12-8-2013

Endogenous rise and collapse of housing prices

Jiaqi Ge
Iowa State University, jge@iastate.edu

Follow this and additional works at: http://lib.dr.iastate.edu/econ_las_workingpapers

Part of the Economics Commons

Recommended Citation
http://lib.dr.iastate.edu/econ_las_workingpapers/49

This Working Paper is brought to you for free and open access by the Economics at Iowa State University Digital Repository. It has been accepted for inclusion in Economics Working Papers (2002–2016) by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
Endogenous rise and collapse of housing prices

Abstract
Based on interviews with local real estate agents, this paper develops a spatial model of the housing market to help us understand what caused the US housing to rise and collapse. We study key factors for their role in the rise and collapse of housing prices, such as speculation and lenient financing. The dynamic simulation findings in the paper demonstrate in concrete terms how lenient bank lending practices combined with speculation can lead to increased volatility in housing prices, including sharp rises followed by sudden collapses. The exploratory work in this paper will help us understand housing price volatilities and make policy advice.

Keywords
housing market, agent-based model, price volatility

Disciplines
Economics

This working paper is available at Iowa State University Digital Repository: http://lib.dr.iastate.edu/econ_las_workingpapers/49
Endogenous Rise and Collapse of Housing Prices

Jiaqi Ge

Working Paper No. 13013
July 2013
Revised on December 2013

IOWA STATE UNIVERSITY
Department of Economics
Ames, Iowa, 50011-1070
Endogenous Rise and Collapse of Housing Prices

Jiaqi Ge 1
Department of Economics
Iowa State University
jge@iastate.edu

December 8, 2013

Abstract

Based on interviews with local real estate agents, this paper develops a spatial model of the housing market to help us understand what caused the US housing to rise and collapse. We study key factors for their role in the rise and collapse of housing prices, such as speculation and lenient financing. The dynamic simulation findings in the paper demonstrate in concrete terms how lenient bank lending practices combined with speculation can lead to increased volatility in housing prices, including sharp rises followed by sudden collapses. The exploratory work in this paper will help us understand housing price volatilities and make policy advice.

1I would like to thank Dr. Leigh Tesfatsion, Dr. Steve Kautz, Dr. Catherine Kling, Dr. Joseph Herriges, and Dr. John Schroeter at Iowa State University for their advice, comments, and help. I would also like to thank Tom Randall Real Estate Team, Hunziker & Associates, and an anonymous realtor for their inputs.
1 Introduction

The housing market is very unique. It has several characteristics that other commodity markets do not have. For example, the housing market is highly leveraged, i.e. most houses are bought with mortgage loans. Moreover, in a typical mortgage contract, the house bought with the help of the loan are used as the collateral for the loan. Another unique feature of the housing market is its mixed nature of consumption and investment good. A house is more than a shelter: it preserves and appreciates in value, which makes it an investment good whose return often beats that on stocks. The third is the long search period and negotiation process in the purchase of a house. Finally a house cannot be separated from its location. Unlike most commodities or financial products, there is no such thing as a standard housing product. Every house is unique because every location is unique. All these characteristics have made the housing market different from a standard commodity or financial market.

The U.S. housing price started to rise around year 2000. Average housing price has almost doubled in just a few years. It kept rising until reaching its peak at the end of 2006. When housing prices collapsed, the consequences are devastating: Nearly $11 trillion in household wealth has evaporated because of it. America went into the most severe recession since the big depression in the 1930s. Likewise, Japan’s property price rose quickly in the late 1980s. It collapsed in the early 1990s, causing economic slowdown in years after. The same large-scaled rise and fall in housing prices take place in European countries like Norway, Spain and Ireland. Housing price rise and collapse is even more severe in emerging economies such as South Korea, Russia, China, India, causing large interruptions in the economy each time.

Even though real estate makes up a significant portion of national economy and is the biggest household wealth, so far much less economic research has been done on the housing market than on the stock market, the bond market or the foreign-exchange market. The current literature has failed to provide us with a good understanding of this essential market (Mayer 2011 [9]). This lack of understanding is confirmed in the final report by the national commission on the causes of the recent financial and economic crisis in the United State [3]. We have paid a high price for our ignorance. This study will fill the gap.

This study makes two major contributions. First, it develops an agent-based computational model of the housing market. The model will incorporate all the unique characteristics and complications mentioned above. It is also flexible enough to be used as a platform by other researchers for future housing market analysis. For example, the model can be adapted and used to study a local housing market or to analyze a particular policy scenario. Second, this study attempts to answer the following research questions: will housing prices rise and
collapse endogenously without an external shock? If so, under what conditions? Our answer is yes to the first question. To the second question, our answer is it will happen when banks are engaged in lenient financing and there is speculation in the market.

We first propose an analytical framework to demonstrate endogenous housing prices rise and fall. With simplified assumptions, we show the two market dynamics that together will lead to cycles in the housing market. The two dynamics are determined by speculation and leniency respectively. This analytical framework helps us identify the two key treatment factors in the computational experiment. Moreover, since the analytical framework is based on simplified assumptions and fails to capture some of the features of the housing market, we move on to develop a more flexible and complex agent-based model that incorporates all the unique characteristics of the housing market. We then use the agent-based model to analyze the effect of the treatment factors identified in the analytic framework on housing price volatility, home-ownership rate, foreclosure rate, mortgage rate, bank’s expected profit etc.

Some of our key findings from the agent-based model are listed below. First, we find that under certain circumstance, housing prices will rise and collapse endogenously without any external shocks. We find that lenient financing or high leverage combined with speculation is responsible for the endogenous rise and fall of housing prices. Moreover, we find that it is possible to set the down payment rate at an optimal level and achieve both affordable housing and market stabilization. Finally, we find that with lenient financing, banks and financial institutions will have incentives to adopt lenient financing even without the securitization of mortgage loans.

The paper is organized as follows. Section 2 presents a review of related literature. Section 3 presents the analytical framework of housing market. Section 4 presents the model logic of the agent-based housing market model. Section 5 presents the treatment factors and experimental design. Section 6 presents the results. Section 7 presents the concluding remarks. For more details of the agent-based model, the UML diagrams of the model’s class structure and activity flow can be found in Appendix C; A detailed structural framework can be found in Appendix B.

2 Related Literature

There are different ways to model the housing market. Theoretically, Poterba [10] developed an asset-market model of the housing market and estimated how changes in the expected
inflation rate affect the real price of houses. Stein [13] used a simple model of trade in
the housing market to show that price volatility and trading volume can be explained by
minimum down payment requirement through repeated buyers. Iacoviello [7] developed a
monetary business cycle model and found that collateral effects dramatically improve the
model with heterogeneous agents to investigate the properties of the wealth distribution and
the portfolio composition regarding housing and equity holdings. Finally, Sommervoll et
al. [12] developed a heterogeneous agent model illustrating the connection between adaptive
expectations and housing market fluctuations.

Empirically, Case and Shiller [2] measured the extent of housing bubble in the U.S. housing
market long before it became obviously dangerous. Interestingly, in 1989 Mankiw and Weil [8]
predicted that real housing price would fall substantially in the next two decades by looking
at the historical relation between housing demand and housing prices. Glaeser et al. [6]
investigated housing supply and found that the price run-ups of the 1980s were almost
exclusively experienced in cities where housing supply is more inelastic.

As for agent-based computational models 2, Markose et al. [?] developed agent-based models
to study the inter-relationship between property market and the value of financial securities.
Torrens [19] developed a multi-leveled agent-based model for individual’s housing choices,
but without the issue of mortgage. Geanakoplos et al. [5] proposed an agent-based model of
the housing market with a sophisticated mortgage structure. Agent-based models of housing
market tend to fall in two categories: those have a detailed spatial landscape but no financial
sector, and those have a sophisticated financial sector but no spatial landscape. Our work
will fill the gap: our model has both a spatial landscape and a financial sector.

Regarding the role of leverage in business cycles, Geanakoplos [4] found that a small shock
in one sector can cause wide-spread crises across sectors with independent payoffs because
of the leverage constraints that connects all sectors. In another study by Thurner et al. [18],
the authors argued that it is leverage, not interest rate, that causes fat tailed distribution of
return and clustered volatility. They show that even a small negative shock in the market
will trigger large price drop, because leveraged investors are forced to sell the assets to stay
within the leverage limit. Similarly, our study will show that high leverage in the housing
market is responsible to cause the endogenous rise and collapse of housing ps.

