Endogenous Labor Supply, Rigid Factor Prices And A Second Best Solution

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The conventional wisdom asserts that distortions due to factor price rigidities should be eliminated by subsidies. However, we argue that the success of this policy rests upon the fact that the return to the factor is a pure rent. If factor supply is endogenous, then subsidies to employers and taxes on factor owners are needed to support the optimal solution. Since we believe this policy combination works only by assuming away the problem, we then devise a model to study the optimal policies in a second best world. Our results show that, in general, both factor subsidies (or taxes) and export subsidies (or taxes) are needed to achieve this constrained optimum.
In recent years there has emerged a large literature which formulates optimal policy interventions when distortions occur in the domestic economy. For example, it is well known that if factor immobility and factor price rigidity prevent an efficient allocation of resources, then the optimal policy intervention is a subsidy that restores full resource utilization, thereby achieving a Pareto Optimum allocation of resources.

However, it should be noted that the efficacy of this policy depends upon the assumption that the supply of the factor (the price of which is rigid) is not price responsive. Under these circumstances, the return to that factor is a pure economic rent, and any tax or subsidy on that factor will have no adverse effect (provided resources are fully utilized). On the other hand, if the supply of the factor is price responsive, then the subsidy alone cannot achieve an efficient allocation because differences remain between the price the factor is paid and its marginal value product. Therefore, a subsidy designed to restore full employment will lead to an overproduction of that factor if its supply is determined by its factor price.

In this paper we plan to discuss optimal policy intervention when the supply of labor is endogenous and when factor prices are rigid downward. The first section of the paper presents the basic model, derives the efficiency conditions and discusses policies that can help achieve a Pareto Optimum allocation. However, it is also argued that these policies are not likely to be feasible, so in the second section we derive the second best solution for a simplified version of the
basic model. Finally, the third section presents an economic interpretation of these conditions in terms of the specific policies that should be pursued and concludes with a numerical example.
I. The Basic Model

The model we use is similar to previous models used to study the problem of domestic distortions, the main difference being that we assume the supply of labor is endogenous. Thus, we assume there are two goods, $C$ and $M$, and that each is produced using only labor. The marginal product of labor is assumed to be positive, but diminishing in each sector:

$$Q_i = F_i(N_i^i); F_i' > 0, F_i'' < 0; i = c, m;$$

$N_i^i = \text{labor input to sector } i.$

Further, we assume that the total number of workers and that the number of workers in each sector (due to immobility) is fixed. However, since each worker freely determines his supply of labor, labor supply is not fixed, but rather is determined endogenously. Specifically, we assume that each worker maximizes, subject to a budget constraint, a (quasi-concave) utility function that depends on his consumption of each good and the amount of labor supplied (leisure consumed):

$$U_i = U_i^i(C_i^i, M_i^i, -L_i^i); i = c, m.$$

In (2), $C_i^i$ ($M_i^i$) represents consumption of $C$($M$) by a worker of type $i$, and $L_i^i$ represents labor supply by that type of worker (to sector $i$).

Finally, we assume the country is "small" and can freely trade without affecting world prices. Choosing $M$ as the numeraire and letting $P$ be the (constant) world price of $C$, balanced trade implies:

$$P F_c(N_c) + F_m(N_m) = P(C_c + C_m) + (M_c + M_m)$$

Equations (1), (2) and (3) can be used to derive the efficiency conditions for this economy. Note, however, that we are not free to use a single utility function (or community indifference curve) to derive these conditions since different types of workers might be called upon to supply different quantities of labor (depending on their marginal value product). Rather, we derive these conditions by tracing out the utility possibility frontier; that is, by maximizing
utility of workers of type c subject to a utility constraint for workers of

type m. The Lagrangean for this problem is:

\[ \eta = U^c(c^c, m^c, -L^c) + \lambda_1 [U^m(c^m, m^m, -L^m) - \bar{u}^m] \]

\[ + \lambda_2 [PF_c(N^c) + F_m(N^m) - P(c^c + c^m) - (m^c + m^m)] + \lambda_3 [L^c - N^c] + \lambda_4 [L^m - N^m] \]

Optimizing (4) with respect to \(c^i\), \(m^i\), \(L^i\) and \(N^i\) yields the standard efficiency

conditions:

\[ \frac{U^c}{U^c} = \frac{U^m}{U^m} = P \]

\[ \frac{U^c}{U^m} = \frac{PF_c(N^c)}{F_m(N^m)}; N^c = L^c \]

\[ \frac{U^m}{U^m} = \frac{F_m^{-1}(N_m)}{N_m}; N^m = L^m \]

In (5) through (7), subscripts on \(U^i\) indicate partial differentiation; thus, \(U^c_m\)

is the marginal utility of good \(M\) to type \(c\) workers, and \(U^m_L\) is the marginal utility

of leisure to type \(m\) workers.

