Physical characteristics of dynamic vertical–horizontal-rocking response of surface foundations on cohesionless soils

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Abstract
A fundamental experimental research programme on the dynamic behaviour of surface foundations on sand in general planar motion and the use of centrifuge modelling in soil–structure interaction studies is presented. Pursued with the dual purpose of extending the conventional formulation of dynamic test programmes as well as generating a physical database with sufficient parametric variations of the key aspects, an extensive experimental study to explore the dynamic soil–structure interaction problem in the small-amplitude regime is described. Through a comparison of the generated data with those from sequential vertical-centric and horizontal load tests, a novel hybrid-mode test concept by way of eccentric excitations is substantiated in terms of its economy and efficiency in capturing the force–response characteristics of the system. Synthesised in the frequency domain for direct qualitative and quantitative insights, multiple forced-response records of the foundation models on sand subjected to random vertical, horizontal and rocking excitations are summarised. As an illustration of the engineering relevance of the experimental database, a critical evaluation of the commonly used homogeneous half-space dynamic foundation solution pertaining to cohesionless soils is also provided.

Keywords
soil/structure interaction, dynamics, vibration, centrifuge modelling, sands

Disciplines
Geotechnical Engineering

Comments

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R. Y. S. PAK*, J. C. ASHLOCK†, S. KURAHASHI‡ and M. SOUDKHAH§

INTRODUCTION

A rational understanding of the forced vibration response of a rigid footing on soil is fundamental to many dynamic problems in geotechnical and earthquake engineering (Richart et al., 1970). Not only will it advance dynamic foundation design relevant to industrial applications, a mechanistic resolution of the underlying physical aspects will be helpful to a proper evaluation of dynamic soil–structure interaction effects in earthquake engineering, vibration isolation, dynamic plate-load tests and various wave-based in situ sounding methods (Moore, 1985; Clough & Penzien 1993). Prompted by the engineering significance of this class of dynamic problems, there have been a number of rigorous mathematical solutions as in Reissner (1936), Robertson (1966), Kobori et al. (1971), Luco & Westmann (1971) and Pak & Gobert (1991) as well as approximate methods such as Dobry & Gazetas (1986) and Wolf & Meek (1994). While these elastodynamic solutions have been the cornerstone for many design formulae and have been shown to be useful in a number of circumstances, their performance in problems involving sand or gravelly soils has generally been less than favourable. In a number of experimental field studies for both surface and embedded foundations for instance, significant discrepancies between theory and experiment were found in both response magnitude and resonance frequencies in vertical against lateral motions (e.g. Fry 1963; Novak & Beredugo, 1972; Stokoe & Richart, 1974; Wong et al., 1977; Lin & Jennings, 1984; Crouse & Hushmand 1989; Crouse et al. 1990; Gazetas & Stokoe, 1991) even under special uniform conditions (e.g. Erden, 1974). Because of the inevitable lack of full control or knowledge of soil properties and the spectrum of loading, however, these observed deviations from the fundamental continuum theory have often been accepted as part of the normal difficulty of real geotechnical engineering rather than as a sign of some critical deficiencies in current conceptual paradigms.

For an in-depth exploration of the physical nature of the dynamic soil–structure interaction problem and the corresponding analytical ramifications, a comprehensive experi-

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mental database which can illuminate the underlying issues in a fundamental physical setting would clearly be helpful. In this paper, a systematic experimental investigation of the dynamic footing problem by the centrifuge modelling approach is reported. Focused on surface foundations, a set of footing models was designed and tested on a large uniform sand sample. By means of an experimental set-up wherein three forms of directional dynamic excitations can be applied, hundreds of ambient and forced vibration tests in both the traditional and a novel hybrid-mode format (Pak et al., 2006) were conducted to explore and cross-validate the key features of the foundation response in general planar motion. As a significant extension of the results of Pak & Guzina (1995), a detailed presentation of the dynamic test data on the response of rigid square foundations under both symmetric and asymmetric excitations for a range of prototype configurations is provided. A critical comparison of the experimental results with the commonly used homogeneous half-space dynamic foundation solution is also included for an advanced understanding of this class of problems.

