EDDY CURRENT PROBE DESIGN AND MATCHED FILTERING FOR OPTIMUM FLAW DETECTION

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INTRODUCTION

Eddy current signals obtained from variations in the probe lift-off are in general much larger in amplitude than the useful flaw signals. Small flaw signals can, however, be detected in the presence of lift-off noise if a large enough phase angle exists between them. Figure 1(a) shows how this phase discrimination can help in lift-off noise suppression. Here, the oscilloscope traces the complex impedance of the probe. The impedance plane has been rotated so that the lift-off noise lies entirely in the horizontal channel. Now if we choose to look only at the signal in the vertical channel of the scope, or the Q channel (in phase quadrature with lift-off), there will be no lift-off noise. This, however, is not a very realistic picture. Figure 1(b) is obtained when we try to detect much smaller

![Diagram](image)

Fig. 1. Phase discrimination used to suppress lift-off. (a) An ideal case. (b) A real case.
flaws (in this case a closed crack of length 20 mils in aluminum). We see that the trace of the liftoff noise has a curvature and that there are also fluctuations along the Q channel axis. Both of these effects eventually limit the detectability of small flaws. Since this contribution of liftoff to the Q channel is in practice larger than circuit noise, we define the detection figure of merit for an EC probe as

\[ D = \frac{(\Delta Z_f) \sin \beta}{(\Delta Z_{lo})_Q} \]  

Another factor contributing to noise in the Q channel besides \((\Delta Z_{lo})_Q\) is probe tilt \((\Delta Z_t)_Q\) which also accounts for the fluctuations in the liftoff curve, but will not be included explicitly here.

In this paper we will be investigating the frequency dependence of \(D\), leading to the conclusion that it becomes independent of \(\omega\) in the large \(a/\delta\) regime. We will also obtain an approximate ranking for different probe types based on their detection figure of merit.

FIELD CALCULATIONS

We will be evaluating the flaw signal and the liftoff noise as a change in the impedance of the probe, expressed in terms of the following surface integral

\[ \Delta Z = \frac{1}{I^2} \oint (\vec{E} \times \vec{H}' - \vec{E}' \times \vec{H}) \cdot ds \]  

where the primed fields are those occurring in the presence of a flaw (or liftoff) and the unprimed fields are for the unperturbed work piece. The surface of integration is taken to coincide with the work piece surface to be scanned (Fig. 2). In order to evaluate the probe impedance change, we need to find the unperturbed and perturbed electromagnetic fields on the surface of the test piece. These fields are functions of the probe geometry; we have calculated them for three geometries of interest: (i) One dimensional straight wire parallel to the test surface. This is a simple case to study which also approximates a large single turn coil operating close to the surface. A superposition of such coils with different radii models a large pancake coil near the surface. (ii) Vertical and horizontal dipoles which can be combined to give any dipole orientation or a rotating dipole. The rotating dipole represents the rotating magnetization in an FMR probe, and the arbitrarily oriented
dipole approximates the field of a small coil in any orientation. (iii). Circular loop with axis normal to the surface. This is an extension of the vertical dipole, and its finite size can act as a spatial frequency filter. This is useful in analyzing the effect of the probe size on detection quality.

We have obtained the following expressions for the spatial frequency (or Fourier transform) spectra of the magnetic field tangential to the work piece. For a one dimensional wire,

$$ h(k) = \frac{1 - \Gamma}{2} e^{-kz_0} $$

(3)

for a circular loop of radius $R$,

$$ h(k) = (1 - \Gamma) \frac{m}{2\pi R} J_1(kR) e^{-kz_0} $$

(4)

and for an arbitrarily oriented or rotating dipole

$$ h(k) = \frac{1 - \Gamma}{4\pi} \left( km_z - k m_x y y \right) e^{-kz_0} $$

(5)

Here, $\Gamma(k) = [Z_s(k) - Z_0]/[Z_s(k) + Z_0]$ is the reflection coefficient of the surface. In each case the magnetic field may be obtained as a function of the spatial coordinates by taking the inverse Fourier transform of $h(k)$. We will see in the next few sections that there is no need for this inverse transform, and that the probe impedance may be obtained using the magnetic fields in the spatial Fourier domain.

