A Risk-Return Analysis For The Midwest Farmer-Feeder

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Abstract
Risk has always been an important dimension of the agricultural sector, and considerable effort has been expended to incorporate risk dimensions in decision models for the farm firm. With the dramatic fluctuations in commodity prices of the 1970's, the Midwest farmer has been confronted with increasing risk, particularly if cattle feeding has been a part of his farm organization. Fluctuations in feed costs, feeder cattle and fed cattle prices have resulted in wide variations in profit per head. In addition, new technology in feeding systems and housing is available and feeders must decide if that technology is feasible and if it should be adopted.

Disciplines
Agribusiness | Behavioral Economics | Business Administration, Management, and Operations | Income Distribution

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A RISK-RETURN ANALYSIS

FOR THE MIDWEST FARMER-FEEDER*

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A RISK-RETURN ANALYSIS FOR THE MIDWEST FARMER-FEEDER

Risk has always been an important dimension of the agricultural sector, and considerable effort has been expended to incorporate risk dimensions in decision models for the farm firm. With the dramatic fluctuations in commodity prices of the 1970's, the Midwest farmer has been confronted with increasing risk, particularly if cattle feeding has been a part of his farm organization. Fluctuations in feed costs, feeder cattle and fed cattle prices have resulted in wide variations in profit per head. In addition, new technology in feeding systems and housing is available and feeders must decide if that technology is feasible and if it should be adopted.

This paper utilizes a multiple objective linear programming model (MOLP) to analyze the key variables affecting the profit levels and income variability of the Midwest farmer-feeder. The variables analyzed include crop and livestock enterprise organization as well as cattle feeding systems, rations, housing systems, and types of cattle. The trade-off between risk and return for a typical farmer-feeder is also developed. The discussion will describe the application of a new method of handling risk in empirical modeling, and the practical interpretation of the numerical results generated. The methodology and conceptual framework for the analysis are developed in the next two sections. Transformation of this conceptual framework into a numerical model and the results of
the numerical analysis follow, along with conclusions and implications for Midwest farmer-feeders.

Methodology

Several studies have used quadratic programming to measure the trade-off between risk and return through the development of an efficiency (expected income-variance) frontier (Markowitz; Freund; McFarquha; Loftsgard and Heady; Scott and Baker)\(^1\). Such a frontier allows one to select the point at which variance is minimized for each level of income. Quadratic programming routines, however, have practical limits in terms of size and are expensive to solve. Therefore, modified linear programming procedures have been proposed to handle risk and approximate the expected income-variance (E-V) frontier. Hazell developed a linear alternative to quadratic programming, MOTAD (Minimization of Total Absolute Deviations), which minimizes the absolute deviations from expected income. Separable programming has been used by Thomas et al. to approximate the E-V frontier. Chen and Baker used a marginal risk constraint (MRC) to handle risk in a linear programming model. Given the income and variance of various enterprises, the Chen and Baker formulation assumes that the farmer maximizes expected return, provided the marginal contribution of each activity to the total variance does not exceed the expected unit income divided by a risk aversion parameter. Chen and Baker incorporate expected gross margins in the objective function and historical gross margins in the covariance matrix of the marginal risk constrained programming model.

Each of these approximation procedures has some serious shortcomings. Most require the calculation of gross margins for all risky activities to develop the covariance matrix. Yet, farm planning models developed by agricultural economists typically do not use gross margins for all
activities. Rather, marketing and transfer activities and equations are used where possible to aggregate sales and purchases in the model, thus simplifying input data manipulation and reducing matrix size.

An alternative approach recently developed by Libbin, Johnson, and Boehlje was instead used to approximate the efficient E-V frontier. The advantages of this approach are that it does not require the calculation of gross margins for all activities, and solution costs are lower than other approximation procedures. Thus, a larger and possibly more realistic model of the problem can be constructed and the estimation of the efficiency frontier appears to be as accurate as other approximation procedures.

