Computational Geometry for Optimal Workpiece Orientation

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Abstract
Workpiece orientation is formulated as an optimal design problem based on a discrete approximation of design surface geometry, the kinematic capabilities of the process machine tool, and processing cost. The primary process application addressed is three-and four-axis numerically controlled (NC) milling, although the techniques presented may be applied to machines with more general articulation. Recent developments in applied spherical geometry are employed to formulate a constrained problem, and furthermore, a nonlinear optimization problem. For three-axis milling applications, a weight is assigned to each surface normal of the discrete model corresponding to the actual area it represents. Workpiece/machine orientation is optimized such that the angle between the weighted normals and the milling tool axis is minimized. This formulation is augmented, for four-axis milling, to incorporate limitations of the rotational degree of freedom, into the optimization formulation. The influence of tool geometry is also discussed and incorporated within constrained orientation algorithm.

Keywords
Weight (Mass), Machinery, Machine tools, Degrees of freedom, Algorithms, Computational geometry, Design, Optimization, Approximation, Geometry

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Comments
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Introduction

Workpiece orientation is an important consideration for many manufacturing processes. For example, in NC milling operations a particular workpiece orientation may allow machining of the entire part in one setup, while in other orientations, portions of the workpiece may not be accessible by the milling tool, thus requiring reorientation and refixturing of the workpiece. For robotics applications, such as inspection, assembly, welding, and painting, workpiece orientation is critical for fully exploiting the capabilities of the machine. This paper presents a general approach to optimal workpiece orientation based on the geometry of the design part, and the geometry and kinematics of the machine tool. Although the primary application addressed in this paper is the optimal workpiece orientation for three- and four-axis NC milling, the techniques presented can be extended to accommodate other more generally articulated machine processes.

To remove material, NC milling machines rotate a cutting tool about its axis of symmetry. The machine is programmed to move the rotating tool along prescribed trajectories so that the remaining material has the shape of the designed part. A three-axis milling machine provides control of tool translation in three orthogonal directions; one along the tool axis, and two others in a plane perpendicular to it. However, for parts with complex geometry, a three-axis tool articulation may not be sufficient to provide accessibility to all portions of the workpiece in one setup. Four- and five-axis NC milling machines provide additional tool accessibility by allowing for rotation of the tool axis itself. In four-axis milling the tool-axis may rotate about an orthogonal axis, while a five-axis machine provides additional rotational degrees of freedom about two orthogonal axes in a plane perpendicular to the tool axis. Of course, these additional degrees of freedom increase machine complexity and cost. The ability to determine, from part geometry, the necessity of four- or five-axis machining capabilities for a specific workpiece, would help manufacturers in reducing the cost of production by choosing the appropriate machine type.

In general, workpiece setup is a time consuming and labor-intensive process. Although a complex-shaped part could be milled with multiple setups on a three-axis machine, it may be more cost-effective to mill it in one setup on a four- or five-axis milling machine. The ideal orientation of a workpiece with respect to a milling tool is one in which the tool axis is aligned with the normals of the surface. This results in the largest machined regions for a given tool size thus reducing milling time, cusp heights, and hand finishing. However, such an ideal tool orientation is not possible at every point on the surface since physical milling machine constraints restrict tool orientation to certain angular domains of the unit sphere. This paper presents a method to find the workpiece/machine orientation which minimizes the angle between design surface normals and a ball-end or fillet-end milling tool axis and minimizes the number of tool motions in rough machining. This technique is applicable to both three- and four-axis milling machines. Recent research results in applied spherical geometry are employed to first determine whether the workpiece can be milled completely in a single setup on a three-axis milling machine. If not, the physical limits on the rotational degree of freedom for a four-axis milling machine is incorporated within the optimal problem formulation.

The general orientation problem requires a complete geo-
metric model of the design part, which may include multiple sculptured and planar surfaces as input. A discrete approximation of these bounding surfaces (Drysdale and Jerard, 1987) is used to form a Gaussian map (GMap) of the workpiece (Chen and Woo, 1992). The GMap is then augmented by assigning a weight to each normal corresponding to the relative area of the design surface which it represents. Since the discretization algorithm is based only on local surface curvature, this modification provides a means to incorporate surface size information with the orientation data provided with the GMap. The goal is to use this augmented GMap to find an optimal workpiece orientation subject to the capabilities and limitations of the machine tool.

