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A Technique for Determining Flaw Characteristics from Ultrasonic Scattering Amplitudes

Abstract
We report an approximate technique for determining the characteristics of flaws in elastic media from a knowledge of their ultrasonic scattering amplitudes. The technique is rigorously valid in the weak scattering limit. Good results have been obtained for strongly scattering flaws. In particular, we tested the technique for a 2-1 oblate spheroidal void in Ti, and for various strongly scattering spherical defects. For these tests the technique yields good results for the volume of the flaws. In the case of the oblate spheroid, satisfactory results were obtained for the calculated ratio of major to minor axis, indicating that the technique is sensitive to the shape of the flaw.

Keywords
Nondestructive Evaluation

Disciplines
Materials Science and Engineering
A TECHNIQUE FOR DETERMINING FLAW CHARACTERISTICS FROM ULTRASOUND SCATTERING AMPLITUDES

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ABSTRACT

We report an approximate technique for determining the characteristics of flaws in elastic media from their ultrasonic scattering amplitudes. The technique depends on the known size, shape, and orientation of the flaws. For these tests the technique yields good results for the volume of the flaws. In the case of the oblate spheroid, satisfactory results were obtained for the calculated ratio of major to minor axes, indicating that the technique is sensitive to the shape of the flaw.

INTRODUCTION

We present an inversion algorithm for determining the size, shape, and orientation of flaws in elastic media from ultrasonic scattering amplitudes. The algorithm is based on the extended quasi-static approximation of J.E. Gubernatis, and is rigorously valid in the weak scattering limit. We have tested the algorithm for some strongly scattering flaws, and obtained good results as we report below.

The extended quasi-static approximation leads to two pieces of independent information which can be used to determine the characteristics of the flaw. The first piece of information is the dependence of the long wavelength scattering amplitude on the orientation of the flaw. The second piece of information is the spectral distribution of the reflected ultrasound as a function of orientation. In this paper we will concentrate on the spectral information. However, the two kinds of information are complementary, and using both of them together may be able to learn something about the material composition of the flaw, as well as its size, shape, and orientation. This possibility is discussed in the final section, and a technique is proposed which may make it possible to determine the density of the flaw.

A major thrust of the current Non-Destructive Testing program is to use adaptive learning techniques to determine flaw characteristics in an empirical way. Approximate inversion techniques such as that proposed in this paper could be used to preprocess the scattering amplitudes before its introduction to the adaptive learning network. Since the inversion algorithm will produce functions which look at least roughly like the actual flaw, such preprocessing may greatly enhance the adaptive learning approach.

The structure of this paper is as follows. We start with this introduction; then provide a general description of the theory. The third section is a discussion of a simplified form of theory which is available for all ellipsoidally shaped flaws. The fourth section contains empirical tests of the inversion algorithm for strongly scattering flaws. In particular, we have tested the procedure both for a 2-1 oblate spheroidal defect in Ti and for spherical cavities and inclusions in various materials.

Our tests up to the present date indicate that the volume of the flaw is given rather accurately by our technique. We conclude the paper by giving a general discussion of the technique and indicate some of the work we intend to pursue in the future.

General Theory

The theory presented in this section is designed to provide information on flaw characteristics in the scattering regime. In this case the wavelength of the ultrasound is approximately equal to or longer than the size of the object. This regime is quite important for NDT applications where the size of the flaws may be quite small and as a result imaging techniques may be unavailable. The algorithm which we will propose is a procedure for approximately determining the Fourier transform of the characteristic function, \( \gamma(\mathbf{r}) \), of the flaw.

A pulse echo type experiment is assumed as shown in Fig. 1. Here a longitudinally polarized plane wave is incident on the flaw, and the directly backscattered longitudinal scattering amplitudes are determined. The longitudinal to longitudinal pulse echo scattering amplitudes can be written for an arbitrarily shaped flaw as

\[
A_L \rightarrow L(k) = a(\mathbf{R},(u)) S(2\mathbf{E}) k^2
\]

where \( S(2\mathbf{E}) \) (called the shape factor) is the Fourier transform of the characteristic function of the flaw. The wavevector of the incident wave is denoted by \( \mathbf{E} \) and \( a(\mathbf{R},(u)) \) is a function to be calculated to yield the correct scattering amplitudes \( A_L \) for an arbitrary \( \mathbf{R} \). Here \( (u) \) denotes the material parameters of the flaw and the host material.

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Pulse-echo measurement

Fig. 1 The geometry of a typical L-L, pulse-echo experiment. The matrix material has one set of material parameters, $p_1, v_1, \lambda_1$; while the flaw is considered to consist of a second homogeneous material with parameters $p_2, v_2, \lambda_2$.