EXPERIMENTAL SET-UP

Centrifuge, soil and container

The experimental testing programme was carried out using the 440-ton centrifuge at the University of Colorado at Boulder (Ko, 1988). As in Lenke et al. (1991), Pak & Guzina (1995), Pak & Ashlock (2000) and Pak et al. (2003), a fine dry silica F-75 Ottawa sand with a specific gravity of 2.65 was used. Specimens were prepared by pluviation to a fine dry silica F-75 Ottawa sand with a specific gravity of 2.65 was used. Specimens were prepared by pluviation to a uniform bulk density of 1730 kg/m³, void ratio of e = 0.53 and relative density of Dr = 86%. To simulate the half-space condition experimentally, a large rectangular container having a plan dimension of 1.2 m by 1.0 m and a depth of 0.61 m was used. As a useful measure to mitigate the wave reflection and dynamic boundary effects owing to the finite soil model (Pak & Guzina, 1995), a 35 mm thick panel of duct seal, an oil-based putty, was placed on the inner walls of the container (see also Coe et al. (1985) and Lenke et al. (1991)). As reported in Pak et al. (2008), the effective inhomogeneous free-field small-strain shear modulus profile for the soil model can be described by

\[ G(z) = 53.8 \text{MPa} \left( \frac{Nz}{\text{m}} \right) \]  

and a Poisson’s ratio of 0.25, with z being the depth and N the centrifugal g-level at the model scale.

Scaled-model foundations

To obtain an experimental database for exploring the dynamic soil–foundation interaction problem parametrically, four solid block footings of the same square footprint were fabricated (see Fig. 1). Of increasing weight and height, they will be referred to as footings B0, B13, B23 and B33 respectively. In each experiment, a single model footing was placed on the sand using a guidance system before spinning, then tested at the nominal centrifugal acceleration levels of 33, 44, 55 and 66g. The models were intended to cover a significant range of practical bearing pressure and footing size by virtue of the scaling relationships in centrifuge modelling (see Appendix). Accordingly, the results of testing four model footings at the four g-levels correspond to 16 different combinations of prototype footing half-width bpr and average contact pressure pr (see Table 1).

For measuring force and acceleration, threaded instrumentation holes were prepared in a central vertical plane, with two out-of-plane holes located ± 18 mm from the top-centre of the three smaller foundations. The footing dimensions, instrumentation hole locations and inertial properties are given in Fig. 2 and Table 2. The instrumentation configurations used in this study and the resulting foundation inertial properties are summarised in Table 3.

Excitation and instrumentation systems

Owing to the inverse scaling relation between frequency and g-level in centrifuge modelling and the desire to achieve a full characterisation of the dynamic foundation response in the frequency domain, the means to generate and control a broad range of frequencies is critical to the investigation. For vertical loads, this was accomplished by a Bruel & Kjær (B&K) 4809 electromagnetic exciter which has a peak force rating of 44.5 N and a 60 g moving armature. For horizontal excitation forces, a smaller B&K 4810 mini-shaker with a peak force rating of 10 N and an 18 g moving element was found to be effective. The applied force and resulting foundation accelerations were measured using Kistler Model 9001 and 9001A quartz piezoelectric load washers and two to four stud-mounted PCB Piezotronics Model 303-B67 and 352-C67 ceramic shear accelerometers. Both load cells have a mass of 3 g and a measurement range of 7.5 kN. Each was integrated into a stinger assembly consisting of a small hemispherical button and a load-distribution housing (see Fig. 1). The load stingers were mounted at selected holes on the foundation for the direct measurement of the applied excitation force. The charge signals from the load cells were amplified and conditioned by Kistler Type 5010A and Type 5004 charge amplifiers. The accelerometers used have a measurement range of ± 50g, a resolution of 0.0003g rms, low cross-sensitivities of less than 1.5% and a mass of approximately 2.5 g when accounting for approximately 50 mm of the attached wires. The power to the accelerometers and signal conditioning were provided by a PCB Model 483A signal conditioner mounted on the centrifuge arm. To ensure uni-axial loading of the footings, special care was taken to minimise the transverse frictional resistance at the contact point. This was accomplished by the use of a

![Fig. 1. Scale model footings and instrumentation](image-url)

Table 1. Average prototype footing contact pressures pr (kPa) for instrumented model footings

<table>
<thead>
<tr>
<th>Footing</th>
<th>Prototype footing half-width bpr (m)</th>
<th>0.9 m</th>
<th>1.2 m</th>
<th>1.5 m</th>
<th>1.8 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>10.0</td>
<td>13.4</td>
<td>16.7</td>
<td>20.1</td>
<td></td>
</tr>
<tr>
<td>B13</td>
<td>43.6</td>
<td>58.1</td>
<td>72.7</td>
<td>87.2</td>
<td></td>
</tr>
<tr>
<td>B23</td>
<td>84.0</td>
<td>112.1</td>
<td>140.1</td>
<td>168.1</td>
<td></td>
</tr>
<tr>
<td>B33</td>
<td>126.7</td>
<td>169.0</td>
<td>211.2</td>
<td>253.4</td>
<td></td>
</tr>
</tbody>
</table>
small oil-filled fluid bearing in vertical tests, and Teflon lubricant applied to the stinger’s button for horizontal excitation. In all tests except those for the light footing B0, a DC current was used on occasion to control the equilibrium position of the moving armature to minimise the level of static pre-stress and friction.

