The application of generic flaw sizing techniques to specific components generally involves difficulties associated with geometrical complexity and simplifications arising from a knowledge of the expected flaw distribution. This paper is concerned with the case of ultrasonic flaw sizing in turbine engine rotor components. The sizing of flat penny shaped cracks in the web geometry will be discussed and new crack sizing algorithms based on the Born and Kirchhoff approximations will be introduced. Additionally we propose a simple method for finding the size of a flat, penny shaped crack given only the magnitude of the scattering amplitude. The bore geometry is discussed with primary emphasis on the cylindrical focusing of the incident beam. Important questions which are addressed include the effects of diffraction and the position of the flaw with respect to the focal line. The appropriate deconvolution procedures to account for these effects will be introduced. Generic features

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of the theory will be compared with experiment. Finally, the effects of focussed transducers on the Born inversion algorithm are discussed.

1. INTRODUCTION

The characterization and sizing of flaws in jet aircraft engine components provide an interesting and detailed focus for the development of inverse scattering algorithms, such as the inverse Born approximation (IBA) [1,2]. In particular, our attention has focussed on: 1) the development of the IBA and related algorithms for sizing cracks; 2) the effects of diffraction, refraction and focussing on flaw size estimates; 3) finite aperture limitations of the data resulting from the geometry of the part; [3,4,5] and 4) the influence of nearby surfaces on the sizing of flaws [5]. In this paper we will detail the first two topics: sizing cracks and the effects of diffraction, focussing and refraction. Finite aperture and near surface effects are discussed in other papers of these proceedings [4,5].

The two particular parts of the jet engine which are considered are the turbine rotor component bore and web. Our goal is the development of inverse scattering methods which can accurately size and characterize internal flaws for such parts. An initial program target is to develop the ability to size cracks which are 15 mils (380 \mu m) in diameter in a flat 1/4" plate of nickel based super alloy (IN-100). It is assumed that the crack lies at least 30 mils below the surface of the part. The primary new challenges introduced by the web problem are crack sizing and the effects of a finite aperture. The new features introduced by the bore primarily have to do with its curved surface which introduces focussing of the ultrasonic interrogating beam. These problems are discussed most naturally within the context of the diffraction theories [6] which have recently been developed and applied within the DARPA/AF Quantitative Nondestructive Evaluation program. Of course, diffraction and refraction are generic features which influence the use of inverse scattering theories in all cases. Thus, a significant portion of this paper details the general effects and the corrections to the scattering amplitude for use in inverse scattering methods. The structure of this paper is as follows. Section 2 discusses the development of sizing algorithms for cracks. These include: 1) the 1-D version of the IBA [7,8]; 2) a similar algorithm based on the Kirchhoff approximation; and 3) a method based on the existence of a strong peak in the scattering amplitude for a penny shaped crack at \( kR = 1.05 \), where \( k \) is the wavevector of the incident, longitudinal ultrasonic wave and \( R \) is the radius of the crack. The effects of crack roughness on the sizing algorithms will also be discussed. In Section 3, we discuss the effects of focussing,
diffraction and refraction on the 1-D IBA for 1) plate geometries
with both focussed and unfocussed transducers, 2) weakly spherically
curved surfaces, and 3) weakly cylindrically curved surfaces (e.g.,
the inner surface of a turbine rotor component bore). A simple model
describing these effects on the IBA will be introduced. The impor­tant effects of the near- and far-field conditions on the reference
signal will be discussed. The fourth section briefly considers the
use of the IBA with a spherically focussed transducer. We will show
that the use of focussed transducers may allow a simple, well con­ditioned correction to the IBA for near-field effects. This coupled
with their high signal-to-noise ratio highlights the utility of
focussed transducers for quantitative sizing of flaws. The paper is
concluded by a brief discussion.

