6-1951

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Keywords
Ames Laboratory

Disciplines
Atomic, Molecular and Optical Physics | Physics | Quantum Physics

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CLOUD CHAMBER MEASUREMENT OF ELEC­TRON PAIRS FOR DE­TER­MINATION OF SYNCHRO­TRON SPECTRUM

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June 1951

Ames Laboratory
CLOUD CHAMBER MEASUREMENT OF ELECTRON PAIRS FOR
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Richard H. Stokes and L. Jackson Laslett

From the Department of Physics
Iowa State College

I. ABSTRACT

The x-ray spectrum of the Iowa State College synchrotron operating at 65 MeV. has been measured. The method consisted of the observation, in a magnetic cloud chamber, of the momentum of each member of electron pairs produced in the air filling of the chamber. Using the theoretical values of cross section for pair production and the results of recent experiments, the energy spectrum of the x-ray quanta was computed. Due to the pronounced forward directional characteristic of electron pairs produced at extreme relativistic energies, discrimination of one energy region with respect to another was avoided and no solid angle corrections were necessary. The results are in agreement with the energy spectrum predicted by the Bethe-Heitler theory.

During observation of a portion of the data, the synchrotron beam intensity was monitored and related to the reading of a Victoreen thimble ionization chamber in a single ended lead cylinder one-eighth inch thick. This procedure enabled a value to be obtained for the flux of quanta and energy, related to the reading of the Victoreen chamber in r units. A quantum flux of $6.6 \times 10^4$ quanta per square cm. per r unit was obtained for the energy flux. These values are in agreement with the possible comparisons with independent work and will

This paper is based on PhD thesis by Richard H. Stokes submitted in June, 1951.
enable absolute values of the cross section for nuclear gamma reactions to be made.

The method used for measurement of the stereoscopically projected electron tracks is described in the Appendix and expressions are derived for obtaining the magnitude of the electron momentum from the measured quantities.

II. INTRODUCTION

A. Statement of Problem

Since 1941, the development of the betatron and synchrotron accelerators have made possible studies of the interaction of nuclei with high energy x-radiation. In these accelerators the electron beam usually is intercepted by a target of high atomic number to produce a continuous x-ray or bremsstrahlung spectrum which is the useful output of the machine. To interpret most experiments in which these electron accelerators are used, it is necessary to know the distribution in energy of the quanta and to have a measure of the number of quanta in a given energy interval so that absolute determinations of the cross section for nuclear processes can be made. Aside from these considerations, the fundamental process of decelerating electrons to produce continuous x-radiation is of interest. The target conditions under which betatrons and synchrotrons usually operate are those giving greatest x-ray output and these are somewhat incompatible with the situation in which a direct comparison with the theory is possible. Some comparison with the theoretical results can be made, however.

Early in 1950, the Iowa State College synchrotron was operating successfully at energies up to 70 Mev. A cloud chamber system which had been under construction for some time was being completed and it was thought that a cloud chamber study of the synchrotron x-ray spectrum could be made. It was expected that the process of electron pair production could be observed in the cloud chamber under rather ideal conditions, which would allow the theory of pair production to be used directly in deducing the x-ray energy spectrum from a measurement of the energy distribution of electron pairs. Two recent experimental investigations of the pair production process at high energies were used as a guide to the reliability of the theory under various conditions.
B. Review of Literature

The bremsstrahlung process has been described theoretically by Bethe and Heitler (1) and the results are given by Heitler (2) and by Rossi and Greisen (3). These calculations make use of the Born approximation, which is valid for light elements and relativistic energies, and use the Thomas-Fermi atomic model to describe the screening of the nucleus by the electronic charge distribution. The spectra predicted by the Bethe-Heitler theory for thin targets are presented in the form of curves with the product of cross section and quantum energy plotted against the quantum energy. These values have been integrated over the angles of both the decelerated electron and the outgoing quantum. Calculations have also been made by Parzen (4) and by Bess (5) without use of the Born approximation. In general, where comparisons are possible, these give nearly the same result for the cross section, but indicate differences in the angular distribution of the quanta. Koch and Carter (6) have measured the bremsstrahlung spectrum of a 19.5 Mev. betatron by observing pairs produced in a cloud chamber filled with air. At this low energy, rather large corrections must be made to compensate for the energy discrimination of the cloud chamber system. Their results indicate some disagreement in their mid-energy region. Powell, Hartsough and Hill (7) have recently measured the spectrum of the Berkeley 322 Mev. synchrotron. Using a cloud chamber to observe pairs produced in a lead foil .001 inch thick, they obtained results agreeing with the Bethe-Heitler theory.

