Some Kinematic and Dynamic Studies of Rigid Transport Wheels for Agricultural Equipment

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SUMMARY

The investigation was confined to a study of the force and motion relations of rigid-cylindrical transport wheels and of the soil on which they operate. The analytical methods of theoretical mechanics, and laboratory and field experiments were used.

Equations were developed for the relative effect of speed, diameter and effective obstruction height on impact. The mechanics of a wheel rolling on a non-elastic friable medium was developed and the fact of slippage established both rationally and experimentally. The nature of the soil motions and soil displacements caused by a rigid wheel was studied. It was found that the soil adjacent to the track left by a rigid wheel is moved ahead and that this change in position of soil particles is attained by curved rather than straight line motion.

The effect of area upon the supporting capacity per unit area was studied for four conditions. A wide variation was found for this relationship. Apparently this variation was caused by differences in the cohesion and internal friction of the soils.

An apparatus for studying the rolling resistance and slippage of individual transport wheels was designed and built. With this apparatus the possibility of using variable load trials was studied. The results obtained by the use of this method were unreliable. The effect of speed was investigated for two conditions, meadow and tilled soil. For these soil conditions and speeds up to 5 miles per hour, the effect of speed appeared to be of minor importance compared to the effects of diameter and width. The effect of repeated trials in the same track was investigated, and the rolling resistance was found to vary approximately as the -0.2 power of the number of the trial for trials on meadow and as the -0.5 power on tilled soil.

The effects of load (300 to 1200 lb.), diameter (16 to 60 inches) and width (2.5 to 20 inches) on rolling resistance and slippage were investigated for three surfaces—meadow, tilled soil and a layer of dry loose sand on concrete. The rolling resistance was found to vary approximately as the 0.6 to 1.3 power of the load, the -0.5 to -0.7 power of the diameter and the -0.5 to 0.5 power of the width. These variations are explained qualitatively by certain combinations of wheel dimensions and soil conditions.

The association of soil moisture, volume weight and resistance to penetration with rolling resistance was studied. A very high positive correlation, 0.97, was obtained between penetration readings of the “penetrometer” used in these studies and rolling resistance, and a high negative correlation, -0.87, was obtained between volume weight and rolling resistance.
Some Kinematic and Dynamic Studies of Rigid Transport Wheels for Agricultural Equipment

By EUGENE G. MCKIBBEN

Farm equipment transportation represents a very large capital investment and a large annual reinvestment and consumes annually a great amount of energy. The equipment required to handle efficiently a 160-acre farm with the crop rotation very frequently used in Iowa (corn, corn, small grain and hay) has 30 to 40 transport wheels where horses are the principal source of power. Where the tractor has largely displaced the horse the number of transport wheels may be reduced as much as a fourth or a third. On the basis of even the smaller number, that is 20 transport wheels per farm, the total for Iowa's 214,928 farms (66) would be approximately 4,300,000. If the average life of farm machines is assumed to be 15 years (16), an annual replacement of about 300,000 wheels is indicated.

The amount of transportation is equally impressive. By simple calculation it is easily shown that 0.4125 of a ton mile of transportation is required for each acre of use for each 100 lb. of machine weight per foot of effective operating width.

On the basis of the equipment and operating practices commonly used in Iowa the transportation of field equipment may

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be estimated at a minimum of 1½ ton miles per acre for all cropped land. This estimate does not include the use of a tractor. Neither does it include that part of the weight of tillage implements which is required to maintain proper depth of penetration. Further, it does not include the field use of wagons or the use of such machines as the corn binder, corn picker and hay loader which are found on only a part of the farms. The field use of wagons represents ½ to 1 ton mile per acre, and the use of heavier machines, such as the corn picker, may easily raise field transportation, including the use of wagons, to 4 ton miles per acre. The use of a tractor is equivalent to a total of about 8 ton miles per acre. Probably that part of the tractor's weight carried on the front wheels, which are more nearly simple transport wheels, accounts for 1½ to 2 ton miles per acre. Thus, for Iowa's 22,738,377 acres (66) of crop land, something more than one-third of which is probably operated by tractors, the annual transportation of field equipment may be conservatively estimated at 50 million ton miles, exclusive of the weight carried on traction wheels and tillage elements.

Transport wheels for agricultural equipment have received little attention in comparison with those used for transportation on established roadways. The following appear to be some of the more important reasons for this neglect:

1. Each type of field equipment presents a more or less individual transportation problem.
2. As noted by Keen (32) and McKibben (40) the condition of the soil over which each machine must operate varies widely, not only from locality to locality, but from one part of the field to another and from day to day.
3. For most agricultural field conditions the structural characteristics of the soil vary in an erratic and more or less unpredictable manner from the surface downward. This point is well illustrated by Davies' (19) work on consolidation.
4. Bernstein (8) has called attention to the fact that there is a definite conflict between those soil conditions needed for optimum crop production and those needed for efficient machine operation.
5. The users of agricultural equipment have been comparatively unorganized, and consequently there has been little interest in possible improvements which would result in a relatively small saving for individual units, even though the saving might be very large in the aggregate.

Recent developments, however, make the study of the field transportation of agricultural equipment particularly timely:

1. The introduction of the small general purpose tractor is requiring the design of special field equipment for its use
and the redesign of certain field machines which have become considered as more or less standardized.

2. The introduction of efficient and reasonably priced pneumatic traction tires for farm tractors has reopened the question of agricultural field equipment transportation and operating speeds (38) (43) (60).

REVIEW OF LITERATURE

The motion and force relationship of transport wheels may be considered from many viewpoints. The following brief, and far from complete, review of the rather extensive literature is arranged according to such viewpoints rather than chronologically.

TRANSPORT WHEEL DESIGN

One of the more important viewpoints is that of the designer and builder. Zimmerman (70), Joseph B. Reynolds (56) and Pippard (53) (54) have published rather complete and very fundamental discussions of the structural characteristics and of the requirements for the balanced design of transport wheels. In addition, Zimmerman (71) has presented an excellent discussion of methods of testing their strength characteristics.

KINEMATICS OF A WHEEL ROLLING ON AN ELASTIC SURFACE

Osborne Reynolds (57) investigated the situation of a loaded wheel rolling on an elastic surface from the standpoint of theoretical mechanics. He developed a rational theory of rolling friction which he verified by laboratory demonstrations. He found that when the material of the wheel was relatively rigid with respect to the material on which it was being rolled the forward travel during one revolution would be less than the circumference of the wheel. The comparable situation of a rigid wheel rolling on granular or friable media, such as sandy soils or cultivated fields, has apparently received little attention.

THE DRAFT OF HORSE-DRAWN VEHICLES

The rolling resistance of vehicle wheels as reflected by the draft of horse-drawn vehicles and by the power requirements of motor vehicles has been extensively investigated by highway and agricultural engineers. Morin (46) (47) (48) was one of the earlier engineers to give consideration to the draft of vehicles. He conducted trials of two and four-wheeled vehicles, with rigid wheels having diameters from 0.9 to 2 meters (35 to 79 inches). He reported the rolling resistance to vary directly with the load and inversely as the first power of the diameter. With respect to the effect of rim width he concluded that on a solid road or pavement the resistance was independent of the width but that on a compressible surface the resistance decreased as the width of the rim increased, the rate depending upon the nature of the surface. Over a speed range of 2 to 6 miles per
hour he found an increase in draft with increase of speed. The amount of this increase apparently depended primarily upon the roughness of the road. For stone block pavement the resistance varied as the square root of the speed.

Mairs (39), Wooley (69) and others (45) at the Missouri Agricultural Experiment Station have studied the effects of height of wheel and width of tire on the draft of farm wagons for a number of soil conditions both on the road and in the field. Somewhat similar trials were recently conducted by Boelter (10) in Germany. Boelter also considered the effect of speed, using speeds of 1.03 and 2.06 meters per second (3.7 and 7.4 miles per hour), and of antifriction axle bearings. He found slight increases in draft at the higher speed. The decreases in draft resulting from antifriction bearings were relatively important only on good roads.

Baker (5) conducted a series of trials to determine the effects of road surfaces on the tractive force required for horse-drawn vehicles. The results of a rather complete set of similar trials are reported by McCormick (37).

**TRACTIVE RESISTANCE OF MOTOR VEHICLES**

Since the development of automotive transportation the tractive resistance of such automotive vehicles, including the rolling resistance on different road surfaces, has been carefully investigated by Agg (1) (2) (3), Paustian (52), Holt (31), Kennelly, (33), Graf (23) and other highway and automotive engineers.

**ROAD IMPACTS OF MOTOR VEHICLES**

The specific problem of the effects of road roughness and the resulting impacts has been considered by Hogentogler (26), Spencer (63), Smith (61) and Buchanan (11) (12). As road surfaces have become harder and smoother and pneumatic automotive transport wheels have been improved, however, the factors of rolling resistance and impact have tended to become subordinated to those of wind resistance and vehicle control.

**PNEUMATIC TIRES FOR AGRICULTURAL EQUIPMENT**

On the other hand, in the field of agricultural field transportation the item of rolling resistance is still one of major importance. McCuen and Silver (38) have reported the results of comparative trials of rigid and pneumatic tired wheels on farm wagons and corn pickers. They found that the use of pneumatic tires effected a material reduction in rolling resistance on rough or soft surfaces. Similar results are reported by Meyer (42) and Schirmer (59) in Germany.

**TRACKS FOR FIELD WAGONS**

Meyer and Schirmer also have run tests on field wagons
equipped with transport tracks somewhat similar to those used for traction. They found that for a given load tracks materially reduced the rolling resistance on soft surfaces, especially at higher loads, but that they increased the rolling resistance on harder surfaces, particularly at lighter loads.

FORMULAS FOR ROLLING RESISTANCE

In 1913 Bernstein (8), in Germany, attempted to set up an equation for the rolling resistance of rigid right-cylindrical wheels on agricultural soils. He started with the assumption that the depth of penetration of a given area is proportional to the load supported and that as the area is increased the supporting capacity per unit area is decreased. His final equation was

\[ W = \frac{0.57 G^{3/2}}{\sqrt{2a_1 + a_2 b} r^{3/4}} \]

where \( W \) is the rolling resistance in kilograms; \( r \), the radius in centimeters; \( b \), the tire width in centimeters and \( a_1 \) and \( a_2 \) soil constants. In his book published in 1930 Kühne (34) discusses this equation briefly and suggests the reduction of the number of constants by introducing the ratio \( \frac{a}{a_1} = 0.27(2\pi r) \).