Spherical Geometry

Using spherical geometry, points are defined on the d-dimensional unit sphere  S^d. Let  E^{d+1} denote the d + 1 dimensional Euclidean space and  p = (x_1, x_2, ..., x_{d+1}) denote a point in  E^{d+1}, then  S^d = {p : |p| = 1}

This application deals with  S^2, the sphere of unit radius in  E^3. In this paper, a general familiarity with the terminology associated with spherical geometry is helpful. Terms such as, great circle, small circle, hemisphere, convex hull (CH), spherical polygon, spherical convex hull (SCH), and normal (N^-), are used throughout this paper and are described in the computational geometry literature (e.g., Preparata and Shamos, 1985) or in recent research in numerically controlled milling (Chen and Woo, 1992; Haghpassand, 1994).

The following section reviews some necessary terminology to aid the presentation of the algorithms in this work.

Definitions

Central Projection, CP. The central projection of a point  (x_1, x_2, x_3) in  S^2 with  x_3 > 0 is defined as  (x_1/x_3, x_2/x_3, x_3) in  E^3 or the point  (x_1/x_3, x_2/x_3, 1) in  E^3. Central projection is a two-to-one mapping since the antipodal point  (-x_1, -x_2, -x_3) maps to the point  (x_1, x_2, x_3) in the plane of  x_3 = 1 (and thus  (x_1/x_3, x_2/x_3) in  E^3). A point  (x_1, x_2, x_3) with  x_3 = 0 is mapped onto the line at infinity in  E^3.

Central projection can be used to convert a problem in  S^2 to an equivalent planar problem in  E^3. Well known algorithms from computational geometry can then be applied on the plane, and the solution can be mapped back onto the sphere by inverse central projection.

Line. A line  l in  S^2 is a great circle, the intersection of the unit sphere with a plane containing the origin. The intersection of this plane with  E^3 is a line  L, the image of  l under central projection.

Distance, D. The distance between two distinct points  a and  b in  S^2, is the length of the line segment joining them. It is defined by the metric,

\[ D(a, b) = \cos^{-1}(a \cdot b) \]

Line Segment, LS. The line segment LS(a, b) that joins two distinct points  a and  b in  S^2, is the shortest arc on the great circle that contains those points. This segment is unique, except for the case when  a and  b are antipodal points.

Gaussian map, GMap. The intersections of N surface normals, and the unit sphere will generate a set of N points in  S^2, called the GMap. The GMap for a sculptured surface can be approximated by tessellating the surface and mapping the normals of each facet.

Visibility Map, VMap. To machine a surface with a ball or fillet-end (toroidal) milling tool, the axis of the tool must not deviate from the surface normals by more than 90 deg. For a point to be visible (accessible) by a tool, tool orientation is limited by the tangent plane at that point. For a fillet-end milling tool, this visibility constraint will be increased by an angle  \( \beta \) (Chen and Woo, 1992).

Optimal Orientation

Given GMaps of the design part surfaces, the plane  x_3 = 1 may be selected as the plane of the CP, where the two coordinates in  E^2 are parallel to the  x_1 and  x_2 axes. One of the disadvantages of CP is that the projection of points which lie on the equator of the unit sphere (plane of  x_3 = 0) will map to a line at infinity in  E^2. Since the transformation of points near the equator may exceed the limits of machine precision, a rotation transformation is necessary to move all points in the GMap as far from the equator as possible. To address this problem, Chen and Woo (1992) suggest a technique to find two successive transformations by first projecting all of the GMap points and their antipodes onto the  x_3 = 0 plane, and finding the largest empty gap between the unitized projected points. The angle between the bisector of this gap and the  x_2-axis constitutes the first rotation, which moves most of the points away from the equator. The second rotation is formulated similarly after the transformed points are projected onto the  x_3 = 0 plane. After application of the second rotation, all of the points in the GMap are transformed away from the equator. Since this algorithm is simple and computationally efficient (ONN time) it has been implemented without modification for this application. With the initial set of points P of the GMap the transformations R1 and R2 are applied to obtain the transformed set of P" in  S^2. After CP, the set of points  P" in  E^2 can be partitioned into subsets  P"^- and  P"^+ based on the sign of  x_3. In other words, P is divided into northern and southern hemisphere subsets. In  E^2, the convex hulls of  P"^- and  P"^+, denoted as CH+(•) and CH(•^-), are constructed in ON(N logN) time (Preparata and Shamos, 1985). Next, the intersection of CH(•) + and CH(•^-) is constructed in ON(N) time (Preparata and Shamos, 1985). The relationship between CH(•) + and CH(•^-) is described by one of two possibilities.

