Diffraction of Ultrasonic Waves by Elliptical Cracks in Metals

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Abstract
Ultrasonic spectrum analysis is used to study the frequency and angular behavior of diffracted longitudinal waves from elliptical cracks with arbitrary orientation in diffusion bonded titanium alloy. The aspect ratio of the cracks (ratio of major to minor axis) ranged from 1 to 8 and the bandwidth of the input signal was such that the scattering diameter was \(ka \geq l\). The experimental data are analyzed based on a recently developed elastodynamic diffraction theory (by Achenbach et al.). Some of the key parameters of the scattering data are identified to address the inverse problem.

Keywords
Nondestructive Evaluation

Disciplines
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DIFFRACTION OF ULTRASONIC WAVES BY ELLIPTICAL CRACKS IN METALS

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ABSTRACT

Ultrasonic spectrum analysis is used to study the frequency and angular behavior of diffracted longitudinal waves from elliptical cracks with arbitrary orientation in diffusion bonded titanium alloy. The aspect ratio of the cracks (ratio of major to minor axis) ranged from 1 to 8 and the bandwidth of the input signal was such that the scattering diameter was ka > 1. The experimental data are analyzed based on a recently developed elastodynamic diffraction theory by Achenbach et al. Some of the key parameters of the scattering data are identified to address the inverse problem.

INTRODUCTION

It has been recognized for some time that quantitative ultrasonic flaw characterization requires the understanding of interaction of ultrasonic waves with discontinuities in solids. The physical phenomenon describing this interaction is generally referred to as scattering, but in the region of ka > 1 (where k is the wave number and a is some dimension of the discontinuity) we speak of this interaction as diffraction. For the diffraction of ultrasonic waves by planar cracks Adler and Lewis used Keller's geometrical diffraction theory to analyze their experimental results. Achenbach, Gautesen, and McMaken extended geometrical theory to the three-dimensional diffraction by cracks in solids where wave motions are governed by a scalar and a vector wave equation. This analysis for diffraction of elastic waves from circular cracks has been evaluated and compared to experimentally obtained results by Achenbach, Adler, Lewis, and McMaken. In the same paper they obtained a simple formula for the inverse problem. Part of their results will be presented here. In addition, the diffraction of elastic waves by elliptical cracks of various eccentricities will be addressed.

THEORY

Elliptical Crack - Let's consider an elliptical crack in a linear elastic isotropic solid with major axis b and minor axis a. The orientation of the crack is described by angles α and β. α is measured from the major axis of the ellipse and β is measured from the plane of the crack to the z axis; e.g., if α = 90° the plane of the crack is in the xy plane. The longitudinal wave is incident to the crack from point $r = (x, y, z)$ and with displacement amplitude $u_l$. The diffracted L wave with amplitude $u_d$ is observed in space at a point which describes $(r_p)$. The problem is then to relate the angular and frequency dependence of $u_d$ and $u_l$ to the α, β, b, and $\mu$. McMaken, Achenbach, and Gautesen has obtained a closed form solution of the diffracted L waves from elliptical cracks in the far field. The diffracted field amplitude $u_d$ is represented by the integral over a crack surface $S$

$$u_d = \int_S r_3 j_k \omega u_j d\sigma$$

where $r_3 j_k$ is the part of the Green's function corresponding to waves of longitudinal motion and $u_j$ is the crack opening displacement. Evaluating (1) for the geometry described in Fig. 1, one obtains

$$u_d = i \exp \left\{ \left( \frac{C_L}{c} \right) \left( 1 - 2(p_1^2 + p_2^2) \right) \left( \frac{C_T}{c} \right) \left[ \frac{2 - (\hat{x}_1^2 + \hat{x}_2^2)}{1 + \hat{x}_2^2} \right] \right\} \frac{b_p \psi_1 (\omega, \Delta \sigma)}{r_k |x|}$$

where

$$k_L = \frac{\omega}{c_L},$$

$$p_r = (p_1, p_2, p_3)$$

defines the direction of propagation of incident wave,

$$\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3)$$

defines the point of observation,

$$C_L, (c_T)$$

is the velocity of longitudinal (transverse) waves in the solid,

b is the length of the major axis,

a is the length of the minor axis,

$\psi_1 (x)$

is the Bessel function of order one, $r = \left( \frac{(x_1 - p_1)^2 + (x_2 - p_2)^2 + (x_3 - p_3)^2}{4} \right)^{1/2}$.

