comparison of Theory and Experiment for Ultrasonic Scattering from Spherical and Flat Bottom Cavities

Bernard R. Tittmann
Rockwell International

Follow this and additional works at: http://lib.dr.iastate.edu/cnde_yellowjackets_1975

Part of the Materials Science and Engineering Commons, and the Structures and Materials Commons

Recommended Citation
http://lib.dr.iastate.edu/cnde_yellowjackets_1975/14

This 4. Ultrasonic Scattering 2 is brought to you for free and open access by the Interdisciplinary Program for Quantitative Flaw Definition Annual Reports at Iowa State University Digital Repository. It has been accepted for inclusion in Proceedings of the ARPA/AFML Review of Quantitative NDE, June 1974–July 1975 by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
comparison of Theory and Experiment for Ultrasonic Scattering from Spherical and Flat Bottom Cavities

Abstract
Earlier in the day you heard several talks on the analytical techniques to treat defect scattering. I would like to try to discuss what amounts to two procedures which should hopefully help in not only testing some of these results but also to reduce these techniques to practice, that is to say, with a view toward obtaining defect signatures in the field. The thread that should hopefully be running through my talk is how to develop procedures for applying present scattering results to laboratory field practice.

Disciplines
Materials Science and Engineering | Structures and Materials
COMPARISON OF THEORY AND EXPERIMENT FOR ULTRASONIC
SCATTERING FROM SPHERICAL AND FLAT BOTTOM CAVITIES*

Bernie Tittmann
Science Center, Rockwell International
Thousand Oaks, California

Earlier in the day you heard several talks on the analytical techniques
to treat defect scattering. I would like to try to discuss what amounts to
two procedures which should hopefully help in not only testing some of these
results but also to reduce these techniques to practice, that is to say, with
a view toward obtaining defect signatures in the field. The thread that
should hopefully be running through my talk is how to develop procedures
for applying present scattering results to laboratory field practice.

The first technique I want to discuss is a procedure for the theoretical
and experimental treatment of ultrasonic pulses of very wide frequency band
width. This is some work that John Richardson and I have been doing.

Before I begin, by way of introduction, I would like to show you some
results of a complementary program in which we have compared the results of
the elastic theory for a spherical cavity with experiment.2 Figure 1 is a plot
of relative power in db as a function of the angle θ. Angle θ is defined
here as the angle relative to the forward scattering direction. This result
is for 10 MHz longitudinal waves incident on a spherical cavity 800 microns
in diameter.

As you can see in this representative example, at 10 MHz we get reasonably
good agreement between theory and experiment, and we have many such graphs
for different cases including mode conversion. So, we're confident in this
approach and the question that arises now is "How can we apply it to a
practical situation"?

In practice, much or most of the ultrasonic work is carried out with
short pulses or pulses with a very wide frequency band width rather than long,
nearly monochromatic pulses. The short pulses are especially useful in the
spectrum analysis and rapid signal processing of signals. However, present
theory assumes plane monochromatic waves and, as is shown for example in Fig. 2,
disagrees drastically with observations for short pulses.

I would like then to suggest a procedure that might shed light on this
problem. The first step is illustrated in Fig. 3 when we look at the rf
signal in the photograph in Fig. 3a. This is the rf wave form sent through a
block of titanium without a scattering center being present. So, it is the
incident rf wave form.

We can obtain the Fourier transform of this signal; its absolute value
is shown in Fig. 3b. We find that, in fact, we get a structure that has a band
width of about 2 1/2 MHz and is centered at 7 1/2 MHz for this particular

* Research sponsored by ARPA/AFML Center for Advanced NDE
Fig. 1. Angular dependence of longitudinal scattered wave power for 10 MHz longitudinal waves incident on a 800 μm diameter cavity embedded in Ti-alloy by the diffusion bonding process.
Fig. 2. Theoretical predictions made for incident monochromatic waves are compared to data obtained with typical laboratory field equipment.
Fig. 3. Pulse incident on scattering center as obtained with laboratory field equipment. 3(a) rf signal as seen on cathode ray oscilloscope 3(b) Absolute value of Fourier Transform of rf signal from 3(a) after digitizing and transforming signal on computer.
transducer. What we have essentially done is carried out steps 1 and 2 of Table I.

Now, scattering theory gives us $A(\omega, \theta)$, which are the complex amplitudes for the wave scattered in the $\theta$ direction with incident wave of unit amplitude. The Fourier transform of the scattered pulse, then, would be proportional to the product of the complex amplitude times the Fourier transform of the unscattered pulse as shown in Table I.

