Stability, Random Disturbances, and the Exchange Rate Regime

Walter Enders
Iowa State University

Harvey E. Lapan
Iowa State University

Follow this and additional works at: http://lib.dr.iastate.edu/econ_las_staffpapers

Part of the Econometrics Commons, Economic History Commons, Economic Policy Commons, International Economics Commons, and the Policy History, Theory, and Methods Commons

Recommended Citation
http://lib.dr.iastate.edu/econ_las_staffpapers/115
Stability, Random Disturbances, and the Exchange Rate Regime

Abstract
Economists have long debated the relative merits of fixed versus flexible exchange rate systems. One of the major issues in this debate concerns the ability of the exchange rate system to isolate a nation from external disturbances: see for example, papers by Mundell (1960), Stern (1963), Tower and Courtney (1974), and Enders (1977), Fischer, (1977) added another dimension to the controversy by examining the relative stability of real consumption and prices, assuming that the economy is subject to stochastic disturbances. His principle conclusion for the small open economy is that if disturbances are external, then a flexible exchange rate regime is better in that foreign disturbances have no impact on the domestic economy.

Disciplines
Econometrics | Economic History | Economic Policy | International Economics | Policy History, Theory, and Methods
Stability, Random Disturbances, and the Exchange Rate Regime

by

Walter Enders and Harvey E. Lapan

No. 68
Economists have long debated the relative merits of fixed versus flexible exchange rate systems. One of the major issues in this debate concerns the ability of the exchange rate system to isolate a nation from external disturbances: see for example, papers by Mundell (1960), Stern (1963), Tower and Courtney (1974), and Enders (1977). Fischer (1977) added another dimension to the controversy by examining the relative stability of real consumption and prices, assuming that the economy is subject to stochastic disturbances. His principle conclusion for the small open economy is that if disturbances are external, then a flexible exchange rate regime is better in that foreign disturbances have no impact on the domestic economy. Under a fixed exchange rate system, these disturbances lead to variances in both the domestic price level and real consumption. However, if the disturbances are internal -- due to either disturbances in real output or real demand -- a fixed exchange rate leads to a constant price level, whereas under flexible rates the internal price level is variable. Furthermore, if the disturbances are due to real output variations, then fixed exchange rate systems perform better in that the variance of consumption is smaller than for a flexible exchange rate regime.¹

The questions posed by Fischer are both interesting and important in assessing the merits of fixed and flexible exchange rate systems. However, his answers are hardly surprising since his assumption of no capital mobility implies that a flexible exchange rate regime is equivalent to a closed economy. As such, foreign disturbances do not
affect the economy, while domestic disturbances have their full impact on the domestic economy. Since it is the fixity of the exchange rate which opens the economy, foreign disturbances will cause domestic repercussions and part of any domestic disturbance will leak abroad. Furthermore, in the small open economy with fixed exchange rates, Fischer's assumption of no non-traded goods means that price fluctuations can only originate abroad.

However, the assumption of a small open economy operating under fixed exchange rates does not necessarily imply that domestic prices are insensitive to domestic disturbances. Rather, only the prices of traded goods are insensitive to such disturbances. Thus, in considering the relative price stability of an open economy under alternative exchange rate regimes, it seems necessary to consider how the prices of non-traded goods, as well as those of traded goods, are affected by disturbances. Furthermore, as relative price changes will affect resource allocation, large random variations in relative prices could lead to (ex post) unwarranted resource adjustments. Hence, the stability of the price level is not the only criterion to be examined in comparing exchange rate regimes; it is also important to examine the stability of relative prices.

As noted above, Fischer's assumption of flexible exchange rates and no capital mobility is equivalent to assuming that the economy in question is closed. In conjunction with the other assumptions of Fischer's model, this implies that no intertemporal transfer of goods is possible, so that real consumption is identical to real output. This conclusion, however, is not a necessary consequence of a flexible exchange rate. If capital were mobile, individuals could transfer real wealth and consumption intertemporally by changing their holdings of foreign assets. Thus, it is important to compare the relative stability of the two exchange rate regimes in the presence of capital mobility.

Thus, the purpose of this paper is to extend Fischer's work in
two directions: 1) to consider relative price stability in the presence of non-traded goods; and 2) to consider how capital mobility affects the stability of consumption. In Section II we consider the case of non-traded goods; Section III then focuses on the role of capital mobility. Some conclusions and directions for further research are presented in the final section.

II. Non-traded Goods and Stability

The basic model we use is the simple monetarist model as employed by Fischer. On the real side, we assume that two goods are produced domestically: $Q_1$, a traded good; and $Q_2$, a non-traded good. Each of these goods may be subject to random (multiplicative) disturbances:

1) \[ Q_i(t) = \bar{Q}_i \pi_i \quad i = 1, 2 ; \quad E(\pi_i) = 1 ; \quad \text{Var}(\pi) > 0 \]

$\bar{Q}_i$ represents the long run production allocation, and $\pi_i$ the disturbance. $P_i^f(t)$ is the foreign price of the traded good at time $t$, $e(t)$ is the price of foreign exchange, and $P_2(t)$ is the domestic price of the non-traded good. Thus:

2) \[ P_1(t) = e(t)P_1^f(t) \]

3) \[ Y(t) = P_1 \bar{Q}_1 \pi_1 + P_2 \bar{Q}_2 \pi_2 \]

in equation 3), $Y(t)$ is the nominal value of total output. Where it is unambiguous, the time argument will be suppressed. Define $\bar{Y}$ as long-run expected income: \[ \bar{Y} = E[ P_1 \bar{Q}_1 \pi_1 + P_2 \bar{Q}_2 \pi_2 ] \]

where $E$ is the expectation operator. Following the monetarist approach, we assume that the long-run demand for money (wealth) is a constant multiple of $\bar{Y}$:

4) \[ M = k \bar{Y} \]

Desired nominal expenditures, $S(t)$, depend upon long-run nominal income and a proportion ($\alpha$) of the excess of the sum of current income and wealth over the long run levels: \[ S(t) = \alpha(M + W) \]
5) \[ S(t) = \bar{Y} + \alpha [\hat{Y}(t) + M(t-1) - \bar{Y} - \bar{M}] \]
\[ = (1-\alpha-s)\bar{Y} + \alpha [\hat{Y} + M(t-1)] \] ; \[ s = \alpha k ; \quad 0 < \alpha < 1 \] .