The assumptions made and the simplifications introduced, however, during the process of deriving this equation prevent its claim of rationality from having much weight. The following somewhat simpler empirical equation used by Meyer (41) appears to be more serviceable:

\[ F = C \frac{G^m}{d^n b^p} \]

where \( F \) represents the rolling resistance; \( G \), the load; \( d \), the wheel diameter; \( b \), the rim width; \( m, n, \) and \( p \), constants depending upon the soil characteristics and the load and wheel dimensions irrespective of the units of measurement used and \( C \), a constant depending upon the soil and the units of measurement used. From such an equation the engineer can readily visualize the manner in which the rolling resistance varies with changes in load, wheel diameter or rim width. Using wheels 80 to 140 centimeters (31 to 55 in.) in diameter, 6 to 12 centimeters (2.4 to 4.7 in.) wide and loads from 250 to 1125 kilograms (550 to 2480 lb.) per wheel on a number of field soils, Meyer found values of \( m \) from 1.2 to 1.5; for \( n \), from 0.8 to 1.3 and for \( p \), from 0.25 to 0.55.

SOIL MECHANICS

While very little has been done to directly relate the performance of transport wheels with the principles of soil mechanics, there is a large body of information on soil mechanics which is closely related to the wheel problem. Griffith (24) (25), Terzaghi (64), Nichols (51), Keen (32) and Doner (21) have publications on the physical and dynamic properties of soils. Mullis (49), Hogentogler (27) (29), Davis (20) and Casagrande (14) have studied the supporting capacity and shear-
ing strength characteristics of soils. These are soil characteristics which are closely related to rolling resistance.

SOIL CONSTANTS

There recently has been extensive study of soil physical constants such as mechanical composition, moisture equivalent, liquid limit, plastic limit, shrinkage ratio, slaking time, settling volume, etc.; of the methods of determining these constants and of the relation of such constants to the stress-strain and other structural characteristics of soils. The papers published by Eno (22), Nichols (50), Hogentogler (28) (30), Wintermeyer (68), Casagrande (13), Thoreen (65) and Rutledge (58) are representative of the work being done in this field. The American Association of State Highway Officials has included in its handbook of standard specifications (4) a number of such soil constants, along with detailed instructions for their determination.

VOLUME WEIGHT OF SOILS

The volume weight of a soil, calculated on a dry basis, is closely related to its porosity and consequently its compressibility. Thus, the volume weight may offer possibilities of relating the rolling resistance to some quantitative soil measurement. Curry (15) studied a number of methods of obtaining volume-weight and recommends the large cylinder method. Lebebev (35) reported success with a smaller cylinder of special construction.

RESISTANCE OF SOILS TO PENETRATION

Various methods of measuring resistance to penetration have been used by a number of investigators with the hope of obtaining a single constant which would be indicative of the structural character of the soil being studied. Bernstein (8) reports the use of an instrument of rather elaborate design in his attempt to develop a rational and quantitative equation for rolling resistance. Keen (32) used an impact type of instrument for the studying of the uniformity of soil with respect to area. Davies (17) (18) (19) reported the use of a spring loaded instrument, which he called a "compactometer," for studying soil tilth and consolidation. Berglund (7) used a hydraulic instrument for measuring the resistance to penetration during his investigation of methods of measuring tilth. Proctor (55) reported the use of a penetration instrument, called a plasticity needle, for controlling the compaction of soil on large construction projects.

PRESSURE DISTRIBUTION IN SOIL

The matter of the compaction of the soil under transport wheels is of interest both from the engineering viewpoint of the energy lost and from the agronomic viewpoint of the destruct-
tion of soil structure and possible damage to crop roots. It seems possible that the proper consideration of the more classical theories of pressure distribution might be of aid in obtaining a fundamental solution to this phase of the transport wheel problem. Michell (44) and Love (36) have considered such pressure distribution problems from the viewpoint of pure mathematics. Bell (6) and Griffith (25) have made engineering applications of these fundamental pressure distribution theories to engineering structures and have presented the solution for a number of cases. Biot (9) has solved this problem for certain types of discontinuities in the material under stress.

EXPERIMENTAL

The investigations were confined to the consideration of rigid right-cylindrical transport wheels, with the purpose of learning as much as possible about the motion and force relations of such wheels and about the motion and force relations of the soil upon which they operate.

In addition to the customary review of the literature three rather definite methods were used.

a. Mathematical analysis
b. Laboratory trials
c. Field trials

MECHANICS OF A RIGID WHEEL STRIKING A SOLID OBSTRUCTION

When a relatively rigid wheel strikes a comparatively rigid obstruction as shown at P of fig. 1, the resulting impact may produce excessive stresses. This is certain to be the case if the forward velocity V is appreciable. Many agricultural equipment transport wheels have been ruined in this way.

The determination of the magnitude of such impact forces, under field conditions, would be difficult and expensive. It is possible, however, by mathematics and theoretical mechanics to obtain a conception of this force and of the relative importance of the factors influencing it.

ANALYSIS

From the principle of the conservation of energy and a consideration of $V_z$, the vertical component of the velocity $V$, equations may be derived for the height, H, from which a wheel would have to fall to receive the same impact. This is a conception which has meaning to every engineer and even to laymen who have had any experience with physical equipment.

**NOTATION AND ASSUMPTIONS** (see fig. 1)

\[ \theta \] Angle between vertical radius OQ and radius striking obstruction OP.

\[ E_k \] Kinetic energy in foot pounds per pound resulting from from vertical velocity, $V_z$. 

F. Force exerted against wheel rim by obstruction.
g. Acceleration of gravity in feet per second per second.
h. Effective height of obstruction in inches, that is height at time wheel leaves level road surface at Q.
H. Effective falling height in feet, height from which wheel would have to fall to receive same impact.
O. Center of wheel.
P. Point of wheel rim striking obstruction.
Q. Last point of wheel rim contact with road surface before passing over obstruction.
r. Radius of wheel in inches.
R. Ratio of radius of wheel r to effective height of obstruction h.
S. Speed in miles per hour parallel to the road surface.
V. Velocity in feet per second of center of wheel O parallel to the road surface.
V₁. Velocity in feet per second of center of wheel at time it leaves road surface.
V₂. Vertical component of V₁ in feet per second.

It is assumed that the road surface is horizontal and that speed S and velocity V are constant. Also, the change in angular velocity about point O is neglected. For most cases this change of angular velocity would not be important.

**DERIVATION OF FORMULAS**

From proportional triangles,
\[ V_2 = V \sqrt{\frac{2hr - h^2}{r - h}} \]  \hspace{1cm} 1

From the standard equation of kinetic energy and the relation between velocity in feet per second and miles per hour,
\[ E_k = \frac{V_2^2}{2g} = \frac{V^2}{2g} \left( \frac{2hr - h^2}{r - h} \right) = \frac{0.0334 S^2}{2g} \left( \frac{2hr - h^2}{r - h} \right) \]  \hspace{1cm} 2

Since equation 2 is in foot pounds per pound of weight, it is a direct measure of the height the wheel would have to fall to store the same kinetic energy, however, since the impact force which imparts this energy is at the angle θ instead of directly beneath the wheel,
\[ H = \frac{0.0334 S^2}{2g} \left( \frac{2hr - h^2}{r - h} \right) \frac{(r - h)^2}{(r - h)^2} = \frac{0.0334 S^2}{2g} \left( \frac{2hr - h^2}{r - h} \right) \frac{(r - h)^2}{(r - h)^2} \]  \hspace{1cm} 3

If Rh, from \( R = \frac{r}{h} \) is substituted for r in equation 3,
\[ H = \frac{0.0334 S^2}{2g} \left( \frac{2R - 1}{R} \right) \frac{R}{(R - 1)^3} \]  \hspace{1cm} 4

(See fig. 2)

Since \( \tan \theta = \frac{\sqrt{2hr - h^2}}{r - h} \) and \( \sec \theta = \frac{r}{r - h} \), equation 3
may be written \( H = 0.0334 S^2 \tan^2 \theta \sec \theta \) \[5\]

For smaller values of \( \theta \) the secant is approximately 1, so that the error resulting from discarding the secant and writing equation 5 as \( H = 0.0334 S^2 \tan^2 \theta \) is less than 5 percent for values of \( \theta \) up to about 18 degrees and for corresponding values of \( R \).

The drawbar which maintains the constant velocity \( V \) is also subject to increased stress. The increased stress in that member would be of the order of \( F \sin \theta \), the exact value depending upon the rigidity of the drawbar and the inertia of the wheel and vehicle being drawn.

CONCLUSIONS

These equations indicate the following facts with regard to the impact force \( F \):

1. It is proportional to the square of the speed \( S \).
2. It is increased by increasing the effective height \( h \) of the obstruction or by decreasing the radius \( r \) of the wheel.
3. It is a function of the ratio \( R \) of the wheel radius \( r \) to the effective height of the obstruction \( h \).
4. It is a function of the angle \( \theta \) between the vertical radius \( OQ \) and the radius \( OP \) striking the obstruction.
5. It is proportional to the product of the secant and the square of the tangent of this angle \( \theta \).
6. It is approximately proportional to the square of the tangent of \( \theta \) for small values of the angle \( \theta \) and the corresponding large values of the ratio \( R \).

It should be kept in mind that while this interpretation indicates the relative magnitude of the force resulting from this impact, as this force is affected by the factors considered, the absolute magnitude is finally determined by the modulus of elasticity and other impact characteristics of the wheel and the obstruction. By assuming a value for these moduli the possible magnitudes of the force might be approximated.

The relative effect on effective falling height of changes in speed and ratio of wheel to obstruction height is shown in fig. 2.

