity of plant now located at \( L_j \in \lambda (\text{km}) \).

The TPC function for a plant located at \( L_j \in \lambda (\text{km}_1) \) is

\[(9) \ \text{TPC}_{1j} = \alpha_1 D(j) + \beta_1 X(\cdot j\cdot) - \beta_2 D(j)[X(\cdot j\cdot) - Z(\cdot j\cdot)] + \beta_2 D(j) [X(\cdot j\cdot) - Z(\cdot j\cdot)] \]

where

\( D(j) = 1 \) if \( X(\cdot j\cdot) > Z(\cdot j\cdot) \)

\( D(j) = 0 \) if \( X(\cdot j\cdot) \leq Z(\cdot j\cdot) \)

---

1 Dennis Lifferth and I collaborated in developing this procedure.
If this plant operates at or below its initial capacity, \( D(j) = 0 \) and \( \text{TPC}_j = \beta_1 \cdot j \cdot j' \cdot j' \). If the plant operates at a higher level, \( D(j) = 1 \) and \( \text{TPC}_j = \alpha_1 + \beta_1 \cdot j \cdot j' + \beta_2 \cdot \left[ (X \cdot j') - Z(j') \right] \). The marginal cost of using existing capacity (\( \beta_1 \)) is less than the marginal cost of using new capacity (\( \beta_2 \)). The TPC for a new plant located at \( L_t \) is

\[
\text{TPC}_{t} = \alpha_2 + \beta_3 \cdot X(t')
\]

And \( \beta_3 > \beta_1, \alpha_2 > \alpha_1, \beta_3 < \beta_2 \)

Substituting \( X(t') = \sum \sum X(ijh) \) into (9) and (10), the problem to be solved can be written: Determine \( m_1, m_2, \lambda(km), X(ijh) \) and \( D(j) \) to maximize

\[
\sum \sum \sum \left\{ P(ijh) - C(ijh) - \left[ \beta_1 - \beta_1 \cdot D(j) + \beta_2 \cdot D(j) \right] \right\} X(ijh) - \left( \beta_1 - \beta_2 \right) \sum \sum \sum D(j)Z(j') - \alpha_1 \sum \sum D(j)
\]

This objective function is nonlinear because it contains the product \( D(j)X(ijh) \); but \( D(j) \) is a function of \( X(ijh) \), and vice versa. The value of \( D(j) \) cannot be specified until the \( X(ijh) \) are known but the solution values of \( X(ijh) \) are not determined until \( D(j) \) has been specified. Therefore the procedure of ORA(1, t) cannot be used to solve this problem. This nonlinear combinatorial problem can be solved by the following five-stage heuristic procedure.

Stage 1

In this stage, set \( D(j) = 1 \) for all existing plants. For these plants, \( \pi(ijh_{ij}) \) in (2) is determined from

\[
\pi(ijh_{ij}) = P(ijh_{ij}) - C(ijh_{ij}) - C(ij') - \beta_2
\]
and \( \alpha(j) \) is determined from

\[
\alpha(j) = \alpha_1 + (\beta_1 - \beta_2) Z(j) > 0
\]

For new plants, \( \alpha(j) = \alpha_2 \) and \( \beta(j) = \beta_3 \). Now (11) is the same as (1).

Use \( \text{ORA}(1, t) \) to solve this problem. After applying \( \text{ORA}(1, t) \), divide the various \( \lambda(km) \) into three sets.

- \( L(N) \) = sets of \( \lambda(km) \) containing only sites where no plants now exist.
- \( L(E1) \) and \( L(E2) \) contain sites where plants now exist.
  - \( L(E1) \) = sets of \( \lambda(km) \) containing sites where plants now exist; for every \( \lambda(km) \) in this set, \( X(j) > Z(j) \) for every existing plant.
  - \( L(E2) \) = sets of \( \lambda(km) \) containing sites where plants now exist; for every \( \lambda(km) \) in this set, \( X(j) \leq Z(j) \) for some existing plants.

(Each \( \lambda(km) \) is a set of plant sites. \( L(N) \), \( L(E1) \) and \( L(E2) \) are sets of sets of plant sites.) For those \( \lambda(km) \) that contain only sites where no plants now exist, i.e., for \( \lambda(km) \in L(N) \), go to Stage 5. \(^1\) For those \( \lambda(km) \) that are in \( L(E1) \), go to Stage 4 and then to Stage 5. For \( \lambda(km) \in L(E2) \), go to Stage 2, then Stage 3, then Stage 4, then Stage 5.