Calculations on the process of electron pair production also have been made in the paper of Bethe and Heitler (1) and a survey of results is given in Heitler (2) and in Rossi and Greisen (3). The Born approximation is used and, where necessary, the Thomas-Fermi atomic model is used to describe the screening. Total cross section as a function of quantum energy is calculated and presented in graphical form. In the low energy range, experimental investigations (8) have indicated good agreement. The total cross section for various elements has been measured by Walker (9) at 17.6 Mev. and by Lawson (10) at 88 Mev. Both of these investigations indicate good agreement with the theory at low atomic numbers, but differences up to 15% are found in heavy elements. Due to the use of the Born approximation in the theory, such errors are expected and are clearly shown by the experiments.

C. Description of Experiment

The experiment that seemed best in the light of the above considerations was as follows: A single pulse of synchrotron x-radiation was passed through a magnetic cloud chamber immediately following the chamber.
expansion. Electron pairs formed in the gas of the chamber were photographed and, from curvature measurements on each member of the pair, the quantum energy was determined. The tracks formed under these conditions are not distorted by the expansion, nor is there appreciable diffusion of the ion tracks before vapor is condensed upon them. Thus a sharp, distortion-free track is recorded, from which a reliable value of curvature may be obtained. Air was used as the non-condensible gas in the chamber and as the target for producing pairs. Having a low atomic number target permits direct use of the cross section for pair production, and in the energy range of interest allows the screening to be almost entirely neglected. Thus, any errors due to the Thomas-Fermi model are minimized. Also, a gas target has the advantage of making pairs easily identifiable and avoids the effects of turbulence present near foil targets. Since the target was distributed in space, a somewhat higher x-ray pulse intensity was usable before overlapping tracks made identification of pairs difficult. This gave a greater yield of pairs per expansion of the chamber.

Due to the high energy range and the consequent strong forward-directed characteristic of the pairs, most of the tracks of the electrons and positrons lay nearly in a plane normal to the magnetic field. Even with a relatively shallow portion of the chamber illuminated, this allowed observation of a sufficient length of track in all cases so that a certain minimum percentage error of curvature measurement could be maintained, without the necessity of rejecting some tracks, which would have resulted in an undesirable energy discrimination.

III. APPARATUS

A. Cloud Chamber

The cloud chamber system is shown in Figures 1, 2 and 3. A region 9 inches in diameter and 1 inch high is illuminated and is useful for observing tracks. Accurately regulated air pressure is used to compress the chamber to between 2 and 3 pounds per square inch above atmospheric pressure. The expansion is effected by opening a large capacity valve which rapidly reduces the volume to atmospheric pressure. The actuating air enters the lowest section of the chamber below a rubber diaphragm which is deflected somewhat upward when the chamber is in the compressed condition. Line air pressure is used and is regulated by two stages of pressure reducers, the one nearest the chamber being a special low pressure Hoke reducer, modified for the present application. Expansion of the chamber causes the air in the illuminated section to pass through a black velvet covering and the chamber floor, which consists
Fig. 1--Cloud chamber assembly.
Fig. 2--Cloud chamber system.
Fig. 3--Cloud chamber.
of a three-eighth inch thick brass perforated plate. The plate and
the black velvet act as a high impedance isolating the lower turbulent
section from the upper section which must be as free from turbulence
as possible. The release valve is of the self sealing type, using the
unbalanced area principle to make the sealing force proportional to
the pressure. A solenoid is used to trip the valve which is auto-
matically reset after expansion. Tests have shown that there is a con-
stant delay of about one-fiftieth of a second between the electrical
impulse to the solenoid and the time that the valve is fully open.

Ions are swept from the upper section of the chamber by an electric
field of 70 volts per cm. applied between expansions. A circular piece
of Nesa conducting glass forms the top of the chamber, and the clearing
field is produced by an 1100 volt potential between this glass and the
chamber floor.

Illumination of the chamber is provided by a General Electric
FT-422 flashtube having an 18 inch lighted length. This tube is at
the focus of a parabolic cylindrical reflector having an attached
collimator. A 10 kilovolt pulse from a transformer initiates the
flashtube, discharging a 200 microfarad capacitor charged to 2000 volts.
An inductance in series with the discharge circuit was used to
lengthen the light flash, avoiding difficulties from the failure of
the photographic reciprocity law at short time intervals.