Multiple objective linear programming (MOLP) problems have been developed in several recent studies (Philip; Evans and Steuer; Isermann; and Ecker and Kouada). Alternatively labeled the vector maximization problem, MOLP has the highly desirable property of allowing simultaneous maximization of several linear objectives. The MOLP problem can be stated as:

\[ \text{(1) Max } Cx \]

subject to: \[ Ax \leq b \]
\[ x > 0 \]

where \( C \) is a \( k \times n \) matrix of \( k \) linear objectives in \( n \) variables, and \( x \) is the vector of \( n \) decision variables. The \( A \) matrix and \( b \) vector are the commonly used coefficient matrix and right-hand-side of conventional linear programming (LP). Kouada and Ecker prove that efficient or Pareto optimal solutions of the MOLP problem can be generated by solving the following problem:

\[ \text{(2) Max } e'y \]

subject to: \[ Cx - Ty = Cx_0 \]
\[ Ax \leq b \]
\[ x, y \geq 0 \]
where $e$ is a $k$-dimensional vector (k-vector) of ones, $s$ is a $k$-vector of artificial variables, and $x_0$ is any feasible solution to problem (1). The MOLP procedure used for this article combines first-order minimization conditions of unconstrained quadratic function and a reformulation of problem (2) compatible with the minimization conditions.

The quadratic programming model provides the conceptual base for the procedure used here. Briefly, the quadratic programming (QP) problem can be written as:

\[ \text{(3) Max } -\frac{1}{2} x'Wx \]
\[ \text{subject to: } \pi' x \geq \alpha, \ 0 \leq \alpha < \infty \]
\[ Ax \leq b \]
\[ x \geq 0 \]

where $W$ is an $n \times n$ covariance matrix, $\pi$ is an $n$-vector of expected prices and costs (or expected net incomes), and $\alpha$ is the level of income which is parameterized from 0 to the maximum value consistent with $Ax \leq b$. Because $W$ is a positive semidefinite matrix, the first order conditions of the optimization problem imply that a minimum will be reached.

Because the gradient of the unconstrained $x'Wx$ function in each of the $n$ coordinate planes is zero when the function is minimized, and increases at an increasing rate away from the minimum, $\frac{1}{2} x'Wx$ can be approximated by minimizing the absolute value of the $n$-vector $Wx$. Thus, problem (4) can be used to approximate the QP problem (3).

\[ \text{(4) Min } |Wx| \]
\[ \text{subject to: } \pi' x \geq \alpha, \ 0 < \alpha < \infty \]
\[ Ax \leq b \]
\[ x \geq 0 \]

Reformulating problem (4) by the theorem of Ecker and Kouada, and rewriting $\min |Wx|$ as $\max -|Wx|$ yields:
(5) \( \max \epsilon \)'s

subject to: \( -|wx| - Is = -|wx_0| \)

\[ \pi'x \geq \alpha \]

\[ Ax \leq b \]

\[ x, \delta \geq 0 \]

The first constraint of problem (5) can be rewritten as \(|wx| + Is = |wx_0|\). But, problem (5) is still not in a form amenable to conventional LP packages because of the absolute value designation on \(|wx|\). The additional of two \(n\)-vectors of artificial variables will allow the problem to be reformulated as in problem (6) such that each \(w_jx\) is converted to its absolute value (\(w_j\) is the \(i\)th row of \(W\)).

(6) \( \max \epsilon' \)'s

subject to: \( wx + It - Is = 0 \)

\[ It + Iu + Is = |wx_0| \]

\[ \pi'x \geq \alpha \]

\[ Ax \leq b \]

\[ x, \delta, t, u \geq 0 \]

where \(t\) and \(u\) are both \(n\)-vectors of artificial variables. For a particular \(w_jx\), either \(t_j\) or \(u_j\) will be activated, but both will not enter the basis because of the maximization of \(\epsilon' \)'s.

