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# Rational ecosystems-based fisheries management

Rajesh Singh

Iowa State University, [rsingh@iastate.edu](mailto:rsingh@iastate.edu)

Quinn Weninger

Iowa State University, [weninger@iastate.edu](mailto:weninger@iastate.edu)

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## **Keywords**

rational expectations, multiple-species, individual transferable quotas

## **Disciplines**

Agricultural and Resource Economics | Aquaculture and Fisheries

# Rational ecosystem-based fisheries management

Rajesh Singh  
Iowa State University

Quinn Weninger\*  
Iowa State University

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## Abstract

Ecosystem-based fisheries management (EBFM) seeks to incorporate the full complexity of marine ecosystems into the design of fisheries management policies and regulations. This paper presents a model of a multiple-species, and spatially-temporally heterogeneous fishery that is managed under a tradable quota regulation. Past studies of fisheries management in this setting have, for purposes of simplification, resorted to (1) restrictive assumptions for multiple-species harvesting technologies, (2) myopic and ad hoc behavioral rules for the fishing sector, and (3) assumptions for regulatory control that do not exist in real world fisheries. We derive a rational expectations, ecological-economic equilibrium outcome for harvests, discards, and fishery rent under a costly but endogenous targeting technology and a real-world regulatory instrument. Our results inform crucial and previously unavailable mappings from ecological, economic, and regulatory fundamentals to outcomes of management interest. Ignoring these feedbacks in the design of regulations will vitiate EBFM goals.

**JEL Classification:** Q2

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\*Correspondence: Quinn Weninger, Department of Economics, Iowa State University, Ames IA 50011 - 1040, USA; Phone: 515-294-8976; E-mail: [weninger@iastate.edu](mailto:weninger@iastate.edu). We thank the Lenfest Oceans Program, contract number 00027004 for financial support.

# 1 Introduction

Fisheries management is currently in transition from reliance on single-species management principles to a more holistic approach that has been termed ecosystem-based fisheries management (EBFM).<sup>1</sup> EBFM encourages policies that incorporate the complexities of marine ecosystems, e.g., interactions among multiple fish species, spatial and temporal heterogeneity, chemical and biological processes, multiple ecosystem services. Application of the EBFM paradigm has tackled complexity in coupled ecological-economic marine ecosystems with the help of quantitative computer-based models that due to abundance computing power are capable of tracking a virtually unbounded number of sentient and non-sentient ecosystem elements at fine spatial and temporal scale.<sup>2</sup> These models are used to simulate ecological and economic outcomes of management interest (e.g., spatial-temporal fishing mortality, rent generation) under alternate model parameterizations and competing management scenarios in a process called management strategy evaluation.<sup>3</sup>

To date, policy designs that follow the EBFM approach (i) assume simplistic multiple-species harvest technologies that rule out crucial elements of commercial fishing, (ii) assume ad-hoc, myopic rules for commercial fishing behavior, and (iii) misspecify management regulations. These naive assumptions and miss-specifications invalidate the management strategy evaluation process as it is currently practiced. This paper introduces a model and a solution approach to remedy this problem.

The central focus of our paper is the spatial-temporal evolution of the marine ecosystem under a decentralized production environment. Our motivation is the following. In practice, a fishery regulator sets quotas after observing the initial state of the ecosystem, i.e., the spatial distribution of the multiple-species fish stock, market demand for fish, factor input prices, and the technology available to the commercial sector. The regulator however controls the seasonal quota only. A particular quota choice initiates a transition to an end-of-season state of the ecosystem that is jointly determined by natural ecological forces and equilibrium fishing mortality. To meet long term, i.e., intra-seasonal, management goals, the regulation must understand the *mapping* from initial conditions and a particular quota, the end-of-season outcome. This paper provides this mapping.

We consider a spatially heterogeneous, multiple-species (commercial) fishery that is managed with an individual transferable quota (ITQ) regulation. Following Singh and Weninger

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<sup>1</sup>EBFM is described in Patrick and Link (2015) as one that “Recognizes the combined physical, biological, economic and social tradeoffs for managing the fisheries sector as an integrated system, specifically addresses competing objectives and cumulative impacts to optimize the yields of all fisheries in an ecosystem.”

<sup>2</sup>The Atlantis ecosystem model (Fulton et al., 2007) is a computer-based “simulation modeling approach for marine ecosystems that includes oceanographic, chemical (nutrient cycling), ecological (competition and predation), and anthropogenic processes in a three-dimensional, spatially explicit domain. Atlantis is intended as a strategic management tool to evaluate hypotheses about ecosystem response, to understand cumulative impacts of human activities, and to rank broad categories of management options.” Fishing behavior is assumed to myopically respond to profit opportunities which are linked in *ad hoc* ways to stock abundance and capital costs. Bioeconomic models that exploit rational economic behavior under real world regulatory instruments have not been developed.

<sup>3</sup>Prellezo, et al., 2009 review 13 bioeconomic models which have been developed primarily for evaluating management of European fisheries. Plagányi, 2007 reviews a larger number of models that seek to improve EBFM. Lehuta et al., (2016) discuss complex systems models and management principles.

(2009, 2015), we specify a dual model of the harvest technology that links factor input allocations (e.g., costs of vessel capital services, labor, fuel, bait) and multiple-species stock abundance to multiple-species harvests. The technology we consider emphasizes the endogenous but costly control that fishermen exhibit over the mix of species they harvest.<sup>4</sup> Our model links endogenous targeting to the absolute and relative abundance of the multiple-species fish stock, a feature that is essential component of bioeconomic fisheries models.

In our model, the regulator announces species-specific seasonal quotas that cap the quantity of fish that can be legally landed during a set regulatory cycle, viz., a fishing season. In order to maximize seasonal private profit, fishermen trade quotas in markets, allocate factor inputs, and harvest quotas across diverse regions and across multiple periods within a fishing season. We derive the rational expectations equilibrium, which consists of endogenous quota trading prices, spatial-temporal stock abundance, harvests, landings, and discards of multiple fish species, capital deployment across fishery regions and across periods, and fishery rent under the ITQ-regulation.<sup>5</sup>

The rational ecological-economic equilibrium outcomes we derive satisfy an equi-marginal principle whereby the profit per unit of each species quota is equalized across space and within the regulatory cycle. In equilibrium, there can be no quota rent *hot spots*. Equilibrium quota prices effectively direct and concurrently determine spatial-temporal capital deployment, harvesting operations, and stock conditions. We study the equilibrium outcomes under (i) a range of ecological conditions including regional stock heterogeneity in initial conditions as well as their growth characteristics, (ii) landing price differentials across species and time periods, and (iii) variations in the cost of capital across time within a season.

For a two-species fishery with a given stock and fish market conditions, the equi-marginal principle across various regions allows us to partition a regionwise two-dimensional quota space, which uniquely maps regional quotas to equilibrium harvest and landings and identifies the set of *implementable* quotas that are fully landed without discards. A simple sum of these regionwise implementable sets generates an aggregate fishery-wide set that foretells the regulator’s feasible options. Together, these partitions facilitate an intuitive and insightful analysis of our results that is a key novelty of our approach. This approach can be trivially extended to any number of regions or periods. However, we believe that the results that highlight the key inter-regional and inter-temporal margins are most simply achieved by limiting the analysis to two regions and two subperiods. Therefore, for expositional simplicity, our results focus on a two-species, two-region and two-subperiod fishery.

The results showcase the key spatial, temporal (within-season), and cross species connections that map fundamentals to outcomes of management interest. For example, a decrease in the seasonal quota of a single fish species will in general affect harvests, landings, discards and thus stock growth for all species in all regions of the fishery during all harvest periods. Similar effects follow a change in any economic or ecological condition, irrespective of whether it is limited to a particular region or applies fishery-wide, or whether it is temporary or holds for

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<sup>4</sup>See Branch and Hilborn, 2008; Singh and Weninger, 2017; Weninger et al., 2018 for empirical evidence of endogenous targeting in commercial fisheries.

<sup>5</sup>Berck and Perloff (1984) derive a rational expectations entry equilibrium in an open access fishery. Analytical and computational challenges that arise under rational expectations assumptions may explain why few other papers take this approach.

the entire season. These contemporaneous and dynamic *general equilibrium* feedbacks operate through (1) cost channels due to the joint technology, (2) ecological channels including own- and cross-species-dependent stock growth effects and spatial fish migration, and (3) the trade in quotas that links all spatial and within-season harvest operations.

Our paper makes several contributions to the literature. The first is its pragmatic *necessity*: the model that we offer in this paper is an essential but currently unavailable component of EBFM. The array of ecosystems models currently in use impose restrictive and empirically unsupported assumptions on multiple-species fishing technologies. The Fleet and Fisheries Forecast (F-cube) model, for example, assumes the mix of harvested species as fixed within fleet métiers. Second, currently available models ignore regulatory and market constraints that exist in managed fisheries: The regulatory instrument in popular EBFM models is the quantity of fishing *effort* allocated by fishermen (see Pelletier, et. al., 2009; Ulrich et. al., 2012; Marchal and Vermard, 2013). Third, these models rely on myopic, ad hoc rules for predicting fishing behavior: Standard assumptions are that fishermen allocate effort across time and space in response contemporaneous *accounting* profit opportunities, at rates that must be specified by the model user. Finally, quota market equilibrium prices are ignored. These restrictive technological assumptions, ad hoc behavioral rules, and missing quota prices can mislead the management strategy evaluation process, with potentially serious negative consequences for long term outcomes in managed fisheries.

Our second contribution is to a broader fisheries management literature. Studies of natural spatial-temporal stock diffusion processes, e.g., Brock and Xepapadeas (2010); Costello and Polasky, 2008; Sanchirico and Wilen (1999, 2001, 2005); Smith et. al., (2009), have featured *steady state* analysis under either open access conditions, i.e., no assigned property rights to the fishery resource, or under fully delineated rights, i.e., a sole owner who controls all aspects of harvest activity.<sup>6</sup> Property rights in real world fisheries lie between these two extreme: ITQ regulations, as noted, grant harvest rights but do not specify where or when fish are harvested.<sup>7</sup> Our model combines multiple-species and spatial and temporal heterogeneity and explicitly studies *dynamic* outcomes.

The remainder of the paper is organized as follows. Section 2 presents our model. Combining multiple-species, space and time, increases the dimensionality and complexity of our model. Operationalizing the no hot spot principle across multiple species, locations, and within-season time steps is computationally challenging. Numerical methods are used to solve our model. We present results for a representative regulatory cycle, under various initial ecological-economic

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<sup>6</sup>Clark (1980) recognized early that first-best harvest outcomes are not generally replicated under an ITQ regulation if a fishery has heterogeneous stock abundance. Costello and Deacon (2007) and Valcu and Weninger (2013) characterize second-best management of a temporally heterogeneous, single-species fishery with a time-independent quota regulation. Sanchirico and Wilen (1999, 2001, 2005) examine implications of managing spatially heterogeneous (also single-species) fisheries under open access and input-control regulatory approaches. Studies of multi-species, ITQ-regulated fishing under spatial and temporal stock heterogeneity have, to our knowledge, not appeared in the literature.

<sup>7</sup>Clark (1980), Boyce (1996), Costello and Deacon (2007), Valcu and Weninger (2013), and others the incomplete property right inherent in an ITQ regulation fosters a form of inefficiency where the spatial-temporal distribution of individual species' harvests deviate from their first-best counterparts. The equilibrium outcomes we derive also differ from the first best. However, our focus is not on efficiency loss due to incomplete property rights, but rather their implications for managing a multiple-species fishery with a quota regulation.

conditions, and for varying seasonal quotas.<sup>8</sup> Section 3 characterizes ecological-economic equilibrium outcomes. The final section 4 summarizes our main findings and their implications for EBFM and discusses directions for future research.

## 2 The Model

Consider a spatially heterogeneous, multiple species fishery. We use subscript  $s = 1, 2, \dots, S$  to denote  $S$  distinct regions. There are  $i = 1, 2, \dots, I$  fish species. Regional heterogeneity derives from variation in the marine habitat and therefore stock growth conditions and carrying capacity, economic characteristics, e.g. distance to fishing ports and therefore costs of accessing the resource, or both. A fishing region may support some or all species. We describe spatial-temporal stock growth conditions in detail below.

The bulk of what follows will consider a single representative regulatory cycle or fishing *season*. Much of the analysis is concerned with the spatial and within-season temporal distribution of harvesting activity. We therefore divided the single season of length  $T$  into subperiods, indexed  $t = 1, 2, \dots, T$ .

The fishery is managed with species-specific individual transferable quotas (ITQs). A regulator announces a quota vector denoted by  $Q_0 \equiv \{Q_{i0}\}_{i=1}^I$  that caps seasonal *landings* of each species. We assume the regulation either permits or, due to unobserved at-sea fishing operations, does not prevent at-sea discards. Quota is neither spatially- or subperiod-specific, and therefore the regulator cannot control where or when within the season species  $i$  fish is landed. Quotas are traded in frictionless markets, i.e., there are no trading transactions costs.

Fishermen have access to a common harvesting technology. Each active fisherman employs a single unit of vessel capital which carries a subperiod capital opportunity cost, which we denote  $\rho_s$  in regions  $s$ ; capital costs are assumed constant within season. Variable costs depend on quantities harvested and the absolute and relative abundance of the individual species stocks. Stock abundance and species mix can vary across regions and subperiods and therefore so can variable costs. The assumption of a common technology allows us to think of a *representative* fishermen in region  $s$ . Harvest, landing, and discard choices are assumed identical for all fishermen operating in the same region and also indexed with subscript  $s$ .