2See Appendix A for an introduction to agent-based computational method.
3 An Analytical Framework of Housing Market

This section presents an analytical demonstration of how housing prices will rise and collapse endogenously. We identify two factors that are central to the rise and collapse of housing prices: down payment requirement and speculative demand. In Section 3.1 we will start with analyzing the relationships between mortgage rate, down payment requirement, and housing price. In Section 3.2 we will show characteristics of non-speculative and speculative demand respectively. Last in Section 3.3 we will use a phase diagram to show that low down payment requirement combined with speculative demand can cause housing price to rise and collapse endogenously.

3.1 Collateral and Mortgage Rate

We will start by connecting mortgage rate, down payment requirement, and housing price through the linkage of collateral. In lending contracts, collateral is a borrower’s pledge of specific property to a lender to protect the latter from loan default. Collateral in a typical mortgage loan contract is the house being acquired with the loan. We define collateral rate, $cr$, as the minimum of the value of collateral over the value of loan and $1 + r^F$, where $r^F$ is the risk-free return. It has an upper limit at $1 + r^F$ because any collateral value beyond that is irrelevant to the lender. In a mortgage loan, value of collateral equals the value of the house bought with the mortgage. Formally, $cr$ in a mortgage loan is defined as

$$cr \equiv \min \left\{ \frac{\hat{p} + \hat{p}}{(1 - \text{MinDown})\hat{p}}, 1 + r^F \right\} = \min \left\{ \frac{1 + \hat{p}/\bar{p}}{1 - \text{MinDown}}, 1 + r^F \right\}$$

(1)

where $\bar{p}$ is the original purchase price, MinDown is the minimum down payment rate required by the bank, $\hat{p}$ is change in price. For simplicity, we assume that the value of loan equals the purchase price minus down payment. Therefore a positive relationship between collateral rate and percentage change in price only exist if the percentage change in price is below the cutoff $r^F - (1 + r^F)\text{MinDown}$. Above the cutoff, collateral rate is fixed at $1 + r^F$.

We assume perfect competition among lenders/funds for mortgage loans. Therefore the non-arbitrage condition requires that the return on the mortgage loan equals the risk-free return,

$$\text{prob}^d(cr) \cdot cr + (1 - \text{prob}^d(cr)) \cdot (1 + m) = 1 + r^F$$

(2)

where $m$ is mortgage rate, and $\text{prob}^d(cr)$ is default probability, which is a decreasing function of $cr$. Equation 2 thus defines the relationship between mortgage rate $m$ and collateral rate.
\( cr \). Take total differentiation with respect to \( cr \) we get,
\[
\frac{dm}{dcr} = -\text{prob}^d \cdot (1 - \text{prob}^d) + \frac{\partial \text{prob}^d}{\partial cr} \cdot (1 + r^F - cr) \\
(1 - \text{prob}^d)^2 < 0
\]  

There exists a negative relationship between mortgage rate and collateral rate. Because collateral rate is a non-decreasing function of percentage change in price as previously stated, there exists a non-positive relationship between mortgage rate and percentage change in price. The relationship is constant when percentage change in price is above the cutoff, \( r^F - (1 + r^F) \cdot \text{MinDown} \), and it is negative when percentage change in price is below the cutoff. Figure 1 show the relationship between mortgage rate and percentage change in housing price below and above the cutoff.

![Figure 1: Mortgage rate (m) and collateral rate (c) vs. price change](image)

**3.2 Demand for Housing**

Since the total cost of a house is \((1 + m) \cdot p\), non-speculative net housing demand, \( D^n \), is decreasing in both \( m \) and \( p \).
\[
\frac{\partial D^n}{\partial p} < 0 \text{ and } \frac{\partial D^n}{\partial m} < 0
\]  

Zero net demand therefore requires that mortgage rater \( m \) and housing price \( p \) move in the opposite direction. Speculative net housing demand, \( D^s \), is an increasing function of past housing price appreciation.
\[
\frac{\partial D^s}{\partial \dot{p}} > 0
\]
When price is increasing, speculative net demand is positive; when price is stabilized, speculative net demand is zero; and when price is decreasing, speculative net demand is negative, meaning there is net supply from speculative homeowners.

3.3 The Phase Diagram

Figure 2 shows the phase diagram of the system. The solid curve is the zero non-speculative net demand curve. If there is a decrease in the risk-free market return \( r^F \), mortgage rate will decrease from \( m_0 \) to \( m_1 \). Housing price will rise in response to the reduced mortgage rate. It will increase from \( p_0 \), the initial price, and overshoot beyond \( p_1 \), the price at which non-speculative net demand is zero at \( m_1 \), because at \( p_1 \) there is still positive speculative net demand as price has been increasing. Price will keep rising until the sum of speculative and non-speculative net demand equals zero. The light blue up arrow depicts the overshoot of housing price due to speculators. Then price will fall because as price is stabilized, speculative net demand becomes zero but non-speculative demand is negative. Price will keep falling and go below \( p_1 \) for the same reason it goes above \( p_1 \). Again, when the price is stabilized below \( p_1 \) it will rise again. In short, housing price will fluctuate around \( p_1 \) due to the existence of speculative demand. The extent of the fluctuation depends on the relative size of speculative and non-speculative demand. This is the first dynamic that does not involve the response of mortgage rate from lenders.

Figure 2: Phase diagram of housing price (p) and mortgage rate (m)
Potentially, there is a second, and more substantial dynamic. Whether the second dynamic will actually take place depends on the level of down payment. The dashed line above the non‐speculative zero net demand curve is the cutoff we talked about before. A higher down payment corresponds to a higher cutoff and vice versa. Due to the existence of speculators, price will overshoot beyond $p_1$, but if the cutoff is high and lies above the overshoot, mortgage rate stays fixed at $m_1$. However, if the cutoff is so low that when price overshoots $p_1$ it goes beyond the cutoff, mortgage rate will respond to change in housing price, which triggers the second dynamic. Hence only when price overshoots above the cutoff curve, will the second dynamic take effect, which is depicted by the dark blue arrow. Once in the area above the cutoff, housing price will no longer fluctuate around $p_1$. Instead, it collapses and goes all the way down to where the dark blue arrow points. Price will keep falling and mortgage rate keep rising until it falls under the cutoff again. Once below the cutoff, mortgage rate will start to fall again. The cycle will then repeat itself.

We have demonstrated in the above framework how housing prices can rise and collapse endogenously in the market by establishing a linkage between mortgage rate and change in housing price through down payment requirement. Knowing this is important because it helps us understand the underlying dynamics that give rise to the rise and collapse of housing prices. It also helps us identify the two key treatment factors in our computational model: the degree of leniency and the degree of speculative demand. However, the above framework makes a simplification of the housing market in many ways. For example, it does not consider the fact that a house is not a standard product and its location will largely affect its value. In fact, a housing market cannot be separated from the landscape in which it sits. The natural, historic and neighborhood characteristics in a sub-region together determines the value of a house in that sub-region; while price movements in the housing market will in turn alters the neighborhood characteristics and amenity of the sub-region in a city. All these complexities are beyond the grip of a simplified analytic framework and call for a more flexible agent-based computational model of the housing market, which is presented in the next section.

4 The Agent-based Housing Market Model

The housing market is very unique: it is less liquid, spatial, highly leveraged, and can be both consumption and investment good. To this day, there are few models of housing market that take into account all of these complications. The agent-based model proposed in this paper can capture all the complexities mentioned above. For the sake of space, we will only show a diagram and a flow chart to explain the workings of the model. For more details, two
UML diagrams of the model’s class structure and activity flow can be found in Appendix C, and a structural framework of the housing market model can be found in Appendix B.

As previously stated, a housing market cannot be separated with the landscape in which it sits. In this model, the housing market sits on a two dimensional landscape that contains 25 regions and has a downtown, suburbs and rural areas as in Figure 3. Two regions are neighbors if they share a common border. Each region is assigned an exogenous location quality, which is represented by the number in each region in Figure 3). The location quality captures exogenously factors that affect quality of life in that region, such as distance to natural sites and distance to downtown. In this model, we assume that location qualities are symmetric and suburbs have better natural qualities than the city center and rural areas. Apart from location quality, each region also has an endogenous neighborhood quality. Neighborhood quality captures endogenous factors that affect quality of life in that region, such as public facilities, public school quality and crime rate. It is endogenous because it depends on the residents living in that region, which are endogenous.