In equation 5), \( \hat{Y} \) is current income, including not only nominal output, \( [ Y(t) ] \), but also net transfers and capital gains. Since we ignore capital movements in this section, \( \hat{Y} \) differs from \( Y \) only due to stochastic disturbances in the money supply which result in net transfers:

6) \( \hat{Y} = Y(t) + \epsilon(t) \) ;

in equation 6), \( \epsilon(t) \) is the stochastic disturbance in the money supply resulting in net transfers. Further:

7) \( M(t) = M(t-1) + B(t) + \epsilon(t) \)

where \( B(t) \) is defined to be the nominal balance of trade which is equal to:

8) \( B(t) = P_1Q_1(t) - P_1C_1(t) \) ; \( C_1 = \) consumption of good \( i \)

Finally, in order to determine the price of the non-traded good we must specify the commodity demand functions. For simplicity, we assume:

9) \( C_1^D = \left[ \frac{\beta S(t)}{P_1(t)} \right] \) ; \( C_2^D = \left[ \frac{(1-\beta) S(t)}{P_2(t)} \right] \)

where \( C_i^D \) is the demand for good \( i \).

Equations 1 - 9 serve to determine \( P_1(t) \), \( P_2(t) \), \( M(t) \), \( C_1(t) \), and \( C_2(t) \), given the values of \( P_1^*(t) \), \( M(t-1) \), and the stochastic disturbances. Upon simplification:

10) \( P_2 = \frac{1-(1-\beta)/a_0Q_2(t)}{(1-\alpha-s)\bar{Y} + \alpha[P_1Q_1 + M(t-1) + \epsilon(t)]} \) ; \( a_0 = (1-\alpha + \alpha \beta) \)

11) \( C_2 = Q_2 \)

12) \( C_1(t) = \frac{\beta/a_0 P_1(t)}{(1-\alpha-s)\bar{Y} + \alpha[P_1Q_1 + M(t-1) + \epsilon(t)]} \)

13) \( M(t) = \frac{1-(1-\alpha)[M(t-1) + \epsilon(t) + P_1Q_1]}{\beta(1-\alpha-s)\bar{Y}/a_0} \)

We are now ready to study the impact of stochastic disturbances on price and consumption stability.
A. Foreign Price Disturbances

Assume that the only disturbance is due to foreign price movements. As such, \( \varepsilon(t) = 0, \pi_1(t) = \pi_2(t) = 1 \) for all \( t \).

1) **Flexible Exchange Rates**

Under flexible exchange rates and in the absence of capital mobility, \( B(t) = 0, \) so that \( M(t) = M(t-1) \) and \( C_1(t) = Q_1(t) \). From (10) and (12), it is clear that \( p_1 \) and \( p_2 \) are constant, so that the economy is completely isolated from foreign price disturbances. Only the exchange rate is altered by foreign price movements. This result is identical to that of Fischer.

2) **Fixed Exchange Rates**

Set the exchange rate equal to unity so that \( p_1(t) = p_\Phi(t) \). From equations (3), (5), and (13), it is apparent that the variations in the price of the traded good affect nominal income, expenditures, the money supply, and real consumption.

From equation (13) since \( \varepsilon(t) = 0 \) and \( Q_1(t) = Q_\Phi \),

\[
M(t) = [(1-\alpha)[M(t-1)+p_\Phi(t)Q_\Phi] - \beta(1-\alpha-s)Y]/a_0 \\
= Q_\Phi \left(1-\alpha\right) \left[ \sum_{k=-\infty}^{t} (1-\alpha)^{t-k} p_\Phi(k) \right] - \beta(1-\alpha-s)Y \left[ \sum_{k=-\infty}^{t} (1-\alpha)^{t-k} \right]
\]

Assuming that foreign price disturbances are serially uncorrelated, with mean \( E(P_\Phi) \) and variance equal to \( \text{Var}(P_\Phi) \), we have:

\[
E[M(t)] = [(1-\alpha)Q_\Phi E(P_\Phi) - \beta(1-\alpha-s)Y]/a_0 = kY \text{ since } E(P_\Phi)Q_\Phi = \beta Y.
\]

Further, the variance of the money supply is given by:

\[
\text{Var}(M) = \left[ Q_\Phi^2 (1-\alpha)^2 / (a_\Phi^2 - (1-\alpha)^2) \right] \text{Var}(P_\Phi), \text{ or }
\]

\[
[\text{Var}(M)/E(M)^2] = [\beta(1-\alpha)^2 / sk(a_\Phi + (1-\alpha))] [\text{Var}(P_\Phi)/E(P_\Phi)^2]
\]

Associated with this variance in foreign prices and domestic wealth is a variance in the price of non-traded goods and in the
consumption of the traded good. From equation 10), and recognizing that $P^*_1(t)$ and $M(t-1)$ are uncorrelated, we find:

18) $\left[ \frac{\text{Var}(P_2)/E(P_2)^2}{\text{Var}(P^*_1)/E(P^*_1)^2} \right] = \left[ \frac{\alpha \beta}{\alpha \eta + 1 - \alpha} \right] \left[ \text{Var}(P^*_1)/E(P^*_1)^2 \right]$ 

Note that as a percent, the variance in the price of the non-traded goods is less than that of traded goods, indicating that foreign price movements cause movements in relative prices. These relative price changes may cause changes in the resource allocation pattern which are unwarranted from a long-run perspective. Furthermore, since $M(t)$ is serially correlated with $M(t-1)$ — even though $P^*_1(t)$ is serially uncorrelated — it follows that the stochastic changes in $P_2(t)$ and in relative prices are both serially correlated. In other words, a stochastic and transitory movement in $P^*_1(t)$ will give rise to changes in $P_2(t)$ that last for more than one period. For example, relative prices may lie below their long-run level for several periods, giving rise to pressure for changes in resource allocation.\(^7/\)

Turning to the effect of foreign price disturbances on real consumption, it is clear that the consumption of the non-traded good is unaffected (assuming no change in resource allocation). However, from equation 12) it is equally clear that foreign price disturbances do cause fluctuations in the consumption of the traded good. From 12):