In deriving Eqs. (3) through (5), we have used both general electromagnetic theory techniques and the much simpler magnetostatic potential approach. These of course yield the same results in the regime of interest where the electromagnetic wavelength in air is much larger than the dimensions of the probe, the flaw, and the liftoff. In view of this we will be using the magnetostatic potential for most flaw signal calculations.

EVALUATION OF LIFTOFF

Probe impedance change due to liftoff is evaluated using Eq. (2). We let the $xy$ plane coincide with the plane to be scanned. The surface element $ds$ points in the $z$ direction, and in Eq. (2) only the field components tangential to the surface contribute. We
This transformation, when substituted in Eq. (2), results in,

$$
\Delta Z = -\frac{1}{I^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta Z_s(k) \overrightarrow{H}(k) \cdot \overrightarrow{H}^*(-k) \, dk_x \, dk_y,
$$

where we have eliminated the electric field terms through the introduction of the surface impedance $Z_s(k)$. Equation (7) relates the impedance change of the probe $\Delta Z$ to the surface impedance change of the test piece $\delta Z_s(k)$. The surface impedance is calculated for the perpendicular polarization ($\perp$ perpendicular to the plane of incidence) of the incident Fourier components, because in the quasistatic limit that is the only polarization present.

In order to formulate the liftoff problem in terms of a surface impedance change, we first let the surface of integration coincide with the surface of the metal (Fig. 2), where $Z_s(k)$ at this surface is a known function of the characteristics of the metal. Next we let the metallic surface recede from the surface of integration by a small distance $\ell$. The new impedance at the surface of integration

![Fig. 2. Formulation of liftoff in terms of a surface impedance change.](image-url)
is a function of $l$ and can be evaluated by standard transmission line formulas\(^2\) Taking the distance $l$ to be small compared to $z_0$, we can expand the probe impedance in powers of $l$

\[
\Delta Z_{l_0} = \left( \frac{\partial}{\partial l} \Delta Z_{l_0} \right) l + \frac{1}{2} \left( \frac{\partial^2}{\partial l^2} \Delta Z_{l_0} \right) l^2 + \ldots \quad (8)
\]

For terms up to second order in $l$, the probe liftoff signal over a good conducting surface is given by

\[
\Delta Z_{l_0} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [1 + (i - 1)k\delta] \times \left[ l - l^2k + \frac{3}{2} l^2k(1 - i)k\delta \right] \hat{h}_i(k) \cdot \hat{h}_i^*(k) \, dk \, dk_y , \quad (9)
\]

where $\delta$ is the skin depth of the work piece. For a one dimensional straight wire probe this expression can be evaluated analytically to give,

\[
\Delta Z_{l_0} = i \omega l_0 \left[ \frac{1}{z_0} \left( 1 - \frac{1 - i}{2} \frac{\delta}{z_0} \right) - \frac{l^2}{2z_0^2} \left( 1 - 5 \frac{1 - i}{2} \frac{\delta}{z_0} \right) \right] . \quad (10)
\]

The dominating term here is of course the first order term in $l$, the phase angle of which determines the direction of the $I$ channel in Fig. 1. The second order term has a different phase angle, and therefore has a component in the $Q$ channel. To see this explicitly we factor out the first order liftoff $(\Delta Z_{l_0})^1$

\[
\Delta Z_{l_0} = \frac{l}{z_0} (\Delta Z_{l_0})^1 \left( \frac{z_0}{l} - 2 \right) + (1 - i) \frac{\delta}{z_0} \quad (11)
\]

The component of liftoff in the $Q$ channel can be seen from Eq. (11) to be

\[
(\Delta Z_{l_0})_Q = \frac{l\delta}{z_0^2} (\Delta Z_{l_0})^1 \quad (12)
\]

