Changes in Supply Functions
And Supply Elasticities
In Hog Production

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SUMMARY AND CONCLUSIONS

Demand relationships for many agricultural products have been examined extensively. Supply analysis has received much less attention by agricultural research workers. Yet a knowledge of both demand and supply functions is required for an adequate understanding of the price mechanism. This study explores supply functions for hogs, particularly in relation to recent increased fluctuations in hog prices.

Recurring cycles in the price and production of hogs suggest the validity of a general cobweb theory underlying the hog market. According to the cobweb theory, a decline in demand elasticity and/or an increase in supply elasticity leads to relatively wider price fluctuations, other things being equal. The major hypothesis advanced in this study is that part of the recent increased fluctuations in hog prices are attributable to increases in the supply elasticity for hogs. Objectives of the study are to obtain evidence on the magnitudes and directional shifts in supply elasticities for hogs over time. Interest also centers on developing forecasting equations. To allow estimates of structural changes over time, the analysis is divided into two periods; one period extends from 1924 to 1937, the other from 1938 to 1956.

The total liveweight of hogs supplied is a direct function of the number of hogs marketed and their average marketing weight. Major changes in total hog supplies result from changes in hog numbers rather than in marketing weights. Numbers of hogs marketed are, in turn, determined primarily by the number of sows that farrowed in preceding time periods.

Single-equation least-squares methods were employed in analyzing spring and fall farrowings in the United States and North Central Region for the periods 1924-37 and 1938-56. Factors which appeared important in explaining spring farrowings were (in order of importance) the hog-corn price ratio at breeding time, production of oats, barley and grain sorghum as a percentage of corn production in the previous year, and various measures of the relative profitability of hogs and beef cattle at breeding time. Coefficients of determination (R^2 values) of 0.90 or greater were obtained for all spring farrowing equations. Estimated elasticities of supply (i.e., changes in farrowings in response to hog prices at breeding time) for the United States increased from 0.50 in the 1924-37 period to about 0.62 in the 1938-56 period. For the North Central Region, the corresponding increase in supply elasticity was from 0.58 to 0.74. Hence, these results support the hypothesis of an increase in supply elasticity for hogs over time.

Factors which significantly influenced fall farrowings were the number of sows farrowing in the spring, production of oats, barley and grain sorghum, and the comparative profitability of hogs and beef cattle. Coefficients of determination (R^2 values) were considerably lower for fall farrowings than for the spring farrowings. The supply elasticities for fall farrowings were relatively low (between 0.28 and 0.41) and did not change appreciably over time.

Estimates of supply elasticities also were obtained using an expected price model. Again, the response in spring farrowings to changes in hog prices expected in the future marketing period increased over time. The magnitudes of the elasticities computed from expected prices were comparable to those computed with respect to hog prices at breeding time.

In addition to changes in hog numbers, total hog supplies vary somewhat from changes in marketing weights. Simple three-equation simultaneous-equation models were used in estimating the responsiveness of farmers to price during 6-month marketing periods (i.e., by varying marketing weights). The within-marketing-period supply elasticities derived from this model were, as expected, relatively low—between 0.04 and 0.08; no appreciable changes occurred over time. Price and income elasticities of demand computed from the three-equation model showed a sharp decrease from the 1924-37 to the 1938-56 period. While the magnitudes of the changes over time probably are overestimated by this model, the direction of change is consistent with the hypotheses advanced.

In summary, the study provided support for the hypothesis of an increase over time in the supply elasticity for hogs, at least with regard to the number of sows farrowing in response to hog prices at breeding time. A decrease in the demand elasticity for hogs over time also was estimated. Therefore, recent observed wide fluctuations in hog prices may be explained, in part, by both an increase in the supply elasticity and a decrease in the demand elasticity for hogs.
A knowledge of supply responses and relationships for individual and aggregate agricultural commodities is of importance for farmers, economists, marketing organizations, national farm program administrators and consumers. Supply relationships are of immediate concern to outlook workers and other agricultural specialists who furnish information on which farmers and consumers. Supply relationships are of immediate concern to producers and other agricultural organizations, national farm program administrators, and decision-makers. With more perfect knowledge, farmers might organize their resources for greater individual profits. A knowledge of supply functions would allow marketing firms to anticipate more accurately the timing and magnitude of future commodity supplies, leading to marketing efficiencies and lower consumer prices. Agricultural supply relations and elasticities also are vital for policy decisions, particularly those dealing with production control programs and price support levels for various farm products.

While many descriptive theoretical formulations of supply response are available in the literature, relatively little research effort has been directed toward obtaining empirical estimates of supply relationships. Agricultural price analysts have concentrated heavily on the demand function for farm products, making the convenient assumption that the quantity supplied may be regarded as predetermined. For many farm commodities such a procedure has resulted in useful short-run predictions of price. Yet, more knowledge on the supply side is required if reasonably accurate representations of structural demand-supply interrelationships are to be obtained.

Pioneering work in the field of supply analysis began in the 1920's. The usual statistical technique employed in early supply studies was multiple regression, often by the short-cut graphic method. These analyses were hampered by the fact that the data were inadequate both in accuracy and in the period of time covered. As a result, the forecasts and relationships derived were frequently found misleading, and supply analysis generally fell into disrepute in the 1930's. Only since World War II has interest again revived in empirical supply studies.

Price instability for several farm products has led to a further interest in supply phenomena. The hog market, in particular, has shown wide price swings in the past several years. One measure of the variability of prices is the coefficient of variation (C). Table 1 indicates that in the months of heaviest hog marketings (October through April), year-to-year variations in deflated hog prices increased in the postwar period compared with the prewar period.

In the prewar period, data for 1931-34 were omitted because of the abnormally depressed hog prices throughout these years. From the prewar to the postwar period, the coefficient of variation increased from 16 percent to 25 percent, while in the 4 years 1953-57 the coefficient reached a high of 28 percent. The coefficient of variation for May through September (the remaining marketing months) showed no change from the prewar to postwar period. Again, however, greater variability occurred in the 4 years 1953-57, as is evidenced by an increase in the C value to 21 percent. Many farmers, economists and legislators were especially puzzled by the low hog prices in the fall and winter of 1955-56. The present study is an attempt to test hypotheses explaining the recent increased price fluctuations in the hog market.

Changes in Supply Functions and Supply Elasticities in Hog Production

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1 Project 1135, Iowa Agricultural and Home Economics Experiment Station, Center for Agricultural and Economic Adjustment cooperating.

2 Several recent empirical supply studies are:


4 Pioneering work in the field of supply analysis began in the 1920's. The usual statistical technique employed in early supply studies was multiple regression, often by the short-cut graphic method. These analyses were hampered by the fact that the data were inadequate both in accuracy and in the period of time covered. As a result, the forecasts and relationships derived were frequently found misleading, and supply analysis generally fell into disrepute in the 1930's.

5 For examples of some early contributions in supply analysis see:


Days 1933-34 October-April 2.73 17.16 18
1946-57 May-September 2.73 19.49 18
1953-56 September-May 3.60 17.93 21

* Hog prices deflated by the Index of Wholesale Prices.

1 Omitting three depression years 1933-34.

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ECONOMIC THEORY

The "cobweb theorem" provides the basic theoretical framework for the empirical results to be presented. Briefly, the cobweb theorem is an attempt to explain recurring cycles in the production and price series for particular commodities. Traditional economic theory assumes that, under static conditions of pure competition, market price tends to be established at the intersection of the demand and supply curves. However, where a considerable time lag occurs between the price change for a commodity and the resulting supply response, the cobweb relationship may lead to widely fluctuating prices and quantities.

Three possible cases of the cobweb theorem are distinguished:

Case 1. Continuous fluctuation. This case is represented geometrically by the left diagram in fig. 1. Assume quantity $Q_1$ is produced in time period 1 and placed upon the market. The resulting price is established at $P_1$. However, the low price $P_1$ results in supply of only $Q_2$ in time period 2. With only $Q_2$ supplied, price is established at the relatively high price $P_2$. Producers respond to the price $P_2$ by producing $Q_3$. But with the quantity $Q_3$ supplied, price once more falls to $P_3$. Price $P_3$ is the same as the original price $P_1$, and the pattern then is repeated in following time periods. When the demand curve is the exact reverse of the supply curve (i.e., when the two curves have identical slopes at any chosen price) this same pattern theoretically will repeat indefinitely. Thus, in the simple case of linear demand and supply functions, the continuous case occurs when both functions have the same absolute slope.

Case 2. Divergent fluctuation. This case, represented by the center diagram in fig. 1, occurs when the absolute slope of the demand function is greater than that of the supply function. Beginning with a quantity $Q_1$ and corresponding price $P_1$, the series of reactions trace out a pattern of successively larger fluctuations in price and quantity.

Case 3. Convergent fluctuation. The right diagram in fig. 1 represents the case of successively converging prices and quantities. Starting from quantity $Q_1$ and price $P_1$, the quantities and prices show successively smaller fluctuations as they approach the equilibrium point at the intersection of the demand and supply functions. In this situation the absolute slope of the supply function is greater than that of the demand function.

Three conditions are required for the cobweb theory to explain the functioning of a commodity market: (a) Producers must base output in period $t+1$ entirely on prices in period $t$; (b) production plans, once made, cannot be changed until the following time period; (c) price must be determined by the quantity supplied. It appears that the demand and supply structure for hogs in the United States approximately meets the conditions outlined. It is necessary, however, to investigate each of the conditions in detail as it pertains to hog production and marketing.