19) $\text{Var}(C_1) = \left( \frac{\beta}{\alpha \eta} \right)^2 \left[ (1-\alpha) \text{Var}(1/P^*_1) + \alpha^2 \text{Var}(M) \left[ \text{Var}(1/P^*_1) + E(1/P^*_1)^2 \right] \right]$ 

Thus, in the case of foreign price disturbances, a flexible exchange rate isolates the small economy. Under fixed exchange rates, these foreign disturbances spill over into the domestic economy causing serially correlated disturbances in the price of non-traded goods and in relative prices. It is these autocorrelated disturbances in relative prices that are of interest because of the likely effects on resource allocation patterns.
B. Domestic Monetary Disturbances

Fischer implicitly assumes that the monetary authority has complete control over the money supply (aside from changes due to the balance of payments). However, there are likely to be unplanned changes in the money supply because of the Central Bank's inability to control this variable, and—in the absence of a debt instrument—unplanned government budget deficits or surpluses will affect current income and the money supply. We now turn to the effects of such monetary disturbances under alternative exchange rate regimes.  

1) Flexible Exchange Rates

While \( \varepsilon(t) \) is the stochastic to the money supply in period \( t \), we assume that:

\[
\varepsilon(t) = \theta(t) - \theta(t-1) ; \quad \mathbb{E}[\theta(t)] = 0, \quad \text{Var}(\theta) > 0
\]

In equation 20) \( \theta(t) \) is assumed to be serially uncorrelated and \( \theta(t-1) \) represents the realized value of the random variable in \( (t-1) \). Thus, money creation (destruction) by the Central Bank in \( t \) consists of two parts: 1) \( \theta(t-1) \) is a planned change to offset errors of the previous period, 2) \( \theta(t) \) represents the inability of the government to perfectly forecast or control expenditures and/or receipts.  

Under flexible exchange rates (and no capital mobility) \( B(t) \) is identically equal to zero. Thus:

\[
M(t) = M(t-1) + \varepsilon(t) = M(t-1) + \theta(t) - \theta(t-1) = M(t-2) + \theta(t) - \theta(t-2) \\
= M + \theta(t)
\]

where \( M \) is the steady state level of the money supply. Since no real disturbances are present:

\[
\text{Var}(C_1) = \text{Var}(C_2) = 0
\]

However, the variance of the money supply causes domestic prices and the exchange rate to fluctuate. From equations 10) and 12):
23) \[ \frac{\text{Var}(P_1)}{E(P_1)}^2 = \frac{\text{Var}(P_2)}{E(P_2)}^2 = \alpha^2 \frac{\text{Var}(\theta)}{\{(1-\alpha)\overline{Y}\}^2} \]

Thus, the variance in domestic prices is directly proportional to the variance in the domestic money supply. However, as is readily seen from equations 10) and 12), relative prices are unaltered by this disturbance:**

24) \[ \text{Var}(P_1/P_2) = 0 \]

2) **Fixed Exchange Rates**

As in the flexible exchange rate case, we assume that Central Bank creation of money in t consists of two components: one stochastic \( \theta(t) \), the other planned to correct errors of the previous period. However, under fixed exchange rates, an unplanned change in the money supply in period \( (t-1) \) does not result in an equal increase in the money supply in period t: some of the unplanned change leaks abroad. Thus, we modify 20) as follows:**

25) \[ \epsilon(t) = \theta(t) - \left[ \frac{1-\alpha}{\alpha_0} \right] \tilde{\theta}(t-1) \]

where \( (1-\alpha)/\alpha_0 < 1 \), is the increase in the money supply which does not leak abroad. Assuming that only monetary disturbances exist, equations 13) and 25) can be rearranged to form:

26) \[ M_t = \left( \frac{1-\alpha}{\alpha_0} \right) Q_t + \left( \frac{1-\alpha}{\alpha_0} \right) \sum_{k=-\infty}^{t-1} \tilde{P}_t \overline{Q}_t \left( \frac{1-\alpha}{\alpha_0} \right)^{t-k} - \frac{\theta(1-\alpha)\overline{Y}}{\alpha_0} \sum_{k=-\infty}^{t-1} \left( \frac{1-\alpha}{\alpha_0} \right)^{t-k} \]

Thus:

27) \[ E[M(t)] = k\overline{Y} = M \]

28) \[ \text{Var}(M) = \left[ \frac{1-\alpha}{\alpha_0} \right]^2 \text{Var}(\theta) \]

By assumption, the domestic price of the traded good is constant, as is consumption of the non-traded good. However, the variations in \( M(t) \) give rise to variations in the price of non-traded goods and to variations in the consumption of the traded good. From equation 13):
29) \[ M(t-1) + \epsilon(t) + P_1Q_1 = [a_0M(t) + \beta(1-\alpha-s)\bar{Y}]/(1-\alpha) \]

Substitution of equation 29) into equations 10) and 12) allows one to calculate the following variances:

30) \[ \text{Var}(P_2) = \frac{\alpha^2(1-\beta)^2}{(\alpha_0\bar{Q}_2)^2} \text{Var}(\theta) \]

so that:

\[ \frac{\text{Var}(P_2)}{E(P_2)^2} = \left(\frac{\alpha}{\alpha_0\bar{Y}}\right)^2 \text{Var}(\theta) \]

31) \[ \text{Var}(C_1) = \left(\frac{\alpha\beta}{\alpha_0P_1}\right)^2 \text{Var}(\theta) \]

Comparing these results to the flexible exchange rate case, it is seen that flexible exchange rates yield larger variances in the prices of both traded and non-traded goods. Under flexible rates, however, consumption of both the traded and non-traded good is unchanged. Furthermore, flexible exchange rates eliminate any variation in relative prices and any undesirable resource reallocation effects.

C. Real Domestic Disturbances

In this section we consider the effects of stochastic disturbances in real output on prices and consumption under the alternative exchange rate systems. As with other disturbances, we assume that the output disturbances are serially uncorrelated; however, the disturbances between sectors may be correlated.