<table>
<thead>
<tr>
<th>0.35</th>
<th>0.56</th>
<th>0.65</th>
<th>0.56</th>
<th>0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.56</td>
</tr>
<tr>
<td>0.65</td>
<td>1.00</td>
<td>0.65</td>
<td>1.00</td>
<td>0.65</td>
</tr>
<tr>
<td>0.56</td>
<td>0.85</td>
<td>1.00</td>
<td>0.85</td>
<td>0.56</td>
</tr>
<tr>
<td>0.35</td>
<td>0.56</td>
<td>0.65</td>
<td>0.56</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Figure 3: The 5x5 landscape

There are five types of market participants: the real estate agent, the developer, buyers, homeowners, and the bank. We further distinguish buyers and homeowners as investors and non-investors. Investors buyers buy a property in hope of profiting from housing price appreciation. Regular buyers, on the other hand, obtain utility from living in the house. Each period in the model represents a month in real time. In each period, speculative and non-speculative buyers are created to enter the market in search of a house. Meanwhile, existing homeowners in the city decide whether to list their houses for sale. The number of non-speculative buyers generated in each period equals 5% of the total number of non-speculative homeowners, which is also the probability that a non-speculative homeowners will decide to sell the house for exogenous reasons (job move, divorce etc.). Therefore in this model we have a balanced city where the number of (non-speculative) buyers equals the
number of (non-speculative) sellers. While the numbers of non-speculative buyers and seller are given, the numbers of speculative buyers and sellers are treatment factors thus will vary in the model. Figure 4 illustrated how the model works and the relationship between the five market participants. A detailed structural framework of the housing market model can be found in Appendix B.

![Figure 4: The housing model illustration](image)

The model procedure can be summarized in the following flow chart:

Step 1 At the beginning of period $t$,
- **The Real Estate Agent**: announces housing prices for all regions in the previous period.
- **The Bank**: announces mortgage rate and lending criteria, and lists foreclosures at the real estate market.

Step 2 **The Developer**: given prices in the last period, builds new houses in each region and submits the asking prices and lists the houses at the real estate agent.
- **A Homeowner**: given prices in the last period and mortgage terms, chooses whether to default on the house. If chooses to default, the property goes into foreclosure and homeowner exits the market. Otherwise, chooses whether to sell the property. If chooses to sell, submits an asking price and lists the house at the real estate agent. Otherwise, make monthly payment and enter period $t + 1$ as a homeowner.
- **A Buyer**: searches each region in the affordable choice set, identifies the house that gives her the highest expected utility (non-speculator) or return (speculator), and
choose whether to submit a bid on the house. Submits a bid to the real estate agent if chooses so.

Step 3 The Real Estate Agent: collects all the bids and asks, and settles the final market price for each region.

Step 4 The Developer: sells houses according to the market price. Unsold units become housing inventory and will be listed in period \( t + 1 \).

The Bank: sells foreclosures according to the market price.

A Homeowner: if the asking price is lower than the market prices, sells her property and exits the market. Otherwise enter period \( t + 1 \) holding the property.

A Buyer: if her bidding price is higher than the market price, buys a house and enters period \( t + 1 \) as a homeowner. Otherwise choose whether to wait for another period or exit the market.

Step 5 End of period \( t \). Enter period \( t + 1 \).

5 Treatment Factors and Experimental Design

In this section we will discuss treatment factors in the model and the experimental design based on the treatment factors.

5.1 Treatment Factors: leniency index and Speculation Index

We identify two key treatment factors in the model that we think is central to the rise and collapse of housing prices: leniency index, denoted by \( L \), and speculation index, denoted by \( S \). Leniency index is defined as one minus the down payment rate. It measures the degree of leniency. We have shown in the analytical framework that low down payment requirement can lead to the rise and collapse of housing prices. Therefore we predicted that high leniency index will lead to large housing price volatility.

Leniency index is closely related to the idea of leverage, defined as the ratio between debt and collateral. In fact, there is a one-to-one positive relationship between leniency index \( L \) and leverage,\[
\text{leverage} = \frac{1}{1-L} - 1 \Leftrightarrow 1 - \frac{1}{1+\text{leverage}}
\]

When down payment requirement is 100% so leniency index is zero, leverage is also zero; when down payment requirement is 0% so leniency index is one, leverage is infinity. Because
of this positive one-to-one relationship between leverage and leniency index, we can also say that we predict high leverage will lead to large housing price volatility.

During the latest crises, banks and financial institutions were found to be engaged in aggressive lending practice, which sometimes means zero down payment rate and zero mortgage payment in the first few years. The rationale behind banks’ aggressive lending behavior has been the center of discussion and a focus has been made on the role of securitization. The reason is that securitization shifts the risks away from the initial lender thus reduces the lender’s incentive to play safe. We agree that securitization is one of the reasons for aggressive lending, but we argue it is not the only reason. In this paper we will show that even without securitization, banks may still be willing to expand lending at the expense of increasing risks.

The second treatment factor, speculation index, is defined as the percentage of speculators in total homeowners. It measures the degree of speculation in the market. Speculators are the ones who buy or sell a property for profit rather than for consumption. They are trend-followers: they buy when the price is rising, and sell when the price is falling. As demonstrated in the analytic framework, we predict that more speculation will lead to large housing price volatility. We should clarify that by making speculation a treatment factor, we control the number of potential speculators searching for profitable investment opportunities in the housing market. However, we do not directly control the number of speculators buying or selling in the market. Potential speculators will stay inactive if they fail to find any profitable opportunities.

The degree of speculation can differ by culture and the availability of alternative investment opportunities. For example in most East Asian cultures, it is a tradition for people to invest in properties with their spare money; while another culture has no such tradition. Also, in countries that lack a mature financial market, people are more prone to invest in the real estate market because there are fewer investment opportunities elsewhere. In short, the degree of speculation is affected by exogenous factors such as culture and the availability of alternative investment opportunities, thus we include it as a treatment factor in the model.

5.2 Experimental Design

Since we have two treatment factors, our experimental design is a two dimensional matrix. We also identify two key functional values in the model. The first one is housing price volatility, defined as the standard deviation of housing prices in a region over its mean.
The second functional value is the number of non-speculative homeowners at the end of simulation. It measures the degree of home-ownership. The higher the number of non-speculative homeowners, the higher the degree of home-ownership.

Leniency index is defined as one minus the minimum down payment rate by the bank. It is bounded between zero and one. Although it can take a value down to zero, meaning 100% down payment rate, it is unusual that an up-front payment in full is required for buying a property. We believe a more realistic range for leniency index would be between 0.6 to 1, or between 40% to 0% down payment requirement. That will be the densely-sampled region for leniency index.

Speculation index is defined as the percentage of speculative buyers in total homeowners in each period. For example, in a city with 100,000 homeowners, a 0.05 speculation index means that in each month 5,000 speculative buyers are looking for investment opportunities. The higher the index, the more widespread speculation is in the housing market. Speculation index is bounded below at zero, but not bounded above. We put a very generous upper bound of 0.25 on speculation index, and we believe that a more realistic range for speculation index would be between 0 and 0.15. That will be the densely-sampled region for speculation index.

Rennen et al. [?] show that a design for computer experiments should satisfy two criteria: space-filling and non-collapsing. The first criteria, space-filling, means that the experiment should be designed to obtain information from the entire parameter space, which in our case is a 1 by 0.25 square. The second criteria, non-collapsing, means there should be no irrelevant parameters which value does not affect the function value. Our experimental design satisfies both criteria. Figure 5 shows our experimental design. For each cell, we will run the model 20 times. For each run, we start from an empty landscape. Buyers are generated each period to start house hunting. As some buyers become homeowners the landscape is gradually filled. Each run is consisted of 500 periods, excluding 200 burnout periods. We then calculate mean and standard deviation for the two function values: price volatility and number of non-speculative homeowners for each cell.

<table>
<thead>
<tr>
<th>$s$</th>
<th>$l$</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.825</th>
<th>0.85</th>
<th>0.875</th>
<th>0.9</th>
<th>0.925</th>
<th>0.95</th>
<th>0.975</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.050</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.075</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Densely Sampled Region**

Figure 5: Experimental design for two treatments: speculation ($s$) and leniency ($l$)
6 Results

In this section we are going to show simulation results from the housing market model. In subsequent Section 6.1 we present illustrated results for single runs. In Section 6.2 will present the report on full experimental design results, and finally in Section 6.3 we compare historical and simulated data of the U.S. housing market.

6.1 Illustrated Results for Single Runs

In section 6.1, we are going to show the housing price, foreclosure rate, mortgage rate, and bank’s expected return from one simulation run, with a specific speculation index and leniency index. We show simulation results for 500 periods, which represents 500 months or around 42 years in real time. Figure 6 shows the housing price under low (0.8) and high (0.9) leniency index. We fix the speculation index at 0.05. The results show that with low leniency index, housing price fluctuates slightly; while with high leniency index, housing price fluctuates wildly. The results confirm our prediction that high leniency index leads to large price volatility.