In regard to condition (a), a few empirical results are available which indicate the nature of price expectation models used by farmers. However, the presence of commodity cycles in themselves is evidence that many farmers use current prices as the basis for projection or forecasting. In one of the few empirical studies available, Schultz and Brownlee concluded that Iowa farmers formulated price expectations for hogs largely on the basis of current prices, at least for the time period investigated. A more

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![Fig. 1: Nature of fluctuations in price and production under specified elasticity situations.](image-url)
realistic hypothesis is that farmers' price expectations are based not only on the current price but also on prices observed in previous years. The most recent price, however, probably carries the greatest influence, while the weight attached to each previous price declines as the time lag increases. On the basis of the rather limited evidence available, the first condition for a cobweb relationship in hog production (i.e., that farmers base price expectations on current prices) seems approximately satisfied.

The nature of the hog production process indicates that conditions (b) and (c) also are reasonably fulfilled. Once sows are bred for farrowing, relatively little can be done to increase future production. Greater effort might be directed toward saving more pigs per litter, and hogs can be carried to slightly heavier marketing weights, but these adjustments affect total supplies to only a relatively small extent. Somewhat greater flexibility is available in reducing supplies, since bred gilts may be sold before farrowing. Heavy price discounts on "piggy" sows, however, tend to minimize this possibility, at least after the second month of pregnancy. A more serious limitation in applying the cobweb theory to hog production may be that hog supplies depend heavily on corn prices as well as on hog prices. However, hog prices in the heaviest marketing period of late fall and winter reflect, in part, the new corn supply and hence the expected price of corn during the next year. Condition (c) implies no interdependence or simultaneity between the price received and the quantity supplied; i.e., quantity is assumed to be predetermined. While farmers do vary marketing weights in response to short-run price changes, the resulting influence in the total hog supply picture probably is relatively minor.

The above discussion suggests the possibility of a cobweb pattern of price and production in the United States hog market. Further evidence of this relationship is provided in fig. 2, where the hog-corn price ratio in October, November and December is measured along the vertical axis, and the number of spring farrowings (in units of 1,000 sows) is measured along the horizontal axis. Since the corn supply is a major factor in hog production, hog-corn price ratios, rather than hog prices alone, are used in fig. 2. October, November and December are the main months in which sows are bred for spring farrowings. The gestation period for hogs is approximately 4 months, while the feeding period required to raise hogs to market weight is another 6 to 8 months. Hence, the pigs raised from sows bred one fall usually are sold the next fall, some 10 to 12 months later. The prices at which hogs of the previous spring pig crop are marketed then are known prior to breeding time for the next spring pig crop. If the cobweb theorem is an accurate description of the hog market, relatively high hog prices one fall would lead to a large num-

![Fig. 2. Relation of hog farrowings and hog prices, 1934-56.](image-url)
ber of farrowings the next spring. Pigs from this large spring crop would be marketed the following fall, driving hog prices downward. Low hog prices would induce a smaller number of spring farrowings, which in turn would lead to higher hog prices the following fall, etc.

Figure 2 provides strong indications that, with some modification, such a process has in fact taken place in the United States. The low hog-corn price ratio in the fall of 1934 \( (P_{34}) \) induced only 5,467,000 spring farrowings in the spring of 1935 \( (Q_{35}) \). This low number of spring farrowings resulted in a short supply in the fall of 1935 and a relatively high hog-corn price ratio \( (P_{35}) \). The higher hog-corn ratio \( (P_{35}) \) encouraged a larger number of spring farrowings in 1936 \( (Q_{36}) \), which, in turn, resulted in a lower hog-corn ratio \( (P_{36}) \) in the fall, etc. There is sufficient regularity in the clockwise rotation to indicate an underlying cobweb relationship. At times the pattern appears to be shifted out of its regular course by some outside force. For example, the effects of World War II and the Korean conflict seem to disrupt the regularity of the cobweb pattern. Of course, other factors, such as the quantity of small grain production and the prices of competing farm products, undoubtedly play a role not accounted for by this simple model. Nevertheless, it is suggested that the cobweb relationship is the appropriate theoretical framework for explaining price and quantity fluctuations in the hog market of the United States.

**Hypotheses and Objectives**

The major hypothesis advanced in this investigation is that part of the recent fluctuations in hog prices can be traced to shifts in the supply elasticity for hogs. Specifically, it is hypothesized that the elasticity of supply for hogs has increased in recent years. As illustrated by the cobweb theory, an increase in supply elasticity (a flattening of the supply curve) leads to wider price fluctuations, other things remaining equal. Of course, an increase in supply elasticity does not necessarily mean that the hog market will be characterized by increasingly wider fluctuations. Starting from the convergent case, an increase in supply elasticity might not cause a shift to the continuous or divergent fluctuation cases; the relationship of the demand and supply curves still could fall well within the convergent case, with only the convergence delayed. A secondary hypothesis is that the demand for hogs has become more inelastic in the past few years. Under the cobweb hypothesis, a demand curve with greater absolute slope than formerly also could lead to wider fluctuations in hog prices. It is hypothesized that the combination of these two forces—increased supply elasticity and decreased demand elasticity—explains in part the recent behavior of the hog market.

It is fairly obvious that the production function for hogs has shifted upward in recent years, causing a corresponding downward shift in the marginal cost curve (assuming prices of inputs constant). Use of improved feeding, breeding and management practices now allows greater output per unit of resource input than was possible a few years ago. However, there is no a priori reason why this shift in the production function should cause a shift toward greater elasticity in the marginal cost curve, and hence in the supply function. While the marginal cost curve is shifted down and to the right, making it appear flatter, elasticity (a percentage change concept) may remain constant or even decrease. Yet, an appraisal of changes in the farm economy suggests the plausibility of an increase in the supply elasticity for hogs in recent years. The hypothesis of increased supply elasticity for hogs implies that farmers are in a position of increased flexibility with respect to hog production. That is, producers now can shift more readily between enterprises with the occurrence of relative price changes. Improvements in building facilities and equipment, as well as in technical managerial skills, have made possible this type of between-enterprise flexibility. Changes in pork production methods also might contribute toward increases in supply elasticity. The time required to raise hogs to market weight has shortened in recent years, because of widespread adoption of new advances in swine nutrition, breeding and sanitation. Thus the impacts of price changes are felt more rapidly in increases or decreases in output. Also, some producers now use a multiple-farrowing system where pigs may be farrowed several times each year, or in some cases during every month of the year. Such a farrowing scheme allows much greater intra-year output adjustment to price changes than is possible under a rigid one- or two-litter-per-year system.

The reasoning behind the hypothesis of a lower demand elasticity for hogs lies in changes in consumer preferences for meat. Shepherd et al. have shown an upward shift in the demand curve for beef and a downward shift in the demand curve for pork over time. In recent years pork apparently has become a less acceptable substitute for beef, poultry and other products.

The objectives of the study flow directly from the hypotheses outlined above. A main objective is to empirically test the hypotheses of changes in supply and demand elasticities over time. Evidence on the directional shifts in elasticity was obtained, as well as point estimates of the magnitudes of these elasticities. Also, forecasting equations were developed to predict hog supplies in future time periods. Since the demand-supply relationships for hogs are not independent of other livestock products, auxiliary information is presented regarding these other products.

**Choice of Estimational Procedures**

A number of alternative procedures are available for deriving supply relationships in agricultural production. One general classification of procedures deals with the supply response of individual "typical" farm firms. Survey data from a sample of farms may provide information on the factors influencing supply

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Another group of procedures attempts to estimate the aggregate supply function directly, usually from annual, quarterly, monthly or daily time series data. One problem encountered in the aggregate approach is that individual firm adjustments, which may offset or cancel one another, tend to be obscured. A second problem lies in the choice of appropriate statistical techniques in analyzing time series data. Nevertheless, the aggregate method is used in this study because primary interest is in aggregate relationships.

Since the present study employs statistical analysis of the production function, the question arises: Should single-equation least-squares methods be used, or are simultaneous equations appropriate? The appropriate method of statistical estimation is determined by the degree of identification of the equations in the model. It is impossible to derive unique estimates of the coefficients of an equation which is under-identified. When an equation is just-identified, the coefficients can be estimated by an indirect use of least squares. In this case, it is possible to make two simple unique transformations. One transforms structural equations into reduced-form equations, each containing one endogenous variable, which can be estimated by least squares; the other transforms the least-squares estimates of the coefficients back to estimates of the structural coefficients. Because of its simplicity, this method has been used in most applications of simultaneous equations. When an equation is over-identified, more difficult problems of statistical estimation arise. The preferred method for obtaining structural coefficients in this case is the maximum-likelihood method. The maximum-likelihood procedure provides means of arriving at a reconciliation of the finite number of alternative estimates obtained in the over-identified situation. Logically, the “full-information” maximum-likelihood method, which utilizes all of the information in the model, is considered superior for the estimation of over-identified equations. However, this procedure is formidable from a computational standpoint. Hence, the “limited-information” maximum-likelihood method, which utilizes only part of the available information, is employed in this study for the estimation of over-identified equations. Details of the computational procedure followed are set forth by Friedman and Foote6 and are summarized in matrix notation by Chernoff and Divinsky.7


ANALYSIS OF SPRING AND FALL HOG FARROWINGS IN THE UNITED STATES AND NORTH CENTRAL REGION

The total liveweight production of hogs in the United States depends directly upon the number of hogs marketed and their average marketing weight. For reasons mentioned earlier, average marketing weights vary relatively little from year to year; the major changes in hog supplies result from changes in the number of hogs marketed. The number of hogs marketed is, in turn, determined largely by the number of sows which farrowed in preceding time periods. Thus, the first and perhaps most important step in studying hog supply is an analysis of spring and fall farrowings. The analysis is carried out at two levels of aggregation: One analysis pertains to the United States as a whole; the other relates to the North Central Region. Since, depending on the year, 70 to 80 percent of the spring pig crop (December through May) and 60 to 70 percent of the fall pig crop (June through September) are produced in the 12-state North Central Region, this area is singled out for special study.

