Government Revenue From Inflation in Open Economies with Fixed Exchange Rates

Walter Enders
Iowa State University

Dennis R. Starleaf
Iowa State University

Follow this and additional works at: http://lib.dr.iastate.edu/econ_las_staffpapers

Part of the Economic History Commons, Economic Policy Commons, Policy History, Theory, and Methods Commons, Social Policy Commons, and the Statistical Models Commons

Recommended Citation
http://lib.dr.iastate.edu/econ_las_staffpapers/142

This Report is brought to you for free and open access by the Economics at Iowa State University Digital Repository. It has been accepted for inclusion in Economic Staff Paper Series by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Government Revenue From Inflation in Open Economies with Fixed Exchange Rates

Abstract
In a closed and stationary economy, real government revenue from the issue of fiat money is maximized when the rate of issue is such that the elasticity of the demand for real money balances with respect to the rate of inflation is unity. However, Friedman [1971] has shown that this result for a stationary economy does not carry over to a growing economy. In the case of a growing economy, the optimum rate of inflation for the maximization of government revenue from the issue of fiat money is lower than for a stationary economy.

Disciplines

This report is available at Iowa State University Digital Repository: http://lib.dr.iastate.edu/econ_las_staffpapers/142
Government Revenue From Inflation in Open Economies with Fixed Exchange Rates

by

Walter Enders and Dennis R. Starleaf

No. 41

September 1976
In a closed and stationary economy, real government revenue from the issue of fiat money is maximized when the rate of issue is such that the elasticity of the demand for real money balances with respect to the rate of inflation is unity. However, Friedman [1971] has shown that this result for a stationary economy does not carry over to a growing economy. In the case of a growing economy, the optimum rate of inflation for the maximization of government revenue from the issue of fiat money is lower than for a stationary economy.

In this paper, we extend Friedman's analysis to consider government revenue from money issue in open economies with fixed exchange rates. We show that, in the absence of an agreement to maximize combined revenue from money issue, attempts by governments to individually maximize revenue will bring about a rate of inflation which is much greater than the rate at which combined government revenue is maximized.

The Model

Assume a two-country world in which the exchange rate between the currencies of the two countries is permanently fixed at unity. The nominal stock of high powered or base money \( M \) in each country is equal to the amount of domestic credit \( D \) issued by the government or central bank plus the country's net international reserves \( R \):

\[
M_i = D_i + R_i; \; i = 1, 2
\]  

(1)

where the subscripts indicate the country in question. A country's net international reserves consist of its holdings of international reserve assets (e.g., gold, SDR's) and monetary claims against the government of the other country less the other country's holding of monetary claims against the domestic government;
\[
\sum R_i = \bar{R}
\]

where \(\bar{R}\) is the nominal world stock of international reserve assets and is assumed fixed.

Let

\[P_i = \text{price level}\]

\[N_i = \text{population}\]

\[Y_i = \text{nominal income}\]

\[\frac{Y_i}{P_i N_i} = \text{per capita real income}\]

\[\frac{M_i}{P_i N_i} = \text{per capita stock of real base money}\.

Also let \(g\) with a subscript represent the relative rate of growth of the subscript variable, e.g.,

\[g_{D_1} = \frac{d(\ln D_1)}{dt} = \frac{1}{D_1} \frac{dD_1}{dt} = \text{the relative rate of growth of domestic credit in country } i.

To keep the model simple, we assume that there are no transportation costs or other barriers to trade between the two countries, all goods are traded, and competition prevails. These assumptions together with the fixed unitary exchange rate imply that \(P_1 = P_2\) and \(g_{P_1} = g_{P_2}\).

Since we are interested in equilibrium positions after all adjustments in response to inflation have taken place, we regard \(g_p\) as both the actual and the expected rate of price change in both countries.

The demand for real base money balances per capita in each country is given by

\[m^d_i = f_i(y_i, g_p).
\]

These demand functions are derived in part from demand functions for per capita real money balances under the assumption that commercial bank reserve
requirement ratios are fixed. Like Friedman, we have omitted interest rates from these demand functions for simplicity, although to the extent that real interest rates are affected by changes in the rate of inflation their influence upon the demand for base money should be taken into account.\(^2\)

Multiplication of both sides of equation (3) by the price level and population yields the aggregate nominal demand for base money:

\[
M_1^d = PN_1 \cdot f_1(y, \gamma_p).
\] (4)

Take the time derivative of the logarithm of equation (4) to obtain

\[
g_{M_1}^d = g_{P} + \gamma_{y_1} g_{y_1}
\] (5)

where \(\gamma_{y_1}\) is the elasticity of the demand for real base money per capita with respect to real income per capita. In obtaining (5), we have assumed the time derivative of \(g_{P}\) to be zero, so equation (5) is valid only for alternative steady-state rates of inflation.