Tracks were recorded with a 35 mm. automatic camera having an
f/3.5 lens. The camera was built around a University of Illinois
design which allowed each frame to be clamped accurately in a repro-
ducible position. A three-dimensional description of the tracks was
obtained by using a commercial stereoscopic attachment on the camera
lens. To obtain curvature measurements, the tracks were projected
through the optical system of the camera used to record them. A
translucent screen arrangement similar to one used by Brueckner et
al (11) allows the curvatures to be measured.

B. Magnetic Coils

A magnetic field was produced in the cloud chamber by a pair of
coils similar to the Helmholtz type. The mean radius of the coils
was 30 cm. and the cross section of the windings was 14.25 cm.,
radially, by 15.25 cm., axially. These are approximately in the ratio
of the square root of 31 over 36, a uniformity condition suggested by
the field expansions of Ruark and Peters (12). Ference, Shaw and
Stephenson (13) have shown that a separation between coils less than
that given by the Helmholtz condition is useful in producing a field
of great uniformity over a considerable volume. With the measurements of Ference as a guide, calculations were made using the expansions of Ruark. These showed that a 5% reduction of the separation was best for uniformity over the useful volume of the cloud chamber. The calculated field was uniform to one-half per cent over this volume and had a constant of 16.95 gauss per ampere. Fluxmeter measurements confirmed the calculations excellently, both as to magnitude and uniformity. A 60 kilowatt D. C. generator produced a field of 2500 gauss with a current of 150 amperes. The field of the generator was connected to an amplidyne which in turn was energized by a small relay and battery.

Figure 4 shows the method of constructing the magnetic coils and Figure 5 shows a coil after the winding operation was completed. Each coil was constructed of 550 turns of 0.25 inch diameter, 0.065 inch wall, copper tubing. The tubing had glass fiber insulation bonded with silicone varnish and was wound on brass coil forms. Silicone varnish was applied between successive layers and the varnish was cured after the winding was completed by constructing a brick furnace around each coil and passing current through the conductors. The windings were thus bonded into a solid mass, preventing movement due to magnetic forces. The elimination of all organic insulation in the coils enables them to be operated at high current values in spite of the higher temperatures involved. Cooling of the coils was done by passing soft water at 500 pounds per square inch through the hollow conductors in 12 parallel paths. The paths were electrically separated by rubber hose sections.

C. Circuits and Collimation

The control system of the cloud chamber is shown in block diagram form in Figure 6 and in detail in Figure 7. One of the main problems was that of synchronizing the cloud chamber expansion with the pulse of x-radiation from the synchrotron. As shown in the block diagram, after the magnetic field had been turned on, the cloud chamber interrogates the synchrotron and receives an answer in the form of a pulse that is in phase with the time of the x-radiation. This pulse, after delays in both the synchrotron and the cloud chamber circuits, enabled the chamber to be just expanded when the x-ray pulse occurs. The quality of the data is very sensitive to this timing and the delays must be reproducible. Delays in non-critical points in the circuit were produced by simple arrangements similar to those described by Getting (14), using OA4 gas triodes and relays.

To eliminate difficulties with a high background of pair electrons, a collimated beam of radiation was allowed to enter the chamber through
Fig. 4--Magnetic coil construction.
Fig. 5—Magnetic coil.
Cloud Chamber Control Circuit — Block Diagram

Fig. 6
2.0 - Magnetic Field Control
5.0 - To Cloud Chamber Table
5.5 - Interconnection - Expansion Power Supply
6.0 - Flash Tube
7.0 - Camera
7.5 - Mechanical Shutter - Source
6.5 - Incand Light Source
4.0 - Warning Light - Synch Control Desk
8.0 - Mech Counter

Relay armatures normally left
Relays are Struthers-Dunn (4200 ~, 31 ma.)
Tubes are 6A4-G unless otherwise marked

Cloud Chamber Control Circuit

Fig. 7
a thin window of low atomic number. The collimator was a stack of lead bricks 18 inches thick with a five-eighth inch diameter hole. The collimator was 110 inches from the synchrotron target and the beam entered the chamber at 170 inches through a .005 inch beryllium window. No exit window was used and no electrons were observed where the beam left the chamber through the three-eighths inch thick glass wall.