Now, I'd like to discuss two ways in which to use this information to try to arrive at what we call the synthesized angular dependence. Instead of referring to the expressions on Table I, let me do it in words using the next slide, Fig. 4. You see four rf wave forms. The one in the left-hand corner Fig. 4a, you have seen already; it is the unscattered rf wave form. The other three are rf wave forms for 3 different angles. This is information at 10 MHz. Figure 4b is for $\theta = 0$ or the forward scattering case; Fig. 4c is $\theta = 90$ or the side scattering case. Figure 4d is for $\theta = 180$ or the back scattering case.

The interesting thing to observe is the fact that the rf wave form, in fact, changes shape. The back scattered wave form is somewhat longer than the forward scattered wave form.

In the experiment, we take the rf wave form, amplify it, rectify it and then look at the envelope and take the peak height of the envelope as the experimental data point. It is clear that this is not entirely satisfactory, because as we have seen in Fig. 4 the rf pulse changes shape, but let's look and see how bad it really is.

To do this, then, we use two methods in the theoretical synthesis. Method A amounts to taking the area under the square of the pulse. That is to say we sum the power intensities for each frequency component with a weighting factor deduced for the differential cross-section at each frequency. The other method is to take the rf wave form, fit a parabola to the rectified signal, and then peak detect it, that is to say, measure the peak height of the parabola fitted to the envelope.

Figure 5 shows the angular dependence, again, for the case discussed above, an 800 micron diameter spherical cavity at 7.5 MHz. The so-called synthesized line, is the one that represents the area under the square of the pulse, and the so-called simulated line represents the peak height of the theoretically detected signal.

You can see there is some disagreement between the two in the back scattering direction because the rf wave form has a substantially different pulse shape than the forward scattering case. But still the disagreement isn't very great, and within the precision of the data.

We are using this technique and have looked at several different frequencies. Figure 6 shows the results at 5 MHz for the monochromatic and synthesized result and an 800 micron sphere. We have done the same thing for the mode converted shear wave as shown in Fig. 7, and the data points agree reasonably well with the synthesis calculation.
Table I. Procedure for Comparing Scattering Theory with Data Obtained on Typical Laboratory Field Equipment.

(1) Measure $f_i(t)$ standard transmitted pulse at $\theta = 0$.

(2) Spectrum analyze pulse and obtain Fourier transform

$$\tilde{f}_i(\omega) \overset{\text{def}}{=} \int_{-\infty}^{\infty} dt \exp(-i\omega t) f_i(t)$$

(3) Scattering theory gives us $A(\omega, \theta)$ the complex amplitude per unit solid angle for wave scattered in $\theta$ direction with incident wave of unit amplitude.

(4) The Fourier transform of scattered pulse

$$\tilde{f}_s(\omega, \theta) = \alpha A(\omega, \theta) \tilde{f}_i(\omega)$$

METHOD A: Sum power intensities for each frequency component with a weighting factor deduced for the differential cross-section at each frequency.

$$\sigma(\theta) = \frac{|\alpha|^2 \int_0^\infty d\omega |A(\omega, \theta)|^2 |\tilde{f}_i(\omega)|^2}{\int_0^\infty d\omega |\tilde{f}_i(\omega)|^2}$$

$\sigma(\theta)$ is area under square of scattered signal.

METHOD B: Calculate

$$f_s(t, \theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \exp(i\omega t) \tilde{f}_s(\omega, \theta)$$

rectify, fit parabolas to envelope, and take peak height.
Fig. 4. rf waveforms of incident and scattered signals as calculated for the case of longitudinal wave scattering from a 800 μm diameter spherical cavity.
Fig. 5. Data and calculations on the angular dependence of ultrasonic pulses scattered from an 800 μ diameter spherical cavity at nominally 7.5 MHz.
Fig. 6. Data and calculations on the angular dependence of ultrasonic pulses scattered from an 800 μ diameter spherical cavity at nominally 5.0 MHz.
Fig. 7. Angular dependence of mode converted shear wave pulses for 800 μm diameter void at 5 MHz.
Well, if we plot some of these results on one graph, Fig. 8, we can quickly realize that, in fact, there is a substantial difference between the results for the three frequencies, and this gives us an opportunity, really, to consider this in some way as a defect signature. If you will, a measurement of perhaps three or so points in the regime between the \( \theta = 40^\circ \) and \( \theta = 80^\circ \) would certainly allow us to distinguish between those three curves and give us an idea of what the size of that sphere is.