The interpretation of these conditions is standard. (5) states that the

marginal rate of commodity substitution (MRS) should be the same for all consumers,

and equal to the foreign rate of transformation (\(P\), the world price ratio); (6)

and (7) assert that the MRS between good \(i\) and leisure of type \(i\) should be equal

to the marginal rate of transformation (MRT) between these commodities. In addition,

the dual variables can be given economic meaning. For example, (\(\lambda_3/\lambda_2\)) and (\(\lambda_4/\lambda_2\))

represent the shadow price of labor (of type \(c\) and \(m\) respectively) in terms of

the numeraire good \(M\).

One important aspect of the problem formulated in (4) is that the solution

depends upon the value chosen for \(\bar{u}^m\). As we vary \(\bar{u}^m\) to trace out the utility

possibility frontier, we change not only the distribution of commodities, but also

the amount of labor (and its shadow price) of each type that will be used, and

hence the amount of each good that will be produced. For example, if leisure is a

normal good in the utility function, an increase in \(\bar{u}^m\) leads to a decrease (increase)
in the amount of labor of type m (c) that will be used, and hence to a decrease (increase) in output of good M (C) and an increase (decrease) in the shadow price of labor of type m (c).

Viewed in the context of a competitive model, this means that the equilibrium wage rates depend not only upon preferences, commodity prices and technology, but also on the fixed transfer payments that each type of worker receives. Thus, the optimal policies needed to correct factor price rigidity in some sector depend upon how transfer payments to each (type of) worker are determined.

For example, suppose for some given transfer to each type of worker that the competitive equilibrium wage rates (as determined by the dual variables) are $\hat{W}_c$ and $\hat{W}_m$ to workers of type c and m respectively. However, assume that due to some institutional constraints, there is a wage floor, $\bar{W}_c$, in sector C, and that $\bar{W}_c > \hat{W}_c$. In the absence of government intervention, this factor price rigidity would result in unemployment and a misallocation of resources. What is the optimal policy to be pursued under these circumstances?

The standard policy prescription (when the supply of labor is fixed) would be to give a wage subsidy to employers in sector C, and to finance this subsidy via profit taxes or lump sum taxes on workers. However, while this policy might restore full employment to sector C, it does not achieve an efficient allocation of resources because the marginal rate of substitution between C and $L^C (\bar{W}_c)$ differs from the marginal rate of transformation between these goods.

As is apparent, the particular first best solution could be obtained via a wage subsidy to producers in C and an income tax on workers in C. If the wage subsidy were $(\bar{W}_c - \hat{W}_c)$, and the income tax were $(\bar{W}_c - \hat{W}_c)$, and if transfer payments were distributed in the same way as before, then this combination of policies would restore full employment in C and would permit fulfillment of the efficiency conditions (5-7). Moreover, the income tax would just finance the subsidy to employers.
However, it is unlikely that this policy combination would be effective since it reduces the workers' net wage to $\tilde{w}_C$. In fact, the policy works only because it assumes away (for all real purposes) the wage floor; $\tilde{w}_C$ does not represent a price that anybody in the economy pays or receives. Therefore, while a combination of selective subsidies and income taxes could yield the first best solution, it is hard to believe such a policy is practical; rather it would probably lead to an increase in the wage floor ($\check{w}_C$).