### 2.1 A constrained planning problem

If the fishery were owned by a single individual, harvests and landings across species, regions, and subperiods would be chosen to maximize fishery rent for the whole season. While making these choices, this *sole owner* understands that any subperiod's fishing mortality will impact a subsequent subperiod's stock abundance. Additionally, the sole owner can also exercise market power in the input and output markets. The only constraint it faces is that the season's total fish landings do not exceed its quotas. The sole owner's choices, however, diverge from the equilibrium quantities in an ITQ *competitive* equilibrium: While trade in quotas ensures that fish are efficiently harvested and landed across regions and across subperiods such that the value of a marginal quota is maximized, the fishermen trading in these markets know that their

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<sup>8</sup>Note that we do not restrict our analysis to steady state conditions and therefore an infinite set of ecological, economic and management scenarios could be considered.

individual choices in any subperiod are not going to alter the stock conditions in subsequent subperiods. In addition, the fishermen also take all prices as given.

It is in this light that an ITQ equilibrium can be replicated by a *constrained* planning problem. Our fictitious planner is *constrained* to take aggregate stock conditions, factor input prices, and the consumer demand for fish as given, in the sense that the planner assumes these variables are not influenced by her choices. However, when it comes to quota utilization, the planner allocates quota across space and within-season subperiods in the most value- and cost-efficient manner. We emphasize that the constrained planner construct is intended to replicate the competitive equilibrium in the decentralized fishery production environment. Posing the problem in this way simplifies the derivation and presentation of necessary and sufficient equilibrium conditions.

The planner’s optimal within-season choices comprise (a) how much of the seasonal quota of various species to utilize across subperiods and regions, (b) the number of vessel capital to employ across subperiods and regions, and (c) the harvest, discard and landing vectors for each unit of capital/vessel (in each region and subperiod).<sup>9</sup>

Note that each unit of vessel capital deployed in the planning problem mirrors the entry of an individual fisherman and per vessel harvest, landing, and discard quantities correspond to the choices of this fisherman in a decentralized ITQ equilibrium. Hereafter, vessels and fishermen will be used synonymously. In what follows, we use small case letters to describe individual fishermen’s variables and capital letters to describe aggregate variables. Let  $h_{ist}$ ,  $l_{ist}$  and  $d_{ist}$  denote non-negative harvests, landings, and discards of species  $i$  fish for a representative fishermen in region  $s$  and subperiod  $t$ . Note that  $d_{ist} = h_{ist} - l_{ist}$ . We use  $N_{st}$  to denote the units of capital deployed in region  $s$ , subperiod  $t$  and  $\rho_{st}$  to denote the capital cost.

Let  $p_{ist}$  denote the market price of species  $i$  landings in region  $s$ , subperiod  $t$ . The  $I$ -dimension vectors of harvests, landings, discards and prices are denoted  $h_{st}$ ,  $l_{st}$ ,  $d_{st}$ , and  $p_{st}$  respectively. We allow for the case where the landings price depends on its own as well as its substitute species’ aggregate landings/supply;  $L_{st} \equiv \{L_{ist}\}_{i=1}^I$ . We will also for simplicity consider the case where the landings price is fixed. This dependence is made explicit as needed for clarification.

## 2.2 Fishing technology

Models of commercial fishing technologies are rooted in early work by Gordon (1954) and Schaefer (1954) (see Hannesson (1983) for a review). The standard Gordon-Schaefer (G-S) model posits harvest as linear in the fishing *effort*, a proxy for the composite of fixed and variable inputs involved in the harvesting process. Two multi-species extensions of G-S model commonly used in the fisheries literature lie on the extremes. In the first, e.g., Flaaten (1991), each species’ harvest is independent of the harvests and this inputs allocated to other species. This non-jointness assumption overlooks the pervasive feature of commercial fisheries in which multiple fish species are concurrently intercepted by the nets, baited hook and other gears.<sup>10</sup>

<sup>9</sup>Notice that the harvest-mix chosen may include individual species’ harvests that exceed permissible seasonal landings. Such overages must be discarded to comply with the regulation. The constrained planner does not care about this waste given her assumed goal of maximizing *seasonal* profit given quota  $Q_0$ .

<sup>10</sup>The fisheries management literature uses the term *mixed fisheries* to described the joint production of multiple fish species (see Ulrich et al., 2012 and reference therein).

In the second, a common harvest technology yields multiple species in *fixed* proportions.<sup>11</sup>

The properties of nonjointness and fixed output proportions have been tested and rejected using empirical data (e.g., Squires 1987a, 1987b, 1988; Branch and Hilborn, 2008; Singh and Weninger 2017; Weninger et al., 2018). Bycatch and discarding cannot occur a non-joint technology or under a technology that exhibit free output disposability (Turner, 1995; Singh and Weninger, 2009). Fixed output proportions rules out endogenous targeting of individual species. Neither assumption is adequate for understanding multiple-species fishing behavior.

Following Singh and Weninger (2009), we specify below a dual, multiple-species technology that is joint-in-inputs, exhibits the property of weak output disposability, and explicitly incorporates multiple-species stock abundance.<sup>12</sup> We assume costs attain a minimum when the mix of harvested species aligns with the mix of stocks in the region and subperiod of fishing. A two species example motivates this property. Suppose region and subperiod  $(s, t)$  stock of species 1 is more abundant than species 2. It should then be less costly to harvest a relatively higher quantity of species 1 than, say, equal amounts of both species. The cost savings arise because by targeting a mix that mirrors the relative stock abundance, costly searching and/or gear and bait modifications that may otherwise be required to intercept additional species 2 or avoid the more abundant species 1 are saved. Note also that if the regulator sets quotas that do not match their relative stock abundance, e.g., suppose the quota for species 1 is relatively small despite its relative abundant stock, the marginal cost of harvesting species 1, evaluated at the regulated quota mix, could be negative. In this scenario, increasing species' 1 harvest beyond its quota level and discarding the excess catch may actually lower costs since costly actions to avoid species 1 will not be required (see Singh and Weninger (2009) for additional discussion and Singh and Weninger (2017) for empirical evidence).

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<sup>11</sup>The F-cube model (Ulrich et al., 2008, 2009) assumes the fishing mortality exerted on a specific fish stock by a fleet *métier* (e.g., a common vessel type and size, gear type, fishing area, time of the year) is proportional to the effort exerted. The implied technology exhibits fixed output proportions and satisfies the property of input-output separability.

<sup>12</sup>Models of dual harvesting technologies appear in Smith (1968), Squires (1987), Kirkely and Strand (1988), Weninger (1998), among others. Smith (1968) includes, as an argument of the cost function, a measure of total capital allocated to the fishery. In this case, costs are assumed to be non-decreasing in total capital, reflecting a potential externality where large number of vessels on the fishing ground can impede each other and raise the cost of harvesting fish.

A functional form that exhibits the stock-dependent *costly targeting* property is<sup>13,14</sup>

$$c(h, \phi(X, H)) = \left( 1 + \sum_{j=1}^I \gamma_j \left( \frac{h_j}{\sum_{k=1}^I h_k} - \frac{X_j}{\sum_{k=1}^I X_k} \right)^2 \right) \sum_{j=1}^I \phi^j(X_j, H_j) h_j^\nu, \quad (1)$$

where the function  $\phi$  is non-increasing in abundance  $X_{st}$ , and non-decreasing in aggregate harvest  $H_{st}$ .<sup>15,16</sup> We assume in addition that  $\nu > 1$  such that variable costs are increasing and convex in the harvest of each species.

When  $\gamma \geq 0$  but finite, which we assume, the first bracketed term on the right-hand side of (1) captures the targeting component of the technology. This term increases as the harvest shares and stock shares diverge. Thus as the fisherman targets a harvest share vector that differs from the mix of stocks in the sea, harvesting costs rise.

Observe that if  $\gamma = 0$  the first right-hand term in (1) takes the value of unity for all  $h$ ; harvest costs then are given by the second additively separable term  $\sum_{j=1}^I \phi^j(X_j, H_j) h_j^\nu$ . Thus when  $\gamma = 0$  the technology is non-joint and species-specific cost (and harvest) functions exist. Alternatively when  $\gamma \rightarrow \infty$ , the first term in (1) is infinite whenever the species mix differs from the stock abundance mix. When  $\gamma \rightarrow \infty$  the technology exhibits fixed output proportions, i.e., adjusting the species mix away from from the stock abundance mix (endogenous targeting) is infinitely costly. Implications of extreme technological assumptions are discussed in section 3.1.

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<sup>13</sup>An alternative specification is

$$c(h, \phi(X, H)) = \sum_{j=1}^I \left( 1 + \gamma \left( \frac{h_j}{\sum_{k=1}^I h_k} - \frac{X_j}{\sum_{k=1}^I X_k} \right)^2 \right) \phi^j(X_j, H_j) h_j^\nu,$$

For the special case  $I = 2$  both specifications are equivalent.

<sup>14</sup>Note that

$$\frac{h_1}{h_1 + h_2} - \frac{X_1}{X_1 + X_2} = \frac{h_2}{h_1 + h_2} - \frac{X_2}{X_1 + X_2}.$$

Hence targeting costs are symmetric across the two species. This need not be the case in general. See Singh and Weninger (2009) for a detailed discussion of the cost function for more than two species and asymmetric targeting costs.

<sup>15</sup>Discrete time models of fisheries exploitation are complicated by the fact that stock growth is a continuous process, while an ITQ regulation specifies seasonal landings limits. Our model strikes a compromise between notational simplicity and accuracy. For example, if there is no within subperiod stock growth, which we assume to be the case, per vessel variable costs may be more accurately defined as,

$$c(h_j, X_j) = \int_0^{h_j} c(z, X_j - N_j z) dz,$$

where  $X_j$  is the beginning subperiod stock abundance and  $z$  is a variable of integration. Smith (1968) and others consider the possibility that variable harvesting costs depend also on the quantity of capital deployed, i.e.,  $\phi = \phi(X_j, H_j, N_j)$  with  $\partial\phi(\cdot)/\partial N_j > 0$ . An extension of the model to consider capital congestion effects is reserved for future work.

<sup>16</sup>The function  $\phi$  may be region specific. The results we present are qualitative and they remain unaltered under this modification. For brevity, however, we abstract from this particular form of heterogeneity in this paper.

### 2.3 A recursive equilibrium

The constrained planning objective is to maximize seasonal profit by choosing a cross-regional and within-season sequence of vessel capital deployment, harvests, landings, and discards for each representative vessel:  $\{N_{st}, \{l_{ist}, d_{ist}\}_i\}_{st}$ . The problem can be described as

$$\max_{\{N_{st}, \{l_{ist}, d_{ist}\}_i\}_{st}} \sum_{t=1}^T \sum_{s=1}^S N_{st} \left( p_{st} \cdot l_{st} - c \left( \underbrace{h_{st}}_{\equiv l_{st} + d_{st}}, \phi(X_{st}, H_{st}) \right) - \rho_{st} \right)$$

To ensure an efficient allocation of quota across space and time, the planner *internalizes* the laws of motion for all species-specific quotas and subperiods  $t$  to  $t + 1$ :

$$Q_{it+1} = Q_{it} - \sum_{s=1}^S L_{ist} = Q_{it} - \sum_{s=1}^S N_{st} l_{ist} \text{ for all } i, \quad (2)$$

where the second equality follows from  $\sum_{s=1}^S L_{ist} = \sum_{s=1}^S N_{st} l_{ist}$ . To solve this problem, the planner must know the sequence of stock abundance across space and time.

It is assumed that the stock abundance of any species in any region in subperiod  $t + 1$  potentially depends on the escapements of all species across all the regions. Formally, the stock vector across all species and regions follows

$$X_{t+1} = \Gamma(E_t) \quad (3)$$

where  $E_{ist} = X_{ist} - H_{ist}$ .<sup>17</sup> To illustrate, let there be two species (1 and 2) and two regions (1 and 2). A simple specification is:

$$X_{11t+1} = E_{11,t} + r_{11} E_{11t} \left( 1 - \frac{E_{11t}}{K_{11}} - \alpha_{11} E_{21t} \right) + \kappa_{11} \left( \frac{E_{12t}}{K_{12}} - \frac{E_{11t}}{K_{11}} \right); \quad (4)$$

$r_{11}$  is an own-stock growth/recruitment parameter;  $\alpha_{11}$  captures cross-species competition (if positive) or predation (if negative) on species 1 by species 2 in region 1;  $\kappa_{11}$  captures net regional migration of species 1, and  $K_{ij}$  is species and region-specific stock carrying capacity.

We now impose the idea of a rational expectations equilibrium to solve the constrained planning problem. Such an equilibrium requires that the sequence of stock abundance be consistent with the aggregate harvest choices. Since  $H_{ist} = N_{st} h_{ist}$  is the aggregate harvest of species  $i$  in region  $s$  in subperiod  $t$ , it follows that  $E_{ist} = X_{ist} - N_{st} h_{ist}$ . The equilibrium then requires that abundance  $X_{t+1}$  be consistent with the growth specification (4) and the constrained planner's harvest choices. Under this equilibrium construct, the regional and temporal sequence of harvests, landings, and discards can be solved for since the problem is fully specified.