The last remaining undefined argument in problem (6) is \(x_0\), which by theorem can be any solution which lies within the feasible region of system (4) (Ecker and Kouada). Clearly the optimal solution \(\bar{x}\) to problem (7):

(7) \( \max \pi'x' \)

subject to: \( Ax \leq b \)

\[ x \geq 0 \]

is feasible for any \(x\) level of problem (6). Thus, substituting \(\bar{x}\) for \(x_0\) in problem (6) yields a problem solvable by conventional LP methods. The
entire approximated E-V frontier can then be generated in two computer jobs: one to determine $\bar{x}$ from problem (7) and the second to solve problem (6) which can be accomplished with standard parametric programming procedures.

The Conceptual Model

The multiple objective function linear programming procedure was applied to the long run whole-farm planning problem of the Midwest farmer-feeder. The model developed here uses expected net present value as the income measure in the objective function. Compared with annual measures of income, expected net present value accounts for the time value of money and enables the simultaneous comparison of the investment returns and use of limited resources for multiple investment alternatives. Furthermore, model size is an issue with all formulations, and the net present value approach enables the analyst to incorporate a multiperiod planning horizon in the model without explicitly including activities and constraints for each time period. Finally, Hart argues that the only decision in dynamic analysis that must be implemented is the first decision, and new information will become available to use in decision for future periods. So the strategy of model development should be to include those factors that will influence the "best" first decision. Properly structured, the net present value formulation will generate the "best" first decision for expected future prices and production relationships. Thus, the model formulation used here is a compromise among the investment models developed by Baumol and Quandt and Weingartner; the multi-period firm growth models as used by Loftsgard and Heady and Boehlje and White; and risk models as
proposed by Markowitz, Freund, Hazell, Scott and Baker, and Chen and Baker.

Based on these concepts and the MOLP procedure, the Midwest farmer-feeder is assumed to simultaneously minimize the absolute value of the gradient of the quadratic risk function in each coordinate plane subject to parametrically increasing net present value and the firm's production capacity. Consistent with problem (4), the MOLP mathematical formulation of this model is described by equations (8) through (12).

After the reformulations of problems (6) and (7), the model can be solved with conventional linear programming procedures.

\[
\text{(8) Min } |D_j \cdot x_j|, \quad 1 \leq j \leq n_1
\]

\[
D = \frac{W_{ij} \cdot j}{(1+r)^{-t}}
\]

where:

\(D_j = j\text{th } n_1\)-vector of the discounted covariance matrix \(D\)

\(x_j = \) level of marketing activity \(j\)

\(W = n_1 \times n_1\) covariance matrix of deviations from trends in prices of the \(j\) activities

\(r = \) discount rate for pure time preference for money (5.845%)

\(t = \) year of the planning horizon, \(1 \leq t \leq T\)

subject to:

\[
\sum_{t=1}^{T} (1+r)^{-t} \sum_{j=1}^{n_1} P_j x_j - \sum_{k=1}^{n_2} C_k + \sum_{\ell=1}^{n_3} C_{\ell} x_{\ell} \geq \alpha
\]

where:

\(P_j = \) market price per unit of activity \(j\), \(P_j > 0\) for selling activities.

\(P_j < 0\) for purchasing activities.

\(C_k = \) cash operating expenses per unit of activity \(k\)
\( x_{2k} \) = level of production or storage activity \( k \)

\( \text{CO}_k \) = capital outlay per unit of activity \( k \)

\( x_{3\ell} \) = level of investment activity \( \ell \)

\( \alpha \) = parameterized income level, \( 0 \leq \alpha < \infty \)

(10) Production Capacity and Storage Constraints

\[
\sum_{k=1}^{n_2} a_{ik} x_{2k} \leq b_{\ell}, \quad 1 \leq i \leq m
\]

where:

\( a_{ik} \) = amount of resource \( i \) used in the production of activity \( k \)

\( b_{\ell} \) = quantity of \( \ell \)th resource available.