There is an alternative way to achieve a first best solution. Since the labor supply (in each sector) depends upon the transfers received by those workers, a redistribution of income will lead to a change in the equilibrium wage rates. Assume, as before, that for a given distribution of (non-wage) income the equilibrium wage rate in $C$ lies below the wage floor, $\check{w}_C$. If transfers to workers in $M$ are reduced and transfers to workers in $C$ are increased, then (assuming leisure is a normal good) the competitive wage will rise in $C$ and fall in $M$. Thus, it is possible that a sufficient redistribution of income could restore a competitive equilibrium. However, it will not be the same equilibrium as would have occurred had there been no wage floor, and thus it will represent a different point on the utility possibility frontier. If, in addition, we assume there is some social welfare function that chooses a best point on the utility possibility frontier, then it is unlikely that this redistribution of income will permit attainment of the optimum optimorum. Moreover, a redistribution, as outlined above, could run into two other problems: (i) since it lowers the competitive wage in $M$, some wage floor constraint may become binding in that sector; and (ii) the process of transferring income to workers in $C$ rewards them for the wage floor. This certainly will cause resentment on the part of workers in $M$ and may induce either (or both) groups of workers to strive for larger wage floors.

To sum up, we have seen that a Pareto Optimum allocation could be achieved
in two separate ways: (i) by a combination of wage subsidy and tax in sector C; (ii) or by a redistribution of income. However, we have argued that neither of these policies is likely to be successful in that the tax-subsidy effectively assumes away the problem, while the redistribution of income alters the actual Pareto Optimum point and may meet with resentment (or wage floors) from workers in the other sector. Thus, we conclude that it might not be possible to achieve the first best solution if the supply of labor is endogenous. In the next section of this paper, we describe how a second best solution can be derived for this problem.
II. A Simplified Model and a Second Best Solution

As previously noted, the equilibrium prices and output for the economy described in Section I are not independent of the distribution of income (or utility). Thus, the appropriate policy responses to distortions will, in general, depend upon this distribution of income. Since we have no a priori way of determining the "proper" allocation of utility among workers, we either must formulate policy responses for all possible distributions of income (and utility), or else find some way around this distributional problem. The assumption that we shall employ in the remainder of this paper is that there is only one type of worker and only one good produced in the domestic economy. While this assumption is a strong one, we believe the results it allows us to derive give a proper feeling for the second best solution in the more general case.

Specifically, we assume there are only workers of type c and hence that only good C can be produced domestically. However, the country can trade by importing M and exporting C and it is assumed that the terms of trade, P, are constant. Production of C is as described by (1) and the preferences of type c workers are given by (2). Assuming workers maximize utility and firms maximize profits under competitive conditions, there is a unique equilibrium wage rate ($W^*$), a unique output of C, and unique consumption of each good. Clearly, this equilibrium yields a Pareto Optimum allocation.

Next, suppose there is a wage floor, $\bar{W}$ (measured in numeraire units), so that wages in C($W$) must be at least as large as $\bar{W}(\bar{W} > W^*)$. As argued previously, the first best solution could be achieved by a combination of wage subsidies to employers and income taxes on workers. However, for reasons discussed earlier, this policy does not seem realistic; it works only by assuming away the problem. If this policy cannot be implemented, then we must look for a second best solution.

The government is assumed to be able to control domestic prices and unemployment
through export taxes (or subsidies) and wage subsidies (or taxes) to employers; however, the take-home pay of workers, \( W \), cannot be reduced below the wage floor, \( \bar{W} \). Using the tools at its disposal, the government attempts to maximize social welfare, assuming individual consumers behave as price-taking utility maximizers. The constraints placed on government actions are:

\[
\begin{align*}
(8) & \quad PF_c(L \cdot (1-u)) - PC - M \geq 0 \\
(9) & \quad u \geq 0; \ u = \text{the unemployment rate} \\
(10) & \quad W - \bar{W} \geq 0 \\
(11) & \quad \left( \frac{U_L}{U_m} \right) - W(1-u) = 0; \ W^e = W(1-u)
\end{align*}
\]

In (11), \( U_L \) represents the marginal utility of leisure; in (10) it is assumed \( W > W^* \), where \( W^* \) is the competitive equilibrium wage rate.

Equation (8) simply reflects the balance of trade constraint; (9) and (10) need no elaboration, but some further discussion of (11) is required. Clearly, \( \left( \frac{U_L}{U_m} \right) \) is the consumer's MRS between leisure and consumption of good \( M \), and under individual optimizing behavior, this is equated to the relative prices of the commodities. However, if some unemployment occurs, the expected wage rate (assuming jobs are randomly distributed) is \( W(1-u) \). We assume that workers base their labor supply decisions on this expected wage rate.