Stage 2

If \( X(j) \leq Z(j) \) for some existing plant, the solution to \( \text{ORA}(1, t) \) in Stage 1 is not consistent with the original specification of the problem because all \( D(j) = 1 \) in the original specification and

\(^1\)To anticipate a bit: It will be proved later that (11) will be maximized by a set of plant sites that contains every site where a plant now exists. Therefore, one does not need to do any computations with \( \lambda(km) \in L(N) \).
D(j) = 1 implies \( X(\cdot j\cdot) > Z(\cdot j\cdot) \). TPC\(_j\) and \( \pi(\ijh) \) need to be recomputed for each existing plant operating at or below initial capacity, and NR needs to be recomputed. For each \( \lambda (\km) \in \mathbb{L}(E2) \), define

\[
\lambda (\km_{-}) = \text{set of } L_j \in \lambda (\km_{-}) \text{ whose receipts in the ORA(1, t) solution equal or fall short of their initial capacities.}
\]

\[
\lambda (\km_{+}) = \text{set of } L_j \in \lambda (\km_{+}) \text{ whose receipts in the ORA(1, t) solution exceed their initial capacities.}
\]

\[
\overline{NR} | \lambda (\km)(1, t) = \text{maximum value of NR yielded by the solution to ORA(1, t) for } \lambda (\km)
\]

The corrected values of TPC\(_j\), \( \pi(\ijh) \), and NR are obtained by using (9) with \( D(j) = 0 \) for every \( L_j \in \lambda (\km_{-}) \). They are

\[
\text{TPC}_j = \beta_1 Z (\cdot j\cdot)
\]

\[
\pi'(\ijh) = P (\cdot j\cdot) - C (\cdot j\cdot) - C (\cdot i\cdot) - \beta_1 - \pi(\ijh)
\]

\[
\beta_2 - \beta_1 > \pi(\ijh)
\]

\[
\overline{NR} (2) | \lambda (\km) = \overline{NR} | \lambda (\km)(1, t) + \sum_{j \in \lambda (\km_{-})} \beta_1 Z (\cdot j\cdot)
\]

\[
+ \beta_2 [X (\cdot j\cdot) - Z (\cdot j\cdot)] - \beta_1 X (\cdot j\cdot)] + \alpha_1 n_1
\]

where \( n_1 \) is the number of plants in \( \lambda (\km_{-}) \). Letting \( X(\ijh;lt) \) be the quantity shipped from \( S_j \) through \( L_j \) to \( M_h \) in the ORA(1, t) solution in Stage 1, the new value of NR can also be expressed as

\[
\overline{NR} (2) | \lambda (\km) = \sum_i \sum_{j \in \lambda (\km_{-})} \pi'(\ijh) X(\ijh;lt)
\]

\[
+ \sum_i \sum_{j \in \lambda (\km_{+})} \pi(\ijh) X(\ijh;lt)
\]

\[
+ \sum_i \sum_{j \in \lambda (\km_{-})} \pi(\ijh) X(\ijh;lt)
\]

\[
+ \sum_{j \in \lambda (\km_{+})} \alpha(j) + \alpha_2 m_2
\]

where \( \alpha(j) \) comes from (13).
Stage 3

Because $\beta_3 > \beta_1$ and $\beta_2 > \beta_1$, for a given $\lambda (\text{km})$ it is always possible to reduce TPC by reducing shipments to $L_{j}\lambda (\text{km}_2)$ and $L_{j}\lambda (\text{km}_1^+)$, and increasing shipments to $L_{j}\lambda (\text{km}_1^-)$, provided receipts at $L_{j}\lambda (\text{km}_1^-)$ do not rise above $Z(\cdot j\cdot)$. This rerouting will affect total transport cost, and may affect total gross revenue received at final markets.

Stage three determines reroutings that will increase NR. To be certain of obtaining the pattern of shipments that maximizes TRN and satisfies the constraints, we must investigate all possible combinations of rerouting of shipments. In many problems this is not practical because of the great number of possible combinations of reroutings. The best that can be done in these situations is to use some heuristic rule for selecting a few sets of reroutings for investigation. The rule used here is to investigate rerouting the material from one source at a time while imposing the restrictions that receipts at any $L_{j}\lambda (\text{km}_1^-)$ cannot rise above capacity of that plant and receipts at $L_{j}\lambda (\text{km}_1^+)$ cannot fall below capacity at that plant. Stage 4 will remove these restrictions.

Consider rerouting shipments to reduce receipts at $L_{g}$, $(L_{g}\lambda (\text{km}_1^+))$ or $L_{g}\lambda (\text{km}_2)$, and to increase receipts at $L_{v}\lambda (\text{km}_1^-)$, and suppose $S_d$ supplies $L_g$. Let $R(dgv)$ be the amount of $S_d$'s raw product that is to be rerouted from $L_g$ to $L_v$. The procedure for determining $R(dgv)$ depends upon whether $L_g$ is in $\lambda (\text{km}_1^+)$ or in $\lambda (\text{km}_2)$. If $L_{g}\lambda (\text{km}_2)$, (16) and (17) are used.

(16) If $X(g\cdot) - X(d\cdot\cdot) \geq 0$, then $R(dgv) = X(d\cdot\cdot)$

(17) If $X(g\cdot) - X(d\cdot\cdot) < 0$, then $R(dgv) = X(g\cdot\cdot) < X(d\cdot\cdot)$

If the inequality in (16) is satisfied, all of $S_d$'s raw product can be
rerouted to $L_v$ without reducing receipts at $L_g$ below zero. If the first inequality in (17) is satisfied, rerouting all of $S_d$'s raw material will reduce receipts at $L_g$ below zero.

If $L_g \in (km_1^+)$, the value of $R(dgv)$ is determined from (18) and (19).