It is easier to approach this problem as a dynamic program. Let  $'$  denote next subperiod

<sup>17</sup>This specification assumes that all discarded fish die. Escapement can be specified alternatively as  $E_{ist} = X_{ist} - L_{ist} - \omega_i D_{ist}$ , where  $\omega_i$  denotes the species  $i$  survival rate for discarded fish.

variables. The state vector includes current quota holdings and stock abundance:  $\{Q, X\}$ . The Bellman equation for the dynamic program is:

$$V_\tau(\{Q_i\}_i, \{\{X_{is}\}_i\}_s) = \max_{\{N_s, l_{is}, d_{is}\}} \left\{ \begin{array}{l} \sum_{s=1}^S N_s (p_s \cdot l_s - c(h_s, \phi(X_s, H_s))) \\ - \sum_{s=1}^S N_s \rho_s + V_{\tau+1}(\{Q'_i\}_i, \{\{X'_{is}\}_i\}_s) \end{array} \right\} \quad (5)$$

subject to,

$$0 \leq Q'_i = Q_i - \sum_{s=1}^S N_s l_{is}; \quad (6a)$$

$$X' = \Gamma(E) \quad (6b)$$

Let the current state be denoted as  $Y \equiv \{Q, X\}$ . A recursive equilibrium consists of a set of landings,  $l_\tau(Y) \equiv \{\{l_{is\tau}(Y)\}_i\}_s$ , discards,  $d_\tau(Y) \equiv \{\{d_{is\tau}(Y)\}_i\}_s$  (which imply harvests  $h_\tau(Y) = l_\tau(Y) + d_\tau(Y)$ ), vessel deployments,  $N_\tau \equiv \{N_{s\tau}\}_s$ , landings prices,  $p(L(Y))$ , and law of motion for  $X'_\tau(Y)$  for  $\tau = 1, \dots, T-1$  such that

1. Given  $X'_\tau(Y)$  and  $p(L(Y))$ ,  $l_\tau(Y)$ ,  $d_\tau(Y)$ ,  $N_\tau(Y)$  for  $\tau = 1, \dots, T$  solve the dynamic program (5) subject to (6a) and (6b).

2. Aggregate laws of motion are consistent with policy functions:

$$(a) \quad L_\tau \equiv N_\tau \cdot l_\tau, H_\tau = N_\tau \cdot h_\tau,$$

$$(b) \quad Q'_\tau(Y) = Q_\tau - L_\tau(Y),$$

$$(c) \quad X'_\tau(Y) = \Gamma(X_\tau - H_\tau(Y)), \text{ for } \tau = 1, \dots, T.$$

### Characterizing the equilibrium

The Kuhn-Tucker necessary condition for the region  $s$  optimal capital allocation  $N_s$  is,

$$p_s \cdot l_s - c(h_s, \phi(X_s, H_s)) - \rho_s - \sum_i \frac{\partial V_{\tau+1}(\{Q'_i\}_i, \{\{X'_{is}\}_i\}_s)}{\partial Q'_i} \cdot l_{is} \leq 0. \quad (7)$$

If  $N_s > 0$  equation (7) holds with equality.

The necessary condition for species  $i$  and region  $s$  landings is,

$$p_{is} - c_{h_{is}}(h_s, \phi(X_s, H_s)) - \frac{\partial V_{\tau+1}(\{Q'_i\}_i, \{\{X'_{is}\}_i\}_s)}{\partial Q'_i} \leq 0, \text{ for } i = 1, \dots, I, \quad (8)$$

where  $c_{h_{is}}(\cdot)$  denotes partial differentiation with respect to the subscripted argument. If  $l_{is} > 0$  equation (8) holds with equality.

Letting  $\lambda_i$  denote the multiplier on the quota constraint for species  $i$ , the optimal choice for the quota to be landed in the current period and that to be carried forward, i.e.,  $Q'_i$  obtains:

$$\frac{\partial V_{\tau+1}(\{Q'_i\}_i, \{\{X'_{is}\}_i\}_s)}{\partial Q'_i} = \lambda_i. \quad (9)$$

This condition states that the current species  $i$  quota shadow price is equal to its marginal value if carried forward. An application of the Envelope theorem for current  $Q_i$  obtains

$$\frac{\partial V_{\tau}(\{Q_i\}_i, \{\{X_{is}\}_i\}_s)}{\partial Q_i} = \frac{\partial V_{\tau+1}(\{Q'_i\}_i, \{\{X'_{is}\}_i\}_s)}{\partial Q'_i} = \lambda_i. \quad (10)$$

This condition implies that an optimal intertemporal quota allocation equates its marginal value in every period. Otherwise, the seasonal profit can be increased by allocating quotas towards subperiods in which its marginal value is higher. Note that in a decentralized equilibrium  $\lambda_i$  will be equal to the equilibrium market (lease) price for a unit species  $i$  quota.

Combining the preceding three equations we have for all  $\tau \leq T$  and  $s = 1, \dots, S$ :

$$p_{is} - c_{h_{is}}(h_s, \phi(X_s, H_s)) - \lambda_i \leq 0. \quad (11)$$

If  $l_{is} > 0$  equation (11) holds with equality. This is a standard result equating the marginal net profit from a unit of quota to its shadow price.<sup>18</sup> What is worth noting though is that the condition holds for all species in all regions and within-season subperiods.

Equation (11) affirms that marginal revenue obtains from landings whereas marginal costs depend on harvested quantities. Since landings can not exceed harvests, and discards are nonnegative, optimal discards are characterized by

$$-c_{h_{is}}(h_s, \phi(X_s, H_s)) \leq 0. \quad (12)$$

If  $d_{is} > 0$ , equation (12) hold with equality. The condition for optimal discards can be stated alternatively as,  $d_{is}c_{h_{is}} = 0$ .

Notice that if  $c_{h_{is}} < 0$ , costs can be lowered by harvesting more species  $i$  fish; variable profits rise even if the extra catch is discarded at sea. Also, since  $p_{is} \geq 0$  for all  $i$  for non-binding quotas,  $\lambda_i = 0$ , condition (11) becomes,

$$p_{is} - c_{h_{is}}(h_s, \phi(X_s, H_s)) = 0; \quad d_{is} = 0. \quad (13)$$

This condition states simply that since harvest can be landed to earn positive revenues, there are no discards when the aggregate quota is slack.

Finally, combining (10) and (9) with (7) yields

$$\underbrace{p_s \cdot l_s - c(h_s, \phi(X_s, H_s)) - \sum_i \lambda_i l_{is}}_{\pi_s} \leq \rho_s. \quad (14)$$

<sup>18</sup>It is also possible that despite quota being available harvesting of a species at a particular location is prohibitively expensive;  $l_{is} = 0$  in all such cases.

If  $N_s > 0$  equation (14) holds with equality.

Equation (14) is a capital entry/exit condition in a decentralized ITQ fishery: variable profit or capital quasi-capital rent in our case, net of quota rent,  $\pi_s$ , must equal the capital cost. If  $\pi_s > \rho_s$ , capital continues to enter until  $\pi_s = \rho_s$ . No capital enters, or all of it exits, in a region  $s$  if  $\pi_s < \rho_s$ .

### Solving the constrained planner's problem

The recursive equilibrium is obtained as the solution to  $S \cdot T$  optimal capital conditions from equation (7),  $I \cdot S \cdot T$  conditions on optimal landings from equation (8), and  $I \cdot S \cdot T$  conditions for optimal discards from equation (12). In addition there are  $I$  quota constraint conditions. These equations, along with the equilibrium consistency requirements described in section (2.3), solve for (i)  $2 \cdot I \cdot S \cdot T$  harvest and landing choices, (ii)  $S \cdot T$  capital deployments, and (iii)  $I$  quota shadow prices.

In the simple case with  $I = S = T = 2$  there are 8 harvest and landings choices, 4 capital deployment choices, and 2 quota prices that must be determined. Interactions between the many variables are complex. In the next sections we derive additional insights under some special cases to illustrate implications of the model for fisheries management.

### Intensive and extensive cost margins

Consider a region  $s$  in which  $N_s > 0$ . From (12), it follows that either  $h_{is} = l_{is} > 0$  or  $c_{h_{is}}(h_s, \phi(X_s, H_s)) = 0$ . Then, (11) implies

$$p_s \cdot l_s - \lambda \cdot l_s = h_s \cdot c_{h_s}.$$

Since  $N_s > 0$ , the capital quantity necessary condition, (14), obtains;

$$h_s \cdot c_{h_s} = c(h_s, \phi(X_s, H_s)) + \rho_s,$$

which essentially reflects an efficiency condition that a competitive equilibrium brings about by linking the *intensive* and *extensive* margins. The right-hand side denotes the cost of an extra vessel (extensive margin) that would harvest  $h_s$ . The left-hand side denotes the cost of additional  $h_s$  by spreading it over existing vessels (intensive margin). This marginal condition can be summarized as:

Proposition 1: *Variable costs incurred by an active vessel is given by*

$$c(h_s, \phi(X_s, H_s)) = \frac{\rho_s}{\nu - 1}. \quad (15)$$

Proof: In appendix 6 we show that  $h_s \cdot c_{h_s} = \sum_i h_{is} c_{h_{is}}(h_s, \phi(X_s, H_s)) = \nu c(h_s, \phi(X_s, H_s))$ . At an optimum that follows (11) - (13), a fisherman's sales revenue net of quota lease cost equals  $h_s \cdot (p - \lambda) = h_s c_{h_s} = \sum_i h_{is} c_{h_{is}}$ . Since the targeting component of the cost function is

homogeneous of degree zero, its contribution to  $\sum_i h_{is} c_{h_{is}}$  is equal to zero. Only the direct (non-targeting) marginal harvest costs matter for the total and net revenue contributions. Notice that the direct component of the cost function has a common exponent  $\nu$  on all individual species' harvests. Then, the harvest revenue net of variable costs is  $(\nu - 1)$  times the latter. Conditions for optimal capital entry/exit ensure that the net harvest profit is equal to the capital cost.

Proposition 1 anchors the fishing costs *for all subperiods* in a fishing season. If the dockside prices are common across regions and are constant within a season, (11) further commands that the marginal costs of each species' harvest  $c_{h_{is}}$  be identical across regions and subperiods. An interesting implication is that a fish species with a sufficiently low landing quota may be discarded throughout the season if its stock conditions warrant discard. Despite the harmonization of the total costs and each species' marginal costs across time and regions, the equilibrium still allows for cross-species harvest and stock variations across time and region, as will be illustrated in the next section.

### 3 Results

This section reports equilibrium harvests, landings, discards, capital allocations, and rent outcomes under varying economic and beginning-of-season ecological conditions and for varying quotas. Note that an infinite number of initial conditions and quotas are possible, with each leading potentially to different equilibrium outcomes. Given our space limitations we select initial conditions and quotas that illustrate some critical aspects of the equilibrium outcomes of the model.

To further simplify the presentation, we consider a two- species, two-region and two-subperiod fishery example. Within a fishing season, quota utilizations across different subperiods are facilitated through quota trade and carryovers. The quotas *utilized* within a subperiod operate as if they are the quota *constraints* for that subperiod insofar as they must be consistent with optimality conditions (11) - (13) and the capital allocation condition in Proposition 1. In effect, each subperiod can be studied independently by taking its initial stock conditions and the remaining unfished quota as given. The full season equilibrium requirement is that (i) the quota prices across all subperiods must be equal and (ii) the stock abundance in the second subperiod must be consistent with the escapement growth and dispersion from the first subperiod.

With  $I = 2$ ,  $\frac{h_j}{\sum_{k=1}^2 h_k} - \frac{X_j}{\sum_{k=1}^2 X_k}$  is the same for both  $j$ . Hence, letting  $\gamma = \gamma_1 = \gamma_2$ , the cost function (1) becomes

$$c(h, \phi(X, H)) = \left( 1 + \gamma \left( \frac{h_1}{h_1 + h_2} - \frac{X_1}{X_1 + X_2} \right)^2 \right) (\phi^1(X_1, H_1) h_1^\nu + \phi^2(X_2, H_2) h_2^\nu).$$

The results we present below are qualitative in nature. However, for graphical presentations, we rely on a specific parameterization of the model. Our benchmark case assumes a symmetric ecology, i.e., common stock growth parameters across species, and common economic conditions across species and regions, i.e.,  $p_{is} = p$  for all  $i, s$ . We specify the function  $\phi$

as  $\phi^s(X_s, H_s) = \frac{\eta_s H_s}{X_s}$  where  $\eta_s > 0$  is a fixed parameter. Parameter values for our benchmark case are reported in table 1.

| Cost and prices        | Stock growth                 |
|------------------------|------------------------------|
| $\gamma = 25$          | $r_{is} = r = 1$             |
| $\nu = 1.25$           | $\alpha_{is} = \alpha = 0.2$ |
| $\eta = 2.5$           | $\beta = 0.05$               |
| $\rho_s = \rho = 0.15$ | $\kappa_i = \kappa = 0.1$    |
| $p_{is} = p = 1$       | $K_{ij} = K = 0.08$          |

Table 1: **Benchmark model parameters.**

After obtaining the steady state of our benchmark case for a season with  $T = 2$ , ecological and economic heterogeneity across species, regions, and subperiods are introduced sequentially and one at a time to highlight the critical predictions of our model. For both species, let  $x^*$  and  $Q^* = H^*$  denote the steady state stock abundance and quota that maximize the present value of the fishery, under the parameters in table 1 over an infinite (seasonal) planning horizon. Let  $N^*$  denote the equilibrium capital employed in each region. Below we consider scenarios in which initial stock abundance and/or seasonal quotas deviate from these steady state values.