Equation (2) is programmed for various crack geometries to obtain some representative configurations of the diffracted field to choose the experimental parameters. For an ellipse with major axis 2500μ and minor axis 612μ and for a normally incident L wave, the amplitude (frequency ranges from 2 to 14 MHz) spectra is calculated for three different polar angles—0°, 52°, and 60°. The observation point moves gradually from major to minor axis (Fig. 2). There are at least three conclusions which can be drawn from the 3D plots shown on Fig. 2: (a) along a fixed axis the periodicity in the frequency will decrease with increased α; (b) for a given α the periodicity will increase going from major to minor axis; and (c) the scattered amplitude will increase as the observation point is approaching the minor axis from the major...
axis. The effect of different aspect ratio (ratio of major to minor axis) of the elliptical crack on the diffracted field is shown on Fig. 3. For $b/a = 1, 2, 4, 8$ the amplitude vs. frequency and azimuth angle (changing from along the major to minor axis) for a fixed polar angle of 60° is given. Since the $b$, the major axis, for all four cases was 2500 μm, the amplitude spectra is the same along the major axis. The periodicity (separation between consecutive frequency maxima) will increase along the minor axis and it is largest for the elliptical crack with the largest aspect ratio. The energy of the scattered energy along the major axis is always less than along the minor axis and the difference of the energy scattered along these axes will increase with aspect ratio. The elliptical crack with aspect ratio of 8 shows very little energy scattered along the major axis. The special case is the elliptical crack with aspect ratio of 1 is the circular crack. In a different formalism by using geometrical theory of diffraction analytical expressions for the diffraction of longitudinal waves from penny-shaped cracks has been obtained.

This analysis accounts for the liquid-solid interface which is in line with our experimental technique. It also leads to a simple inversion procedure, therefore the results will be repeated here briefly. Penny-shaped Cracks. An incident L wave diffracted from a penny-shaped crack produces diffracted rays at the two extreme edges of the crack (see Fig. 4). The diffracted rays are refracted at the solid-liquid interface and combined at point $B$. The diffracted field $u_L$ at that point is given as

$$u_L = F(\alpha_0) \exp\left[\frac{i\omega}{c_L}(S_{CL} + S_{CL}) + \frac{i\pi}{4}\right] U_0$$

$$F(\alpha_0) = H_1 \exp\left[-i\left(\frac{\omega a}{c_L}(\cos\theta - \sin\alpha_0)\right)\right]$$

$$+ H_2 \exp\left[i\left(\frac{\omega a}{c_L}(\cos\theta - \sin\alpha_0)\right)\right].$$

Comments on the Inverse Problem. In the previous sections expressions were obtained for the diffracted field of ultrasonic waves from cracks when the size, shape, and orientation of the crack are known. This is the so-called direct problem. The aim of the ultrasonic scattering studies by flaws is to characterize them from the measured scattered field which is the inverse problem. The fact that the frequency distribution of the diffracted field leads to simple inversion procedure has been recognized for special cases earlier and can be shown from the theoretical analysis. From Eq. (3) the absolute magnitude (modulus) in the far field can be written as

$$u_L = F(\alpha_0) \exp\left[\frac{i\omega}{c_L}(S_{CL} + S_{CL}) + \frac{i\pi}{4}\right] U_0$$

$$F(\alpha_0) = H_1 \exp\left[-i\left(\frac{\omega a}{c_L}(\cos\theta - \sin\alpha_0)\right)\right]$$

$$+ H_2 \exp\left[i\left(\frac{\omega a}{c_L}(\cos\theta - \sin\alpha_0)\right)\right].$$
\[
|F| = \left\{ |H_1|^2 + |H_2|^2 + 2|H_1||H_2|\sin[2(\varphi/a_c)] \right\}^{1/2}.
\]