To investigate the hypothesis of an increased supply elasticity for hogs, the analysis is further divided into two time periods. Comparisons between these time periods provide estimates of changes in structural relationships. A logical division with respect to time might be into prewar and postwar periods. Most available agricultural demand analyses are based on the interwar period from about 1920 to 1941. Few analyses include several postwar years along with the prewar period, omitting the war years because of disturbances due to government interference in pricing, rationing, etc.8 In the latter procedure, however, changes in structural relationships over time may be obscured. On the other hand, a separate postwar analysis must be based on rather scanty data. As a compromise, the time periods selected for study are 1924-37 and 1938-56 (omitting war years 1942, 1943 and 1944). In terms of relatively homogeneous periods, this appears to be a reasonable division. By 1958 the United States had recovered from the depths of the depression. Also, the agricultural sector no longer felt the major effects of the drouth years 1934 and 1936.

The nature of the production process for hogs indicates that a single-equation least-squares model is appropriate in estimating spring and fall farrowings. Because of the 4-month gestation period for hogs, the number of sows farrowing cannot be changed quickly in response to price changes during the farrowing period. Most producer decisions regarding the number of sows to farrow are made at or before breeding time, preceding the farrowing period. Therefore, numbers of sows farrowing may be regarded as a function of predetermined variables, known in advance of the farrowing months. Two qualifications should be noted: First, since the farrowing periods

8 The reasons for omitting the war years in the supply analysis are less apparent, since producers supposedly react to market prices whether they are administered or not. However, in this part of the study, the earlier war years are omitted because increased wartime production may have resulted from patriotic motivations, etc., rather than from response to measurable phenomena.
are defined as 6 months in length and the gestation period is only 4 months, prices at the beginning of the period might influence the number of farrowings at the end of the period. Second, bred sows may be sold during the gestation period if the outlook is for unfavorable hog prices. These factors, while recognized, are believed to be of insufficient importance to destroy the assumption that farrowings are essentially predetermined.

**SPRING FARROWINGS IN THE UNITED STATES**

Regression equations 1 and 2 estimate spring farrowings in the United States for the period 1938 to 1956 (omitting war years 1942, 1943 and 1944). Standard errors of the regression coefficients are given in parentheses below the coefficients.

1. \[ Y = -5.970 + 392X_1 + 60X_2 - 105X_3 \quad R^2=0.92 \]
   \[ (34) \quad (11) \quad (54) \quad d = 1.55 \]

2. \[ Y = -7.490 + 418X_1 + 66X_2 + 578X_4 \quad R^2=0.93 \]
   \[ (36) \quad (11) \quad (292) \quad d = 1.02 \]

The variables are defined as follows:

1. \( Y \) = Estimated first difference in the number of spring farrowings, United States (in 1,000 litters). The spring farrowing period extends from December, year t-1, through May, year t.
2. \( X_1 \) = United States hog-corn price ratio as an average of October, November and December, year t-1; computed as the ratio of average hog prices in dollars per hundredweight to average corn prices in dollars per bushel.
3. \( X_2 \) = First difference of oats, barley and grain sorghum production as a percentage of corn production, United States. That is, \( S_{t-1} - S_{t-2} \), where \( S \) denotes oats, barley and grain sorghum production as a percentage of corn production (production in tons). This variable is coded by adding a constant of 15.0 to remove negative values.
4. \( X_3 \) = Margin or difference between the average price (in dollars per hundredweight) of 500-800 pound good-choice stocker and feeder cattle at Omaha and the average price (in dollars per hundredweight) of choice-prime slaughter steers of all weights at Chicago during October, November and December, year t-1, deflated by the Index of Wholesale Prices (1910-14 = 100).
5. \( X_4 \) = Ratio between the average price (in dollars per hundredweight) of 500-800 pound good-choice stocker and feeder cattle at Omaha and the average United States hog price (in dollars per hundredweight) during October, November and December, year t-1.

In both equations, the hog-corn price ratio \( (X_1) \) is the most important variable in predicting changes in spring farrowings, as judged by the standard partial regression coefficients. It appears that the absolute level of this ratio strongly influences the direction and magnitude of changes in farrowings. When hog prices are favorable relative to corn (a high hog-corn price ratio), farrowings tend to increase from the previous level and vice versa.

The hog-corn ratio reflects to a considerable extent the supply of corn available for feeding. However, Brandow\(^{11}\) notes a separate influence on hog supplies exerted by the production of oats, barley and grain sorghum. When these grains comprise a relatively large proportion of the total feed grain supply, hog production tends to increase and vice versa. The variable expressing this relationship \( (X_2) \) is next in importance in explaining changes in spring farrowings.

Beef cattle feeding probably is the chief competitive farm enterprise with hogs in the major hog-raising areas. According to theory, the relative profitability of cattle and hogs should influence the number of sows farrowing. The third variables in equations 1 and 2 represent two possible methods of expressing this influence. The regression coefficient for the deflated price margin on beef cattle \( (X_3) \) is negative, indicating that as margins increase, the number of sows farrowing the following spring decreases and vice versa. For example, when cattle margins are relatively high, resources apparently are shifted from hog production to beef cattle production. In equation 2, the price ratio between feeder cattle and hogs \( (X_4) \) indicates the relative attractiveness of beef cattle versus hog production. When feeder cattle prices are relatively high, farmers tend to reduce cattle production and increase hog production.\(^{12}\)

Figures 3 and 4 show the actual spring farrowings compared with those predicted from equations 1 and 2. Admittedly, comparing the predicted and actual farrowings over the time period used in developing the regression equation is not a completely satisfactory test of the value of the equation for predictive purposes.\(^{13}\) Recognizing the limitations of this test, the regression equations correctly indicate the direction of change in spring hog farrowings, with the single exception of the 1945 prediction for 1946 in fig. 3.

Some idea of the precision of the estimates is given by computing the standard error of the estimate. This figure provides a measure of the amount by which the estimates of farrowings deviate from the observed farrowings in the years studied. For equation 1, the standard error of the estimate is 275,000 litters or approximately 3.36 percent of the mean number of farrowings each spring. Of course, the standard error of a forecast is somewhat larger. The standard error of the estimate for equation 2 is 256,000 litters or 3.13 percent of the mean number of sow farrowings.

The Durbin-Watson\(^{14}\) test for serial independence of the residuals also is computed, although the relatively low number of observations increases the probability of obtaining a "consistent" sign in this way, the method appears highly arbitrary. More investigation is needed on this relationship.

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\(^{12}\) While not shown here, a slaughter cattle-hog price ratio is nearly as effective as the feeder cattle-hog price ratio in predicting changes in sow farrowing. Because of the high correlation between feeder cattle and slaughter cattle prices, the regression coefficient for the slaughter cattle-hog price ratio also has a negative sign. This result appears inconsistent with logic. In almost all the analyses undertaken, some form of beef cattle-hog price ratio is significant; however, the signs are sometimes positive, sometimes negative. Since feeder and slaughter cattle prices are highly correlated, either a feeder cattle-hog ratio or slaughter cattle-hog ratio produces a significant regression coefficient. Thus, it is possible to argue that producers are influenced in some instances by feeder cattle prices and in others by slaughter cattle prices. While it is always possible to obtain a "consistent" sign in this way, the method appears highly arbitrary. More investigation is needed on this relationship.

\(^{13}\) A somewhat better test might be to test one year at a time. For example, the data for 1938-53 could be used to develop a regression equation containing the same variables used in equations 1 and 2. Then, an estimate for 1956 could be made and compared with the actual 1956 value. This could be done, however, for only a few recent years in the time series.

Fig. 3. Actual spring farrowings in the United States compared with predictions from equation 1.

Fig. 4. Actual spring farrowings in the United States compared with predictions based on equation 2.

Fig. 5. Actual spring farrowings in the United States compared with predictions based on equation 3.

ability of obtaining an inconclusive test result. The $d$ statistic for equation 1 is 1.55, which falls in the inconclusive range. However, the $d$ statistic for equation 2 is 1.02, indicating that the hypothesis of serial independence in the residuals is rejected. When plotted, the residuals for equation 2 show a slight cyclical effect, probably accounting for the significant test result.