### 3.1 Within-subperiod equilibrium

With the background above, we first examine how quota constraints map to endogenous multiple-species fishing mortality within a single subperiod. To proceed, we first partition the quota space into *zones* that (a) demarcate quota combinations for which landings constraints either bind or are slack and (b) demarcate quota combinations for which the harvest of either of the species is discarded.

Figure 1 shows the eight quota zones that correspond to the constraints in equations (11) and (12). Zones I-VIII in figure 1 show aggregate or fishery-wide outcomes. Quantities on the axes are scaled as a percentage of the fishery value-maximizing steady state quota. Line segments  $0A\bar{A}$  and  $0C\bar{C}$  delineate discard zones (Singh and Weninger, 2009). In zones I, II, and III the species 1 quota exceeds the unconstrained optimal harvest of species 1, i.e., the species 1 quota is slack. Zones III and IV contain quotas for which optimal harvests satisfy the necessary condition  $c_{h_{2s}}(h_s, \phi(X_s, H_s)) = 0$  with positive discards of species 2 fish. This occurs because the species 2 quota in zones III and IV is relatively small, i.e., costs can be reduced by harvesting in excess of  $Q_2$  and discarding the overage. In zone II discards are zero; while the species 2 quota is relatively low in this zone, both species quotas are large (relative to quotas in zones III and IV). For quota in zone II, it is profitable to harvest a mix of species that more closely aligns with stock abundance and leave a portion of the species 1 quota unutilized, i.e.,  $\lambda_1 = 0$  for  $Q \in$  zone II. For the same reasons (but with species numbers reversed), species 1 discards are positive for  $Q \in$  zones V and VI, and there are no discards when  $Q \in$  zone VII. In zone VIII, both the quotas bind while neither species is discarded. Importantly, observe that the diamond shaped zone VIII is the implementable harvest set identified in Singh and Weninger (2009). The implication is that only in zone VIII is the mapping from quotas to harvests, and landings with no discards, one-to-one.

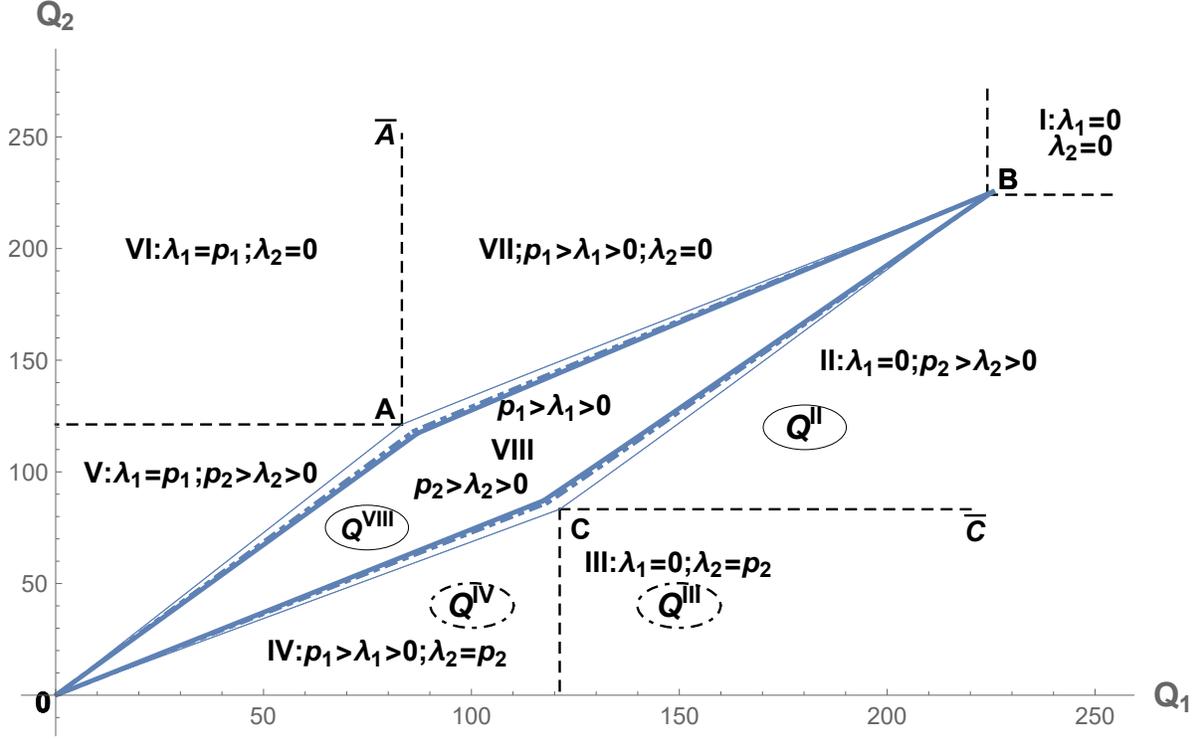


Figure 1: **Equilibrium in an ITQ-Regulated Fishery.** Quotas are denoted as a percentage of steady state values. Zones I - VIII identify quota combinations  $\{Q_1, Q_2\}$  for which landings constraints either bind or are slack and/or for which individual species harvests are either retained or discarded at sea.

Figure 1 shows implementable sets under three scenarios for the beginning season stock abundance. The innermost set demarcated by thick lines assumes that the fish stock is spatially homogeneous with each region's abundance equal to the steady state stocks,  $X^* = \{\{x^*, x^*\}, \{x^*, x^*\}\}$ . Hereafter, we refer to this stock abundance scenario as the spatially homogeneous fishery or SP-HOM for short. The outermost set, demarcated by a thin line, assumes that regional stock abundance at the start of the season is,  $X = \{\{1.2x^*, 0.8x^*\}, \{0.8x^*, 1.2x^*\}\}$ . In this case, region 1 is relatively more abundant in the species 1 stock while region 2 is symmetrically more abundant in species 2 stock; no region has an *absolute* abundance advantage but each region has a *comparative* advantage in a particular species' stock. Hereafter we will refer to this stock scenario as the spatial comparative advantage fishery or SP-CA for short. The implementable set demarcated with a dashed lines assumes initial regional stock abundance quantities,  $X = \{\{1.2x^*, x^*\}, \{0.8x^*, x^*\}\}$ . In this scenario region 1 exhibits an absolute abundance advantage. Also, both regions have a comparative advantage in one of the species stocks but these magnitudes are smaller compared to the previous scenario. Below we refer to this scenario as the spatially comparative and absolute abundance fishery, SP-CAA for short.

The reader will notice that while our stock abundance differ considerably, the fishery wide

implementable sets in figure 1 almost converge. This is by construction: in each of the three scenarios, aggregate abundance is  $\{2x^*, 2x^*\}$ . The regional differences become salient in the respective regional partitions shown in figure 2 below.

It is worth reiterating that the equilibrium quota trading prices,  $\lambda_1$  and  $\lambda_2$  vary with the quota set by the regulator, initial stock conditions, landings and factor input prices, and model parameters, but are common across regions. Similarly, because dockside prices are assumed common across regions, if either species' harvest is discarded its quota price will equal to its landings price, i.e.,  $\lambda_i = p_i$ , and there will be discards in both regions.<sup>19</sup> Since the equilibrium quota prices are common across the whole fishery including all its regions, zones I-VIII of the fishery wide quota partition uniquely map with similar zones in regional partitions, as illustrated in figure 2.

Figure 2a corresponds to the SP-CA stock abundance scenario, with relatively high species 1 abundance and thus lower species-specific costs in region 1. As a result, quotas for which the species 1 constraint is slack (zones II and III) and quotas for which species 2 is discarded (zones III and IV) are relatively large. The situation is reversed in region 2 where species 2 is more abundant and less costly to harvest. Overall, the divergence between the regional implementable sets highlights the assumed species-specific comparative advantages. Both sets however cover the same area, which indicates that neither has an absolute stock advantage.

Figure 2b shows zonal demarcations corresponding to the SP-CAA abundance scenario; a comparative advantage for species 1 in region 1, albeit diminished relative to the SP-CA scenario. Observe regional sets that exhibit more overlap than in figure 2a. A larger area  $OA_1B_1C_1$  in region 1 indicates its absolute stock size advantage.

We next present a complete characterization of harvest and discard behavior for quotas in zones I-VIII from figure 1. Reference to the three stock scenarios support the discussion. To ease notation and where no confusion can arise, this section adopts the notational convention  $\phi^s = \phi(X_s, H_s)$ .

**Zone I: Neither species' quota binds; no discards.** For quotas set in zone I,  $\lambda_i = 0$  for  $i = 1, 2$ . Then,  $c_{h_{is}}(h_{1s}, h_{2s}, \phi^s) = p_i$  for all  $i, s$ . These conditions along with Proposition 1 determine the unconstrained per vessel harvests  $\{h_{is}\}$  and number of vessels  $\{N_s\}$  within each region. The unconstrained aggregate harvests at  $B$  in figure 1 obtain by simple addition of harvest vectors corresponding to points  $B_1$  and  $B_2$  in figure 2. Correspondingly, any fishery-wide quota that lies in zone I in figure 1 will, in equilibrium, map to points  $B_1$  and  $B_2$  in figure 2.

**Zone II: Species' 2 constraint binds; no discards.** If a quota is set in zone II,  $\lambda_1 = 0$ , and  $\lambda_2 > 0$ . In the absence of discards,  $\lambda_2 < p_2$ . Here,  $c_{h_{1s}}(h_{1s}, h_{2s}, \phi^s) = p_1$  for  $s = 1, 2$ ;  $c_{h_{21}}(h_{11}, h_{21}, \phi^1) = c_{h_{22}}\left(h_{12}, \frac{Q_2 - N_1 h_{21}}{N_2}, \phi^2\right) = p_2 - \lambda_2 \in (0, p_2)$ . These conditions along with proposition 1 obtain per vessel harvest vectors  $\{h_{is}\}$  and number of vessels  $\{N_s\}$  in each region.

A simple aggregation obtains total regional and fishery-wide harvest vectors. Each point  $\lambda_2 \in (0, p_2)$  lies on the line segment  $B_1C_1$  and  $B_2C_2$  in figure 2. If the regionally allocated

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<sup>19</sup>These arguments hold for any number of regions and species; assuming  $I = 2$  facilitates a graphical representation.



quotas of species 2 (read on  $y$  – axis) are at or above  $B_1$  and  $B_2$ , they do not bind, i.e.,  $\lambda_2 = 0$ . They bind below  $B_1$  and  $B_2$  and as they decrease further  $\lambda_2$  rises until  $\lambda_2 = p_2$ ; points  $C_1$  and  $C_2$  correspond to this pair of species 2 quotas. The line segment  $BC$  in figure 1 aggregates these regional segments: for each  $\lambda_2 \in (0, p_2)$ , there is a unique point on the three line segments  $\{B_1C_1, B_2C_2, BC\}$ . Conversely, a point on  $BC$  can be uniquely mapped to points on regional zonal boundaries. Thus, if a fishery wide quota lies in zone II, the total harvest and landing of both species lies on the line segment  $BC$  and corresponding regional harvests lie on  $B_1C_1$  and  $B_2C_2$ , with  $H_{21} + H_{22} = Q_2$ . Obviously,  $Q_1 > H_{11} + H_{12}$ .

Quota point  $Q^{II} = \{1.8Q^*, 1.2Q^*\}$  in figure 1 falls in zone II. For the SP-CA stock scenario (figure 2a), the two regional harvests are at point  $H^{II}$  in the respective partitions.<sup>20</sup>

**Zone III: Species' 2 constraint binds; positive species 2 discards.** For quotas set in zone III we have  $\lambda_1 = 0$  and  $\lambda_2 = p_2$ . Here,  $c_{h_{1s}}(h_{1s}, h_{2s}, \phi^s) = p_1$  for all  $s$ ;  $c_{h_{21}}(h_{11}, h_{21}, \phi^1) = c_{h_{22}}(h_{12}, h_{22}, \phi^2) = 0$ ;  $Q_2 < H_{21} + H_{22} = N_1h_{21} + N_2h_{22}$ . These conditions along with Proposition 1 obtain per vessel harvest vectors  $\{h_{is}\}$  and number of vessels  $\{N_s\}$  in each region. Notice from the above conditions that in zone III, equilibrium harvest quantities are independent of  $\{Q_1, Q_2\}$ .

Quota  $Q^{III} = \{1.5Q^*, 0.4Q^*\}$  in figure 1 falls in zone III. For the SP-CAA stock scenario (figure 2b), the two regional harvests and landings fall at points  $H^{III}$  and  $L^{III}$  in the respective regional partitions. Species 2 discard occurs as a result. Notice that under SP-CAA, region 1 harvests a larger share of both species due to its larger stock.

**Zone IV: Quotas of both species bind; positive species 2 discards.** In zone IV,  $\lambda_1 \in (0, p_1)$  and  $\lambda_2 = p_2$ . Here  $c_{h_{11}}(h_{11}, h_{21}, \phi^1) = c_{h_{12}}\left(\frac{Q_1 - N_1h_{11}}{N_2}, h_{22}, \phi^2\right) = p_1 - \lambda_1 \in (0, p_1)$ ;  $c_{h_{21}}(h_{11}, h_{21}, \phi^1) = c_{h_{22}}(h_{12}, h_{22}, \phi^2) = 0$ . Once again, these conditions along with proposition 1 obtain per vessel harvest vectors  $\{h_{is}\}$  and vessel allocations  $\{N_s\}$  for region  $s$ . A simple aggregation obtains fishery-wide harvest vectors. Each point  $\lambda_1 \in (0, p_1)$  lies on the line segment  $0C_1$  and  $0C_2$ , with  $\lambda_1 = 0$  at  $C_1$  and  $C_2$  and  $\lambda_1 = p_1$  at 0 (figure 2). The line segment  $0C$  in figure 1 aggregates these two regional segments: for each  $\lambda_1 \in (0, p_1)$ , there is a unique point on the three line segments  $\{0C_1, 0C_2, 0C\}$ . Conversely, a point on  $0C$  can be uniquely mapped to the points on its regional counterparts.

