The paradox of interest rates of the Greenback Era: A reexamination

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Disciplines
Behavioral Economics | Economic Theory | Growth and Development

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Oleksandr Zhylyevskyy

Abstract

The two leading explanations for the counterintuitive behavior of interest rates during the Greenback Era (1862-1878) – the resumption expectations model of Calomiris (1988) and the capital flow argument of Friedman and Schwartz (1963) – are inconsistent with each other in terms of their treatment of financial arbitrage. A methodology to identify unexploited arbitrage opportunities in financial data is proposed. Observable returns strongly suggest that the money market of the Greenback Era did not systematically admit arbitrage, except possibly around the times of the Gold Corner of 1869 and the Panic of 1873, which implies that Calomiris provides a more plausible explanation.

Keywords: Greenback Era, money market, arbitrage opportunity, interest rate paradox

JEL classification: N21

1. Introduction

The “Greenback Era” (1862-1878) in U.S. financial history is a source of continued fascination among economists. This period is marked by the first time since the War of Independence when paper notes served as legal tender currency, the first time ever when the federal government levied personal income taxes, and the first time when the system of national banks that has survived in a modified form until today came into existence.1 Lessons of the Greenback Era are particularly informative from an empirical perspective, since the wealth of available data (e.g., Mitchell, 1908; Macaulay, 1938) allows for rigorous testing of economic theories.

This paper reinvestigates one of the most stunning phenomena of the Greenback Era, which is commonly known as “the paradox of interest rates.” Different from the existing literature, the paper analyzes the behavior of short-term money market returns rather than long-term bond yields, focusing explicitly on whether the money market admitted arbitrage opportunities. While the “no arbitrage” assumption is customary in the analysis of present-day financial markets, its applicability to American markets of the nineteenth century remains a hotly debated issue. This paper is the first one to empirically address the topic of market efficiency in the context of the money market rather than to perform an analysis of the gold market efficiency, which has been done – and redone – many times in the literature. Knowledge of whether financial markets admitted arbitrage opportunities is crucial for determining which of the two leading explanations of the paradox – the resumption expectations model of Calomiris (1988) or the capital flow argument of Friedman and Schwartz (1963) – is more plausible. The paper has other novel aspects. Most notably, it analyzes whether the money market systematically admitted arbitrage by applying tools of modern asset pricing theory. Also, it utilizes rich return data, a part of which appears to have been overlooked in the prior research. The paradox of low nominal interest rates during rampant inflation in the Civil War North had long been known to economic historians (e.g., Mitchell, 1903, pp. 367-368).2 A counterintuitive behavior of the rates becomes even more puzzling when one considers the Greenback Era as a whole. Figure 1 shows that the monthly wholesale price index in the U.S., which is believed to be the most reliable price index for the period (Kindahl, 1961, p. 34), more than doubled...
in the course of the Civil War. Afterwards, the index gradually declined and returned to its prewar levels in the late 1870s. On average, the annual inflation rate during the Civil War exceeded 25%. Then, between 1866 and 1878, the country experienced prolonged deflation at the average annual rate of more than 6%. If money market participants had correctly anticipated the price dynamics during the Greenback Era, one would expect to observe high nominal interest rates between 1862 and 1865 and low rates afterwards, in line with the textbook Fisher equation, \( i = r + \pi' \), where \( i \) and \( r \) are the nominal and real interest rate, respectively, and \( \pi' \) is the expected inflation rate.

However, the actual pattern of the money market rates is strikingly different. Figure 2 plots monthly averages of annualized commercial paper rates in New York City, which represent nominal rates on 60 to 90-day uncollateralized loans made to businesses with good or excellent credit histories. Other money market rates (not shown in the figure) such as the New York collateralized call loan rate and the Boston first-class bankable paper rate exhibit a similar pattern. Between 1862 and 1865, the nominal commercial paper rate fluctuated between 4.5% and 9.2%, with the average rate of 6.5% being well below the annual inflation rate of 25%. Between 1866 and the Panic of 1873, the average commercial paper rate increased to 8%, despite concurrent deflation at the average annual rate of 5.6%. During the recessionary years of 1874 to 1878, the average commercial paper rate declined to 5.3%, while deflation accelerated to 7.8%.

Succinctly put, between 1862 and 1865, the ex post real money market rates were negative and large in absolute magnitude, while between 1866 and 1878, they were positive and large. In the latter case, it is doubtful that real rates in excess of 13% were entirely due to a risk premium, because the premium on the money market instruments had to be at least 8%, assuming a risk-free return of 5%.\(^3\) Such a high risk premium is only characteristic of much riskier equity securities. The essence of the paradox is that large negative real returns until 1865 and large positive real returns afterwards persisted for several years, which implies that each case is likely an outcome of a money market equilibrium. From the standpoint of the rational expectations theory, these equilibria may seem strange, as they indicate that inflationary expectations were out of line with the actual price dynamics for prolonged periods of time.

Understandably, the original explanation of the paradox is that investors had erroneous inflationary expectations (Mitchell, 1903, pp. 369-370). However, Mitchell qualifies his claim by stating that this explanation, while potentially correct, is insufficient and that the low nominal rates during the Civil War might, in fact, have resulted from a weak bargaining power of lenders and diminished demand for credit (Mitchell, 1903, p. 375).

Friedman and Schwartz (1963) offer a distinctly different explanation, which presupposes arbitrage opportunities (Roll, 1972, p. 494). In their view, the pattern of the interest rates can be attributed to speculative capital flows. For example, the low nominal rates during the Civil War may have resulted from a massive capital inflow from abroad, as foreign investors took advantage of buying cheap (in terms of gold) greenback-denominated assets in anticipation of the greenback’s eventual appreciation in terms of gold. Thus, the expected effective yield on such investments was high but primarily reflected a large expected capital gain rather than interest payments. Strictly speaking, the argument of Friedman and Schwartz seems to apply more readily to investments in U.S. government bonds rather than in money market securities that have short duration.