If there is unemployment, then the labor supply, \( L \), differs from the amount of time actually spent working \( (1-u)L \). The question arises as to how the time \( (uL) \) should be treated; should it count as leisure in the utility function, or should it have the same disutility as work? We adopt the latter approach; if desired, the time \( (uL) \) can be thought of as time spent in job search, and hence time which is not available for leisure.

Under these assumptions, the Lagrangean for the optimization problem is:
(12) \[ \eta = U(C, M, -L) + \lambda [PF_c(L(l-u)) - PC - M] + Q_1 u \]
\[ + Q_2 \left( \frac{U_L}{U_m} - W(1-u) \right) + Q_3 [W - W] \]

(13) \[ \lambda, Q_1, Q_3 \geq 0 \]

Optimizing with respect to \( W, u, C, M \) and \( L \) yields:

(14) \[ \frac{\partial \eta}{\partial W} = Q_3 - Q_2 (1-u) = 0 \]

(15) \[ \frac{\partial \eta}{\partial u} = Q_1 - \lambda PF_c'L + Q_2 W = 0 \]

(16) \[ \frac{\partial \eta}{\partial C} = U_{C} - \lambda P + Q_2 \left[ \frac{U_{Cl}}{U_m} - \frac{U_{L}}{U_m} \cdot \frac{U_{mc}}{U_m} \right] = 0 \]

(17) \[ \frac{\partial \eta}{\partial M} = U_{m} - \lambda + Q_2 \left[ \frac{U_{Lm}}{U_m} - \frac{U_{L}}{U_m} \cdot \frac{U_{mn}}{U_m} \right] = 0 \]

(18) \[ \frac{\partial \eta}{\partial L} = -U_{L} + \lambda PF_c'(1-u) + Q_2 \left[ -\frac{U_{LL}}{U_m} - \frac{U_{L}}{U_m} \cdot \frac{U_{mc}}{U_m} \right] = 0 \]

From (14) through (18) it is apparent that all of the multipliers (with the exception of \( Q_1 \)) must be positive, providing the wage constraint is binding.

While (16) through (18) might not seem informative, manipulation of these equations reduces them to a tractable form. Specifically, multiply (16) by \( U_m \), (17) by \( U_c \) and subtract (17) from (16):

(19) \[ \lambda U_m [P^d - P] = Q_2 \left[ -U_{Lc} + \frac{U_{L}}{U_m} \cdot U_{mc} + \frac{U_{c}}{U_m} \cdot U_{Lm} - \frac{L U c}{U_m^2} \cdot U_{mn} \right] = Q_2 A \]

where \( A \) is defined as the bracketed term on the RHS in (19). In (19), \( P^d \) represents the domestic price of \( C([P^d - P] \) is the export subsidy), and, under the behavioral assumptions, \( P^d = [U_c/U_m] \).

Proceeding in a similar fashion with (16) and (18), and (17) and (18) respectively, yields:

(20) \[ \lambda PU_m (1-u) [P^d F_c' - W] = Q_2 \left[ -\frac{U_{L}}{U_m} \cdot U_{Lc} + \frac{U_{L}}{U_m} \cdot U_{mc} + \frac{U_{c}}{U_m} \cdot U_{LL} - \frac{L U c}{U_m^2} \cdot U_{mL} \right] = Q_2 B \]
In (20) and (21) we substituted for the constraint, \([U_L / U_m] = W(l-u)\). Again, B and D are defined by the bracketed terms on the RHS of each equation.

While (19) through (21) may not appear to be an improvement on the earlier equations, a moment's reflection will indicate that the terms A, B, and D measure the complementarity or substitutability between the goods. For example, A represents the change in the MRS between M and leisure as consumption of C and M is varied, holding utility constant; thus, the sign of A reflects whether good C and leisure are complements or substitutes. Similar interpretations hold for B and D.