Regression equation 3 is computed for spring farrowings in the United States during the earlier period, 1924-37. Variables $Y$, $X_1$ and $X_2$ are the same as those defined earlier. Variable $X_3$ is similar to $X_3$; it is the average price margin (in dollars per cwt.) between feeder cattle and slaughter cattle prices at Chicago from August to December, year $t-1$, deflated by the Index of Wholesale Prices (1910-14=100). Chicago feeder cattle prices are used because the Omaha series does not extend back to 1924. However, the sign of the regression coefficient is positive for $X_3$, the opposite of $X_3$ in equation 1. Economic logic indicates that as cattle margins increase, making cattle production more favorable, hog production should decrease. Perhaps in the earlier time period cattle margins were viewed more as an indicator of profitability of livestock production in general, rather than in a strictly competitive role with hogs. The extended depression period might have contributed to such psychology on the part of producers. A more likely explanation is that, when margins are high, feeder cattle prices also are usually high, discouraging beef cattle production. Again, more study is needed of the supply interrelationships between beef cattle and hogs.

As shown in fig. 5, regression equation 3 indicates the correct direction of change in hog farrowings in every year. The standard error of the estimate for equation 3 is slightly larger than those for the later time period - 355,000 litters per year or about 4.11
percent of the mean number of farrowings. The Durbin-Watson d statistic is 1.42, again an inconclusive test result.

**SPRING Farrowings in the North Central Region**

As mentioned previously, 70 to 80 percent of the spring farrowings in the United States normally occur in the 12-state North Central Region. The importance of the North Central Region in the total hog supply picture, regression equations 4 and 5 are computed for this region alone, for the two periods 1938-56 (omitting years 1942, 1943 and 1944) and 1924-37, respectively.

(4) $\hat{Y} = -6.770 + 400X_1 + 50X_2 + 726X_3$  \hspace{1cm} R^2=0.93  \hspace{1cm} (33) (9) (195)  \hspace{1cm} d = 2.01$

(5) $\hat{Y} = -6.621 + 316X_1 + 22X_2 + 894X_7$  \hspace{1cm} R^2=0.90  \hspace{1cm} (35) (10) (248)  \hspace{1cm} d = 1.75$

The variables are defined as follows:

$\hat{Y}$ = Estimated first difference in the number of spring farrowings, North Central Region (in 1,000 litters). The spring farrowing period extends from December, year t-1, through May, year t.

$X_1$ = Chicago hog-corn price ratio as an average of October, November and December, year t-1; computed as the ratio of average hog prices in dollars per hundredweight to average corn prices in dollars per bushel.

$X_2$ = As defined previously.

$X_7$ = Margin or difference between the average prices (in dollars per hundredweight) of all feeder cattle and slaughter cattle at Chicago as an average for the months August through December of year t-1, deflated by the Index of Wholesale Prices (1910-14=100).

In equation 4 for the later time period, the hog-corn ratio ($X_1$) remains the most important explanatory variable, followed by $X_2$ and $X_0$, respectively. Once again the coefficient for variable $X_0$ (the feeder cattle-hog price ratio) is positive and 3.73 times as large as its standard error. Figure 6 shows the actual farrowings for the North Central Region compared with those predicted by equation 4. The direction of yearly changes is predicted correctly for every year except 1946. Regression equation 4 for United States spring farrowings also failed for this year (see fig. 3). The standard error of the estimate for regression equation 4 is 199,000 litters per year or approximately 3.20 percent of the mean number of spring farrowings in the North Central Region. The calculated value of the Durbin-Watson d statistic is 2.01, indicating support for the assumption of serial independence of the residuals.

The coefficients of equation 5 for the North Central Region (1924-37) are similar to those obtained in equation 3 for the United States. Again, $X_7$ (deflated cattle margins), while large relative to its standard error, has a sign inconsistent with economic logic. Figure 7 shows that regression equation 5 correctly indicates the direction of change in farrowings for every year. The regression equation for 1924-37 again has a larger standard error of the estimate than the equations for 1938-56. The standard error of the

---

15 The states included in this region are: Ohio, Indiana, Illinois, Michigan, Wisconsin, Minnesota, Iowa, Missouri, North Dakota, South Dakota, Nebraska and Kansas.

16 As pointed out previously, the slaughter cattle-hog price ratio is nearly as effective as the feeder cattle-hog price ratio in these equations. If interest is primarily in prediction rather than in estimation of structural relationships, some criterion such as the highest R² value might be used in selecting between these two variables.
estimate for equation 5 is 332,000 litters per year or 5.20 percent of the mean number of spring farrowings in the North Central Region from 1924-37. The Durbin-Watson d statistic for equation 5 is 1.75, indicating that the assumption of serial independence in the residuals is not rejected.

**FALL FARROWINGS IN THE UNITED STATES**

The fall farrowing period as defined by the United States Department of Agriculture extends from June 1 to Nov. 30. Regression equation 6 is computed for fall farrowings in the United States for the period 1937-56 (omitting years 1941, 1942, 1943 and 1944).

(6) \[ \hat{Y} = 159.91 + 0.29X_1 + 0.78X_2 + 3.98X_3 + 8.14X_4 \]

\[ R^2 = 0.92 \]

\[ d = 2.62 \]

Regression equation 7 becomes the prediction equation when variable \( X_2 \) is dropped from equation 6. The variables in the equations are:

\( \hat{Y} \) = Estimated number of fall farrowings, United States (in 1,000 head). The fall farrowing period extends from June through November, year t.

\( X_1 \) = Year

\( X_2 \) = Number of male pigs registered, Iowa

\( X_3 \) = Number of pigs registered, United States

\( X_4 \) = Number of pigs registered, United States, per 1,000 head
$X_1 =$ Number of spring farrowings, United States (in 1,000 head); i.e., from December, year $t-1$, through May, year $t$.

$X_2 =$ United States hog-corn price ratio as an average of March, April, May and June, year $t$; computed as the ratio of average hog prices in dollars per hundredweight to average corn prices in dollars per bushel.

$X_3 =$ Quantity of oats, barley and grain sorghum produced (in 100 tons), United States, year $t$.

$X_4 =$ Ratio of the average price (in dollars per cwt.) of slaughter steers, all grades, at Chicago to the average price of corn (in dollars per bushel) at Chicago during March, April, May and June, year $t$.

The hog-corn price ratio at breeding time (March, April, May and June) for fall farrowing has a non-significant regression coefficient in equation 6. Thus, while the hog-corn price ratio at breeding time is the most important variable influencing spring farrowings, the corresponding factor does not significantly influence fall farrowings. More important than the hog-corn price ratio in determining fall farrowings are the number of spring farrowings, anticipated feed grain supplies and the competitive position of hogs with cattle. Many producers plan during the fall months for production over the entire year ahead. That is, plans are made for a certain number of sows to farrow in the spring, then the same sows are carried over and farrow again in the fall. Since many farmers follow this two-litter system, the number of fall farrowings apparently is influenced more by the hog-corn ratio in the previous fall than by this ratio at breeding time for fall pigs (March, April, May and June). In this situation, the decision to farrow sows for the fall period is a “routine” or “automatic” decision not appreciably influenced by prices at breeding time.

In fitting equation 7, the actual quantity of small grain production ($X_3$) in year $t$ was used. Of course, the magnitude of this variable is quite uncertain at the time decisions are made to breed sows for early fall farrowings. As indicated above, however, this decision often is made rather automatically. Later on, when more evidence is available on potential grain supplies and other factors, a portion of the bred sows may be sold. The practice of breeding sows, with the alternative of selling them before farrowing if conditions appear unfavorable, provides added flexibility under uncertainty and apparently is used by a number of hog producers. Forecasts from equation 7 probably would be made in June, at which time reasonably accurate estimates of the current year small grain production are available.

In equation 7 the relative profit position of beef cattle and hogs is expressed through a slaughter cattle-corn price ratio. According to equation 7, relatively high cattle prices at breeding time for fall pigs are associated with a greater number of fall farrowings. Again, either a slaughter cattle-corn price ratio or a feeder cattle-corn price ratio is effective in raising the $R^2$ value in the regression equation for fall farrowings. Perhaps farmers are mainly influenced by feeder cattle prices. If so, a feeder cattle-corn ratio variable might be defended as follows: Prospectively high feeder cattle prices require a greater outlay and increase the risk associated with the beef cattle enterprise. Resources then are shifted into increased hog production. Conversely, when feeder cattle prices are relatively low, risk in cattle feeding is lessened and resources are diverted from hogs to cattle production.

Figure 8 compares the actual fall farrowings in the United States with the predicted farrowings from equation 7. With the exception of 1951, the prediction is in the correct direction in every year. The standard error of the estimate is 177,000 litters or 3.48 percent of the mean number of fall farrowings in the 1937-56 period. The calculated $d$ statistic for equation 7 is 1.70. Once again the hypothesis of serial independence of the residuals is not rejected.
Regression equation 8 is computed for fall farrowings in the United States, based on data for the period 1924-36. Variables $Y$, $X_1$, and $X_4$ are defined the same as for equations 6 and 7. Variable $X_5$ expresses the influence of feed grain supplies; it is measured as the change in corn production (in 100-ton units) from year $t-1$ to year $t$. Again, the hog-corn ratio at breeding time for fall farrowings ($X_2$) has a nonsignificant regression coefficient and therefore has been excluded from equation 8. As shown by the $R^2$ value of 0.75 the explanation of variance in the dependent variable (fall farrowings) by the chosen independent variables is less satisfactory than in equations 6 and 7 for the later 1937-56 period. Part of the explanation for this difficulty appears to be the uncertainty of, and wide fluctuations in, feed grain supplies during the later years of the 1924-36 period. For example, in fig. 9 large prediction errors occur in 1933, 1934 and 1936, years in which feed grain supplies shifted drastically from the level of the previous year. Also, regression equation 8 predicted the wrong direction in fall farrowings for the three years 1929, 1933 and 1936. The standard error of the estimate $-346,000$ litters or 8.04 percent of the mean is larger than in previous equations. The Durbin-Watson $d$ statistic for equation 8 is 2.45, which indicates an inconclusive test result. If equation 11 were relevant for forecasting purposes, it would be desirable to refine it further. However, the purpose of studying the earlier time period (1924-36) is to estimate regression and elasticity coefficients for the important variables. Comparisons of supply elasticities computed from the regression equations are presented later.