(18) If $X(\cdot g^*) - X(d^*) \geq Z(\cdot g^*)$ and $X(\cdot v^*) + X(d^*) \leq Z(\cdot v^*)$,
then $R(dgv) = X(d^*)$

(19) If $X(\cdot g^*) - X(d^*) < Z(\cdot g^*)$ or $X(\cdot v^*) + X(d^*) > Z(\cdot v^*)$, then
$R(dgv) = \min [X(\cdot g^*) - Z(\cdot g^*), Z(\cdot v^*) - X(\cdot v^*)]$

If $R(dgv)$ is rerouted away from $L_g$ to $L_v$, the resulting change in NR is $\Delta NR(dgv) | \lambda (km)$. The method of determining this change depends upon $L_g$ and $X(\cdot g^*) - Z(\cdot g^*)$. If $L_g \in (km_2^+)$, then

(20) $\Delta NR(dgv) | \lambda (km) = [\pi(dvh_v) - \pi(dgh_g)] R(dgv) + (\beta_3 - \beta_1) R(dgv) \leq 0$

If $L_g \in (km_1^+)$ and $X(\cdot g^*) - R(dgv) = Z(\cdot g^*)$, then

(21) $\Delta NR(dgv) | \lambda (km) = [\pi(dvh_v) - \pi(dgh_g)] R(dgv) + (\beta_2 - \beta_1) R(dgv) + \alpha_1 > 0$

If $L_g \in (km_1^+)$ and $X(\cdot g^*) - R(dgv) > Z(\cdot g^*)$, then

(22) $\Delta NR(dgv) | \lambda (km) = [\pi(dvh_v) - \pi(dgh_g)] R(dgv) \leq 0$

Identify each $S_d$ that supplies a plant in $\lambda (km_1^+)$ or $\lambda (km_2^+)$. For each such $S_d$, determine (20), (21) or (22), whichever is appropriate, for all $L_v \in \lambda (km_1^-)$. For each such $S_d$, this provides $N(l^-)$ values of (20), (21) or (22) where $N(l^-)$ is the number of plants in $\lambda (km_1^-)$. Arrange these values as an $N(l^-)$ element row vector, $[\Delta NR(dg) | \lambda (km)]$. After computing (20), (21) or (22) for every $S_d$ supplying a plant in $\lambda (km_1^+)$ or $\lambda (km_2^+)$, we have a matrix $[\Delta NR | \lambda (km)]$ in which the number of rows equals the number of sources that ship their raw product to a
plant in $\lambda (km_1)$ or a plant in $\lambda (km_2)$.

Find the largest element of this matrix. If the largest element is negative, the processing costs saved by sending more raw material through plants in $\lambda (km_1)$ and less through plants in $\lambda (km_2)$ are more than offset by increases in transportation cost, and no product should be rerouted. If the largest element is positive, suppose it occurs for $d = d'$, $g = g'$ and $v = v'$. Then reroute $R(d'g'v')$ accordingly, i.e., reroute this amount of $S_{d'}$'s raw material away from the plant at $L_{g'}$ to the plant at $L_{v'}$.

If $R(d'g'v') = X(d'\cdots)$, $S_{d'}$ is now dropped from further stage three computations and remaining elements of $[\Delta NR | \lambda (km)]$ for $L_{g'}$ and $L_{v'}$ must be adjusted to reflect the reduced receipts at $L_{g'}$ and the increased receipts at $L_{v'}$. This requires recomputing (16) through (22) for $g = g'$ and $v = v'$.

If $L_{g'} \in \lambda (km_1)$ and $R(d'g'v') = [X(g'\cdots) - Z(g')] < X(d'\cdots)$, the volume handled at $L_{g'}$ after rerouting equals $Z(g')$, and plant site $L_{g'}$ is dropped from further stage-three computations. If $L_{g'} \in \lambda (km_2)$ and $X(g') = R(d'g'v') < X(d'\cdots)$, the volume handled at $L_{g'}$ after rerouting equals zero, and plant site $L_{g'}$ is dropped from further stage-three computations. If $R(d'g'v') = [Z(v'\cdots) - X(v'\cdots)] < X(d'\cdots)$, the volume at $L_{v'}$ after rerouting equals the initial capacity at $L_{v'}$, and site $L_{v'}$ is dropped from further stage-three computations. In each of these three situations, the remaining elements in row $d'$ of the matrix $[\Delta NR | \lambda (km)]$ must be recomputed. To recompute these elements, obtain the adjusted values of $R(d'gv)$ from (16) through (19) by setting $d = d'$ and replacing $X(d'\cdots)$ by $X'(d'\cdots) = X(d'\cdots) - R(d'g'v')$, and then use $X'(d'\cdots)$ in place of $X(d'\cdots)$ in (20) through (22).
Now find the largest element in the reduced, adjusted matrix
\[ \Delta NR | \lambda (km) \]. Relabel so that this element occurs for \( d = d', \ g = g', \ v = v' \) and reroute \( S_d \) appropriately. Then revise the matrix again.
Proceed in this way until all plants in \( \{ g(\lambda (km - \gamma) \) \) have been eliminated
from the matrix (i.e., until no existing plant is operating below its
initial capacity) or until the revised reduced matrix of \[ \Delta NR | \lambda (km) \]
has no positive elements.