Furthermore, the terms A, B, and D are (proportional to) cofactors of the Bordered Hessian derived from the individual utility maximization problem. Thus, they represent the slope of the compensated demand curve—that is, the substitution term of the Slutsky-Hicks equation. Specifically, it can be shown that:

\[
A = \left[ \frac{\partial C}{\partial We} \right]_{\bar{u}} \quad ; \quad A < 0
\]

\[
B = \left[ \frac{\partial C}{\partial Pd} \right]_{\bar{u}} = \left[ \frac{\partial C}{\partial Pd} \right]_{\bar{u}}
\]

\[
D = \left[ \frac{\partial C}{\partial Pd} \right]_{\bar{u}} < 0
\]

In (22) through (24), \(\Delta\) is the determinant of the Bordered Hessian and must be negative if the utility function is quasi-concave. The terms \(\left[ \frac{\partial C}{\partial We} \right]_{\bar{u}}, \left[ \frac{\partial C}{\partial Pd} \right]_{\bar{u}}, \text{ etc.}\), represent the substitution term of the Slutsky-Hicks equation; thus, \(\left[ \frac{\partial C}{\partial Pd} \right]_{\bar{u}} < 0\). Naturally, the normal symmetry relations hold for the cross-price effects.

Letting \(P_m = 1\) and substituting (22) through (24) into (19) through (21)
yields:

\begin{align*}
(19') \quad & \lambda U_m^2 [P^d - P] = -\Delta Q_2 \left[ \frac{\partial C}{\partial W e} \right]_u = \Delta Q_2 \left[ \frac{\partial L}{\partial P d} \right]_u \\
(20') \quad & \lambda U_m^2 P (1-u) (P F_c' - W) = \Delta Q_2 \left[ \frac{\partial C}{\partial P_m} \right]_u = \Delta Q_2 \left[ \frac{\partial M}{\partial P d} \right]_u \\
(21') \quad & \lambda U_m^2 (1-u) (P F_c' - W) = -\Delta Q_2 \left[ \frac{\partial C}{\partial P d} \right]_u < 0
\end{align*}

If desired, \( \lambda \) and \( Q_2 \) can be solved for by substitution of (19') through (21') into (16) through (18).

Equations (14), (15), (19'), (20') and (21') characterize the second best solution. In Section III, we shall analyze the properties of this solution.
III. Properties of the Second Best Solution

It is well known that if one of the conditions for an efficient allocation of resources cannot be met, then it is not generally optimal to insure fulfillment of the other conditions (Lipsey and Lancaster (1956-7)). In the context of the model of Section II, this implies that free trade will not, in general, be optimal if the first best solution is unattainable.

Specifically, the three Pareto Optimal conditions for the model of Section II are that:

(i) the marginal rate of substitution (MRS) between M and leisure (which equals the expected wage rate) should be equal to the marginal rate of transformation (MRT) between M and leisure (via production and trade, and equal to PF_c'),

(ii) the MRS between C and leisure (equal to W/P) should equal the MRT between C and leisure (F_c'),

(iii) the MRS between C and M (P^d) should equal the MRT (FRT) between C and M (P).

If, as a result of wage rigidities, the first best solution is unattainable, then at least two of these three conditions cannot be met. Nevertheless, it is possible that at least one of these conditions could be fulfilled (either P = P^d or F_c' = W); the question is whether this is ever desirable.

The optimal policy responses to the distortion are readily ascertainable from (19') through (21'). From (21'):

(28) W > PF_c' (L(l-u)) if W > W*

where, in (28), W* is the competitive equilibrium wage rate. As a result of the wage distortion, the MRS of M for L will exceed the MRT between M and L. The appropriate policy with respect to the domestic price level (P^d) depends upon the relationship between C and leisure. From (19'):

(29) sign (P^d - P) = sign (∂C/∂W^e)_u

Free trade is optimal only if leisure and C are independent goods (as defined by the compensated demands). If C and leisure are substitutes, then an export
subsidy \((P^d > P)\) is desirable, whereas if they are complements, an export tax is desirable \((P^d < P)\).

If \(P^d \leq P\), then from (28) it is apparent that \(W > P^d F'\), so that the MRS of \(C\) for leisure \((W/P^d)\) will exceed the MRT between these goods \((F'\)). However, if \(C\) and leisure are substitutes, then \(P^d > P\), and the sign of \([P^d F' - W]\) depends on the relationship between \(C\) and \(M\). From (20'):

\[
(30) \text{sign } [P^d F' - W] = -\text{sign } [\partial C/\partial M]_U
\]

If \(C\) and \(M\) are independent, then no wage subsidy or tax is appropriate, and efficiency condition (ii) will be met. However, if they are substitutes (complements), then a wage subsidy (tax) is the appropriate policy response.