(11) Marketing Constraints

\[
\sum_{k=1}^{n_2} a_{jk} x_{2k} = x_{1j}, \quad 1 \leq j \leq n_1
\]

where:

\( a_{jk} \) = amount of marketable output \( j \) produced by or stored through

(or marketable input \( j \) consumed by) activity \( k \)

(12) Investment Limitations

\[
\sum_{k=1}^{n_2} a_{k\ell} x_{2k} \leq x_{3\ell}, \quad 1 \leq \ell \leq n_3
\]

\[
\sum_{\ell=1}^{n_3} x_{3\ell} \leq b_3
\]

where:

\( a_{k\ell} \) = the amount of purchased capital facilities \( \ell \) required by activity \( k \)

\( b_3 \) = exogenous facilities size limit

The covariance matrix, \( \mathcal{W} \), for prices rather than gross margins is included in the MOLP objective functions for two reasons. First, price is the only stochastic variable considered in the analysis. Furthermore, various production activities produce the same product which is aggregated
and then sold through a single marketing activity. Since a separate objective function must be added to the model for each activity that includes a stochastic element, aggregating and selling like products through a single activity reduces the size of the covariance matrix dramatically.

The total variation of price over time is a function of a discernible trend plus random deviations about that trend. The relevant variance and covariance data, and consequently W, are taken after the trend effect has been deleted by use of simple regression leaving only the random deviations from trend (Chen). Crop yield variances were not considered in the model because of the difficulty of choosing among individual farm yields or average county or regional yields. It is suggested by Freund that farm yield variances may be nearly four times the magnitude of county yields. Yield variances could be entered into the model by adding a partition to the D matrix corresponding to associated crop production activities.

The Numerical Model

A multiple objective linear programming model that includes crop and livestock production activities, buying and selling activities, and investment activities was constructed to represent the conceptual model and test the trade-off between risk and return for farmer-feeders. The model is structured for a typical Midwest farm that includes 400 tillable acres. The labor resource consists of eighteen months of labor divided among four three-month time periods. Quarterly time periods are specified according to the seasonality of crop production.

Alternative cropping activities include continuous corn and a corn-soybean rotation on Classes I and II land. Corn can be harvested as silage or high moisture corn grain. The grain may be artificially dried in the harvest period. Buying and selling activities are included for high
moisture grain in the harvest period and for dry corn throughout the year, but corn silage cannot be purchased or sold. Dry corn and soybeans may be stored from the period of production to any of three additional quarterly sale periods. Soybeans cannot exceed 50% of the total tillable acres. Land not used in production can be cash rented.  

Cattle-feeding activities were identified on the basis of the type of production facility used, ration fed, sex of cattle, and time of year. Two basic types of production facilities are considered in the model—an open lot and confinement. The open lot can be equipped with shelter or windbreak fence. The confinement units are classified according to the climate control technology used. The specific production facilities included in the model are: 1) open lot with windbreak fence; 2) open lot with shelter; 3) cold confinement, flush gutter; and 4) warm confinement, flush gutter. For a complete discussion on these systems, see Boehlje and Trede and Petritz. Cattle feeding facilities (including feed storage facilities) may be constructed up to a level of 300 head capacity.  

Five different cattle feeding programs are included in the model: 1) Purchase steer calves and background on total roughage rations for placement in yearling steer program (125 days), 2) Purchase steer calves and feed to market weight (325 days), 3) Purchase heifer yearlings and feed to market weight (180 days), 4) Purchase steer yearlings (or obtain from backgrounding program) and feed to market weight (165 days), and 5) Purchase steer yearlings and custom feed in commercial feedlot (165 days). Feed and nonfeed input requirements were developed by quarters.  

Three different finishing rations are included for each type of cattle and production facility. The rations tested are a high-concentrate, wet-corn ration; high-concentrate, dry-corn ration; and a high-silage, wet-corn ration. No seasonal differences in feed requirements are considered (Self).
Placements may be made in any quarter. The scheduling of those placements is accomplished by the model with consideration given to seasonal variability in feeder cattle and finished cattle prices.