Obtaining the new value of NR involves the following four steps,
(a) Each origin whose raw material is sent to two or more plants is
now renumbered. If \( S_5 \)'s raw material is divided among three plants,
for example, \( S_5 \) becomes three "origins," all raw material from each
"origin" goes to one plant. (b) The values of \( X(ijh_j) \) obtained in
Stage 1 must be adjusted to reflect the reroutings of Stage 3; call
these adjusted values \( X_3(ijh_j) \). (c) Values of \( TPCl_j \) must be recomputed
for those plants in \( \lambda (km_1^+) \) whose volumes have been reduced to their
initial capacities. (d) The values of \( \pi(ijh_j) \) must be adjusted to
reflect the changes made in step (c). Call these new values \( \pi_3(ijh_j) \).
Clearly \( \pi_3(ijh_j) = \pi(ijh_j) \) or \( \pi'(ijh_j) \) for all other plants. The new
value of NR is
\[
(23) \quad \bar{NR}(3) | \lambda (km) = \sum_{i} \sum_{j \in \lambda (km)} \sum_{h \in \lambda (km_1^+)} \pi_3(ijh_j) X_3(ijh_j) - \sum_{j \in \lambda (km_1^+)} \alpha(j) - \alpha_2 m_2
\]
where \( \lambda (k'm_1^+) \) is the set of existing plants whose receipts still
exceed their initial capacities and \( m_2 \) is the number of plants in
\( \lambda (km_2) \in \lambda (km) \).

This series of operations completes the stage-three computations
for one set of plant locations: \( \lambda (km) \). It must now be repeated for
each of the other sets of \( m \) plant locations, then be carried out for
all sets of $m+1$ sites, and so on. This series may not provide the revenue-maximizing routing for any $\lambda(km)$. Further increases in revenue might be possible by making simultaneous reroutings; e.g., rerouting the quantity $q_i$ supplied by $S_d$ away from $M_h$ to $M_v$ and rerouting an equal quantity supplied by $S_e$ away from $M_v$ to $M_u$. The number of such possible reroutings for each $\lambda(km)$ is $I(I-1)m(m-1)/4$. Suppose $I = 40$ and $m = 20$. The number of combinations to be evaluated is then nearly 160,000. To evaluate 160,000 possible combinations a large number of times -- once for each $\lambda(km)$ -- would be prohibitively expensive.

Stage 4

Suppose two existing plants are to expand their capacities, according to the routings used to derive (23). If some raw product shipped to these plants is rerouted so that their total volume remains the same but only one plant needs to expand its capacity, TPC is reduced. This rerouting may or may not increase NR, depending upon its effect on gross revenue and transport costs.

Stage-four computations answer the question: Can NR be increased above the value in (23) by rerouting shipments so that fewer existing plants must expand their capacities? The procedure for answering this question depends upon the relation between $\beta_2$ and $\beta_3$. If $\beta_2 < \beta_3$, use Stage 4A. Stage 4A does not allow reroutings away from existing plants to new plants; it only considers routings away from one existing plant to other existing plants. If $\beta_2 > \beta_3$, use Stage 4B, which allows rerouting from existing plants to new plants.

Stage 4A

Assume the plant sites are numbered so that, according to (23),
plants at \( L_1, L_2, \ldots, L_Y \) are the existing plants that are required to expand their capacities. These are the only plants to be considered. And the only sources to be considered are those that supply plants at \( L_1, L_2, \ldots, L_Y \). For each of the \( L_y \) \((y = 1, 2, \ldots, Y)\) it is necessary to determine the way of reducing its volume down to its initial capacity that reduces TRN the least. For each source supplying an \( L_y \) it is necessary to determine which one of the other plants receives the source's raw material if the plant at \( L_y \) reduces its volume of operation. Define

\[ S_i = \text{i-th source that ships raw material to plant at } L_y, \quad y_i = y_1, y_2, \ldots, y_{ly} \]

\[ X(y_i) = \text{amount shipped from } S_i \text{ to } L_y \text{ in (23).} \]

Consider rerouting \( S_i \)'s raw material away from \( L_y \) to \( L_w \). Assume all of \( S_i \)'s raw material can be so rerouted without reducing volume at \( L_y \) down to \( Z(\cdot y^*) \). The resulting change in TRN is obtained from (24), which is similar to (22).

\[ (24) \Delta \text{TRN}(y_iyw) = [\pi(y_iwh^w) - \pi(y_iyh^y)]X(y_i) \leq 0 \]

For each \( S_i \), (24) is computed \( Y - 1 \) times, once for each \( L_w \). If \( X(y_i) \) is rerouted away from \( L_y \), (25) determines the plant site to which \( X(y_i) \) should be sent to minimize the loss in TRN.

\[ (25) \Delta \text{TRN}(y_iyw_1) = \max_w \Delta \text{TRN}(y_iyw) \]

Plant site \( w_1 \) is the one to which \( X(y_i) \) should be rerouted. For a given \( L_y \), (25) must be computed for every source sending raw material to \( L_y \). Assume values obtained from (25) for the various \( S_i \) can be ordered as

\[ (26) \Delta \text{TRN}(y_{i_1}yw_1) \geq \Delta \text{TRN}(y_{i_2}yw_2) \geq \Delta \text{TRN}(y_{i_3}yw_3) \geq \ldots \geq \Delta \text{TRN}(y_{i_y}yw_y) \]

where it may happen that \( w_i = w_j \) for \( i \neq j \). The quantity \( X(\cdot y^*) \)
- \( Z(y) \) is to be rerouted away from \( L_y \). To accomplish this with minimum loss in \( \text{TRN} \), the reroutings are determined from the first (largest) terms in (26).