**Data acquisition systems and measurement approach**

Force and acceleration measurements were recorded using a pair of Spectral Dynamics model 20-42 dynamic signal analysers with a 20 kHz bandwidth, controlled by way of the SigLab virtual instrument suite in Matlab. Each analyser can provide measurement and digitisation of four input signals and can generate two independent analogue output signals. To obtain the foundation’s frequency response over the entire bandwidth of interest with efficiency, random force excitations having uniform spectral distributions were used. In a typical test, the time-domain signals were recorded and synthesised in the form of auto/power-spectral densities (ASD), coherence functions (COH) and, most importantly, transfer/frequency–response functions (FRF) which relate the acceleration response at different locations to the forcing. Referred to as the ‘accelerance’ function (Pak & Guzina, 1995), the particular FRF of interest in this study is defined as the ratio of the Fourier transforms of the acceleration to the force, and can be evaluated effectively as

\[ A(f) = \frac{G_{fs}(f)}{G_{ff}(f)} \]  

where \( f \) is the frequency, and \( G_{fs}(f) \) and \( G_{ff}(f) \) are the cross- and auto-spectral densities of the particular system output acceleration \( a(t) \) of interest and the applied input

---

**Table 2. Physical properties of model footings (width \( 2b = 55.0 \) mm)**

<table>
<thead>
<tr>
<th>Footing name</th>
<th>Height: mm</th>
<th>Top holes ( e ): mm</th>
<th>Side holes ( h_0 ): mm</th>
<th>Side holes ( \Delta h ): mm</th>
<th>Mass: g</th>
<th>Mom. of inertia: kg mm²</th>
<th>Centroid height: mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0</td>
<td>7.5</td>
<td>18.0</td>
<td>–</td>
<td>–</td>
<td>63.23</td>
<td>16.23</td>
<td>3.745</td>
</tr>
<tr>
<td>B13</td>
<td>45.0</td>
<td>18.0</td>
<td>5.0</td>
<td>10.0</td>
<td>380.70</td>
<td>159.30</td>
<td>22.47</td>
</tr>
<tr>
<td>B23</td>
<td>90.0</td>
<td>18.0</td>
<td>20.0</td>
<td>20.0</td>
<td>764.40</td>
<td>704.20</td>
<td>44.95</td>
</tr>
<tr>
<td>B33</td>
<td>137.5</td>
<td>17.875</td>
<td>45.375</td>
<td>20.625</td>
<td>1173.00</td>
<td>2147.00</td>
<td>68.70</td>
</tr>
</tbody>
</table>

**Table 3. Inertial configurations of model footings with instrumentation**

<table>
<thead>
<tr>
<th>Inertial configuration name</th>
<th>Footing</th>
<th>Load cell hole no.</th>
<th>Accel. hole no.</th>
<th>Mass: g</th>
<th>Mom. of inertia: kg mm²</th>
<th>Centroid coords.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( x_C ): mm ( y_C ): mm</td>
</tr>
<tr>
<td>B0-a</td>
<td>B0</td>
<td>1, 2</td>
<td>3, 4, 5</td>
<td>94.5</td>
<td>24.15</td>
<td>–1.821 7.46</td>
</tr>
<tr>
<td>B13-a</td>
<td>B13</td>
<td>5, 6</td>
<td>3, 8, 9</td>
<td>411.1</td>
<td>190.05</td>
<td>–0.326 24.467</td>
</tr>
<tr>
<td>B13-b</td>
<td>B13</td>
<td>7, 11</td>
<td>1, 3, 6</td>
<td>411.0</td>
<td>197.95</td>
<td>–0.110 23.645</td>
</tr>
<tr>
<td>B23-a</td>
<td>B23</td>
<td>5, 6</td>
<td>3, 8, 9</td>
<td>792.1</td>
<td>786.31</td>
<td>–0.017 46.825</td>
</tr>
<tr>
<td>B23-b</td>
<td>B23</td>
<td>7, 11</td>
<td>1, 3, 6</td>
<td>792.1</td>
<td>774.53</td>
<td>–0.057 46.246</td>
</tr>
<tr>
<td>B33-a</td>
<td>B33</td>
<td>8, 11</td>
<td>2, 5, 7</td>
<td>1194.3</td>
<td>2241.71</td>
<td>–0.039 69.847</td>
</tr>
</tbody>
</table>

