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Endogenous formation and collapse of housing bubbles

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Endogenous formation and collapse of housing bubbles

Abstract
This paper develops an agent-based spatial model of the housing market. This paper develops an analytical framework and an agent-based spatial model of the housing market. We show that low down payment requirement will cause housing bubbles. With low down payment requirement, a small decrease in the housing price will cause mortgage rate to rise in response to the lowering value of collateral. The rising mortgage rate will further suppress demand and turn the market into a downward spiral. The agent-based model is based on our interviews with local real estate agents. The exploratory work in this paper will help us better understand the housing market, give policy advice, and eventually prevent another damaging housing bubble.

Keywords
housing market, housing bubble, agent-based model

Disciplines
Economics
Endogenous Formation and Collapse Of Housing Bubbles

Jiaqi Ge

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Abstract

This paper develops an agent-based spatial model of the housing market.

1 Introduction

The U.S. housing price bubble started around year 2000. At the height of the housing bubble, mortgage loans could be easily obtained on very lenient terms. Housing price has doubled in just a few years, and when it burst, price has dropped by nearly a third nation wide, and more than a half in the sand states (Arizona, California, Florida, and Nevada). The economy went into the most severe recession since the big depression in the 1930s.

What could have caused the damaging bubble? Some blame speculators; Others blame government sponsored entities like Freddie Mac and Fannie Mae for extending loan to riskier (sub-prime) lenders; Still others blame financial institutions for relentless lending. However, many agreed that the bubble has to do with lenient financing. However, how exactly does lenient financing lead to a housing bubble remains unclear.

This paper proposes a structural and an agent-based model to analyze the cause of a housing bubble.

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1We would like to thank Tom Randall Real Estate Team, Hunziker & Associates, and an anonymous realtor for their helpful comments and inputs.
We found that lenient financing does cause bubbles in the housing market. However, the main cause is not relentless lending, inclusion of riskier buyers, or speculative demand that are associated with leniency, as many have speculated. Even though those factors might have exacerbated the process. We believe that the key lies in the collateral, which value is related with housing price. Hence in both structural and agent-based models of ours, even without risky lenders and irrational lending, housing bubbles seem inevitable under lenient financing.

2 Review of Literature

There are mainly three groups of papers in the housing literature. The first group focuses on speculative behavior in the housing market. Examples are Shiller[11] and Riddel [10]. The second group of papers, mostly done after the U.S. housing crises, look at the relationship between the housing market and sub-prime lending. Examples are Goetzmann et al. [5] and [1]. The third group studies land use choice and housing supply elasticities. Examples are Goodman et al. [7] and Glaeser et al [4]. However, according to a review by Mayer [8], the current literature is not satisfying in explaining and predicting a housing bubble. While the housing market makes up a significant part of national economy and household wealth, far less economic research has been done on the property market than on the stock market, the bond market, or the foreign-exchange market.


John Geanakoplos is among the first few to acknowledge the role leverage or equivalently, collateral rate plays in the business cycle. In his paper on leverage cycle [2], he looks at how a small shock in one sector can cause wide spread crises across sectors which payoffs are independent, because the most highly leveraged investors who are forced out of the market first happen to be the most optimal ones. In another paper by Thurner et al. [12], the authors argue that it is leverage that causes fat tails and clustered volatility. They use an agent-based model to show that even a small negative shock in the market will trigger large price drop, because leveraged investors are forced to sell the assets to stay within the maximum leverage rate.

Our paper also focuses on the collateral rate, which is the inverse of leverage. We are going to show
how leverage or collateral in the housing market can cause regular business cycles. We will start
with the analytical model and then move on to the agent-based model.