2. INTERMEDIATE FREQUENCY CRACK SIZING

Ultrasonic methods for sizing cracks have been introduced over
the last several years. Most of these techniques have concentrated
on the use of the very low or very high frequency portion of the
spectrum [9,10,11]. In this preliminary study we will suggest
several methods which use the intermediate frequency information.
The impetus for these studies comes from 1) the large success of
using similar methods for sizing voids and inclusions and 2) the
need for a reliable method of estimating crack size when high
frequency data is either unavailable or not usable. Two inverse
scattering methods are introduced. The first is the one dimensional
inverse Born approximation. The second is also a one dimensional
method based on the Kirchhoff [12] (or Physical Optics) Approximation,
which we call the Inverse Kirchhoff Approximation (IKA). (We call
a method one dimensional when it is designed to treat each scattering
record independently.) We will also discuss an ad hoc inversion
method which uses particular features of the scattering cross-section
to determine the crack size (when it is known a priori that the crack
is flat and penny shaped). Tests of the inversion algorithms are
presented which use theoretically calculated longitudinal to longi­
tudinal (L→L) backscattered displacement amplitudes for a perfectly
smooth flat penny shaped crack.

Before proceeding to the inversion results, a brief discussion
of the direct scattering problem is in order. Figure 2.1 shows the
L→L backscattered impulse response function for a smooth flat penny
shaped crack whose normal is oriented at an angle $\theta$ (45°) with
respect to the incident direction. Figure 2.1A shows the impulse
response obtained from a detailed numerical calculation using the
method of optimal truncation (a T-matrix-like approach). In Fig.
2.1B the impulse response function was calculated using the Kirchhoff
approximation. This approximation replaces the field at each point
of the crack surface by the incident and specularly reflected
amplitudes appropriate to an infinite plane. The Kirchhoff result
is easily analyzed. At time $t = -2R/c \sin \theta$ there is a negative
Fig. 2.1. L→L backscattered impulse response function from a flat penny shaped crack oriented at 45° with respect to the incident direction. A) Numerical results. B) Kirchhoff approximation.
square root singularity which corresponds to the incident wave striking the crack's leading edge, where \( R \) denotes the radius of the crack, and a second positive square root singularity at \( t = +2R/c \sin \theta \). The waveform is purely antisymmetric (and thus zero at \( t = 0 \)). The more exact, detailed numerical calculation is very similar from time \( t = -\infty \) to \( t = 2R/c \sin \theta \), however, a strong return occurs later, at approximately \( t = R/c \sin \theta + 2R/c \). Here \( c_R \) is the Rayleigh wave (RW) velocity on a stress free planar surface. This return arises in the following way. The incident L wave impinges upon the leading edge of the flaw. Consequently, a Rayleigh wave is launched. When this wave strikes the far edge an L wave is launched (i.e., the process is L\( \rightarrow \)RW\( \rightarrow \)L). This leads to the time estimate given above and is completely consistent with the exact results of Mal [13] for an L-wave normally incident on a flat penny-shaped crack. For the smooth crack the Rayleigh contribution to the scattered L-wave is very sharp. However, for a slightly roughened crack, this contribution is considerably reduced in maximum amplitude and is broadened.

The inversion algorithms to be introduced will focus on the antisymmetric portion of the impulse response function, which is more or less directly related to the geometry of the crack. The Rayleigh wave (and other) contributions are not properly accounted for and will produce unwanted distortions in the reconstruction. Generally, the orientation of the crack with respect to the incident direction is unknown, and so from a single shot one can only hope to determine the quantity \( R \sin \theta \). This is the same estimate as obtained from the flash point method [10,11] when abundant high frequency information is available.

Below we will give several algorithms for determining \( R \sin \theta \) from a single pulse echo scattered signal. Simplifying assumptions have been made in order to reduce the problem to a dependence on a single scattering record. These approximations are not exact and their validity must be tested experimentally before they can be used confidently. In the future we hope to be able to test the algorithm for other circumstances such as rough flat cracks and elliptical cracks.