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Fig. 2. Instrumentation positions and centroidal location \( C \) of model foundations
force excitation $F(t)$ (see Bendat & Piersol, 1986). As is customary in frequency-domain presentation of dynamic data and theories (see Wolf, 1985), the FRFs are presented in complex notation, with the real (Re) and imaginary (Im) parts denoting the physical in-phase and 90° out-of-phase components, respectively. They can be converted into a magnitude (Mag) and a phase angle ($\phi$) by way of the formulae $\text{Mag} = \sqrt{(\text{Re})^2 + (\text{Im})^2}$ and $\phi = \tan^{-1}(\text{Im}/\text{Re})$. The phase angle and its frequency-domain representation can, however, be very sensitive to noise and the user’s analytical assumptions regarding its continuity. To minimise the effects of extraneous noise and uncorrelated components in the response measurements, an ensemble average of 30 transfer functions was taken in each test, with Hanning windowing employed to reduce the effects of spectral leakage.

In the dynamic testing, 4096 samples and a sampling rate of 2.56 times the measurement bandwidth were the standard, giving 1600 alias-free frequency lines. Typically, a bandwidth of 5 kHz was used, yielding a sampling interval of $\Delta t = 78.125 \mu$s and a frequency resolution of $\Delta f = 3.125$ Hz. For enhanced resolution of the low- to mid-frequency region, a bandwidth of 2 kHz was also employed on occasion, leading to a longer sampling duration of $T = 0.8$ s for each time-history, with a finer frequency spacing of $\Delta f = 1.25$ Hz.

**DYNAMIC TEST METHODOLOGY**

To explore the vertical, horizontal and rocking response characteristics of the foundations experimentally, three dynamic loading configurations were employed for enhanced collaboration and experimental validation: (a) the vertical-centric (VC) test, (b) the horizontal-centric (HC) test and (c) the vertical–eccentric (VE) test. A composite schematic diagram of the three loading configurations is shown in Fig. 3. In a VC test, the force is applied vertically at the centre of the foundation with the goal of provoking only the symmetric vertical response. In the HC test, the force is applied in a horizontal direction on the central plane of the footing, resulting in a loading which is equivalent to a vertical–eccentric force in conjunction with a rocking moment with no horizontal resultant. Because of the lateral rocking foundation impedance coupling and inertial effects, however, significant composite dynamic vertical–horizontal rocking motions will generally be produced as a result of the VE loading. Depending on the chosen point of force excitation, the test can provoke synchronous participation of multiple vibration modes of interest in targeted proportions.

As the purpose of this study was to characterise fully the three degrees of freedom of the foundations in general planar motion, the applied force and the resulting horizontal and vertical accelerations at three or more non-collinear points were measured and synthesised as the directional accelerance functions described earlier. To distinguish these directional FRFs, the notation ‘AA/BB’ (AA, BB = VC, HC, VE) will be used, where the prefix and suffix denote the response and excitation types, respectively. Focused on the case of small-amplitude vibration response and soil modulus condition in this experimental programme, the scope and contents of the database generated are detailed in the following sections.

**EXPERIMENTAL RESULTS**

To provide a comprehensive database for the family of multi-modal foundation vibration problems addressed in this study, over 200 VC, 100 HC and 400 VE forced-vibration tests were performed, with ambient vibration tests performed periodically to monitor the conditions of the soil sample and testing environment. The key experimental findings can be grouped as below.

**Ambient vibration**

To interpret forced vibration test results properly, it is useful first to ascertain the ambient condition of the test environment, which is seldom totally quiescent. This is particularly true in centrifuge chambers, where turbulence-induced vibration of the hinged platform and continuous air pressure fluctuations as a result of the spinning motion are practically unavoidable and differ only in degree. Although suchambient excitations may be small, they can generate model motions through direct wind loading on the foundation as well as the induced centrifuge arm–platform–container vibration. To monitor the level of such disturbances and utilise the measurements to enhance the insights of the dynamic study, the acceleration response of all model footings before any contact with the exciters during centrifuge spinning were recorded and processed in the form of ambient-acceleration auto-spectral densities (ASD). An example of such ASD measurements from an HC accelerometer on footing B33 under the ambient condition at 66g is shown in Fig. 4(a). Included in the figure are the force and acceleration ASD measurements for an immediate follow-up forced-vibration HC test. As illustrated by the results, the ASD of the foundation’s HC acceleration response under ambient conditions is primarily in the low-frequency range (below 500 Hz), with a sharp resonant peak at $f = 170$ Hz. In the corresponding forced-vibration HC test which will be discussed later, the peak frequencies of the HC acceleration and force ASDs are 172.5 Hz and 96.3 Hz, respectively (Figs 4(a) and 4(b)). In Figs 5(a) and 5(b) where the resulting HC/HC acceleration function is shown, one can see a sharp fundamental peak frequency of 167.5 Hz. This frequency corresponds well to the 170 Hz ambient ASD peak seen in Fig. 4(a). Also shown in Fig. 5(c) is the