Another theoretical explanation has been proposed by Calomiris (1988), who rejects the capital flow argument of Friedman and Schwartz and builds upon the original descriptive analysis of Mitchell. In Calomiris’ model, expectations about the resumption of specie payments imply expected future values of the exchange rate and the aggregate price level. Government money-supply policy effectively pegs the nominal rate of interest and the equilibrium rate of expected inflation. The terminal condition for the price level on the resumption date together with the equilibrium rate of inflation determine a unique path of the price level recursively. The nominal money stock adjusts endogenously (through the issue of national bank notes, the creation of bank deposits, and gold flows) to the level demanded given the nominal interest rate and the price level.\(^5\) Thus, if the assumptions underlying Calomiris’ model are valid, actual

\(^3\)Even if one were to exclude from the computation of this average abnormally high and potentially unreliable quotations during the Gold Corner of 1869 and the Panic of 1873, the average rate would still have increased to 7.6%.

\(^4\)The assumed risk-free rate of 5% likely represents an accurate approximation to the true risk-free rate. British consols, which were the safest financial asset of the period, yielded roughly 3%, and historical evidence suggests that the best quality American financial securities commanded a premium of about 2% over comparable British securities (Field, 1983, pp. 420-421). An estimate for the American risk-free rate is obtained by summing the British consols’ yield and this premium.

\(^5\)More specifically, the supply of money is perfectly elastic (see Calomiris, 1988, Figure 4 on p. 217).
nominal interest rates are consistent with rational expectations in the sense that the interest rate corresponds to the outcome of an equilibrium determined by the resumption expectations, which eventually turned out to be correct.

A critical assumption in Calomiris’ model is absence of arbitrage opportunities in the financial market (Calomiris, 1988, pp. 195, 199). This assumption is the cornerstone of modern asset pricing theory (e.g., Black and Scholes, 1973; Harrison and Kreps, 1979; Harrison and Pliska, 1981), but its applicability to financial markets of the nineteenth century has been contested. In particular, a well known paper on the nineteenth-century American and British technology argues that arbitrage opportunities systematically emerged in the American economy and international capital flows reflected arbitrage activities of foreign investors (Field, 1983, p. 422). The possibility that the nineteenth-century financial market in New York City was inefficient is also raised by Michie (1986) in his analysis of institutional aspects of the London and New York stock exchanges. He finds that trading rules in New York tended to discourage speculative operations and may have prevented the financial market from operating efficiently (Michie, 1986, pp. 176, 181).

Several empirical studies have examined the issue of arbitrage in the context of the operation of the gold standard in the late nineteenth century. Clark (1984) argues that the gold points were often violated and arising profit opportunities were not eliminated quickly. However, Officer (1986) identifies flaws in Clark’s methodology and shows that there were few gold point violations and they did not persist. Although these studies do not address the question of whether unexploited arbitrage opportunities existed during the Greenback Era, they indicate the need to rigorously test for the presence of arbitrage rather than rule it out a priori.

The objective of this paper is to examine the Greenback Era return series, focusing explicitly on the critical issue of whether the money market admitted arbitrage opportunities. The analysis sheds further light on the nature of the interest rate paradox and helps to evaluate which of the two competing theories – the resumption expectations model of Calomiris or the capital flow argument of Friedman and Schwartz – is better suited to explain the paradox. More specifically, the paper uses tools of modern asset pricing theory to develop a methodology for identifying the presence of unexploited arbitrage opportunities in financial data. Besides a few mild technical conditions necessary to apply the theory, the only substantive economic assumption imposed here is one of frictionless markets: money market investors are allowed to sell assets short and transaction fees are negligibly small. The essence of the developed methodology is to provide a convenient way to aggregate the data by constructing a series of realizations of a scalar stochastic discount factor (Cochrane, 2001, pp. 8-9). The market admits no arbitrage opportunities if and only if there exists a positive stochastic discount factor, and checking positivity of corresponding realizations is straightforward.

Applying the methodology to the financial data from the Greenback Era, the paper finds that the observable return series strongly suggest that, except possibly around the times of major market shocks such as the Gold Corner of 1869 and the Panic of 1873, the money market of the period did not admit arbitrage opportunities on a systematic basis. This result complements the findings of Calomiris and others that the gold market of the Greenback Era was efficient and suggests that an overall description of the American financial market of the second half of the nineteenth century as a market with no systematic arbitrage opportunities is more appropriate than the description proposed by Field (1983). Moreover, the result implies that the resumption expectations model of Calomiris likely provides a more plausible explanation of the interest rate paradox than the capital flow argument of Friedman and Schwartz.

The rest of this paper proceeds as follows. Section 2 outlines the methodology. Section 3 describes the data series. Section 4 applies the methodology to the data and discusses the results. Section 5 concludes.

2. Methodology

This section begins by discussing the substantive economic assumptions underlying the analysis. It then introduces the notation used in the paper, outlines the relevant theory, and describes the methodology for testing of whether unexploited arbitrage opportunities are present in financial data.

Calomiris argues that the gold market during the Greenback Era was efficient in the sense that the series of gold prices “is probably best described as a random walk” (p. 203). However, he does not check whether the money market or financial market, as a whole, admitted arbitrage when investment strategies involving assets other than gold are taken into account.
2.1. Assumptions

Most importantly, the money market of the Greenback Era is viewed here as a frictionless market in the sense that (1) the investors can sell assets short and (2) transaction fees are negligibly small. The first condition effectively means that borrowing to finance speculative operations is allowed, for which there is supporting historical evidence, except in the aftermath of the Gold Corner of 1869 and the Panic of 1873, when regular financial transactions virtually ceased. The reason for the second condition is more technical. It is needed to rule out nonlinear pricing, which would arise from scale economies if transaction fees are present. The condition should approximately hold in the context of a money market, because most transactions on it occur between banks, who traditionally do not charge each other fees, while transactions involving nonbanking institutions such as, for example, a sale of commercial paper by a business firm are typically conducted in large volume, making any nonlinearities arising from brokerage fees negligibly small.