**Fall Farrowings in the North Central Region**

The 12-state North Central Region produces a somewhat smaller percentage of the total United States fall pig crop than of the spring pig crop; the percentage historically has been between 60 and 70 percent. From 1950-56, however, the percentage of total fall farrowings produced in the North Central Region has increased to between 70 and 75 percent.

Regression equations 9 and 10 are computed for the 1937-56 (omitting 1941, 1942, 1943 and 1944) and 1924-36 periods, respectively.

$$Y = -941.89 + 0.23X_5 + 5.46X_5 + 8.20X_4 \quad R^2 = 0.89$$

$$Y = -390.11 + 0.32X_5 + 0.84X_6 + 8.51X_4 \quad R^2 = 0.71$$

The variables are defined as follows:

$Y$ = Estimated number of fall farrowings, North Central Region (in 1,000 head). The fall farrowing period extends from June through November, year $t$.

$X_5$ = Number of spring farrowings, North Central Region (in 1,000 head) i.e., from December, year $t-1$, through May, year $t$.

$X_6$ = Quantity of oats, barley and grain sorghum produced (in 100 tons), United States, year $t$.

$X_7$ = Ratio of the average price of slaughter steers, all grades, at Chicago to the average price of corn (in dollars per bushel) at Chicago during March, April, May and June, year $t$.

$X_8$ = Change in corn production (in 100 tons), United States, from year $t-1$ to year $t$.

The logic of the variables has been explained previously and will not be repeated. Figures 10 and 11 show that the predictions for the 1937-56 period are more accurate, both in direction and in magnitude, than those for the 1924-36 period. Regression equation
9 predicts the direction of change correctly in every year except 1940 (fig. 10), while equation 10 predicts the incorrect direction of change four times in the earlier 13-year period (fig. 11). Again, equation 10 is not further refined because interest in the earlier time period centers on measuring the influence of the major independent variables rather than on forecasting. The comparative precision of equations 9 and 10 is revealed by their standard errors of estimate. For equation 9, the standard error of the estimate is 380,600 litters or 13.1 percent of the mean number of farrowings. The calculated d statistic for equation 9 is 1.27, which falls in the rejection region. That is, the hypothesis of serial independence in the residuals is rejected. For equation 10 the d value is 2.50, providing an inconclusive result.

**Elasticities of Supply from Farrowing Equations**

Elasticity of supply is defined as the percentage change in quantity associated with a 1-percent change in price. Equation 11 gives the various mathematical formulas used in computing the elasticity of supply...
The supply elasticities presented below measure the percentage change in the number of farrowings associated with a 1-percent change in the average hog price at breeding time. For spring farrowings the supply elasticities measure the percentage change in number of farrowings (Q) from December, year t-1, through May, year t, associated with a 1-percent change in average hog price (P) in October, November and December, year t-1; i.e., at breeding time for spring farrowings. However, a somewhat different procedure is used in computing supply elasticities for fall farrowings. Regression coefficients for hog prices in March, April, May and June, year t, are non-significant in predicting fall farrowings (from June through November, year t); supply elasticities based on these coefficients would be rather meaningless. Hence, as in the case of spring farrowings, elasticities for fall farrowings, year t, are computed with respect to average hog prices in October, November and December, year t-1. The rationale for this procedure is that decisions are made in the fall, year t-1, apparently for both spring and fall farrowings, year t. Computational details of this procedure are presented later.

An example of computing the supply elasticity for spring farrowings is given next for regression equation 1. Variable \( Y \) is the estimated year-to-year change in spring farrowings; i.e., \( Y = (Y_t - Y_{t-1}) \). Variable \( X_1 \) is the hog-corn ratio in the previous fall; i.e.,

\[ X_1 = \frac{\text{Price of hogs}}{\text{Price of corn}} = \frac{P_h}{P_c} \]

Thus, equation 1 may be rewritten as equation 12. The partial derivative of quantity with respect to hog price \( \frac{\partial Y_t}{\partial P_h} \) is given in equation 13. The definition of elasticity of supply

\[
(11) \quad E_s = \frac{\text{Percentage change in quantity}}{\text{Percentage change in price}} = \frac{\Delta Q \times P}{\Delta P \times Q} \times \frac{P}{Q} = \frac{\partial Q}{\partial P} \times \frac{P}{Q}
\]

In this study the last formula \( \frac{\partial Q}{\partial P} \times \frac{P}{Q} \) is used in computing elasticities. All elasticities are evaluated at the means of the variables.

and the computation of the elasticity at the means of all variables are presented in equation 14. Thus, at the mean, a 0.64-percent change in the number of spring farrowings is associated positively with a 1-percent change in the average price of hogs in October, November and December of the previous fall. Several equations (for example, equation 2) include both a hog-corn price ratio and a cattle-hog price ratio. For these equations, the partial derivative of farrowings with respect to hog price contains two terms. Otherwise, the elasticities of supply are computed in the manner previously illustrated.

For reasons mentioned above, elasticities of supply for fall farrowings are computed with respect to hog prices during the previous fall rather than at breeding time for fall pigs. However, the average hog price (or hog-corn ratio) in October, November and December is not included directly in the regression equations predicting farrowings for the next fall. Thus, two regression equations are combined to obtain elasticities for fall farrowings. To illustrate, the supply elasticity for equation 7 is computed. In equation 7, the number of spring farrowings (\( X_1 \)) is used as an independent variable in predicting fall farrowings (\( Y \)). However, the number of spring farrowings is estimated, in turn, as \( Y_t \) in equation 1. Substituting the estimate of spring farrowings (\( Y_t \)) from equation 1 for the actual number of spring farrowings (\( X_1 \)) in equation 7 gives equation 15. By this substitution, fall farrowings are expressed as a function of average hog prices (i.e., through the hog-corn ratio) in the preceding October, November and December. The partial derivative of fall farrowings (\( Y \)) with respect to the average price of hogs in the previous fall (\( P_h \)) is given in equation 16. Equation 17 indicates the computation of the supply elasticity at the means of the variables.

\[
(11) \quad E_s = \frac{\Delta Y_t \times P_h}{\Delta P_h \times Y_t} = \frac{392}{P_c} \times \frac{Y_t}{1.16} \times \frac{15.48}{8,173} = 0.64
\]

\[ (12) \quad Y_t = -5,970 + 392 \frac{P_h}{P_c} + 60X_2 - 105X_3 \]

\[ (13) \quad \frac{\partial Y_t}{\partial P_h} = \frac{392}{P_c} \]

\[ (14) \quad E_s = \left( \frac{\partial Y_t}{\partial P_h} \right) \times \frac{P_h}{Y_t} = \left( \frac{392}{P_c} \right) \times \frac{15.48}{8,173} = 0.64 \]

\[ (15) \quad Y = 237.96 + 0.25X_1 + 4.00X_3 + 8.46X_4 \]

\[ (16) \quad \frac{\partial Y}{\partial P_h} = \frac{0.28(392)}{P_c} = \frac{111.78}{P_c} \]

\[ (17) \quad E_s = \frac{Y}{P_h} \times \frac{P_h}{Y} = \frac{111.78}{1.16} \times \frac{15.48}{5,085} = 0.29 \]

\[ (18) \quad Y = 237.96 + 0.25X_1 + 4.00X_3 + 8.46X_4 \]
Table 2 summarizes the estimates of supply elasticities for the various combinations of geographical areas, time periods and farrowing seasons analyzed in regression equations 1 through 10 (excluding equation 4). For spring farrowings, higher elasticities are found for the North Central Region than for the United States. For fall farrowings, the elasticity is slightly higher in the 1938-56 period, while for the North Central Region the elasticity is slightly higher in the 1924-37 period.

An important consideration, of course, is whether the elasticities between time periods are actually different or whether the observed differences might easily have occurred by chance. Fairly complicated statistical procedures are available for placing confidence limits on elasticity estimates. For the purposes here, however, a comparatively simple procedure appears sufficient to provide a rough approximation to the standard error of the elasticity figures. Upper and lower limits are computed for each elasticity, taking into account the standard errors of the regression coefficients on which the elasticities are based. Elasticities based on plus or minus one standard error of the regression coefficients are computed for the spring farrowing months. For the United States, the upper and lower limits are 0.70 and 0.58 for equation 1, 0.69 and 0.51 for equation 2, and 0.45 and 0.55 for equation 3. The intervals for equations 1 and 3 do not overlap, providing some evidence for the hypothesis of an increase in supply elasticity over time. However, the elasticity intervals for equations 2 and 3 slightly overlap, because of the relatively wide interval for equation 2. The elasticity computed from equation 2 is subject to greater variation because it is derived from two regression coefficients, each of which is estimated with some error. Similar evidence exists for the hypothesis of an increase in supply elasticity for spring farrowings over time in the North Central Region. The upper and lower elasticity limits for equation 4 are 0.83 and 0.65, while the limits for equation 5 are 0.64 and 0.51. As mentioned earlier, more sophisticated statistical tests for comparing elasticities could be employed. However, the procedure used provides a useful idea of the relative magnitudes of the elasticities and the errors with which they are estimated. The differences in point estimates over time are sufficiently large and consistent for the United States and North Central Region to provide somewhat greater confidence in the results than might be indicated by statistical significance tests alone.

