Application of Optimization Methods to Crack Profile Inversion Using Eddy Currents

John R. Bowler
Iowa State University, jbowler@iastate.edu

Wei Zhang
Iowa State University

Aleksandar Dogandžić
Iowa State University, ald@iastate.edu

Follow this and additional works at: http://lib.dr.iastate.edu/ece_pubs

Part of the Non-linear Dynamics Commons, and the Signal Processing Commons

The complete bibliographic information for this item can be found at http://lib.dr.iastate.edu/ece_pubs/37. For information on how to cite this item, please visit http://lib.dr.iastate.edu/howtocite.html.
Application of Optimization Methods to Crack Profile Inversion Using Eddy Currents

Abstract
A numerical scheme for finding crack shapes from eddy current measurements has been developed based on a standard iterative inversion approach in which a nonlinear least squares objective function quantifying the overall difference between predictions and measurements is minimized. In this paper, steepest descent and conjugate-gradient methods for minimizing the objective function are investigated and compared. Cramér-Rao lower bounds on the crack parameters are derived to quantify the accuracy of the estimated crack shape. Cramér-Rao bounds are also used to indicate improvements in the design of eddy-current nondestructive evaluation systems.

Keywords
Eddies, electric measurements, nondestructive testing, optimization

Disciplines
Electrical and Computer Engineering | Non-linear Dynamics | Signal Processing

Comments

Rights
Copyright 2003 American Institute of Physics. This article may be downloaded for personal use only. Any other use requires prior permission of the author and the American Institute of Physics.
APPLICATION OF OPTIMIZATION METHODS TO CRACK PROFILE INVERSION USING EDDY CURRENTS

J. R. Bowler, Wei Zhang, and Aleksandar Dogandžić
Iowa State University, Center for Nondestructive Evaluation, 1915 Scholl Road, Ames, IA 50011, USA

ABSTRACT. A numerical scheme for finding crack shapes from eddy current measurements has been developed based on a standard iterative inversion approach in which a nonlinear least squares objective function quantifying the overall difference between predictions and measurements is minimized. In this paper, steepest descent and conjugate-gradient methods for minimizing the objective function are investigated and compared. Cramér-Rao lower bounds on the crack parameters are derived to quantify the accuracy of the estimated crack shape. Cramér-Rao bounds are also used to indicate improvements in the design of eddy-current nondestructive evaluation systems.

INTRODUCTION

A key aim of eddy-current nondestructive evaluation is to quantify flaws in conductors using changes of the probe impedance due to defects. The task of predicting the impedance from a knowledge of the probe and flaw is a direct or forward problem. Correspondingly, the aim of the inverse problem is to determine the shape and size of the flaw from probe measurements recorded as a function of probe position and excitation frequency. An optimization approach to inversion seeks the flaw by minimizing an objective function which quantifies the overall difference between the impedance predictions and the measurements. Starting with a initial estimate, the optimization proceeds iteratively. In each step, if the agreement is unsatisfactory, the flaw is updated and a new prediction made. The process continues until the objective function becomes less than a reasonable tolerance or no longer decreases.

This paper is organized as follows. In the next section the forward problem is outlined. Then the objective function is defined and a comparison between the optimization methods of steepest descent and conjugate gradient are made. The crack profile has been found through each of these optimization procedures. Finally, a statistical concept, the Cramér-Rao bound (CRB), is introduced to quantify the accuracy of the inversion. The CRB is also used to seek the optimum measurement frequency for inversion and the best value of probe liftoff.

FORWARD PROBLEM

In the idealized eddy-current inspection system that is investigated here, a coil, excited by a time-harmonic current interacts with a crack, as shown in Fig.1, giving rise to perturbations in the field whose effects are observed through changes of probe impedance. The forward problem predicts the coil impedance variation with position at one or more excitation frequency by first computing the field at the crack using a boundary element method [1]. In the inverse
problem, the crack shape is found from the impedance measurements [2].