\[
\sum_{i=1}^{T} X(y_i) > X(y') - Z(y') \quad \text{and} \quad \sum_{i=1}^{T-1} X(y_i) < X(y') - Z(y').
\]

Then rerouting raw material of \( S_{y_1}, S_{y_2}, \ldots, S_{y_T} \) away from \( L_y \) will reduce volume at \( L_y \) below initial capacity, but rerouting material of \( S_{y_1}, S_{y_2}, \ldots \) and \( S_{y_T} \) away from \( L_y \) will leave volume at \( L_y \) above initial capacity. The amount to be rerouted away from \( L_y \) is somewhere between these two values. Determining the one(s) of the sources \( S_{y_T}, S_{y_T+1}, \ldots \) whose raw material is to be rerouted away from \( L_y \) and the quantity to be rerouted is accomplished by (27) through (35).

For fixed \( y \), compute (27) and (28) for \( s = T, T+1, \ldots, I_y \)

\[
(27) \rho_{1y_s} = \min \left\{ \frac{X(y') - Z(y') - \sum_{i=1}^{T-1} X(y_i) - \rho_{1y_s} X(y_s)}{X(y_s)} \right\}
\]

\[
(28) \Delta \text{TRN}(y_s yw_s)
\]

Then determine

\[
(29) \rho_{1y_s} \Delta \text{TRN}(y_{M1} yw_{M1}) = \max_{s \geq T} \left\{ \rho_{1y_s} \Delta \text{TRN}(y_s yw_s) \right\}
\]

If

\[
(30) X(y') - Z(y') = \sum_{i=1}^{T-1} X(y_i) + \rho_{1y_M1} X(y_{M1})
\]

then go to (36) to determine change in net revenue. If (30) is not satisfied, determine \( \rho_{2y_s} \Delta \text{TRN}(y_{M2} yw_{M2}) \) from (31) through (33)

\[
(31) \rho_{2y_s} = \min \left\{ \frac{[X(y') - Z(y') - \sum_{i=1}^{T-1} X(y_i) - \rho_{1y_M1} X(y_{M1})]}{X(y_s)} \right\}
\]

\[
(32) \rho_{2y_s} \Delta \text{TRN}(y_s yw_s)
\]

\[
(33) \rho_{2y_s} \Delta \text{TRN}(y_{M2} yw_{M2}) = \max_{s \geq T} \left\{ \rho_{2y_s} \Delta \text{TRN}(y_s yw_s) \right\}
\]
If
\[ (34) \ X(\cdot y') - Z(\cdot y') = \sum_{i=1}^{T-1} X(y_i) + \sum_{i=1}^{T-1} \rho_{1y_M1}^i X(y_{M1}) + \sum_{i=1}^{T-1} \rho_{2y_M2}^i X(y_{M2}) \]
go to (36) to determine change in net revenue. If (34) is not satisfied, determine
\[ \rho_{3y_M3} \Delta TRN(y_{M3},y_{M3}). \] Continue until
\[ (35) \ X(\cdot y') - Z(\cdot y') = \sum_{i=1}^{T-1} X(y_i) + \sum_{g=1}^{G} \rho_{g\gamma_{Mg}}^i X(y_{Mg}) \]
(35) shows the amounts to be rerouted away from \( L_1 \): \( X(y_1) \) to \( L_{w_1} \), \( X(y_2) \) to \( L_{w_2} \), ..., \( X(y_{T-1}) \) to \( L_{w_{T-1}} \), \( \rho_{1y_M1}^i X(y_{M1}) \) to \( L_{w_M1} \), ..., \( \rho_{g\gamma_{Mg}}^i X(y_{Mg}) \) to \( L_{w_Mg} \). Carrying out these reroutings reduces TRN by
\[ (36) \ \Delta TRN(\cdot y') = \sum_{i=1}^{T-1} \Delta TRN(y_i, y_{i+1}) + \sum_{g=1}^{G} \rho_{g\gamma_{Mg}}^i \Delta TRN(y_{Mg}, y_{Mg+1}) \]
To determine which (if any) existing plant should reduce its volume of operations down to the level of its initial capacity compute (36) for \( y = 1, 2, ..., Y \) and determine
\[ (37) \ \Delta NR(p) \mid \lambda(km) = \max_y [\Delta TRN(\cdot y') + \alpha(y)] \]
where \( \alpha(y) \) comes from (13). If \( \Delta NR(p) \mid \lambda(km) < 0 \), it is not possible to increase raw material producers' income above the value provided by (23). If \( \Delta NR(p) \mid \lambda(km) > 0 \), producers' net revenue can be increased by reducing shipments to \( L_p \) down to \( Z(\cdot p') \). The amounts from the \( S_{p_1} \) that should be rerouted and the plant sites to which they should be sent are given in (26) and (35) with \( y = p \) and \( y_1 = p_1 \). After these reroutings are performed, the plant at \( L_p \) and the sources still supplying that plant are deleted from further consideration.