When we are talking about signatures, I would like to show a slide, Fig. 9, shown by Bruce Thompson earlier, where he showed the angular dependence for four different shapes. This is a solution for only the scalar potential equation and not the vector potential equation. The vector potential solution hasn't been generated yet in a way for easy comparison with experiment, but we hope to have that soon. At any rate, it is clear that with a loving eye toward the horizon we hope to distinguish between these various shapes simply on the basis of an angular dependence with perhaps only a few experimental points necessary. The question of a unique inversion is, of course, entirely different and much more difficult.

I would like to go on to the second topic, which is a comparison of Ermolov's theory with a variety of experimental procedures, some work that Dick Elsely and I have been doing. I would like to now point out that while we have been looking at the angular dependence before, now we will be looking at changes in scattering amplitude with transducer-scatterer separation. Figure 10, as a reminder, is a graph obtained by having a transducer look at the end of a rod in a water tank and changing the distance between the transducer and the rod. As you know, far away we get the typical \( \propto 2 \) type dependence for the reflected power, which is generally called the far-zone, and then, when we come very close, the response has sharp nulls and peaks, and we call this the near-zone.

This problem has been treated by many theories and one technique enjoying some popularity currently is that by Ermolov\(^5\). Figure 11 shows the results of a calculation by Dick Cohen\(^4\) for the signal amplitude as a function of the frequency from 0 to 15 MHz for the scattering from a disk. We can recognize the far-zone regime, and then as we go to higher frequencies, the pattern breaks into nulls and peaks corresponding to the near zone. Of course, if the transducer is comparable to the size of the defect, then, these nulls are washed out.

We have been looking at some Al samples made by N. Paton and have found that we get qualitatively the same kind of results as shown in Fig. 12. There are three things wrong with this kind of comparison, however. First of all, there are too few data points to make any concrete comparison. That's because both commercial-type transducers and ultrasonic equipment available were confined to very discrete frequencies, 1 MHz, 21/4, 5, 10, 15 MHz. Secondly, we were using three different brands of transducers and as we know, they have different bandwidth characteristics and different center-frequencies. Finally, the fact that there is a substantial frequency content in the pulses would tend to smooth out the nulls and peaks that we would expect from Ermolov's theory. Ermolov's theory is after all, a monochromatic theory that treats the disk as a rigid motionless disk in a fluid medium.
Fig. 8. Angular dependence of scattered pulses for several different frequencies.
Fig. 9. Theoretical polar plots for scattering from acoustically soft prolate spheroids (scalar potential solution only).
Fig. 10. Typical plot of on-axis signal amplitude versus separation between transducer and scattering center (a flat-topped rod in a waterbath). Apparent in the plot is the near-zone characterized by sharp nulls and peaks and the far-zone characterized by a $1/r^2$ power dependence.
Fig. 11. Plot of theoretical amplitude of acoustic wave reflected from a disc of varying radius for a fixed transducer diameter and transducer-disk separation $R$. 

TRANS. DIAM. = 1.27 cm

$R = 2.88$ cm
Fig. 12. Plot of experimentally observed signal scattered from flat-bottom hole in 2032 Al block for same values of parameters used in Fig. 11.
To shed a little bit more light on this problem, we performed an experiment in a water bath with a 5/64 inch rod, 3/4 inch transducer at 1 MHz, and plotted out on a quasi-continuous basis the signal amplitudes scattered from that end of the rod as a function of the separation.

Shown on Fig. 13 are the theoretical positions for the first null and for the first peak and the agreement is reasonably good. What happens if we make a normalized plot? This is a plot which we now refer to as a Cohen Nomogram, Fig. 14. The normalized abscissa $a$ is divided by the square root of $R\lambda$ where $a$ is the transducer radius, $R$ is the separation between transducer and flat-bottom hole and $\lambda$ is the wavelength. We see that the continuous curves are for three different ratios of $b$ where $b$ is the radius of the rod; for a small value of this ratio, we get well defined nulls and peaks while when this ratio becomes large, the nulls and peaks wash out. Shown also on Fig. 14 are some data obtained for $a = .1$ and $b/a = .16$ in reasonably good agreement with Ermolov.

What happens when we go to the more popular case of a flat-bottom hole in a solid specimen, the solid specimen being immersed in a water bath, and the transducer being a fixed distance away from the solid specimen, let's say three inches? This case is shown in Fig. 15, again, with normalized coordinates. We have a $b/a$ ratio of .2 and .33, and again, we get reasonably good agreement with the Ermolov treatment.

Well, Ermolov's theory, as stated before, treats a rigid motionless disk in a fluid. When we got to solid media, in principle, this theory should not apply, because in the solid medium you expect the solution of the vector potential equation to become important. In order to test this idea, we performed some experiments in a solid medium taking great pains to eliminate the liquid interface or liquid bond by using a wax bond, so that now, there is only one solid medium between the flat bottom hole and the transducer. These measurements were made in some very special samples that were furnished to us, courtesy of John Moore of the B-1 Division, and these specimens are remarkably uniform. For example the attenuation data gave excellent exponential curves and repeated well from sample to sample as did the measured velocity of sound.