![Fig. 3. VC, HC and VE loading–response configurations](image-url)
coherence function between the acceleration and the load-cell measurement in the forced-vibration test. Such functions are generally affected by the presence of ambient vibration as well as the relative level and frequency distribution of the forced excitation. Depending on the type of tests being run, the coherence function can also show slightly different characteristics in VC, HC and VE test measurements. As will be illustrated by the results in the ensuing sections, however, the high number of ensemble averages employed was effective in providing a sufficiently sharp resolution of the FRFs in most cases.

**Vertical-centric (VC) excitation**

The symmetric vertical response of footings is often useful as a benchmark vibration mode in dynamic soil-foundation interaction. Some typical accelerance functions from such VC excitations of the heaviest footing B33 are shown in Fig. 6(a) at prototype scale for the four centrifugal levels of 33, 44, 55 and 66\(g\), corresponding to the prototype foundation half-widths \(b_{pr}\) of 0.9 m, 1.2 m, 1.5 m and 1.8 m respectively. These VC/VC FRFs are characterised by a broad peak in the range 10–20 Hz, beyond which the complex magnitude of the response reaches a horizontal asymptote proportional to the reciprocal of the mass \(m_{pr} = N^3 m\) of the prototype footing, where \(N\) is the centrifugal \(g\)-level and \(m\) is the mass of the model footing (see Table 2 and Appendix 1). One can also see that the resonant regime for the footing gradually shifts to higher frequency as the prototype footing size and mass decrease. The same trends are observed for the lighter footings B23, B13 and B0, an example of which is shown in Fig. 6(b). As summarised in Fig. 7 where the VC accelerances of all four footings having the same prototype width but different prototype masses are plotted, the peak response for a lighter foundation clearly becomes broader and higher in frequency, and has a higher asymptotic value as \(f \to \infty\).

**Horizontal-centric (HC) excitation**

For the characterisation of dynamic lateral motions of a foundation, horizontal excitation has been most commonly used in experimental studies. Such results were also obtained in this study by way of the HC test format. In prototype scale, representative horizontal accelerances at different points on footings B33 and B13 are shown in Fig. 5. HC/HC accelerance and coherence function at model scale from forced horizontal vibration test of footing B33 at 66\(g\)

![Fig. 4. Auto-spectral densities of (a) HC acceleration (b) and HC force during ambient and forced horizontal vibration tests of footing B33 at 66\(g\) at model scale](image)

![Fig. 5. HC/HC acceleration and coherence function at model scale from forced horizontal vibration test of footing B33 at 66\(g\)](image)

coherence function between the acceleration and the load-cell measurement in the forced-vibration test. Such functions are generally affected by the presence of ambient vibration as well as the relative level and frequency distribution of the forced excitation. Depending on the type of tests being run, the coherence function can also show slightly different characteristics in VC, HC and VE test measurements. As will be illustrated by the results in the ensuing sections, however, the high number of ensemble averages employed was effective in providing a sufficiently sharp resolution of the FRFs in most cases.

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The symmetric vertical response of footings is often useful as a benchmark vibration mode in dynamic soil-foundation interaction. Some typical accelerance functions from such VC excitations of the heaviest footing B33 are shown in Fig. 6(a) at prototype scale for the four centrifugal levels of 33, 44, 55 and 66\(g\), corresponding to the prototype

![Fig. 6. Representative VC/VC prototype-scaled accelerances (mm/s²/kN) in vertical-centric tests for (a) footing B33 and (b) footing B13 at 33, 44, 55 and 66\(g\)](image)
Fig. 8 for HC forcing. In contrast to VC tests, the HC/HC accelerance generally has two prominent features: a very sharp resonance peak at low frequency associated with the primary rocking mode (see the close-ups in Fig. 8), followed by a gentler peak in the mid-frequency range. Similar to the VC/VC accelerances, these transfer functions are seen to exhibit a gradual shift towards higher frequencies with decreasing footing size and mass. While being dependent on the location of forcing and measurements, they all have horizontal asymptotes that are inversely related to the footing's mass and moment of inertia. Owing to the difficulty of installing a horizontal accelerometer or force sensor on the side of the thin plate footing, an HC test series was not pursued for footing B0. The effect of footing mass and static contact pressure on the HC test response for a given prototype footing width is illustrated in Fig. 9 for footings B33, B23 and B13 at the same g-level. As shown in this figure, the low-frequency peak increases in frequency with decreasing footing mass, moment of inertia and static contact pressure.