(a) leniency index=0.8

(b) leniency index=0.9

Figure 6: Housing price with low and high leniency index (conditioning on speculation index=0.05)

Figure 7 shows housing price and the corresponding foreclosure rate, defined as the percentage of foreclosures in total homes. If housing price dives so deep that the outstanding loan is more than the value of the house plus the default cost (so the house is not only under water, but deeply under water), the homeowner will choose to default. The results show that when
leniency index is low, foreclosure is very rare; while when leniency index is high and when the price is falling quickly, foreclosure is wide-spread across the region (more than 10%).

Figure 8 shows housing price and mortgage rate under low and high leniency index. With low leniency index, mortgage rate is low and does not change much over time. With high leniency index, however, mortgage rate remains low when the housing price is stabilized or in the rise, but jumps when housing price starts to collapse. The reason is what we have shown in the analytical framework in Section 3. When a large down payment is required, a decrease in housing price is unlikely to trigger the mortgage rate to increase in response. But when little or none down payment is required, a small decrease in housing price will trigger the mortgage rate to increase to compensate for the depreciation in the value of collateral. An individual lender is regarded too small to affect the market price by raising mortgage rate. It rationally raises mortgage rate to protect itself from increased default risk. However, if every lender does so, demand for housing will be suppressed, which will in turn worsen the market condition and drag down housing price.
Figure 9 shows housing price and bank’s expected profit under low and high leniency index. With low leniency index, bank’s expected profit is very stable over time. With high leniency index, on the other hand, for most of the time bank’s expected profit is higher than that under low leniency index. However, higher profits comes with a price. In three occasions in the simulation over a period of over 42 years, the bank would suffer big losses. Since in the current model we do not explicitly model competitions among banks so there is only one representative bank, we assume that the bank is well-capitalized and will never go out of business.

If the bank is forward-looking long enough into the future, they might prefer to have a more prudent lending policy just to avoid big losses doomed in the future. However, the bank might not be all forward-looking: the average CEO tenure in the financial industry is only five years [14]. Compared with a five-year tenure, we see that in the simulation the first loss does not occur until after 10 years of high profits, and it only happens three times over a period of 42 years. At all other times, banks enjoy high profits which more than doubles that under a tight lending policy.

We now show that the simulation results for the second treatment factor, speculation index. Figure 10 shows housing price without speculation (speculation index=0) and with speculation (speculation index=0.05). We fix the leniency index at 0.9. The results confirm our conjecture that speculation fueled with lenient financing will lead to large price volatilities in the real estate market.

Figure 11 shows the corresponding foreclosure rate without and with speculation. We found that foreclosure rate is high with speculation and is close to zero without speculation. However it does not mean that only speculative homeowners will default. Both speculative and regular homeowners is more likely to default if housing price becomes more volatile. The high
6.2 Report on Full Experimental Design Results

In section 6.2 we show results of the computational experiment, in which we systematically change the two treatment factors, leniency index and speculation index, and simulate the two functional values-housing price volatility and number of non-speculative homeowners. For each combination of treatment factors, we run 20 simulations. We also run 100 simulations for five randomly selected cells and found that our results are robust to sample size: the difference between the mean values from 20 and 100 simulations is less than 5%. To save space, we only display the mean functional values in the densely-sampled region. The mean and standard deviation in the entire parameter space for both functional values can be found in Appendix D.
In Figure 12, we present a heat map of mean housing price volatility in the more densely-sampled region. High-valued cell is in red and low-valued cell in blue. The mean and standard deviation of housing price volatility for the entire parameter space can be found in Figure 19 in Appendix D. Cells are more in blue in the northwestern corner of the heat map, where leverage is low and speculation is low, meaning low housing prices volatility and a stabilized market. Cells are more in red in the southeast corner of the heat map, where leverage is high and speculation is high, representing large housing price volatility. We show that housing price volatility is increasing with leverage and speculation. Lenient financing which allows high leverage will cause large cycles in the housing price given that speculation exists in the market.

<table>
<thead>
<tr>
<th>S</th>
<th>L</th>
<th>0.6</th>
<th>0.7</th>
<th>0.75</th>
<th>0.8</th>
<th>0.82</th>
<th>0.825</th>
<th>0.85</th>
<th>0.875</th>
<th>0.9</th>
<th>0.925</th>
<th>0.95</th>
<th>0.975</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.023</td>
<td>0.025</td>
<td>0.022</td>
<td>0.021</td>
<td>0.026</td>
<td>0.017</td>
<td>0.017</td>
<td>0.016</td>
<td>0.015</td>
<td>0.017</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td>0.032</td>
<td>0.031</td>
<td>0.030</td>
<td>0.026</td>
<td>0.029</td>
<td>0.033</td>
<td>0.062</td>
<td>0.111</td>
<td>0.181</td>
<td>0.204</td>
<td>0.244</td>
<td>0.235</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.050</td>
<td>0.042</td>
<td>0.039</td>
<td>0.038</td>
<td>0.040</td>
<td>0.046</td>
<td>0.060</td>
<td>0.123</td>
<td>0.223</td>
<td>0.219</td>
<td>0.228</td>
<td>0.241</td>
<td>0.222</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.075</td>
<td>0.042</td>
<td>0.046</td>
<td>0.051</td>
<td>0.055</td>
<td>0.067</td>
<td>0.085</td>
<td>0.162</td>
<td>0.265</td>
<td>0.247</td>
<td>0.228</td>
<td>0.189</td>
<td>0.213</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.046</td>
<td>0.054</td>
<td>0.059</td>
<td>0.066</td>
<td>0.081</td>
<td>0.097</td>
<td>0.178</td>
<td>0.273</td>
<td>0.258</td>
<td>0.246</td>
<td>0.224</td>
<td>0.214</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td>0.054</td>
<td>0.068</td>
<td>0.068</td>
<td>0.085</td>
<td>0.099</td>
<td>0.115</td>
<td>0.194</td>
<td>0.248</td>
<td>0.241</td>
<td>0.230</td>
<td>0.219</td>
<td>0.215</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 12: Housing price volatility, densely sampled region

In Figure 13, we present a heat map of mean number of non-speculative homeowners in the more densely-sampled region. Again, high-valued cell is in red and low-valued cell in blue. The mean and standard deviation of the number of non-speculative homeowners in the entire parameter space can be found in Figure 20 in Appendix D. Cells are more in red in the northeast corner of the heat map, where speculation index is low and leniency index is high: home-ownership rate is highest when we have a lenient lending criteria but few speculative buyers. Moving from left to right, as leniency index increases, the number of non-speculative homeowners continue to increase until it is 0.85. After that, the gain in home-ownership is only marginal. Moving down, as the number of speculators increases, the

<table>
<thead>
<tr>
<th>S</th>
<th>L</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.82</th>
<th>0.825</th>
<th>0.85</th>
<th>0.875</th>
<th>0.9</th>
<th>0.925</th>
<th>0.95</th>
<th>0.975</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>3482</td>
<td>6667</td>
<td>8098</td>
<td>9476</td>
<td>9966</td>
<td>10150</td>
<td>10474</td>
<td>10850</td>
<td>10841</td>
<td>10562</td>
<td>10346</td>
<td>10151</td>
<td></td>
</tr>
<tr>
<td>0.025</td>
<td>5146</td>
<td>6707</td>
<td>7635</td>
<td>8735</td>
<td>9346</td>
<td>9953</td>
<td>10491</td>
<td>11534</td>
<td>11886</td>
<td>11300</td>
<td>11322</td>
<td>8636</td>
<td></td>
</tr>
<tr>
<td>0.050</td>
<td>4881</td>
<td>6479</td>
<td>7430</td>
<td>8520</td>
<td>9226</td>
<td>9898</td>
<td>10446</td>
<td>10526</td>
<td>10522</td>
<td>10970</td>
<td>9577</td>
<td>9451</td>
<td></td>
</tr>
<tr>
<td>0.075</td>
<td>4695</td>
<td>6161</td>
<td>7168</td>
<td>8444</td>
<td>8984</td>
<td>9440</td>
<td>9390</td>
<td>9322</td>
<td>8659</td>
<td>9523</td>
<td>8884</td>
<td>9219</td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>4398</td>
<td>5939</td>
<td>6919</td>
<td>7968</td>
<td>8537</td>
<td>8964</td>
<td>8100</td>
<td>6319</td>
<td>7194</td>
<td>7436</td>
<td>7022</td>
<td>7705</td>
<td></td>
</tr>
<tr>
<td>0.150</td>
<td>3875</td>
<td>5531</td>
<td>6409</td>
<td>7177</td>
<td>8205</td>
<td>8184</td>
<td>5707</td>
<td>4659</td>
<td>4214</td>
<td>4726</td>
<td>4858</td>
<td>4817</td>
<td></td>
</tr>
</tbody>
</table>

Figure 13: Number of non-speculative homeowners, densely sampled region
number of non-speculative homeowners decreases. Speculators has driven up the price and crowded out some of the non-speculative homeowners.