Values of \( X(\cdot y') \) must be increased for the remaining plants whose volumes have been increased by rerouting material. Then (24) through (37) are applied to the reduced set of plants. The process continues until (37) is negative. The new values of \( X(ijh) \), call them \( X_4(ijh) \), must be determined. TPC functions must be adjusted for those existing plants formerly in \( \lambda(k'm_1) \) whose receipts have been reduced to their
Initial capacities, and values of $\pi(i_{jh})$ must be adjusted accordingly to obtain $\pi_i(i_{jh})$. The value of $NR$ is

$$NR(4) | \lambda(km) = \sum_i \sum_{j \in \lambda(km)} \pi_i(i_{jh})X_j(i_{jh}) - \sum_{j \in \lambda(k''m_1+)} \alpha(j)$$

where $\lambda(k''m_1+)$ is the number of existing plants whose volumes exceed their initial capacities. (38) represents the heuristic maximum $NR$ attainable at the origins if plants at the $m$ sites in $\lambda(km)$ are used.

Operations (24) through (38) must next be carried out with all other sets of $m$ plant sites, $m+1$ sites, $m+2$ sites, and so on.

**Stage 4B**

Stage 4B is the same as Stage 4A, with one exception: In 4A, the only plants considered as possible $L_w$ were existing plants operating above their initial capacities. In 4B, new plants are also treated as possible $L_w$.

**Stage 5**

This stage determines the heuristic optimum pattern of shipments and number, size and location of plants. It is necessary to determine, for each $\lambda(km)$, the computational stage that provided the heuristic maximum $NR$. Define for each $\lambda(km)$

$$NR(i) | \lambda(km) = \text{heuristic maximum } NR \text{ obtained from stage } i$$

for fixed $\lambda(km)$; $i = 1, 2, 3, 4$

To find which stage provided the heuristic maximum $NR$ for fixed $\lambda(km)$, determine

$$NR | \lambda(km) = \max_i NR(i) | \lambda(km)$$

$NR(1) | \lambda(km)$ is obtained from (6); $NR(2) | \lambda(km)$, from (15) or (15a); $NR(3) | \lambda(km)$ from (23); and $NR(4) | \lambda(km)$ from (38). $NR(i) | \lambda(km)$ will not have been computed for every $i$, and it is not proper to include
both (6) and (15) in determining (40). For example, if \( \lambda(km) \in L(E2) \) -- see discussion of stage 1 for definition of \( L(E2) \) -- \( NR(1) | \lambda(km) \) should not be considered, but \( NR(2) | \lambda(km) \) should be; \( NR(3) | \lambda(km) \) and \( NR(4) | \lambda(km) \) should be considered in determining (40) only if stages three and four lead to rerouting some shipments. The one set of \( m \) plant sites that provides the heuristic maximum \( NR \) is found from

\[
NR(m) = \max_k NR(km)
\]

Finally, the heuristic optimum number, size and location of plants and pattern of shipments is determined from

\[
NR = \max_m NR(m).
\]

Use of Existing Facilities

It will now be shown that (42) will be provided by a set of plant sites that contains every existing plant. This follows from the fact that adding one more existing plant to a set of plant sites, without changing the number of sites where no plants now exist in the set, cannot reduce net revenue but may increase net revenue.

Suppose that \( \lambda(km') \) contains one more existing plant than does \( \lambda(km'') \), and both contain the same sites where no plants now exist. Then net revenue cannot be smaller for \( \lambda(km') \) than for \( \lambda(km'') \), and may be larger. Every shipping pattern available in \( \lambda(km'') \) is available in \( \lambda(km') \). Therefore TRN cannot be less for \( \lambda(km') \) than for \( \lambda(km'') \), and may be greater. Total fixed processing cost need not be increased by adding an existing plant to the analysis because the added plant can be operated at or below its initial capacity. This option is available and considered in stages 3 and 4. If operating the added plant at a level above its initial capacity yields a smaller net revenue than operating at or below its initial capacity, the stages 3 and 4 solutions
will operate the plant at or below its initial capacity. The results of stages 3 and 4 will indicate that the added plant should be operated at a level above its initial capacity only if doing so yields a larger net revenue than operating at a lower volume.

This result has an important computational implication, namely that in order to find the optimum number, size and location of plants, and optimum routings, one needs to consider only those sets of plant sites that contain every site where a plant now exists. That is, no computations need to be performed for sets of $\lambda(km)\epsilon L(N)$.

In the studies reported in Lifferth [7], Ladd and Lifferth [6] and Baumel et al. [1] the restriction that every existing plant operate at or above its initial capacity was imposed. In that study, the values of $\beta_3$ and $\alpha_2$ relative to $\beta_2$ and $\beta_1$ and to $\alpha_1$ were such that this restriction turned out to be redundant. Application of the procedure presented here automatically resulted in solutions that satisfied the restriction. If desired, this restriction can be easily added to the procedure presented here. Simply change the rule for ending stage 3 from: Quit when either: (a) the matrix $[\Delta NR | \lambda (km)]$ has no more positive elements or (b) no existing plant operates below its initial capacity, whichever occurs first, to: Stop when no existing plant operates below capacity. Applying this rule may result in doing some reroutings that reduce NR. The reroutings to make are still determined by selecting the largest elements of $[\Delta NR | \lambda (km)]$. If all elements are negative, the largest is the one closest to zero and is the one that reduces NR the least.