3 An Analytical Demonstration of Housing Bubbles

3.1 Collateral and Mortgage Rate

The return on the mortgage loan equals the exogenous market return.

\[ pr(r, c) \cdot c + (1 - pr(r, c)) \cdot (1 + r) = 1 + \tilde{i} \tag{1} \]

where \( pr(r, c) \) is default or short-sell probability of a lender at mortgage rate \( r \), \( c \) is the collateral value per dollar of loan, and \( \tilde{i} \) is exogenous market return. The above equation defines the relationship between collateral rate and mortgage rate. The short-sell or default probability at \( r \), \( pr(r, c) \), could vary across borrowers by their credibility, down payment, and the characteristics of the property. So each loan has a different mortgage rate because each is associated with different default or short-sell risk. The collateral rate \( c \) is defined as,

\[ c := \min \left\{ \frac{V(\text{asset})}{V(\text{mortgage})}, 1 + \tilde{i} \right\} = \min \left\{ \frac{1}{1 - \frac{\text{down}}{p}} + \frac{\dot{p}/p}{(1 - \frac{\text{down}}{p})}, 1 + \tilde{i} \right\} \tag{2} \]

where \( p \) is original purchase price and \( \dot{p} \) is change in price. For simplicity, we ignore subsequent monthly payments and assume that the amount of loan equals purchase price minus down payment. Collateral rate has to be bigger than or equal to zero (positive collateral) and less than or equal to \( 1 + \tilde{i} \) (full collateral). When collateral rate equals \( 1 + \tilde{i} \), mortgage rate is lowest and equals the exogenous market rate. Take total differentiation with respect to \( c \) in the Eq 1 gives us the relationship between mortgage rate \( r \) and collateral rate \( c \),

\[ \frac{dr}{dc} = \frac{\frac{\partial pr}{\partial c} \cdot (1 + r - c) - pr}{1 - pr - \frac{\partial pr}{\partial r} \cdot (1 + r - c)} \tag{3} \]

The nominator is negative. The sign of the denominator is undetermined. The denominator is positive when \( pr(r, c) \) is relatively small and \( \frac{\partial pr}{\partial r} \) is not too large. Since \( c = \min \left\{ \frac{1}{1 - \frac{\text{down}}{p}} + \frac{\dot{p}/p}{(1 - \frac{\text{down}}{p})}, 1 + \tilde{i} \right\} \), \( \frac{dr}{dc} \leq 0 \) means \( \frac{dr}{dp} \leq 0 \). There exists a negative relationship between interest rate and change in price when \( c \) is within certain range. Under normal lending criteria, \( c \) usually exceeds \( 1 + \tilde{i} \). The value of the loan is usually smaller than the value of the house because of the initial down payment and the following monthly payments. Hence in normal times, we do not observe the mortgage rate changes very often, except when it’s linked to \( \tilde{i} \), the exogenous baseline rate and \( \tilde{i} \) changes. The lenient lending criteria, however, requires very little down payment and monthly payment at the
beginning. Hence with lenient lending criteria, collateral rate is more likely to fall in the range where a negative relationship exists between mortgage rate and housing price. A fund manager may well know that if she raises the rate, the homeowner is more likely to sell the property or payoff the loan earlier by refinancing. But it is not a problem to the fund manager as long as the zero net return condition still holds. What the fund manager does not consider is if all funds like hers raise the mortgage rate at the same time, it will induce more selling, and more selling will make the housing price and thus the collateral rate lower, which will in turn require an even higher rate. Individual funds are too small to affect the market price by raising its own mortgage rate. It rationally raise the rate to protect itself from increased default risk. However, if every fund does so, the aggregate effect of raising rate together will change the market condition and increase the default risk even more.

3.2 Demand for Housing

3.2.1 Non-speculative Housing Demand

Since the total cost of a house is \((1 + r) \cdot p\), non-speculative net housing demand, \(D^n\), is decreasing in both \(r\) and \(p\).

\[
\frac{\partial D^n}{\partial p} < 0 \text{ and } \frac{\partial D^n}{\partial r} < 0
\]  

(4)

Zero net demand therefore requires that mortgage rate \(r\) and housing price \(p\) move in the opposite direction.
### 3.2.2 Speculative Housing Demand

Speculative net housing demand, \( D^s \), is an increasing function of past housing price appreciation.

\[
\frac{\partial D^s}{\partial \dot{p}} > 0
\]  

(5)

When price has been increasing, speculative net demand is positive; when price has been stabilized, speculative net demand is zero; and when price has been decreasing, speculative net demand is negative, meaning there is more supply than demand.