The IBA has surprisingly proven to be quite useful in sizing cracks despite the fact that it was originally derived for the case of weakly scattering volumetric flaws. In a rather general form the algorithm as derived in the volumetric case is

\[
\gamma(\mathbf{r}) = \text{const.} \int \frac{d^3k}{k^2} \frac{A(k)}{e^{-2ik \cdot \mathbf{r}}}.
\]  

(2.1)
Here $A(\mathbf{\hat{k}})$ denotes the $L\rightarrow L$ backscattered displacement amplitude as a function of the incident wavevector, $\mathbf{\hat{k}}$. The reconstructed characteristic function (defined to be 1 inside the flaw and zero otherwise) is denoted by $\gamma(\mathbf{r})$. For spherically shaped flaws Eq. 2.1 can be rewritten as:

$$\gamma(\mathbf{r}) = \text{const.} \int_0^\infty dk \frac{\sin 2kr}{2kr} A(k).$$

(2.2)

![Diagram](image)

Fig. 2.2. $L\rightarrow L$ backscattered impulse response function for a spherically shaped, weakly scattering flaw.

Figure 2.2 shows the impulse response function for a spherically shaped, weakly scattering flaw. The essential feature of this impulse response function is its symmetric shape. Consequently, the scattering amplitude, which is given by a Fourier transform of the impulse response, is purely real. For more strongly scattering flaws $A(k)$ becomes complex. However, we have found empirically that volumetric flaws with a center of inversion symmetry can be accurately reconstructed if we replace $A(k)$ in Eq. 2.2 by $\text{Re}A(k)$. 
The use of the 1-D IBA (Eq. 2.2) in sizing cracks requires a modification to account for the antisymmetric form of the impulse response. A reasonable guess is

\[ \gamma(r) \approx \text{const.} \int_0^{\infty} \frac{\sin 2kr}{2kr} \text{Im} A(k). \quad (2.3) \]

We expect \( \gamma(r) = 1 \) for \( r < R \sin \theta \) and zero otherwise. Figure 2.3 shows the reconstructed characteristic function using the numerical calculations for a penny shaped crack at \( \theta = 30^\circ \) and 90°. As can be seen, the characteristic function is fairly well reconstructed according to our expectations. Figure 2.4 shows the estimated value of \( R \sin \theta \) from the Born reconstruction. As can be seen, the agreement is excellent for \( \theta > 30^\circ \). For \( \theta < 30^\circ \) the 1-D IBA gives estimates which are systematically too large. This results from the band limited nature of the input scattering amplitudes which were available only for \( kR < 10 \). The restricted bandwidth causes the estimates for \( \theta < 30^\circ \) to be effectively diffraction limited. The 1-D IBA for cracks (Eq. 2.3) can also be expressed in the time domain where its antisymmetric treatment of the impulse response function becomes clear:

\[ \gamma(t) = \text{const.} \frac{1}{2r} \int_{-\infty}^{\infty} dt R(t) \log \left| \frac{r+ct/2}{r-ct/2} \right| \quad (2.4) \]

Here \( R(t) \) denotes the impulse response function. This expression has also been discussed by R. K. Elsley [14] on the basis of Eq. 2.3 which was developed by one of us [15].

The use of the 1-D IBA for cracks is entirely ad hoc. The successful attempt to describe the flat penny shaped crack was based on its usefulness in describing volumetric flaws. A more systematic approach to crack sizing starts from the Kirchhoff approximation (which has already been exploited in the high frequency limit by Achenbach et al.). We define a new two dimensional characteristic function \( \gamma_2(r_{11}) \) for flat cracks which is one or zero on the plane of the crack. The Kirchhoff scattering amplitude is given by

\[ A(k_{11}, r_{11}) = \text{const.} i k_{11} \int d^2 r_{11} \ e^{-2i k_{11} \cdot r_{11}} \gamma_2(r_{11}) \quad (2.5) \]

Here \( k_{11} \) and \( r_{11} \) are respectively the projection of the wavevector and the radial vector onto the plane of the crack and \( k_{11} \) is the projection of the wavevector onto the crack normal. Equation 2.5 is in the form of a Fourier transform and can be readily inverted formally to give the relationship
Fig. 2.3. Reconstructed characteristic functions for a penny shaped crack for $\theta = 30^\circ$ and $90^\circ$ using the 1-D IBA.
Fig. 2.4. Estimated tangent plane distance ($R \sin \theta$) for a penny shaped crack as a function of $\theta$ as calculated in the 1-D IBA.