**KINEMATICS OF A RIGID WHEEL ROLLING ON A FRIABLE OR GRANULAR MEDIUM**

There are at least two important differences between the situation of transport wheels used on field equipment in agriculture and those used in transportation on high grade road surfaces. First, field soils which form the “road surface” for agricultural equipment wheels may for practical purposes be classed as non-elastic. This non-elasticity is particularly true with respect to tensile stresses. Second, the magnitude, with respect to the bearing capacity of field soils, of the loads carried by wheels on field equipment is such that there is a relatively large permanent deformation of the soil in the wheel track and of the soil in the vicinity of the wheel track. Thus, the laws which govern the operation of a rigid rimmed wheel on an elastic surface do not apply.

In the case of a loaded wheel resting on a plane surface the wheel is flattened and the surface depressed, as shown in fig. 3a. If the wheel is relatively rigid the flattening of the wheel may be neglected and the situation will become essentially as shown in fig. 3b.

When, as the result of the application of a horizontal force at the axis of the wheel, rolling starts, at least two cases must be considered. First, the case of an elastic surface and second, the case of a surface of non-elastic friable material.

**ROLLING ON AN ELASTIC SURFACE**

This case has been rather carefully and completely treated by a number of authors of whom Osborne Reynolds (57) was one of the earliest. The situation in this case is shown in fig. 3c. The distance from A to B along the wheel periphery is greater than the horizontal distance between the same points. Since this increased distance is attained by elastic extension of the surface, the wheel may be thought of as measuring off its
circumference on an extended surface, with the result that during one revolution the forward travel of the wheel will be less than the length of its circumference. The effect will be the same as that of a wheel of decreased radius and correspondingly shorter circumference.

From the standpoint of the kinematics of the wheel this means that the instantaneous center of rotation of the wheel is located above the lower rim and, of course, on the vertical line through the center of the wheel. Thus all points on the wheel rim which lie below a horizontal line through this instantaneous center of rotation have a backward motion, and the complete motion of any point on the rim surface is a curvate trochoid instead of a simple cycloid. This permits the backward motion, beneath the wheel, of the elastic surface over which it is rolling and prevents accumulation of compressed material ahead of the wheel.
ROLLING ON A NON-ELASTIC, FRIABLE SURFACE

In this case the material of the surface on which the wheel is rolling is being continuously compressed and depressed ahead of and below the wheel. Since the material is not elastic under tension, there is no movement backward under the wheel as in the case of an elastic surface. Thus, the results of this compression are accumulative, causing cracks to appear in the track behind the wheel (note fig. 7). The wheel may be thought of as measuring off its circumference on a compressed or shortened surface, with the result that during one revolution it will travel a distance greater than the length of its circumference.

Under conditions of light to medium loads this conception can be illustrated by marking the soil ahead of the wheel with some material which will adhere to the surface of the wheel rim. The marks on the wheel rim will be found to have a closer spacing than those made on the soil. It also may be readily demonstrated by rolling any small wheel, such as a roller skate wheel, on sand or any granular material. From the standpoint of kinematics, this means that the center of rotation of the wheel with respect to the soil is located below the lower rim as at C' of fig. 3d.

KINEMATIC ANALYSIS

The following more rigorous analysis further verifies these statements and indicates that on a yielding, friable medium the instantaneous center of rotation of a rigid wheel must be outside the wheel, that is below the lower rim, in order to attain rotation. Thus on such a medium the effective radius must be greater than the wheel radius, and the bottom of the wheel must slip or have a forward motion (note D of fig. 4b).

The same analysis also may be applied to certain tillage implements. It may help to explain the unsatisfactory results obtained by the extensive use of the disc harrow on certain soils. This analysis cannot be applied to a wheel rolling on a medium which is elastic under tensile stresses.

Referring to fig. 4a, if the axle friction is assumed to be zero, the velocity V of the center of the wheel C to be constant, the wheel rim and the surface AA' to be rigid and the rotation to be clockwise, the following statements may be made concerning the motion of the wheel (for the time being the line BB' of fig. 4a is to be disregarded):

1. The contact between the wheel and the surface AA' is a line through the point C' and perpendicular to the plane of the wheel.
2. This line of contact is the instantaneous axis of the rotation of the wheel with respect to the surface AA'.
3. The direction of motion of any point P on the wheel is
perpendicular to the line drawn from the point to the instantaneous center of rotation $C'$.

4. The magnitude of the motion of any point $P$ on the wheel will be $\frac{Vd}{r}$ where $V$ is the velocity of point $C$, $d$ is the distance from $P$ to $C'$ and $r$ is the radius of the wheel.

5. If the point $P$ is located on the circle $CC'$, its motion is parallel to the wheel radius upon which it is located.
6. If the point P is located inside of the circle CC' (note P of fig. 4a), it has motion with a counter-clockwise component perpendicular to the radius upon which it is located.

7. If the point is located outside of the circle CC' (note P of fig. 4a), it has motion with a clockwise component perpendicular to the wheel radius upon which it is located.

8. The angular velocity \( \omega \) is equal to \( \frac{V}{r} \).

9. The path of a point on the rim of the wheel is a cycloid QC'O' whose equation in parametric form is \( x = r(\theta - \sin \theta) \) and \( y = r(1 - \cos \theta) \), where \( x \) is the distance to the right of the line through CC', \( y \) is the height above line AA', and \( \theta \) is the angle of rotation in radians.

If instead of a rigid surface a non-elastic, friable surface is assumed, line BB' of fig. 4a becomes the soil surface and the line AA' becomes the bottom of the wheel track. Those points of the wheel rim outside of the circle CC' and in contact with the soil, that is the rim face from C' to S' and the rim edges from S to S', will have clockwise components of motion perpendicular to the wheel radii upon which they are located. This clockwise component of motion means that counter-clockwise soil force reactions will be encountered which will tend to retard rotation of the wheel.

Only the very limited area of the rim edges which are within the circle CC' will encounter clockwise soil force reactions. Thus the rotation of the wheel will be retarded with the following results:

1. The instantaneous center of rotation C' will move downward to some point below the wheel rim or track bottom, as shown in fig. 3d, 4b and 7.

2. The position of C' will be such that \( \frac{V}{r'} \) will equal the reduced angular velocity, \( r' \) being the new distance C to C'.

3. The area of the wheel rim within the circle CC' and in contact with the soil will be increased and the area outside the circle CC' and in contact with the soil will be decreased.

4. The path of a point on the wheel rim will be the prolate trochoid of fig. 4b, Q'D'O', instead of the cycloid of fig. 4a, QC'O'. The equation of this trochoid in parametric form is \( x = r\theta - r' \sin \theta \) and \( y = r - r' \cos \theta \).

5. The lowest point of the wheel rim, D of fig. 4b, will have a motion parallel to \( V \), the velocity of the center of the wheel and of magnitude \( \frac{V(r' - r)}{r'} \). It is this motion which accounts for the cracks observed across the track left by a wheel operating on yielding soil and for the major portion
Fig. 4a. (top) Rigid wheel on an ideal rigid surface.

Fig. 4b. (bottom) Rigid wheel on a non-elastic, friable surface.
of the reversed sigmoidal motion of soil particles on the surface directly in the path of the wheel (note fig. 6 and 7). The prolate trochoid shown in fig. 7 was drawn on the basis of the measured value of the effective radius. The similarity between these curves and the paths of the soil particles in contact with the wheel rim, as well as the occurrence of characteristic tension cracks in the wheel tracks, is experimental confirmation of this kinematic theory.

In this analysis at least two items have been neglected, the axle friction and the slight forward motion of soil at $S'$ of fig. 4b. Under most conditions these items would probably have little effect, and in any case whatever influence they would have would tend to retard the rotation of the wheel and thus still further increase slippage.

CONCLUSIONS

1. The mechanics which have been developed for the rolling of a rigid wheel on an elastic surface cannot be applied to such a wheel when rolling on a non-elastic, friable medium.
2. The non-elasticity of such a medium, particularly under tensile stresses, causes the compression of the material ahead of the wheel to be cumulative, with the result that the wheel may be thought of as measuring off its circumference on a shortened surface.
3. The distance traveled under these conditions during one revolution is greater than the length of the rim circumference.
4. These results can be explained in the terms of the principles of kinematics by analyzing the motions of certain regions of the wheel with respect to the soil.

MOTION OF SOIL PARTICLES UNDER A RIGID TRANSPORT WHEEL (A LABORATORY INVESTIGATION)

By use of the apparatus shown in fig. 5, a qualitative study was made of the soil particle motions resulting from the rolling of a simple transport wheel. A 24-inch wheel with a 2-inch rim was used and the path of soil particles was traced by the use of BB shot, aluminum foil and white sand. Five trials were made, using two types of soil (Cecil Clay and Norfolk Sand) with 150 and 250 lb. loads on the clay and 100, 150 and 250 lb. loads on the sand. Qualitatively similar results were obtained in all cases. Figures 6 and 7 show the results obtained with Norfolk Sand, moisture 9 percent, and a 250 lb. load. The soil motions shown by the side views of fig. 6 and 7 were obtained by means of a soil box with a plate glass side and BB shot. The displacements shown in the top and rear views of fig. 6 were obtained by the use of aluminum foil.

3This investigation was made under the direction of Prof. M. L. Nichols, head of the Agricultural Engineering Department of the Alabama Polytechnic Institute.
The trochoid shown on fig. 7 was constructed on the basis of the experimentally determined effective radius. The cycloid shown indicates the form of the path which would have been taken by points on the wheel rim if there had been no slippage.

Fig. 5. Apparatus for studying the motions of soil particles under a rigid transport wheel (agricultural engineering laboratory, Alabama Polytechnic Institute)

Fig. 6. Soil displacements caused by rigid transport wheel.
Note the similarity between the prolate trochoid and the curves generated by the surface soil particles (fig. 7).

Thus it is evident that a rigid transport wheel causes a permanent soil displacement parallel as well as perpendicular to the direction of its travel. The soil surrounding the track left by a transport wheel has been moved ahead across the field as well as in a vertical plane. Further, this change in position of soil particles is attained by curved rather than straight line motion.