In addition, the money market investors of the Greenback Era are assumed to strictly prefer more wealth to less. This restriction of strictly – rather than weakly – insatiable preferences is technical and is related to the strict positivity of a linear pricing operator (Harrison and Kreps, 1979, pp. 384-387). It likely adequately represents reality, because money market investors are usually professionals who trade for profit. Notably, in contrast to Calomiris (1988), the investors are not required to be risk neutral. In fact, no specific assumption about risk attitudes needs to be imposed at all. Also, unlike in the model of Calomiris, this paper does not take a stance on how resumption expectations may affect the aggregate price level. No specific assumptions as to whether the aggregate price level is flexible or predetermined are imposed here.

2.2. Price and return processes

Let $t$ index calendar months starting with some initial month $t_0$, which represents January 1857 in the empirical application, as explained in Section 3. For each month $t$, denote by $t'$ the third consecutive month after $t$. This choice is motivated by a typical time to maturity of money market securities, so that realizations of return processes can be computed with minimum interpolation.

All of the following price and return processes refer to quotations in New York City, except as noted otherwise. Let $(g_t)_{t \geq t_0}$ stand for the price process of one gold dollar quoted in legal tender currency. Specifically, before the suspension of specie payments in December 1861 and after their resumption in January 1879, $g_t$ represents the price of one gold dollar at the beginning of month $t$ in terms of gold dollars, $g_t \equiv 1$. During the Greenback era, $g_t$ is the price of one gold dollar in terms of greenbacks. Observe that since the gold dollar can be used as a means of payment, the inequality $g_t \geq 1$ always holds.

Next, let $\{1 + i_{t,t'}\}_{t \geq t_0}$ stand for the nominal gross return process of the call loan index,\(^7\) where $i_{t,t'}$ represents the net amount payable at $t'$ on one dollar invested in the index at $t$. Likewise, let $\{1 + \bar{i}_{t,t'}\}_{t \geq t_0}$ be the nominal gross return process of the commercial paper index. Given the findings of Sushka and Barrett (1984) on the integration of the national capital market, the New York money market investors must also be allowed to (potentially) trade in the Boston bankable paper index. Its nominal gross return process is denoted as $\{1 + \bar{i}_{t,t'}\}_{t \geq t_0}$.

Furthermore, since there is historical evidence that gold flows between the U.S. and the U.K. responded to changes in interest rates, the New York traders must be additionally allowed to invest in British money market securities. Let $\{1 + \bar{i}_{t,t'}\}_{t \geq t_0}$ be the nominal gross return process of the London banker’s bill index defined in terms of the British pound. Following the literature (e.g., Kindahl, 1961; Friedman and Schwartz, 1963), the dollar-pound spot exchange rate is approximated as $4.8665 \cdot g_t$, legal tender dollars per £1, where 4.8665 is the mint ratio of the gold dollar to the pound (the U.K. was on the gold standard throughout the period). Then, the same process defined in terms of legal tender dollars is $\left\{\frac{g_t}{4.8665} \cdot \left(1 + \bar{i}_{t,t'}\right)\right\}_{t \geq t_0}$.

For convenience, all gross returns are arranged in one vector as:

$$x_{t,t'} = \left(1 + i_{t,t'}^1, 1 + i_{t,t'}^2, 1 + i_{t,t'}^3, \frac{g_t}{4.8665} \cdot \left(1 + \bar{i}_{t,t'}\right)\right)' .$$

---

\(^7\)This paper uses indices rather than specific assets, because available return data are in the form of monthly averages of quotations for a mix of securities from a given asset class. Implicitly, index investing presumes that a trader is able to diversify his or her portfolio. Historical evidence supports this assumption. For example, banks of the Greenback Era usually held highly diversified call loan portfolios and gradually retired older loans when making new ones.
Finally, let \((S_t)_{t\geq 0}\) be the railroad stock price process, where \(S_t\) is the value of the railroad stock index at \(t\). This process will be used to check validity of the constructed stochastic discount factor.

2.3. Arbitrage and stochastic discount factor

Formally, an arbitrage opportunity is an investment strategy with the following three features. First, the strategy has zero cost in the sense that any asset purchases are financed by borrowing. Second, it never results in a loss, although it might be worthless at maturity. Third, it has a strictly positive expected payoff. Absence of unexploited arbitrage opportunities in the money market implies existence of specific relationships in financial data, as explained below.

Let \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)\) be a filtered probability space, where \(\Omega\) is the set of states of the world, filtration \((\mathcal{F}_t)_{t\geq 0}\) represents the evolution of information available to investors at the beginning of each month, \(\sigma\)-field \(\mathcal{F}\) is interpreted as \(\mathcal{F} = \bigcup_{t\geq 0} \mathcal{F}_t\), and \(P\) is the objective probability measure.\(^\text{8}\) The price and return processes are assumed to be adapted to the filtration, so that for every \(t\), the prices \(g_t\) and \(S_t\) are measurable with respect to \(\mathcal{F}_t\), while the vector of gross returns \(x_{t,s}\) is measurable with respect to \(\mathcal{F}_t\).

Under the imposed assumptions about frictionless trading and strictly insatiable preferences, by the first fundamental theorem of asset pricing (see Harrison and Kreps, 1979; Harrison and Pliska, 1981), the money market admits an a

\[ E \left[ \frac{1}{g_{t'}} \cdot x_{t,s} | \mathcal{F}_t \right] = \frac{1}{g_t} \cdot I \text{ for every } t, \]

where \(E^* [\cdot | \mathcal{F}_t]\) is the conditional expectation under \(P^*\) given the information represented by \(\mathcal{F}_t\) and \(I\) is the vector of ones having the same dimension as \(x_{t,s}\).