\[
\gamma_2(r_{11}) = \text{const. } i \int \frac{d^2 k_{11} A(k_1, k_{11})}{k_1} e^{2i k_{11} \cdot r_{11}}
\]  

(2.6)

A complete set of backscattered amplitudes for all directions of incidence is (within the Kirchhoff approximation) infinitely redundant since $\gamma_2(r_{11})$ is a two dimensional function, while $A(k_1, k_{11})$ is a three dimensional function. To be specific, we require $k_1/k_{11} = \text{const.}$; or, in terms of the angle $\theta$ introduced earlier, we write $k_{11} = k \sin \theta$, $k_1 = k \cos \theta$ for fixed $\theta$. For the case of a penny shaped crack, Eq. 2.6 can be further simplified

\[
\gamma(r_{11}) = \text{const. } i \int dk_{11} \text{Jo}(2k_{11} r_{11} \sin \theta) A(k_1, k_{11}).
\]  

(2.7)

The evaluation of this equation requires only the scattering amplitude for a single direction of incidence and a knowledge of $\theta$. 
Since $\theta$ is generally unknown we can rewrite Eq. 2.7 with $p = r_{11}\sin \theta$

$$\gamma(p/\sin \theta) = \text{const. } i \int dk J_0(2k\rho) A(k, \theta). \quad (2.8)$$

$A(k, \theta)$ is a measured quantity and hence is known. If sufficient bandwidth is available the characteristic function decreases abruptly at

$$p = R \sin \theta. \quad (2.9)$$

That is, Eq. 2.8 serves to evaluate the distance from the center of the flaw to the front surface tangent plane. This method is called the 1-D inverse Kirchhoff approximation (IKA) and is expected to be most accurate for circular and elliptical flat cracks.

The results of inverting the flat penny shaped cracks scattering amplitude are shown in Fig. 2.5. Compared to the inverse Born results these estimates for the characteristic function are sharper; that is, they show more resolution for the same maximum bandwidth ($k_{\text{max}}R = 10$). Also, there is an abrupt, sharp change in $\gamma_2(r_{11})$ at the correct radius. On the other hand, the Rayleigh wave plays an apparent role in $\gamma_2(r_{11})$ causing a large undershoot. If one knows a priori that the flaw population consists of cracks the negative portions of the characteristic function could be excluded, and a more accurate size estimate obtained.

A time domain version of the 1-D IKA can also be found by Fourier transforming Eq. 2.7:

$$\text{Re} \gamma_2(r) = \text{const. } \left\{ \int_{-\infty}^{\infty} \frac{R(t)dt}{\sqrt{t^2-4\tau^2/c^2}} - \int_{-\infty}^{\infty} \frac{R(t)dt}{2r/c \sqrt{t^2-4\tau^2/c^2}} \right\}. \quad (2.10)$$

Again the antisymmetric nature of the inversion algorithm is explicit.

We find that the 1-D IKA has more resolution than the 1-D IBA. However, the inverse Kirchhoff method appears more sensitive to the presence of Rayleigh waves and probably other "noise" components of the signal. Both methods give encouraging preliminary results. It is probably desirable to pursue further theoretical and experimental studies.

Finally, we have observed certain features in the scattering cross section of the penny shaped flat crack which yields the radius immediately. Figure 2.6 shows the magnitude of the L→L backscattered
Fig. 2.5. Inverse 1-D Kirchhoff results for the characteristic function of a penny shaped crack at $\theta = 30^\circ$ and $90^\circ$. 
amplitude $|A|$ for $\theta = 30^\circ$, for a smooth crack. Furthermore, the numerical calculations show that for all incident angles there is a maximum in $|A|$ at $k_R = 1.05$. This feature leads to an immediate estimate of the flaw radius, $R$, if it is known a priori that the crack is flat and penny shaped. The radius estimate is