The virtue of writing the scattering amplitudes in the form of Eqn. 2.1 is that several approximate theories yield very simple forms for the factor $a(t_0l, l)$. In particular, we will use the form of $a(k, l)$ which can be derived from the extended quasi-static approximation. In that approximation one takes account of the long wavelength elastic deformation of the flaw correctly, and hence obtains the angular features of the scattering correctly in this limit. For the extended quasi-static approximation $a(k, l)$ is independent of $l$ and depends only on $k$ and $l$.

The major approximation in using $a_{QSA}$ is that we assume that it depends only on $k$ and not on $l$. The characteristic function is given explicitly in terms of the shape function as

$$\gamma(k) = \text{const.} \int d^3 k \ e^{2i k \cdot r_A L} / (k^2 a_{QSA}(k, l))$$

(2.4)

**Simplified Theory for Ellipsoidally Shaped Flaws**

In the last section we described an approximate procedure for determining the size, shape, and orientation of an arbitrary three dimensional flaw. In order to use this inversion technique, one requires pulse-echo measurements from all incident directions $k$. The characteristic function is then obtained (Eqn. 2.4) as an inverse Fourier transform which involves integrating over both $|k|$ and $k$. For the class of ellipsoidally shaped flaws, one can obtain all relevant information about the flaw by inverting each pulse-echo record independently as discussed below. This avoids the angular integration over $k$ in the inverse Fourier transform, and significantly simplifies the application of the algorithm.

In order to illustrate how this simplification comes about, let us consider the weak scattering limit. Then the theory of the last section is rigorously valid and Eqn. 2.2 becomes

$$S(2k_0) = \text{const.} A_{L+L}(k_0) / k^2$$

(3.1)

We have used the fact that $a(k, l)$ is a constant in the weak scattering limit as a function of $k$. For an ellipsoid we know that $S(2k)$ is given by the following equations

$$S(2k) = \sin(2 k r_e) - 2 k r_e \cos(2 k r_e)$$

(2.2)

and

$$r_e = (a_2 \cos \theta \sin^2 \phi + a_y \cos \phi + a_z \sin^2 \phi)^{1/4}$$

(3.2)

Here the axes of the ellipsoid are $a=(a_x, a_y, a_z)$, and $\theta$ and $\phi$ define the direction of $k$ in spherical coordinates. The angular dependence of the shape factor comes in strictly through the function which we have called $r_e(\theta, \phi)$. In a pulse-echo measurement, the incident direction $k$ is kept fixed, and $r_e$ is a constant for that set of data. We note for a fixed incident direction, Eqn. 3.2 has the same form as a Fourier transform of a sphere with an effective radius $r_e$. For each incident direction $k$, we obtain $r_e$ in the following way. First we obtain $S(2k_0)$ from Eqn. 3.1. Then we extend $S(2k)$ to be spherically symmetric in $k$-space. Thus we obtain the three-dimensional Fourier transform of a sphere of radius $r_e$. This Fourier transform is then inverted to yield

$$S(2k_0) = \text{const.} A_{L+L}(k_0) / k^2$$

(3.3)
the effective radius for that direction. The resulting, calculated, effective radius will vary according to Eqn. 3.3. An important consequence of Eqn. 3.3 is that pulse-echo measurements along the axis of an ellipsoid yield the axis length directly. For example, a measurement along the $a_X$ axis yields an effective radius equal to $a_X$. Hence one can obtain the length of the ellipsoid axes directly from three measurements if one knows the orientation of the ellipsoid.

So far we have been discussing the weak scattering limit for the sake of illustration. The appropriate extension to the strong scattering case is straightforward. Eqn. 2.2 is

$$S(2k) = \text{const. } A_{L+L}(k)/[k^2 a_{QSA}(k)] \quad (3.4)$$

For a given incident direction $a(k, \mu)$, is just a constant since it doesn't depend on $|k|$ in the quasi-static approximation. With this approximation we recover Eqn. 3.1 and can proceed in an approximate way with the entire procedure which was given above. Of course, for a strongly scattering flaw, our analysis is only approximate and must be checked empirically. In the next section we provide some empirical tests of the strong scattering limits.

Results: Test of Inversion Procedure

Here we report our tests of the proposed inversion algorithm. These results are given in two parts. First we report tests for spherical voids. Then we report preliminary results for the case of a 2:1 oblate spheroidal cavity in Ti. The most direct test of our inversion procedure is shown in Fig. 2. Here we show the results of inverting experimentally obtained ultrasonic scattering amplitudes for a spherical void in Ti. 3 For this case the band width of the data was $0.5 < kR < 4$, where we use $R$ to denote the radius of the sphere. As can be seen, the inversion algorithm does a good job of determining the flaw radius (even in this experimentally realistic case with a rather restricted bandwidth).