1—No Intersection. In the case of a null intersection of CH(•) + and CH(•^-), a single three-axis orientation will allow tool access to the entire design surface.

Three-Axis Constrained Solution. In this case, to find a feasible setup direction, Chen and Woo (1992) proposed a method which first finds a separating line L between CH(•^-) and CH(•+) in  E^2 using a planar algorithm (Megiddo, 1983). The inverse central projection of L yields a line  l in  S^2, and the normal  N^- of  l is a feasible although not necessarily optimal solution. In other words, if  N^- is aligned with the spindle axis of a three-axis milling machine then all of the surfaces of the workpiece can be accessed by a ball-end tool.

Three-Axis Optimal Solution. Since a GMap characterizes only the visibility of various surfaces, there is no means to account for relative surface area, and therefore, the machining cost cannot be minimized with the approach described by Chen and Woo (1992). For example, suppose a design part model is composed of large planar surface combined with relatively small areas modeled by sculptured surfaces. With the above technique, the single normal which represents a very large portion of the total area is treated with the same importance as a normal representing a small facet of the sculptured surface. Indeed, the numerous normals from the discretized sculptured surface could dominate the solution to such an extent that the normal representing the large planar surface could be oriented in an awkward position relative to the tool axis.

One approach to this problem is to assign to each normal, a weight relative to the surface area of its facet. The normals...
are then written in sorted order relative to area. A subset $P_x$ of the normals with the largest areas is considered to generate a smaller subset of the normals which represent relatively large areas of the surface. This final list $P_x$ is generated from $P_x$ by comparing the distance between pairs of entries $P_x$ to some small deviation $\delta$. If two normals lie within $\delta$ of one another, the one with the smallest area is eliminated and its weight is added to the other normal. Thus, the list of weighted normals is reduced to some relatively small number (generally < 100).

Using the weighted normals to build an augmented $G_M$ map, a general three-axis separating line solution can be optimized with respect to tool contact orientation and surface area by penalizing the distance from the tool axis $t$ to each weighted normal $W_i$. As depicted in Fig. 1, this problem is formulated for the ball-end tool as,

\[
\text{Minimize: } \left( (W_{i1})^2 + (W_{i2})^2 \right) \forall W_i \in P_x
\]

Subject to:
\[
V_{i1} \geq 0.0; \forall W_i \in \text{SCH}(+), \text{or } \text{SCH}(-)
\]

where the independent variables are the two angular orientations about the $x_1$ and $x_2$ axes.

**Definition: $\beta$-Reduced Visibility.** The tool cutting surfaces are symmetric about the tool axis and can be approximated by a few simple mathematical models. The angle $\phi$ between the normals of two extreme tangent planes of the tool cutting surface is referred to as visibility magnitude. Thus, a tool axis can deviate from the surface normals from 0 to a drilling tool up to $\pi/2$ for a ball-end tool. For a fillet-end tool that can operate only with a portion of the tool-end, $V$ is reduced from $\pi/2$ by $\beta$ as in Fig. 2.

**Application of Fillet-End Tool.** A tool with visibility angle less than $\pi/2$ reduces accessibility to the design surface. As the result of this additional constraint, the three-axis optimization formulation may be restated as following:

\[
\text{Minimize: } \left( (W_{i1})^2 + (W_{i2})^2 \right) \forall W_i \in P_x
\]

Subject to:
\[
V_{i1} \sin (\beta) \geq 0.0; \forall W_i \in \text{vertices of SCH}(+), \text{or } \text{SCH}(-)
\]

2—Intersection. If $\text{CH}(\times)$ and $\text{CH}(\times)$ intersect, then no single orientation will allow access to all surfaces with a three-axis mill, but the part could be milled with multiple setups on a three-axis milling machine. Tang et al. (1992) present a method for finding a line which intersects a maximal subset of spherical convex hulls. The normal of this line can be aligned with the mill axis to provide access to many surfaces. However, the solution is not optimal and may yield a result which violates the limits of tool articulation.

An alternative approach is to construct $V_M$ maps corresponding to $\text{SCH}(\times)$ and $\text{SCH}(\times)$, then apply a spherical adaptation of an algorithm for finding the distance between two planar convex polygons (Edelsbrunner, 1988; Schwartz, 1981). However, this algorithm requires construction of the convex hull of the union of the two polygons. Since, in this case the $G_M$ maps contained in $\text{SCH}(\times)$ and $\text{SCH}(\times)$ are not hemispherical, the union of the $V_M$ maps might comprise the entire sphere. Thus, implementation of this planar algorithm directly on $S^2$ is not practical.