First order liftoff has a dominating term proportional to $\omega$ (for small $\delta/z_0$), but the liftoff contribution to the $Q$ channel has a factor $\delta$ which makes it proportional to the square root of the
As will be seen later, the flaw signal in the large $a/\delta$ regime will have the same frequency dependence, making the $D$ factor in that regime independent of $\omega$. The second order liftoff contribution to the $Q$ channel can also be seen in the liftoff curves calculated for circular coils by Dodd and Deeds (Fig. 3). Numerical evaluation of

\[(\Delta Z_{l})_Q \sim \sqrt{\omega}\]

Fig. 3. Normalized complex impedance of a circular coil of mean radius $\bar{r}$. Dashed lines are liftoff curves at constant frequency, showing their contribution to the $Q$ channel. (After Dodd and Deeds.)
the Dodd and Deeds theory by Bahr\(^3\) has demonstrated the \(\sqrt{\omega}\) dependence of the \(Q\) channel liftoff, noted above, for an air coil with axis normal to the surface.

Probe tilt can also be a significant noise source in eddy current detection systems. A small change \(\theta\), in the angle of the probe with respect to the test piece gives a signal that can be formulated in terms of a surface impedance change and evaluated by Eq. (7). If we expand the tilt signal \(\Delta Z_t\) in powers of the tilt angle \(\theta\), the first order term in \(\theta\) will vanish. The second order term will have a contribution to the \(Q\) channel which is again proportional to \(\sqrt{\omega}\),

\[
(\Delta Z_t)_Q \sim \sqrt{\omega},
\]

leading again to the same frequency independence of the \(D\) factor in the large \(a/\delta\) regime.

**FLAW SIGNAL EVALUATION**

In the large \(a/\delta\) regime, where \((a)\) is the depth of a surface crack, the flaw signal can be calculated using an unfolding technique similar to the method used by Michael and Collins\(^4,5\). The application of this technique to a rectangular surface flaw of length \(2c\) and crack mouth opening \(\Delta u\), gives the result,\(^6\)

\[
\Delta Z_f = 2c \sum_{\delta}^0 - \frac{2}{\delta} (1 + i) c^2 \sum_{\delta}^1 + i \frac{2}{\delta^2} c^2 \Delta u \sum_{\delta}^1 ,
\]

where the summation terms are functions of the aspect ratio \(a/c\), and have no frequency dependence. For a closed crack \(\Delta u = 0\), and the frequency dependent part of \(\Delta Z_f\) goes as the inverse of the skin depth or

\[
\Delta Z_f \sim \sqrt{\omega} ,
\]

which confirms our previous assertion that the \(D\) factor saturates at high \(a/\delta\). In Fig. 4 this behavior is shown graphically.

**PROBE PERFORMANCE EVALUATION**

Using what was developed in previous sections we can evaluate the liftoff impedance change for any orientation of a magnetic dipole. The problem is set up by substituting Eq. (5) into Eq. (7), and changing the surface integral into polar coordinates. The angular integration is trivial and results in,
Fig. 4. The $D$ factor saturates at large values of $a/\delta$. The frequency scale is for the 7-15 mil target flaw in the RFC program.

\[
\Delta \zeta_0 = (2|z|^2 + |x|^2 + |y|^2)F(\ell, \omega) ,
\]

where

\[
F(\ell, \omega) = i \omega \mu_0 \int_0^\infty [1 + (i - 1)k\delta] \\
\times \left[ \ell - \ell^2k + \frac{3}{2} \ell^2k(1 - i)k\delta \right] \frac{1 - \Gamma(k)}{4\pi} k^3 \, dk .
\]

Without explicit evaluation of this integral it is apparent that a vertical dipole ($m_z$) has twice the sensitivity to liftoff than a horizontal dipole. This will serve as a basis for probe performance ranking along with the probe sensitivity to surface flaws.

