ABSTRACT

It is well known that when a finite ultrasonic beam of a given spatial distribution is incident at the Rayleigh angle to a liquid-solid interface, the spatial distribution of the reflected field may be altered significantly. The "energy redistribution" is due to the interference between the specularly reflected beam and a surface wave which has leaked back to the water. The "shape" of the reflected field depends on the so-called Schoch displacement (which is characteristic of the interface) and on the width of the ultrasonic beam. It has also been observed that significant energy is scattered back to the transmitter at the Rayleigh angle. Experimental results will be presented on the evaluation of the parameters effecting the backscattered amplitude. The backscattered Rayleigh angle phenomena are also applied to measured surface wave velocities of anisotropic materials such as casts and welds.

INTRODUCTION

Consider an ultrasonic beam incident to a liquid-solid interface. The mathematical analysis to calculate the reflection coefficient is well known for this problem. It is generally assumed, however, that the incident plane wave is infinitely extended in space. This assumption makes the mathematics simple but neglects the physically
realizable finitude of the beam which obscures significant physical mechanisms such as the so-called "Schoch displacement." Schoch\(^1\) by using schlieren photography demonstrated what he called the ultrasonic equivalent of the Goos-Hänchen effect:\(^2\) the center of the reflected beam is displaced by a certain amount along the interface if the angle of the incident beam is nearly equal to the Rayleigh angle. This "Schoch displacement" is attributed to the fact that the ultrasonic beam is "finite," bounded, and not infinitely extended, as is usually assumed. Schoch\(^1\) developed a theory by representing both incident and reflected field with a Fourier integral pair. He included mutual phase relationship of partial waves in the reflected beam by expanding the phase shift upon reflection into a power series. The first derivative of this phase shift is identified as the "Schoch displacement." Although this theory is successful because it predicts the beam displacement, it is not complete, and some physical significances are hidden in the analysis. As was pointed out by a number of people\(^3\)-\(^7\) the displaced beam is associated with the excitation of the surface waves on the interface. Experimental studies also revealed that there are two reflected beams at some liquid-solid interfaces. Bertoni and Tamir\(^8\) introduced a new theory to explain the Rayleigh angle phenomenon. Their theory postulates the existence of a specularly reflected wave and a reradiated surface wave which they call "leaky waves" because they leak energy back to the liquid. Bertoni and Tamir\(^8\) obtained an analytical solution for an incident beam with Gaussian distribution. Their solution is valid at the interface.

Breazeale, Adler and Scott\(^9\) modified the approximations for points in the liquid half space. They have verified experimentally the modified Bertoni and Tamir theory. Investigation of NDE problems using ultrasonic leaky waves were carried out by Adler and Scott\(^10\) to study the effect of near surface defects on the energy distribution of the reflected field. A thin layer on a substrate modifies also the leaky velocity as was shown by Adler and McCathern,\(^11\) resulting in a dispersion phenomenon which depends on the layer thickness.\(^12\) An analytical model to treat the effect on the leaky waves due to thin layers has recently been developed by Nayfeh, Chimenti, Adler and Crane.\(^13\) They show that the Schoch displacement becomes frequency dependent in a nonlinear fashion for the layered medium. Recent experimental observations\(^14\)-\(^18\) indicate the presence of a small but measurable signal which scatters back to the transmitter at the Rayleigh angle. In Fig. 1 both forward and backward scattering of leaky waves are illustrated schematically. The objective of this investigation is to analyze experimental results on this non-specular phenomena.

THEORETICAL CONSIDERATIONS

Bertoni and Tamir\(^8\) developed an analytical model for calculating the spatial distribution of the reflected field at a liquid-
solid interface. According to their predictions, for a beam incident at the Rayleigh angle the reflected beam profile depends on the dimensionless parameter $2W_O/\Delta_S$, where $2W_O$ is the projected beam width on the interface and $\Delta_S$ is the so-called Schoch displacement given by the relationship

$$\Delta_S = \frac{2\lambda\rho}{\pi} \left( \frac{r(r-s)}{s(s-1)} \right)^{\frac{1}{4}} \left( \frac{1 + 6s^2(1-q) - 2s(3-2q)}{s - q} \right)$$

(1)

where: $s = \left( \frac{C_S}{C_R} \right)^2$; $r = \left( \frac{C_S}{C_W} \right)^2$; $q = \left( \frac{C_S}{C} \right)^2$.