$$R = \frac{1.05}{k_0}.$$ 

Here $k_0$ is the wavevector of the peak in the $L+L$ cross section. We were concerned that this feature might disappear for a roughened crack. However, upon examining the recent results of Opsal [16] for slightly roughened cracks, we found that this feature remains intact. Figure 2.7A shows the $L+L$ backscattered cross section for a roughened crack with $\theta = 30^\circ$. As can be seen from the figure, the maximum at $k_R = 1.05$ remains. This maximum is even more pronounced in the longitudinal to transverse scattering amplitude which is plotted in Fig. 2.7B. We therefore expect that the measurement
Fig. 2.7. Magnitude backscattered amplitudes, $|A|$, for $\theta=30^\circ$ for a rough penny shaped crack. A) $L\rightarrow L$ and B) $L\rightarrow T$. 
of this feature will provide a straightforward determination of the size of a penny shaped crack. An especially important point is that phase information is not necessary. It would be of interest to see if similar features occur in the scattering cross sections of more realistically shaped cracks.

3. DIFFRACTION, REFRACTION, FOCUSING AND THE IBA

Diffraction, refraction and focusing all play important roles in implementing the use of the IBA for the bore and web geometries. Two of the authors (RBT and TAG) [6] have developed approximate corrections which describe these effects on the propagation of elastic waves from a circular transducer in immersion, through a planar or weakly curved surface and into the bulk of a sample. In this section we use these corrections to determine the modifications necessary in using the IBA. We will first consider the effects of diffraction on the inversion of a spherical flaw in a flat plate. Then we will consider the use of the IBA for spherically and cylindrically (weakly) curved surfaces. The presence of surface curvature causes the incident beam to focus and introduces new and interesting features to the problem.

We consider the experimental set-up shown in Fig. 3.1. The sample is a flat plate of thickness, d, which is immersed in a water bath and contains a weakly scattering spherical flaw. Ultrasonic

Fig. 3.1. Experimental configuration for sizing flaws in a plate.
illumination is provided by a circular planar transducer (piston source) whose axis of symmetry is perpendicular to the surface of the sample and passes through the center of the flaw. The hypothetical experiment is conducted in pulse-echo (backscattering) mode. The distance from the plane of the transducer face to the sample's front surface is denoted \( z_0 \) and \( z_1 \) is the distance from the sample surface to the flaw's center. The effects of diffraction on the scattering amplitude depend on two transfer functions, \( C(s_F) \) and \( D(2s_R) \). The parameters \( s_F \) and \( s_R \) are the dimensionless quantities \( s_F = (\lambda_0 z_0 + \lambda_1 z_1)/a^2 \) and \( s_R = (\lambda_0 z_0 + \lambda_1 d)/a^2 \) where \( \lambda_0 \) and \( \lambda_1 \) are the wavelength of the ultrasound in water and the sample material, respectively, and the radius of the transducer is denoted by \( a \). The transfer function \( C(s) \) describes the on-axis pressure distribution of the transducer. The transfer function \( D(s) \) describes the response of the transducer to its own transmitted signal reflected off a normally oriented infinite plane. These functions are defined in detail in the paper by Thompson and Gray [6] in these proceedings.

This experiment proceeds as follows. First, the complex frequency domain scattered displacement amplitude \( S(\omega) \) from the flaw is obtained. The flaw signal is roughly the flaw impulse response convolved with the transducer pulse. Thus a reference spectrum, \( R(\omega) \), such as the reflection from either the front or back surface of the sample at normal incidence, is also obtained. This signal is essentially the transducer response, so the flaw impulse response can be approximated by simple divison

\[
U(\omega) = S(\omega)/R(\omega)
\]

which corresponds to deconvolution in the time domain. \( U(\omega) \), with suitable desensitization such as Wiener filtering, has been considered to be an adequate approximation to the scattering amplitude for use with the IBA. For planar geometries we will show this approximation is adequate as long as \( s_R \geq 1 \) (i.e., far-field measurements). However, \( U(\omega) \) is more accurately related to the scattering amplitude (dropping constants) through

\[
U(\omega) = \frac{A(\omega) C^2(s_F)}{i\omega D(2s_R)}
\]

Again, see the paper by Thompson and Gray (these proceedings) [6].