A covariance matrix was calculated for the prices of corn (both purchase and sale), soybeans, soybean meal, feeder cattle, fed cattle, and cash rents. Deviations from trends of quarterly prices for the 1967-75 period were used to determine this covariance matrix. Expected cash flow levels for the marketing activities were calculated by discounting the prices and costs of the 1974 predicted values from the regression analyses. Costs of production activities were determined at 1974 actual levels. The specific procedure used in obtaining a solution to the multiple objective linear programming model is summarized by Libbin, Johnson and Boehlje.

Numerical Results

A summary of the numerical results is contained in tables 1 and 2 and in figure 1. The following discussion will first emphasize the characteristics of the risk-return efficiency frontier. Then the enterprise organization of the farm including cropping patterns, livestock production and resource use will be reviewed. Finally, details of the cattle feeding activities including investment in facilities, type of cattle, scheduling of sales and input purchases will be discussed.

The Efficiency Frontier

Figure 1 summarizes the efficiency frontier generated by the multiple objective linear programming procedure. The measure of return on the horizontal axis is the present value of the net income stream for 15 years discounted at a rate of 5.845% which represents the pure time preference for money. Risk is measured on the vertical axis as the discounted variance of income during the 15 year planning horizon. Consistent with theoretical
expectations, the efficiency frontier is convex, thus requiring increasing risk to obtain higher levels of income. For example, a movement from point 10 to point 9 on the efficiency frontier results in less than a 1% decline in return (at present value), but a 12.5% reduction in risk (discounted variance). In contrast, to move from point 3 to point 2 on the frontier results in an 18.7% reduction in return, and a 67.4% reduction in risk. So at lower levels of risk aversion, a small amount of income must be foregone to obtain significant reductions in risk. In contrast, at high levels of risk aversion, much larger income sacrifices are required to reduce risk.

**Enterprise Organization**

The enterprise organization for various points on the efficiency frontier of figure 1 are summarized in table 1. The organization of cropping and livestock enterprises as well as resource use changes dramatically along the frontier. With respect to the cropping program, almost all of the land is rented out or remains idle at point 1 which represents the highest level of risk aversion of those points summarized in table 1. Almost 87 acres of Class I land are idle rather than rented out at this point because variance is associated with the land rental activity, and the variance of income from land rented is higher for Class I land compared to Class II land.

As one moves along the frontier to points of lower risk aversion, soybean production enters into the optimal cropping program at the maximum amount of 100 acres for each land class. Once soybean production enters into the optimal plan, it remains at the maximum level allowed with only one minor exception throughout the range of decreasing risk aversion. As risk aversion decreases and more cattle are fed, corn silage acreage expands from 6.3 acres on Class I land (point 2) to 140.5 acres at point 8
on the frontier (40.5 acres on Class I land and 100 acres on Class II land). Adjustments in the type of cattle fed and the ration results in a decline in silage production after point 8 on the frontier.

Because of the combination of market opportunities, and risk and return characteristics of the various enterprises, the acreage of corn produced for grain adjusts to the acreage allocated to soybean production and corn silage production as risk aversion decreases.

Corn acreage enters the optimal plan at point 4 in table 1 but varies in terms of total acreage with decreasing risk aversion. Thus, the soybean enterprise appears to be the most desirable cropping activity at all levels of risk and is relatively stable in size in the optimal plans. The corn-silage enterprise increases in acreage as cattle numbers increase with decreasing risk aversion. The corn-grain enterprise appears to be the residual claimant on land and varies along the frontier depending upon the quantity of soybeans and silage produced.

The data in table 1 indicate that cattle production is included in the optimal enterprise organization for all levels of risk aversion. Although a very diversified cattle feeding program is included for the highest level of risk aversion (point 1) with a combination of 38.6 yearling steers, 48.3 yearling heifers and 12 calves, calves are phased out and heifers dominate the cattle feeding program at point 2 on the frontier. As risk aversion decreases further, finishing calves dominates the cattle program until point 8 on the frontier where the maximum feedlot capacity of 300 head is encountered. As risk aversion declines further beyond this point, an increasing proportion of the feedlot space is allocated to yearling steers rather than calves. At the final
point on the frontier (point 10) approximately 123 head of yearlings are being fed in the feedlot and 177 head of calves.