1) Perfect Correlation Between Sectors

First, assume that the output disturbances between sectors are perfectly correlated so that: \( \pi_1(t) = \pi_2(t) \).

a) Flexible Exchange Rates

Clearly, output equals consumption for both traded and non-traded goods:

32) \[ \frac{\text{Var}(C_1)}{E(C_1)^2} = \text{Var}(\pi_1) \]

Under flexible exchange rates, the money supply is unchanging. As such,
disturbances in real output are fully absorbed by price variations.
From equations 10) and 12):

33) $\frac{\text{Var}(P_1)}{E(P_1)}^2 = \frac{\text{Var}(1/T)}{E(1/T)}^2$

Since the real output disturbances are perfectly correlated, the price movements in the two sectors are also perfectly correlated:

24) $\text{Var}(P_1/P_2) = 0$

While the real output disturbances cause movements in absolute prices, relative prices are unaltered.12/

b) Fixed Exchange Rates

Under fixed exchange rates, the variance of the consumption of the non-traded good is equal to the variance of output. Disturbances in the output of the traded good, however, result in money supply changes (as opposed to price changes). Thus, consumption of the traded good will be less variable than output, but relative prices will be more variable than in the flexible rate case. From 13):

35) $\text{Var}(M) = \left[\frac{1}{a_0}(1-a)\right]^2 \text{Var}(\tau)$

$\frac{\text{Var}(M)}{E(M)}^2 = \left[\frac{1-a}{1-a} \right] \text{Var}(\tau)$

Further, from equations 10) and 12):

36) $\frac{\text{Var}(C)}{E(C)}^2 = \frac{a_0(1-a)}{a_0+1-a} \text{Var}(\tau) < \text{Var}(\tau)$

37) $\frac{\text{Var}(P_2)}{E(P_2)}^2 = \frac{(1-a)^2 \text{Var}(X) + (a_0/(a_0+1-a)) \text{Var}(\tau) [\text{Var}(X) + X^2]}{[a_0(1-a)X]^2}$

where: $X = (1/\tau)$ ; $\overline{X} = E(1/\tau)$ ; $\text{Var}(X) = \text{Var}(1/\tau)$ ;

since $E(\tau) = 1$, $E(1/\tau) \neq 1$.

Comparing these results to the flexible exchange rate case, it is seen that the variance of consumption of the traded good is smaller under fixed exchange rates. No direct comparison of the variance of the price of non-traded goods is possible without specifying the
distribution of $\pi$. However, since $P_1$ is constant under fixed rates, it immediately follows that the variance of relative prices of the two goods is larger under fixed rates. Further, since the induced disturbances in the money supply are autocorrelated, the disturbances in the price of the non-traded good (under fixed rates) are autocorrelated. For example, a random disturbance to real output that increases the price of the non-traded good above its long-run level at time period $t$ implies that the expected price at $t+1$ is above the long-run level. Thus, if individuals have rational expectations, these autocorrelated disturbances will cause pressure for resource reallocations, a phenomenon that does not occur under flexible rates.

2) Independent Real Disturbances

Next, suppose that the real output disturbances are independent, although the distributions are identical.

a) Flexible Exchange Rates

Under flexible exchange rates the money supply is constant. Thus, variations in the consumption of each good are equal to the variations in the output of that good:

39) $\text{Var}(C_i)/E(C_i)^2 = \text{Var}(\pi_i) \quad i = 1, 2.$

From equations 10) and 12), the price level in each period -- and its mean and variance -- can be calculated directly:

40) $\text{Var}(P_i)/E(P_i)^2 = \text{Var}(1/\pi_i)/E(1/\pi_i)^2 \quad i = 1, 2.$

Since the disturbances are independent, so are the price movements. Hence, relative prices fluctuate in response to the real output changes:

41) $\text{Var}(P_2/P_1)/E(P_2/P_1)^2 = [\text{Var}(1/\pi_2)/E(1/\pi_2)^2]\text{Var}(\pi_1)+1] + \text{Var}(\pi_1)$

$\text{Var}(P_1/P_2)/E(P_1/P_2)^2 = [\text{Var}(1/\pi_1)/E(1/\pi_1)^2]\text{Var}(\pi_2)+1] + \text{Var}(\pi_2)$

Note that these relative and absolute price movements are not serially correlated and that the actual price in any period is not known until output is determined. Since the best forecast of any price
for the next period is the long-run price, these relative price changes should not cause pressure for a reallocation of resources. Thus, from a perspective of resource allocation, these relative and absolute price movements should not be important.

b) Fixed Exchange Rates

Since real disturbances in the non-traded sector do not affect current money demand (due to the unitary price elasticity of demand), the variance of $M(t)$ is calculated as in the previous section:

$$35') \text{Var}(M) = \left[ \frac{(P_1 \bar{Q}_1 (1-\alpha))^2}{(a_0^2-(1-\alpha)^2)} \right] \text{Var}(\pi_1)$$

The variance of consumption of the non-traded good is the same as the variance in real output; for the traded good we find (from equation 12):

$$42) \frac{\text{Var}(C)}{E(C)^2} = \frac{\alpha \beta}{a_0 + 1 - \alpha} \text{Var}(\pi_1) < \text{Var}(\pi_1) \text{ as } \alpha < 1.$$ 

Thus, real consumption of the traded good is less variable under fixed than flexible rates. Turning to the price of the non-traded good; from equations 10 and 13):

$$43) \frac{\text{Var}(P_2)}{E(P_2)^2} = \frac{\text{Var}(1/\pi_2)}{E(1/\pi_2)^2} + \frac{\alpha \beta}{a_0 + 1 - \alpha} \frac{\text{Var}(1/\pi_2)}{E(1/\pi_2)^2} \text{Var}(\pi_1)$$

Comparing equations 43 and 40, it is seen that the price of non-traded goods is more variable under fixed than flexible rates provided that $\text{Var}(\pi_1) > 0$. Without a specific price index, we cannot conclude which regime gives rise to greater variations in price levels.

The comparison of the variance of relative prices is more difficult in this case since $\text{Var}(1/P_2)$ is not readily calculated. Under fixed rates, the variance of relative prices $(P_2/P_1)$ is given by equation 43 since $P_1$ is constant. For flexible rates:

$$44) \frac{\text{Var}(P_2/P_1)}{[E(P_2/P_1)]} = \left[ 1 + \frac{\text{Var}(1/\pi_2)}{[E(1/\pi_2)]^2} \right] \text{Var}(\pi_1) + \left( \frac{\text{Var}(1/\pi_2)}{[E(1/\pi_2)]^2} \right)$$

Comparing 44 to 43, it is clear that $(P_2/P_1)$ has a larger variance under flexible rates; however, the relative variance of $(P_1/P_2)$ cannot
be ascertained from this information.