Bertoni and Tamir showed, through a series of plots of the reflected beam profile for various values of $2W_O/\Delta_S$, that the reflected field will have different shapes from the incident field if $2W_O/\Delta_S < 1$. For values of $2W_O/\Delta_S > 2$, however, the reflected beam is displaced only by an amount $\Delta_S$, as predicted by Schoch without any beam distortion or "energy redistribution." It appears plausible to assume that backscattering will take place mostly in the region of $2W_O/\Delta_S < 1$ where the energy "redistributes" itself on the interface, thus providing a mechanism to reradiate (leaky) waves in the backward direction. It is further assumed and attempts are made in this paper to prove experimentally that smaller values of $2W_O/\Delta_S$ will produce larger backscattered amplitudes from liquid-solid interfaces. Recent analysis by Norris theoretically predicts this assumption.
EXPERIMENTAL SYSTEM

The experimental setup is a standard system used for more general experiments consisting of an ultrasonic two-transducer goniometer system connected to a turntable which rotates the sample. For back-scattering experiments, we used only one transducer acting as transmitter and receiver. The orientation of the transducer is so critical that an accurate rotation system is necessary.* The sample and the transducer were immersed in a large tank filled with water. The distance between the scattering surface and the front face of the transducer can be adjusted, and was about 12 cm.

An ultrasonic system operated in pulse-echo mode: the rf signal is obtained from a pulse generator (Arenberg or Panametrics) and its length is 7.2 μs. The scattered signal is amplified and the video signal is then visualized on the screen of an oscilloscope (Fig. 2). A manual attenuator is added in the circuit of the receiver to adjust the amplitude of the backscattered echo on the CRT. The transducers we used in these experiments are commercial transducers with diameter ranging from 5 to 50 mm and the central frequencies of 4.5 or 6.8 MHz. We feel that the beam patterns of the transducers are such that the main lobe describes a Gaussian profile. The electronic disposition previously described can be modified for the studies with two transducers.

*The angle of incidence ($\theta_i$) is measured with respect to the normal of the interface and is, of course, equal to the backscattering angle ($-\theta_b$).
Fig. 3. Grain structure of centrifugally cast stainless steels with the radial direction vertical. From left to right: CPF 8 (low ferrite), CPF 8 (high Ferrite), CPF 8M (low ferrite), and CPF 8M (high ferrite). The sample on the far right was not suitable for the present work because it contained regions of large, roughly equixed grains.
DIFFERENT samples have been used for these experiments. For backscattering measurements we used samples of different materials and glass, which we considered as isopropic. Physical and acoustical properties of these specimens are given in Table 1: the density $\rho$, the bulk velocities and the Rayleigh's velocity. These former quantities have been experimentally measured. In the last column are given the Schoch's displacement ($\Delta_S$) calculated for $f = 4.5$ MHz. The dimensions of these samples are much larger than the incident wavelength ($\lambda$) and can be considered as infinite. The samples are held by a mechanical support such that the optically polished scattering surface contains at any time the axis of rotation.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$C_L$ (m/s)</th>
<th>$C_T$ (m/s)</th>
<th>$C_R$ (m/s)</th>
<th>$\Delta_S$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>6455</td>
<td>3125</td>
<td>2960</td>
<td>6.71</td>
</tr>
<tr>
<td>Glass (B 69.56)</td>
<td>3.7</td>
<td>5425</td>
<td>3156</td>
<td>2892</td>
<td>7.6</td>
</tr>
<tr>
<td>Titanium</td>
<td>4.5</td>
<td>6430</td>
<td>3163</td>
<td>2435</td>
<td>10.9</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>7.8</td>
<td>5740</td>
<td>3126</td>
<td>2892</td>
<td>17.6</td>
</tr>
<tr>
<td>Brass</td>
<td>8.6</td>
<td>4350</td>
<td>2102</td>
<td>2004</td>
<td>7.7</td>
</tr>
<tr>
<td>Copper</td>
<td>8.8</td>
<td>4700</td>
<td>2371</td>
<td>2211</td>
<td>10.2</td>
</tr>
</tbody>
</table>