FIELD INVESTIGATIONS

APPARATUS FOR STUDYING ROLLING RESISTANCE AND SLIPPAGE OF INDIVIDUAL TRANSPORT WHEELS

Using a Model C Heider tractor as the structural foundation and as the source of power, an apparatus for studying the rolling resistance and slippage of individual wheels under actual field conditions was designed and built (see fig. 9 and 22). Adjustments for keeping the frame level and the trolley track support chain vertical were provided as shown in fig. 9.

The dynamometer was provided with three tension springs with extension constants of 81.3 lb. per inch. With an extension of 5 inches this allowed for three ranges of operation, that is 0 to 400 lb. with only the middle spring connected, 0 to 800 lb. with the two outside springs connected and 0 to 1200 lb. with three springs connected. A fourth spring (not shown) with an extension constant of 34.6 lb. per inch was substituted for the middle spring for lighter loads, 0 to 150 lb. An adjustable hydraulic damper (a modified double acting automobile shock absorber) was used to prevent excessive oscillation when operating on rough surfaces.

The recording arm was designed to take a standard automatic pencil, and the record was made on an 81/2 x 11 inch chart (standard notebook size). Figure 8 shows one of these charts. This chart was driven by a music wire wrapped around a threaded drum. By staking the end of the wire to the ground a chart motion proportional to the distance of wheel travel was obtained. By the use of an adjustable chart drive lever this ratio of chart motion to wheel travel was adjusted so that 1 inch of chart motion corresponded to 10 feet of wheel travel. Thus, the 10 inches of chart scale allowed for 100-foot trials.

A record of distance per revolution (see fig. 8), from which slippage was calculated, was obtained by the use of a second pencil operated by a cam on the test wheel axle.

The test wheel was mounted on a live axle which in turn was provided with ball bearings. Spacers were used to keep the test wheel centered behind the dynamometer when the rim width was changed by adding extension rims. The frame dimensions and vertical adjustment were sufficient to permit the use of test
Fig. 7. Motions of soil particles under a rigid transport wheel. (Note the cracks at the bottom of the wheel track, the location of the instantaneous center of rotation and the similarity between the prolate trochoid and the curves generated by the soil particles.)
wheels from 16 to 60 inches in diameter with rims up to 20 inches wide. On soft surfaces, however, there was not sufficient clearance to permit the satisfactory trial of wheels smaller than 20 to 24 inches.

The load on the wheel was adjusted by the addition or removal of weights from the arm extending to the rear, the addition or removal of weights from the trolley and by changing the position of the trolley. The trolley was provided with two locks, one for locking it to the track for constant load trials and one for locking it to the trolley drive chain for variable load trials. This chain was operated by the same mechanism which moved the dynamometer chart. Thus, if desired, the wheel load as well as the chart motion could be made proportional to the distance traveled.

The weights available provided for loads up to 1200 lb. It was thus possible to give the test wheel any load from this maximum down to zero. For very light loads, however, it was necessary to use a heavy trolley weight well to the left of the track support. On rough surfaces the pendulum effect produced by this overhanging weight prevented satisfactory results with loads of less than about 300 lb.

The tractor transmission, as rebuilt to permit the installation of pneumatic tires, provided for a speed range of 3 to 6 miles per hour at rated engine speed. By reducing the engine speed certain of the trials were made at speeds as low as 1.5 miles per hour.
Both theoretical analysis and field experience indicated that the only critical frame adjustment was the longitudinal horizontal adjustment for keeping the trolley track support chain vertical. During all trials a second operator was employed whose first duty was to maintain this adjustment.

Fig. 9. Apparatus for studying the rolling resistance and slippage of individual wheels.
This type of apparatus was selected in preference to a simpler two wheeled cart for the following reasons:
1. It requires less wheel and weight equipment and less manipulation of such equipment.
2. It permits a greater number of trials on a given field area, thus reducing the difficulty of maintaining reasonably uniform soil conditions.

WHEELS USED IN FIELD TRIALS

Figures 10a and 10b show the wheels and rims used during the field trials. Those shown in fig. 10a are 16, 24, 36, 48 and 60 inches in diameter. The disc shown in fig. 10b is 24 inches in diameter and \( \frac{3}{4} \) inch thick.
Fig. 11. Apparatus used for obtaining volume weight samples.

VOLUME WEIGHT SOIL SAMPLER

The soil sampler shown in fig. 11 was used in the volume weight determinations recorded in table 6 and on fig. 38, pages 373 and 375. In making these determinations the sampler was driven into the soil 6.5 inches or until the ring contacted the soil surface. The top inch of soil was removed with the trowel. The soil was removed from one side with a spade and the trowel inserted under the sampler. Thus a sample 0.04 cubic foot in vol-
ume was obtained. This sample was weighed. Using this weight and the results of standard moisture determinations, at 110 degrees Centigrade, the dry weight per cubic foot was calculated.

**IMPACT "PENETROMETER"**

The simple device shown in fig. 12 was used to obtain a single valued soil constant which it was thought might be associated with the rolling resistance (see table 6 and fig. 36, pages 373 and 374). Its weight is 15 lb., 5 lb. each for the hammer, A, the guide tube and penetrator, B, and the surface gauge, C. Its use consisted of placing the instrument on the surface to be tested with the guide tube vertical, of lifting the hammer 3 feet and allowing it to drop and of reading the penetration at the top of the surface gauge. The design and procedure are quite arbitrary. It gave, however, consistent results.

**TRIAL FIELD**

The field used for the rolling resistance trials is the north 5 acres of that part of the S.E. 1/4 of the S.W. 1/4 of Section 21, T. 83N, R. 24W which lies south and east of the railroad track. Figure 13 shows the soil type boundary and 1-foot contour locations for this field. Volume weight, centrifuge moisture equivalent, lower plastic limit, lower liquid limit, plasticity index and pH determinations were made for 11 samples located as shown in fig. 13. The mean results of these determinations which were made in duplicate are shown in the table of fig. 13. Mechanical composition determinations by the hydrometer method were made of samples 4 and 10, the finest and coarsest grained samples. The results are shown in fig. 14. All soil determinations were made according to the standards of the American Association of State Highway Officials (4).

**VARIABLE LOAD STUDIES**

When the rolling resistance dynamometer was designed, it was hoped that the study of the effects of varying wheel load, diameter and width could be materially speeded up by the use of variable load trials. Therefore, the first field trials were comparative runs to determine the feasibility of such a plan. A variable load (50 to 1050 lb.) trial was made and immediately followed by 250, 500 and 1000 lb. constant load trials. The results for both meadow and tilled soil are given in table 1, page 352. Note that the differences are rather large compared to the standard deviations given in table 5, page 372. Apparently the rolling resistance corresponding to a given load did not develop until some time after the application of the load. This result checks with those shown by the curves of fig. 43, page 379.

Since it thus appeared that variable load trials 100 feet in
Fig. 12. Impact "pennrometer"
length were unreliable, and since the difficulty of soil uniformity and varying grade made it impracticable to increase the length of the trials, the effect of load was studied by means of constant load trials of different magnitudes, 300, 600, 900 and 1200 lb., loads being used in most cases.
During the variable load studies, as well as during all other field investigations, all trials were run tangent to the contours, thus keeping the grades to a minimum. In addition, the grade for each set of trials was determined with a surveyor's level and the necessary corrections made. The rolling resistance values reported have been corrected for grade.

Fig. 14. Grain diameter accumulation curves.
TABLE 1. ROLLING RESISTANCE WITH CONSTANT AND VARIABLE LOADS WITH 36 x 2.5-INCH WHEELS.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Load (lb.)</th>
<th>Constant load trial (lb.)</th>
<th>Variable load trial (lb.)</th>
<th>Difference (lb.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Meadow</td>
<td>250</td>
<td>40.1</td>
<td>27.0</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>68.2</td>
<td>47.4</td>
<td>20.8</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>116.2</td>
<td>94.5</td>
<td>21.7</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>46.4</td>
<td>41.2</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>75.1</td>
<td>49.1</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>113.8</td>
<td>108.3</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>43.2</td>
<td>29.1</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>64.4</td>
<td>56.1</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>105.5</td>
<td>96.2</td>
<td>9.3</td>
</tr>
<tr>
<td>**Tilled</td>
<td>500</td>
<td>103.5</td>
<td>97.6</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>251.4</td>
<td>223.3</td>
<td>28.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>100.7</td>
<td>100.3</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>262.0</td>
<td>220.9</td>
<td>41.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>120.3</td>
<td>113.5</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>271.3</td>
<td>267.1</td>
<td>14.2</td>
</tr>
</tbody>
</table>

* Timothy clover meadow (1 ton per acre.)
** Webster loam.
Soil moisture, dry basis, 22 percent.
Dry weight per cubic foot, 67 lb.
Plowed 8 inches deep, double disced and harrowed.
Clarion loam.
Soil moisture, dry basis, 6 percent.
Dry weight per cubic foot, 62 lb.
Speed, 2 miles per hour.

EFFECT OF SPEED ON ROLLING RESISTANCE

A number of paired trials were made to determine the effect of speed upon rolling resistance. These trials were made on both the meadow and the tilled soil. The results for the trials on the meadow are shown in table 2 and those for the tilled soil in table 3. While in both cases the mean difference is in favor of the lower speed, it is quite small, only 0.18 lb. for the meadow and 2.18 lb. for the tilled soil. These differences are not statistically significant (67).

TABLE 2. ROLLING RESISTANCE, POUNDS, AT HIGH AND LOW SPEEDS WITH 36 x 2.5-INCH WHEEL AND 1000-POUND LOAD ON MEADOW.