Now we pick a row in the heat map of price volatility and fix speculation index at 0.05. In Figure 14 we show the relationship between leniency index and housing price volatility, conditioning on speculation index equals 0.05. The result shows that there exists a positive relationship between leniency index and housing price volatility. It also shows that the relationship is not necessarily linear. Volatility stays at a low level as long as leniency index is between 0 and 0.85, that is, down payment rate is higher than 15%; when down payment rate is less than 15%, price volatility increases quickly from less than 5% to more than 15% as down payment rate decreases from 15% to 10%. When down payment rate is further lowered to less than 10%, price volatility stays at a high level of above 15%, but does not further increase. This implies that we do not need to make down payment rate prohibitively high to prevent large housing price volatilities. If we set the down payment rate at the right level, which is at about 15% in this case, we can both avoid large price volatility and have high home-ownership rate.

We then pick a column in the heat map at leniency index equals 0.9. In Figure 15 we show the relationship between speculation and housing price volatility, conditioning on leniency index equals 0.9. Like we have predicted, speculation will lead to larger price volatility. Speculation is a treatment factor in this model for the reasons we describe in section 5: it is affected by things like culture or housing market history, which are exogenous in our model. However, Culture and housing market history is not all exogenous: people learn and adapt; culture can be formed. Non-investors can be turned into investors if they constantly see their friends making profit in the housing market. Slowly, the change in behavior is ingrained in the collective memory and becomes part of the culture. Since we now know that speculation will cause larger price volatility, it would be interesting to see what will happen if we endogenize it. However this is beyond the scope of this paper, so we will leave it for future study.
6.3 Comparison of Historical and Simulated Data

In this subsection, we demonstrate that our simulation results can resemble some characteristics of housing price history. We make a qualitative comparison between the quarterly U.S. home price index (Shiller index) [1] between 1970 and 2012, and simulation results over the same time span. In the simulation, before the year 2000, down payment rate is set at a moderate 15% and market interest rate is set at 4%. At the beginning of January, 2000, we introduce lenient financing into the system and decrease down payment from 15% to 4%. We also decrease the market rate from 4% to 1%, to represent the decreased interest rate, due to the abundance of hot money in the U.S. at that time. Other than that one time shock, we let the model run without interference. Figure 16 shows a comparison between the history of U.S. home price index and simulated price index. We use the earliest price in 1970 as baseline.

The two graphs have shared some common patterns. When we began to deregulate the
financial market and adopt more lenient lending criteria around the year 2000, housing price starts to rise. Housing price keeps rising until all of sudden, without any obvious external reasons, it collapsed. Our simulation model has captured that important aspect of the reality. However, we are cautious not to overstate the implications of the two graphs’ similarities. To actually validate the model using empirical data on the housing market, we need to carefully calibrate model parameters such as region characteristics, income, and market interest rate. All these parameter values will affect simulation results. Since the purpose of this paper is to provide a computational model of the housing market and use it to demonstrate the endogenous rise and collapse of housing prices, we will leave model validation for future research.

7 Concluding Remarks

The recent crisis in the U.S. housing market has had devastating consequences. About four and half million American families have lost their homes to foreclosures or were on the edge of going foreclosure. America has since gone into the most severe recession since the big depression in the 1930s. The housing market receives less attention from researchers than the stock and foreign exchange market despite its heavy weight and influence in the economy. Even today we are still lacking in understanding of this important market. The housing market is very unique: it is less liquid, highly leveraged, spatial are both investment and consumption good. To this day, there are few housing market models that take into account all of these complications.

This study develops an agent-based spatial model of the housing market that incorporates all of the issues above. We attempt to answer the following questions: Will housing prices rise and collapse endogenously in a housing market? If so, under what conditions? Our results show that housing prices will rise and collapse endogenously without any external shock, although a shock may magnify the process. We find that the two necessary conditions for the endogenous rise and collapse of housing prices are low down payment requirement or leniency, and speculation. When banks are engaged in lenient financing and there is speculation in the market, the rise and collapse of housing prices are inevitable. Our simulation results also show that when housing prices start to fall, foreclosure is wide-spread and mortgage rate increases, all of which are similar to the observations in the recent U.S. housing market crises.

In addition, we find that a bank may opt for lenient lending practices even without the securitization of mortgage loans. Although the securitization of mortgage loans will lead
to more aggressive lending from the banks, we believe this is only part of the underlying reasons. Our simulation results show that, even without securitization of mortgage loans, banks may be willing to adopt a lenient lending policy, knowing they may face losses in the future, for the benefit of higher-than-normal profits for extended period of time. It is especially true when the average CEO tenures in the financial sector is much shorter than the average length of boom period, and a CEO’s payoff depends only on short-run profits. Since the securitization of mortgage loans is not the only reason for banks’ aggressive lending behavior, simply restricting securitization alone is no guarantee that a housing market crises will not come back in the future. A more fundamental reform of financial institutions is needed.

We also find that policies can be made so that it encourages home-ownership and achieves housing market stability at the same time. One of the reasons for government sponsored entities (GSEs) like Frannie Mae and Freddie Mac to lower lending criteria is to extend lending to low income household and encourage home-ownership. However, our simulation results show that at 15% down payment rate, home-ownership rate reaches its high: further lowering down payment requirement will no longer increase home-ownership rate. Rather, it will cause the housing prices to rise so high that it hinders rather than helps home-ownership among low income households. In other words, lowering lending criteria can make housing less affordable, not more. We find that it is possible to achieve both affordable housing and market stability by setting the down payment requirement at an optimal level, which is 15% in our simulation.

Finally, we show that our agent-based model is able to generate price patterns similar to those in the U.S. housing market. We believe our agent-based model has incorporate the key elements in the housing market and is able to capture the dynamics that give rise to large housing price volatility. However, we do not claim that our model is able to match real world data on a value base. Neither is data replication the purpose of this study. The empirical validation of the model is beyond the scope of this study, and we will leave it for future research.

To sum up, we have shown that housing prices can rise and collapse by itself without any external economic shock. The two necessary conditions for the endogenous rise and collapse of housing prices are lenient financing and speculation. We find that banks have incentives to engage in aggressive lending even without the securitization of mortgage loans. We also find that we can achieve both affordable housing and housing market stability by setting the down payment requirement at an optimal level. Moreover, we show that the agent-based model is able to generate price patterns similar to those in the U.S. housing market. Future work includes the empirical validation of the model and the endogenization of speculative
References


Appendices

A An Introduction to Agent-based Computational Modeling

Agent-based computational modeling is a research methodology that simulates natural, operational and social systems as automated computational agents interacting in a virtual environment. It sees the world as composed of individual agents, be it electronics or human beings, and the observed phenomenon the outcomes of those agents’ interaction. As a result, agent-based models are capable of modeling systems that are complex and heterogeneous. Agent-based computational model is a major methodology in fields like engineering, national defense, and epidemiology. In areas like power market, it has become the dominant research methodology. In social sciences such as sociology and economics, agent-based computational models are increasingly used to study complex social or economic systems.

Agent-based computational economics (ACE) is a field in economics that studies the dynamic economic systems as virtual worlds of interacting agents [15]. The defining characteristic of ACE models is their constructive grounding in the interactions of agents [17]. In ACE models researchers obtain high-level aggregate outcomes from individual agent’s interactions on the ground. In other words, researchers are “growing economies from the bottom up” [16]. Because of that, agent-based modeler is able to relax many of the assumptions in equation-based models, and adopt a more flexible and/or realistic model setting.

B The Structural Framework of Housing Market

In this section We are going to present the structural framework of housing market that we used to develop programming code and generate simulation results. Before we proceed, we list below some key notations which will be used through out the rest of the section.