Vertical–eccentric (VE) excitation

Following the hybrid-mode test concept in Pak et al. (2006), multiple series of VE tests were performed. The VE tests provide an enriched form of response data for the dynamic soil–foundation interaction problem and form a basis to evaluate the effectiveness of hybrid simultaneous vertical–horizontal rocking vibration testing. A typical set of vertical and horizontal accelerances at prototype scale are shown in Fig. 10 for footing B33. One can see that the VC/VE accelerance is essentially identical with the VC/VC accelerance shown in Fig. 6(a): it is characterised by a single, gentle peak in the mid-frequency range followed by a horizontal asymptote which can be shown to coincide with the reciprocal of the foundation’s static mass. Comparable with the basic features of HC/HC measurements in Fig. 8(a), the lateral rocking mode’s HC/VE accelerances in Fig. 10(b) exhibit a strong low-frequency fundamental peak, followed by a broader crest at a higher frequency. It is worth noting that the horizontal acceleration-to-force ratios shown in Fig. 8(a) under HC forcing are noticeably higher than those of Fig. 10(b) under VE forcing. This is attributed to the expected effectiveness of a horizontal force in producing horizontal motion. The VE test, however, induces a horizontal response of sufficient strength to provide a clear picture of the transfer function as well.

Representative VC/VE and HC/VE accelerances from all
four g-levels are shown in Figs 10–12 for footings B33, B23 and B13 at prototype scale. The peaks and asymptotes of these vertical and horizontal responses exhibit the same trends with decreasing prototype footing size and mass as were described earlier in the cases of VC and HC tests. For example, the accelerances of the lighter footing B23 (Fig. 11) are similar in shape to those of the heavier footing B33, but have higher peak frequencies and asymptotic magnitudes. These trends continue for the smaller footing B13 (Fig. 12), which also has a broader first HC/VE peak than the two larger footings. Comparing the horizontal responses of footing B13 measured under vertical–eccentric against horizontal forcing (e.g. Figs 12(b) and 8(b)), one will find that the hybrid-mode VE test configurations generally provide higher-quality lateral rocking accelerance measurements than the HC tests in terms of their smoothness and definition. As mentioned earlier, HC tests were not performed on the lightweight footing B0. Its rocking response characteristics can, however, be distilled from the difference between the VC and VE responses shown in Figs 13(a) and 13(b) respectively.

EXPERIMENTAL AND ANALYTICAL IMPLICATIONS

Performance of dynamic test methodologies

As described in the preceding sections, the dynamic vertical–horizontal rocking responses are well characterised by the VC, HC and VE test formats. To clarify their correspondence, comparisons between vertical responses to VC against VE forcing as well as the horizontal responses under HC against VE forcing are shown in Figs 14 and 15 for footings B33 and B13 at 33g. Consistent with theoretical expectations, the major features such as the location and progression of the broad peak of the vertical (VC) accelerance are seen to be practically identical under VC and VE forcing for both of these footings. For the coupled lateral rocking modes under HC against VE forcing, comparable characteristics can also be observed such as the frequency of the sharp resonance. Similar trends were observed for the intermediate footing B23. While the HC/HC and HC/VE functions have slightly different profiles at higher frequencies, it can be shown that this is largely owing to the difference in the location and direction of the applied force excitations in the two types of tests. At other g-levels, similar agreement was also observed between the VC, HC and VE test responses of the three footings. Owing to the low profile of the thin plate B0, HC tests were not performed on it as noted previously. However, the agreement between the VC responses measured under VC and VE excitations is also excellent for this footing (see Fig. 16).

With the strong correspondence of the VC and HC tests to VE tests, the advantages of the VE test format should be appreciated. First, the hybrid-mode approach can deliver the essence of all three fundamental modes of vibra-
tion in a single test. Second, by being able to extract the multi-mode data simultaneously from the foundation in one test, the common question and concern of cumulative effects of load history from sequential single-mode tests in a soil–structure interaction problem can be greatly alleviated. Third, as demonstrated in the case of footing B0, there are occasions where certain test configurations or combinations (e.g. a HC test on a thin plate) are difficult to implement, and a VE load test can be a competent and efficient option.

**Theoretical implications of experimental data**

Using the experimental data discussed above, new or existing analytical theories can be examined for their validity and limitations in modelling the dynamic behaviour of surface foundations on cohesionless soils under the most basic of physical conditions. To this end, the performance of the classical continuum theory for a rigid footing resting on a homogeneous elastic half-space (e.g. Luco & Westmann, 1971; Pak & Gobert, 1991), which has been the basis of many current formulas and engineering interpretation of dynamic soil–structure behaviours, is of particular interest. Computed rigorously for the square foundation geometry, the frequency-dependent foundation impedance functions $K_{ij}(\omega)$ used in the ensuing assessment are given in Table 4 for completeness.