Several reasons for hypothesizing an increase in the supply elasticity for hogs were mentioned earlier. Technological changes appear especially important in explaining this shift in "price responsiveness" on the part of farmers. Many producers now have the specialized facilities and technical knowledge required for successfully farrowing large litters in the winter months. For example, automatic heating and watering facilities, farrowing stalls and other specialized equipment now are quite common on Midwest farms, while technical information directed toward producers undoubtedly results in more efficient swine management. Therefore, when hog prices in the fall months are favorable, producers possess the physical and managerial resources to easily increase winter farrowings (i.e., during the spring farrowing period, December to May). An increased supply elasticity also implies that, as hog prices fall, producers restrict hog production relatively more than formerly. Ordinarily, a restriction in hog production is likely to be accompanied by a shift of resources to other enterprises. Perhaps the recent favorable capital position of farmers has contributed toward a willingness to shift, when hog prices are relatively low, from hog production into higher risk enterprises such as cattle feeding. The importance of technology in supply response is indicated in comparing elasticities for the United States with those for the North Central Region. Greater technological change in hog production undoubtedly has occurred in the North Central Region compared with the United States as a whole. As expected, the point estimates of supply elasticities are higher for the North Central Region in both time periods studied (table 2).

For fall farrowings, the statistical procedure for estimating elasticity intervals reveals no difference between time periods in the supply elasticities for either the United States or the North Central Region. Also, the elasticities for fall farrowings are considerably lower than those for spring farrowings. Elasticities for fall farrowings probably are relatively low partly because of the time lag between the price and output variables; conditions often change markedly in the interim. As before, the elasticities of supply are higher for the North Central Region than for the United States as a whole.

Elasticities of Supply From a Model Using Expected Prices

In the preceding analysis it is assumed that hog producers, in planning spring farrowings for year t, react to prices prevailing in year t-1; i.e., at breeding
time. However, an alternative hypothesis is that hog producers react, not to the price at breeding time, but rather to the price they expect when the hogs are to be sold. Nerlove\textsuperscript{19} points out that expected prices may depend only to a limited extent on last year's price. He proposes a simple model representing expected price as a weighted moving average of past prices, where the annual weights decline backward in time. The procedure of representing expected price by price lagged 1 year, then, is a special case of this general hypothesis in which the weight attached to last year's price is 1 and the weight attached to all other past prices is zero.

Nerlove assumes the simple model in equation 18. Variable \( Y_t \) is output in year \( t \), \( P_t^* \) is the expected price for year \( t \) and \( u_t \) is a random residual. One possible hypothesis is that farmers revise their expected price in proportion to the error they made in predicting last year's price. This hypothesis, advanced by Nerlove, is stated mathematically in equation 19. The \( \beta \) term is called the coefficient of expectation. Equation 19 is solved for \( P_t^* \) to give equation 20. Since

\[(19) \quad P_t^* - P_{t-1}^* = \beta (P_t - P_{t-1}^*) \]

\[(20) \quad P_t^* = \beta P_{t-1}^* + (1 - \beta) P_{t-1} \]

\[(21) \quad Y_t = \frac{a_0 + a_1 P_{t-1} + (1 - \beta) Y_{t-1} + v_t}{a_1} \]

the relationship in equation 18 is valid for year \( t-1 \) as well as year \( t \), all time subscripts are changed to \( t-1 \) and equation 18 is solved for \( P_{t-1}^* \) in equation 21. Substituting \( P_{t-1}^* \) from equation 21 into equation 20, and the resulting expression for \( P_t^* \) from equation 20 into equation 18 results in equation 22. Equation 22 expresses output as a function of last year's price and quantity, while \( v_t \) is a new residual term. The coefficients of equation 22 are estimated by least squares, and from these estimates are derived the estimates of \( a_0, a_1 \) in equation 18 and the coefficient of expectation, \( \beta \).

In this study, a similar but somewhat more complex model is used in deriving the response of spring farrowings to expected prices. In addition to the expected price of the single commodity (hogs), it is desirable to include expected prices of the main inputs and alternative products. Thus, prices for corn (\( P_c \)) and beef cattle (\( P_b \)) now enter the model rather than hog prices alone. As indicated by previous results, producers apparently respond to the hog-corn price ratio \( \left( \frac{P_h}{P_c} \right) \) and the beef cattle-hog price ratio \( \left( \frac{P_b}{P_h} \right) \).

Thus, the model illustrated in equation 26 expresses output (the number of spring farrowings in year \( t \)) as a function of these two price ratios expected to prevail when the spring pigs are sold (October, November and December, year \( t \)). The expectational model for each price ratio is shown in equations 24 and 25; it is the same model assumed in equation 19 for a single price: Producers are assumed to revise their expected price ratios

\[(23) \quad Y_t = a_0 + a_1 \left( \frac{P_h}{P_c} \right)^* + a_2 \left( \frac{P_b}{P_h} \right)^* + u_t \]

\[(24) \quad \left( \frac{P_h}{P_c} \right)^* = \beta \left( \frac{P_h}{P_c} \right) - \beta \left( \frac{P_h}{P_c} \right)_{t-1} \]

\[(25) \quad \left( \frac{P_b}{P_h} \right)^* = \beta \left( \frac{P_b}{P_h} \right) - \beta \left( \frac{P_b}{P_h} \right)_{t-1} \]

in proportion to the error they made in predicting last year's ratios. Of course, other expectational patterns might be hypothesized. To keep the computations manageable, the same coefficient of expectation (\( \beta \)) is assumed for both the hog-corn price ratio and the beef cattle-hog price ratio. Starting from the model indicated by equations 23, 24 and 25, an algebraic transformation similar to that used previously for one price results in equation 26, whose coefficients are fitted by least squares.\textsuperscript{20} Again, from the estimates of these coefficients, the estimates of \( a_0, a_1 \) and \( a_2 \) in equation 23 are obtained.

\[(26) \quad Y_t = a_0 + a_1 \beta \left( \frac{P_h}{P_c} \right) + a_2 \beta \left( \frac{P_b}{P_h} \right) + (1 - \beta) Y_{t-1} + v_t \]

The empirical estimates derived from the expected price model are summarized in table 3.

\textsuperscript{19}Nerlove, Marc. Estimates of elasticities of supply of selected agricultural commodities. Jour. Farm Econ. 38: 496-509. 1956.

\textsuperscript{20}Details of this transformation are presented in Appendix A.

<table>
<thead>
<tr>
<th>Quantities estimated</th>
<th>United States ( 1924-37 )</th>
<th>United States ( 1938-56^\dagger )</th>
<th>North Central Region ( 1924-37 )</th>
<th>North Central Region ( 1938-56^\dagger )</th>
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</thead>
<tbody>
<tr>
<td>( a_o )</td>
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<tr>
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<tr>
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<tr>
<td>( R^2 )</td>
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<td>0.63</td>
<td>0.53</td>
<td>0.73</td>
</tr>
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</table>

\textsuperscript{*}Figures in parentheses below the estimates are standard errors.

\textsuperscript{†}Omitting years 1942, 1943 and 1944.

\textsuperscript{‡}Supply elasticities indicate response of spring farrowings, year \( t \), to the hog price expected to prevail in October, November and December, year \( t \). Elasticities are computed at the means of all variables.
farrowings, year t, to the hog price expected to prepare similar in magnitude to those based on lagged directly.

It appears that the assumption that farmers closely magnitude of the coefficient of expectation prices (compare tables 2 and 3). From this comparison, 'With level. However, the assumption that farmers closely immediately from the regression coefficient 1 - \( \beta \). This implies that the expected price in year t is identical with the observed price in year t-1 (i.e., \( P^* t = P_{t-1} \)). These results, then, support the proposition that prices and quantities of hogs are generated by a cobweb mechanism. Specifically, support is provided for the crucial condition that, for the cobweb theory to be applicable, producers must base future output on current prices.

THREE-EQUATION DEMAND AND SUPPLY MODELS FOR HOGS, BASED ON 6-MONTH MARKETING PERIODS

As mentioned previously, the total liveweight of hogs slaughtered in the United States is a direct function of the number of hogs slaughtered and their average slaughter weight. Numbers of hogs marketed are determined primarily by the number of sows farrowing in previous periods and secondarily by a technological factor, number of pigs saved per litter. The latter factor (pigs saved per litter) has shown a definite upward trend over time and hence can be predicted with reasonable accuracy from year to year. Minor fluctuations about the long-time trend in the number of pigs saved per litter appear to be related primarily to exogenous factors such as weather and disease. The preceding analysis of spring and fall farrowings, then, is important from the standpoint of forecasting; major changes in future hog marketings can be predicted from changes in the number of sows farrowing. Also, within the entire hog supply process, the most important changes in price responsiveness over time are expected to result from decisions on the number of sows to farrow.