B.1 Classification of Variables

Indices and Index Sets
• \( t \): index for discrete time points, month \( t=[t, t+1) \)
• \( B_t \): set of buyers at time \( t \), indexed by \( b \)
• \( G \): set of 25 regions, \( g=0,1,2,...,24 \)
• \( H_t \): set of homeowners at time \( t \), indexed by \( h \)
• \( A_t = B_t \cup H_t \): set of all buyers and homeowners (decision-making private agent) at time \( t \), indexed by \( i \)
• \( C_t \): set of bank’s clients at time \( t \), \( C_t \subseteq A_t \), indexed by \( i \)

**Exogenous Variable**

• \( c_1, c_2, c_3 \): parameters for construction cost for developer
• \( DC^h \): default cost of homeowner \( h \)
• \( \text{LocQ}^g \): location quality of region \( g \)
• income\(^i\): monthly income attribute of agent \( i \)
• \( M \): loan maturity measured in months set by bank set at 20 year or 240 months for all loans at current study
• MaxDTI: maximum debt-to-income ratio permitted by the bank
• MaxTOM: maximum time-on-market permitted for any Buyer \( b \)
• MinDown: minimum down payment rate required by the bank
• \( N^g \): the set of neighboring (bordering) regions for region \( g \)
• \( r^F_t \): annual risk-free return rate at time \( t \)
• \( T_{Dev} \): number of periods it takes the developer to build a house
• \( \omega^i \): savings attribute of agent \( i \)

**Endogenous Variable**

• \( \bar{c}_t \): mean collateral rate of all clients of the bank at time \( t \)
• \( c_{oi} \): collateral rate of client \( i \) at time \( t \)
• \( \text{DTI}^i(g) \): debt-to-income ratio of agent \( i \) in at time \( t \) if agent \( i \) buys a property in region \( g \)
• \( \text{ER}^i(g) \): annual return rate expected by speculative agent \( i \) at time \( t \) from the purchase and reselling of a house in region \( g \) in a year
• \( \text{EU}^b_t(g) \): monthly utility from living in a house in region \( g \) expected by regular buyer \( b \) at time \( t \)
• \( F^b_t \): the set of regions considered by buyer \( b \) for purchase of a house at time \( t \), including the null region
• \( L^i_t(g) \): loan value of agent \( i \) at time \( t \) if agent \( i \) buys a property in region \( g \)
• \( m_t \): annual mortgage rate required by bank at time \( t \)
• \( M_i^t \): remaining loan maturity of agent \( i \) at time \( t \)
• \( m_{pi}^t(g) \): monthly mortgage payment of agent \( i \) at time \( t \) if agent \( i \) buys a property in region \( g \)
• \( \text{NbhdQ}_g^t \): neighborhood quality of region \( g \) at time \( t \)
• \( p_g^t \): housing price at the beginning of month \( t \) in region \( g \)
• \( \text{prob}_D^t \): default probability of the lending pool for month \( t \) estimated by the bank
• \( \text{TOM}_i^t \): time-on-market of agent \( i \) at time \( t \)

B.2 The Mortgage Contract

A mortgage contract between the bank and agent \( i \) for a property in region \( g \) at time \( \tau \) is made up of the following components,

• eligibility requirements:
  – maximum debt-to-income ratio, MaxDTI
  – minimum down payment rate, MinDown

• value of loan for agent \( i \) at time \( \tau \) in region \( g \), \( L_i^\tau(g) \)
• loan maturity in months, \( M \)
• annual mortgage rate set by bank for agent \( i \) at time \( \tau \), \( m_i^\tau \)
• collateral for the loan, which is a property in region \( g \)

Loan maturity is set to be 240 periods or months at the current study. In each period the remaining loan maturity of agent \( i \) is deducted by one.

\[
M_i^t = M = 240 \tag{7}
\]

\[
M_i^t = \begin{cases} 
N/A & \text{if } t < \tau + M \\
M_i^{t-1} - 1 & \text{if } \tau < t \leq \tau + M \\
0 & \text{if } t > \tau + M 
\end{cases}
\]

Agent \( i \)'s mortgage rate equals bank’s mortgage rate at time \( \tau \), \( m_\tau \), and it does not change throughout the mortgage maturity. Agent \( i \)'s Outstanding loan in region \( g \) at time \( t \), \( L_i^t(g) \), is defined in the following equation,

\[
L_i^\tau(g) = (1 - \text{MinDown}) \cdot p_g^\tau \tag{8}
\]

\[
L_i^t(g) = \begin{cases} 
N/A & \text{if } t < \tau \\
(1 + m_\tau/12) \cdot (L_i^{t-1} - m_{pi}^t(g)) & \text{if } \tau \leq t \leq \tau + M \\
0 & \text{if } t > \tau + M 
\end{cases}
\]
where \( p_g^\tau \) is housing price in region \( g \) at time \( \tau \), and \( mp^i_t(g) \) is agent \( i \)'s monthly payment in region \( g \) at time \( t \). It is a function of the initial loan value \( L^i_\tau(g) \), mortgage rate \( m_\tau \), total loan maturity \( M \), and time \( t \). \( mp^i_t(g) \) is defined in equation 9.

\[
mp^i_t(g) = \begin{cases} 
N/A & \text{if } t < \tau \\
mp(L^i_\tau(g), m_\tau) = \frac{(m_\tau/12) - L^i_\tau(g)}{1 - (m_\tau/12)} & \text{if } \tau \leq t \leq \tau + M \\
0 & \text{if } t > \tau + M 
\end{cases} \tag{9}
\]

Agent \( i \) is eligible for a mortgage contract in region \( g \) at time \( \tau \) if and only if all of the following conditions are met:

\[
p_g^\tau - L^i_\tau(g) = \text{MinDown} \cdot p_g^\tau \leq \omega_i \tag{10}
\]

\[
\frac{mp^i_t(g)}{\text{income}^i} \equiv DTI^i_t(g) \leq \text{MaxDTI} \tag{11}
\]

where \( \text{income}^i \) is agent \( i \)'s monthly income and \( \omega_i \) is agent \( i \)'s savings. Both are attributes of agent \( i \) and remain the same throughout time. The first condition says that the minimum down payment has to be less than or equal to savings. The second condition says that the debt-to-income ratio has to be less than or equal to the maximum debt-to-income ratio.

### B.3 Bank

We assume perfect competition among banks (funds) for the loans. The market arbitrage condition thus require that the expected return on the loans equals the exogenous market return, \( r_t^F \),

\[
\text{prob}_t^D \cdot \bar{co}_t + (1 - \text{prob}_t^D) \cdot (1 + r_t) = 1 + r_t^F \tag{12}
\]

where \( \text{prob}_t^D \) is the default probability estimated by bank, \( \bar{co}_t \) is the mean collateral rate, \( m_t \) is the mortgage rate, and \( r_t^F \) is risk-free return rate at time \( t \). \( \text{prob}_t^D \) is defined as follows,

\[
\text{prob}_t^D = \frac{\sum_{i \in C} I(\text{client } c \text{ went into foreclosure})}{\sum_{i \in C} 1} \tag{13}
\]

where \( C \) is the set of all clients borrowing from the bank, \( I \) is the indicator function, and \( \bar{co}_t \) is the mean collateral rate, which is defined below,

\[
\bar{co}_t = \frac{\sum_{i \in C} co^i_t \cdot L^i_t}{\sum_{i \in I} L^i_t} \tag{14}
\]

where \( co^i_t \) is collateral rate of client \( i \), which is defined below,

\[
co^i_t = \max \left\{ \frac{p^i_t}{L^i_t}, 1 + m_\tau(i) \right\} \tag{15}
\]
where $p_{i,g}^t$ is value of client $i$’s property at time $t$, $L_i^t$ is agent $i$’s outstanding loan value at time $t$ and $m_{r(i)}^t$ is the mortgage rate client $i$ is paying when client $i$ took out the loan in period $\tau(i)$. Bank’s mortgage rate at period $t$, $r_t$ thus is,

$$m_t = \frac{1 + r_t^F - \text{prob}_t^D \cdot \bar{c}_t}{1 - \text{prob}_t^D} - 1$$

(16)

### B.4 Regular Buyer

For regular buyer $b$, its objective is,

$$\max_{g \in F^b} U^b_t(g) = U\left(NbhdQ^g_t, LocQ^g_t, \text{Composite}^b_{t,g}; \beta^b_t\right)$$

(17)

where $F^b$ is the feasible choice set of regions for buyer $b$. It includes the null option, that is, the option of not buying. We assume $U^b(\text{null})=0$. $NbhdQ^g_t$ is the neighborhood quality in region $g$, $LocQ^g_t$ is the location quality in region $g$. $\text{Composite}^b_{t,g}$ is the composite good buyer $b$ can consume if buyer $b$ buys a property in region $g$ at time $t$. $\beta^b_t$ is buyer-specific preference parameter vector in the utility function. $\text{Composite}^b_{t,g}$ is defined as,

$$\text{Composite}^b_{t,g} = \left(1 - DTI_{b,g}^t\right) \cdot \text{income}^b_t$$