Labor utilization increases from the most risk averse to the least risk averse point on the frontier by almost eight-fold. Labor usage consistently increases from point 1 through point 8 in table 1 as the size of both the cropping and livestock programs increase. The slight decline in labor utilization for points 9 and 10 reflect shifts from the calf to a yearling steer feeding program which requires fewer hours of labor.

To support the cattle feeding enterprise at the higher levels of risk aversion, corn must be purchased. At high levels of risk aversion, lower variability in total income occurs by renting out the land and buying the corn to feed to cattle rather than producing grain on the farm. In the middle of the efficiency frontier (points 4 to 8), sufficient corn is produced to feed the expanding livestock operation and still have some available for cash sales. At points 9 and 10, corn is both bought and sold. In both cases, corn is sold in the September-October-November period and then purchased in the March-April-May period in the following year to support the cattle feeding operation. This schedule of sales and purchases takes advantage of seasonal differences in the purchase and sale price of corn.

Although most of the changes and adjustments in enterprise organization over the frontier are consistent and gradual, one should note the major adjustments that occur from point 8 to point 10. Point 10 represents the traditional linear programming solution to such a problem with no concern for the issues of risk. The adjustments in enterprise organization between these two points include a shift from approximately 140 to 100 acres of corn silage and from 60 to 100 acres of corn for grain.
With respect to the livestock operation, the adjustments include changing from 300 head of calves to 177 head of calves and 123 head of yearling steers. Major changes also occurred in the purchase and sale of corn as noted earlier. These adjustments resulted in only a $7,054 increase in net present value (a 1.2% increase), but a 25.9% increase in discounted variance. So optimal enterprise organizations that include risk may be quite different than those which ignore the dimensions of risk, even though the income levels may not be significantly different.

**Cattle Feeding**

Additional detail concerning the cattle feeding enterprise is summarized in table 2. At the highest level of risk aversion (point 1), a combination of open-lot shelter and open-lot windbreak fence feedlot facilities are purchased for the cattle feeding operation. A 100 head size facility is purchased. With the exception of point 2 on the efficiency frontier, the investment in feedlot facilities increases as higher levels of risk are assumed until the maximum of a 300 head feedlot is purchased at point 8. The maximum size unit is also purchased at points 9 and 10 on the frontier. In all cases except point 1, an open-lot with a shelter is preferred to any of the other technologies including the open-lot with a windbreak fence and various types of confinement facilities.

The type of cattle fed has been noted earlier—a combination of yearling heifers and calves are fed at high levels of risk aversion, calves at medium levels of risk aversion, and a combination of calves and yearling steers at low levels of risk aversion. At point 1 on the frontier, a dry-corn ration is used for all cattle production. Most of the heifers
are placed on feed in December, whereas the yearling steers are purchased in September. Heifers continue to be placed on feed in December for points 2 and 3, but at point 3 calves are also purchased and placed on feed in the December feeding period. At points 4 and 5 increased numbers of calves are placed on feed in December, and the yearling heifers are eliminated from the cattle feeding program. All of the yearling heifers and calves placed in the lot for points 2, 3, 4, and 5 are fed a wet-corn ration.

At point 6 on the frontier, changes occur in both the scheduling of placements and the ration fed. Although the cattle feeding operation includes all calves, 125 head are placed in the feedlot in December and fed a wet-corn ration, whereas 37.8 head are placed in the feedlot in January and 87.1 in September and fed a high silage ration. At the following two points on the frontier, calves placed in September and fed a silage ration increase in numbers, and those placed in December and fed a wet-corn ration decline. Finally, at the lower levels of risk aversion, calf numbers decline and yearling steer placements increase. The steers are purchased in March and fed a dry-corn and hay ration, whereas the majority of calves are still placed in the lot in September and fed the high silage ration.