Summary

In summary, the five stages of the ORA(2, t) procedure are:
(1) Assume all cost functions are known and apply ORA(1, t).

(2) Adjust values of TPC.j, π(ijh) and NR to be consistent with the pattern of shipments from Stage 1.

(3) If profitable, reroute raw material away from new plants and away from existing plants that operate above capacity, according to Stage 1 results, toward existing plants that operate below their initial capacities. In performing the reroutings, consider only one raw material source at a time.

(4) If profitable, reroute raw material among existing plants to reduce the number of existing plants that operate above their initial capacities. In performing the reroutings, consider only reducing the volume at one plant at a time.

The essential features of this heuristic procedure are first to simplify the problem and solve, and then remove the simplifying assumptions and determine the effects of a limited set of reroutings and make the changes that improve the value of the objective function. These basic ideas have a number of other applications, some of which will be mentioned briefly in the next section.

OTHER HEURISTIC PROCEDURES

The first procedure to be discussed in this section is not a heuristic procedure. It is possible to prove that the procedure provides an optimum solution. The procedure is presented simply as an introduction to later problems. In this problem, the TPC for a plant at L_j is

\[ TPC_j = α + βX(ij) \]

and the problem is to: Determine m, λ(km) and X(ijh) to minimize
\[(44) \sum_{i \in \lambda} \sum_{j \in \lambda} C(ijh)X(ijh) + \alpha m + \beta X\]

This is almost the same as the problem in ORA(1, t). The first step is to compute for all \(S_i\), \(L_j\) and \(M_h\) the minimum transport cost for serving \(S_i\) through a plant at \(L_j \in \lambda\):\[
(45) C(ijh\_j) = \min_{h} [C(ijh\_j) + C(jh\_j)] = C(ijh\_j) + \min_{h} C(jh\_j)
\]

The minimum transport cost for serving \(S_i\) through some plant in \(\lambda\) is

\[(46) \min_{j} C(ijh\_j)\]

For a given \(\lambda\), the minimum attainable total transport cost (TTC) is

\[(47) \overline{TTC}_{\lambda} = \sum_{i \in \lambda} \sum_{j \in \lambda} C(ijh\_j)X(ijh\_j)\]

The minimum attainable TTC for \(m\) plants is

\[(48) \overline{TTC}_{m} = \min_{k} \overline{TTC}_{\lambda}\]

The set of plant sites and routings that minimizes (44) is

\[(49) \overline{TC} = \min_{m} [\overline{TTC}_{m} + \beta X + \alpha m]\]

Economies of Size in Transport

Suppose the per unit transport cost from \(L_j\) to \(M_h\) is

\[(50) C(jh\_j) = \gamma(jh) + \Gamma X(jh); \gamma(jh) > 0, \Gamma < 0\]

and the problem is again to determine \(m\), \(\lambda\) and \(X(ijh)\) to minimize (44).

One procedure for solving this problem is as follows. First, assume each \(C(jh\_j)\) is known and is

\[(51) C(jh\_j) = \gamma(jh) + \Gamma X/m\]

Then use (45), (46) and (47) for each \(\lambda\). Now (50) will not be consistent with all plant volumes. Because \(\sum_{h} X(jh\_j) = X(jh\_j) = X(j\_j)\) and \(\sum_{h} X(jh\_j)^2 = X(j\_j)^2\), the actual TTC of using the routings called \(h\)
for by (45), (46) and (47) is

\[
\text{TTC} | \lambda (\text{km}) = \sum_{i} \sum_{j} C(ij\cdot) X(i\cdot\cdot) + \sum_{j} \gamma(jh\cdot) X(\cdot j\cdot) + \Gamma \sum_{j\in\lambda} X(\cdot j\cdot)^2
\]

Suppose that, according to (45), (46) and (47), \( S_k \) sends its raw material to \( L_u \) and thence to \( M_h \), and suppose that all raw material sent to \( L_u \) is sent (after processing) to \( M_v \). What happens to \( \text{TTC} | \lambda (\text{km}) \) if raw material from \( S_k \) is rerouted from \( L_u \) to \( L_u' \) and then to \( M_v' \)?

This rerouting changes \( \text{TTC} | \lambda (\text{km}) \) by the amount

\[
\Delta \text{TTC}(gu) | \lambda (\text{km}) = [C(d\cdot\cdot) + \gamma(\cdot\cdot\cdot)]X(\cdot\cdot\cdot) - [C(d\cdot\cdot) + \gamma(gh\cdot)]
\]

\[
X(\cdot\cdot\cdot) + 2\Gamma[X(\cdot\cdot\cdot) - X(\cdot\cdot\cdot)]X(\cdot\cdot\cdot)
\]

It is easily seen that

\[
\Delta \text{TTC}(gu) | \lambda (\text{km}) > 0 \text{ if } X(\cdot\cdot\cdot) < X(\cdot\cdot\cdot)
\]

This is true because: (a) each \( X(\cdot\cdot\cdot) > 0 \), (b) \( C(d\cdot\cdot) + \gamma(\cdot\cdot\cdot) > C(d\cdot\cdot) + \gamma(gh\cdot) \) because of the way \( \text{TTC} | \lambda (\text{km}) \) was derived, and (c) \( \Gamma < 0 \).