(18)

where $\text{income}^b_t$ is buyer $b$’s monthly income, and $DTI_{t,b,g}^t$ is buyer $b$’s debt to income ratio if she purchases a property in region $g$. $\text{Composite}^b_{t,g}$ thus is the composite good buyer $b$ can afford after paying the monthly payment for mortgage. During month $t$, regular buyer $b$ searches each region in the feasible choice set, and chooses the region that gives her the highest expected utility. Call that chosen region $g_t(b)$. Regular buyer $b$’s bid at period $t$ for a house in region $g$, bid$_t^b(g_t(b))$, is defined in equation 19,

$$\text{bid}_t^b = \text{bid}_t^b(g_t(b)) = (1 - \eta + \delta \cdot \text{TOM}_t^b) \cdot P_{t-1}^{gr(b)}$$

(19)

where $g_t(b)$ is the region chosen by buyer $b$ at time $t$, $\eta$ is a buyer’s initial discount on current price, set to be 10% at current study. $\delta$ is the percentage increase in price each period the buyer’s bid is not accepted, set to be 2% at current study. $P_{t-1}^{gr(b)}$ is the last period price in the buyer $b$’s chosen region $g_t(b)$. $\text{TOM}_t^b$ is buyer $b$’s time-on-market at period $t$. The bigger the time-on-market, the higher the bidding price. For one more period on market, the bidding price is increased by $\delta$ percent. Once the price is settled by the real estate agent, regular buyer $b$’s action in period $t$, $a_t^b$ is defined below.

$$a_t^b = \begin{cases} 
\text{buy a house and become a homeowner} & \text{if } \text{bid}_t^b \geq P_{t}^{gr(b)} \\
\text{enter period } t+1, \text{TOM}_{t+1}^b = \text{TOM}_t^b + 1 & \text{if } \text{bid}_t^b > P_{t}^{gr(b)} \text{ and } \text{TOM}_t^b > \text{MaxTOM} \\
\text{exit market} & \text{if } \text{bid}_t^b > P_{t}^{gr(b)} \text{ and } \text{TOM}_t^b \geq \text{MaxTOM}
\end{cases}$$
where MaxTom is the maximum time on market. A buyer will continue to increase her bid until the time she has been on the market exceeds the maximum waiting period. Then the buyer will leave the housing market.

### B.5 Speculative Buyer

We define $\mathbf{P}_t^g$ as the housing price history over the last 12 periods (months) or a year in region $g$ at period $t$. $\mathbf{P}_t^g$ is defined as,

$$
\mathbf{P}_t^g = \{p_t^g, p_{t-1}^g, \ldots, p_{t-12}^g\} \quad (20)
$$

$N^g$ is the set of neighboring or bordering regions of region $g$, and $n(g)$ is the number of the bordering regions of region $g$, which differs across regions. In other words, $n(g)$ is the number of elements in $N^g$: $n(g) = |N^g|$. Formally, set $N^g$ can be defined as,

$$
N^g = \{g_1, g_2, \ldots, g_{n(g)}\} \quad (21)
$$

The information set of region $g$ at time $t$, $I_t^g$, includes the price history of region $g$ and price of all its neighboring regions over the last 12 periods:

$$
I_t^g = \{\mathbf{P}_t^g, \mathbf{P}_{t}^{g_1}, \mathbf{P}_{t}^{g_2}, \ldots, \mathbf{P}_{t}^{g_{n(g)}}\} \quad (22)
$$

We then define $ER_b^t(g)$ as the expected return rate of speculative buyer $b$ for the purchase of a house in region $g$ in period $t$ and resell the house in a year, i.e., in period $t+12$. $ER_b^t(g)$ for all $b(g) \neq \text{null}$ is defined as follows,

$$
ER_b^t(g) = ER\left(I_t^g; w^b\right) = (1 - w^b) \cdot \frac{p_t^g}{p_{t-12}^g} + w^b \cdot \frac{1}{n(g)} \sum_{g' \in N^g} \frac{p_{t}^{g'}}{p_{t-12}^{g'}} - m_t - HC \quad (23)
$$

\forall b(g) \neq \text{null}

Speculative buyer $b$’s expected return rate on a housing investment in region $g$ depends on prices in region $g$ in the past. It also depends on prices in region $g$’s neighboring regions in the past. The parameter $w^b$ is the weight speculative buyer $b$ put on the mean price appreciation in the neighboring regions of region $g$ relative to the price appreciation in region $g$ itself. $w^b$ is an attribute of speculative buyer $b$, $w^b \in (0, 1)$. The expected net return also depends on the mortgage rate at time $t$, $m_t$, and the annual cost of holding a house as percentage of property value. We call it holding cost, $HC$, which includes property tax, transaction cost, depreciation, and (minus) rental income as percentage of property value. $HC$ is set at 2% at current study. For a speculative buyer, her objective is to maximize net return rate. Speculative buyer $b$’s objective function thus is,

$$
\max_{g \in F^b} ER_b^t(g) \quad (24)
$$
where $F^b$ is the feasible choice set of regions for speculative buyer $b$, including the option of not buying a house or $g$=null. We assume $ER(null) = m^F_t \cdot \text{MinDown}$, that is, the risk-free return rate at time $t$ multiplied by the minimum down payment required by the bank, $\text{MinDown}$, for all buyer $b$ and at all time $t$. The reason it is so lies in the magnifying effect of leverage. Investing in the housing market gives a speculator access to the credit market to which she has not access if she invest her down payment money in the risk-free deposit. Because of leverage, returns on the real estate investment is magnified. During month $t$, speculative buyer $b$ searches each region in the feasible choice set, and chooses the region that gives her the highest expected return. Call that chosen region $g_t(b)$. Regular buyer $b$’s bid at period $t$ for a house in region $g$, $\text{bid}^b_t(g_t(b))$, is defined in equation 19,

$$\text{bid}^b_t(g_t(b)) = \begin{cases} (1 + \frac{ER^b_t}{12}) \cdot P^{g_t(b)}_{t-1} & \text{if } g_t(b) \neq \text{null} \\ N/A & \text{if } g_t(b) = \text{null} \end{cases} \tag{25}$$

where $g_t(b)$ is the region chosen by speculative buyer $b$ at time $t$ that gives her the highest expected return rate. Once the price is settled by the real estate agent, For any speculative buyer $b$ who has submitted a bid to a a region, i.e., $g_t(b) \neq \text{null}$, her action at time $t$, $a^b_t$ is,

$$a^b_t = \begin{cases} \text{buy a house and become a homeowner} & \text{if } \text{bid}^b_t \geq P^{g_t(b)}_{t-1} \\ \text{exit market} & \text{if } \text{bid}^b_t < P^{g_t(b)}_{t-1} \end{cases}$$

B.6 Regular Homeowner

Once a regular buyer $b$ has succeeded in the purchase of a house in her chosen region $g(b)$, she becomes a regular homeowner $h$ in region $g(b)$. At the beginning of each period, homeowner $h$ can hold, list, or default on her property. A regular homeowner $h$’s action at period $t$, $a^h_t$, thus is,

$$a^h_t = \begin{cases} \text{default} & \text{if } P^h_{t-1} + DC^h_t < L^h_t \\ \text{hold property} & \text{if } h \text{ does not default } & \text{a rare event does not occur} \\ \text{list property,} & \text{a rare event occurs} \\ \text{ask}^h_t(g(h)) = (1 + \eta - \delta \cdot \text{TOM}^h_t) \cdot P^{g(h)}_{t-1} & \text{if } h \text{ does not default } & \text{a rare event occurs} \end{cases}$$

where $g(h)$ is the region in which homeowner $h$ homeowner $h$ owns a property. $\eta$ is the targeted margin over current price, which is set to be 10% at current study. $\delta$ is the percentage decrease in price each period a seller’s bid is not accepted, which is set to be 2% at current study. $P^h_{t-1}$ is the price in the region where homeowner $h$’ owns property in period $t - 1$. $\text{TOM}^h_t$ is seller $h$’s time one market at period $t$. The way a regular homeowner sells a house is symmetric to the way she buys a house.
There is a small probability (set to be 5% at current study) that any homeowner will want to sell the house for exogenous reasons such as job move or divorce. We call them rare events. When a rare event occurs, homeowner $h$ will list her property. $L^h_t$ is homeowner $h$’s outstanding loan at time $t$ and $DC^h$ is the default cost of homeowner $h$. A regular homeowner will default if the price of her property and the default costs fall below the value of the outstanding loan. Default costs are costs associated with default and foreclosure, such as legal cost, loss of credibility, and mental stress. Default cost of homeowner $h$ is assumed to be proportional to the monthly income of homeowner $h$ at the current study.