In order to further test our inversion procedure, we considered the case of spherical voids in stainless steel and Ti, and the case of an aluminum inclusion in Ti. 4 For these cases we can obtain the exact theoretical scattering amplitudes from the calculations of Ying and Truell. 5 Using these results for $A_{L+L}(k)$, we determined $S(2k)$ from Eqn. 2.2 for a bandwidth, $0 < kR < 10$. The result for the case of a spherical void in Ti is shown in Fig. 3. Quite good definition of the radius and hence the volume is obtained. These results are characteristic of the results for the other cases mentioned above. Finally, in order to test the stability of the routine to noise, we introduced random Gaussian noise into the scattering amplitudes. The inversion procedure proved to be quite insensitive to random noise. In Fig. 4 we show the result for the Ti spherical cavity when the signal contains 50% Gaussian random noise. Note that the inclusion of noise did not appreciably effect the determination of the flaw radius.

![Fig. 2](image-url)  
Fig. 2 Calculated characteristic function for a 400 micron radii sphere obtained by inverting experimental ultrasonic scattering amplitudes with a bandwidth of $0.5 < kR < 4$, with $R$ the radius of the sphere.

![Fig. 3](image-url)  
Fig. 3 Calculated characteristic function for a spherical void of radius $a_0$ in Titanium. The result was obtained by inverting theoretical scattering amplitudes. The bandwidth used above was $0 < k a_0 < 10$.

![Fig. 4](image-url)  
Fig. 4 The relatively weak effects of noise on the inversion procedure is illustrated. Here we show the same calculation as in Fig. 3 for a void in Ti but have included a random component in the signal (the random component is on the average 50% of the exact signal). Note that the determination of the radius of the sphere is hardly affected by the inclusion of this extremely large random error.
We, also, report preliminary results for inverting the scattering amplitudes from a 2-1 oblate spheroid using the simplified theory of section II. Here the scattering amplitudes used in the inversion procedure were theoretical results provided by W. Visscher who used a series expansion technique capable of providing numerically exact results. 6 Our preliminary analysis consisted of considering only two different pulse-echo measurements. The first calculation was for a beam incident along the symmetry axis (θ = 0°). The second calculation was for a beam incident in the plane of the spheroid (θ = 90°).

The results obtained for the length of the axes are:

<table>
<thead>
<tr>
<th></th>
<th>calculated</th>
<th>exact</th>
</tr>
</thead>
<tbody>
<tr>
<td>major axis</td>
<td>850</td>
<td>800</td>
</tr>
<tr>
<td>minor axis</td>
<td>360</td>
<td>400</td>
</tr>
</tbody>
</table>

The calculated ratio of axes was .42 while the exact result is .50. Finally, we note that the computed volume is equal to the exact volume of the spheroid within our calculational accuracy.

Discussion and Conclusions

We have proposed a method for determining the size, shape and orientation of flaws in elastic media. This inversion procedure is rigorous in the weak scattering limit. For strongly scattering flaws the validity of the inversion procedure was uncertain. Hence, we performed empirical tests for strongly scattering flaws. We have concentrated on voids since they are strong scatterers and are important in various fracture processes. For spherical voids in Ti, we obtained strikingly good results in determining the flaw radius and volume. For these tests we used both experimentally measured and theoretically generated scattering amplitudes in different examples. We have also reported preliminary results for a 2-1 oblate spheroidal void in Ti. Here our preliminary calculations indicated excellent agreement for the flaw volume, and satisfactory agreement for the relative length of the axes. We have not yet been able to test the procedure for its sensitivity to the orientation of the flaw. If future tests over a wider range of flaw shapes continue to give good determinations of the flaw volume, this will have important consequences for determining the material parameters of the flaw as will be discussed below. Currently, work is in progress with several groups to obtain experimentally and theoretically generated scattering amplitudes for a wide range of axially symmetric flaws: prolate and oblate spheroids and pill boxes (both voids and inclusions will be considered).7 These efforts should result in an extensive empirical test of the inversion algorithm and give a good idea of its range of validity.

We have concentrated in this paper on determining the characteristic function of a flaw from a determination of the shape factor. As discussed in another talk in these proceedings, there is additional information about the shape of the flaw in the long wavelength factor argg(k(z)). 8 Together with the inversion algorithm which we have proposed in this paper, it appears likely that not only the size, shape, and orientation of simple volume flaws may be obtained, but also we may be able to determine the material composition of the flaw. An example of one way the two techniques may be used together to determine the density of the material in the flaw region is discussed below.