### 3.3 The Phase Diagram

Figure 2 shows the phase diagram of the system. When the exogenous market rate \( r \) decreases, as

![Figure 2: Phase Diagram of Housing Price (p) and Mortgage Rate (r)](image)

an response mortgage rate decrease from \( r_0 \) to \( r_1 \). Housing price starts to climb in response to the reduced mortgage rate. The increase of price does not stop when it reaches \( p_1 \), because since price has been increasing, there is still positive speculative demand at \( p_1 \). Price will keep rising until total demand (speculative and non-speculative) equals zero, \( D^n + D^s = 0 \). Then price will start to decrease and it will again go pass \( p_1 \) due to the speculative demand. If we only have the above dynamics, housing price will fluctuate around its new equilibrium \( p_1 \). And the extend of the fluctuation depends on the number of investors relative to the number of non-investors.

However, there is a second, more substantial dynamics. Whether this dynamics will happen depends on the level of down payment. The dashed line above the non-speculative demand curve is the curve under which \( \dot{r} = 0 \). A higher down payment requirement means that the dashed curve is further
away from the demand curve. If speculative demand drives the price above the dashed line (as is the case in Figure 2), the region where creditors will respond to decreasing price with increasing mortgage rate, the second dynamic is triggered and housing price will no longer fluctuate at the high level, it collapses instead. Once in the region above the dashed curve, price will keep falling and mortgage rate keep rising until it’s under the dashed curve again. Then mortgage rate stop increasing and price is again stabilized, but at a much lower level. After price has stabilized, mortgage rate will go back to the low $r_1$ level. And the same cycle start to repeat itself. Under lenient financing, due to the low down payment requirement, the $\dot{r} = 0$ curve (dashed) is very close to the non-speculative demand curve (solid). As a result, with even a very small number of investors, housing price will fall into the upper right region, collapse, and has repeated cycles afterwards.

4 The Agent-based Model

One of the purposes of the agent-based model is to expand the basic dynamics in the analytical model to a richer representation of real-world scenarios. To start, our housing market sits on a two dimensional landscape that contains multiple regions. Our general model has a landscape that is a five by five square, as shown in Figure 3. Each region has exogenous attributes called location quality, and endogenous attributes called neighborhood quality. The numbers in Figure 3 is the exogenous location quality that we use in for the model.

The model has five types of market participates: the real estate agent, the developer, buyers, homeowners, and the bank. We further distinguish buyers and homeowners as investors and non-investors. Investors buyers buy a property in hope of profiting from housing price appreciation. Regular buyers, on the other hand, obtain utility from living in the house. Figure 4 is a class diagram of the model. It outlines the model’s class structure and demonstrates relationships between
different types of agents.

![Class Diagram For The Housing Model](image)

In each period, new buyers enter the market in search of a house. At the same time, existing homeowners decide whether to put their houses on the market for sale. One period in the model represents a month in real time. The model works as follows:

**Step 1** At the beginning of period \( t \),
- **The Real Estate Agent**: announces housing Prices for all regions in the previous period.
- **The Bank**: announces mortgage rate and lending criteria.

**Step 2** **The Developer**: given prices in the last period, builds new houses in each region.
- **A Homeowner**: given price in the last period and mortgage terms, decides whether to default on the house. If chooses to default, the property goes into foreclosure and homeowner exits the market. Otherwise, chooses whether to sell the property. If chooses to sell, submits an asking price on the house. Otherwise, make monthly payment and enter period \( t + 1 \) as a homeowner.
- **A Buyer**: searches a subset of all regions, submits a bid for an affordable house that gives her the highest utility (non-investor) or net return (investor).

**Step 3** **The Real Estate Agent**: collects all the bids and asks, and settles the final market price for each region.
Step 4 **The Developer**: sells houses according to the market price. Unsold units become housing stock, and are put for sale in period $t+1$.