Thus, none of the efficiency conditions will be met unless two of the goods are independent of each other. In the general case, a combination of export subsidies (or taxes) and wage subsidies (or taxes) should be used. The specific policy responses are summarized in Table I.

Our second best solution differs from the usual results in two ways. First, unless \(C\) and leisure are independent, free trade is not appropriate; when the supply of labor is assumed exogenous, as in the standard models, free trade is always the optimal policy. Secondly, either a wage tax or subsidy (or no intervention) can occur in our model; the standard models conclude that a wage subsidy is needed.

These results can be explained intuitively in the following way. As a result of the wage floor, there will be an excess supply of labor; people will economize on leisure, thereby consuming either too much \(C\) or \(M\) (or both). The optimal policies should seek to counteract this misallocation. If \(C\) and leisure are complements (substitutes), then too little (much) of good \(C\) is consumed as a result of the wage floor. Thus, the optimal policy entails decreasing (increasing) \(P^d\), thereby decreasing the quantity of labor supplied and increasing
(decreasing) the amount of C consumed. This change in $P^d$ serves to partially offset the adverse effects of the artificial wage rate.

For the case in which C and leisure are substitutes, the question arises as to how much $P^d$ should be increased; that is, as to the sign of $[P^d F'c - W]$. If C and M are substitutes (complements), increases in $P^d$ lead to increases (decreases) in the compensated demand for M. Thus, if they are substitutes, the increases in $P^d$ can lead to overconsumption of M, so that it is not desirable to increase $P^d$ "too much" ($P^d F'c < W$). On the other hand, if they are complements (so that M and leisure are substitutes), then increases in $P^d$ not only lead to reductions in labor supply and consumption of C, but also serve to decrease the overconsumption of M (due to the wage floor). In this case, larger export subsidies are beneficial, and it is optimal to couple these with wage taxes ($P^d F'c > W$).

Our discussion so far has tacitly assumed that full employment is desirable; if there is not full employment, then there is no real trade-off between C (or M) and leisure. The role that unemployment plays in our model is to reduce the expected wage ($W(1-u)$) to workers. Thus, unemployment has some of the same implications as an income tax, though it also entails the cost of lost output due to idle labor (or lost leisure due to search). Intuitively, then, we would expect unemployment to become desirable only when the minimum wage ($\bar{W}$) is set considerably above $W*$; that is, when the costs of maintaining full employment (through the misallocation in consumption of C, M and leisure) exceed the costs of unemployment.

Rewriting equation (15):

\[
\frac{\partial \pi}{\partial u} = Q_1 - \lambda P F'_c L + Q_2 W = 0
\]

Since $Q_2$ is a measure of the costs of the artificial wage rate ($Q_3 = Q_2(1-u)$), it is apparent that the larger these costs, the greater is the incentive to reduce
the effective wage rate \( W(1-u) \) and hence to create unemployment. From (21'):

\[
(31) \quad \frac{Q_2}{\lambda} = \frac{U_m^2(1-u)(W - PF'_c)}{\Delta [\frac{\partial C}{\partial P_d} - \frac{\partial L}{\partial W}]_u}
\]

Furthermore, it can be shown that:

\[
(32) \quad \frac{U_m^2}{\Delta} = \left[ \frac{\partial C}{\partial P_d} - \frac{\partial L}{\partial W} \right]_u + \left[ \frac{\partial C}{\partial W} \right]_u < 0
\]

Also, from (19') and (21'):

\[
(33) \quad \frac{[\frac{\partial C}{\partial P_d} - \frac{\partial L}{\partial W}]_u}{[\frac{\partial C}{\partial W}]_u} = \frac{(P - P^d)/(1-u)(W - PF'_c)}
\]

Substituting (31), (32) and (33) into (15) yields:

\[
(34) \quad \frac{\partial \eta}{\partial u} = Q_1 - \lambda \left[ PF'_c \cdot L - W \cdot \left( P - P^d \right) - \frac{\partial L}{\partial W} \right]_u (W - PF'_c) = 0
\]

where \( \frac{\partial C}{\partial W} \) at \( u = 0 \). If the bracketed expression on the RHS in (34) is negative at \( u = 0 \), then \( Q_1 = 0 \), indicating that some unemployment is desirable.