<table>
<thead>
<tr>
<th>Speed</th>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean and standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8–5.0 mph</td>
<td>134.6</td>
<td>134.9</td>
<td>134.9</td>
<td>129.8</td>
<td>132.5</td>
<td>121.1</td>
<td>131.30</td>
<td></td>
</tr>
<tr>
<td>2.2–2.7 mph</td>
<td>134.6</td>
<td>134.2</td>
<td>137.4</td>
<td>129.1</td>
<td>133.2</td>
<td>129.4</td>
<td>131.48</td>
<td></td>
</tr>
<tr>
<td>Difference</td>
<td>0.0</td>
<td>0.7</td>
<td>-2.5</td>
<td>9.7</td>
<td>-0.7</td>
<td>-8.3</td>
<td>0.18 ± 2.38</td>
<td></td>
</tr>
</tbody>
</table>

Beginning 400 feet from north end and 30 feet from east side of trial field, running north and working east.
Timothy clover meadow (1 ton per acre).
Webster loam.
Soil moisture, 24 percent.
Dry weight of soil per cubic foot, 67 lb.
TABLE 3. ROLLING RESISTANCE, POUNDS, AT HIGH AND LOW SPEEDS WITH 36-INCH WHEELS ON TILLED SOIL

<table>
<thead>
<tr>
<th>Trial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Mean and standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rim width, in.</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>7.5</td>
<td>7.5</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Load lb.</td>
<td>500</td>
<td>300</td>
<td>600</td>
<td>900</td>
<td>900</td>
<td>600</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>3.9—4.1 mph</td>
<td>102.5</td>
<td>68.9</td>
<td>144.9</td>
<td>223.5</td>
<td>126.8</td>
<td>94.6</td>
<td>50.4</td>
<td>114.51</td>
</tr>
<tr>
<td>2.0—2.4 mph</td>
<td>101.8</td>
<td>57.2</td>
<td>149.9</td>
<td>217.7</td>
<td>126.9</td>
<td>89.6</td>
<td>43.2</td>
<td>112.33</td>
</tr>
<tr>
<td>Difference</td>
<td>0.7</td>
<td>1.7</td>
<td>—5.0</td>
<td>5.8</td>
<td>—0.1</td>
<td>5.0</td>
<td>7.2</td>
<td>2.18 ±1.58</td>
</tr>
</tbody>
</table>

Beginning at 675 feet from north end and 170 feet from east side of trial field, running east and working south.
Plowed 8 inches deep, double disced and harrowed.
Webster loam.
Soil moisture, 11 percent.
Dry weight of soil per cubic foot, 63 lb.

In the case of the tilled soil, however, it is reasonable to think that the increased speed does in fact increase the rolling resistance, though not to a very great degree for the speed range tried. Because of the following reasons this phase of the investigation was not extended:

1. It appeared that the effect of speed on rolling resistance was a relatively minor factor within the range of speeds suited to rigid wheels.
2. The engine power was insufficient for well controlled trials at the higher speeds, particularly on the tilled soil.
3. Speeds above about 4 miles per hour gave evidence of greatly increasing the problem of test equipment maintenance.

REPEATED TRIALS IN THE SAME TRACK

In order to get a better understanding of the rolling resistance reduction to be gained by having equipment wheels “track,” trials were repeated in the same track on both meadow and tilled soil. The results of these trials are shown graphically in fig. 15 and 16. By use of logarithmic coordinate paper it was found that the rolling resistance varied approximately as the —0.2 power of the number of the trial for trials on the meadow and as the —0.5 power for the tilled soil.

![Fig. 15. Repeated trials in the same track of 36 by 2.5-inch wheel on meadow. Speed, 2 miles per hour. Beginning at 400 feet from north end and 20 feet from east side of trial field, running north and working west. Timothy clover meadow (1 ton per acre). Webster loam. Soil moisture, 24 percent. Dry weight of soil per cubic foot, 67 lb.](image-url)
Fig. 16. Repeated trials in the same track of 36 by 4-inch wheel on tilled soil. Speed, 2 miles per hour. Beginning at 690 feet from north end and 170 feet from east side of trial field, running east and working south. Plowed 8 inches deep, double disced and harrowed. Webster loam. Soil moisture, dry basis, 12 percent. Dry weight of soil per cubic foot, 61 lb.

Fig. 17. Rolling resistance trials on meadow. (See fig. 18 to 21.)

ROLLING RESISTANCE TRIALS IN FIELD ON TIMOTHY MEADOW (see fig. 17)

Four sets of trials were made on timothy clover meadow to study the effects of wheel load, diameter and width on the roll-
ing resistance and slippage. The results are shown in fig. 18 to 21. The trials were made under the conditions given below the figures. Each value plotted as a point on the graphs represents the mean of a 100-foot trial.

Fig. 18. Rolling resistance and slippage of 16, 24, 36, 48 and 60 by 4-inch wheels with various loads on meadow. Estimate of standard deviation of rolling resistance determinations, 5.4 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 810 feet from north end and 490 feet from east side of trial field, running northwest and working northeast. Clarion loam. Timothy clover meadow (1 ton per acre). Soil moisture, dry basis, 11 percent. Dry weight of soil per cubic foot, 84 lb.
Fig. 19. Rolling resistance and slippage of 16, 24, 36, 48 and 60 by 4-inch wheels with various loads on meadow. Estimate of standard deviation of rolling resistance determinations, 5.4 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 300 feet from north end and 120 feet from east side of trial field, running south and working west. Timothy clover meadow (1 ton per acre), Webster loam. Soil moisture, dry basis, 15 percent. Dry weight per cubic foot, 70 lb.
Fig. 20. Rolling resistance and slippage of 24 by 0.25, 1, 2.5, 4, 7.5 and 14-inch wheels with various loads on meadow. Estimate of standard deviation of rolling resistance determination 5.4 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 300 feet from north end and 80 feet from east side of trial field, running south and working west. Timothy clover meadow (1 ton per acre). Webster loam. Soil moisture, dry basis, 14 percent. Dry weight of soil per cubic foot, 73 lb.
Fig. 21. Rolling resistance and slippage of 36 by 2.5, 4, 7.5 and 14-inch wheels with various loads on meadow. Estimate of standard deviation of rolling resistance determinations, 5.4 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 645 feet from north end and 385 feet from east side of trial field, running northwest and working northeast. Timothy clover meadow (1 ton per acre). Clarion loam. Soil moisture, dry basis, 12 percent. Dry weight of soil per cubic foot, 93 lb.
ON TILLED SOIL (see fig. 22)

Four sets of trials were made on tilled soil to study the effect of wheel load, diameter and width on rolling resistance and slippage. The trials were run under the conditions listed below the figures. Each value plotted on the chart represents the mean of a 100-foot trial. These results are shown graphically in fig. 23 to 26.

SOIL UNIFORMITY TRIALS

In order to obtain information on the error of determination and the uniformity of the soil with respect to area, occasional pairs of trials with the 36 by 2.5-inch wheel and a 1000-lb. load were scattered through the sets run on the meadow and tilled ground.

The plan of these trials was such that the variation between adjacent trials was primarily due to errors of determination, while soil variability was the primary cause of variation between the means of pairs of adjacent trials.

The results of these trials are given in fig. 27, page 364. This graph indicates rather more variability than one would like and probably explains the failure of certain of the results to plot as smooth curves.

The standard deviation of individual trials with respect to the mean of pairs of adjacent trials is 2.7 lb. for the meadow and 3.8 lb. for the tilled soil. With respect to the mean of a set of trials the standard deviation of the means of pairs of adjacent trials is 3.8 and 7.2, respectively.

If analysis of variance notation is used, the “variance” with-
in pairs of adjacent trials is 7.3 for the meadow and 14.4 for the tilled soil, and the “variance” between means of classes is 28.9 and 103.7, respectively.

Fig. 23. Rolling resistance and slippage of 24, 36, 48 and 60 by 4-inch wheels with various loads on tilled soil. Estimate of standard deviation of rolling resistance determinations, 7.0 lb. (see table 3). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 300 feet from the north end and 80 feet from east side of trial field, running southeast and working southwest. Plowed 8 inches deep, double disced and harrowed. Webster loam. Soil moisture, dry basis, 12 percent. Dry weight per cubic foot, 60 lb.
Fig. 24. Rolling resistance and slippage of 24 by 0.25, 1, 2.5, 4, 7.5 and 14-inch wheels with various loads on tilled soil. Estimate of standard deviation of rolling resistance determinations, 7.0 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 625 feet from north end and 170 feet from east side of trial field, running east and working south. Plowed 8 inches deep, double disced and harrowed. Webster and Clarion loam. Soil moisture, dry basis, 12 percent. Dry weight of soil per cubic foot, 51 lb.
Fig. 25. Rolling resistance and slippage of 36 by 2.5, 4, 7.5 and 14-inch wheels with various loads on tilled soil. Estimate of standard deviation of rolling resistance determinations, 7.0 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 300 feet from north end and 110 feet from east side of trial field, running south and working west. Plowed 8 inches deep, double disc ed and harrowed. Webster and Clarion loam. Soil moisture, dry basis, 9 percent. Dry weight per cubic foot, 64 lb.
Fig. 26. Rolling resistance and slippage of 48 by 4, 8, 14 and 20-inch wheels with various loads on tilled soil. Estimate of standard deviation of rolling resistance determinations, 7.0 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Speed, 2 miles per hour. Beginning at 500 feet from north end and 60 feet from east side of trial field, running south and working west. Plowed 8 inches deep, double disked and harrowed. Webster loam. Soil moisture, dry basis, 14 percent. Dry weight of soil per cubic foot, 55 lb.
Fig. 27. Rolling resistance trials with 36 by 2.5-inch wheel and 1000-lb. load run in pairs. These trials were interspersed as shown among the trials which supplied the data for fig. 18 to 21 and 23 to 26. The standard deviation of individual trials with respect to the mean of pairs of adjacent trials is 2.7 lb. for the meadow and 3.8 lb. for the tilled soil. With respect to the mean of a set of trials the standard deviations of the means of pairs of adjacent trials are 3.8 and 7.2, respectively.
In order to study the qualitative effects resulting from the combination of a soil surface layer of very low supporting ability and a sub-layer of relatively high supporting capacity, four sets of trials were run on 2 inches of loose dry sand on a concrete floor and two sets with 4 inches of sand. The laboratory set-up is shown in fig. 28. The depth-gauge rake shown in fig. 29 was used to maintain uniform depth and looseness from trial to trial. The results of these trials are shown graphically.
by fig. 30 to 35. Each value shown on the graphs represents the mean of two 30-foot trials.

Table 4, page 372, shows the sieve analysis and moisture content of the sand used. This table indicates a statistically (62) significant reduction in the grain size of the sand during the trials. A comparison of trials with the same loads and wheels at the beginning and end of the study indicated, however, that the corresponding change in rolling resistance was not serious.