**The Bank**: sells foreclosures according to the market price.

**A Homeowner**: if the asking price is lower than the market prices, sells her property and exits the market. Otherwise enter period $t+1$ as a homeowner.

**A Buyer**: if her bidding price is higher than the market price, buys a house and enters period $t+1$ as a homeowner. Otherwise choose whether to wait for another period or exit the market.

Step 5 End of period $t$. Enter period $t+1$.

Figure 5 is an activity diagram of the housing model. It illustrates how market participants interact. A red line represents an information flow, and a black line represents an action flow.

## 5 Results

We start from an empty land. As the model runs the land is gradually filled (Each run of the simulation is uniquely linked to a random seed.). The first 150 periods are deleted as burn-outs. Below we show the simulation results for 500 periods, which represents 500 months or around 42 years in real time. In the first 100 out of 500 periods, both runs have a normal lending criteria. Then starting from period 101, one is shifted to a lenient criteria while the other remains under normal lending criteria. Details of normal and lenient lending criteria are listed in Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>mortgage rate ($c^1 = 1 + \bar{i}$)</th>
<th>mortgage rate ($c &lt; 1 + \bar{i}$)</th>
<th>minimum down payment</th>
<th>maximum debt to income ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.33</td>
</tr>
<tr>
<td>Lenient</td>
<td>0.01</td>
<td>0.08</td>
<td>0.01</td>
<td>0.50</td>
</tr>
</tbody>
</table>

1 collateral rate, equals housing price over outstanding loan.

Each lending criteria is composed of three components: mortgage rate, minimum down payment, and maximum debt to income ratio. The last is simply the ratio between monthly payment and monthly income, as we assume away any other forms of household debt. In the case of lenient lending, mortgage rate is adjustable after the first two years. Mortgage rate can rise if the collateral rate $c$ is less than one (if housing price is less than outstanding loan). Figure 6 shows the housing price under normal and lenient lending criteria.

Under normal lending criteria with fixed mortgage rate, price is quite stable, although it has small
Figure 5: The Activity Diagram For The Housing Model
perturbations around the mean; While under a lenient lending criteria, we see regular large boom-and-bust cycles in the housing price. Figure 7 is an anatomy of housing bubbles, in which we shows how housing price corresponds with other market indicators.

Total number of foreclosure corresponds with the collapse of a housing bubble. Total foreclosure is highest when price is decreasing. It happens when the rate has jumped and homeowners decide to put their properties for sell to avoid paying the high rate. Since a large number of homeowner is selling, the average time on market before a property is sold is longer. When a household cannot sell a house in a given time, it chooses to default. Or, for an investor homeowner, if the price is so low that the outstanding loan she owns is bigger than the value of the house plus default cost (so the house is deeply under water), she also chooses to default.

The leverage level, defined as outstanding loan over property price and also equals the inverse of collateral rate, moves in the opposite direction as housing price. Since the house is also used as the collateral for the loan, when housing price drops, the value of collateral also drops and leverage level increases.

Under normal lending criteria, mortgage rate is fixed at 5% and does not depend on the housing price. Under lenient lending criteria, however, mortgage rate is adjustable after two years and
(a) Housing Price and Number Of Foreclosures

(b) Housing Price and Leverage Level

(c) Housing Price and Mortgage Rate

(d) Housing Price and Total Return On Loans

Figure 7: An Anatomy of Housing Bubble
depends on the housing market condition. Like we have shown in Figure 1, when the collateral rate fall below 1, a negative relationship exists between housing price and mortgage rate. To maintain a certain return on the loan, mortgage rate has to rise when housing price starts to fall. Our simulation results confirm that average mortgage rate is higher when housing price is decreasing and lower when housing price is increasing.

Aggregate return on the loans rises when price rises, and it drops when price drops. However, there is a lag between the turning point of the housing price and the time when total return on loans start to fall. It’s not until the price has been falling for a while that total return starts to fall. Moreover, the total returns under lenient lending is higher than under normal lending until the bubble burst, because the increase in the volume of loans more than offset the rate earned on each loan. So it is not necessarily irrational for financial institutions to adopt a lenient lending criteria at the beginning. What those financial institutions do not foresee is by doing so, the aggregate risk in the system is adding up.