Kohn and Rice 9 have shown that the long wavelength scattering amplitudes can be used to determine the mass defect, ΔM, of the flaw. ΔM is the difference in mass of the flaw and the mass it would have if there were host material in the flaw region. Together with an accurate knowledge of the flaw volume, V, one may infer the density of the flaw material

\[ \rho_{\text{flaw}} = \rho_{\text{host}} + \frac{\Delta M}{V} \]  

If sufficient accuracy can be obtained in the inversion algorithm which determines V and the determination of ΔM, then one should be able to infer the material composition of the flaw. It should be stressed that such a determination depends on an absolute measurement of the scattered power.

In conclusion, we note that the general formalism (Eqn. 2.1 to 2.4) applies in principle to not only volume type flaws but also to crack like defects.10 Again, the correspondence should be exact for weakly scattering flaws. We have not had the necessary data to test the algorithm yet for the strongly scattering case. However, we hope to provide an empirical determination of this technique for the case of cracks in the near future.

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6. W. Visscher kindly made available his calculations for the L-L scattering amplitudes for a 2-1 oblate spheroid prior to publication. Similar results were also kindly provided by V.V. Varadan.

7. The persons involved are: 1) B. Tittmann and R. Elsley at the Rockwell International Science Center and V.V. Varadan and V.J. Varadan at Ohio State University.


DISCUSSION

Norman Bleistein (Denver Applied Analytics): Are your data far field data?

Jim Rose (Cornell): I don't really know the answer to that question. Certainly the theoretical data are far field. I don't really know about the experimental data.

Norman Bleistein: That is very important though, because if you are in a far field regime, there is another justification for ending up with your Fourier Transform results.

Jim Rose: I have not worked in the high frequency regime in establishing a justification for this procedure. What I have done is to justify the low frequency regime and to extend it to higher frequencies.

Gordon Kino (Stanford University): Is the high frequency regime any different from just using straight pulse-echo and getting the reflection which should be proportional to the radius of curvature?

Jim Rose: Of course, if you are in the very high frequency regime, you are absolutely correct. If you come down, as Achenbach has indicated, it is probably not too bad a procedure if you put in these corrections to get down to $KA = 1$. The kind of flaws I am interested in are flaws where you have a band width of $KA$ of about 5.

James Krumhansl (National Science Foundation): What you say about the imaging part is absolutely true.

Gordon Kino: I am not saying imaging; I am just saying straight pulse-echo.

David Lee, Chairman (Applied Mechanical Research Laboratory): I have a question for the speaker. Is there something a little bit inconsistent in the fact that your model is accurate for wave lengths that are long compared to the scatterer and yet, the Fourier Transform has in it sharp information about gamma which results from higher frequencies?

Jim Rose: Right. I think the way to answer this is that the Born approximation is not good in the frequency domain. However, it gets the first peak and first valley rightly. It is off quantitatively in a differential sense, but it has got a peak and a valley where a peak and a valley ought to be and it has got another peak up where the second peak ought to be. Again, they are off in frequency. I see this as an integrated, average way to find the position of the first peak. You know that you can essentially take the position of the first peak from the sphere and guess the radius darn well. If you just look at the position of the first peak itself, that is a differential quantity. Here I have got an integral average. That's how I would understand it. The kernel is multiplied by an interference pattern and that is the same as averaging.

David Lee, Chairman: Forgive me, but one more question on this. Of course, the numerical inversion model like Fourier Transforms is one of the archtypical field force problems; one suspects that your insensitivity to noise arises because of whatever you did had the effect of a low pass filter.

Jim Rose: Absolutely. That's the beautiful thing about the Born approximation for elastic waves. In the long wave length limit, you get information on the size of the object. That is where you get your best information; the opposenedness, I think, comes out because you have got a limited band width of higher frequencies and I have got long wave lengths.

Phillip Hodgetts (Los Angeles Div., Rockwell): Did I hear you say that you injected random noise in the experiment to duplicate noise in titanium?

Jim Rose: No, I took Gaussian random noise into my theoretical data that I had generated and, not knowing anything more about it, I simply added a random amount to each number, 50 percent.

Phillip Hodgetts: The reason I am asking is that down in the real world where I work, where we look at titanium with our present crude methods, what we call noise is really a misnomer because it is absolutely repeatable and it comes up all the time.

Jim Rose: I think the point again is that my information is really in the position of the first peak in some averaged way. Now, you can move things around differentially by putting in small cracks and influence the low frequency behavior, but their main influence is at high frequency where $K$ is of the size of a little flaw. I am getting the information out basically at lower frequencies. So I think I just average out there.

Phillip Cook (University of Houston): There is literature in radar which says that if you will send towards the void a ramp-shaped pulse and detect that ramp-shaped pulse in the time domain, then you can get information which is related to the area of the void. I think it all carries over very closely to this theory. It is all based on the long wave length approximation, and is based upon the face that the surface is radiating.