**OBJECTIVE FUNCTION**

Assume that the crack profile is described by an $n \times 1$ parameter vector $p$. We estimated $p$ by minimizing the following nonlinear least squares objective function:

$$
\xi(p) = \sum_{i=1}^{M} |Z_{obs,i} - Z_i(p)|^2 = [z_{obs} - z(p)]^H [z_{obs} - z(p)]
$$

(1)

where $z_{obs} = [Z_{obs,1}, ..., Z_{obs,4}, ..., Z_{obs,M}]^T$, $z(p) = [Z_1(p), ..., Z_{H}(p), ..., Z_M(p)]^T$ are vectors containing the measured and predicted impedance at a specific excitation frequency at $M$ observation points. Here, the superscript $H$ denotes Hermitian (conjugate) transpose and the superscript $T$ denotes the transpose. The nonlinear least squares estimate of the crack parameter vector $p$ is also its maximum likelihood (ML) estimate for the following measurement model:

$$
z_{obs} = z(p) + e,
$$

(2)

where $e$ is the additive white Gaussian noise with covariance $E[ee^H] = \sigma^2 I$. Here $E[\cdot]$ denotes expectation and $I$ denotes the identity matrix [3].

**Descent Methods**

We use descent methods to minimize the objective function. An estimate of $p$ at iteration $k$ is updated as follows:

$$
p_{k+1} = p_k + \alpha_k d_k
$$

(3)

where the vector $d_k$ is called the descent direction along which we search for the minimization of the objective function and $\alpha_k\|d_k\|$ is the step size. Usually $d_k$ is composed of a combination of objective function gradients. Descent methods only employ an admissible direction in each iteration which means $\xi(p_{k+1}) \leq \xi(p_k)$ as $\alpha_k \rightarrow 0$. An iteration algorithm for optimization thus involves two steps:

1. determine a search descent direction.
2. search along the line to determine a step size, $\alpha_k$ that minimizes $\xi$ along that line.
Gradient of the Objective Function

Assume that the crack lies in a known plane and that the crack geometry can be represented by the equation of the line of the crack edge written as \( \nu(p) = 0 \). The first derivatives with respect to \( p_j, (j = 1, 2, \ldots, n) \) are:

\[
\frac{\partial \xi(p)}{\partial p_j} = -2\text{Re}\left\{ \sum_{i=1}^{M} [Z_{\text{obs},i} - Z_i(p)]^* \frac{\partial Z_i(p)}{\partial p_j} \right\}, \quad j = 1, 2, \ldots, n,
\]

which can be written as:

\[
\frac{\partial \xi(p)}{\partial p} = -2\text{Re}\left\{ \frac{\partial z(p)^H}{\partial p} \cdot [z_{\text{obs}} - z(p)] \right\}
\]

where \( \frac{\partial z(p)^H}{\partial p} = \left( \frac{\partial z(p)}{\partial p^T} \right)^H \) and

\[
\frac{\partial z(p)}{\partial p^T} = \begin{pmatrix}
\frac{\partial Z_1(p)}{\partial p_1} & \frac{\partial Z_1(p)}{\partial p_2} & \cdots & \frac{\partial Z_1(p)}{\partial p_n} \\
\frac{\partial Z_2(p)}{\partial p_1} & \frac{\partial Z_2(p)}{\partial p_2} & \cdots & \frac{\partial Z_2(p)}{\partial p_n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial Z_M(p)}{\partial p_1} & \frac{\partial Z_M(p)}{\partial p_2} & \cdots & \frac{\partial Z_M(p)}{\partial p_n}
\end{pmatrix}
\]

Here, \( \frac{\partial Z_i(p)}{\partial p_j} \) is the partial derivative of the predicted impedance at the \( j \)th observation point with respect to the \( j \)th parameter. The derivative can be found from the functional gradient of the impedance with respect to a variation in the flaw function \( \nu(p) \) along the direction normal to the crack edge, see [1] and [2]. Also, "Re" denotes the real part and * denotes the complex conjugate.

Linear Search

We wish to find \( \alpha_k \) which minimize \( \xi(p_k + \alpha_k d_k) \). An explicit formula for the step-size \( \alpha_k \) in our problem is derived as in [4] using a Taylor series expansion:

\[
\alpha = \frac{P}{Q}
\]

where

\[
P = \text{Re}\left\{ \sum_{i=1}^{M} [Z_i(p_{k-1}) - Z_{\text{obs},i}]^* \frac{\partial Z_i(p_{k-1})}{\partial \alpha} \right\}
\]

\[
Q = \sum_{i=1}^{M} \left| \frac{\partial Z_i(p_{k-1})}{\partial \alpha} \right|^2
\]

\[
\frac{\partial Z_i(p_{k-1})}{\partial \alpha} = \sum_{j=1}^{n} \frac{\partial Z_i(p_{k-1})}{\partial p_j} (d_{k-1})_j
\]

Here \( p_{k-1} \) and \( d_{k-1} \) are fixed in the \( k \)th iteration.