6 Conclusions

This paper develops a analytical and an agent-based spatial model of the housing market. We have shown, both in the analytical and agent-based models, that collateral or leverage plays is the key factor in the formation of a housing bubble. Under lenient financing which requires zero to very little down payment, mortgage rate has to rise in response to a lower housing price, and thus lower collateral rate to maintain the return on the loan. While an individual loan is too small to affect the market condition, all loans raising rates together when the price starts to fall will have tremendous effect and kick the market into a downward spiral.

We incorporated in our agent-based model important aspects of the housing market that are largely ignored in the existing housing literature, such as the spatial aspect of the market, the role of time on market on buyer’s decision making, the inclusion of both investor and non-investor buyers, and homeowner’s default behavior. Our model design is based on our interviews with real estate agents. We are show that we also able to generate in the simulation similar patterns as in the real data.

Simulation results from the agent-base model show that lenient financing leads to housing bubbles and wide-spread foreclosures when housing prices start to fall. Leverage level, or the inverse of collateral, moves inversely with housing price by construction. Moreover, when we first implement leniency the average mortgage rate drops, because a low mortgage rate in the first few years is part of the adjustable mortgage rate (AMR) included in lenient lending terms. However when market collapses, some lenders fail to get refinance with the low rate because they can’t provide enough
collateral as the value of the property has shrunk. As a result, the average mortgage rate raises as price plummets, even though the lenient lending term is still in place. The homeowners who cannot afford the higher rate and fail to sell the property in time are forced into foreclosures.

After the crises, people attribute the housing bubble and collapse to aggressive lending and securitization of loans by financial institutions. Many argue that the twisted incentives cause by securitization prevented the financial institutions from behaving rationally. Interestingly, our simulation results show that there is a lag between the turning point of the housing price and the time when total return on loans start to fall. It’s not until the price has been falling for a while that total return starts to fall. Therefore, even if the loans are not securitized and the financial institutions are able to respond to falling return promptly, they are not going to prevent the bubble from collapsing. Moreover, the total returns on the loans before the bubble collapses could be higher because the increase in the volume of loans more than offset the rate earned on each loan. So it is not necessarily irrational for financial institutions to adopt a lenient lending criteria and expand lending. What those financial institutions do not consider is the effect of its own behavior on the aggregate risk in the market.

To sum up, we have shown in both a analytical model and an agent-base model that lenient lending with little down payment and monthly payment in the beginning will cause endogenous formation and collapse of a bubble in the housing market. Even without irrational behavior, speculation, or external shock in the fundamentals, a housing bubble will arise with lenient financing. The reason is with lenient financing, collateral rate is more likely to fall below one, where mortgage rate respond negatively to collateral rate. Future research includes a case study on the housing market of Washington DC. We will test our agent-based model’s empirical performance and its predicting power using real estate transaction data for D.C.. We are also going to visualize the model’s spatial dynamics using animations and GIS mapping.

References


Appendices

A The Structural Framework of the Housing Market

We are going to present a structural framework of the housing market. We will show for each type of agent their objectives, choice variables, and available actions. The structural framework corresponds to the computer code that is used to generate simulation results.

A.1 Non-investor Buyer

Here we omit the notation \( t \) for time period.

A.1.1 Objective

For non-investor household \( i \), its objective is,

\[
\max_{g \in G^i} U^i (\text{nbhd}^g, \text{loc}^g, (1 - \text{lev}_i^g))
\]

subject to

\[
\text{lev}_i^g \leq \text{lev}
\]

wealth, \( i \geq \text{down} \cdot p^g \)

where \( G^i \) is the available choice set of regions for household \( i \), including the option of not buying a house or \( g=\text{null} \). Assume \( U(\text{null})=0 \). \( \text{lev}^g \) is the leverage the household takes on if it decides to buy a house in region \( g \), formally,

\[
\text{lev}_i^g := \frac{\text{mp}(p^g, r)}{y^i}
\]

where \( y^i \) is income of household \( i \).