However, the price movements under flexible exchange rates are not serially correlated, so that there should be no change in resource allocation. In contrast, price movements of the non-traded good are correlated in a fixed exchange rate regime. Thus, disturbances in the real output of the traded good in \((t-1)\) not only affect the price of the non-traded good in \((t-1)\), but also affects the money supply and consequently, the non-traded goods price in \(t\). These autocorrelated price movements may lead to temporary changes in resource allocation.

III. Capital Mobility and Stability

In Section II, the absence of capital movements in the flexible exchange rate case meant that the economy under consideration was effectively closed. Hence, real disturbances led to a greater variability of consumption in a flexible exchange rate regime. External and monetary disturbances both led to a greater variability of consumption when the exchange rate was fixed. However, in the presence of capital movements, a country with a flexible exchange rate can transfer consumption intertemporally. We now introduce foreign assets into domestic portfolios in order to determine how the presence of capital movements alters the results of Section II.

In order to avoid index number problems, we assume that only one commodity (which is traded) is produced and consumed. Since this assumption eliminates relative price changes, we assume that the goal of policy makers is simply to reduce the variability of real consumption. Setting \(Q_2\) equal to zero, equations 1 - 3 can be rewritten as:

45) \(Q(t) = \bar{Q}_2 \tau(t)\)
46) \(P(t) = e(t)P^*(t)\)
47) \(Y(t) = P(t)Q(t)\)

In rewriting these equations, the subscript "l" has been omitted.
since we are only considering one good. This procedure will be followed throughout.

In introducing a second asset, we follow the approach used by Lapan and Enders (1977) and assume that individuals hold both domestic and foreign currency. Denote the proportion of foreign currency held to total asset holdings by "m". Thus,\[13/\]

\[ M(t) = (1-m)W(t) \]
\[ e(t)F(t) = mW(t) \]

in equation 48), \( W \) is equal to total asset holdings or wealth \( (W = M + eF) \), \( F \) is the foreign currency value of domestic holdings of foreign currency.

The introduction of foreign assets into domestic portfolios means that exchange rate changes will cause asset holders to experience capital gains or losses equal to \( e(t)F(t-1) - e(t-1)F(t-1) = [e(t) - e(t-1)]F(t-1) \). As current nominal income \( (\bar{Y}) \) includes these capital gains and losses, equation 5) can be rewritten as:

\[ S(t) = \bar{Y} + \alpha[Y(t) + \epsilon(t) + (e(t)-e(t-1))F(t-1) + M(t-1) - \bar{Y} - k\bar{Y}] \]
\[ = (1-\alpha-s)\bar{Y} + \alpha[Y(t) + \epsilon(t) + e(t)F(t-1) + M(t-1)] \]

Given the presence of two assets, it is necessary to modify equation 7):

\[ W(t) = W(t-1) + B(t) + \epsilon(t) + [e(t)-e(t-1)]F(t-1) \]

where, as in equation 8), \( B \) is defined to be:

\[ B(t) = Y(t) - S(t) \]

Equation 50) states that wealth in any period is equal to last period's wealth plus the trade balance plus net transfers to individuals plus the capital gains or losses on foreign asset holdings. Equation 51) follows directly from equation 8), once it is recognized that in a
one good world, consumption of good 1 is equal to total expenditures. Notice that the balance of payments (BP) can be determined directly from equation 50). The balance of payments is equal to current holdings of domestic currency minus the sum of last period's domestic currency holdings and net transfers:

52) \( BP = M(t) - M(t-1) - \varepsilon(t) = B(t) - e(t) [F(t) - F(t-1)] \)

Under a system of flexible exchange rates BP is identically equal to zero, while \( e(t) = e(t-1) \) under a system of fixed rates. Equations 45 - 49, 51, and either 50 or 52 determine the values of all the endogenous variables given the magnitudes of the predetermined variables, \( \pi(t) \), \( \varepsilon(t) \), \( P^*(t) \), and knowledge of whether the exchange rate is fixed or flexible.

A. Real Output Disturbances

We depart from the format of Section II and consider real output disturbances first. Intuitively, one would expect the introduction of capital movements to make the greatest difference in this case. In the absence of capital movements in a flexible rate regime, all output must be consumed. However, the introduction of capital movements means that intertemporal transfers of consumption are possible. It is assumed that the only disturbance is due to random changes in output so that \( \varepsilon(t) = 0 \), and \( \text{Var}(P^*) = 0 \). For simplicity we assume that \( P^* = 1 \). As in Section II, we assume that the random disturbance is serially uncorrelated.

1) Fixed Exchange Rates

Under a fixed exchange rate regime, a constant foreign price level acts to fix the level of domestic prices, \( P(t) = \overline{P} \). As such, the variance of real consumption \( [S(t)/P(t) = C(t)] \) is found as follows:

From equations 48-51:
\[ W(t) = (1-\alpha)Y(t) - (1-\alpha-s)\bar{Y} + (1-\alpha)W(t-1), \text{ so that} \]
\[ \text{Var}(W) = \frac{(1-\alpha)^2\bar{Q}^2\pi^2}{1 - (1-\alpha)^2}, \]

Since: \[ \frac{S(t)}{P} = C(t) = (1-\alpha-s)\bar{Y} + \alpha[\bar{Q}\pi(t) + \frac{W(t-1)}{P}] \]
\[ \text{Var}(C) = \frac{\alpha^2Q^2\pi^2}{P^2} + \frac{\alpha^2W^2}{\pi^2} = \frac{\alpha^2Q^2\pi^2}{1 - (1-\alpha)^2}, \text{ and } \text{Var}(C)/\text{Var}(Q) < 1. \]

This result is identical to that of Fischer. The presence of capital movements in a fixed exchange rate system does not act to alter the variance of consumption. Even in a one asset world, fixed exchange rates allow for intertemporal transfers of consumption. The variance of consumption is less than the variance of output since individuals can accumulate assets if \( \pi(t) > 1 \) and disaccumulate assets if \( \pi(t) < 1 \).