Another approach is to measure the shortest distance between two $V_M$ maps directly. But in order to prevent the tool and the workpiece from collision, a ball-end tool may not have maximum visibility of $\pi/2$ and the point visibility will be reduced by $\beta$. Thus, the construction of $V_M$ map from $G_M$ map that is a spherical cap, will be in $O(M \cdot N)$ time, where $M$ depends on the accuracy of the linearization of the $V_M$ map’s boundary, and $N$ represents the total number of $V_M$ map’s vertices. Therefore, the distance between two $V_M$ maps can be calculated in $O(M \cdot K)$ time, where $K$ is the quadratic computation time for “the distance calculation between two polygons” algorithm in $S^2$.

**Definition: Spherical digon.** A spherical digon of angle $\alpha$ is a subset $D$ of $S^2$ bounded between two half-great-circles with same endpoints $\pm m$ and whose tangent vector at $m$ form an angle $\alpha$ (Berger, 1987).

**Minimum Digon Algorithm.** Formulation of the optimal workpiece orientation problem requires a method for finding the smallest digon necessary to cover a spherical convex polygon $P$. Based on previous similar applications of spherical geometry to automated machining problems (Chen and Woo, 1992), one may expect that this problem could be approached by first mapping the spherical polygon onto $E^2$ through CP, then applying a known planar algorithm for finding the width of a planar polygon, such as the “rotating calipers” approach of Houle and Toussaint (1988). However, since CP is a projective mapping, it distorts distance and angle and cannot be applied directly to problems involving proximity or measurement of magnitude.

Another approach would be to modify algorithms developed for $E^2$ to operate directly in $S^2$. For example, the rotating calipers algorithm of Houle and Toussaint defines the width of a polygon as the minimum vertex-to-edge distance. However, on a spherical polygon the minimum vertex-to-edge distance is a necessary but not sufficient condition for the polygon width.
Theorem A. It has been shown that one of the bounding lines of the minimum spanning digon of a spherical convex polygon \( P \) will lie on an edge \( E_i \in P \) and the other will either pass through a vertex \( V_j \notin E_i \) or will lie on another edge \( E_j \neq E_i \) (Haghpassand, 1994). The minimum digon algorithm uses each edge of the intersection of \( CH(+) \) and \( CH(-) \) as one of the bounding half-great-circles of digon, then searches other edges to find the minimum digon in \( O(N) \) time (Haghpassand, 1994).

Four-Axis Constrained Solution. The approach presented in this paper is to find a line segment in \( S^2 \) which specifies the minimum domain of tool orientation necessary to mill the part with one set up on a four-axis milling machine in \( O(N) \) time. Thus, we seek the smallest digon in \( S^2 \) such that a mill axis, which spans its equator will have access to all surfaces.

Since the intersection of \( CH(+) \) and \( CH(-) \) is constructed, the corresponding intersected edges on both \( SCH'(i) \) are known. Two pointers are assigned for each pair of intersecting edges. For example, as shown in Fig. 3, the two pointers \( P1 \) and \( P2 \) identify the edge intersection point \( a \) and the pointers \( P1r \) and \( P2r \) identify the intersection point \( b \). The constrained four-axis orientation algorithm proceeds by considering the edge of \( SCH(1) \) pointed to by \( P1 \), and employs the minimum digon algorithm described above to find the minimum digon of the polygon pointed by \( P2 \). To find the digon angle to cover \( SCH(2) \), the algorithm proceeds by incrementing \( P1 \) on \( SCH(1) \) and \( P2 \) on \( SCH(2) \). The above process will be stopped once all the edges of \( SCH(1) \) are used. Then, the entire process is resumed by assigning the values of the pointers \( P2 \) to \( P1 \) and \( P1r \) to \( P2r \). The minimum of all subsolutions is the constrained four-axis line segment solution with the magnitude \( \phi = D(N_1^*, N_2^*) \), as shown in Fig. 4.

\[
\alpha = \pi/2 - \cos^{-1}(\sin(\beta)/\cos(D(a,b)_2))
\]

where the \( LS(a,b) \) should satisfy the following condition.

\[
0.0 \leq \beta \leq \left( \pi - D(a,b) \right)/2
\]

Rough Cutting. For rough cutting, we can generate a list \( P_x \) similar to \( P_w \) to select several three-axis tool setups along which we can remove the maximum excess material that exists between design part and the raw block. This can be done by measuring the distance along each normal from the design part surface to the intersecting point of the normal and the raw stock without intersecting the workpiece surface. These magnitudes can be incorporated in a manner similar to the method presented above for surface area.