By a well known result, there exists an almost surely positive Radon-Nikodym derivative associated with \(P^*\), denoted below as a random variable \(\zeta_{t,s}\), that allows to change measures in (1) from \(P^*\) to \(P\):

\[ E \left[ \frac{\zeta_{t'}}{g_{t'}} \cdot x_{t,s} | \mathcal{F}_t \right] = \frac{1}{g_t} \cdot I, \]

where \(E [\cdot | \mathcal{F}_t]\) is the conditional expectation under \(P\) given \(\mathcal{F}_t\).

Rearranging (2):

\[ E \left[ M_{t,s'} \cdot x_{t,s} | \mathcal{F}_t \right] = 1, \]

where \(M_{t,s'} \equiv \frac{\zeta_{t'}}{g_{t'}}\) is an almost surely positive random variable that is commonly known in the finance literature as the stochastic discount factor (e.g., Cochrane, 2001, pp. 8-9).

Applying \(E [\cdot | \mathcal{F}_0]\) to both sides of (3), by the law of iterated expectations:

\[ E_0 \left[ M_{t,s'} \cdot x_{t,s} \right] = 1, \]

where \(E_0 [\cdot] \equiv E [\cdot | \mathcal{F}_0]\) for brevity.

Hansen and Jagannathan (1991) and Cochrane (2001) propose a particularly convenient representation of \(M_{t,s'}\) as an affine function of the shocks to the returns, \(M_{t,s'} = E_0 M_{t,s'} + (x_{t,s'} - E_0 x_{t,s'})' b\), where \(b\) is a vector of constants such that (4) holds. Solving for \(b\):

\[ M_{t,s'} = E_0 M_{t,s'} + [1 - E_0 M_{t,s'} \cdot E_0 x_{t,s'}]^{-1} (x_{t,s'} - E_0 x_{t,s'}), \]

where \(\Sigma_0 = E_0 (x_{t,s'} - E_0 x_{t,s'}) (x_{t,s'} - E_0 x_{t,s'})'\) is the covariance matrix of the gross returns.

When markets are incomplete, \(M_{t,s'}\) need not be unique. However, to rule out arbitrage, it is sufficient to find one strictly positive stochastic discount factor that prices the assets (Harrison and Kreps, 1979, Corollary on p. 392). Thus, the essence of this paper’s methodological approach is to compute realizations of the stochastic discount factor (5) and check whether they are positive.

\(^{8}\)This discussion is intended for interested readers. Others may want to skip the formalism and proceed to the stochastic discount factor representation (5) on p. 5.
2.4. Empirical strategy

If \( E_0 \mathbf{x}_{t}, \Sigma_{t} \) and \( E_0 \mathbf{M}_{t} \) in (5) were known, computing a realization of the stochastic discount factor \( \mathbf{M}_{t} \) would be straightforward, because the only other variable in (5), vector \( \mathbf{x}_{t} \), is a recorded realization of the gross returns.

Under mild assumptions, it is possible to estimate \( E_0 \mathbf{x}_{t} \) and \( \Sigma_{t} \). To simplify the notation, the gross returns are denoted more compactly as \( I_{i,t}^{1}, I_{i,t}^{2}, I_{i,t}^{3} \), and \( I_{i,t}^{4} \), where \( I_{i,t}^{i} = 1 + r_{i,t} \) for \( i = 1, 2, 3 \) and \( I_{i,t}^{4} = \frac{e_{t}^{*}}{\mathbf{E}} \left( 1 + r_{i,t} \right) \), so that \( \mathbf{x}_{t} = \left( I_{1,t}^{1}, I_{2,t}^{2}, I_{3,t}^{3}, I_{4,t}^{4} \right)^{\prime} \). Let the sample size be \( T \) and average gross returns be denoted as \( \bar{I}_{1}, \bar{I}_{2}, \bar{I}_{3}, \bar{I}_{4} \), where
\[
\bar{I}_{i} = \frac{1}{T} \sum_{t=0}^{T} I_{i,t}^{i} \quad \text{for every } i.
\]
Then, define vectors \( \mathbf{m}_{t} \) and \( \mu_{t} \) as:
\[
\mathbf{m}_{t} = \left( \mathbf{x}_{t}, [I_{1,t}^{1} - \bar{I}_{1}]^{2}, [I_{2,t}^{2} - \bar{I}_{2}]^{2}, [I_{3,t}^{3} - \bar{I}_{3}]^{2}, [I_{4,t}^{4} - \bar{I}_{4}]^{2} \right),
\]
\[
\mu_{t} = \left( \mathbf{E}_0 \mathbf{x}_{t}^{\prime}, \Sigma_{1}, \Sigma_{1}^{2}, \Sigma_{1}^{3}, \Sigma_{1}^{4}, \Sigma_{2}, \Sigma_{2}^{2}, \Sigma_{2}^{3}, \Sigma_{2}^{4}, \Sigma_{3}, \Sigma_{3}, \Sigma_{4}, \Sigma_{4}^{2}, \Sigma_{4}^{3}, \Sigma_{4}^{4} \right)^{\prime},
\]
where the last ten elements in \( \mu_{t} \) represent the upper triangle of \( \Sigma_{t} \) in a vectorized form.

In line with assumptions commonly imposed on asset returns in the finance literature (e.g., Cochrane, 2001, p. 11), suppose that the gross return process \( \{ \mathbf{x}_{t} \}_{t \geq 0} \) is ergodic stationary. Because the sample data span a time period that is only slightly longer than two decades (see Section 3), it is not possible to ascertain stationarity of the return series. However, augmented Dickey-Fuller tests (Hamilton, 1994, pp. 516-530) suggest that the series are covariance stationary.