This result implies that the amplitude of the diffracted field is modulated with respect to \( \varphi/a_c \) with period
\[
P = \pi/a \cos \theta - \sin \theta_0
\]
where \( \theta \) is the scattering angle in the solid, \( a \) is the radius of the crack, and \( \theta_0 \) is the orientation of the crack. For two different scattering angles the amplitude spectra can be used to evaluate the size and orientation of the penny-shaped crack by using Eq. (5). In the first approximation the diffracted field can be inverted also by expanding the Bessel function in Eq. (2). Along the major and minor axes the diffracted field is modulated in the same fashion as obtained in Eq. (5).

**EXPERIMENT**

**Experimental System.** The present configuration of the ultrasonic data acquisition and processing system is illustrated in Fig. 5. The SCR pulser produces a fast rise-time high voltage (162 volts) negative spike with an exponential return to zero. This wide band electrical pulse excites an untuned, highly damped ceramic transducer with center frequency of 10 MHz. The ultrasonic pulse (pulse length \( \sim 1 \) usec) which is produced contains a broad band of frequencies. Ultrasound scattered by the target is received by either (1) the transmitting transducer (pulse-echo) or (2) a receiving (identical) transducer (pitch-catch). The electrical pulse produced by this receiving transducer is amplified by a wide bandwidth gain stage. A stepless gate is used to select a portion of the received signal for further analysis. Signals falling outside the gated regions are highly attenuated. An oscilloscope displays both the entire receiver output and the section of waveform passed by the gate.

**Fig. 5. Experimental System.**

The frequency content of the gated waveform is presented on an analog spectrum analyzer. The gated pulse may also be captured and stored through use of the digital acquisition system. A transient recorder samples the ultrasonic signal at 100 MHz, and stores the amplitude at discrete times in its digital memory. The minicomputer controls the acquisition of the ultrasonic pulse and then transfers the digitally represented signal from the recorder to the minicomputer memory. The signal may also be permanently stored by writing it onto magnetic tape. Processing of the ultrasonic signal (Fast Fourier Transform, correlation and deconvolution) is performed on the minicomputer. An electrostatic plotter provides a visual display of pertinent information. For the Fourier Transform both amplitude and phase spectra can be calculated. In this experiment the amplitude spectra are used only. The digital amplitude spectra is monitored by the analog spectra (i.e., the spectrum offered by the spectrum analyzer).

**Experimental Technique and Procedure.** The technique used is shown in Fig. 6. The sample, which is a disk (2.5 x 10 cm) titanium alloy with a flat surface, is immersed in water. The transmitter launches a longitudinal wave to the liquid-solid interface at some angle. For nonnormal incidence both L and T waves are produced in the metal. The cavity can be insonified either by the L wave or by the T wave with incident angle \( \alpha \). At the cavity the waves are scattered and mode converted. The scattered waves are received and analyzed separately due to their separation in time. The various types of waves present are illustrated schematically on Fig. 7 by the so-called time mapping. The interface corresponds to \( t = 0 \) where \( t \) is the time in usec. A is the incident pulse whose position is shown 3.5 usec before it hits the interface. B and D are the T and L waves at \( t = 1.5 \) usec after they entered the sample. With the illustrated configuration only the L wave will interact with the cavity. At a later time, at \( t = 4.5 \) usecs E and G are the scattered T and L waves and are clearly separated due to their differing velocities. C and E are the positions of T and L waves at \( t = 4.5 \) usecs not interacting with the defect.
Fig. 6. Experimental Technique

Fig. 7. Time Mapping of Transverse and Longitudinal Waves before and after Interaction with Defect. (A) Incident Wave in Water; (B) and (C) Transverse Waves; (D) and (E) Longitudinal Waves; (F) Scattered Transverse; (G) Scattered Longitudinal.