\( \beta \)-Reduced Visibility. Theoretically, when a successful constrained solution is obtained, all the workpiece surfaces are accessible by the cutting tool with an infinite length. However, a more practical approach is to reduce the point visibility from \( \pi/2 \) by an angle \( \beta \) to account for the geometry of the tool chuck as shown in Fig. 6. Therefore the ball-end may be represented as a fillet-end tool with \( \beta \)-reduced visibility. Based on theorem B, the constrained line segment solution may be modified to incorporate \( \beta \)-reduced visibility for a fillet-end tool into the VMaps (Haghpassand, 1994).

Four-Axis Optimal Solution. The constrained four-axis solution line segment defined by \( N_1 \) and \( N_2 \), may be computed as described above. In order to minimize the machining cost and to satisfy the machining kinematic constraints, we seek an optimal formulation, which will orient the entire constrained four-axis digon inside the tool digon. Any actual four-axis milling machine must have the angular mobility of at least \( \phi = D(N_1, N_2) \) to mill all of the surfaces in one setup. However if the mill has \( \phi > D(N_1, N_2) \), an optimal solution is formulated as follows: Given the subset of weighted surface normals \( P_w \) or excess material normals \( P_e \) described above, the optimization problem is stated as,
Minimize: $D^*(\text{Digon}(T), W')$
Subject to: $D^*(\text{Digon}(T), N_1) \leq 0.0$
$D^*(\text{Digon}(T), N_2) \leq 0.0$
$D(N, N^T) = 0.0$

where:
- $T$: The tool axis orientation
- $\text{Digon}(T)$: The tool's rotational digon, which represents maximum angular extent of the tool axis
- $N^T$: The tool's fourth axis rotation (perpendicular to the tool's spindle axis)
- $N$: The normal of the $LS(N_1, N_2)$
- $W$: A finite number of the extreme weighted normals
- $D^*$: The distance from a point to the plane containing either of the two half-great-circles of tool's rotational digon in $E^3$

$D$: The spherical distance

Design Variable: The angular rotation $\theta$ about $N^T$

This formulation will thus find the optimal workpiece orientation with respect to stock material volume and surface area, which also best exploits the capabilities of the machine.

**Application**

Workpiece orientation is an important consideration for many conventional and unconventional manufacturing processes. Component design can be successfully incorporated with optimal orientation planning, to fully exploit the capabilities of existing machines, in order to promote successful and robust designs. While this research is a contribution to the constrained three, four-axis optimal workpiece orientation planning, the results can be applied to many other areas as described by Albert and Hern (1989). These applications range from applications like visualization and electronics surface mounting, to conventional manufacturing applications like inspection (Albert, 1990), welding, painting and component assembly.

**Example**

The above algorithms are encoded in C and are compiled and executed with the Borland C ++ compiler version 3.1 on an Intel 80486 Microprocessor with the 80487 co-processor running at 33 MHz. A nonlinear optimization software package supplied by Northrup International, is used in the optimization formulations. We have implemented the algorithms on a workpiece that needs machining process on all its surfaces and is approximated by the model as shown in Fig. 7(a). The construction of the GMap as in Fig. 7(c) is achieved by using the normals to the discretized model as shown in Fig. 7(b). Thus, the four-axis constrained solution, i.e., $LS(N_1, N_2)$ is obtained as shown in Fig. 8. The augmented GMaps and the corresponding four-axis solution line segment with $\beta = 0.1$ radian are shown in Fig. 9, where the maximum tool orientation is determined by the end points $t_1$ and $t_2$. The optimally oriented workpiece is shown in Fig. 10. The four-axis optimization process has an execution time of 0.49 second with $\beta = 0.1$ radian and 0.28 second with $\beta = 0.05$ radian. The three-axis optimal algorithms are implemented on the same model as in Fig. 7(a). However, some of the original machining process on that model were eliminated. For example, the surfaces with normals along negative $X_2$-axis and with normal along positive $X_2$-axis are excluded from the machining process. Thus, the resultant $SCH$ of the GMap, shown in Fig. 11 has a constrained orientation as shown in
Conclusions

The algorithms presented in this paper provide a method for finding orientation for three-, and four-axis milling applications. In both cases a feasible solution is found first based on design part and tool geometry. It is then optimized with respect to surface area, tool contact orientation, and, in the four-axis case, with respect to the limits of the angular mobility of the machine. Implementation of these algorithms and their applications to actual part production is currently underway.

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