Clearly, if the fishery-wide quota lies in zone IV, i.e., the total harvest of both species lies at its *vertical* projection on the line segment  $0C$ . The corresponding regional harvests lie on  $0C_1$  and  $0C_2$ , with  $H_{11} + H_{12} = Q_1$  while  $Q_2 = L_{21} + L_{22} < H_{21} + H_{22}$ .

Quota point  $Q^{IV} = \{1.5Q^*, 0.4Q^*\}$  in figure 1 falls in zone IV. For the SP-CAA stock scenario (figure 2b), the two regional harvests and landings are at points  $H^{IV}$  and  $L^{IV}$  in the respective regional partitions. Species 2 discard occurs as a result. Unlike zone III, the quota of species 1 is now fully utilized.

**Zone V, VI, and VII** Equilibrium outcomes in zones V, VI, and VII mirror those in zones IV, III, and II respectively, with species' quotas and stocks reversed. We do not repeat the results to conserve space.

<sup>20</sup>  $L^{II} = H^{II}$ , though not shown in the figure to avoid clutter.

**Zone VIII: Both species' quotas bind: no discards.** In zone VIII, quota shadow prices satisfy  $\lambda_i \in (0, p_i)$ . Here  $c_{h_{11}}(h_{11}, h_{21}, \phi^1) = c_{h_{12}}\left(\frac{Q_1 - N_1 h_{11}}{N_2}, h_{22}, \phi^2\right) = p_1 - \lambda_1 \in (0, p_1)$ ;  $c_{h_{21}}(h_{11}, h_{21}, \phi^1) = c_{h_{22}}\left(h_{12}, \frac{Q_2 - N_1 h_{21}}{N_2}, \phi^2(X_2, H_2)\right) = p_2 - \lambda_2 \in (0, p_2)$ .

Quota point  $Q^{VIII} = \{0.75Q^*, 0.75Q^*\}$  in figure 1 falls in zone VIII. For the SP-CA stock scenario (figure 2a), the two regional harvests are at points  $H^{VIII}$  inside the implementable sets of respective regional partitions. Both species quotas are fully utilized with no discards:  $L^{VIII} = H^{VIII}$ .

### Regional capital deployment

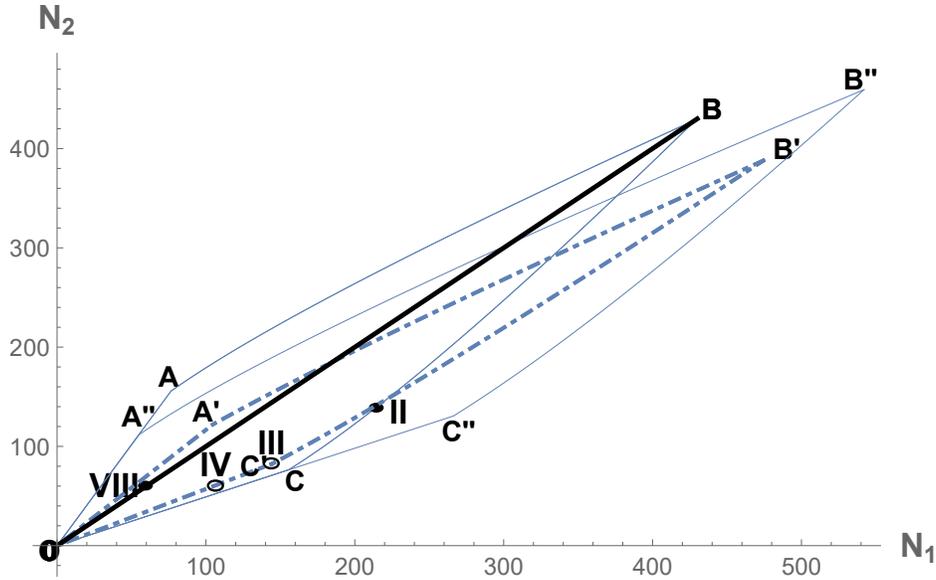


Figure 3: **Equilibrium Capital Allocation.** Horizontal (vertical) axes denote capital allocations as a % of  $N^*$  in regions 1 (2). The solid line  $OB$  represents SP-HOM with  $X = \{\{x^*, x^*\}, \{x^*, x^*\}\}$  and  $p_1 = p_2 = 1$ . The set  $OABC$  corresponds to SP-CAA with  $X = \{\{1.2x^*, x^*\}, \{x^*, 1.2x^*\}\}$  and  $p_1 = p_2 = 1$ ;  $OA'B'C'$  corresponds to  $X = \{\{1.2x^*, x^*\}, \{0.8x^*, x^*\}\}$  and  $p_1 = p_2 = 1$ ;  $OA''B''C''$  corresponds to  $X = \{\{1.2x^*, x^*\}, \{x^*, 1.2x^*\}\}$ ,  $p_1 = 1.5$ , and  $p_2 = 1$ .

Equilibrium capital allocations corresponding to quota zones in figures 1 and 2 are shown in figure 3: the allocation set demarcated by  $OABC$  corresponds to the SP-CA stock abundance while  $OA'B'C'$  corresponds to SP-CAA abundance.

When regions are ecologically and economically symmetric, equilibrium capital will also be symmetric. The solid line  $OB$  in figure 3 conforms to this case.

Consider the SP-CA abundance scenario, i.e., neither region has an absolute advantage. The set  $OABC$  indicates that a higher quota of species 1 (2) attracts more capital in region 1 (2) in equilibrium. For quotas  $Q^{II}$  and  $Q^{VIII}$  in figure 1 and SP-CA stock conditions, the

equilibrium capital allocations are indicated at points II and VIII, respectively. On the other hand, for the SP-CAA stock abundance scenario, the set  $OA'B'C'$  in figure 3 shows that more capital is allocated to region 1. For  $Q^{III}$  and  $Q^{IV}$  from figure 1, equilibrium capital occurs at points III and IV, respectively, in figure 3.

### Non-joint and fixed output proportions technology

Recall that the results in figure 1 assume a technology that is joint-in-inputs with positive targeting costs. We next consider how the harvest mapping changes under the technological assumptions that dominate the multiple-species fisheries literature.

When the technology is non joint in inputs individual species harvests are independent and there is no longer any motive to discard fish. Since each species are independently harvested, whether their quota binds or not is independent of the quota of the other. Refer once again to figure 1. As  $\gamma \rightarrow 0$  segment  $OC$  rotates clockwise toward the horizontal axis causing discard zone III and IV to vanish. Simultaneously, segment  $OA$  rotates counterclockwise toward the vertical axis, eliminating discard zone V and VI. The no-discard zone VIII takes a rectangular shape with width (height) that is determined by the value of  $Q_1$  ( $Q_2$ ) at which the marginal profit from harvesting additional species 1 (2) fish falls to zero.

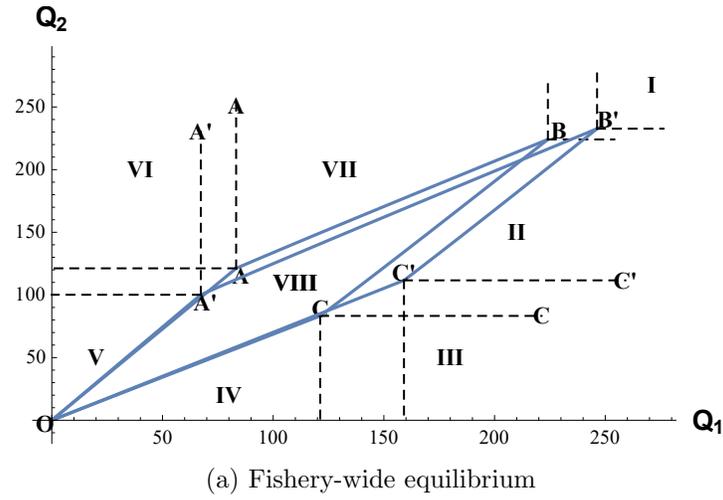
Next consider a fixed output proportions technology. When  $\gamma \rightarrow \infty$ , i.e., harvest occurs in fixed proportions, no-discard zone VIII collapses to a ray. Any quota that does not lie on this ray entails discard of one of the species, unless none of the quotas bind. Therefore, only zones I, IV, and V exist.

As we have argued, the harvesting technology is more realistically represented by an intermediate value of  $\gamma$ . A counterfactual assumption on the harvest technology will lead to flawed predictions. Suppose, for example, a quota pair chosen by the regulator falls within the implementable set under a non-joint technology, but lies outside the implementable set of a realistically proximate joint technology. The harvest and landing predictions under the former will be very different than that expected under the latter. The prediction of subsequent stock abundance will also diverge. Over multiple subperiods in a season, these errors can be further amplified.

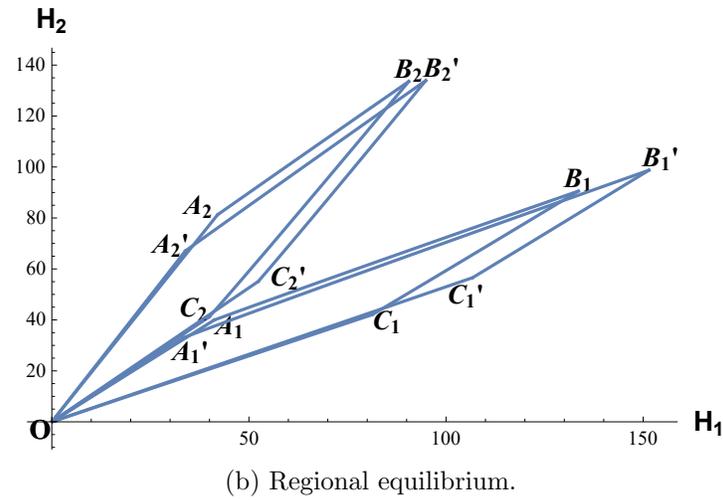
A technology misspecification can be equally detrimental from a regulatory perspective. A non-joint technology offers an expanded implementable set and rules out discards completely, whereas the implementable set under fixed proportions is fully restricted to lie on a ray: the former offers maximum flexibility for setting regulatory quotas, whereas the latter offers none. Under a non-jointness assumption, the quotas *optimally* chosen by a regulator may lead to unintended discards. Under a fixed-proportions assumption, a regulator fearful of discards may opt for overly restrictive quotas.

### Landings prices

We next consider the role of landings prices in the mapping from quota regulations to equilibrium outcomes. For this exercise we assume  $p_1 = 1.5$  and  $p_2 = 1$ . The initial stock conditions are set to the SP-CA scenario. All other parameters are unchanged. Figure 4 contrasts changes in harvest/discard outcomes relative to the symmetric price case studied above.



(a) Fishery-wide equilibrium



(b) Regional equilibrium.

Figure 4: **Equilibrium Harvests and Landings Prices.** Figure shows the impact of a 50 % increase in the landing price of species 1 fish, relative to the baseline parameters. Quotas and Harvests are reported as percentage of benchmark model steady state values. Stock conditions follow the SP-CA scenario with  $X = \{\{1.2x^*, 0.8x^*\}, \{0.8x^*, 1.2x^*\}\}$ .

There are four notable effects of an increase in the price of species 1 landings. First, the set of quotas for which the species 1 quota is slack, i.e., zones II and III is now smaller. Second, and vice versa, zones VI and VII, i.e., cases where the species 2 quota is slack become larger. These changes result from the increased profitability of species 1 which, under a joint technology, increases the quantity of both species for which variable profits offset capital costs.

The discard set of species 2, the area under  $0C'C'$  is enlarged, while the discard set for species 1,  $0A'A'$  shrinks. Changes in regional zonal partitions (figure 4b) mimic fishery-wide changes (figure 4a) closely. Region 1, with higher relative abundance of the now higher price species 1 spans a larger portion of quota space, some of which was unutilized under the lower benchmark price.

It is evident that the set of quotas in zones IV and V falling under line segments  $0C$  and  $0A'$ , respectively, are invariant to a change in the price of species 1 fish. Recall that both quotas bind along these segments. For one species, harvest equals quota, whereas for the other (over-quota) species, marginal harvest cost is equal to zero. A change in fish price of species 1 only raises its quota price for quotas set in zones IV and V.<sup>21</sup>

We note that a rise in the price of species 1 increases capital quasi rent particularly in region 1 where its stock is relatively abundant. The set  $OA''B''C''$  in figure 3 illustrates the resultant shift in equilibrium capital.

### 3.2 Equilibrium across regions and subperiods

In this section we characterize equilibrium spatial and temporal harvests, landings, discards, and quota utilization. With multiple subperiods temporal quota utilization is endogenous and satisfies the temporal arbitrage condition in equation (10). In the two region case, the quota market clearing equation becomes,

$$\sum_{t=1}^2 (L_{1it} + L_{2it}) = \sum_{t=1}^2 (N_{1t}l_{1t} + N_{2t}l_{2t}) \leq Q_i, \quad i = 1, 2.$$

It is worth reiterating that given dockside fish prices and quota prices, equations (11) - (13) along with proposition 1 continue to determine equilibrium harvests and landings. With  $S = T = 2$ , the quota market clears across the two regions and the two subperiods.