![Graphs showing rolling resistance and slippage](image-url)

**Fig. 30.** Rolling resistance and slippage of 24, 36, 48 and 60 by 4-inch wheels with various loads on 2 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 4.7 lb. (see table 5). Exponents of equations for broken line curves selected by plotting data on logarithmic coordinate paper. The data given are the mean of two 30-foot trials. Speed, 1.5 miles per hour. Moisture content of sand, 0.3 percent.
Fig. 31. Rolling resistance and slippage of 24 by 0.25, 1, 2.5, 4, 7.5 and 14-inch wheels with various loads on 2 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 4.7 lb. (see table 5). Exponents for equations of broken line curves selected by plotting data on logarithmic coordinate paper. Data given are the mean of two 30-foot trials. Speed, 1.5 miles per hour. Moisture content of sand, 0.3 percent.

STANDARD DEVIATION OF ROLLING RESISTANCE AND SLIPPAGE DETERMINATIONS

Table 5, page 372, gives the best estimates available of the standard deviations for the rolling resistance and slippage determinations on the different soil conditions studied. This table indicates that very little confidence should be placed in differences of less than 10 to 15 lb. for trials of a given set on the meadow or 15 to 20 lb. for trials on the tilled soil.
Fig. 32. Rolling resistance and slippage of 36 by 2.5, 4, 7.5 and 14-inch wheels with various loads on 2 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 4.7 lb. (see table 5). Exponent for equation of broken line curve selected by plotting data on logarithmic coordinate paper. Data given are the mean of two 30-foot trials. Speed, 1.5 miles per hour. Moisture content of sand, 0.3 percent.
Fig. 33. Rolling resistance and slippage of 60 by 4, 8, 14 and 20-inch wheels with various loads on 2 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 4.7 lb. (see table 5). Exponent for equation of broken line curve selected by plotting data on logarithmic coordinate paper. Data given are the mean of two 30-foot trials. Speed, 1.3 miles per hour. Moisture content of sand, 0.3 percent.
Fig. 34. Rolling resistance and slippage of 24, 36, 48 and 60 by 4-inch wheels with various loads on 4 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 2.7 lb. (see table 5). Exponents of equations of broken line curves selected by plotting data on logarithmic coordinate paper. Data given are the mean of two 30-foot trials. Speed, 1.5 miles per hour. Moisture content of sand, 0.3 percent.
Fig. 35. Rolling resistance and slippage of 24 by 0.25, 1, 2.5, 4, 7.5 and 14-inch wheels with various loads on 4 inches of sand on concrete. Estimate of standard deviation of rolling resistance determinations, 2.7 lb. (see table 5). Exponents of equations of broken line curves selected by plotting data on logarithmic coordinate paper. These data are the mean of two 30-foot trials. Speed, 1.5 miles per hour. Moisture content of sand, 0.3 percent.
TABLE 4. SIEVE ANALYSIS OF SAND USED IN ROLLING RESISTANCE TRIALS, (Percent Retained on Sieves)

<table>
<thead>
<tr>
<th>Sieve no.</th>
<th>Before trials</th>
<th>After trials</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>Average</td>
</tr>
<tr>
<td>8</td>
<td>7.7</td>
<td>7.9</td>
<td>7.6</td>
</tr>
<tr>
<td>14</td>
<td>18.2</td>
<td>13.4</td>
<td>20.3</td>
</tr>
<tr>
<td>28</td>
<td>49.8</td>
<td>39.7</td>
<td>47.6</td>
</tr>
<tr>
<td>48</td>
<td>21.6</td>
<td>33.7</td>
<td>21.7</td>
</tr>
<tr>
<td>100</td>
<td>2.4</td>
<td>4.9</td>
<td>2.4</td>
</tr>
<tr>
<td>Pan</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Moisture before trials, 0.32 percent.
Moisture after trials, 0.28 percent.

TABLE 5. ESTIMATE OF STANDARD DEVIATION OF ROLLING RESISTANCE AND SLIPAGE DETERMINATIONS

<table>
<thead>
<tr>
<th>Wheel (in.)</th>
<th>Load (lb.)</th>
<th>Soil</th>
<th>1Rolling resistance (lb.)</th>
<th>2Slip-page percent</th>
<th>Adjacent trials</th>
<th>2Over a set of trials</th>
<th>Standard Deviation</th>
<th>Degrees of freedom</th>
<th>Standard Deviation</th>
<th>Degrees of freedom</th>
<th>Rolling resistance</th>
<th>Slip-page</th>
<th>Rolling resistance</th>
<th>Slip-page</th>
</tr>
</thead>
<tbody>
<tr>
<td>36x2.5</td>
<td>1000</td>
<td>Concrete</td>
<td>15.0</td>
<td>0.3</td>
<td>10</td>
<td>1.6</td>
<td>0.14</td>
<td>26</td>
<td>5.4</td>
<td>0.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x2.5</td>
<td>1000</td>
<td>Meadow (1 ton per acre)</td>
<td>109.5</td>
<td>2.4</td>
<td>14</td>
<td>2.7</td>
<td>0.30</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x2.5</td>
<td>1000</td>
<td>Plowed</td>
<td>360.8</td>
<td>21.5</td>
<td>7</td>
<td>16.6</td>
<td>0.65</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x2.5</td>
<td>600</td>
<td>Tilled</td>
<td>287.9</td>
<td>17.8</td>
<td>9</td>
<td>3.8</td>
<td>0.56</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x2.5</td>
<td>300</td>
<td>Tilled</td>
<td>128.8</td>
<td>13.4</td>
<td>8</td>
<td>2.8</td>
<td>0.43</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x7.5</td>
<td>300</td>
<td>Tilled</td>
<td>55.1</td>
<td>10.9</td>
<td>10</td>
<td>3.9</td>
<td>1.11</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36x14</td>
<td>300</td>
<td>Tilled</td>
<td>46.4</td>
<td>11.8</td>
<td>2</td>
<td>3.6</td>
<td>0.30</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24x0.25</td>
<td>1200</td>
<td>2 in. of sand on concrete</td>
<td>45.9</td>
<td>10.0</td>
<td>2</td>
<td>3.0</td>
<td>0.28</td>
<td>14</td>
<td>7.0</td>
<td>0.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1Mean of all values used in calculations.
2Each set of trials covered an area of 50 to 100 feet wide and 100 feet long and was conducted within a period of 36 hours.
3Timothy clover meadow plowed 8 inches deep, double disced and harrowed.

TRIALS WITH 36 BY 2.5-INCH WHEEL AND 1000-LB. LOAD ON VARIOUS SOIL CONDITIONS

In order to study the effect of different soil conditions on rolling resistance and the association of moisture content, volume weight, and "penetrometer" reading with rolling resistance, duplicate trials with the 36 by 2.5-inch wheel and a 1000-lb. load were run on as many soil conditions as possible. The results of these trials are given in table 6.
<table>
<thead>
<tr>
<th>No.</th>
<th>Soil type</th>
<th>Weight per cu. ft.</th>
<th>Moisture percent</th>
<th>Rolling resistance (lb.)</th>
<th>Slippage percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Concrete</td>
<td>--</td>
<td>--</td>
<td>15.0</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>Cinder Road</td>
<td>--</td>
<td>--</td>
<td>37.8</td>
<td>2.2</td>
</tr>
<tr>
<td>3</td>
<td>Clarion Loam</td>
<td>82</td>
<td>13</td>
<td>42.2</td>
<td>2.4</td>
</tr>
<tr>
<td>4</td>
<td>Clarion Loam</td>
<td>87</td>
<td>7</td>
<td>65.8</td>
<td>2.4</td>
</tr>
<tr>
<td>5</td>
<td>Webster Loam</td>
<td>78</td>
<td>13</td>
<td>81.4</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>Clarion Loam</td>
<td>84</td>
<td>10</td>
<td>91.4</td>
<td>2.4</td>
</tr>
<tr>
<td>7</td>
<td>Clarion Loam</td>
<td>69</td>
<td>27</td>
<td>117.8</td>
<td>--</td>
</tr>
<tr>
<td>8</td>
<td>Webster Loam</td>
<td>70</td>
<td>12</td>
<td>118.0</td>
<td>--</td>
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<tr>
<td>9</td>
<td>Webster Loam</td>
<td>73</td>
<td>14</td>
<td>133.4</td>
<td>--</td>
</tr>
<tr>
<td>10</td>
<td>Clarion Loam</td>
<td>80</td>
<td>10</td>
<td>123.3</td>
<td>1.4</td>
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<tr>
<td>11</td>
<td>Clarion Loam</td>
<td>76</td>
<td>7</td>
<td>134.1</td>
<td>1.9</td>
</tr>
<tr>
<td>12</td>
<td>Sand</td>
<td>81</td>
<td>11</td>
<td>151.4</td>
<td>1.5</td>
</tr>
<tr>
<td>13</td>
<td>Sand</td>
<td>75</td>
<td>20</td>
<td>161.4</td>
<td>1.5</td>
</tr>
<tr>
<td>14</td>
<td>Sand</td>
<td>78</td>
<td>17</td>
<td>164.4</td>
<td>1.5</td>
</tr>
<tr>
<td>15</td>
<td>Sand</td>
<td>99</td>
<td>--</td>
<td>192.2</td>
<td>6.8</td>
</tr>
<tr>
<td>16</td>
<td>Clarion Loam</td>
<td>67</td>
<td>9</td>
<td>217.8</td>
<td>13.8</td>
</tr>
<tr>
<td>17</td>
<td>Webster Loam</td>
<td>58</td>
<td>12</td>
<td>232.8</td>
<td>14.6</td>
</tr>
<tr>
<td>18</td>
<td>Webster Loam</td>
<td>63</td>
<td>11</td>
<td>231.4</td>
<td>15.8</td>
</tr>
<tr>
<td>19</td>
<td>Clarion Loam</td>
<td>62</td>
<td>6</td>
<td>256.9</td>
<td>15.4</td>
</tr>
<tr>
<td>20</td>
<td>Webster Loam</td>
<td>56</td>
<td>15</td>
<td>258.7</td>
<td>16.1</td>
</tr>
<tr>
<td>21</td>
<td>Webster Loam</td>
<td>53</td>
<td>14</td>
<td>265.4</td>
<td>17.0</td>
</tr>
<tr>
<td>22</td>
<td>Webster Loam</td>
<td>60</td>
<td>12</td>
<td>266.5</td>
<td>16.9</td>
</tr>
<tr>
<td>23</td>
<td>Clarion Loam</td>
<td>76</td>
<td>7</td>
<td>280.5</td>
<td>17.8</td>
</tr>
<tr>
<td>24</td>
<td>Webster Loam</td>
<td>64</td>
<td>9</td>
<td>317.9</td>
<td>19.8</td>
</tr>
<tr>
<td>25</td>
<td>Webster Loam</td>
<td>62</td>
<td>11</td>
<td>360.8</td>
<td>20.1</td>
</tr>
</tbody>
</table>

*Mean of two 100-ft. trials.
**Mean of 10 trials.
Fig. 36. "Penetrometer" reading and rolling resistance for 36 by 2.5-inch wheel and 1000-lb. load on various soil conditions (see table 6).