OPTIMIZATION METHODS

Steepest descent and conjugate-gradient are two descent methods which are used widely [5]. They both have their own advantages and disadvantages. In order to investigate which of the two methods has the better performance for the inversion problem that is considered in this work, the corresponding algorithms have been implemented and the results of inversion calculations using the different approaches compared for their speed, their rate of convergence and accuracy.

744
Method of Steepest Descent

In this method, the descent step $d$ is chosen as the negative of the gradient vector, $-\nabla\xi(p)/\nabla p$. The iteration becomes

$$p_{k+1} = p_k - \alpha_k \left( \frac{\partial\xi}{\partial p} \right)_k$$

(11)

Because the gradient direction is normal to the crack edge, we need to transform it to the orthogonal coordinates first and update $x$ and $y$ coordinates of the crack points respectively:

$$x_{k+1} = x_k - \alpha_k \left( \frac{\partial\xi}{\partial p} \right)_k \sin(\theta)$$

(12)

$$y_{k+1} = y_k - \alpha_k \left( \frac{\partial\xi}{\partial p} \right)_k \cos(\theta)$$

(13)

where $\theta$ denotes the angle between the normal direction and $y$ direction.

In general, the iteration procedure is in 5 steps:

1. Select starting crack profile $p_0$.
2. Compute the impedance predictions from the solution of the forward problem.
3. Compute gradient of the objective function and the new descent direction $d$.
4. Perform a line search for the best step-size $\alpha$.
5. Update the flaw profile and go to (2).

Conjugate Gradient Method

Using the method of Fletcher and Reeves [5], the descent direction at the current point, $d_{k+1}$, can be found as linear combination of the negative gradient at the current point, $g_{k+1} = (\partial\xi/\partial p)_{k+1}$, and the previous descent directions in the form

$$d_{k+1} = -g_{k+1} + \frac{g_k^T g_{k+1}}{g_k^T g_k} d_k$$

(14)

The five iteration steps of the conjugate gradient algorithm are nearly the same as those of the steepest descent method except that the algorithm for computing the descent direction is different. For the first iteration, because there is no previous step direction, $d_0$ is set to be $g_0 = g_0/\|g_0\|^{1/2}$ (i.e., normalized $g_0$).

Discussion on the Optimization Performance

Our inversion is implemented on crack D2 [6] at a frequency of 250Hz using a normal coil. The impedance calculation is performed using a grid of $32 \times 16$ boundary elements and the results are compared with the experimental data acquired by Harrison, Jones and Burke on spark eroded notches in an aluminum alloy, see [6]. The crack profiles found by the two optimization methods are shown in Fig.2. We performed an investigation on their convergence speeds and precision including: (1) convergence value, $\xi(p)_{min}$, (2) error percent, Err%, which is defined as $\sum_{i}[Z_{obs}(z) - Z_i]$, $(3)$ number of iterations needed to achieve convergence, $n$, $(4)$ average time for one iteration, $t(s)$. The results are listed in Table 1. Although the results listed here are just for one single crack input, we find that the others are similar.
TABLE 1. Performances of two optimization methods.

<table>
<thead>
<tr>
<th>Performance</th>
<th>$\xi(p)_{\text{min}}$</th>
<th>$\text{Err%}$</th>
<th>n</th>
<th>t(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steepest Descent</td>
<td>0.162658</td>
<td>2.33</td>
<td>25</td>
<td>3.90</td>
</tr>
<tr>
<td>Conjugate Gradient</td>
<td>0.166148</td>
<td>2.38</td>
<td>21</td>
<td>3.94</td>
</tr>
</tbody>
</table>

FIGURE 2. Inversion of experimental impedance data to determine the crack shape using two optimization methods.