Beginning with the vertical mode of vibration and covering a significant range of soil bearing pressure, the results

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**Fig. 12.** Representative prototype-scaled accelerances (mm/s²/kN) in vertical-eccentric tests of footing B13 at 33, 44, 55 and 66g: (a) VC/VE and (b) HC/VE

**Fig. 13.** Representative prototype-scaled accelerances (mm/s²/kN) in vertical-eccentric tests of footing B0 at 33, 44, 55 and 66g: (a) VC/VE and (b) VE/VE

**Fig. 14.** Comparison of accelerance (mm/s²/kN) by VC, VE and HC tests: (a) VC/VC against VC/VE and (b) HC/HC against HC/VE responses of footing B33 at 33g, $b_p = 0.9$ m
of such comparisons for footings B33 and B13 are shown in Figs 17(a) and 18(a) at prototype scale as illustrations. Similar trends were observed for the intermediate footing B23. As can be seen from the displays, it is indeed possible to find an equivalent homogeneous modulus $G_{\text{eq.hom.}}$ by which a reasonable match can be realised between the theoretical and experimental VC accelerances for the square foundations, as in Pak & Guzina (1995) for circular footings. However, the required homogeneous shear moduli are strongly dependent on the foundation configuration, varying from approximately 80 MPa for footing B33, to 71 MPa for footing B23, to 57 MPa for footing B13. Confirmed by the multi-directional response data from the experiments, however, is a much more critical problem of the homogeneous half-space theory for the coupled horizontal rocking modes. Figs 17(b) and 18(b) for footings B33 and B13 illustrate well the typical situation in comparing the measured HC response of the footings with the equivalent VC/VE accelerances.

Fig. 15. Comparison of accelerance (mm/s²/kN) by VC, VE and HC tests: (a) VC/VC against VC/VE and (b) HC/HC against VC/VE response of footing B13 at 33g, $b_{pr} = 0.9$ m

Fig. 16. Comparison of accelerance (mm/s²/kN) by VC and VE tests of footing B0 at 33g: VC/VC against VC/VE responses, $b_{pr} = 0.9$ m

Fig. 17. Theoretical homogeneous half-space accelerances (mm/s²/kN) with Poisson’s ratio $v = 0.25$ calibrated to fit measured VC/VE response of footing B33 at prototype scale (ten tests): (a) VC/VE accelerance, (b) HC/VE accelerance

Fig. 18. Theoretical homogeneous half-space accelerances (mm/s²/kN) with Poisson’s ratio $v = 0.25$ calibrated to fit measured VC/VE response of footing B13 at prototype scale (12 tests): (a) VC/VE accelerance, (b) HC/VE accelerance

Fig. 19. Theoretical homogeneous half-space accelerances (mm/s²/kN) with Poisson’s ratio $v = 0.25$ calibrated to fit measured HC/VE response of footing B33 at prototype scale (ten tests): (a) VC/VE accelerance, (b) HC/VE accelerance
lent homogeneous half-space solution which uses the vertical-mode calibration of $G_{ehom}$. As can be seen from the displays, while the overall shape and trend of the theoretical and experimental HC/VE acceleration functions are comparable, both the sharp peak and the higher-frequency portion of the experimental acceleration curves lie noticeably to the left of (i.e. softer than) the theoretical predictions, as noted in Richart et al. (1970) and Hushmand (1983). As occasionally done in past studies (e.g. Hushmand, 1983; Gazetas & Stokoe, 1991), it can be tempting to ignore the vertical mode and determine a separate equivalent shear modulus $G_{ehom}$ so that the homogeneous half-space solution will more closely match the measured lateral rocking mode from physical testing. Such an approach is shown in Figs 19(b) and 20(b) for footings B33 and B13. While the agreement is now significantly improved between the theoretical solution and the measured HC data, the necessary shear moduli in the re-calibrated theoretical solution are found to be about 66, 54 and 40 MPa for footings B33, B23 and B13 respectively. These values are about 20–40% lower than the moduli of 80, 71 and 40 MPa for footings B33, B23 and B13 respectively. These values are about 20–40% lower than the moduli of 80, 71 and 45 MPa needed for a match of the vertical mode of vibration (see Figs 19(a) and 20(a)). The demand of such a serious reduction of the shear modulus of the soil reflects the inherent limitations of the classical homogeneous half-space theory in modelling an inherently inhomogeneous problem, and points to the need of a better conceptual platform beyond the prevalent depth-wise homogenisation or representative modulus approaches for general applications.