B.7 Speculative Homeowner

For speculative homeowner $h$, its objective is,

$$\max_{\text{default, hold, list}} \ ER^h_t = ER^h_t(g(h))$$

where $R^h_t(g(h))$ is speculative buyer $h$’s expected return rate on the property she owns in region $g(h)$ at time $t$. Speculative buyer $h$’s action at period $t$ thus is,

$$a^h_t = \begin{cases} 
\text{default} & \text{if } P^h_{t-1} + DC^h < L^h_t \\
\text{hold the property} & \text{if } h \text{ does not default} \&
   \text{ER}^h_t \geq \text{ER(null)} \\
\text{list the house,} & \text{if } h \text{ does not default} \&
   \text{ask}^h_t(g(h)) = (1 + ER^h_t/12) \cdot P^g_{t-1} \text{ ER}^h_t < \text{ER(null)} \\
\text{ask}^h_t(g(h)) = (1 + ER^h_t/12) \cdot P^g_{t-1} \text{ ER}^h_t < \text{ER(null)} 
\end{cases}$$

Unlike a regular homeowner who lists her house for exogenous reasons, a speculative homeowner will list her house if and only if the expected return on the property is negative. Like a regular homeowner, a speculative homeowner will default if the price of her property and the default costs fall below the value of the outstanding loan.

B.8 Developer

The total cost of building $q$ houses in region $g$ at the beginning of period $t$, $TC^g_t(q)$, is assumed to be a quadratic function in $q$:

$$TC^g_t(q_t) = c_1 h + \frac{c_2}{2} (q_t)^2 + \frac{c_3}{2} (Q^g_{t-1})^2$$

where $c_1$, $c_2$, and $c_3$ are positive parameters for construction cost. $Q^g_{t-1}$ is total number houses already exist on region $g$ at period $t - 1$; while $q^g_t$ is the number of houses just finished being constructed in region $g$ at time $t$. We assume that the more houses exist in
the region, the more expensive to build new constructions due to land scarcity, regulations, etc. $Q^g_t$ for all regions $g \in G$ is updated at the beginning of period $t$,

$$Q^g_0 = 0$$
$$Q^g_t = Q^g_{t-1} + q^g_t \forall t > 0$$

The marginal cost, $MC^g_t(q^g)$, thus is

$$MC^g_t(q^g) = c_1 + c_2 q^g_t + c_3 Q^g_{t-1}$$

The developer’s supply in region $g$ in period $t$, $q^g_t$, is set where price equals marginal cost $T^{Dev}$ periods ago, because the decision was made $T^{Dev}$ periods ago, where $T^{Dev}$ is the number of periods it takes for the developer to build houses.

$$P^g_{t-T^{Dev}} = MC^g_t(q^g_t) = c_1 + c_2 q^g_t + c_3 Q^g_{t-1-T^{Dev}}$$

Hence $q^g_t$ equals,

$$q^g_t = \frac{P^g_{t-T^{Dev}} - c_1 - c_3 Q^g_{t-T^{Dev}}}{c_2} - q_{stock}^g$$

where $stock_t^g$ is housing stock in region $g$ at period $t$. $stock_t^g$ is updated at the beginning of month $t$,

$$q_{stock}^0 = 0$$
$$q_{stock}^g_t = q_{stock}^g_{t-1} + q^g_t - q_{sold}^g_t \forall t > 0$$

where $sold_t^g$ is the number of houses sold in region $g$ during period $t$, which equals

$$q_{sold}^g_t = \min \left\{ q^g_t, \frac{P^g_t - c_1 - c_3 Q^g_{t-T^{Dev}}}{c_2} \right\}$$

The asking price for a newly constructed house $j$ at time $t$, $ask^j_t$, is its marginal construction cost,

$$ask^j_t = c_1 + c_2 \cdot j + c_3 H_t^g$$
$$\forall j = 1, 2, ..., q^g_t$$

Existing housing stocks in region $g$ at time $t$ are listed at $c_1 + c_3 H^g_{t-1}$. 

34
C  UML Presentation Of the Housing Model

Figure 17 is a class diagram of the model. It summarizes the model’s class structure and demonstrates relationships between different types of agents.

Figure 18 is an activity diagram of the housing model. It summarizes how market participants interact at each point. A red line represents an information flow, and a black line represents an action flow.

D  Results From Computational Experiments

Housing price volatility for the complete parameter space are shown in Figure ???. In the red rectangle is the more densely-sampled region. Number of non-speculative homeowners for the complete parameter space are shown in Figure ???. In the red rectangle is the more
densely-sampled region.
Figure 18: The Activity Diagram For The Housing Model
Figure 19: Housing Price Volatility, Complete Range (standard deviation in parenthesis)

<table>
<thead>
<tr>
<th>S</th>
<th>L</th>
<th>0.000</th>
<th>0.200</th>
<th>0.400</th>
<th>0.600</th>
<th>0.700</th>
<th>0.750</th>
<th>0.800</th>
<th>0.825</th>
<th>0.850</th>
<th>0.875</th>
<th>0.900</th>
<th>0.925</th>
<th>0.950</th>
<th>0.975</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.003</td>
<td>0.023</td>
<td>0.025</td>
<td>0.022</td>
<td>0.021</td>
<td>0.026</td>
<td>0.017</td>
<td>0.016</td>
<td>0.015</td>
<td>0.017</td>
<td>0.018</td>
<td>0.018</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>0.025</td>
<td>0.000</td>
<td>0.003</td>
<td>0.032</td>
<td>0.031</td>
<td>0.030</td>
<td>0.026</td>
<td>0.029</td>
<td>0.033</td>
<td>0.062</td>
<td>0.111</td>
<td>0.181</td>
<td>0.204</td>
<td>0.244</td>
<td>0.235</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.050</td>
<td>0.000</td>
<td>0.002</td>
<td>0.042</td>
<td>0.039</td>
<td>0.038</td>
<td>0.040</td>
<td>0.046</td>
<td>0.060</td>
<td>0.123</td>
<td>0.223</td>
<td>0.219</td>
<td>0.228</td>
<td>0.241</td>
<td>0.222</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.075</td>
<td>0.000</td>
<td>0.003</td>
<td>0.042</td>
<td>0.046</td>
<td>0.051</td>
<td>0.055</td>
<td>0.067</td>
<td>0.085</td>
<td>0.162</td>
<td>0.265</td>
<td>0.247</td>
<td>0.228</td>
<td>0.189</td>
<td>0.213</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.100</td>
<td>0.000</td>
<td>0.002</td>
<td>0.046</td>
<td>0.054</td>
<td>0.059</td>
<td>0.066</td>
<td>0.081</td>
<td>0.097</td>
<td>0.178</td>
<td>0.273</td>
<td>0.258</td>
<td>0.246</td>
<td>0.224</td>
<td>0.214</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.150</td>
<td>0.000</td>
<td>0.002</td>
<td>0.054</td>
<td>0.068</td>
<td>0.068</td>
<td>0.085</td>
<td>0.099</td>
<td>0.115</td>
<td>0.194</td>
<td>0.248</td>
<td>0.241</td>
<td>0.230</td>
<td>0.219</td>
<td>0.215</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.200</td>
<td>0.000</td>
<td>0.003</td>
<td>0.059</td>
<td>0.077</td>
<td>0.081</td>
<td>0.085</td>
<td>0.101</td>
<td>0.128</td>
<td>0.204</td>
<td>0.244</td>
<td>0.198</td>
<td>0.212</td>
<td>0.189</td>
<td>0.193</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>0.250</td>
<td>0.000</td>
<td>0.004</td>
<td>0.066</td>
<td>0.073</td>
<td>0.082</td>
<td>0.090</td>
<td>0.100</td>
<td>0.118</td>
<td>0.183</td>
<td>0.232</td>
<td>0.203</td>
<td>0.172</td>
<td>0.189</td>
<td>0.187</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>
Figure 20: Number of Non-speculative Homeowners, Complete Range

<table>
<thead>
<tr>
<th>S</th>
<th>L</th>
<th>0.000</th>
<th>0.025</th>
<th>0.050</th>
<th>0.075</th>
<th>0.100</th>
<th>0.125</th>
<th>0.150</th>
<th>0.200</th>
<th>0.250</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>3482</td>
<td>432</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>33</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 20: Number of Non-speculative Homeowners, Complete Range

39