2) Flexible Exchange Rates

Under a flexible exchange rate regime, \( BP \) is identically equal to zero. As such, equation 49 can be written as:
\[ (1-\alpha)Y(t) + (1-\alpha)e(t)P(t-1) = (1-\alpha-s)\bar{Y} + \alpha M(t-1) + e(t)F(t), \]

Since \( M(t) = M(t-1) = M = (1-m)W(t), W(t) = W(t-1) = \bar{W}, \) and \( e(t)F(t) = \bar{m}W: \)
\[ (1-\alpha)Y(t) + (1-\alpha)m\bar{W} + \frac{P(t)}{P(t-1)} = (1-\alpha-s)\bar{Y} + \alpha(1-m)\bar{W} + m\bar{W} \]
or \[ \frac{1}{P(t)} = \frac{(1-\alpha)}{a_1} \left[ \bar{Q}\pi(t) + m\bar{W}/P(t-1) \right] \quad \text{where: } a_1 = (1-\alpha-s)\bar{Y} + \alpha(1-m)\bar{W} + m\bar{W} \]

Thus: \[ \text{Var}(1/P) = \frac{(1-\alpha)^2Q^2}{a_1^2} - \frac{(1-\alpha)^2m^2\bar{W}^2}{P^2}\text{Var}(\pi) \]

Since \( C(t) = (1-\alpha-s)\bar{Y}/P(t) + \alpha(\bar{Q}\pi(t) + \frac{(1-m)\bar{W}}{P(t)} + \frac{m\bar{W}}{P(t-1)}), \)
\[ = [(1-\alpha-s)\bar{Y} + (1-m)\bar{W}] \cdot \frac{1}{P(t)} + \alpha\bar{Q}\pi(t) + \frac{\alpha m\bar{W}}{P(t-1)} \]
\[ \text{Var}(C) = \frac{(1-\alpha)(a_1-\overline{mW})}{a_1} \left[ Q\pi(t) + \frac{\overline{mW}}{P(t-1)} \right] + \alpha Q\pi(t) + \frac{q\overline{mW}}{P(t-1)} \]

\[ \text{Var}(C) = \left[ \alpha + \frac{(1-\alpha)(a_1-\overline{mW})}{a_1} \right]^2 \left[ Q^2 \text{Var}(\pi) + m\overline{W}^2 \text{Var}(1/P) \right] \]

\[ = \frac{Q^2 \text{Var}(\pi)}{1 + 2mk} \]

WHERE: \( k\overline{Y} \) has been used to approximate \( \overline{w}^{1/4} \)

and \( \text{Var}(c)/\text{Var}(Q) = (1 + 2mk)^{-1} \leq 1 \).

As opposed to Fischer, we find that the variance of consumption may be less
than the variance of output. In particular, the variance of real consumption will
always be less than the variance of real output if domestics hold foreign assets
i.e., if \( m > 0 \). In the case Fischer considers, \( m = 0 \) and the variance of consumption
equals the variance of output. The point to note is that when doemstics hold
foreign assets, there is a mechanism by which it is possible to save or dissave.
Since \((1 + 2mk)\) is an increasing function of \( m \), the variance of consumption is
negatively related to what might be called the "degree of capital mobility,"
i.e., the greater the proportion of foreign assets in domestic portfolios, the
lower is the variance of real consumption. The difference between this result
and that of Fischer is due to the presence of capital movements. While flexible
rates constrain \( \text{BP} \) to equal zero, equation 52) demonstrates that the trade balance
can differ from zero so long as \( F(t) - F(t-1) \neq 0 \), i.e., as long as domestics can
accumulate or disaccumulate foreign assets. When output is less than long run
output \((\pi(t) < 1)\) domestics can reduce their asset holdings in order to acquire
goods. Alternatively if \( \pi(t) > 1 \), domestics can acquire assets.

The presence of foreign assets in domestic portfolios also affects domestic
income when exchange rates change. A random increase (decrease) in output will
act to decrease (increase) domestic prices and the exchange rate. In the presence
of foreign asset holdings, the depreciation (appreciation) of the exchange rate will impose capital losses (gains) on domestics. These capital losses (gains) will depress (stimulate) real consumption. Thus, the capital gains and losses on foreign asset holdings will tend to cause consumption to move counter to the change in output.

For a given variance in real output, the variance of consumption under flexible exchange rates will be greater than the variance of consumption under fixed exchange rates, if:

$$\frac{1}{1 + 2mk} > \frac{\sigma^2}{1 - (1-\alpha)^2} \quad \text{or if: } 1 > \alpha + ms.$$ 

While $\alpha$ and $ms$ are both less than unity, it is possible for their sum to be greater than one. Thus, the variance of real consumption under flexible exchange rates may be less than under fixed exchange rates.

B) Foreign Price Disturbances

Assume that the only disturbance is due to foreign price movements; thus, $\varepsilon(t) = 0$, and $\pi(t) = 1$.

1) Fixed Exchange Rates

As in the case of output disturbances, we would not expect the presence of capital movements to alter the variance of real consumption. For simplicity we set the exchange rate equal to unity so that $P(t) = P^*(t)$ and $\text{Var}(P) = \text{Var}(P^*)$.

From equations 51 and 52:

$$W(t) = (1-\alpha)Y(t) - (1-\alpha-s)\bar{Y} + (1-\alpha)W_{T-1}$$

so that:

$$\text{Var}(W) = \frac{(1-\alpha)^2}{\sigma^2} \text{Var}(P) \times \frac{1}{1 - (1-\alpha)^2}$$

From equation 49):
\[ C(t) = (1-\alpha-s) \frac{\bar{Y}}{P(t)} + \alpha \bar{Q} + \frac{\alpha \bar{W}(t-1)}{P(t)} \]

Since \( W(t-1) \) and \( P(t) \) are independent:

\[ \text{Var}(C) = (1-\alpha)^2 \bar{Q} \text{Var}(1/P) + \frac{\alpha^2 \bar{Q}^2 (1-\alpha)^2}{1 - (1-\alpha)^2} \text{Var}(P) \left[ \text{Var}(1/P) + \left[ \frac{1}{P} \right]^2 \right] \]

As \( \alpha, \bar{Q}, \) and \( P \) are independent of domestic holdings of foreign assets, the variance of consumption is also independent of foreign asset holdings.