Ergodic stationarity implies that \( E_0 \mathbf{x}_{t} \) is the same vector and \( \Sigma_{t} \) is the same matrix across \( t \), so that the subscript \( t \) on \( E_0 \mathbf{x}_{t}, \Sigma_{t} \) and \( \mu_{t} \) can be dropped: \( E_0 \mathbf{x}_{t} = E_0 \mathbf{x}, \Sigma_{t} = \Sigma, \) and \( \mu_{t} = \mu \). Moreover, defining the average of \( \mathbf{m}_{t} \)'s as \( \bar{\mathbf{m}}_{t} \), the law of large numbers (Hayashi, 2000, pp. 101-102) implies that:
\[
\bar{\mathbf{m}}_{t} \equiv \frac{1}{T} \sum_{t=0}^{T} \mathbf{m}_{t} \to^{\mathbb{P}} \mu.
\] (6)

Assuming that Gordin’s condition (Hayashi, 2000, p. 405) also holds, the central limit theorem implies that:
\[
\sqrt{T} \left( \bar{\mathbf{m}}_{t} - \mu \right) \to^{d} N \left( \mathbf{0}, \mathbf{S} \right),
\] (7)
where \( \mathbf{S} \) is the asymptotic covariance matrix.

The empirical application employs the Newey-West estimator of \( \mathbf{S} \) that is robust to heteroscedasticity and autocorrelation up to 2 lags:
\[
\hat{\mathbf{S}} = \hat{\mathbf{f}}_{0}^{2} + \frac{2}{3} \left( \hat{\mathbf{f}}_{1} + \hat{\mathbf{f}}_{2} \right) + \frac{1}{3} \left( \hat{\mathbf{f}}_{2} + \hat{\mathbf{f}}_{3} \right),
\] (8)
where \( \hat{\mathbf{f}}_{i} = \frac{1}{T} \sum_{t=0}^{T} \mathbf{m}_{t} \) for \( v = 0, 1, 2 \).

Together, (6), (7), and (8) provide a way to consistently estimate the expected value of the returns and their covariance matrix, as well as to conduct valid statistical inference.

In contrast to \( E_0 \mathbf{x}_{t} \) and \( \Sigma_{t} \), the expected value of the stochastic discount factor, \( E_0 \mathbf{M}_{t} \), cannot be estimated from the available data. However, as is well known from the finance theory, this expected value is simply the inverse of the gross risk-free return, \( E_0 \mathbf{M}_{t} = \left( 1 + r_{t}^{f} \right)^{-1} \), where \( r_{t}^{f} \) represents the net amount payable (for sure) at \( t' \) on one legal tender dollar invested in the risk-free asset at time \( t \).

In the empirical application, the annual risk-free rate is set to 5%, which is obtained by adding a 2% premium to the approximate average yield of 3% on British consols (see footnote 4). An analysis of the robustness of the results to this assumption is performed in Section 4.

\[\text{An analysis of the robustness of the results to this restriction is provided in Section 4.}\]
Lastly, it is worth noting that if the financial market admits no arbitrage, a properly constructed stochastic discount factor must be able to price not only the four money market indices above, but also other assets including the railroad stock index, which implies that:

\[ E_0 \left[ M_{t,T} \frac{S_T}{S_t} - 1 \right] = 0. \tag{9} \]

Now, suppose that \( \{ M_{t,T} \}_{t=0}^T \) are realizations of a valid stochastic discount factor. Given (9), the sample average \( \frac{1}{T} \sum_{t=0}^{T-1} \left[ M_{t,T} \frac{S_T}{S_t} - 1 \right] \) must be statistically zero. Reinterpreting \( m_t \) and \( \mu \) in (6), (7), and (8) as (scalar) \( m_t = M_{t,T} \frac{S_T}{S_t} - 1 \) and \( \mu = 0 \), the central limit theorem (7) leads to a formal test, provided that ergodic stationarity and Gordin’s condition hold, as explained earlier. Specifically, the null hypothesis that \( M_{t,T} \) constructed according to (5) is a valid stochastic discount factor can be tested using a test statistic \( Q = T \bar{m}_T \bar{S}^{-1} \bar{m}_T \), since under the null, \( T \bar{m}_T \bar{S}^{-1} \bar{m}_T \rightarrow^d \chi^2(1) \).

3. Data

The data series employed in this paper come from Macaulay (1938), as well as other sources discussed below. They run on a monthly basis, starting in January 1857, and are in the form of monthly averages of quotations in New York City, except as noted otherwise. The last month is December 1879. Thus, the sample comprises 276 consecutive months (\( T = 276 \)).

Several comments about the return series and data summary statistics presented in Table 1 are in order. A typical call loan during the Greenback Era required a collateral in the form of stocks and bonds, usually worth 130% of the loan amount. As a rule, such loans were extended by banks to stock and bond brokers to finance speculative operations or distribution of new securities. The loans were callable by lenders at any time, in which case borrowers expected to return the money within a day. Historical evidence suggests that banks operated diversified rolling portfolios of loans by gradually retiring older loans and arranging for new ones in order to adjust reserves to desired levels. As shown in Table 1, call loans on average commanded a rate of 6%. The maximum quotation of 61% was recorded during the Panic of 1873 in September 1873. Relatively low rates, including the minimum quotation of 1.7% in August 1876, are characteristic of the recessionary years 1874 to 1878.

The term “commercial paper” was generically applied to promissory notes on which merchants and manufacturers borrowed funds short-term for use in the ordinary course of business. Such loans had a typical maturity of 60 to 90 days and, unlike call loans, did not require collateral. Each commercial paper issue was accompanied by a financial statement of the borrower, and historical evidence suggests that only companies with good or excellent credit histories were able to draw regularly on the commercial paper market. As can be seen, commercial paper commanded a small premium over call loans with the average rate being almost 7%. The minimum rate of 3.6% was recorded in August 1876 and July 1878, and a historical maximum at 24% occurred during the Panic of 1857. The rates in the aftermath of the Panic of 1873 (not shown in the table) were also high, ranging from 14% to 16.5%. Thus, the pattern of the commercial paper returns follows the one on call loans.