For the anisotropy measurements, a piece of centrifugally austenitic stainless steel pipe (CPF 8) has been analyzed. Typical samples are represented in Fig. 3, which shows the oriented grain structure.
It has been shown that the major axes of the grains are oriented in the radial direction, the minor axis along the axis of the pipe and in a direction tangential to the radius of the pipe. On the basis of this grain structure it appears that these samples present an orthorombic symmetry. These three directions are shown in Fig. 4, which represents a section of cast pipe. The incident surface is again well polished and a mechanical system insures the rotation of the sample around the 2 axis. In that case, the transducer rotates around a vertical axis contained in the front face of the sample (parallel to the plane defined by the 1-3 directions).

**MEASUREMENT TECHNIQUES**

To study the angular variations of the backscattered signal, the following procedure, which avoids the nonlinearities in the gains of the amplifier, has been used. The mean amplitude of the backscattered signal is maintained at a constant level on the screen of the oscilloscope by adjusting the value of the manual attenuator. A normalization of the experimental data with the attenuation value for the backscattered signal obtained at normal incidence gives directly the variation of the normalized field scattered at the interface liquid-solid and captured by the receiver versus the angle of incidence \( \theta_i \). All the data required we fix an origin of the incidence angle: the angular position of the transducer for which we obtain a maximum amplitude of the video signal. The corresponding value of the attenuation is \( \alpha_0 \) (expressed in dB). The sample is then rotated one degree. A smaller value, \( \alpha_p \), is necessary to adjust the amplitude of the output signal at the arbitrary and constant level. Again the position of the emitter-receiver is incremented one degree and the process is repeated. The variation of the differences \( \alpha_b - \alpha_o \) (in dB) versus the angle of incidence (the backscattering angle) is recorded and plotted for comparison with different materials. Several experiments have been done with different transducers to observe the influence of the beam width on the results.

![Fig. 4. Coordinate system with cast pipe.](image)
RESULTS

Velocity Measurements

Typical plots of experimentally measured backscattered amplitude as a function of incident angles are shown in Fig. 5. The vertical scale is in dB and the data are normalized to $\theta_1 = 0^\circ$. The presence of backscattered peaks is observed at $\theta_1 = 31^\circ$ for stainless steel, $42^\circ$ for copper, $67^\circ$ for brass and $31^\circ$ for titanium. These experimental results and the theoretical developments have shown that a small peak appears in the backscattering diagram for $\theta_1 = \theta_R$ (Rayleigh's angle) at the interface liquid-solid. This presence of this maximum in the angular diagrams will be used to determine in an ultrasonic way the velocity of the surface wave. Remember that the velocity $v_R$ is related to the angular position of the peak by the straightforward relation

$$v_R = v_L \left( \sin \theta_i \right)^{-1}$$

where $v_L$ is the velocity in the liquid medium. This equation shows that the measurement of $\theta_1$ (or $\theta_R$) leads directly to the $v_R$ evaluation. This method has an accuracy of about 3% and appears to be a very good technique to measure the dispersion curve of the surface wave velocity. It also has the advantage that is uses only one transducer.

Fig. 5. Ultrasonic backscattering amplitude dependence on angle of incidence for water-stainless steel and water-copper interface.
Table 2.

<table>
<thead>
<tr>
<th>Material</th>
<th>Measured Rayleigh Angle</th>
<th>$N^e_R$</th>
<th>$N^{th}_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>30°</td>
<td>2960</td>
<td>2906</td>
</tr>
<tr>
<td>Glass</td>
<td>31°</td>
<td>2892</td>
<td>2897</td>
</tr>
<tr>
<td>Titanium</td>
<td>30.5°</td>
<td>2935</td>
<td>2961</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>31°</td>
<td>2892</td>
<td>2891</td>
</tr>
<tr>
<td>Brass</td>
<td>48°</td>
<td>2004</td>
<td>1965</td>
</tr>
<tr>
<td>Copper</td>
<td>42°</td>
<td>2211</td>
<td>2250</td>
</tr>
</tbody>
</table>

$C^e_R$ is the calculated Rayleigh velocity from measured angle.