2) **Flexible Exchange Rates**

In section II of this paper and in Fischer's paper, the findings were that flexible exchange rates insulate the domestic economy from foreign disturbances. Any increase (decrease) in the foreign price level would act to decrease (increase) the exchange rate by the same percentage change as the foreign price level changes. As real consumption is independent of the exchange rate, real consumption is invariant to foreign price level disturbances. However, when domestics hold foreign assets, exchange rate changes will cause domestics to experience capital gains or losses. These wealth changes will act to alter both real income and consumption. Only in the case in which domestic holdings of foreign assets are zero, will a flexible exchange rate isolate an economy from foreign disturbances.

Since a flexible exchange rate ensures that \( BP = 0, M(t) = M(t-1) = \bar{M} \) equation 49 becomes:

\[ S(t) = (1-\alpha-s) \bar{Y} + \alpha \left[ Y(t) + e(t)F(t-1) + \bar{M} \right] \]

From 50 and 51:

\[ (1-\alpha) \left[ Y(t) + e(t)F(t-1) \right] = (1-\alpha-s) \bar{Y} + \alpha \bar{M} + e(t)F(t) \]

Since \( e(t)F(t) = \frac{m \bar{M}}{1-m} \), this relationship and equation 54 can be substituted into 53 to obtain:
55) \[ C(t) = \left( (1-\alpha-s)Y + \frac{\alpha \bar{M}}{1-m} \right) \left( (1-\alpha)P(t) \right)^{-1} \]

From 55 it is immediately seen that the variance of real consumption will be zero, if the variance of the domestic price level is zero:

\[ \text{Var}(C) = \frac{\left( (1-\alpha-s)Y + \frac{\alpha \bar{M}}{1-m} \right)^2}{(1-\alpha)^2} \text{Var}(1/P) \]

In order to determine \( \text{Var}(1/P) \), form the following:

\[
\frac{1}{P(t)} = \frac{1}{e(t)P^*(t)} = \frac{F(t)}{e(t)F(t)P^*(t)} = \frac{(1-m)}{m \bar{M}} \left( \frac{F(t)}{P^*(t)} \right)
\]

From equation 54: \( e(t)F(t-1) + Y(t) = (1-\alpha)^{-1} \left[ (m \bar{M} / (1-m)) + \alpha \bar{M} + (1-\alpha-s)Y \right] \), to that:

\[
\frac{F(t)}{F(t-1) + P^*(t)Q} = (1-\alpha) m \bar{M} + \alpha (1-m) \bar{M} + (1-\alpha-s) (1-m) \bar{Y} \]

and: \( F(t) = a_2 F(t-1) + a_2 P^*(t) Q = a_2 \bar{Q} \sum_{j=0}^{\infty} P^*(t-j) \cdot a_2^j \). As such:

\[ 1/P(t) = \frac{(1-m)}{m \bar{M}} a_2 \bar{Q} \left[ 1 + \sum_{j=1}^{\infty} \left( \frac{P^*(t-j)}{P^*(t)} \right) a_2^j \right] \]

Since cov. \( (P^*(t), P^*(t-1)) = 0 \)

\[ \text{Var}(1/P) = \frac{(1-m)^2 a_2^2 a_2^4}{m \bar{M}^2} \left[ \text{Var}(P^*) (\text{Var}(1/P^*) + \left[ E(\frac{1}{P^*}) \right]^2) + \left[ E(P^*) \right]^2 \text{Var}(1/P^*) \right] \]

While this expression is somewhat difficult to interpret, notice that when \( m = 0, a_2 = 0 \). Thus, if \( m = 0 \), \( \text{Var}(1/P) = 0 \) and \( \text{Var}(C) = 0 \). The point is that flexible exchange will only insulate an economy from foreign disturbances if domestic holdings of foreign assets are zero.

Substitution of \( \text{Var}(1/P) \) into the equation for \( \text{Var}(C) \) yields:
\[
\text{Var}(C) = \frac{(1-\alpha-s) + \frac{\alpha M}{1-m}}{(1-\alpha)^2} \left( \frac{(1-m)^2}{mM} \right)^2 \frac{\sigma^2}{a_2^2} \left( \frac{E(P*)^2}{(1-a_2)^2} \right) \text{Var}(1/P*) + \\
\frac{\text{Var}(P*)(\text{Var}(1/P*) + \left[ E\left(\frac{1}{P*}\right) \right]^2)}{1 - \frac{\sigma^2}{a_2^2}} \\
\]

\[
= \frac{m^2k^2Q^2}{(1+mk)^2} \left[ E(P*)^2 \text{Var}(1/P*) + \frac{\text{Var}(P*)(\text{Var}(1/P*) + \left[ E\left(\frac{1}{P*}\right) \right]^2)}{(1+2mk)} \right] \\
\]

WHERE: \( \bar{W} \) has been set equal to \( k \bar{Y} \)

The difference between the variance of consumption under fixed exchange rates and the variance of consumption under flexible exchange rates is given by: 

\[
\left[ (1-\alpha)^2 - \frac{(mk)^2}{(1+mk)^2} \right] \left[ E(P*)^2 \text{Var}(1/P*) \right] + \left\{ \frac{\sigma^2(1-\alpha)^2}{1 - (1-\alpha)^2} - \frac{m^2k^2}{(1+mk)^2(1+2mk)} \right\} \\
\left[ \text{Var}(P*)(\text{Var}(1/P*) + \left[ E\left(\frac{1}{P*}\right) \right]^2) \right] \\
\]

The variance of consumption under flexible rates will be greater than the variance of consumption under fixed rates if: \( 1-\alpha-ms < 0 \). Notice that this condition is the exact opposite of that for real disturbance case. Thus, if all disturbances are either real or due to foreign price disturbances, the choice between fixed and flexible exchange rates is immaterial so long as \( 1-\alpha-ms = 0 \). The choice between fixed versus flexible exchange rates depends upon the sign of \( 1-\alpha-ms \) and upon the source of disturbances. If the initial source of disturbances is external, and if \( 1-\alpha-ms > 0 \) (\( 1-\alpha-ms < 0 \)) a flexible (fixed) exchange will best even out the time path of consumption. Instead, if the initial source of disturbances is due to output variations, and if \( 1-\alpha-ms >0 \) (\( 1-\alpha-ms < 0 \)) a fixed (flexible) exchange rate will best even out the time path of consumption.
III. Summary

Fischer found that flexible exchange rates isolate an economy from external disturbances, but that internal disturbances cause larger price variations under flexible than under fixed exchange rates. However, his model deals with a single, aggregate traded good and a small open economy. In this paper we have shown how his analysis can be extended to deal with non-traded goods and we have found that his results remain valid for external disturbances. When disturbances are internal, no direct comparison of absolute price movements is possible because of the presence of two goods, and hence the need for a price index. When the disturbances are monetary, both prices are more variable under flexible exchange rates; when disturbances are real and independently distributed, the price of traded goods are more volatile under flexible exchange rates, whereas the price of non-traded goods are more volatile under fixed rates.