A second difference is that stock abundance across multiple subperiods is endogenously determined, simultaneously with spatial, within-season and species-specific harvests. A perfect foresight equilibrium requires the dynamic evolution of stock, given by  $X_{t+1} = \Gamma(X_t - H_t)$ , to be consistent with the spatial-temporal harvest profile. In equilibrium, fishermen rationally forecast harvests and accompanying stock conditions for all  $s, t$ .<sup>22</sup> Appendix 7 provides a fixed point algorithm to solve for the equilibrium by using a computer.

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<sup>21</sup>It is easily checked that  $p_1 - \lambda_1$  remains constant on the segment  $0C$  when  $p_1$  changes from 1 to 1.5 and the segment extends up to  $0C'$ . With  $p_1 = 1$ ,  $\lambda_1 = 0$  at  $C$ , but with  $p_1 = 1.5$ ,  $\lambda_1 = 0$  at  $C'$ . It must be the case that  $\lambda_1 = 0.5$  at  $C$  so that  $p_1 - \lambda_1 = 1$  remains as before at  $C$ .

<sup>22</sup>In the model, it is *perfect foresight* that allows fishermen to correctly forecast future stocks. Under uncertainty, a *rational expectations* equilibrium requires that fishermen utilize publicly known stochastic distributions of the model's random variables to form their stock forecasts.

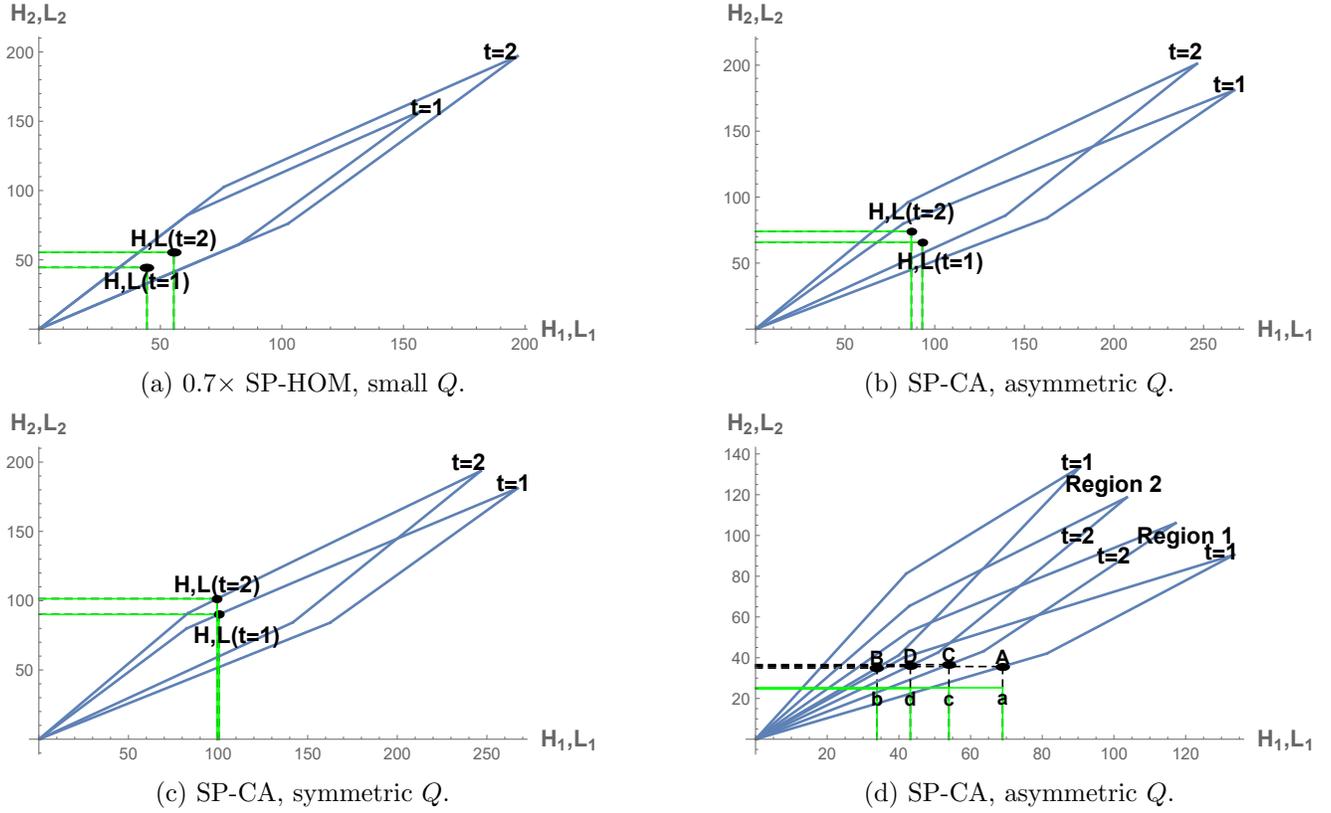


Figure 5: **Equilibrium under multiple-subperiods.** Harvests and landings are the percentage of benchmark model steady state values. Panel (a) assumes stock abundance at 70% of SP-HOM values ( $\{0.7x^*, 0.7x^*, 0.7x^*, 0.7x^*\}$ ), quota  $\{Q^*, Q^*\}$ ; panel (b) assumes initial abundance,  $\{1.2x^*, 0.8x^*, 1.2x^*, 0.8x^*\}$ , quota  $\{1.8Q^*, 1.4Q^*\}$ ; panel (c) assumes initial abundance  $\{1.2x^*, 0.8x^*, 1.2x^*, 0.8x^*\}$ , quota  $\{1.6Q^*, 1.6Q^*\}$ ; panel (d) abundance SP-CA, quota  $\{2Q^*, Q^*\}$ . Subperiods are denoted  $t = 1, 2$ .

In this setting, the shape and location of quota zones discussed above and the implementable harvest sets *evolve* as stock conditions change throughout the season. We next present equilibrium outcomes under alternate management scenarios to further characterize equilibrium outcomes of management interest.

### Stock growth

Figure 5 shows equilibrium harvests and landings under varying stock conditions and quotas. In each case,  $t = 1, 2$  indexes subperiods within the season. Figures 5a, 5b, and 5c focus on temporal effects in a fishery with spatially homogenous stock abundance at the beginning of the season. Figure 5d shows a case of heterogenous regional abundance at the beginning of season. The quota regulations vary to illustrate important properties of the equilibrium outcomes. Specifics are presented below.

Following the notation of the previous section,  $x^*$  and  $Q^*$  denotes species-specific, steady state stocks and optimal subperiod quotas for our baseline fishery. With  $T = 2$ , the seasonal

quota is  $2Q^*$  for each species. The results in figure 5a show fishery-wide outcomes for a regionally homogeneous fishery (effectively a single-region fishery) under a tight seasonal quota. The scenario in figure 5a assumes beginning season stocks equal to  $\{0.7x^*, 0.7x^*, 0.7x^*, 0.7x^*\}$ , i.e., 70% of the SP-HOM stock scenario above. Seasonal quota is assumed set at  $\{Q^*, Q^*\}$ , i.e., 50% of its steady state value. This scenario may represent the case of an low quota chosen to protect an overfished stock.

As evident in figure 5a, equilibrium harvest in the first subperiod is less than the in the second subperiod. The equilibrium foresees the growth of both species and thus the lower harvesting costs at  $t = 2$ . Equilibrium capital and harvest per vessel at  $t = 1$  and  $t = 2$  increase over time while the equilibrium quota price remains constant throughout the season. Note also that the zones that characterize fishing behavior expand reflecting within-season stock growth.

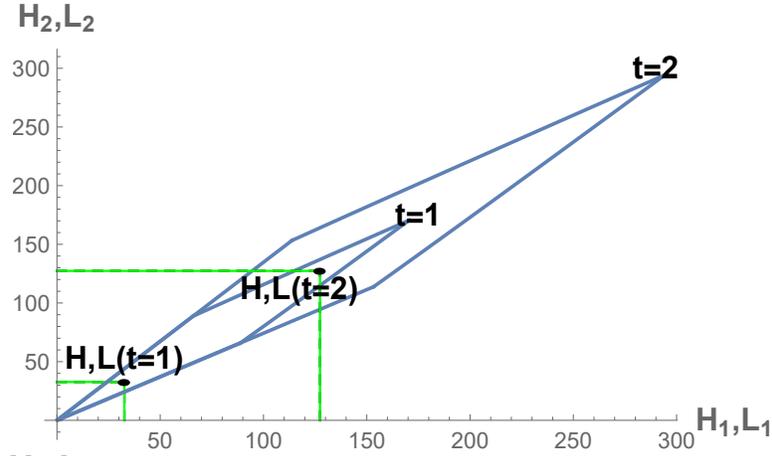
The results in figure 5b consider a scenario with beginning season stocks that deviate from their steady state levels but remain homogeneously distributed across space. Initial abundance follows  $\{1.2x^*, 0.8x^*, 1.2x^*, 0.8x^*\}$ ; the species 1 stock is 20% above its steady state value while the species 2 stock is 20 % below its steady state value. We assume quotas to be  $\{1.8Q^*, 1.4Q^*\}$  that mirror their relative stock abundance.

Figure 5b illustrates how the relative stocks of the two species and their harvests evolve within the season. Note first that the  $t = 1$  implementable harvest set tilts toward the more abundant species 1 stock. The ITQ equilibrium induces a lower harvest of less-abundant species 2, and a higher harvest of more-abundant species 1. Escapement at  $t = 1$  and stock growth between subperiods aligns the two stocks toward their common steady state value as is evident from the rotation of the implementable set in  $t = 2$  counterclockwise toward species 2. At  $t = 2$ , the species 2 harvest increases while the species 1 harvest falls relative to their  $t = 1$  values.

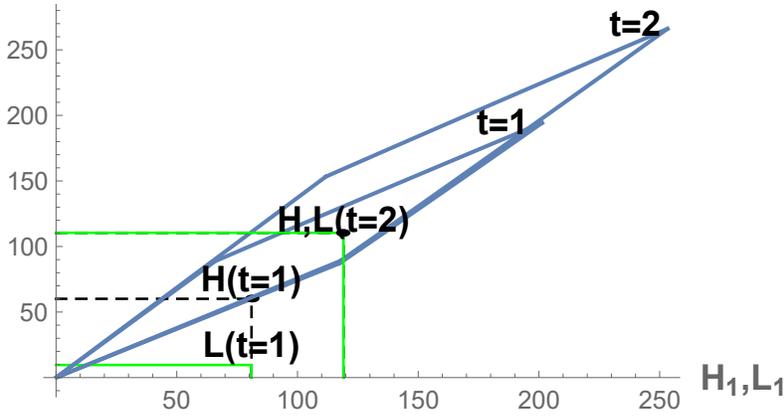
Species-specific quotas either bind or are slack at the seasonal level. Figure 5c assumes initial stock abundance,  $\{1.2x^*, 0.8x^*, 1.2x^*, 0.8x^*\}$ , but with quotas set equally at  $\{1.6Q^*, 1.6Q^*\}$ . This scenario features a stock-quota mismatch whereby the regulator has issued excess species 2 quota. In figure 5c, the species 2 quota is slack throughout the season. Equilibrium harvests in both subperiods equates the landing price to the marginal cost of are species 2 harvests, since with a slack seasonal quota  $\lambda_2 = 0$ . As in the scenario shown in figure 5b, low species 2 harvests at  $t = 1$  allows for species 2 stock growth, which again causes the  $t = 2$  implementable harvest set to rotate counterclockwise.

Figure 5d considers the SP-CA stock scenario  $\{1.2x^*, 0.8x^*, 0.8x^*, 1.2x^*\}$  and quota  $\{2Q^*, Q^*\}$ . This scenario is chosen to illustrate conditions under which discarding occurs. Specifically, when species 2 quota is particularly small, its targeting costs rise. Costs can then be lowered by targeting a mix of species that aligns with their relative abundance and discarding the overage.

Figure 5d shows equilibrium harvests across regions and subperiods; points *A* and *B* denote regional harvests at  $t = 1$ , and *C* and *D* represent regional harvests at  $t = 2$ . Landings are represented with their small case counterparts. In each region and in each subperiod, harvests of species 2 fish exceeds landings, i.e.,  $d_{2st} > 0$ . Note also that with positive discards the species 2 seasonal quota lease price satisfies  $\lambda_2 = p_2$ . Finally, and as above, we also see that intra-regional stock differences diminish over time and both species stocks evolve toward their



(a) SP-HOM, symmetric  $Q$ , rising prices.



(b) SP-HOM, asymmetric  $Q$ , rising prices.

Figure 6: **Temporal Price Effects.** Panel (a) assumes abundance scenario SP-HOM and quota  $\{1.6Q^*, 1.6Q^*\}$ . Panel (b) assumes abundance scenario SP-HOM and quota  $\{2Q^*, 1.2Q^*\}$ . In both cases, landings prices are 20% below and 20% above baseline values in subperiod  $t = 1$  and  $t = 2$ , respectively.

steady state values.

### Price variation

We next consider the effects of exogenously varying market conditions on equilibrium outcomes. We specifically focus on the variation of landings prices within a fishing season. One might conjecture that equilibrium harvest behavior will induce higher harvests in periods with higher fish prices and vice versa. What is less obvious is how the temporal distribution of harvests interacts with season stock growth to determine the seasonal harvest and landings profile.