IV. PROCEDURE

Figure 8 is a photograph of the experimental arrangement, showing the cloud chamber with the collimator and the synchrotron in the background. A mixture of two parts ethyl alcohol and one part water was introduced into the chamber to provide the condensible vapor. Data were taken over a period of one month during which about 4000 photographs were obtained. A typical pair having an 18.6 Mev. total energy is shown in Figure 9. The synchrotron beam pulse intensity was reduced by a factor of 50 to 100 from its peak value and was monitored by an ionization chamber. Since only a single pulse of radiation was used, the ballistic deflection of the ionization chamber monitor was observed. Some difficulty was experienced in the interaction of the cloud chamber magnetic field with the synchrotron. With the synchrotron operating, turning on the cloud chamber field reduced the x-ray beam intensity by an amount dependent upon the exact tuning conditions of the synchrotron. By first tuning with the cloud chamber field off, it was found that a single adjustment of the synchrotron compensation coils would counteract completely the effect of turning the field on. For monitoring that part of the data used to determine the number of quanta, a Victoreen thimble ionization chamber in a one-eighth inch thick lead cylinder with closed end was used. Since the Victoreen chamber measures only large dosages of radiation, it was exposed to the machine when operating continuously. The ionization chamber monitor was used, over a portion of its range known to be linear, to determine the ratio between the x-ray beam intensity at the high level of continuous operation and the low level beam intensity used to produce pairs in the cloud chamber.

Some difficulty was experienced with temperature drifts due either to changes in room temperature or to slight heating of the magnetic coils. Since the chamber must operate without visual monitoring when the x-ray beam is on, the changes in proper expansion ratio with thermal drifts were somewhat troublesome. Usually data could be obtained over intervals of 40 to 50 minutes, before it was necessary to interrupt for expansion ratio adjustment.
Fig. 8--Experimental arrangement.
Fig. 9--Electron-positron pair.
Linagraph Ortho film exposed in lengths of 35 ft. was developed in a tank using D-19. For curvature measurements the tracks were projected through the camera used to record the data. The quantities measured were: the apparent radius of curvature of a segment of the helical electron track as determined by matching a circle at each end and the angle between the chord and a plane normal to the magnetic field. Using these three quantities the momentum of each member of the pair is obtained (See Appendix) and the total energy of the pair is computed. Only pairs originating in the forward portion of the chamber were selected. A small enough portion was used to enable almost all tracks to have a ratio of the chord squared to the radius of curvature greater than one centimeter, corresponding to about 15% maximum allowable percentage error in the curvature determination. Only a negligible number of tracks were such that this condition was impossible to fulfill. A majority of the tracks had chord lengths which gave curvature measurements with an accuracy of 3 to 10%.

V. RESULTS AND CONCLUSIONS

A. Spectrum

After measuring curvatures and computing the total energy of all pairs, the pairs were grouped according to energy into intervals 5 Mev. wide. The number of pairs in the various intervals is shown in Table 1. For the mid-energy, E, of each of these intervals the relative cross section for pair production, $\sigma$, was read from the curve given in Bethe and Heitler (1). The curve used was the one for $\text{H}_2\text{O}$, which gives a slight difference from the unscreened case at the higher energies. In each energy interval the number of pairs is multiplied by E and divided by $\sigma$ giving the relative value of the x-ray energy spectrum. A plot of these results is shown in Figure 10. The curve is the Bethe-Heitler bremsstrahlung energy spectrum for air as obtained from Rossi and Greisen (3). An upper limit of 65 Mev. was used for this curve, corresponding to the operating conditions of the synchrotron as given by a calibration chart having as a basis the measurement of magnetic field strength at the time the electron beam strikes the target.

All data were taken with the cloud chamber magnetic field at nearly the same value. This causes some discrimination at the lowest energies due mainly to a decreasing ability to identify pairs. When pronounced inequalities in the energy division between members of a pair occur, the low energy positron, a small circle, can be missed, either due to overlapping with other tracks, or because it makes a large
angle with a plane normal to the magnetic field. This effect causes the lowest energy experimental point to fall somewhat below the theoretical curve. The lowest energy interval is very greatly affected in this way and no attempt was made to obtain data in this interval.