Let \(w_1 = M_1/(M_1 + M_2)\), the proportion of the world stock of base money which is held by country 1, and \(w_2 = (1 - w_1) = M_2/(M_1 + M_2)\). If the demand for base money equals the supply in each country, the rate of growth of the world demand for base money is

\[
g_{M_1}^d + g_{M_2}^d = w_1 g_{M_1}^d + w_2 g_{M_2}^d
\]

\[
= w_1 (g_{N_1} + \gamma_{y_1} g_{y_1}) + (1 - w_1)(g_{N_2} + \gamma_{y_2} g_{y_2}) + g_{P}.
\] (6)

From equations (1) and (2),

\[
M_1 + M_2 = D_1 + D_2 + \bar{R}.
\] (7)

Hence, the rate of growth of the world supply of base money is
\[ g_{M_1} + M_2 \, \frac{dD_1}{dt} \, \frac{1}{M_1 + M_2} + \frac{dD_2}{dt} \, \frac{1}{M_1 + M_2} = \frac{dD_1}{dt} \, \frac{M_1}{M_1 + M_2} \, \frac{D_1}{D_1} + \frac{dD_2}{dt} \, \frac{M_2}{M_1 + M_2} \, \frac{D_2}{D_2} \]

\[ = g_{D_1} \, w_1 \, s_1 + g_{D_2} \, (1 - w_1) s_2 \]

where \( s_i = D_i / M_i \).

Equating the rates of growth of the world demand for base money, equation (6), and the world supply of base money, equation (8), we obtain

\[ g_p = g_{D_1} \, w_1 \, s_1 + g_{D_2} \, (1 - w_1) s_2 \]

\[ - w_1 (g_{N_1} + \eta y_1 \, g_{y_1}) - (1 - w_1) (g_{N_2} + \eta y_2 \, g_{y_2}) \, \frac{3}{8} \]

Real government revenue per period of time generated by the government of the \( i \)th country through the issue of domestic credit is

\[ GR_i = \frac{dD_i}{dt} \, \frac{1}{P} \]

and for both countries together

\[ GR_1 + GR_2 = \frac{dD_1}{dt} \, \frac{1}{P} + \frac{dD_2}{dt} \, \frac{1}{P} \]

\[ = \frac{M_1}{P} \, g_{M_1} + \frac{M_2}{P} \, g_{M_2} \]

\[ = N_1 \cdot f_1(y_1, g_p) (g_{N_1} + g_p + \eta y_1 \, g_{y_1}) \]

\[ + N_2 \cdot f_2(y_2, g_p) (g_{N_2} + g_p + \eta y_2 \, g_{y_2}) \]

(11)

The governments may cooperate to maximize the sum of their revenues from the issue of domestic credit or each government may act individually to maximize the value of its own revenue.
Case I: Governments Cooperate to Maximize Combined Revenue from the Issue of Domestic Credit

To maximize their combined revenue from the issue of domestic credit, the governments select that rate of inflation which yields a maximum value for equation (11), i.e., the rate of inflation at which

\[
\frac{d(GR_1 + GR_2)}{dg_p} = N_1 \cdot f_1(y_1, g_p)\left(1 + g_{y_1} \frac{d\gamma_1}{dg_p}\right)
\]

\[+ N_2 \cdot f_2(y_2, g_p)\left(1 + g_{y_2} \frac{d\gamma_2}{dg_p}\right)\]

\[+ N_1 \cdot (g_{N_1} + g_p + \gamma_{y_1} g_{y_1}) \frac{df_1(y_1, g_p)}{dg_p}\]

\[+ N_2 \cdot (g_{N_2} + g_p + \gamma_{y_2} g_{y_2}) \frac{df_2(y_2, g_p)}{dg_p}\]

\[= \frac{M_1}{p} \left[1 + g_{y_1} \frac{d\gamma_1}{dg_p} + (g_{N_1} + g_p + \gamma_{y_1} g_{y_1}) \frac{d \ln f_1(y_1, g_p)}{dg_p}\right]
\]

\[+ \frac{M_2}{p} \left[1 + g_{y_2} \frac{d\gamma_2}{dg_p} + (g_{N_2} + g_p + \gamma_{y_2} g_{y_2}) \frac{d \ln f_2(y_2, g_p)}{dg_p}\right]\]

\[= 0 \tag{12}\]

or

\[\frac{w_1}{1 - w_1} \left[1 + g_{y_1} \frac{d\gamma_1}{dg_p} + (g_{N_1} + g_p + \gamma_{y_1} g_{y_1}) \frac{d \ln f_1(y_1, g_p)}{dg_p}\right]
\]

\[+ \left[1 + g_{y_2} \frac{d\gamma_2}{dg_p} + (g_{N_2} + g_p + \gamma_{y_2} g_{y_2}) \frac{d \ln f_2(y_2, g_p)}{dg_p}\right] = 0 \tag{13}\]

where $\gamma_i, g_p$ is the elasticity of the per capita demand for real base money in the $i$th country with respect to the rate of inflation. 4/
Once the combined revenue maximizing (CRM) rate of inflation has been obtained from equation (13), it can be substituted into equation (9) to yield the infinite set of combinations of \( g_{D_1} \) and \( g_{D_2} \) which maximize combined government revenue from domestic credit issue. The \( g_{p}^{g_1} \) line in Figure 1 is the locus of such combinations of \( g_{D_1} \) and \( g_{D_2} \).