Both corn and hay must be purchased for the cattle feeding operation at both extremes of the efficiency frontier. All of the corn and hay used in cattle production at point 1 is purchased, with most of it purchased the December-January-February period. For points 2 and 3 on the efficiency frontier, wet corn is purchased at harvest to support the cattle feeding program. At points 9 and 10 on the efficiency frontier, corn is purchased in the March-April-May period (along with hay) to feed to the yearling steers
in a dry-corn hay ration. Soybean meal is purchased as required by the cattle feeding activities, but since variance values were relatively high for soybean meal in all periods, offsetting covariances may have had an effect on cattle feeding selection.

Again, consistent trends in the cattle feeding program with respect to scheduling, ration fed and type of cattle fed exists as one moves from high to lower levels of risk aversion. However, major adjustments in the cattle feeding enterprise occur at points 9 and 10 of the efficiency frontier, again indicating the important impacts of including risk in the analysis.

Summary and Conclusions

The volatility of prices of the recent past has resulted in more concern for the issues of risk and risk management strategies in farm firm decision models. A procedure to efficiently handle risk in programming models through the use of multiple objective linear programming procedures was applied to the long run planning and short run feeding and scheduling problems of a typical Midwest farmer-feeder. The results provide numerical documentation of the risk-return trade-off faced by a typical producer. As expected, the efficiency frontier is convex, indicating that for both low and intermediate levels of risk aversion, it is possible to reduce income variability without a proportional decrease in returns.

The optimal enterprise organization and cattle feeding program change dramatically along the efficiency frontier. Much of the land is rented out at high levels of risk aversion. As the farmer-feeder becomes less
risk averse, soybeans first enter the optimal cropping program with corn silage acreage expanding as the size of the cattle feeding enterprise increases. Corn grain production appears to be the residual claimant on land and varies along the frontier depending on soybean and corn silage acreage.

Cattle feeding is part of the optimal farm organization at all points on the efficiency frontier because the variance in crop prices is high enough to prevent the farmer from specializing in crop production. In fact, at high levels of risk aversion the cattle feeding enterprise is supported by purchased grain and most of the land resource is rented out. The portfolio approach to choosing an enterprise organization which allows the consideration of offsetting variances and covariances is probably best illustrated by this low risk optimal enterprise organization where the high variability and moderate incomes of cattle fed with all purchased inputs is offset by the lower variability and higher incomes of land rental compared to the production and sale of grain. As risk aversion decreases, more feedlot space is acquired and the cattle feeding enterprise changes from the lower risk yearling heifer and calf feeding programs to a larger proportion of higher risk and return yearling steers. Adjustments in placements from the winter to the early fall and spring periods to exploit higher but more variable prices also occurs as risk aversion declines. However, diversification is maintained even at low levels of risk aversion. Not only is the farmer-feeder diversified in terms of enterprise organization, (with soybean, corn grain and cattle enterprises in the optimal organization), but the cattle enterprise is also diversified and includes both calves and yearling steers bought at
three different times of the year and fed two different rations.

With higher levels of risk aversion, less feedlot space is used. Similarly, the labor and capital requirements for each program decrease. Thus, as risk aversion increases, the farmer-feeder will have excess resources for the farming operation. These resources may be rented or hired out at a relative riskless rate. Thus, off-farm employment of resources should possibly be considered as a strategy to reduce risk as well as increase income. For all levels of risk aversion, the opportunity to feed cattle in a custom feedlot is not exercised.

As noted earlier, major adjustments in enterprise organization and the cattle feeding enterprise occurred from points of very low risk aversion to the linear programming solution which ignores risk completely. These adjustments were accompanied by almost insignificant increases in income, but large increases in variance. So ignoring risk in firm decision models may result in solutions that are dramatically different than those which consider risk, and the income level generated when risk is ignored may not be significantly higher.