A novel empirical feature of this research is the use of the return series for first-class bankable paper in Boston. The series comes from Martin (1898), a comprehensive source of historical data on the Boston stock and money markets, which appears to have been overlooked in the literature. The term “first-class bankable paper” refers to high grade bank-endorsed promissory notes with a typical maturity of 90 to 180 days. For every month in the sample, Martin either lists one prevalent rate on the paper during the corresponding month or provides a range of the respective rate quotations (e.g., 9 to 10% in January 1857). In case a prevalent rate is listed, this rate is used as is. In case a range of the quotations is provided, the midpoint of the range is used as an approximation to the relevant rate (e.g., 9.5% in January 1857).

As can be seen in Table 1, loans procured via the Boston first-class bankable paper on average commanded a rate of 6.4%. The pattern of Boston’s rate resembles the one of New York’s quotations with some exceptions. For example, the minimum rate of 3% was recorded in September 1858, when rates in New York were also low but not at a historical minimum. Also, the rate in the aftermath of the Panic of 1873 (not shown in the table) was a relatively high 9%, but the corresponding rate spike was far less pronounced than in New York. However, similarly to commercial paper in New York, a historical maximum at 30% occurred during the Panic of 1857.

The London banker’s bill series comes from the NBER macrohistory database (http://www.nber.org/macrohistory) and represents a discount rate on drafts drawn in British pounds maturing in 90 days. Observe that the American
returns are on average 2.5 to 3 percentage points higher than London’s rate. According to Field (1983), this positive return difference is expected, because the U.S. had access to superior agricultural “technology” due to the country’s relative land abundance, which resulted in a higher economy-wide interest/profit rate. It is also worth noting that in comparison with the Boston paper rate series, London’s rate exhibits less correlation with the New York returns.

Daily gold dollar prices between 1862 and 1878 are obtained from Mitchell (1908) by averaging high and low daily quotes. As can be seen, the gold dollar on average commanded a premium of 27 cents over the legal tender greenback. The highest price, 2.80 greenbacks, was recorded on July 11th, 1864. In comparison, the high quote on “Black Friday,” September 24th, 1869, was a much lower 1.62 greenbacks (not shown in the table), but the price crashed by nearly 20% to 1.33 greenbacks on that day after the Treasury made public its plan to prevent the gold “corner.”

The last data series in Table 1, the railroad stock index, represents a weighted average of railroad stock prices quoted in legal tender dollars, where the weight of a stock is the number of corresponding shares outstanding at the beginning of each year. Macaulay (1938) made no attempt to correct for the change in the composition of the index over the long run, as some publicly traded railroad companies went bankrupt and others entered the market. However, in the context of the empirical application, which uses the three-month returns on the index, the gradual change in its composition is only a minor nuisance.

4. Results

Figure 3 presents a time series of realizations of the stochastic discount factor $M_{t,r}$ computed according to (5). The figure refers to the baseline case, in which the annual risk-free rate is set to 5%, while the expected value $E_0\mathbf{x}_{t,r}$ and the covariance matrix $\Sigma_t$ are computed using all sample data and are held fixed across $t$, under the assumption that the return series are ergodic stationary. It additionally reports an approximate 90%-level confidence interval for each realization of $M_{t,r}$, which is obtained by simulating realizations of $\bar{\mathbf{m}}_t$ from its approximate distribution, as implied by (7). To facilitate the analysis, calendar months with negative realizations are separately listed in Table 2.

Notably, while performing a test of whether equation (9) holds, the value of the test statistic $Q$ is found to be 0.01, which is less than $\chi^2_{90}(1) = 2.71$, supporting the hypothesis that $M_{t,r}$ constructed according to (5) is, indeed, a valid stochastic discount factor.

Now, as Figure 3 demonstrates, there is little evidence that the money market of the Greenback Era admitted arbitrage opportunities on a systematic basis, except possibly during the months listed in Table 2. Realizations of $M_{t,r}$ are positive in a large majority of cases, and instances when an entire confidence interval is negative (15 months overall) comprise less than 7% of the total 216 months of the Greenback Era. Moreover, focusing explicitly on the timing of the negative realizations, it is easy to see that a predominant fraction of such cases pertain to a major market disturbance such as, specifically, the Gold Corner of 1869 and the Panic of 1873. The very few remaining instances that cannot be linked to the Gold Corner of 1869 or the Panic of 1873, such as, for example, October 1864, appear to have been short-lived.

It is reasonable to ask if the above results are sensitive to the assumption about the value of the risk-free rate, especially since this rate is the only input parameter not estimated from the data. To illustrate that the results are, in fact, robust to the assumption, Figure 4 plots realizations of $M_{t,r}$ for three different levels of the rate, namely, $r^f = 4\%$, $r^f = 5\%$ (the baseline case), and $r^f = 6\%$. To avoid clutter in the figure, confidence intervals are not reported. As can be seen, the three series are similar and, importantly, are in close agreement as to the timing of negative realizations of $M_{t,r}$ (particularly, in the aftermath of the Gold Corner of 1869 and the Panic of 1873). Qualitatively, the results for $r^f = 4\%$ and $r^f = 5\%$ are nearly identical. What may be less obvious from the figure is that increasing the risk-free rate tends to dampen fluctuations of $M_{t,r}$, so that slightly fewer instances of negative realizations of $M_{t,r}$ are detected.

10The first half of 1864 was marked by a rapidly rising gold price. Eventually, the Congress intervened on June 17th by passing a bill to ban gold trading on an open market. However, the bill was repealed on July 2nd in the wake of civil unrest in New York City.

11This event in U.S. financial history is commonly referred to as “the Gold Corner of 1869.”

12To show that a market admits no arbitrage it is sufficient to find just one strictly positive stochastic discount factor.

13In fact, the negative realizations of $M_{t,r}$ in those two episodes should be viewed with some scepticism, because the corresponding computations may be based on quotations from few actual transactions.
in the case of \( r^f = 6\% \). Although this result strengthens the conclusion that the money market did not systematically admit arbitrage opportunities, the rate at 6% may be overstating the true risk-free rate.