The sensitivity of a probe to a surface flaw is proportional to the square of the tangential magnetic field at the position of the flaw. For a horizontal dipole, the maximum tangential field occurs at the point on the surface closest to the dipole, and is given by

\[
|H_t|_{\max}^2 = \left| \frac{m_x}{2\pi z_0^3} \right|^2 .
\]
Here, we have taken the surface to be a perfect conductor. For a vertical dipole the maximum tangential field occurs a distance \( z_0/2 \) away from the axis of the dipole. For this reason, the tangential field is weaker in this case,

\[
|H_t|_{\text{max}}^2 = 0.74 \left( \frac{mz}{2\pi z_0} \right)^2 .
\]  

(17)

We see that the horizontal dipole is 35 percent more sensitive to a surface flaw in a perfect conductor than the vertical dipole. Combining this result with the liftoff argument, and assuming that the results for a practical conductor are not far away from those of a perfect conductor, we conclude that the horizontal dipole gives a detection figure of merit that is better than that of a vertical dipole by a factor of 2.7,

\[
(D)_{\text{horizontal}} = 2.7 (D)_{\text{vertical}} .
\]  

(18)

Real probes of course have more complicated structures, but many real probe field patterns can be approximated by vertical and horizontal dipole fields. In Figs. 5 and 6 the field patterns of some practical probes have been sketched. The probes referred to as "parallel" have field patterns similar to that of a horizontal dipole, and those referred to as "normal" may be approximated by vertical dipoles. As can be seen in Eq. (18), it is advantageous to use horizontal coils [Fig. 5(b)] and tape-head type probes [Fig. 6(b)], over vertical coils and cup-core probes, but more detailed analysis will be required for careful evaluation of specific probe designs.

MATCHED FILTERING

Up to this point we have been mainly concerned with the relative amplitudes of the flaw signal and the liftoff noise. As can be seen from the signal-to-noise relation [Eq. (1)] the phase angle \( \beta \) between the flaw signal and the liftoff noise can play an important role in improving the S/N.\(^7\) The discrimination angle \( \beta \) can be shown to be a function of the field shape of the probe, and can be optimized by proper field shaping. More specifically, different components in the spatial frequency spectrum of the probe field (or the spatial Fourier space) give different discrimination angles as well as different amplitudes. Proper field shaping amounts to choosing to operate in a desirable range of the spatial frequency spectrum. One way of achieving such a localization in the Fourier space is by making the probe periodic. The higher the number of periods, the better the localization. High number of periods on the other hand increase the probe dimension. The signal \( s(t) \) obtained from a localized flaw will thus be extended in time (Fig. 7).
Fig. 5. Field patterns of simple practical coils are similar to (a) vertical dipole field, and (b) horizontal dipole field.
FERRITE CORE PROBES

(a) NORMAL PROBE STRUCTURE

(b) PARALLEL PROBE STRUCTURE

Fig. 6. (a) A cup-core probe has a field pattern similar to a vertical dipole. (b) A tape head probe has a field pattern similar to a horizontal dipole.
Fig. 7. A long periodic probe (a) has an extended periodic output in time (b).

In order to achieve a higher signal-to-noise ratio and a better spatial resolution, it is desirable to compress $s(t)$ in time. A filter that will realize this compression is called a matched filter and is characterized mathematically by its impulse response $h(t)$ which should satisfy the condition

$$h(t) = s(-t) .$$

One practical filter that satisfies this condition is a tapped delay line [Fig. 8(a)]. An analog delay line usually has a minimum allowed frequency which may be too high for a slow scanning probe. There are, however, digital filters performing the same task with no minimum frequency limitation.\textsuperscript{8}
Fig. 8. (a) Matched filter for a fixed frequency input. (b) Filter output of an uncoded waveform. (c) Filter output of a Barker coded waveform.
If we denote the amplitude of each one of the peaks in \( s(t) \) by \( S \), and the number of probes in the array by \( n \), the compressed output will have a peak \( N' = nS \). Any random noise \( N \) in \( s(t) \) will be added incoherently, i.e., \( N' = \sqrt{nN^3} \), resulting in the new signal-to-noise ratio,

\[
\frac{S'}{N'} = \sqrt{n} \frac{S}{N}
\]

(19)

This suggests that operating for example 13 probes in such an array, will improve the signal-to-noise ratio by 11 dB.