$C^{th}_R$ is the theoretical Rayleigh velocity.

In Table 2 we give the values of $\theta$, for different materials experimentally measured, from which we have calculated the Rayleigh velocity ($N^e_R$). In the last column we have listed the theoretical values ($N^{th}_R$) for comparison. The agreement is very good.

$W$ Dependence

To study the relationship between the backscattered amplitude and the beam width ($W_b$), we have carried out the following experiments: for a given liquid-solid interface, ultrasonic beams of several widths were used. The normalized backscattered amplitude is plotted in Fig. 6 for water-stainless steel and for water-titanium interfaces versus the beam diameter $d = 2W_b$. The frequency is 4.5 MHz. The curves show that an inverse relationship exists between

![Fig. 6. The dependence of ultrasonic backscattering amplitude on beam diameter.](image-url)
the backscattered amplitude and the beam width. We note that a 25 dB drop in backscattered amplitude occurs when the beam width is increased by a factor of 8. This result is in qualitative agreement with Bertoni and Tamir's theoretical prediction. 

\[ \Delta \text{ Dependence} \]

In a second experiment the dependence of the backscattered amplitude on the \( \Delta \) has been studied at the same frequency (4.5 MHz) and with a half beam width of 6 mm. The backscattered normalized amplitude at \( \theta_R \) on different materials has been measured. The plot of the data (Fig. 7) versus the Schoch's displacement (Table 1) suggests that a qualitative dependence exists between the backscattered amplitude and \( \Delta_s \). As \( \Delta_s \) becomes larger, the amplitude becomes stronger.

\textbf{Anisotropy Measurements}

We have shown that when a bounded acoustic beam is reflected from a fluid-solid interface, it will be possible to measure the velocity of the surface wave ultrasonically by measuring the angle

\[
10 \log \left( \frac{A_s}{A_2} \right)
\]

\[ \Delta_s \text{ (mm)} \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{The dependence of ultrasonic backscattering amplitude on calculated Schoch displacement.}
\end{figure}
Fig. 8.

Fig. 9. Variation of Rayleigh wave velocities (m/sec) with orientation on a steel case pipe section. Solid line: theory (hexagonal symmetry); dots: backscattering measurements.
for which we observe a maximum in the backscattered amplitude diagram. This procedure has been used to study the dispersion of Rayleigh's velocity in an anisotropic specimen. The insonified surface determined by the two directions 7 and 3 rotates around the 2 axis (Fig. 8) and the transducer is moved to get a maximum signal in backscattering. This position corresponds with the critical angle \( \theta_A \) from which we calculate the velocity of the surface wave. A measurement was taken every 10° and the dispersion of the velocity is plotted in polar coordinates in Fig. 9.

We note the anisotropy of the studied sample underlined by the six symmetrical lobes, the amplitude of which varies from 2.2 to 2.9 km/s. The theoretical plot is obtained by solving the boundary value problem for liquid-anisotropic solid interface.\(^2\)\(^0\) It appears that this method may give an accurate determination of surface wave velocity even for anisotropic materials.

**GENERAL RAYLEIGH ANGLE PHENOMENA**

Throughout the investigation of the backscattered phenomena (previously described) it was noticed that for any angle of incidence leaky waves were produced. These leaky waves may be observed only at the Rayleigh angle, however. This is illustrated in Fig. 10: for any angle of incidence (including zero), a finite beam ultrasonic wave may produce (at a liquid-solid interface) a specularly reflected wave and a leaky wave propagating in both forward and backward directions, and can be observed at the Rayleigh angle. This is the general Rayleigh angle phenomena and clearly the effect shown in Fig. 1 is just a special case for the Rayleigh angle of incidence.

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![Diagram](image)

**Fig. 10.** Illustration of finite beam ultrasonic phenomena at interface for a general angle of incidence.
REFERENCES

20. K. Bolland and L. Adler, to be published.