The second major element determining total hog supplies is average marketing weight. To accurately forecast the total liveweight of hogs supplied, some notion is required of the responsiveness of marketing weights to price and other factors. Average marketing weights are determined jointly with influences prevailing within the slaughter period, such as prices for hogs, other livestock and feed. To aid in forecasting, however, an attempt was made to estimate hog marketing weights from predetermined variables alone. A preliminary regression analysis indicated that hog marketing weights were inversely related to the number of pigs saved in the preceding period and directly related to quantities of corn and other grains available for feeding. While logical, these relationships were not sufficiently stable to serve usefully in prediction.

Because hog prices and marketing weights are to some extent jointly determined, simultaneous equations appear to be an appropriate technique for investigating their interrelationship. While this type of analysis may be of limited value in prediction, it should provide useful estimates of the within-marketing-period elasticities of supply. The following analysis is an attempt to isolate the extent to which farmers respond, within the production year, to price by varying marketing weights alone.

THREE-EQUATION RESULTS FOR THE 6-MONTH MARKETING PERIOD, AUG. 1 TO FEB. 1

August 1 to Feb. 1 represents the period during which most of the spring pig crop moves to slaughter. The following three-equation model is designed to measure the extent to which the average marketing weight, and hence total slaughter, during the period is influenced by hog prices within the period. Variables in the model

\[
Q = b_{11} + b_{12}X_1 + b_{13}X_2 + b_{14}X_3 + u_1
\]

\[
Q = b_{21} + b_{22}P + b_{23}Z + u_2
\]

\[
P = b_{31} + b_{32}Q + b_{33}Y + u_3
\]

are expressed in logarithms and defined as follows:

**Q** = Total liveweight of hogs slaughtered under federal inspection, United States, Aug. 1, year t, to Feb. 1, year t+1 (in units of 100,000,000 lbs.).

**X** = Number of pigs saved from spring pig crop, United States, year t (in units of 1,000 head).

**X** = Total feed grain produced in the United States, year t (in units of 1,000 tons). This variable is classed as predetermined on the basis of being a current variable determined outside of, or exogenous to, the model.

**X** = Time, where "time" takes values from 1 to N. (N is the number of years in the period investigated.)

**P** = Average price of hogs (in dollars per cwt.) received by farmers from Aug. 1, year t, to Feb. 1, year t+1, United States, divided by the Index of Prices Received

21 Assuming that tests of significance are applicable here, which might be debated, a t-test is performed where \( t = \frac{\beta - 1.0}{\sigma} \). The symbol \( \sigma \) denotes the standard error of \( \beta \). The standard error of \( \beta \) is the standard error of \( \beta \) which has been computed by ordinary regression analysis. From statistical theory, \( \beta_{11} \approx \beta + 1.64 \). But \( t_1 \approx 0.8, \) since the standard error of \( \beta \) is constant equals zero. Thus \( \beta_{11} - \beta \approx \sigma. \)

22 Nerlove, ibid., hypothesizes that the value of \( \beta \) ordinarily is less than 1, since farmers are noted for the strength of their convictions and thus will revise their future price expectations by only some fraction of the error made.

23 This general model was used by Fox for annual data on pork demand and supply. See: Fox, K. A. The analysis of demand for farm products. U. S. Dept. Agr. Tech. Bul. 1081. 1955. pp. 31-32.

24 Total hog slaughter was not used since this series is not available on a monthly basis for the entire time period studied. However, little error is expected in using federal inspected slaughter since the multiple correlation coefficients \( r^2 \) between annual changes in total slaughter and changes in federal inspected slaughter are 0.99 for 1924 through 1937 and 0.90 for 1938 through 1956.
by Farmers for Livestock and Livestock Products (1947-49 = 100), United States, during the same period.

\[ Z = \text{Estimate of } Q \text{ based on predetermined variables } X_1, X_2 \text{ and } X_3. \]

In other words, \( Z = \hat{Q} \) from equation 30 in the model.

\[ Y = \text{Per capita disposable personal income (in dollars), average of last two quarters, year } t, \text{ United States, divided by the Index of Consumer Prices (1947-49 = 100), average of last two quarters, year } t, \text{ United States.} \]

\( u_i (i = 1, 2, 3) = \text{Random residuals.} \)

Equation 27 provides an estimate of \( Q (\hat{Q} = Z) \) based on predetermined variables \( X_1, X_2 \) and \( X_3 \). That is, at the beginning of the marketing period (Aug. 1) an estimate can be made of hog slaughter based on variables determined in advance of the marketing period. The predicted quantity (\( Q = Z \)) for each year then is included as a predetermined variable in equation 28. The variable \( Z \) estimates the general level of hog slaughter (\( Q \)) expected during the marketing period. Deviations from the general level of \( Q \) are caused primarily by changes in average marketing weights, which in turn are influenced by hog prices (\( P \)) within the marketing period. Thus, since the variables are in logarithmic form, the coefficient of \( P \) (\( b_2 \)) may be interpreted as the elasticity of supply. Of course, \( b_2 \) is a different type of elasticity than those presented earlier. Previous elasticity estimates indicated the relationship between sow farrowings and hog prices prevailing at or before breeding time are consistent with theory, although the elasticity of supply and demand equations (36 and 37, respectively) are delayed until results are presented for the Feb. 1 to Aug. 1 marketing period.

Equations 30, 31 and 32 are estimated for the Aug. 1 to Feb. 1 marketing period for 1938-56 (omitting years 1942 through 1940). Again, all signs in the supply and demand equations (36 and 37, respectively) are consistent with theory, although the elasticity of supply (\( b_{22} = 0.08 \)) probably is not statistically significant. Discussion of the changes in supply and demand elasticities over time, as indicated by this model, are delayed until results are presented for the Feb. 1 to Aug. 1 marketing period.

### Three-Equation Results for the 6-Month Marketing Period, Feb. 1 to Aug. 1

A major portion of the fall pig crop is marketed during the 6-month period from Feb. 1 to Aug. 1. Equations 40, 41 and 42 are estimated for this marketing period from 1924-37. Variables again are expressed or independent variables appears to account for the relatively large standard errors for \( X_2 \) and \( X_3 \). However, the purpose of equation 30 is to predict \( Q \) as accurately as possible from predetermined variables; the statistical significance of the individual regression coefficients (judged by the ratio of the coefficients to their standard errors) is of secondary importance. As indicated by the \( R^2 \) value of 0.87, a relatively high proportion of the variation in \( Q \) is associated with predetermined variables. This result is consistent with the earlier hypothesis that producers vary total slaughter relatively little once hog numbers are established (i.e., after farrowings). Subsequent changes in total slaughter through variation in marketing weights are expected to be considerably less important. Thus, the elasticity of supply (\( b_{22} = 0.04 \)) is positive but small in magnitude.

Equations 35, 36 and 37 are estimated for the Aug. 1 to Feb. 1 marketing period for 1938-56 (omitting years 1942 through 1940). Again, all signs in the supply and demand equations (36 and 37, respectively) are consistent with theory, although the elasticity of supply (\( b_{22} = 0.08 \)) probably is not statistically significant. Discussion of the changes in supply and demand elasticities over time, as indicated by this model, are delayed until results are presented for the Feb. 1 to Aug. 1 marketing period.

### Equations

#### Supply

\[ \hat{Q} = Z = -0.71 + 0.98X_1 + 0.10X_2 - 0.19 + 0.04P + 1.03Z \]

\[ \hat{P} = 3.80 - 1.55Q + 1.62Y \]

#### Demand

\[ Z = 0.69 + 0.99X_1 - 0.12X_2 - 0.02X_3 \]

\[ 0.14 \quad 0.09 \]

\[ R^2 = 0.87 \]

\[ (0.15) \quad (0.12) \]

\[ 0.83Y \quad 0.98X_1 \]

\[ 0.95 \quad 0.63Q + 0.98Y \]

\[ 0.16 \quad 0.15 \]

\[ R^2 = 0.88 \]

\[ (0.20) \quad (0.11) \]

\[ 0.04Y \]

\[ 0.06X_3 \]

\[ 0.55 - 0.36Q + 1.03Z \]

\[ 0.06 \quad 0.21 \]

\[ R^2 = 0.87 \]

\[ (0.17) \quad (0.04) \]

\[ 0.55 - 0.36Q + 0.83Y \]

\[ 0.06 \quad 0.20 \]

\[ R^2 = 0.88 \]

\[ (0.18) \quad (0.11) \]
pressed in logarithms and are defined as follows:

\[ Q = \text{Total liveweight of hogs slaughtered under federal inspection, United States, Feb. 1 to Aug. 1, year t (in units of 100,000,000 lbs.)} \]

\[ X_1 = \text{Number of pigs saved from fall pig crop, United States, year t-1 (in units of 1,000 head).} \]

\[ X_2 = \text{Total feed grain produced in the United States, year t-1 (in units of 1,000 tons).} \]

\[ X_3 = \text{Time, where "time" takes values from 1 to N. (N is the number of years in the period investigated.)} \]

\[ P = \text{Average price of hogs (in dollars per cwt.) received by farmers from Feb. 1 to Aug. 1, year t, United States, divided by the Index of Prices Received by Farmers for Livestock and Livestock Products (1947-49 = 100), United States, during the same period.} \]

\[ Z = \text{Estimate of } Q \text{ based on predetermined variables } X_1, X_2 \text{ and } X_3. \text{ In other words, } Z = \hat{Q} \text{ from equation 43.} \]

\[ Y = \text{Per capita disposable personal income (in dollars) average of first two quarters, year t, United States, divided by the Index of Consumer Prices (1947-49 = 100), average of first two quarters, year t, United States.} \]

Again, all signs in supply equation 41 and demand equation 42 are consistent with theory. However, the elasticity of supply \((b_{12} = 0.07)\) is small and probably nonsignificant, although no test was made.