For example, assume \( \frac{\partial C}{\partial W} = 0 \), so that \( P^d = P \). Then (34) becomes:

\[
(35) \quad \frac{\partial \eta}{\partial u} = Q_1 - \lambda L \left[ PF'_c - \frac{\partial L}{\partial W} \right]_u (W - PF'_c) = 0
\]

As \( W \) increases, \( L \) increases, and \( PF'_c \) decreases (at \( u = 0 \)); if the compensated elasticity of labor supply remains constant, the bracketed expression decreases. For sufficiently large \( W \) it could become negative at \( u = 0 \), indicating that some unemployment is desirable. Even if \( C \) and leisure are not independent, it is likely that unemployment becomes desirable at sufficiently large wage rates. For example, let:

\[
(36) \quad U = \ln C + \ln M - L
\]

\[
(37) \quad F_c(N) = 2N^{1/2}; \quad N = L(1-u)
\]

In (36), \( C \) and \( M \) are independent, so \( P^d F'_c = W \). Thus, \( C \) and \( L \) must be substitutes and \( P^d > P \) for \( W > W^* \), the competitive wage rate. Therefore, an export subsidy, but no wage subsidy (or tax) is desirable.

To solve this problem assume \( P = 1 \). The competitive equilibrium is:
\[ P^d = P = 1; \, W^* = 1; \, M = C = L = 1 \]

Next, assume a wage floor, \( W > 1 \), is imposed. Optimizing (36) subject to the constraints gives the following solution:

\[ (39) \]

(a) \( \bar{W} \in \left[ 1, \frac{3\sqrt{2}}{2} \right] \):

(i) \( u = 0, \, L = \frac{1}{2} + \frac{W^2}{8} + \frac{W}{8}(8 + W^2)^{1/2} > 1 \)

(ii) \( C = L^{-\frac{1}{2}}, \, M = W \)

(iii) \( P^d = \frac{W^2}{4} + \frac{W}{4}(8 + W^2)^{1/2} > 1 \)

(iv) \( P^d_{Fc} = W \)

(b) \( \bar{W} > \frac{3\sqrt{2}}{2} \):

(i) \( u = 1 - \frac{9}{2W^2} > 0; \, u \in (0, 1) \)

(ii) \( L = 2; \, N = 9/W^2 \)

(iii) \( C = (3/2W); \, M = (9/2W) \)

(iv) \( P^d = 3; \, P^d_{Fc} = W > W(l-u) = \frac{9}{2W} \)

We see that the particular solution corresponds with the properties outlined earlier. Thus, since \( C \) and \( L \) are substitutes, \( P^d > P \) and an export subsidy is needed. Also, since \( C \) and \( M \) are independent, \( P^d_{Fc} = W \), and no wage policy is needed. Moreover, for "small" wage distortions, full employment is desirable; however, as argued earlier, large departures of the wage floor from the competitive equilibrium wage rate require some unemployment in order to reduce the expected wage rate to workers, and hence to hold down labor supply.
IV. Conclusion

Conventional wisdom states that wage subsidies should be used to eliminate the distortions caused by wage rigidities. However, as we have argued, the success of this policy depends upon the crucial assumption that the return to labor is a pure rent. If labor supply is assumed endogenous, then the first best policy must consist of wage subsidies to employers and wage taxes on employees. It should be noted that this policy combination works by assuming away the problem, since the (rigid) wage is not paid or received by any economic agent.

When the first best solution is unattainable, the standard presumption in favor of free trade is lost. The optimal policies in this case depend upon the relationships among the goods and leisure, and could consist of interferences with free trade and wage subsidies or taxes to employers. While the model we have employed is a rather special one, we feel that the qualitative results would hold in the standard two-good model. Thus, the optimal responses to domestic distortions depend upon both the behavioral assumptions and the tools available to the central authorities.
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FOOTNOTES