**SUMMARY**

In this paper, a fundamental experimental investigation of the dynamic behaviour of surface foundations on sand by means of centrifuge modelling is presented. Pursued with the dual purpose of extending the typical format of dynamic test programmes as well as generating a physical database with sufficient parametric variations of the key aspects, an experimental study to explore the dynamic soil–structure interaction problem in the small-amplitude regime is described in detail. Through a comparison of the generated data with those from sequential centripetal vertical and horizontal load tests in multiple series, the novel hybrid-mode test concept by way of eccentric excitations is substantiated in terms of its economy and efficiency in capturing the multi-directional dynamic characteristics of a foundation system. Synthesised in the frequency domain for enhanced physical insights, hundreds of forced response transfer functions of the foundation models undergoing vertical, horizontal and rocking motions are processed with cross-validation. As an illustration of the engineering significance of the new experimental database, its use in assessing the capability and limitations of the conventional homogeneous half-space theory when applied to sand is also demonstrated.

**APPENDIX**

*Key centrifuge scaling relations for dynamic modelling*

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>g-level</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>Length</td>
<td>N</td>
<td>1</td>
</tr>
<tr>
<td>Velocity</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Acceleration</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>Mass density</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mass</td>
<td>$N^3$</td>
<td>1</td>
</tr>
<tr>
<td>Force</td>
<td>$N^2$</td>
<td>1</td>
</tr>
<tr>
<td>Stress</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Strain</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Time (dynamic)</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Frequency</td>
<td>1</td>
<td>N</td>
</tr>
</tbody>
</table>

**Table 4. Foundation impedance functions for a rigid square footing of width 2b on homogeneous half-space with shear modulus G, density $\rho$, and Poisson’s ratio $\nu = 0.25$**

<table>
<thead>
<tr>
<th>$\omega b/\sqrt{G/\rho}$</th>
<th>$K_{ve}/Gb$</th>
<th>$K_{bh}/Gb$</th>
<th>$K_{mm}/Gb^3$</th>
<th>$(K_{mm} = K_{mh})/Gb^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.3909</td>
<td>5.3682</td>
<td>5.9456</td>
<td>0.6797</td>
</tr>
<tr>
<td>0.05</td>
<td>6.3916 + 0.3012i</td>
<td>5.3686 + 0.1881i</td>
<td>5.9412 + 0.0024i</td>
<td>0.6850 + 0.0208i</td>
</tr>
<tr>
<td>0.25</td>
<td>6.3715 + 1.4090i</td>
<td>5.3649 + 0.9336i</td>
<td>5.8267 + 0.0327i</td>
<td>0.7385 + 0.0581i</td>
</tr>
<tr>
<td>0.50</td>
<td>6.2745 + 2.9793i</td>
<td>5.3371 + 1.8632i</td>
<td>5.5244 + 0.2026i</td>
<td>0.8053 + 0.0401i</td>
</tr>
<tr>
<td>0.75</td>
<td>6.0991 + 4.5033i</td>
<td>5.2830 + 2.8060i</td>
<td>5.1680 + 0.5464i</td>
<td>0.8482 + 0.0080i</td>
</tr>
<tr>
<td>1.00</td>
<td>5.8601 + 6.0929i</td>
<td>5.2124 + 3.7745i</td>
<td>4.8361 + 1.0167i</td>
<td>0.8720 + 0.0659i</td>
</tr>
<tr>
<td>1.50</td>
<td>5.1105 + 9.5487i</td>
<td>5.0796 + 5.7958i</td>
<td>4.3010 + 2.1489i</td>
<td>0.8788 + 0.2004i</td>
</tr>
<tr>
<td>2.00</td>
<td>4.9191 + 13.3483i</td>
<td>5.0218 + 7.8738i</td>
<td>3.8944 + 3.4049i</td>
<td>0.8145 + 0.3481i</td>
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<tr>
<td>2.50</td>
<td>4.8619 + 17.2477i</td>
<td>5.0308 + 9.9235i</td>
<td>3.5864 + 4.7404i</td>
<td>0.6673 + 0.4558i</td>
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<tr>
<td>3.00</td>
<td>5.0494 + 21.0002i</td>
<td>5.0402 + 11.9175i</td>
<td>3.3805 + 6.1289i</td>
<td>0.4780 – 0.4708i</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENT

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REFERENCES


Pak, R. Y. S. & Ashlock, J. C. (2000). Fundamental dynamic behavior of foundations on sand. In Soil dynamics and liquefac-


Reissner, E. (1936). Station are axialsymmetricle druch eine elas-
tischen halb raues. Ingenieur Archiv 7, No. 6, 381–396.