It can be seen from the results that the steepest descent calculation converges a little more slowly than the conjugate-gradient calculation. But the former gives greater precision than the latter. Although for some more complex minimization problem, steepest descent may converge much more slowly than conjugate-gradient, in our problem their speeds are similar. Hence, in a comprehensive view, the relatively simple steepest descent method is better for our inversion problem.

CRAMÉR-RAO BOUNDS

We introduce Cramér-Rao bound (CRB) to quantify the inversion accuracy. Cramér-Rao bound is a lower bound on the covariance of all unbiased estimators that, under mild regularity conditions, is attained asymptotically by the maximum likelihood (ML) estimator. (Asymptotic means that the number of observation points is much larger than the number of unknown parameters.) In general, CRB can be used as a measure of the potential performance attainable from the system, providing a benchmark for assessing both the algorithm performance and the system design.

CRB for Crack Profile Estimation

As observed earlier, for measurements containing white Gaussian noise, finding the ML and nonlinear least squares minimization are equivalent. Hence, the proposed estimator asymptotically achieves the following CRB:

$$\text{CRB} = \frac{\sigma^2}{2} \cdot \left\{ \text{Re} \left[ \frac{\partial z(p)^H \partial z(p)}{\partial p \partial p^T} \right] \right\}^{-1}$$

(15)

which follows easily from [3, p.525]. Note that the CRB in (15) is an $n \times n$ matrix whose
diagonal entries are the asymptotic variances of the \( n \) unknown parameters. We consider the following two performance criteria:

1. \( \text{tr}(\text{CRB}) \), which is the sum of the variances of the parameters, and

2. \( \log(\det(\text{CRB})) \), which is proportional to the logarithm of the volume of the asymptotic confidence region of \( p \).

In our problem, for a simple case, we choose \( n \) equally spaced points along the crack edge and let \( \frac{\partial Z_i(p)}{\partial p_j} \) be the gradient of the predicted impedance of the \( i \)th observation point with respect to the crack edge along normal direction at the \( j \)th crack point. Below we examine the dependence of the above two performance criteria on frequency and lift-off.

**CRB vs Frequency**

Assume that the noise variance is constant across all the frequencies from 250Hz to 8KHz. We chose 51 observation points and 9 equally spaced crack points along the edge. The crack profile is as shown in Fig.2. We vary the crack depth and obtain the results in Fig.3 and Fig.4.
FIGURE 5. Performance criterion 1, $\text{tr}(\text{CRB}/\sigma^2)$ vs. liftoff.

FIGURE 6. Performance criterion 2, $\log(\text{det}(\text{CRB}/\sigma^2))$ vs. liftoff.

The figures show clearly that both of the two criteria decrease with increasing frequency. This means that in the frequency domain in which our theoretical model is valid, the higher the test frequency, the more accurate the inversion will be. As for different crack depths, the deeper the crack depth is, the larger the estimation error tends to be, which can be easily explained by the skin depth effect. Hence, in order to get better results, the inversion should be done at as high a frequency as possible and will be more accurate for shallower cracks.

**CRB vs Liftoff**

Here we assume the noise variance is constant across all liftoffs. Furthermore, the simulation conditions such as the number of observation points, number and location of the crack points, crack shape and depths are all the same as before. For a fixed frequency of 250Hz, the performance variation with probe liftoff are shown in Fig.5 and Fig.6.

Here the two criteria increase with liftoff and the logarithm of CRB is nearly linear with liftoff. This indicates that the asymptotic variance will become larger if the liftoff becomes larger and accords with the fact that the induced electrical field will become weaker with the liftoff increasing. Similarly, the profiles of shallower cracks can be estimated better than those that extend deeper into the material for the same liftoff.
CONCLUSION

Setting the unknown parameters as the crack points along the crack edge, inversion of the crack shape according to the experimental impedance data has been made by two typical optimization methods, steepest descent and conjugate gradient. Their performances on optimization precision and speed have been investigated. CRB has been used to give a measure of the shape estimation accuracy. An investigation of the relationships between CRB and frequency and liftoff indicates that to get a good inversion result, it is better to implement the inversion at high frequency with small probe liftoff.

ACKNOWLEDGEMENT

This work was supported by NASA, grant number NAG-1-01040.

REFERENCES