A.1.2 Action

A non-investor buyer \( i \)'s action at period \( t \), \( a^i_t \), is,

\[
a^i_t = \begin{cases} 
\text{bid}^i_t (g(i)) = (1 - \text{margin} + \delta \cdot \text{TOM}_i) \cdot P^g_{t-1} & \text{if TOM}_i < T \\
\text{exit market} & \text{if TOM}_i \geq T
\end{cases}
\]

where \( g(i) \) is the region chosen by household \( i \) at time \( t \), \( \text{TOM}_i \) is household \( i \)'s time on market, \( \delta \) is the percentage increase in price each period the buyer’s bid is not accepted, \( P^g_{t-1} \) is the price
in the last period. Once a non-investor buyer has picked up the region that gives her the highest utility in her choice set, she then puts a bid for a house in that region. A non-investor buyer’s bidding price depends on the number of periods she has been on the market. The longer the buyer has been on the market, the more impatient she is, and the higher the bidding price. A buyer has a maximum waiting time of $T$ periods. A buyer will continue to increase her bid until the time she has been on the market exceeds her maximum waiting period ($t \geq T$).

A.2 Investor Buyer

A.2.1 Objective

For investor household $j$, its objective is,

$$\max_{g \in G^j} \quad ER^j(g)$$

subject to

$$lev^g_j \leq \overline{lev}$$

$$wealth_j \geq down \ast p^g$$

where $G^j$ is the available choice set of regions for investor household $j$, including the option of not buying a house or $g$=null. $lev^g$ is the leverage the household takes on if it decides to buy a house in region $g$, defined the same way as in Eq 7. $ER^j$ is investor buyer $j$’s expected net return on region $g$. Assume $ER^j(null) = 0$.

$$ER^j(g) = ER^j\left(p^g_{t-\text{lag}}, p^k_{t-\text{lag}}, r, cost^b; \omega_j\right)$$

$$\forall k \in N^g \text{ and } \forall \text{lag} = 1, 2, ..., \overline{\text{lag}}$$

where $N^g$ is region $g$’s neighboring regions. Investor $j$’s expected return on a housing investment in region $g$ depends on prices in region $g$ in the past. It also depends on prices in region $g$’s neighboring regions in the past. $\overline{\text{lag}}$ is the maximum length of memory a household has about prices in the past. The parameter $\omega_j$ is the weight investor $j$ put on the region itself relative to its neighboring regions in the formation of expected returns. The expected net return also depends on the mortgage rate, $r$, and the cost of keeping a house (before the purchase is made), $cost^b$, which includes property tax, transaction cost, depreciation, and (minus) rental income.

A.2.2 Action

Investor buyer $j$’s action at period $t$ is,

$$a^j_t = \begin{cases} 
\text{bid}^j_t(g(j)) = \left(1 + \frac{R^j(g)}{12}\right) \cdot P^g_{t-1} & \text{if } R^j(g) > 0 \\
\text{exit market} & \text{if } R^j(g) \leq 0
\end{cases} \quad (11)$$
where \( g(j) \) is the region chosen by household \( j \), and \( R^j(g) \) is investor \( j \)'s expected net return on region \( g \).

## A.3 Non-investor Homeowner

### A.3.1 Action

A non-investor homeowner \( i \)'s action at period \( t \), \( a^i_t \), is,

\[
a^i_t = \begin{cases} 
\text{hold property} & \text{if } pr \geq pr(r, c) \\
\text{ask}^i_t(g(i)) = (1 + \text{margin} - \delta \cdot \text{TOM}_i) \cdot P^g_{t-1} & \text{if } p < pr(r, c) \text{ and } \text{TOM}_i \leq T \\
\text{go into foreclosure} & \text{if } \text{TOM}_i > T 
\end{cases} 
\]

where \( pr \) is the random number uniformly distributed between zero and one, \( pr(r, c) \) is the probability that the household is going to sell her house. It is a function of the mortgage rate \( r \) and collateral rate \( c \). The latter is a ratio between housing price and outstanding mortgage. \( g(i) \) is the region where household \( i \)'s property. \( \text{TOM}_i \) is household \( i \)'s time on market, \( \delta \) is the percentage decrease in price each period the buyer’s bid is not accepted, and \( T \) is the maximum number of time on market.