The results and adjustments in enterprise organization along the frontier seem plausible and appear to be consistent with not only theoretical expectations, but actual behavior of Midwest farmer-feeders. In particular, the optimal organizations generated for the middle of the efficiency frontier are not dissimilar from those exhibited by numerous participants in the Iowa Farm Business Association.

Finally, the multiple objective linear programming procedure used here appears to have considerable appeal in analyzing problems that include risk dimensions. The procedure is less restrictive in terms of model structure and model size compared to other procedures for approximating an efficiency frontier, and thus larger and typically more realistic models
can be constructed and solved with the same level of computer and manpower resources. Furthermore, the approximation error appears to be acceptable and possibly smaller than other estimation procedures. Additional applications of the procedure are required and underway to evaluate the general applicability of multiple objective linear programming to problems that include quadratic functions, but initial results as evidenced here are encouraging.
Table 1. Optimal Cropping and Livestock Program for Various Points on the Efficiency Frontier.

<table>
<thead>
<tr>
<th>Point Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discounted Variance (x10^6)</td>
<td>5.7</td>
<td>42.9</td>
<td>131.4</td>
<td>482.6</td>
<td>975.2</td>
<td>1,310.6</td>
<td>1,625.6</td>
<td>1,802.4</td>
<td>1,985.0</td>
<td>2,268.9</td>
</tr>
<tr>
<td>Cropping Program</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class I (Acres)</td>
<td></td>
<td></td>
<td></td>
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^a The letter in parentheses indicates the ration fed where D denotes dry corn and hay, W denotes wet corn and corn silage and S denotes a high silage ration.

^b Wet corn could be purchased only in the September-October-November period.
Figure 1. E-V frontier generated by MOLP

- □ = Solution Summarized in Tables 1 and 2
- ◇ = Extra Solution Not Summarized

Net Present Value of Income (x 10^3)

Discounted Variance of Income (x10^6)
Footnotes

1 Alternatively, Boussard and Petit incorporated a focus of loss constraint in a linear programming model. They assumed that farmers choose those alternatives that will maximize expected gain provided that the potential loss is within subjectively determined acceptable limits.

2 For example, the model developed in this article requires a $35 \times 191$ (rows x columns) matrix, of which 143 columns are production and transfer activities and 52 are marketing activities. The structure also requires a $52 \times 52$ covariance matrix and a set of five $52 \times 52$ identity matrices which added 52 rows and 156 columns to the model. In total, the matrix size is $191 \times 347$. To construct a similar model in gross margin terms compatible with MRC, a much denser $338 \times 320$ matrix would be required, of which the A matrix is $52 \times 320$ and W is $286 \times 286$. Approximately 16 times the number of matrix elements required for MOLP would be required for MRC. A quadratic programming model would be $87 \times 191$.

3 That W is positive semidefinite is generally true (Graybill), but should be checked to prevent critical errors. At times, rounding errors can cause W to lose this necessary property.

4 In contrast, an MRC frontier of this problem would require approximately 20 computer runs at an estimated cost of $150 each, and QP would require one run per selected frontier point at an estimated cost of $350 each. MOLP procedures required only two runs at a total cost of $320 (13 minutes CPU).

5 All production and marketing activities were specified on a quarterly basis, excepting crop production; thus each should carry a time subscript. Those subscripts were deleted for simplicity in exposition. Consequently, two similar actions (e.g. selling soybeans) which differ by quarter are considered as two separate activities.

6 Because the expected cash flow values of the various production and marketing activities are presented in net present value terms, W must also be discounted to the matrix D. So,

\[
\text{NPV} = \sum_{t=1}^{T} (1+r)^{-t} \cdot \text{Expected Income} = (6.7994) \cdot \text{Expected Income}, \quad T = 15
\]

\[
D = \sum_{t=1}^{T} (1+r)^{-2t} \cdot \text{Variance} = W \sum_{i=1}^{T} (1+r)^{-2t} = W(9.8114), \quad T = 15
\]

7 This resource base is representative of the typical farmer-feeder in the Midwest as evidenced by Iowa Farm Business Association records and other survey data.
References