If the governments agree to pursue the CRM rate of inflation, they must also agree upon a particular point on the \( g_{p}^{g_1} \) locus or a particular distribution of the revenue gains. It is to the interest of the government of country 1 to have an agreed-upon point on the \( g_{p}^{g_1} \) locus which is as far to the left as possible. For example, at point A on the locus, \( g_{D_2} = 0 \) and all of the combined government revenue from the issue of credit goes to the government of country 1. For points on the locus to the left of A, \( g_{D_2} < 0 \) and the government of country 1 gets more than the total combined government revenue from domestic credit issue. Similarly, it is to the interest of the government of country 2 to have the agree-upon point as far to the right on the \( g_{p}^{g_2} \) locus as possible.

Moreover, no point on the \( g_{p}^{g_1} \) locus is simultaneously optimal for both governments acting as individual revenue maximizers. Provided that it believes the rate of credit issue by the other government is invariant with respect to its own actions, at least one of the governments will always have an incentive to push the rate of inflation above the CRM rate. This can be shown in a straightforward manner if it is assumed that each government determines its rate of domestic credit expansion making the Cournot assumption that the foreign rate of credit expansion is fixed.

Case II: Governments Act Individually to Maximize Revenue from the Issue of Domestic Credit

Equation (10) for the government of country 1 can be written as follows:
To maximize its revenue from the issue of domestic credit under the assumption that the rate of credit expansion in the other country is fixed, the government of country 1 selects the rate of inflation at which

\[
\frac{d\GR_1}{dg_p} \bigg|_{g_{D_2}} = g_{D_2} = N_1 \cdot f_1(y_1, g_p) \left( 1 + g_{y_1} \frac{d\eta_1}{dg_p} \right)
\]

\[
+ N_2 \cdot f_2(y_2, g_p) \left( 1 + g_{y_2} \frac{d\eta_2}{dg_p} \right)
\]

\[
+ N_1 \cdot (g_{N_1} + g_p + \eta_{y_1} g_{y_1}) \frac{df_1(y_1, g_p)}{dg_p}
\]

\[
+ N_2 \cdot (g_{N_2} + g_p + \eta_{y_2} g_{y_2} - g_{D_2} s_2) \frac{df_2(y_2, g_p)}{dg_p}
\]

\[
= \frac{d(\GR_1 + \GR_2)}{dg_p} - \frac{M_2}{P} g_{D_2} s_2 \eta_{y_2, g_p} = 0.
\]

(15a)

Suppose that \( g_{D_2} = 0 \), i.e., the government of country 2 is neither increasing nor decreasing its domestic credit. Then the value of \( g_p \) which satisfies
equation (15a) is the CRM rate or that rate at which \( \frac{d(GR_1 + GR_2)}{dt} = 0 \). This means that point A (on the \( g^*_p g^*_p \) locus) in Figure 1 is a point of maximum revenue for the government of country 1 even when it acts as an individual revenue maximizer. On the other hand, since \( \eta_2 g_p < 0 \), if \( g_{D_2} > 0 \) (\( g_{D_2} < 0 \)) the rate of inflation which satisfies (13a) is greater (less) than the CRM rate. Indeed, the larger is the value of \( g_{D_2} \), the greater is the revenue maximizing rate of inflation for the government of country 1. Given \( g_{D_2} \) and [from equation (15a)] country 1's corresponding desired \( g_p \), the optimum value of \( g_{D_1} \) can be calculated from equation (9). The \( g^*_1 g^*_1 \) curve in Figure 1 shows the revenue maximizing rate of domestic credit expansion for the government of country 1 at given values of \( g_{D_2} \).