The Born inversion algorithm for a spherical flaw is given by [7,8]

\[
\gamma(r) = \text{const.} \int_0^\infty d\omega \frac{\sin(2\omega r/v)}{2\omega r/v} A(\omega).
\]

(3.3A)
Here \( v \) is the velocity of longitudinal sound in the sample. If the effects of diffraction are not accounted for, the measured signal will be approximately \( U(\omega) \) rather than the scattering amplitude, \( A(\omega) \). The effect of using the measured signal in Eq. 3.3A rather than the true scattering amplitude can be modeled for a flaw by replacing \( A(\omega) \) in 3.3A by \( U(\omega) \) from Eq. 3.2. Then we find

\[
\gamma(r) = \text{const.} \int_0^\infty \frac{\sin(2\omega r/v)}{2\omega r/v} A(\omega) \left[ \frac{C^2(s_F)}{i\omega D(2s_R)} \right] \quad (3.3B)
\]

The bracketed quantity in Eq. 3.3B is denoted by \( B(\omega) \), or explicitly

\[
B(\omega) = \frac{C^2(s_F)}{i\omega D(2s_R)}. \quad (3.4)
\]

For normal incidence (see [6])

\[
C(s) = 2i e^{-i\pi/2s} \sin\left(\frac{\pi}{2s}\right) \quad (3.5A)
\]

and

\[
D(s) = 1 - e^{-2\omega i/s} \left\{ J_0\left(\frac{2\pi}{s}\right) + i J_1\left(\frac{2\pi}{s}\right) \right\}. \quad (3.5B)
\]

\( B(\omega) \) has an important effect on the IBA if either \( s_R \) or \( s_F \) are less than or equal to one. On the other hand, if \( s_R \) and \( s_F \) are much greater than one for all measured frequencies, \( B(\omega) \) has no effect on the inversion algorithm.

First we consider the case where both the flaw and the reference plane are in the far field. It is useful to note that both \( s_R \) and \( s_F \) are, by definition, inversely proportional to the angular frequency, \( \omega \). Thus, at sufficiently long wavelength \( (\omega \rightarrow 0) \), \( s \) becomes large and in the limit \( C(s), D(s) \rightarrow 1/\pi/s \) as \( s \rightarrow \infty \). Thus, from Eq. 3.4, \( B(\omega) \) approaches a constant independent of frequency. That is, if \( s_F \) and \( s_R \gg 1 \) for all frequencies of interrogation, the only effect of the diffraction correction is to multiply the reconstructed characteristic function by a constant. Since the sizing depends only on the shape of the characteristic function, the radius estimate
is unaffected. However, from a more practical point of view, we will see below in the numerical results that for reasonable values of the experimental parameters, $s_F$ and $s_R > 3/4$ is quite adequate for successful reconstruction.

Next we consider the experimentally common situation in which the flaw is in the near field ($s_F < 1$) and the reference is in the far field $s_R >> 1$. In this case we can rewrite

$$B(\omega) = [4 \frac{t_R}{\pi}] e^{i \pi \omega / t_F} \sin^2 \left( \frac{\pi}{2t_F} \omega \right).$$