A specially designed goniometer is used (Fig. 8) to mount the transmitter and the receiver. The position of both transmitter and receiver in polar angle can be changed separately. A special feature of the goniometer is its flexibility of keeping the polar angle fixed and varying the azimuthal angle. This latter feature is especially important for elliptical cracks because of the asymmetry in the scattered field along the different axes of the ellipse (see Figs. 2 and 3).

The selection of the polar and azimuthal angles were so that the data could also be used by Adaptronics's data base for inversion (see their results in this report).

Data Correction. In order to analyze the experimental results based on the analytical prediction the effect of the transducers and the crack had to be separated. In a linear time invariant system this is done in the frequency domain by dividing the frequency response from a system by the so-called transfer function. In this problem the spectra of the transmitted signal through the material (without the crack) is considered the transfer function.

Figure 9A shows the RF signal transmitted normally through a 2.5 cm thick parallel flat smooth titanium (flawless) disk immersed in water. There is usable energy through the frequency range from 2 to 15 MHz. The amplitude spectrum shown on Fig. 9B is the transfer function of the system. The spectrum of the scattered wave is then divided by this transfer function.

Fig. 8. Multiplane Ultrasonic Goniometer.

Fig. 9. Transfer Function. (A) RF Signal; (B) Amplitude Spectrum.

RESULTS

3-Dimensional Display of the Experimental Data. The diffracted waveforms have been processed, stored, and collected from different points in space and displayed in a 3D fashion to obtain an overall view of the diffracted field due to different cracks. Typical displays are shown on Fig. 10 where the diffracted amplitude vs. frequency and azimuthal angle by 15° intervals from major to minor axes is
shown. The ellipse is 2500µ x 1250µ and the incident wave is normal to the crack. The polar angles were 52° and 60°. The amplitude of the scattered wave is higher toward the major axis as was predicted by the theory (Figs. 3 and 4).

Another feature of the diffracted field is shown on Fig. 11. The amplitude is plotted vs. frequency and polar angle for three cases when the receiver is placed along the minor axis along 45° and along the major axis. The polar angles change from 30° to 60° in 5° intervals. The elliptical crack's dimensions are 2500µ x 612µ. Although these 3D plots are very useful to obtain qualitative features of the diffracted field, the comparison between experiment and theory was carried out in 2-dimensional displays.

Comparison between Theory and Experiment. The experimental data have been normalized (divided by the transfer function, Fig. 9B) and the diffracted amplitude is shown vs. frequency for the penny-shaped crack of 2500µ radii. Two different polar angles, 45° and 55°, are shown on Figs. 12 and 13. The solid curve is calculated from Eq. (3) and the dots are experimental points. The agreement is good for the lower frequencies. For higher frequencies the attenuation is significant in water and theory does not include attenuation.

On Fig. 14 the experimental data is compared to theory for the 2500µ x 1250µ elliptical crack. The data are displayed when the receiver is along the minor and along the major axis. The receiver's position is such that the polar angle is 30°. The agreement with theory is very good for the periodicity but the measured amplitudes are lower at high frequencies than predicted by theory. Similar results are shown for the case of the 60° polar angles (Fig. 15).

Fig. 10. Experimental Amplitude Spectra of Normal Incidence L Wave Scattered from a 2500µ x 1250µ Elliptical Crack in Titanium.

Fig. 11. Experimental Data for L-L Scattering from a 2500µ x 625µ Elliptical Crack.

Fig. 12. Amplitude Spectrum of 45° Scattered Longitudinal Wave.