Suppose, for example, that landings prices are higher in subperiod 1. This price dynamic will induce higher harvests early and lower harvests later in the fishing season. A lower escapement in the first subperiod in turn reduces stocks available in later subperiods thus further reducing late-season harvests. Figure 6 shows these effects formally.

Assume stocks are symmetric across a regions at  $\{x^*, x^*, x^*, x^*\}$ . Figure 6a displays the seasonal equilibrium when the quotas are set symmetrically at  $\{1.6Q^*, 1.6Q^*\}$  but with landings prices for both species 20% below the baseline at  $t = 1$ , and 20% above the baseline at  $t = 2$ . The expansion of the implementable sets in figure 6a highlights the amplified stock growth effects. A lower price at  $t - 1$  not only induces lower harvest and quota utilization, but by increasing stocks size at  $t = 2$  further tilts harvests and quota utilization toward subperiod 2.

Sufficiently large variation in the price of fish within a season can induce discards during subperiods of low prices. Recall that a discard requires  $\lambda_i = p_{it}$ . Suppose discards occur in a subperiod with  $p_{it} = p_i^L = \lambda_i$ . In another subperiod with  $p_{it} = p_i^H > p_i^L = \lambda_i$ , discards fall to zero. Figure 6b illustrates this scenario for an ecologically homogenous fishery, i.e., symmetric initial stocks at  $\{x^*, x^*, x^*, x^*\}$ . We assume quotas at  $\{2Q^*, 1.2Q^*\}$ . Finally, we let  $p_1$  remain constant at its baseline value over the entire season, and assume  $p_2$  varies from 20% below to 20% above its benchmark in subperiods  $t = 1$  and  $t = 2$ , respectively.

In contrast to figure 6a, a relatively lower price for species 2 fish tilts the  $t = 1$  implementable set towards species 1. The harvest of both species, 1 and 2, is lower relative to harvests at  $t = 2$ . Since the quota of species 2 is relatively small, it is optimal for fishermen to wait until  $t = 2$  to bring species 2 harvest to the docks. At  $t = 1$ , much of the harvest of species 2 is discarded. This harvest overage occurs in order to avoid harvesting costs that would otherwise incur if a lower harvest of species 2 were targeted.

### 3.3 Comparative analysis

This section studies the effects of key model parameters on equilibrium outcomes. We continue with the benchmark stock growth, dockside prices, and harvest technology (table 1). We assume the fishery is initially in steady state with stock abundance and quota set to their long run rent-maximizing levels.

Our first experiment varies the intrinsic growth rates of both species stocks in region 1. The purpose is to examine spatial-temporal implications of an asymmetric ecological shock. An example, may be a pollution event that alters the quality of the marine habitat, or perhaps climate change that impacts nutrient availability in a region of the fishery.

A second experiment studies the spatial-temporal implications of shock to the cost of capital, e.g., a government program that subsidizes capital investments (Sumaila, et al., 2010).

The first two experiments assume that the two fish species have identical characteristics within a region. In the third experiment, regions are assumed heterogeneous with different species-specific relative abundance. We then let the price of one species vary across time. This variation impacts the two regions asymmetrically because of their differences in relative stock abundance. As a result, the equilibrium outcomes vary across all three dimensions: species, regions, and time.

#### Regional stock growth

We let the growth rate of both species in region 1, as reflected by parameters  $r_{11}$  and  $r_{21}$ , to vary from 50% below to 50% above the benchmark value of 1. All other parameters remain as reported in table 1. Figure 7a displays changes in capital deployment and harvests in subperiod 1 as regional stock growth varies along the horizontal axes,  $\Delta r_1$ . Figure 7b displays changes

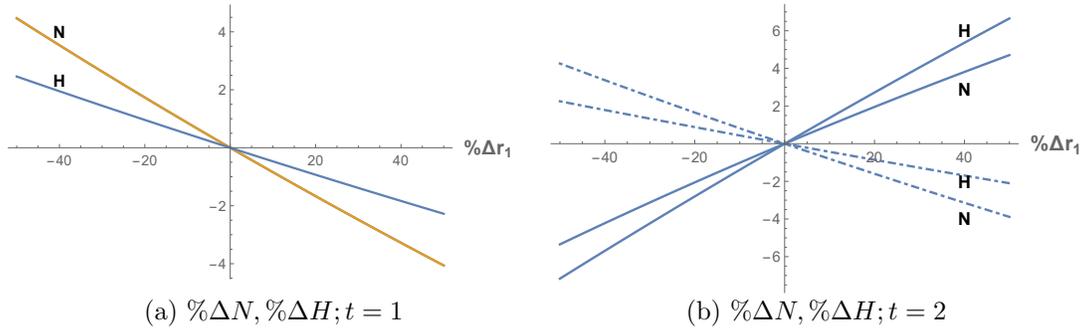


Figure 7: **Regional Stock Growth: Capital and Harvest Effects.** Units are reported as percentage change from benchmark model steady state values. Panel (a) results (subperiod 1) are common across regions. In panel (b), solid lines denote region 1 and dashed lines denote region 2.

in capital and harvests in subperiod 2. The solid lines in the figure show changes in region 1 and the dashed lines in regions 2. There are no regional differences in capital and harvests in subperiod 1 and therefore no dashed lines appear in figure 7a.

Consider positive growth shocks in region 1,  $\Delta r_1 > 0$ . Higher stock growth in region 1 implies higher abundance in region 1 in subperiod  $t = 2$  relative to the benchmark case. It is therefore less costly and more efficient to utilize quota in region 1 at  $t = 2$ . Figure 7b confirms that in subperiod  $t = 2$  both capital and harvest are increase in region 1 and fall in region 2.

What is striking is that, despite higher growth in region 1, both regions respond identically at subperiod  $t = 1$ . Both regions have identical stock conditions at  $t = 1$ , identical dockside prices, and identical quota prices. A competitive equilibrium therefore attracts identical capital which implies identical harvest across the two regions. That is, the quota that is reallocated from subperiod 1 to subperiod 2 is drawn from regions 1 and 2 equally.<sup>23</sup>

Figure 8 shows how equilibrium quota prices change as region 1 becomes more productive. Recall that a single species-specific quota price prevails in equilibrium irrespective of where the growth occurs. Intuitively, if the fishery is more productive ( $\Delta r_1 > 0$ ), the unit quota rent become larger reflecting cost saving and thus profitability with increased stock abundance.

### Capital costs

In the benchmark case the cost of capital,  $\rho$ , is assumed common across regions and subperiods. To examine inter-temporal capital price effects, we now let  $\rho$  in region 1 drop/rise by  $x\%$  of its benchmark value in subperiod 1 and then rise/drop by an equal amount in subperiod 2. The capital price in region 2 is assumed to remain constant at its benchmark value. To focus on spatial-temporal economic effects, the two fish species are assumed to have identical growth characteristics. Since stocks and initial quotas are otherwise symmetric, the results presented

<sup>23</sup>In contrast, a sole owner may leave a higher escapement in the region with higher growth rate to exploit the subsequent cost advantage of a higher stock. It is precisely because a sole-owner internalizes the stock dynamics, which an ITQ equilibrium does not.

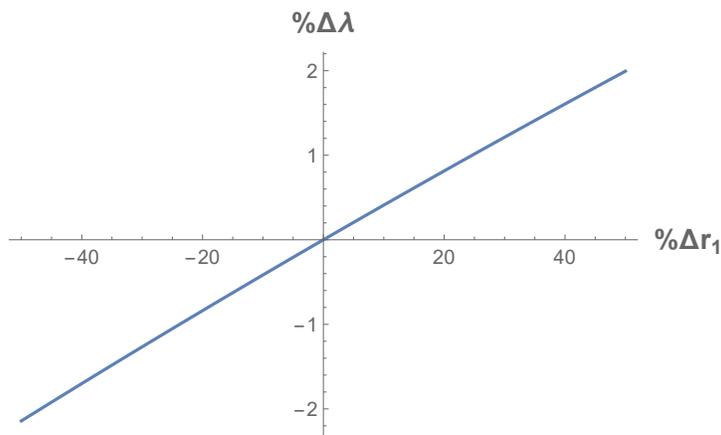


Figure 8: **Regional Stock Growth and Equilibrium Quota Prices.** Units are reported as percentage changes relative to benchmark model steady state values.

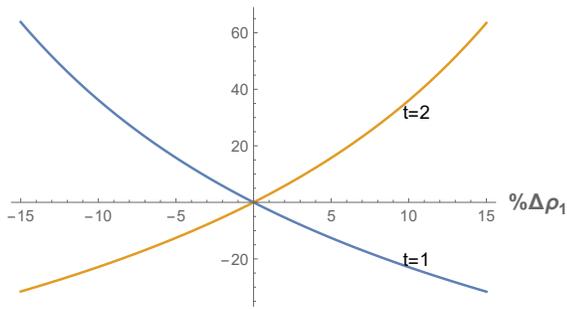
below are common to both species.

Figures 9a and 9b show how equilibrium capital allocations change in region 1 and region 2, respectively, under seasonal and regional capital price shocks. Note that negative values on the horizontal axes in both figures signify a capital price decrease in subperiod 1 followed by a price increase (of the same % magnitude) in subperiod 2.

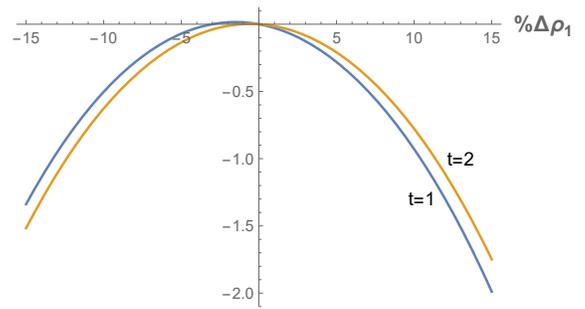
Consider first a rise in  $\rho_1$  at  $t = 1$  (accompanied with an offsetting fall at  $t = 2$ ). We see that  $N$  is lower than the benchmark value at  $t = 1$  in region 1. A lower harvest in region 1 implies higher stock abundance at  $t = 2$ , which combined with a lower cost of capital leads to a bumper rise in the capital deployment in this region at  $t = 2$ . The variation in the capital price does not much impact the capital deployment and harvest activity in region 2 however (effects are in the range of 0%-2%). At  $t = 1$ , region 2 has a relatively low cost of capital and continues to attract capital close to the benchmark quantity. At  $t = 2$ , a sufficiently high activity in region 1 also creates accompanying stock growth effects. As a result, region 2 continues to attract about the same amount of capital as in the benchmark case.

Figure 9c displays the fishery wide and regional quota utilization at  $t = 1$ , with the remaining quota being utilized at  $t = 2$ . There is not much action in region 1. Its utilization remains close to the benchmark. Though not presented above, it is found that region 2's harvest and quota utilization at  $t = 2$  is also close to benchmark values. Thus, the main impact of harvest activity occurs in region 1. When panels (a) and (c) of figure 9 are inspected together, it becomes clear that the aggregate harvest response is more muted in region 1 than is the capital response. Thus, at  $t = 1$ , a fewer units of capital harvest a larger amount of fish than under the benchmark, whereas at  $t = 2$ , more capital harvests less than under the benchmark.

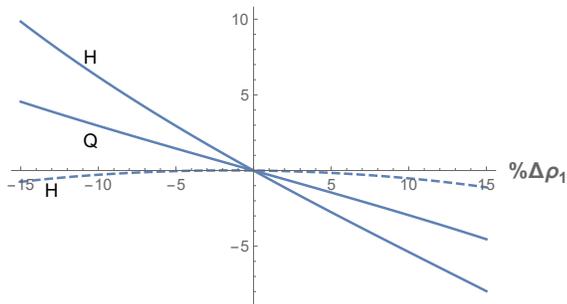
Finally, figure 9d shows changes in the equilibrium quota prices. Despite temporal changes of 15% in  $\rho_1$ , the quota price does not vary beyond 1% of its benchmark value. The capital adjustments in region 1 are consistent with small change in the fishery rent. Whether  $\rho_1$  rises in the first period and falls in the second, or vice versa does not change the direction of response of the quota price. When  $\rho_1$  rises at  $t = 1$ , region 1 is more cost efficient at  $t = 2$  with a



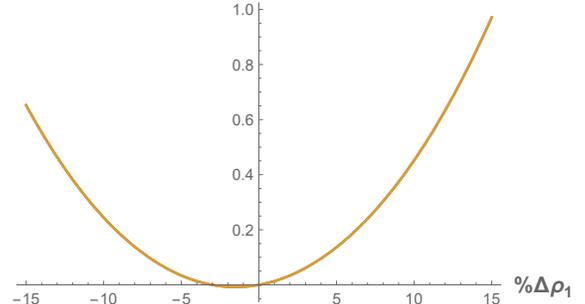
(a)  $\% \Delta N$ , region 1



(b)  $\% \Delta N$ , region 2



(c)  $\% \Delta H, \% \Delta Q; t = 1$



(d)  $\% \Delta \lambda$

Figure 9: **Regional-Temporal Capital Costs.** Units are reported as percentage changes relative to the benchmark model steady state values with price increases (declines) in subperiod 1 matching declines (increases) in subperiod 2.