1 For an excellent recent survey of this literature, see Magee (1973). 
2 For example, see Bhagwati and Ramaswami (1963). 
3 Our conclusions would not be significantly altered if we assumed there were more than one factor of production. 
4 We assume that all workers of the same type have identical and homothetic preferences and are treated identically. However, since different types of workers may face different wage rates, we cannot represent society's preferences with a community indifference curve. 
5 We normalize by assuming there is only "one" worker of each type. This does not alter our result, given that the number of workers in each sector is fixed. 
6 In most models used in this literature, it is assumed that a community indifference curve can represent society's preferences. Thus, a change in the distribution of utility will have no effect on prices, aggregate production or consumption. However, this is not the case in our model. 
7 Given technology as in (1) and profit maximization, firms in each sector will earn positive profits; these may be viewed as a return (or rent) to some fixed factor, such as land. We assume these profits are redistributed to consumer-workers, but that each consumer views these transfers as exogenously determined. 
8 Normally, the wage floor is assumed to be caused by a change in the terms of trade. The proper solution is independent of the cause of this wage floor. 
9 If the supply of labor is fixed, the optimal subsidy is \( \hat{W}^c - \hat{W}^c \), and the financing of the subsidy does not affect labor supply. In our model, the quantity of labor supplied depends on the net wage to workers and on fixed transfer payments. Thus, the subsidy needed to restore full employment depends on the method of financing the subsidy and the slope of the labor supply curve.
We assume all workers have identical and homothetic preferences and are treated identically, so that a community indifference curve can be used to represent these preferences. Alternative methods of determining the appropriate distribution of transfers include: (i) postulating a Social Welfare Function and using transfers as a control variable; (ii) assuming transfers must be allocated in specific proportions to each type of worker. This latter rule is arbitrary and is likely to be inefficient.

Uniqueness is guaranteed by the existence of a community indifference curve; we assume this equilibrium is stable. Also, we again normalize by assuming there is only "one" worker of type c.

We assume that the government cannot levy income taxes; otherwise, the first best solution is attainable. Note that lump sum taxes or transfers will be ineffective (since all workers are identical) unless some of the taxes are given away as a Balance of Trade surplus. This, however, will never be optimal.

For the remainder of the paper, we drop the superscripts that identify the type of worker.

For a similar treatment, see Harris and Todaro (1970). The assumption workers respond to \( W^e \), and not \( W \), is not crucial to our main results. If they base their decisions on \( W \), then unemployment would never be desirable, provided \( F'_c > 0 \).

Alternatively, we could derive consumer demand and (labor) supply decisions as functions of \( P^d \), \( W \), and \( u \), and then derive optimal policies, given these behavioral assumptions. The two methods give the same results; we believe the approach in the text conforms more closely with the literature.

From (14), if \( Q_2 = 0 \), then \( Q_3 = 0 \); from (16), \( \lambda > 0 \) and thus \( Q_1 > 0 \) (\( u = 0 \)). Substitution in (16) through (18) yields \( [U_c/U_m] = P, [U_L/U_m] = PF'_c \). But \( W > W^* \) and \( P^d = P \) implies that labor supply \( L \) exceeds \( L^* \), labor supply in the competitive equilibrium. Therefore, \( PF'_c(L) < W^* < W \), which violates the constraint \( [U_L/U_m] = W \).
Similarly, it can be shown that $\lambda$ and $Q_3$ must be positive.

These results are derived by maximizing $U(C, M, -L)$ subject to:

$$W(l-u) L + T - P^d C - M \geq 0,$$

where $T$ is transfer payments. By totally differentiating the FOC, we get the Bordered Hessian; inverting this yields the slopes of the (normal and compensated) demand curves. The computations are omitted to save space.

Since leisure ($-L$), not labor, is the "good" in the utility function.

Only two of these conditions are independent.

Even though the slope of the normal labor supply curve may be ambiguous, the wage floor must increase $L$ (given $P^d$) since: (i) the compensated supply elasticity is positive; and (ii) the wage floor lowers utility, thereby increasing $L$, provided leisure is a normal good.

If $C$ and Leisure are independent, changes in $P^d$ have no effect on the compensated labor supply, and the wage floor has no effect on the compensated demand for $C$. Thus, it is optimal to let $P^d = P$.

If $C$ and $M$ are independent, $P^d$ has no effect on the compensated demand for $M$, so it is optimal to insure the equality between the MRS and MRT of $C$ for leisure (hence, $P^d_{C'} = W$).

This can be shown by performing a series of elementary row and column operations on the adjoint matrix of the Bordered Hessian derived from the consumer utility maximum problem (footnote 17). The proof is omitted to save space.

For $u > 0$, $W$ decreases as $W$ increases, thereby decreasing compensated labor supply. However, utility decreases as $W$ increases, thereby increasing labor supply. In our example, these effects just offset each other, so that $L$ is constant.
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