Fig. 37. Slippage and rolling resistance for 36 by 2.5-inch wheel and 1000-lb. load on various conditions (see table 6).

CORRELATION OF ROLLING RESISTANCE WITH SOIL MOISTURE, VOLUME WEIGHT, AND "PENETROMETER" READING

The correlation between "penetrometer" reading and rolling resistance is 0.97, and the standard error of estimate for the regression equation shown in fig. 36 is 23.4 lb.

The association between slippage and rolling resistance for the conditions given in table 6 is shown graphically by the scatter diagram of fig. 37, which indicates a rather close relationship.
There was also a high correlation between volume weight and rolling resistance. The coefficient in this case is \(-0.87\), and the standard error of estimate for the regression equation shown in fig. 38 is 52.2 lb.

These highly significant correlation coefficients (67) are evidence of the probable association of both decreased resistance to penetration and decreased volume weight with those changes in soil properties which tend to increase rolling resistance. "Penetrometer" readings and volume weight determinations
have, therefore, considerable promise as means of determining the rolling resistance characteristics of a field soil.

There was a slight correlation between soil moisture and rolling resistance. The coefficient is \(-0.21\). The negative sign of this coefficient would certainly not be true for all ranges of moisture and probably results from the following facts:

1. There was very little rain during the period of this investigation with the result that there was a continuous decrease in soil moisture.
2. The trials on tilled soil, where the rolling resistance was higher because of conditions other than moisture content, were run last when the soil was driest.
EFFECT OF AREA UPON SUPPORTING CAPACITY PER UNIT AREA

Figure 39 shows the use of the penetration discs of fig. 40 to study the effect of area upon the supporting capacity per unit area. By disconnecting the trolley drive wire and substituting a crank turned in synchronism with a properly adjusted metronome, it was possible to load all penetration discs at the uniform rate of 150-lb. per square inch per minute (see fig. 39) per unit area. Further, by the use of certain attachments it was possible
to obtain on the dynamometer chart a graph of the penetration while this load was being applied. The results obtained by the use of 2, 3 and 4-inch discs are shown by fig. 41. Each point of fig. 41 represents the mean of three trials. The differences for the 20-lb. per square inch load were tested for statistical significance by the method of analysis of variance (62). The differences for sand and tilled soil were found to be highly significant; the differences for stubble, significant and the differences for meadow, not quite significant.

It is to be noted that for the dry sand where cohesion is absent and shearing is controlled by friction, the larger discs had a greater supporting value per unit area; while for the timothy clover meadow and the rye stubble where cohesion was an important factor, the smaller discs showed a greater supporting capacity per unit area. For the tilled soil the smallest disc, 2 inches, gave by far the greatest supporting value per unit area. The 4-inch disc had, however, greater supporting value than the intermediate 3-inch size.

It would be of interest to study this soil condition further to see if there were in reality this reversal in the relationship between area and supporting value per unit of area.

Figure 42 shows the relation between load and penetration for a 36 by 2.5-inch stationary wheel. Although no direct application of static measurements to wheel performance has been developed, further studies of this relationship for different wheel diameters and widths on different soils might contribute toward a more fundamental solution of the rolling resistance problem.

Figure 43 shows the continued penetration of a 3-inch disc
after being loaded to 50 lb. per square inch at the rate of 12.5 lb. per square inch per second. These points represent the mean of three trials and are further evidence of the time-lag between application of load and the attainment of equilibrium of penetration previously offered as an explanation for the differences between the constant and variable load trials shown in table 1, page 352.

**DISCUSSION AND INTERPRETATION OF ROLLING RESISTANCE TRIALS**

For the following reasons it seems impracticable to attempt to set up a rational equation for the rolling resistance of a rigid wheel operating on a friable medium:

1. Even if an ideal perfectly rigid wheel and a homogenous isotropic medium are assumed, such a rational derivation would become very involved and would require a number of simplifying assumptions and approximations in order to be serviceable.

2. Under most agricultural field conditions the soil is not homogeneous, but its physical and structural characteristics vary from the surface downward.

3. For many agricultural field conditions the soil is not isotropic, that is, its stress-strain moduli are not independent of direction.

4. The response of certain soil conditions to certain types of wheel variation may even have maxima or minima. (Note the width-rolling resistance curves of fig. 32 and 33 and the curves for tilled soil of fig. 41.)

5. At least a part of the rolling resistance of rigid wheels on agricultural soil is the result of soil deformations which are beyond the range of stable equilibrium. This is indicated

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**Fig. 43.** Continued penetration of 3-inch disc after being loaded to 50 lb. per square inch at the rate of 12.5 lb. per square inch per second.
by permanent deformation and rupture, a situation which is somewhat analogous to the state of turbulence in the field of hydraulics.

Therefore, it seems best to consider the phenomena of rolling resistance primarily from the standpoint of the qualitative effects of certain individual factors, such as those of load, diameter and width.

FACTORS CONTRIBUTING TO ROLLING RESISTANCE

The following factors appear to be responsible for the rolling resistance of a rigid wheel operating on a friable medium:

1. Friction in axle bearing (relatively small for the conditions of this investigation).
2. Displacement of soil. (See fig. 6 and 7.)
   a. Forward.
   b. Lateral.
   c. Downward.
3. Friction between wheel and soil.
   a. Rim face (probably not very important).
   b. Rim edges.
4. Adhesion of soil to wheel.
5. Impact (important at higher speeds and on rougher surfaces).

EFFECT OF LOAD UPON ROLLING RESISTANCE

In order to place the investigation upon a more mathematical basis, the data were plotted on logarithmic coordinate paper to determine approximately the best value for the exponents of $x$ for the equations plotted as broken lines on fig. 18 to 21, 23 to 26 and 30 to 35. These values are shown as $n$, $m$ and $p$ of tables 7 to 9. Table 7 shows that the value of the exponent $n$ varied from 0.59 to 1.26 depending upon wheel and soil conditions. A study of this table and fig. 18 to 21, 23 to 26 and 30 to 35 indicates a tendency for this exponent to be smaller as the wheels are increased in diameter; a tendency for it to be smaller with increased width where such increases in width decrease rolling resistance but larger where such increases in width increase the rolling resistance. Stated more directly, any wheel change which tends to reduce the rolling resistance for a given load also tends to reduce the relative effect of increased load.

While the evidence is not conclusive, it appears that the value of this exponent would be of the order of 1.0 for smooth hard surfaces, greater than 1.0 for softer surfaces which are relatively uniform from the surface downward and less than 1.0 where there is a layer of low supporting capacity with a sublayer of relatively high supporting capacity and where the load and wheel conditions are such as to cause the surface layer of low supporting ability to be of importance. It is of course realized


TABLE 7. VALUES OF EXPONENT, \( n \), IN EQUATION* ROLLING RESISTANCE

\[ = (\text{CONSTANT}) (\text{LOAD})^n. \]

<table>
<thead>
<tr>
<th>Diameter (in.)</th>
<th>Width (in.)</th>
<th>Soil conditions</th>
<th>Approximate value of ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24–60</td>
<td>4</td>
<td>Meadow</td>
<td>0.72</td>
</tr>
<tr>
<td>16–60</td>
<td>4</td>
<td>&quot;</td>
<td>0.81</td>
</tr>
<tr>
<td>24</td>
<td>0.25–14</td>
<td>&quot;</td>
<td>0.95</td>
</tr>
<tr>
<td>36</td>
<td>2.5–14</td>
<td>&quot;</td>
<td>0.76</td>
</tr>
<tr>
<td>24–60</td>
<td>4</td>
<td>Tilled Soil</td>
<td>1.26</td>
</tr>
<tr>
<td>24</td>
<td>0.25–14</td>
<td>&quot;</td>
<td>1.02</td>
</tr>
<tr>
<td>36</td>
<td>2.5–14</td>
<td>&quot;</td>
<td>1.15</td>
</tr>
<tr>
<td>48</td>
<td>4.0–20</td>
<td>&quot;</td>
<td>0.85</td>
</tr>
<tr>
<td>24–60</td>
<td>4</td>
<td>2 in. of sand on concrete</td>
<td>0.74</td>
</tr>
<tr>
<td>24</td>
<td>0.25–14</td>
<td>&quot;</td>
<td>0.74</td>
</tr>
<tr>
<td>36</td>
<td>2.5–14</td>
<td>&quot;</td>
<td>0.76</td>
</tr>
<tr>
<td>60</td>
<td>4.0–20</td>
<td>&quot;</td>
<td>0.59</td>
</tr>
<tr>
<td>24–60</td>
<td>4</td>
<td>4 in. of sand on concrete</td>
<td>0.88</td>
</tr>
<tr>
<td>24</td>
<td>0.25–14</td>
<td>&quot;</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*Equation \( y = kx^{0.72} \) of fig. 18 and similar equations of fig. 19–21, 23–26 and 30–35.

that such a simple exponential equation is only an approximation and in many cases an approximation for only a limited range of loads. This latter fact is particularly true if the curve is bending downward in the range for which the exponential expression is a satisfactory approximation. Engineering experience in the field of the resistance of materials indicates that if the load were increased sufficiently, the curve would finally bend upward. This upward bend would result in a compound curve which could not be approximated by a simple exponential expression. Figure 18 appears to be an example of this situation, and fig. 19, 20, 21, 24 and 26 give slight indication of the same tendency.