## A.4 Investor Homeowner

### A.4.1 Objective

For investor household \( j \), its objective is,

\[
\max_{\text{hold, sell}} \max \{ ER^j, 0 \} \tag{13}
\]

where \( ER^j \) is investor buyer \( j \)'s expected net return on the property it owns.

\[
ER^j = ER^j \left( \overline{P_{t-\text{lag}}, P_{t-\text{lag}}, r, \text{cost}_a; \omega_j} \right) \tag{14}
\]

\( \forall k \in N^g \) and \( \forall \text{lag} = 1, 2, ..., \overline{\text{lag}} \)

It is the same as in Eq 10 except that here we use the cost after purchase, \( \text{cost}_a \) instead of cost before purchase, \( \text{cost}_b \) as in Eq 10. Cost such as transaction cost is sunk once the purchase has been made, hence is not counted in the cost after purchase.
A.4.2 Action

Investor buyer $j$’s action at period $t$ is,

$$a^j_t = \begin{cases} 
\text{hold the property} & \text{if } R^j(g) \geq 0 \\
\text{ask } i \text{ at } (g(j)) = \left(1 + \frac{R^j(g)}{12}\right) \cdot P^j_{t-1} & \text{if } R^j(g) < 0 \text{ and } P^j_{t-1} \geq V(\text{loan}) - \text{default cost} \\
\text{go into foreclosure} & \text{if } P^j_{t-1} < V(\text{loan}) - \text{default cost}
\end{cases}$$

(15)

where default cost is the cost associated with default, such as legal cost, loss of credibility, and emotional cost.

A.5 Creditor

A.5.1 Objective and Action

We assume zero expected net return on the loans, due to perfect competition among funds for the loans. A creditor’s objective is to maintain the expected return on the loans at its own cost of financing (such as Libor), $\bar{r}$, assumed to be around 2.25% in normal times and 1% in lenient times.

$$pr \cdot c + (1 - pr) \cdot (1 + r) = 1 + \bar{r}$$

(16)

where $pr$ is the probability that the household default on the loan or have a short-sale, in which the proceeds from selling the property will fall short of the balance of debts. We assume that when default or short-sale occurs, the creditor is expected to recover, after fees and costs, only half of the loan value. In normal times, because the 5% down payment is required for the purchase, $pr$ is assumed to be as low as 5%. Therefore in normal times, a creditor’s action is to issue mortgage at 5% mortgage rate, and require 5% down payment.

$$0.05 \cdot 0.5 + 0.95 \cdot (1 + r) = 1 + 0.05 \Rightarrow r = 0.05$$

(17)

In lenient times, only 1% down payment is required, and a creditor is allowed to adjust mortgage rate through the mortgage structure or refinancing. A creditor will ask for a lower mortgage rate when there is enough collateral to back up the loan, and increase the rate when the value of collateral falls.

$$a = \begin{cases} 
1 + r = 1 + 0.01 \Rightarrow r = 0.01 & \text{if } p \geq \text{outstanding loan} \\
0.12 \cdot 0.5 + 0.88 \cdot (1 + r) = 1 + 0.01 \Rightarrow r = 0.08 & \text{if } p < \text{outstanding loan}
\end{cases}$$

(18)

When housing price is so that even the household default on the loan, the proceeds from selling the property can payoff the loan and the interest, $pr$ equals zero because the creditor can always recoup the principal and interest. $pr$ is assumed to be around 12% when housing price falls below the value of loan. This probability is higher than 5%, the default probability in normal times, because only 1% is required for the down payment, and the pool of lenders attracted by the lower rate at the beginning are presumably more risky than lenders under normal times.