However, from a microeconomic viewpoint, the movement in relative prices is probably of more significance than absolute price movements. We have shown that unless disturbances are real and independent, relative price variability is greater under fixed than flexible exchange rates. Furthermore, the price movements under flexible rates are not serially correlated, whereas under fixed rates the movement in the price of non-traded goods is serially correlated. Since these serially correlated price disturbances are more likely to give rise to shifting patterns of resource allocation, it follows that flexible exchange rates are more likely to give rise to stable resource allocation patterns and hence, in this sense, more effectively isolate an economy from stochastic disturbances, whether they are internal or external. This conclusion seems to contradict prior thought concerning the effect of the (alleged) greater uncertainty due to flexible exchange rates on resource allocation.
Turning to real consumption, it is apparent that a flexible exchange regime with no capital mobility is equivalent to a closed economy. Hence, real consumption will be more variable under fixed rates if disturbances or external or monetary in nature, but if disturbances are real, flexible exchange rates yield greater variability in consumption. However, if capital mobility is present, this conclusion is no longer true a priori.
1. Obviously if the disturbances are due to fluctuations in real demand, the fixed exchange rate system yields a greater variability in consumption than does a flexible exchange rate system (which has zero variance in real consumption). However, it is difficult to conclude that this result is "bad" since the disturbance to demand may reflect changes in "need" or intertemporal variations in utility functions. To avoid this ambiguity, we will consider stochastic disturbances in the money supply; analytically, this is comparable to stochastic disturbances in demand, but it is easier to interpret the results for this case.

2. From a private perspective, holding any assets accomplishes this; however, from a national perspective only holdings of foreign assets affect the claims on real goods; of course, commodity hoarding could accomplish the same end. Presumably, goods are assumed perishable in the Fischer model.

3. Perhaps a more fundamental question in comparing the two exchange regimes concerns the nature of the stochastic disturbances. Fischer assumes all disturbances are known to be random movements around an unchanging equilibrium; thus, systems--such as fixed exchange rates--which minimize responses to these disturbances, will be desirable. If it is not known ex ante whether the disturbances are transitory or permanent movements in (e.g.) real output, then delayed adjustment is not necessarily desirable. However, this issue--while important is beyond the scope of this paper.

4. We assume that resource allocation is insensitive to current prices--this may be taken as a first-order approximation if changes in relative prices are small and short lived. However, large and serially correlated movements in relative prices may induce short-run changes in resource allocation that entail unwarranted adjustment costs.

5. This formulation differs slightly from Fischer; his expenditure function is: $S(t) = (1-s)P_t Y_t + \alpha(M_{K-1})$. Thus, his function treats transitory movements in income and wealth differentially, and creates something of a problem when capital gains or wealth creation occurs. Either specification yields similar qualitative results.

6. These demand functions are, of course, derived from a Cobb-Douglas utility function. The assumption that the utility function is homothetic (unitary income elasticity) is important for some of our results concerning relative prices; the assumption of unitary own price elasticity allows for an analytic solution but does not significantly alter results, as shown by simulations for alternative CES utility functions.

7. The correlation coefficient between $P_2(t)$ and $P_2(t-1)$ can be shown to be $(1-\alpha)$, indicating that for small marginal propensities to consume out of transitory income, the changes in relative prices induced by a one-time change in foreign prices can be quite long-lived.
8. As noted earlier, Fischer poses a similar question—though in the context of changes in money demand, rather than money supply. However, our primary interest concerns the effect of these disturbances on relative prices, an issue not addressed by Fischer.

9. If there were a stochastic disturbance to the money supply in each $t$, with no attempt in subsequent periods to correct these disturbances, we could not speak of a long-run money supply or its variance since 

$$M_t = (\bar{M} + \sum_{0}^{t} \theta_k)$$

has no finite variance as $t \to \infty$.

10. This conclusion does not depend on the specific demand functions used, but rather on the assumption of unitary income elasticity.

11. Alternatively, we might let: $\varepsilon_t = \theta_t + (\bar{M} - M_{t-1})$ where $\theta_t$ is random and $(\bar{M} - M_{t-1})$ is the planned change in the money supply to restore it to the steady-state level. One problem with this is that—under fixed exchange rates—adjustment takes place through variations in the money supply, and hence the rule given above precludes such adjustments if other disturbances—besides money supply ones—are present. In effect, such policy is comparable to full sterilization and may be destabilizing.

12. Again, this result depends on unitary income elasticity of demand, and not on the price elasticities.

13. Throughout, we assume that $m$ is constant. Allowing $m$ to vary in response to price or income changes would add to the complexity of the model, without changing our major conclusions.

Notice that it possible to incorporate random changes in asset composition into the model. Equation 48 could be written as:

$$M(t) = (1-m)W(t) + \beta(t)$$

where: $\beta$ is a random variable

$$e(t)F(t) = mW(t) - \beta(t)$$

This case is trivial since with a fixed rate: $\text{Var}(C) = 0$ while with flexible rates $\text{Var}(C) > 0$.

We also assume that residents of the foreign country do not hold the currency of the domestic country.

14. As actual wealth is constant, we have set actual wealth equal to desired wealth. Notice, however, that desired wealth need not equal expected wealth.

15. Recall that under fixed exchange rates, $e(t) = e(t-1) = \ldots = 1$ so that $P = P^*$. 
References