Lastly, one may wonder if the results are robust to the restriction of the constant expected value \( E_0 x_{t, \ell} \) and covariance matrix \( \Sigma_t \) across \( t \), which is implied by the unverifiable assumption of ergodic stationarity. While a comprehensive analysis without imposing ergodic stationarity is difficult,\(^{14}\) to show that relaxing the restriction is unlikely to qualitatively change the results, the realizations of \( M_{t, \ell} \) for \( r^f = 5\% \) were recalculated using only data between months \( t_0 \) and \( t - 1 \) to estimate each \( E_0 x_{t, \ell} \) and \( \Sigma_t \) (so that the estimates vary with \( t \)). The series along with the approximate 90%-level confidence intervals is plotted in Figure 5.

The new \( M_{t, \ell} \) series in Figure 5 tends to have more pronounced fluctuations than the baseline series (Figure 3). Expectedly, as each \( E_0 x_{t, \ell} \) and \( \Sigma_t \) is now estimated on a subsample of size less than \( T \), the confidence intervals become wider, which makes inferencing on \( M_{t, \ell} \) particularly difficult for early years of the Greenback Era. Nevertheless, except in very few instances (e.g., December 1862 and April–June 1864), the new and baseline series closely agree on the timing of negative realizations. Qualitatively, Figures 3 and 5 tell the same story that arbitrage opportunities appear not to have been systematic and, if such opportunities ever existed, they only occurred during episodes of a major financial market disturbance.

To summarize, the existing evidence strongly suggests that the money market of the Greenback Era did not admit arbitrage opportunities on a systematic basis, except possibly around the times of major market shocks such as the Gold Corner of 1869 and the Panic of 1873. While arbitrage opportunities in the few other instances cannot be ruled out entirely, they were apparently short-lived. These results provide a verification of the critical “no arbitrage” assumption of Calomiris (1988) that, in part, distinguishes his analysis from the conceptual framework adopted by Friedman and Schwartz (1963). Thus, the resumption expectations model of Calomiris appears to provide a more plausible explanation of the interest rate paradox than the capital flow argument of Friedman and Schwartz.

5. Conclusion

The paradox of low nominal interest rates amid rampant Civil War inflation and of relatively high rates during prolonged deflation after the war is one of the most striking phenomena of the Greenback Era (1862-1878) in U.S. financial history. The paradox had long been known to economic historians, but an adequate explanation eluded the profession for many decades. The two leading competing theories – the resumption expectations model of Calomiris (1988) and the capital flow argument of Friedman and Schwartz (1963) – provide different explanations of the paradox and are irreconcilable because of their opposite approaches to the existence of unexploited arbitrage opportunities in the financial market. Calomiris rules out such opportunities at the outset, while the explanation offered by Friedman and Schwartz is, strictly speaking, valid only if the opportunities persisted, as first noted by Roll (1972).

This paper reinvestigates the interest rate paradox, focusing explicitly on the critical issue of whether the money market of the Greenback Era admitted arbitrage opportunities. Knowledge of whether or not such opportunities existed on a systematic basis is crucial for determining which of the two explanations of the paradox is more plausible. Also, it helps shed further light on the hotly debated topic of efficiency of American financial markets of the nineteenth century. The paper contributes to the literature by analyzing the behavior of short-term money market rates rather than long-term bond yields, which have already been extensively studied, and by looking for evidence of arbitrage opportunities in the interest rate data rather than performing a yet another analysis of gold market efficiency.

The paper has other novel features that distinguish it from the existing literature on the Greenback Era. Most importantly, it uses mathematical tools of modern asset pricing theory to develop a methodology for identifying the presence of unexploited arbitrage opportunities in financial data. The only substantive economic restriction is one of frictionless markets in that the money market investors are allowed to sell assets short and transaction fees are presumed to be negligibly small. Unlike in the prior research, no specific assumption is made here about attitudes of the investors toward risk, they only must strictly prefer more wealth to less. Given that, the first fundamental theorem of asset pricing implies that the absence of arbitrage opportunities is, effectively, equivalent to the existence of specific data relationships, which can be verified. The essence of the methodology is providing a way to aggregate the data by

\(^{14}\)If a series has an arbitrary form of time dependence, it is impossible to consistently estimate population moments from sample data, because the law of large numbers is not guaranteed to hold.
constructing a series of realizations of a stochastic discount factor having a particularly convenient form. Finding just one positive stochastic discount factor is sufficient to rule out arbitrage, and checking positivity of the realizations of the constructed discount factor is straightforward. A novel empirical aspect of the analysis is that it simultaneously employs four different money market return series, one of which – the Boston first-class bankable paper series – appears to have been overlooked in the prior research.

Applying the methodology, the paper finds that the observable return series strongly suggest that the money market of the Greenback Era did not admit arbitrage on a systematic basis, except possibly around the times of major market shocks such as the Gold Corner of 1869 and the Panic of 1873. While arbitrage opportunities in the few other instances cannot be ruled out entirely, they were apparently short-lived. Additional analysis shows that this conclusion is invariant to different assumptions about the value of the risk-free interest rate and suggests that the conclusion may also be robust to relaxing the technical condition of ergodic stationarity.

These results complement the findings of Calomiris and others that the gold market of the Greenback Era was efficient, suggesting that an overall description of the American financial market of the second half of the nineteenth century as a market with no systematic arbitrage opportunities is more appropriate than the description proposed by Field (1983). Moreover, they imply that the resumption expectations model of Calomiris likely provides a more plausible explanation of the interest rate paradox than the capital flow argument of Friedman and Schwartz.

Acknowledgment

I am grateful to John James and T. Wake Epps for thoroughly reviewing an early draft and to Joydeep Bhattacharya, Marco Cagetti, Sergey Khovansky, Harvey Lapan, Ronald Michener, John Pepper, and Monika Piazzesi for helpful comments. I also thank Associate Editor Edward Nelson and an anonymous referee for thoughtful suggestions.