Fig. 13. Amplitude Spectrum of 55° Scattered Longitudinal Wave.

Fig. 14. Amplitude Spectrum of 60° Scattered Longitudinal Wave.

Determination of Crack Size. The periodicity of the amplitude spectra gives very good agreement between experiment and theory which is the basis for inversion. On Table 1 the result of inversion is shown for the penny-shaped crack for different
polar angles. The largest deviation is less than 5% between measured and actual values of the 2500μ circular crack. The result of the inversion for the elliptical crack is given in Table 2. From several angular measurements the averages were taken. The major and minor axes are calculated from Eq. (4) and compared to the actual values. The agreement is good except for the case of the major axis of the 8 to 1 aspect ratio ellipse. The signal along the major axis is too noisy to be meaningfully analyzed in the frequency domain. Additional data for nonnormal cases have also been taken and sent to Adaptronics for their data base (see details in this report).

Correction for Attenuation. The agreement between theory and experiment for the periodicity in the amplitude spectra is very good. The theoretical prediction of the amplitude values for the higher frequencies are consistently larger than is observed experimentally. It appears that one of the reasons is the attenuation in water. At a water column of 15 cm (the distance between the sample surface and the receiving transducer) the attenuation is .033 f^2 dB, where f is in MHz. Because of the square dependence in frequency the effect of attenuation is more dominating at higher frequencies. We have corrected the theoretical curve with attenuation as shown on Fig. 16. The solid line is the theoretical curve without attenuation. The theoretical curve with attenuation correction (dashed line) agrees much better with experimental data points. The effect of the attenuation in the titanium is less significant but it appears that the complete report should include the effect of attenuation in both the solid and in water to calculate the diffracted field due to cracks.

### Table 1

<table>
<thead>
<tr>
<th>Scattering Angle</th>
<th>Measured Radius of the Crack in μ</th>
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<tr>
<td>35</td>
<td>2.18 2530</td>
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<tr>
<td>40</td>
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<tr>
<td>45</td>
<td>1.83 2450</td>
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<tr>
<td>50</td>
<td>1.68 2460</td>
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<tr>
<td>55</td>
<td>1.60 2410</td>
</tr>
<tr>
<td>60</td>
<td>1.47 2500</td>
</tr>
<tr>
<td>65</td>
<td>1.39 2510</td>
</tr>
</tbody>
</table>

is the attenuation in water. At a water column of 15 cm (the distance between the sample surface and the receiving transducer) the attenuation is .033 f^2 dB, where f is in MHz. Because of the square dependence in frequency the effect of attenuation is more dominating at higher frequencies. We have corrected the theoretical curve with attenuation as shown on Fig. 16. The solid line is the theoretical curve without attenuation. The theoretical curve with attenuation correction (dashed line) agrees much better with experimental data points. The effect of the attenuation in the titanium is less significant but it appears that the complete report should include the effect of attenuation in both the solid and in water to calculate the diffracted field due to cracks.

### Table 2

<table>
<thead>
<tr>
<th>(B) in Microns</th>
<th>(A) in Microns</th>
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<tr>
<td>Actual</td>
<td>Measured</td>
</tr>
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<table>
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<td>2500</td>
<td>830</td>
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<td>2500</td>
<td>265</td>
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SUMMARY

Experimental studies of the diffraction of elastic waves from elliptical cracks have been carried out. The diffracted amplitudes were measured in a broad range of frequencies as a function of polar and azimuthal angles for cracks with various eccentricities. Key parameters were identified such as periodicity in amplitude spectra, total scattered power to aid the inversion process. From the experimental data the major and minor axes were determined with good accuracy by using simple expressions obtained from diffraction theory. The diffraction theory calculations accurately predicting the periodicity of the amplitude spectra were observed in the experiment. The amplitude values at higher frequencies are predicted too high. This can be corrected by the attenuation of the diffracted waves in the water column.

ACKNOWLEDGMENT

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