Equations 45, 46 and 47 relate to the Feb. 1 to Aug. 1 marketing period for 1938-56 (omitting years 1942 through 1946).\(^{29}\)

\[
(45) \quad \hat{Q} = Z = 0.39 + 0.98X_1 - 0.04X_2 + (0.11) (0.14)
+ 0.25X_3 \quad R^2 = 0.96
+ (0.05)
\]

\[
(46) \quad \hat{Q} = 0.66 + 0.05P + 0.85Z \quad \text{: Supply}
\]

\[
(47) \quad \hat{P} = 4.52 - 1.61Q + 1.45Y \quad \text{: Demand}
\]

Once more, the signs of all the coefficients in supply equation 46 and demand equation 47 are consistent with economic theory.

### Elasticities Computed From the Three-Equation Models

Table 4 presents the supply and demand elasticities derived from the preceding three-equation systems for 6-month marketing periods. The individual supply elasticities are not measured with sufficient precision statistically to allow a high degree of confidence in interpretation. However, the logically consistent signs and magnitudes of the supply elasticities in all four models permit somewhat greater confidence in these estimates. (If the true supply elasticity were in fact zero, two positive and two negative signs for \(b_{22}\) would be expected, on the average, in the four equations.) It seems fairly safe to state that the within-marketing-period supply is positive but quite inelastic. However, it is impossible to deduce from these estimates whether the within-marketing-period elasticity of supply has changed over time.

The price elasticities of demand presented in table 4 show a marked decrease from the 1924-37 period to the 1938-56 period. However, the demand elasticities for the 1924-37 period appear unreasonably high, at least in comparison with previous estimates for the interwar period. For example, using annual data for the 1922-41 period, Fox obtained price elasticities of demand for pork of \(-1.18\) based on retail prices and about \(-0.65\) based on farm prices.\(^{30}\) The price elasticities obtained in this study should compare more nearly with the latter figure, since deflated farm prices are used. Alternative deflation and trend removal procedures might explain part of the differences between the estimates of this study and others. Also, the purpose of the simple two-equation model is mainly one of estimating supply response through changes in marketing weights. Consequently, total production figures are used. For a study in which demand elasticities are of primary interest, per capita production or consumption figures clearly are more relevant. Failure to incorporate these refinements into the demand equations may account for the unusually high demand elasticity estimates for the 1924-37 period. It appears that a more complex model is required to derive meaningful estimates of both demand and supply elasticities.

While the magnitude of the change in price elasticity of demand from 1924-37 to 1938-56 probably is overestimated in table 4, the results are consistent with the earlier hypothesis of a decrease in demand elasticity over time. The income elasticity figures in table 4 also show a decrease over time, lending support to the hypothesis that pork has become more of a staple food in the diets of American families.

\[ ^* \text{Omitting years 1942 through 1946.} \]

\[ ^{29} \text{Fox, ibid. p. 43-46.} \]

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**Table 4. Elasticities of Supply and Demand Computed From the Three-Equation Models.**

<table>
<thead>
<tr>
<th>Years</th>
<th>6-month marketing period</th>
<th>Elasticity of supply</th>
<th>Price elasticity of demand</th>
<th>Income elasticity of demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1924-37 &amp; Aug. 1 - Feb. 1 &amp; 0.04 &amp; -1.59 &amp; 1.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1924-37 &amp; Feb. 1 - Aug. 1 &amp; 0.08 &amp; -0.69 &amp; 1.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* As shown in Appendix A, structural equations 46 and 47 are derived from reduced-form equations 48 and 49.

\[
(48) \quad P = 0.54 + 1.59Y - 1.50Z \quad R^2 = 0.88
\]

\[
(49) \quad \hat{Q} = 0.08 + 0.09Y + 0.93Z \quad R^2 = 0.96
\]
APPENDIX A

DERIVATION OF ESTIMATES FOR AN EQUATION INVOLVING TWO EXPECTED PRICE RATIOS

Let \( P_e \) and \( P_r \) denote respectively the expected price of a certain crop at the beginning and end of the growing season for each price ratio in the year. The expectational model for the price ratio \( P_e / P_r \) for each price ratio is shown in equations 1 and 2. It is desired to estimate the coefficients \( a_0 \), \( a_1 \), and \( a_2 \) in equation 3, where \( Y_t \) denotes spring farrowings in year \( t \), \( R^* \) and \( S^* \) denote the above expected price ratios in year \( t \), and \( u_t \) denotes the random residual in year \( t \). Equation 4 shows the same relationship for year \( t-1 \). Subtract equation 4 from equation 3 to obtain equation 5. Substitute the right-hand sides of equations 1 and 2 for the quantities in parentheses in equation 5; the result is given in equation 6. Collecting terms, rewrite equation 6 as equation 7.

(1) \( R^* - R^*_{t-1} = \beta(R_t - R^*_{t-1}) \)
(2) \( S^* - S^*_{t-1} = \beta(S_t - S^*_{t-1}) \)
(3) \( Y_t = a_0 + a_1 R^*_{t-1} + a_2 S^*_{t-1} + u_t \)
(4) \( Y_{t-1} = a_0 + a_1 R^*_{t-1} + a_2 S^*_{t-1} + u_{t-1} \)
(5) \( Y_t - Y_{t-1} = a_1 (R^*_{t-1} - R^*_{t-1}) + a_2 (S^*_{t-1} - S^*_{t-1}) + (u_t - u_{t-1}) \)
(6) \( Y_t - Y_{t-1} = a_1 \beta (R_{t-1} - R^*_{t-1}) + a_2 \beta \)
\( (S_{t-1} - S^*_{t-1}) + (u_t - u_{t-1}) \)
(7) \( Y_t - Y_{t-1} = a_1 \beta R^*_{t-1} + a_2 \beta S^*_{t-1} - \beta \)
\( (a_1 R_{t-1} + a_2 S^*_{t-1}) + (u_t - u_{t-1}) \)
(8) \( (a_1 R^*_{t-1} + a_2 S^*_{t-1}) = Y_{t-1} - a_0 - u_{t-1} \)
(9) \( Y_t - Y_{t-1} = a_1 \beta R_{t-1} + a_2 \beta S_{t-1} - \beta \)
\( (Y_{t-1} - a_0 - u_{t-1}) + (u_t - u_{t-1}) \)
(10) \( Y_t = a_0 \beta + a_1 \beta R_{t-1} + a_2 \beta S_{t-1} + \beta \)
\( (1 - \beta)Y_{t-1} + v_t \)
where \( v_t = [u_t - (1 - \beta)u_{t-1}] \)

Rewrite equation 4 as equation 8. To obtain equation 9, substitute the right-hand side of equation 8 for the first quantity in parentheses in equation 7. Rewrite equation 9 as equation 10. Estimate the coefficients \( a_0 \), \( a_1 \), \( a_2 \), and \( (1 - \beta) \) in equation 10 by least squares. From these estimates obtain the estimates of \( a_0 \), \( a_1 \), and \( a_2 \) to substitute in the original equation 3.

APPENDIX B

DERIVATION OF ESTIMATES FOR A JUST-IDENTIFIED TWO-EQUATION SIMULTANEOUS-EQUATIONS MODEL

Assume the model indicated by equations 1 and 2. The variables are defined as in the text; lower case letters are used

(1) \( q = b_{22} p + b_{23} z \)
(2) \( p = b_{32} q + b_{33} y \)

here since the variables are expressed in the form of deviations from the mean. To obtain reduced-form equations, substitute the right-hand side of equation 1 for \( q \) in equation 2. Solve equation 2 for \( p \) in terms of the predetermined variables \( y \) and \( z \) to obtain equation 3. Similarly, substitute the right-hand side of equation 2 for \( p \) in equation 1. Solve equation 1 for \( q \) to obtain equation 4, which expresses \( q \) as a function of the same predetermined variables \( y \) and \( z \).

(3) \( p = \left( \frac{b_{33}}{1 - b_{32} b_{22}} \right) y + \left( \frac{b_{32} b_{23}}{1 - b_{32} b_{22}} \right) z \)
(4) \( q = \left( \frac{b_{23} b_{22}}{1 - b_{32} b_{22}} \right) y + \left( \frac{b_{23}}{1 - b_{32} b_{22}} \right) z \)

Fit equations 3 and 4 by least-squares regression. The resulting coefficients of equations 3 and 4 are themselves combinations of the structural coefficients \( b_{22} \), \( b_{23} \), \( b_{32} \), and \( b_{33} \). Coefficient \( b_{22} \) is estimated as the ratio of the coefficient of \( y \) in equation 4 to the coefficient of \( y \) in equation 3. Coefficient \( b_{32} \) is estimated as the ratio of the coefficient of \( z \) in equation 3 to the coefficient of \( z \) in equation 4. Given estimates of \( b_{22} \) and \( b_{32} \), coefficients \( b_{23} \) and \( b_{33} \) are estimated directly by algebraic substitution.