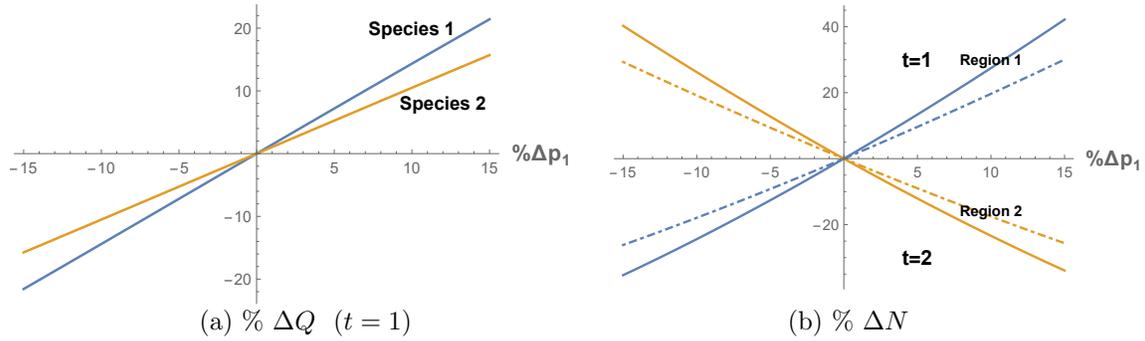


Figure 10: **Landings Prices Under Regional and Species-Specific Stock Asymmetry.** Results are reported as percentage change relative to the benchmark model steady state values. Price increases (declines) in subperiod 1 match declines (increases) in subperiod 2.

lower  $\rho_1$  and higher stocks. This raises the quota price. When  $\rho_1$  falls at  $t = 1$ , much harvest activity takes place in region 1 at  $t = 1$ . A lower escapement however endogenously entails a higher growth rate. The overall impact again is an increase in the quota price.

### Price changes in a heterogeneous fishery

We now deviate somewhat from the above experiments and consider a fishery with regional and species-specific differences in habitat quality. We let region 1 have a higher carrying capacity for species 1, while region 2 has an identical advantage for species 2. In this modified benchmark steady state the relative stock abundance  $\frac{X_{11}}{X_{21}} = \frac{X_{22}}{X_{11}} = 1.4$ .

To examine region-specific, intertemporal price effects, we let the price of species 1 fall (rise) by  $x\%$  in subperiod 1 and then rise (fall) by an equal amount in subperiod 2. The price change we consider is  $\pm 15\%$ .

We note first that the law of one price continues to hold, i.e., the landings price for either species does not vary regionally. Figure 10a shows how within-season price variation for species 1 impacts aggregate harvest and quota utilization of the two species at  $t = 1$  (quota utilization at  $t = 2$  is its reflection over the horizontal axis). Figure 10b shows percent changes in vessel capital deployment across regions and subperiods relative to the benchmark case.

Notice that negative values on the horizontal axis for  $\Delta p_1$  signify that the price of species 1 is lower in the first subperiod by the value indicated on the horizontal axis; the price is higher than the benchmark value by the same percentage amount in the second subperiod. The opposite is the case when this value is positive.

Figure 10b shows that a rise (fall) in the price of species 1 in the first subperiod leads to a rise (fall) in capital deployment; accompanying changes in aggregate harvest and quota utilization are shown in figure 10a. A lower price of species 1 in the first period induces fishermen to wait until the subperiod 2, not only for the higher harvest of species 1, but also for the harvest of species 2 in order to take advantage of the cost complementarities under the joint harvest technology.

The response of the equilibrium quota price is somewhat complicated. Figure 11 shows that a first subperiod drop in the species 1 price increases the equilibrium quota price for species

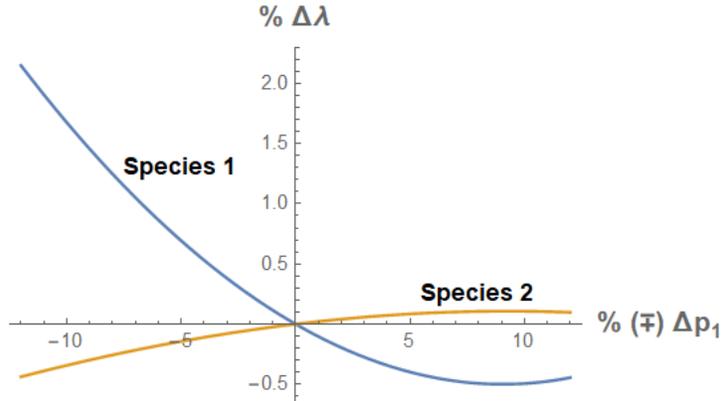


Figure 11: **Price Shock Effects: equilibrium quota prices.** Results are reported as percentage change relative to the benchmark model steady state values. Price increases (declines) in subperiod 1 match declines (increases) in subperiod 2.

1 (which remains constant throughout the season) and causes a decline in the species 2 quota price, although the effect is relatively minor.

To understand this result, first recall that *average* seasonal landings prices are constant in the scenario under consideration. In the first period, stocks are identical to the benchmark case and thus harvest costs are the same as under the benchmark. However, a lower landing price and thus harvest in subperiod 1 implies higher abundance in subperiod 2 and therefore a stock-effect cost savings in the second subperiod. This stock effect lowers average seasonal costs and increases unit quota value.

## 4 Conclusion

This paper derives a dynamic rational expectations equilibrium in a complex ecological and economic environment under a tradable quota regulation. Our model features a general representation of a multiple species ecology and a joint harvesting technology. We characterize a rational equilibrium mapping from initial spatial stock conditions and economic fundamentals to spatial-temporal harvests, landings, and discards by species, quota prices, capital allocations, revenues under a tradable quota regulation. The results demonstrate complex interactions between multiple ecological and economic forces that are operational in a quota-managed fishery.

While we focus on a two-species, two-region and two-subperiod case of our model, the conditions for a recursive equilibrium extend to multiple species, regions and subperiods. Our numerical algorithm can also be applied to higher dimension problems although as is well understood, computational time increases exponentially as species, regions and seasonal subperiods increase.

Equilibrium capital allocations satisfy a condition that equates capital rent to its opportunity cost, and quota utilization in which unit rent is constant across space and time within a

fishing season. These principles combine with the growth and spatial dispersion patterns of fish stocks to determine rational equilibrium outcomes of interest to managers and stakeholders. An important theme present in the model and results is that ecological and economic outcomes are determined concurrently. The implication is that marine ecosystems that support commercial fisheries are simultaneously influenced by ecological and anthropogenic (economic and regulatory) forces and should be managed as such.

We derive a mapping from initial ecological-economic conditions to equilibrium harvests, landings, discards, capital allocation and rent outcomes under varying quotas. Without full understanding of this mapping, well intentioned regulations will fail to meet management goals: in the jargon of the EBFM literature, regulations will be vulnerable to *implementation uncertainty*. Our results indicate that without foresight of the range of ecological-economic consequences, setting multiple-species quotas that meet long term management goals, e.g., stock conservation and generation of rent, may be impossible. Our model predicts ecological-economic equilibrium outcomes *ex ante*, and therefore offers an approach for reducing implementation uncertainty and a path forward for improving management strategy evaluation in complex marine environments and under regulations that are operational in real world fisheries.

This paper has assumed that commercial fishermen are forward looking, fully rational agents that address private profit maximizing objectives within a complex ecological, economic, and regulatory environment. The rationality assumption invites some skepticism and empirical validation. On the other hand, evaluative tools that ignore the disciplining forces present in individual transferable quota markets, and/or rely on *ad hoc* behavioral rules lack internal validity. Our application of recursive, fully rational equilibrium methodology under realistic regulatory instrument is an important advance to implementing ecosystem-based management of marine ecosystems.

An understanding of the equilibrium mapping from regulations to seasonal profits and post-season stocks is the first and the *necessary* step towards a long-term optimal management of an ITQ regulated fishery. A long-term optimal quota management requires finding the optimal quota policy function that responds to the current state of the fishery. This includes both exogenous states, e.g., processes for fish market prices, input and capital prices, and endogenous states, i.e., spatial-species-specific stock abundance and their laws of motions, in turn, as functions of current exogenous and endogenous states. The optimal policy function maximizes the fishery value function that maps current fishery state to its present discounted dollar value. Exploring these policy functions for alternative fishery environments is our goal for future research.

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## 6 Appendix: Proof of Proposition 1

Write the variable cost function as

$$c(h, \phi(X, H)) = \left( 1 + \sum_j \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right)^2 \right) \sum_k \phi^k(X_k, H_k) h_k^\nu,$$

where  $j, k$  is the species' index. Taking its derivative with respect to  $h_i$  and then multiplying by the same  $h_i$  gets

$$\begin{aligned} c_{h_i} h_i &= \nu \left( 1 + \sum_j \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right)^2 \right) \phi^i h_i^\nu \\ &+ 2\gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \left( \sum_{j \neq i} h_j \right) h_i \frac{\sum_k \phi^k(X_k, H_k) h_k^\nu}{(\sum_k h_k)^2} \\ &- 2h_i \sum_{j \neq i} \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) h_j \frac{\sum_k \phi^i(X_k, H_k) h_k^\nu}{(\sum_k h_k)^2}. \end{aligned}$$

The last two terms equal  $\frac{\sum_k \phi^i(X_k, H_k) h_k^\nu}{(\sum_k h_k)^2}$  times

$$\begin{aligned} \Lambda_i &\equiv 2\gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \left( \sum_{j \neq i} h_j \right) h_i - 2h_i \sum_{j \neq i} \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) h_j \\ &= 2h_i \gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \left( \sum_j h_j \right) - 2h_i \sum_j \gamma_j h_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right). \end{aligned}$$

Note that

$$\begin{aligned}
\sum_{\iota} \Lambda_i &= 2 \left( \sum_j h_j \right) \sum_i h_i \gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \\
&\quad - 2 \left( \sum_i h_i \right) \sum_j \gamma_j h_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) \\
&= 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\sum_{\iota} c_{h_i} h_i &= \nu \left( 1 + \sum_j \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right)^2 \right) \sum_{\iota} \phi^i h_i^{\nu} \\
&\quad + \frac{\sum_k \phi^i(X_k, H_k) h_k^{\nu}}{(\sum_k h_k)^2} \sum_{\iota} \Lambda_i \\
&= \nu c(h, \phi(X, H)).
\end{aligned}$$

Suppose, instead, that the cost function takes the following form:

$$c(h, \phi(X, H)) = \sum_j \left( 1 + \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right)^2 \right) \phi^j(X_j, H_j) h_j^{\nu}$$

In this case,

$$\begin{aligned}
c_{h_i} h_i &= \nu \left( 1 + \gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right)^2 \right) \phi^i h_i^{\nu} \\
&\quad + 2\gamma_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \frac{h_i}{(\sum_k h_k)^2} \left( \sum_{j \neq i} h_j \right) \phi^i h_i^{\nu} \\
&\quad - 2h_i \sum_{j \neq i} \gamma_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) \frac{h_j}{(\sum_k h_k)^2} \phi^j h_j^{\nu}.
\end{aligned}$$

The last two terms equal  $\frac{1}{(\sum_k h_k)^2}$  times

$$\begin{aligned}
\Xi_i &\equiv 2 \left( \sum_j h_j \right) \left( \gamma_i h_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \phi^i h_i^{\nu} \right) \\
&\quad - 2h_i \sum_j \gamma_j h_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) h_j \phi^j h_j^{\nu}.
\end{aligned}$$

Once again,

$$\begin{aligned}
\sum_i \Xi_i &= 2 \left( \sum_j h_j \right) \sum_i \gamma_i h_i \left( \frac{h_i}{\sum_k h_k} - \frac{X_i}{\sum_k X_k} \right) \\
&\quad - 2 \left( \sum_i h_i \right) \sum_j \gamma_j h_j \left( \frac{h_j}{\sum_k h_k} - \frac{X_j}{\sum_k X_k} \right) \\
&= 0,
\end{aligned}$$

which, once again, implies

$$\sum_i c_{h_i} h_i = \nu c(h, \phi(X, H)).$$

## 7 Appendix: Computational Algorithm

The equilibrium allocations for  $T \geq 2$  are obtained numerically by following the steps enumerated below. We continue to focus on a fishery with two regions 1 and 2.

1. Fix  $X \equiv \{X_{11}, X_{21}, X_{12}, X_{22}\}$ . Given  $X$ , generate regional and aggregate implementable partitions by following the steps discussed in section 3.1.
2. Given these implementability partitions, compute

$$\{N \equiv \{N_1, N_2\}, H \equiv \{H_{11}, H_{21}, H_{12}, H_{22}\}, \{\lambda_1, \lambda_2\}\}$$

for all possible  $Q \equiv \{Q_1, Q_2\}$ .

3. Repeat step 2 over a plausible domain for  $X$ . These computations also obtain the multiplier values for  $\lambda_i(X, Q) \in [0, p_i]$ .
4. Now consider subperiod  $T - 1$  and  $T$ . The optimal solution to the static problem in any period given  $\{X, Q\}$  has already been solved. Let  $\{X, Q\}$  be the state in subperiod  $T - 1$ . But now the optimal choice problem entails *only a part of  $Q$  to be utilized* and the remaining to be carried forward to  $T$ . Let  $Q'$  be carried over to  $T$ . Then optimal static choice problem in  $T - 1$  gets  $\lambda_i(X, Q - Q')$  and  $\{N, H\}$  for  $T - 1$  as functions of  $\{X, Q - Q'\}$ . In Equilibrium  $X' = \Gamma(X - H)$ . Then in  $T$ ,  $\lambda_i(\Gamma(X - H), Q')$ . The equilibrium requires that

$$\lambda_i(X, Q - Q') = \lambda_i(\Gamma(X - H(X, Q)), Q')$$

Solving this fixed point problem recursively obtains season's optimal quota utilization over multiple subperiods.