Table 1

<table>
<thead>
<tr>
<th>Total Energy of Pair in Mev.</th>
<th>N</th>
<th>$\sigma$</th>
<th>NE</th>
<th>$\Delta % = \frac{1}{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 - 9.9</td>
<td>123</td>
<td>2.90</td>
<td>318</td>
<td>9.0</td>
</tr>
<tr>
<td>10 - 14.9</td>
<td>119</td>
<td>4.35</td>
<td>344</td>
<td>9.2</td>
</tr>
<tr>
<td>15 - 19.9</td>
<td>90</td>
<td>5.20</td>
<td>303</td>
<td>10.5</td>
</tr>
<tr>
<td>20 - 24.9</td>
<td>67</td>
<td>5.90</td>
<td>255</td>
<td>12.2</td>
</tr>
<tr>
<td>25 - 29.9</td>
<td>64</td>
<td>6.50</td>
<td>270</td>
<td>12.5</td>
</tr>
<tr>
<td>30 - 34.9</td>
<td>59</td>
<td>7.10</td>
<td>270</td>
<td>13.0</td>
</tr>
<tr>
<td>35 - 39.9</td>
<td>44</td>
<td>7.50</td>
<td>220</td>
<td>15.0</td>
</tr>
<tr>
<td>40 - 44.9</td>
<td>52</td>
<td>7.80</td>
<td>283</td>
<td>13.8</td>
</tr>
<tr>
<td>45 - 49.9</td>
<td>46</td>
<td>8.10</td>
<td>270</td>
<td>14.7</td>
</tr>
<tr>
<td>50 - 54.9</td>
<td>36</td>
<td>8.30</td>
<td>227</td>
<td>16.6</td>
</tr>
<tr>
<td>55 - 59.9</td>
<td>32</td>
<td>8.55</td>
<td>215</td>
<td>17.7</td>
</tr>
<tr>
<td>60 - 64.9</td>
<td>20</td>
<td>8.80</td>
<td>142</td>
<td>22.3</td>
</tr>
<tr>
<td>65 - 67.4</td>
<td>3</td>
<td>9.00</td>
<td>22.1</td>
<td>57.0</td>
</tr>
</tbody>
</table>

$x^2 = 44.2$

The experimental data were obtained by observing a portion of the center of the synchrotron beam having an angular width small with respect to the total beam width. Comparison of this spectrum with the Bethe-Heitler curve giving the spectrum integrated over all angles may seem improper. Schiff (15) has shown, however, that the x-ray beam width of electron accelerators is usually determined not by the characteristics of the radiation process, but by the multiple scattering in the target before the electron radiates. This produces an effective integration over the angles of the quanta. In the present case calculation and measurements indicate that the angular spread due to multiple scattering is at least as large as the angular width of the beam for an infinitely thin target.
SYNCHROTRON ENERGY SPECTRUM

Fig. 10
When a comparison of the experimental points with the solid curve is made, two facts should be recognized. First, it is clear that one should compare the experimental points with the average value of the curve over the 5 Mev. energy interval. The difference between the averaged value and the value of the curve at the center of the 5 Mev. intervals is small, however, except for the interval 60-65 Mev. In this interval the averaged value is about 15% lower than the value at the center, but still falls within the statistical accuracy of the experimental point. The second point is the effect arising from the finite energy resolution of the cloud chamber. The general magnitude of this resolution has been indicated, but an exact determination is difficult due to the variety of chord lengths contributing to each energy interval. The energy resolution effect causes the greatest distortion at the upper end of the energy spectrum and, if corrected for, would tend to raise slightly the experimental point at 62.5 Mev. The effect of both these corrections would be in the direction of making the spectrum have a sharper falling off at the upper limit. This agrees with the suggestion that if the correct wave functions were used in the theory, the upper end of the spectrum would probably tend to a finite value (Reference 2, page 171).

B. Energy and Quantum Flux

From that part of the data which was accurately monitored, values were obtained for the flux of quanta and energy. The following symbols are used:

- \( N_p \) Observed number of pairs in the range 10-65 Mev. per square cm. per r unit of the Victoreen thimble chamber. Measured value is 9.8 \( \times 10^3 \).
- \( N \) Number of atoms per c. c. in expanded cloud chamber.
- \( l \) Length of path over which observed pairs originated.
- \( E \) Quantum energy in Mev.
- \( \sigma(E) \) Total cross-section for pair production at quantum energy \( E \).
- \( n(E) \) Number of bremsstrahlung quanta per energy interval per square cm. per r unit.
- \( n_1 \) Total number of quanta in range 10-65 Mev. per square cm. per r unit.
$E_{\gamma}$ Total energy of radiation in range 0-65 Mev. per square cm. per r unit.