With regard to the factors contributing to rolling resistance which were listed above, there is every reason to believe that increased load would increase the magnitude of all of them except possibly adhesion and impact. There is, however, no question but that for a given wheel and soil condition, increased load would mean increased rolling resistance.

EFFECT OF WHEEL DIAMETER UPON ROLLING RESISTANCE

Table 8 shows that the value of the exponent \( m \) varied from \(-0.48\) to \(-0.69\) depending upon soil conditions. The relative effects of changes of diameter appeared to be more nearly uniform from one soil condition to another than those of load or width, although there is some evidence that this exponent assumes a larger negative value for softer soil conditions. Unfortunately "effect of diameter" trials were made for only one wheel width (4 inches).
There are rational reasons why an increase in diameter would tend to decrease the contribution of each of the individual causes of rolling resistance listed above, except possibly that of adhesion which it might not materially change, although increased diameter would certainly not increase the effect of adhesion.

Thus it would appear that for a given wheel width and load and given soil conditions increased wheel diameter would always tend to result in decreased rolling resistance.

**TABLE 8. VALUES OF EXPONENT, m, IN EQUATION *, ROLLING RESISTANCE = (CONSTANT) (DIAMETER)^m.**

<table>
<thead>
<tr>
<th>Width (in.)</th>
<th>Load (lb.)</th>
<th>Soil condition</th>
<th>Approximate value of m</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>300—1200</td>
<td>Meadow</td>
<td>-0.48</td>
</tr>
<tr>
<td>4</td>
<td>300—1200</td>
<td>Meadow</td>
<td>-0.50</td>
</tr>
<tr>
<td>4</td>
<td>300—1200</td>
<td>Tilled soil</td>
<td>-0.69</td>
</tr>
<tr>
<td>4</td>
<td>300—1200</td>
<td>2 in. of sand on concrete</td>
<td>-0.56</td>
</tr>
<tr>
<td>4</td>
<td>300—1200</td>
<td>4 in. of sand on concrete</td>
<td>-0.66</td>
</tr>
</tbody>
</table>

*Equation \( y = kx^{-0.48} \) of fig. 19 and similar equations of fig. 18, 23, 30 and 34.

**EFFECT OF RIM WIDTH UPON ROLLING RESISTANCE**

Table 9 shows that the value of the exponent \( p \) varied from -0.41 to 0.45 depending upon wheel diameter and soil conditions. There is evidence indicating that where the soil is relatively uniform from the surface downward with very little tendency for adhesion to the wheel, the sign is negative and that where there is a surface layer of definitely lower supporting value and appreciable adhesion of soil to the wheel rim, the sign tends to be positive (see fig. 31 and 35). This tendency becomes more marked as the wheel diameter is decreased or the load increased.

Changing the rim width has qualitatively different effects upon the individual factors contributing to rolling resistance. Increasing the rim width decreases the work of the downward, and probably the lateral, displacements of the soil, and the loss caused by friction between the soil and the edges of the wheel rim has little effect upon the axle friction and under many conditions little effect upon impact losses. On the other hand, increasing the wheel width might or might not increase the work of the forward displacement of the soil, the friction loss between the soil and the face of the wheel rim and would increase the adhesion of soil to the wheel rim if the soil were adhesive.

The relative magnitude of these opposing effects of increasing the wheel width would depend upon wheel diameter, load and soil conditions. For certain combinations of these factors the width-rolling resistance curve passes through a maximum, thus
TABLE 9. VALUES OF EXPONENT, p, IN EQUATION *
ROLLING RESISTANCE
= (CONSTANT) (WIDTH)
P.

<table>
<thead>
<tr>
<th>Diameter (in.)</th>
<th>Load (lb.)</th>
<th>Soil Condition</th>
<th>Approximate value of p</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>300—1200</td>
<td>Meadow</td>
<td>—0.12</td>
</tr>
<tr>
<td>36</td>
<td>300—1200</td>
<td>&quot;</td>
<td>—0.12</td>
</tr>
<tr>
<td>24</td>
<td>300—1200</td>
<td>Tilled soil</td>
<td>—0.41</td>
</tr>
<tr>
<td>36</td>
<td>300—1200</td>
<td>&quot;</td>
<td>—0.24</td>
</tr>
<tr>
<td>48</td>
<td>300—1200</td>
<td>&quot;</td>
<td>—0.20</td>
</tr>
<tr>
<td>24</td>
<td>300—1200</td>
<td>2 in. of sand on concrete</td>
<td>0.30</td>
</tr>
<tr>
<td>36</td>
<td>300—1200</td>
<td>&quot;</td>
<td>——**</td>
</tr>
<tr>
<td>48</td>
<td>300—1200</td>
<td>&quot;</td>
<td>——**</td>
</tr>
<tr>
<td>24</td>
<td>300—1200</td>
<td>4 in. of sand on concrete</td>
<td>0.45</td>
</tr>
</tbody>
</table>

*Equation y = kx\(^{-0.12}\) of fig. 20 and similar equations of fig. 21, 24, 25, 26, 31 and 35
**For these two sets of trials the wheel width-rolling resistance curves passed through a maximum and therefore cannot be even approximated by a simple exponential expression.

indicating a worst width (width of highest rolling resistance). This possibility is illustrated by fig. 32 and 33 and the curve for the 60-inch wheels shown in fig. 35.

This possible reversal of the response of rolling resistance to changes of wheel width is probably responsible for the erratic results which have been reported by certain investigators.

WHEEL SLIPPAGE

In general, any wheel or soil change which tends to increase rolling resistance also tends to increase the slippage. However, where there is a surface layer of lower supporting value, this may not be true. Under such circumstances increasing the load may cause the firmer subsurface to become dominant and thus tend to decrease slippage as the load is increased. (See fig. 30, 31 and 34.)

SUGGESTIONS FOR FURTHER INVESTIGATIONS

1. The effects of soil moisture, texture and structure upon the performance of transport wheels have not been adequately investigated.
2. The effect of changes in the shape of the rim cross section deserves scientific attention.
3. A comparative study of rigid and pneumatic tired transport wheels would be particularly timely.
4. Any one of the several phases of the problem considered during the present investigations might well be given more complete and intensive study.
CONCLUSIONS

1. The impact resulting when a rigid wheel strikes a solid obstruction is:
   a. Proportional to the square of the speed S.
   b. Increased by increasing the effective height h of the obstruction or by decreasing the radius r of the wheel.
   c. A function of the ratio R of the wheel radius r to the effective height of the obstruction h.
   d. A function of the angle θ between the vertical radius, O to Q, and the radius, O to P, striking the obstruction.
   e. Proportional to the product of the secant and the square of the tangent of the angle θ.
   f. Approximately proportional to the square of the tangent of θ for small values of the angle θ and the corresponding large values of the ratio R.
   g. Of the same order as the impact which would result by falling from a height, H, where

   \[ H = \frac{0.0334 \, S^2 R (2R - 1)}{(R - 1)^3} = 0.0334 \, S^2 \tan^2 \theta \sec \theta. \]

2. The mechanics which have been developed for the rolling of a rigid wheel on an elastic surface cannot be applied to such a wheel when rolled on a non-elastic friable medium.

3. During one revolution a rigid wheel rolling on a non-elastic friable medium will travel a distance greater than the length of its circumference.

4. The rolling of a rigid transport wheel causes a permanent soil displacement parallel to the direction of its travel.

5. The change in position of soil particles caused by the passage of a rigid transport wheel is attained by curved rather than straight line motion.

6. One hundred-foot variable load trials are unreliable for the study of the effect of load upon the rolling resistance of rigid transport wheels because of the time lag between the application of a given load and the attainment of equilibrium penetration with its corresponding rolling resistance.

7. For speeds up to 5 miles per hour and operation on agricultural soils, the effect of speed upon the rolling resistance of rigid transport wheels appears to be of minor importance compared to the effects of diameter and width, although there was some evidence of slightly greater rolling resistance at the higher speeds.

8. The rolling resistance on agricultural soils of rigid transport wheels for trials repeated in the same track tends to be proportional to some power of the number of the trial. (For the trials reported in this investigation the value of the exponent was approximately −0.2 on meadow and −0.5 on tilled soil.)
9. For the range of areas tried (approximately 3 to 12 square inches) increasing the supporting area increases the supporting capacity per unit area for cohesionless soils and decreases it for soils with cohesion. For soils of intermediate characteristics there is some evidence that this relationship passes through a minimum with the lowest supporting value per unit area occurring at some intermediate area.

10. The rolling resistance of rigid transport wheels is closely associated with the soils resistance to penetration. (The simple correlation coefficient for these investigations was —0.97.)

11. The rolling resistance is also associated with volume weight. (The simple correlation coefficient for these investigations was —0.87.)

12. Because of the limited range of soil moisture compared to the range of other soil characteristics it is not possible to draw any conclusions concerning the relationship of soil moisture to rolling resistance.

13. The factors contributing to rolling resistance are:
   a. Friction in the axle bearing.
   b. Forward, lateral and downward displacement of the soil.
   c. Friction between the soil and the face and edges of the rim.
   d. Adhesion of soil to the wheel.
   e. Impact of wheel upon surface irregularities.

14. Within the range of usual operating conditions the effect of changes in wheel load, diameter and width can usually be approximated by a simple exponential equation of the form \( Y = KX^c \), where \( Y \) is the rolling resistance, \( X \) is the load, diameter or width, \( K \) is a constant depending upon the soil and wheel conditions and the units of measurement used, and \( c \) is a constant depending upon soil and wheel conditions but independent of the units of measurement. (For this investigation the values of \( c \) were of the order of 0.6 to 1.3 for the effects of load, —0.5 to —0.7 for the effects of diameter and —0.5 to 0.5 for the effects of width.)
LITERATURE CITED


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(49) Mullis, I. B. What subgrade investigations have shown during the past year. Proceedings of the Highway Research Board, 4:36-40. 1924.