The operation of the matched filter is equivalent to taking the autocorrelation of its input. From this it is apparent that a train of \( n \) equal peaks will produce an output with a central peak and accompanying side lobes in a triangular envelope [Fig. 8(b)]. With the output in this form we have gained in signal-to-noise ratio, but we have lost some spatial resolution, i.e., two close lying flaws cannot be resolved. This problem may be eliminated through the use of different coding schemes. A seven bit Barker code (Fig. 8) requires that in \( s(t) \) the first three peaks be positive, the next two be negative, and the sixth and seventh be positive and negative respectively. The same code is used to multiply the delay line outputs before the summation stage. The resulting filter output is shown in Fig. 8(c) to have equal side lobes that are suppressed 16.9 dB below the central peak. Barker codes are available for up to thirteen bits. A thirteen bit code is \((+++-+-+-++)\) and gives side lobes that are 22.3 dB below the central peak. In case the side lobes are still above the noise level and need to be suppressed further, other coding schemes such as complementary coding are available which will eliminate the side lobes altogether.

We conclude that the phase discrimination \( \beta \) can be improved by designing the probe to be periodic. The compressed output of such a probe will have a further improved \( S/N \) by a factor \( \sqrt{n} \), without any loss of spatial resolution.

SUMMARY AND CONCLUSIONS

In using eddy current probes in the detection of small surface flaws it has been observed that the detectability of the flaw is limited not by the circuit noise but by the signal obtained from variations in the probe liftoff. It has long been realized that the liftoff noise is large in magnitude compared to the flaw signal, and detection systems currently in use tend to maximize liftoff discrimination by detecting the component of the flaw signal in phase quadrature with the liftoff noise. Studies presented in this paper indicate that there always exists a residual contribution in
the Q output channel due to liftoff variations, and this contribution is what limits the detectability of small flaws. The detection figure of merit $D$ [Eq. (1)] has been defined on this basis and is found to reach a constant saturation value at high frequencies (large $a/\delta$ regime). Studies of the $D$ factor in the small $a/\delta$ regime have shown a rising trend with frequency as demonstrated in Fig. 4. A region of practical interest covering the range $0.8 < (a/\delta) < 3$ has not been studied extensively, and possible structures in it are not yet well established.

The detection figure of merit $D$, was found to be larger by a factor of 2.7 for a horizontal dipole probe than a vertical dipole. On this basis we conclude that simple probe geometries with field patterns similar to a horizontal dipole field are advantageous over those approximating a vertical dipole field. Ferrite core probes are predicted to be more sensitive than air core probes, as reported in practice, because they produce larger magnetic fields per unit current at the work surface. For this purpose, the preferred choice of probe for detecting the target flaw in the RFC program appears to be a tape-head or split-toroid type of probe [Fig. 6(b)].

Finally, it was pointed out that the discrimination angle $\beta$ between the flaw signal and liftoff can be increased by probe field shaping through the use of probe arrays. These arrays also allow for signal compression and matched filtering which will improve the signal-to-noise ratio of the detection system.

REFERENCES

6. F.G. Muennemann and B.A. Auld, "Inversion of eddy current signals in a non-uniform probe field," these proceedings.

DISCUSSION

R.B. Thompson (Ames Laboratory): Could you compound a faster signal or the effect of a miniprobe flaw detector by compounding a rotating probe and then counting each time it comes around as one time on the multi array probe?

M. Riaziat (Stanford University): That will definitely give an improved signal-to-noise ratio by adding the peaks each time it goes over because lift-off would add less strongly than the peaks would. There is, however, the problem that you cannot do any field shaping in that fashion, and if we drive the probes together and make a probe array, we can also adjust the beta.