There were 19 such specimens with the flat-bottom hole to surface separation ranging from an eighth of an inch, to about 5 3/4 inches. These are specimens precision machined and are actually ultrasonic standards that were made commercially as standards by a well-known company.

The results are shown here in Fig. 16, again as reflected power in db as a function of separation, and we get qualitatively the typical kind of response. However, the theoretical position of the nulls and peaks do not agree with the experiment. In fact, Ermolov's peak, theoretically indicated by the position of the arrow is almost an inch away from the peak observed experimentally.

Well, you ask yourself what that could be due to. Misalignment, or a lack of precision in the machining of the flat-bottom hole, appears to be unlikely because this misalignment would have to be present in all specimens yet we see the monotonic increase towards a well-defined peak. The
Fig. 13. Chart recording of amplitude of signal scattered from flat top of rod in water bath as separation between rod and transducer is varied.
Fig. 14. Scattering from rod in water bath. Plot of data and Ermolov's predictions in terms of normalized coordinates.
Fig. 15. Comparison of theory and experiment (normalized coordinates) for scattering from flat-bottom hole in Al block immersed in water. The travel path in water is fixed (3 inches) whereas travel path in Al changes with different block lengths. The experimental points were obtained from data published for Ultrasonic Standard Reference blocks.5
Fig. 16. Relative power of signal scattered from 5/64" diameter flat-bottom-holes in series of Ti-6%Al-4%V Ultrasonic Standard Reference blocks. The theoretical positions calculated for the null and peak are indicated by arrows.
data was obtained with two different transducers, one a commercial type, the other one just a PZT-5 disk, so we cannot blame the transducer.

I would like to suggest that perhaps we are seeing some evidence of the presence of mode conversion here, that is to say, that since here we have a solid medium, we can now intercept mode-converted signals. And although we might expect mode conversion to play a somewhat minor role, because we are looking at a flat target, perhaps the edges still generate enough shear waves to interfere at the transducer with the main signal.

I must remind you that this data was obtained with very long monochromatic signals so that there was no possibility to resolve in time the scattered longitudinal and shear wave.

Well, I hope I have conveyed to you some of the difficulties that might be encountered in using present theories in the real world, and I hope I have provided a few answers to some of these difficulties.

References

Acknowledgement

The author is indebted to J. M. Richardson and R. K. Elsley for the extensive computations required in the program and to J. Moore of the B-1 Division for the loan of the Ti-6%Al-4%V Ultrasonic Standard Reference blocks.
MR. STEVE HART (Naval Research Lab): I wonder if you would expect any
difference in the pattern from the end of the flat-bottom hole and from
just a disk shaped defect? And in that connection, I was wondering
why they didn't make disk shaped defects in earlier work with the
diffusion bond? I should think that would have been more realistic.

That's really two questions.

DR. TITTMANN: All of my experiments were for normal incidence. I would
expect a significant difference between the disk and the flat bottom
hole for a study of the angular dependence and I believe we will hear
from Laszlo Adler about just this sort of thing in the next talk.

The present samples made by Rockwell do not contain diffusion bonded
disks but very early samples did, and I had occasion to measure some
of those, and for normal incidence, I found no difference to first order.

DR. THOMAS SZABO (AF Cambridge Research Lab): Could you comment on the
variation in the beam pattern from the transducer affecting these
kinds of measurements? In other words, the theory probably assumes a
plane wave impinging on the cavity and, in reality, the transducer
beams have some other kind of shape. It didn't seem to affect your
measurements too much, and I wonder if you had any comments on it.

DR. TITTMANN: We are aware of the fact that you can look at a transducer as
an antenna having as its radiation pattern a main lobe and many side
lobes. We tried to take great pains to make sure that the defect was
always covered, completely covered, by the main beam, in fact, by a
solid angle that was contained well within the half power beam width
of the main beam. That's an important problem certainly.

DR. E. R. COHEN (Rockwell International Science Center): I think I can answer
your question about the approximation in the wave front at the trans­
ducer in the Ermolov theory. This theory is essentially a Huygen's
construction in which each point of the transducer sends out a spherical
wave. Each point of the disk scatterer then generates its spherical
wave and each of these are then received by the transducer again. The
calculation then adds up all the wave fronts, taking appropriate account
of the phases. The difficulty, of course, is that this is a scalar
wave theory at a single frequency. The extension to the case of elastic
waves with frequency spread is nontrivial.