Here we have made the $\omega$ independence of $s_R$ and $s_F$ explicit by defining $s_R = t_R / \omega$ and $s_F = t_F / \omega$ (note $t_R$ and $t_F$ are not times). Further, we have taken the $s_R \to \infty$ limit for $D(\omega)$. In this limit $B(\omega)$ has two effects on the IBA (see Eq. 3.3B). First, it introduces a linear phase factor $e^{-i \pi \omega / t_F}$. This phase factor has no effect on the determination of the flaw radius since all such linear phase errors are corrected for by the determination of the "zero of time". The other physically significant term in Eq. 3.6, $\sin^2 \left( \frac{\pi \omega / 2t_F}{\omega} \right)$, serves to filter out the high frequency components of the scattering amplitudes. Such filtering occurs since $B(\omega)$ is constant for $\omega < \frac{\pi}{2t_F}$ (i.e., $s_F >> 1$). For larger values of $\omega$, $B(\omega)$ decreases and at $\omega = 2t_F$ (i.e., $s_F = 1/2$) there is a zero (the near field null). This null occurs for circular transducers due to the destructive interference of the waves launched from the center and the edges of the transducer. The near field null limits the maximum usable frequency of the scattering amplitude. As long as the near field null occurs at a sufficiently high frequency that $kR > 2.5$, the constraint on the usable bandwidth is not too important. Here $R$ is the relevant feature of the flaw which is to be resolved. More generally the effects of limited high frequencies on the IBA has been studied by Addison and Elsley [17]. Although they did not consider the causes of the bandwidth limitation (e.g., diffraction), their results shed considerable light on the effect of the near field null upon sizing accuracy of the 1-D IBA.

The final situation, which is encountered frequently in experiments, occurs when the flaw is in the far field ($s_F >> 1$), but the reference is in the near field ($s_R << 1$). Such an occurrence may be realized for a sample in a water bath when the echo from the sample front surface is used as the reference signal. The effects of diffraction in this case indicates why this choice of the reference signal has often led to poor results for the IBA. Our formalism shows how to correct these errors. For this experimental configuration we find that
\[ B(\omega) = \frac{\pi^2}{t_F^2} i\omega. \] (3.7)

As might be expected, this frequency dependent form for \( B(\omega) \) substantially degrades the sizing ability of the IBA. To show this, we have calculated the effects on sizing of a weak scattering sphere by replacing \( A(\omega) \) in Eq. 3.3 by its Born approximation. The results are shown in Fig. 3.2. Here the normalized estimated radius \( r/R \) is plotted versus \((s_R)_{\text{min}}\) for a series of values of \((kR)_{\text{max}}\) and \((s_F)_{\text{max}}\), where the flaw radius is \( R \) and \( k \) is the wavevector. The subscript "\( \text{min} \)" indicates that for the range of frequencies used, the smallest values obtained for \( s_R \) and \( s_F \) were \((s_R)_{\text{min}}\) and \((s_F)_{\text{min}}\). Similarly, \((kR)_{\text{max}}\) is the largest value obtained by Ellis product. These results show that the flaw is dramatically undersized for \((s_F)_{\text{min}} < 1/2\), that is, when the reference signal is measured in the near field. For \((s_F)_{\text{min}} > 1/2\) the estimated radius rapidly assumes its diffraction free values.

In the general case we can approximately correct for the effects of diffraction on an experiment scattering amplitude by inverting Eq. 3.2

\[ A(\omega) = \frac{i\omega D(s_R)U(\omega)}{C^2(s_F)}. \] (3.8)

For the case \( s_R, s_F >> 1 \), the correction term is constant and accurate results are obtained with or without it. For \( s_R >> 1 \) and \( s_F < 1/2 \), (the second case above) Eq. 3.8 is not a satisfactory correction since it involves division by zero which is not a well-conditioned procedure. Nonetheless, the correction has been roughly applied in this case to experimental data with reasonable results [18]. For the final case, \( s_R < 1 \) and \( s_F >> 1 \), Eq. 3.8 provides a well conditioned means of correcting the measured scattering amplitude for the effects of diffraction since \( C(s) \) is never zero and \( D(s) \) is smoothly and slowly varying.

Figure 3.3 shows the results of inversion of experimental data for the case that the flaw is in the far field while the reference is in the near field. The flaw was a 2-1 (400 x 200 \( \mu \)m semiaxes) oblate spheroidal void in a Ti disk. The flaw spectrum \( U(\omega) \) was obtained using the reflection from the sample front surface as a reference signal for a series of distances between the transducer and sample. In all cases, the reference data was near field (0.14 \( \leq s_R \leq 0.70 \)). Results of the 1-D IBA are shown in the figure for
Fig. 3.2. Shows theoretical effects on flaw sizing when the reference is in the near field. Here a denotes the radius of the flaw.
Fig. 3.3. Shows experimental results for flaw sizing when the reference is in the near field. Here $a$ denotes the radius of the flaw.
the uncorrected experimental data and for data corrected using Eq. 3.8. For the uncorrected data, inversion results are inaccurate, as expected, and are quite close to the corresponding theoretical curve from Fig. 3.2, which is redrawn here for comparison. After correction, the estimated radius is given rather accurately.