Using the approximations (See reference 2)

$$\sigma(E) = \frac{1}{\sigma} \left[ \frac{28}{9} \log 4E - \frac{218}{27} \right]$$

and

$$n(E) = \frac{A(130-E)}{E}$$

the value of the constant $A$ is determined from

$$N_p = N_1 \int_{10}^{65} \sigma(E) n(E) \, dE. \quad (1 \text{ = 15.3 cm}.)$$

Then $n_{\gamma}$ as calculated from

$$n_{\gamma} = \int_{10}^{65} n(E) \, dE,$$

has a value of $6.6 \times 10^7$ quanta per square cm. per r unit.

From

$$E_{\gamma} = \int_{0}^{65} E \, n(E) \, dE,$$

the value of $E_{\gamma}$ obtained is $2.2 \times 10^9$ Mev. per square cm. per r unit.

The above values of $n_{\gamma}$ and $E_{\gamma}$ have a statistical error of 20%, but errors in monitoring may increase this.

McMillan (16) has recently obtained results from the Berkeley synchrotron which can be used for a general comparison with the value of $E_{\gamma}$ determined in this experiment. McMillan's Victoreen chamber was in a one-eighth inch thick open-ended cylinder of lead, while our value was obtained with a similar cylinder having an eighth inch thick end. For the peak energies of 320 and 160 Mev., McMillan's values respectively are 3.3 and $2.2 \times 10^9$ Mev. per square cm. per r unit. Our value of $E_{\gamma}$ shows remarkable agreement considering the magnitude of the expected errors.
VI. LITERATURE CITED

VII. ACKNOWLEDGMENTS

Much credit is due Dr. James Palmer, Mr. Robert McKenzie and Mr. I. Coleman for their part in the construction of the equipment. The cooperation of many members of the Department of Physics and in particular of the synchrotron group is gratefully acknowledged.

VIII. APPENDIX

The technique of making measurements and corrections of the electron track curvatures will be described. The method is exact in the sense that tracks making large angles with a plane normal to the magnetic field can be measured and their momentum calculated making no small angle approximations.

The path of a charged particle in a uniform magnetic field is a helix, a segment of which we measure. Figure 11 shows a helical segment denoted by $r'$. We define the following quantities, the first three of which are those measured when projecting the image of the electron track:

- $r'$: Apparent radius of curvature determined by matching a circle to both ends and the midpoint of a segment of the helix,
- $K$: Length of chord of the helical segment used to measure $r'$,
- $a$: Angle the chord makes with plane $P$, normal to magnetic field,
- $s$: Sagitta distance of helical segment or of helical segment projected on plane $P$, these having equal values,
- $a'$: Pitch angle of helix,
- $d$: Length of arc of helical segment projected on to plane $P$.

Using the sagitta formula, but not the sagitta approximation, we can write
\[ K^2 - 8 r's + 4s^2 = 0, \]  
\[ K^2\cos^2 a - 8rs + 4s^2 = 0. \]

Eliminating \( s \) gives
\[ r^2 - rr' (1 + \cos^2 a) + \frac{K^2(1 - \cos^2 a)^2}{16} + r'^2\cos^2 a = 0. \]

Solving and choosing the sign corresponding to segments smaller than semicircles, one obtains
\[ \frac{r}{r'} = \frac{1}{2} \left[ 1 + \sqrt{1 - \left( \frac{K}{2r'} \right)^2} \right] \cos^2 a + \frac{1}{2} \left[ 1 - \sqrt{1 - \left( \frac{K}{2r'} \right)^2} \right]. \]

A plot of this function for various values of \( K/2r' \) is given in Figure 12. This enables the determination of the curvature corresponding to the momentum component in the plane normal to the magnetic field. To obtain the curvature corresponding to the momentum in the original direction of the electron, we note from Figure 11 that
\[ \tan a' = \frac{K \sin a}{d} \]
\[ d = 2br \]
\[ \sin b = \frac{K \cos a}{2r}. \]

This gives
\[ \frac{\tan a'}{\tan a} = \frac{\frac{K \cos a}{2r}}{\sin^{-1} \left( \frac{K \cos a}{2r} \right)}, \]
from which \( a' \) may be determined. The correction factor by which one multiplies the measured radius of curvature \( r' \), to obtain the radius of curvature corresponding to the magnitude of the electron momentum, is
\[ F = \frac{r}{r' \cos a'}. \]
Fig. 11--Helix geometry.

Fig. 12--Curvature correction function.

\[
\frac{r}{r'} = \frac{1}{2} \left[ 1 + \sqrt{1 - \left(\frac{k}{2r'}\right)^2} \right] \cos^2 \alpha + \frac{1}{2} \left[ 1 - \sqrt{1 - \left(\frac{k}{2r'}\right)^2} \right]
\]