Similarly, equation (10) for the government of country 2 can be written as

\[
GR_2 = N_2 \cdot f_2(y_2, g_p)(g_{N_2} + g_p + \eta y_2 g_{y_2}) + N_1 \cdot f_1(y_1, g_p)(g_{N_1} + g_p + \eta y_1 g_{y_1} - g_{D_1} s_1).
\]  

(14b)

To maximize its revenue from domestic credit issue under the assumption that \( g_{D_1} \) is fixed, the government of country 2 selects the rate of inflation at which

\[
\frac{dGR_2}{dt} \bigg|_{g_{D_1} = g_{D_1}^*} = \frac{d(GR_1 + GR_2)}{dt} - \frac{M_1}{s_1} g_{D_1} s_1 \eta_1, s_p = 0.
\]  

(15b)

Then the value of \( g_{D_1} \) and the corresponding desired rate of inflation can be substituted into equation (9) to obtain the revenue maximizing rate of domestic credit expansion for the government of country 2. The \( g^*_1 g^*_1 \)
curve in Figure 1 shows such revenue maximizing values of \( g_{D_2} \) for given and assumed fixed values of \( g_{D_1} \).

Point B in Figure 1 (at the intersection of the \( g_{D_1}^* \) and \( g_{D_2}^* \) curves) gives the stable equilibrium values of \( g_{D_1} \) and \( g_{D_2} \) when both governments act as individual revenue maximizers. It is obvious that this combination of \( g_{D_1} \) and \( g_{D_2} \) results in a rate of inflation which is greater than the CRM rate.

**A Special Case**

In order to provide some notion as to the magnitude of the difference between the CRM rate of inflation and the equilibrium rate which would result if both governments act as individual revenue maximizers, we employ Friedman's special case of the demand for money:

\[
m^d_i = l_i(y_i) \cdot e^{-b_i g_p}.
\]

This function has the following convenient properties:

\[
\frac{d \ln (m^d_i)}{d g_p} = -b
\]

and

\[
\frac{d y_i}{d g_p} = 0.
\]

In order to facilitate comparisons with Friedman's results, we assume that the two countries are identical (i.e., the parameters of their base money demand functions are identical, \( y_1 = y_2 \), \( g_{y_1} = g_{y_2} \), \( N_1 = N_2 \), \( g_{N_1} = g_{N_2} \) ) and \( s_1 = s_2 = w_1 = w_2 = 1/2 \). Table 1 shows both the equilibrium rates of inflation when the governments act as individual revenue maximizers and (in parentheses) the CRM rate of inflation for various alternative values.
<table>
<thead>
<tr>
<th>Assumptions about Population and Real Income Growth (% per year)</th>
<th>Rate of Inflation (% per year) if b is</th>
<th>2</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_N$ $g_y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 0</td>
<td></td>
<td>100</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(50)</td>
<td>(50)</td>
<td>(10)</td>
</tr>
<tr>
<td>0 5</td>
<td></td>
<td>95</td>
<td>90</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(45)</td>
<td>(40)</td>
<td>(5)</td>
</tr>
<tr>
<td>2 0</td>
<td></td>
<td>98</td>
<td>98</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(48)</td>
<td>(48)</td>
<td>(8)</td>
</tr>
<tr>
<td>2 5</td>
<td></td>
<td>93</td>
<td>88</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(43)</td>
<td>(38)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Table 1

Equilibrium Individual Revenue Maximizing Rates of Inflation and (in Parentheses) Combined Revenue Maximizing Rates of Inflation for Alternative Assumptions
of $g_y$, $g_N$, $\eta_y$, and $b$. Note that when the CRM rate of inflation is nonzero, the equilibrium individual revenue maximizing rate of inflation is always at least twice as large.

Conclusions

In this paper, we have been concerned with government revenue from the issue of fiat money in open economies with fixed exchange rates. We have shown that if each government attempts to individually maximize its revenue, under the assumption that the foreign rate of money expansion is fixed, an equilibrium rate of inflation would result which is much greater than the rate at which combined government revenue is maximized. We have analyzed a two-country world, but the model can easily be extended to include more countries. Indeed, it would seem that the larger the number of countries the less likely it is the governments will cooperate to pursue the combined revenue maximizing rate of inflation.
References


Footnotes

1. This involves no essential loss of generality. The model can easily be extended to a multi-country world (see Johnson [1972]). Similarly, it is not necessary that the exchange rate be fixed at unity.

2. We also follow Friedman in assuming that real incomes are invariant to the inflation rate.

3. Essentially the same equation is developed in Johnson [1972].

4. If the two countries are identical, i.e., $y_1 = y_2$, $g_{y_1} = g_{y_2}$, $N_1 = N_2$, $g_{N_1} = g_{N_2}$, and $f_1(\cdot) = f_2(\cdot)$, the combined revenue maximizing rate of inflation is the rate at which

$$\frac{d\eta_i}{g_{y_i}} + (g_{N_i} + \eta_i g_{y_i})\eta_i, g_p = -1$$

(13a)

simultaneously in both countries. This is the same solution obtained by Friedman [1971] for a closed economy. However, if the two countries are not identical, there is no reason to expect that a particular value of $g_p$ can be found at which equation (13a) is satisfied simultaneously for both country 1 and 2.