References

Table 1: Summary statistics for price and return series

<table>
<thead>
<tr>
<th>Data series</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call loans, %(^a)</td>
<td>6.15</td>
<td>4.63</td>
<td>1.70</td>
<td>61.23</td>
</tr>
<tr>
<td>Commercial paper, %(^a)</td>
<td>6.85</td>
<td>2.31</td>
<td>3.60</td>
<td>24.00</td>
</tr>
<tr>
<td>Boston bankable paper, %(^a)</td>
<td>6.43</td>
<td>2.57</td>
<td>3.00</td>
<td>30.00</td>
</tr>
<tr>
<td>London banker’s bills, %(^b)</td>
<td>3.62</td>
<td>1.84</td>
<td>0.91</td>
<td>9.75</td>
</tr>
<tr>
<td>Gold dollar, greenbacks(^c)</td>
<td>1.27</td>
<td>0.27</td>
<td>1.00</td>
<td>2.80</td>
</tr>
<tr>
<td>Railroad stock index, dollars(^d)</td>
<td>30.82</td>
<td>9.39</td>
<td>12.83</td>
<td>45.20</td>
</tr>
</tbody>
</table>

Notes: The table provides summary statistics for the price and return series used in the empirical analysis. All summary statistics are computed using monthly data between January 1857 and December 1879, with the exception of the gold dollar price series, in which case the statistics are computed using daily data from 1862 to 1878. The data for the price and return series are taken from Martin (1898), Mitchell (1908), Macaulay (1938), and the NBER macrohistory database (http://www.nber.org/macrohistory, last accessed on June 30, 2010).

\(^a\) Data series comprises net annualized percentage returns on loans made in legal tender dollars.

\(^b\) Data series comprises net annualized percentage returns on loans made in British pounds.

\(^c\) Data series comprises daily prices of one gold dollar in terms of greenbacks between 1862 and 1878 (a total of 5,170 data points).

\(^d\) Data series comprises values of the railroad stock index in legal tender dollars.

Table 2: Months with negative realizations of stochastic discount factor

<table>
<thead>
<tr>
<th>Month</th>
<th>(\hat{M}_{t,r})</th>
<th>Month</th>
<th>(\hat{M}_{t,r})</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 1864</td>
<td>-0.39</td>
<td>November 1871(^*)</td>
<td>-0.52</td>
</tr>
<tr>
<td>October 1864(^*)</td>
<td>-0.82</td>
<td>December 1871(^*)</td>
<td>-0.47</td>
</tr>
<tr>
<td>March 1869</td>
<td>-0.13</td>
<td>September 1872</td>
<td>-0.31</td>
</tr>
<tr>
<td>April 1869(^*)</td>
<td>-0.35</td>
<td>October 1872(^*)</td>
<td>-1.35</td>
</tr>
<tr>
<td>July 1869(^*)</td>
<td>-0.50</td>
<td>November 1872(^*)</td>
<td>-1.95</td>
</tr>
<tr>
<td>August 1869(^*)</td>
<td>-0.57</td>
<td>January 1873(^*)</td>
<td>-0.38</td>
</tr>
<tr>
<td>October 1869(^*)</td>
<td>-0.97</td>
<td>February 1873</td>
<td>-0.12</td>
</tr>
<tr>
<td>November 1869(^*)</td>
<td>-1.36</td>
<td>March 1873</td>
<td>-0.18</td>
</tr>
<tr>
<td>December 1869(^*)</td>
<td>-0.73</td>
<td>October 1873(^*)</td>
<td>-3.83</td>
</tr>
<tr>
<td>January 1870</td>
<td>-0.35</td>
<td>November 1873(^*)</td>
<td>-3.28</td>
</tr>
<tr>
<td>October 1871</td>
<td>-0.21</td>
<td>December 1873(^*)</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

Notes: The table lists calendar months with negative computed realizations of the stochastic discount factor (the \(\hat{M}_{t,r}\) series) and the corresponding values of \(\hat{M}_{t,r}\) for the baseline case, in which the risk-free rate is set to 5% and the returns are assumed to be ergodic stationary.

\(^*\) The entire approximate 90%-level confidence interval for \(\hat{M}_{t,r}\) is negative in this month.
Figure 1: Wholesale price index (WPI).
Notes: The figure plots monthly values of the wholesale price index (WPI) in the U.S. from January 1861 to December 1878 (the base year of the WPI is 1914). The data for the WPI are taken from the NBER macrohistory database (http://www.nber.org/macrohistory, last accessed on June 30, 2010).

Figure 2: Commercial paper rate, %.
Notes: The figure plots monthly averages of annualized commercial paper rate quotations in New York City from January 1861 to December 1878. The data for the rate are taken from Macaulay (1938).
Figure 3: Stochastic discount factor in baseline case.
Notes: The figure plots computed monthly realizations of the stochastic discount factor (the $\hat{M}_{t,r'}$ series) and the corresponding approximate 90%-level confidence intervals for the baseline case, in which the risk-free rate is set to 5% and the returns are assumed to be ergodic stationary. The $\hat{M}_{t,r'}$ series is represented by the solid black curve. The confidence intervals are represented by the shaded gray area.

Figure 4: Stochastic discount factor under different risk-free rates.
Notes: The figure plots computed monthly realizations of the stochastic discount factor (the $\hat{M}_{t,r'}$ series) under different assumed values of the risk-free rate ($r^f$). The case of $r^f$ at 4% is represented by the dotted curve. The case of $r^f$ at 5% is represented by the solid curve. The case of $r^f$ at 6% is represented by the dashed curve. In each case, the returns are assumed to be ergodic stationary.
Figure 5: Stochastic discount factor under time-varying return moments.

Notes: The figure plots computed monthly realizations of the stochastic discount factor (the $\hat{M}_{t'}$ series) and the corresponding approximate 90%-level confidence intervals for the case of time-varying return moments, when the assumption of the ergodic stationarity of the returns is relaxed. The $\hat{M}_{t'}$ series is represented by the solid black curve. The confidence intervals are represented by the shaded gray area.