Rerouting shipments from a plant with a large volume of receipts to a plant with a small volume of receipts will increase \( \text{TTC} \). Rerouting shipments from a plant with small receipts to a plant with large receipts may, but need not, reduce \( \text{TTC} \).

The procedure for determining if rerouting shipments from a plant with a small volume of receipts to a plant with a large volume of receipts is much like the procedure in stage 2 of ORA(2, t). For each \( \lambda (\text{km}) \), compute (53) for each possible combination of \( S_k \), \( L_g \) and \( L_u \) satisfying \( X(\cdot\cdot\cdot) < X(\cdot\cdot\cdot) \). If any is negative, select the most negative one. Say this occurs for \( d = d', g = g', u = u' \). Reroute \( X(d'\cdot\cdot\cdot) \) away from \( L_g \) to \( L_u' \). Drop \( S_{d'} \) from further computations and recompute (53) for \( g' \) and \( u' \), replacing \( X(\cdot\cdot\cdot) \) and \( X(\cdot\cdot\cdot) \) by \( X(\cdot\cdot\cdot) \).
+ X(d'\ldots) and X(g'\ldots) - X(d'\ldots). Reroute if any values are still negative. Continue until all S_d have been dropped from further consideration or until all values of (53) are zero or positive.

This series of steps must be carried out for each set of m sites, each set of m + 1 sites, each set of m + 2 sites, and so on.

Predetermined Demand At Each Final Market

This problem is like (44), except that now each X(\ldots h) is a known constant and \( \Sigma_{h} X(\ldots h) = \Sigma_{i} X(\ldots i) = X. \)

In the first step, ignore the constraints on the X(\ldots h) and solve exactly like problem (44) for each \( \lambda (km) \). Suppose comparison of the solution with the market requirements shows that markets \( M_1, M_2, \ldots, M_s \) are surplus markets (receipts exceed requirements) and markets \( M_{s+1}, M_{s+2}, \ldots, M_H \) are deficit markets (receipts fall short of requirements).

Suppose \( M_h \) is a surplus market and \( M_v \) a deficit market and \( S_d \) supplies \( M_h \) through \( L_j \). The quantity that can be rerouted from \( S_d \) is \( R(djv) \) and is determined from:

\[
\text{(54) If } \Sigma \Sigma X(ijh) - X(\ldots h) \geq X(\ldots v), \text{ then } R(djv) = X(\ldots v) \\
\text{(55) If } \Sigma \Sigma X(ijh) - X(\ldots h) < X(\ldots v), \text{ or } \Sigma \Sigma X(ijv) + X(\ldots h) > X(\ldots v), \text{ then } R(djv) = \min \left[ \Sigma \Sigma X(ijh) - X(\ldots h); X(\ldots v) - \Sigma \Sigma X(ijv) \right] \]

Now raw material from \( S_d \) can be rerouted to \( M_v \) in either of two ways:
(a) through \( L_j \) to \( M_v \) or (b) through some \( L_u \) (\( L_u \neq L_j \)) to \( M_v \). If (a) is followed, the resulting change in TTC is

\[
\text{(56) } \Delta \text{TTC}(djv) | \lambda (km) = [C(\ldots jv) - C(\ldots jh)]R(djv) \geq 0
\]
If (b) is followed it is necessary to select some $L_{ue}$. This can be accomplished by minimizing over $u$, and the resulting minimum increase in TTC is

$$
\Delta \text{TTC}(duv) \big| \lambda (\text{km}) = \min_u [C(du^*) + C(\cdot uv)] - C(dj^*) + C(\cdot jh)]
$$

For each origin that supplies a surplus market, compute (56) and (57) for $v = s + 1$ and select the minimum. Doing this for all values of $v$ and for all surplus markets provides a matrix $[\min \Delta \text{TTC}(d\cdot v)]$. Each element in the $i$-th row of the matrix shows the minimum increase in TTC if material from one origin supplying a surplus market is rerouted to a deficit market. The smallest element in the matrix identifies the rerouting to be made.

**Nonlinear Processing Cost Function**

Suppose the TPC function at $L_j$ is

$$
(58) \quad \text{TPC}_j = \alpha + \beta X(\cdot j^*) + \theta X(\cdot j^*)^2
$$

Suppose we encounter this quadratic TPC function when we are trying to minimize total costs and suppose also that per unit transport costs are given by (50). Assume (52) was used to determine $\text{TTC} | \lambda (\text{km})$ and consider the same situation considered in deriving (53): Raw material from $S_d$ is rerouted from $L_g$ to $L_u$. The change in TPC is

$$
(59) \quad \Delta \text{TPC}(gu) \big| \lambda (\text{km}) = 2\theta [X(\cdot u^*) + X(\cdot d^*) - X(\cdot g^*)]X(\cdot d^*)
$$

Adding this to (53) provides the change in TC resulting from specified rerouting of $X(\cdot d^*)$. 
REFERENCES