We have dwelt upon the effects of diffraction in planar geometries for two reasons. First, this is the geometry of the turbine rotor component web and is commonly found experimentally. Second, the relatively simple concepts discussed above can be used to great advantage in describing the somewhat more complicated results for the bore geometry. The major new feature introduced by the bore is the effect of focussing. Rather than turning immediately to the relevant effects of cylindrical focussing, we will discuss the effects of a spherically curved surface for two reasons. First, the results will be directly relevant, with only small change, to the spherically focussed circular transducer. Secondly, the formalism is considerably more tractable than that for cylindrical focussing. Thus, analytical explanations can be given for effects of diffraction. We will then show numerically that similar results obtain for the cylindrically focussed sample.

The geometry we consider is shown in Fig. 3.4. Here we assume a sample with a spherically (or cylindrically) curved surface.

![Fig. 3.4. Experimental configuration for sizing flaws in a sample with a spherically curved surface.](image-url)
The flaw is assumed to lie on the axis (or plane) of symmetry of the sample. The reference signal is assumed to be obtained as the back surface reflection from a separate flat plate calibration sample. We therefore assume that $s_R \gg 1$ from now on since it is an available experimental option and its value depends on the thickness of the reference block.

For a sample with a weakly spherically curved surface immersed in water we redefine $s_F$ as

$$s_F = \frac{\lambda_o z_o}{a^2} + \frac{\lambda_1 F z_1}{(F-z_1)a^2}$$

(3.9)

where $F$ is the focal length and we have assumed that the flaw is on axis. Since the reference signal is taken from a planar sample and $s_R$ is as before. Near the focus, $s_F$ becomes very large, resulting in a far-field interpretation of the flaw signal as $|z_1-F| \to 0$.

We define a new function to characterize the on-axis pressure, $C_s(s)$, which can be defined in terms of the $C(s)$ function used above

$$C_s(s) = \frac{F}{F-z} C(s).$$

(3.10)

The apparent singularity at $z = F$ is actually cancelled by $C(s)$ with $s$ defined in Eq. 3.9 as shown by Thompson and Gray [6]. To perform the inversion we use Eq. 3.2 in which $B(\omega)$ is evaluated using Eq. 3.4 with $C(s)$ replaced by $C_s(s_F)$. Explicitly, taking $s_R \to \infty$ we find

$$B(\omega) = 4 \left( \frac{F}{F-z} \right)^2 e^{\frac{-i\pi \omega}{t_F}} \sin^2 \left( \frac{\pi}{2t_F} \omega \right)$$

(3.11)

where we have defined $s_F = t_F \omega$. Near the focal point $s_F$ and $t_F \to \infty$, $z_1 \to F$, and

$$B(\omega) \to \frac{4\pi^2 a}{v^2 z_1}$$

(3.12)

which is a constant independent of the frequency. Consequently, the IBA sizing algorithm is unaffected for $z_1 \to F$. For $z_1 \neq F$, $t_F$ is finite
and we find that the term $\sin^2\left(\frac{\pi \omega}{2t_F}\right)/\omega^2$ again acts to filter out the high frequency component with the maximum usable upper frequency occurring for $s_F = 1/2$.

To test these ideas we have modeled an experiment using a weakly scattering sphere and the above formalism. The flaw is assumed to occur at various distances from the focal point. Figure 3.5 shows the result for a 15 mil (381 μm) diameter flaw in a sample whose surface has a $3\frac{1}{2}$" radius of curvature. A planar, $\frac{1}{4}$" diameter transducer is assumed to be positioned 1" above the surface. We assume that the velocity of longitudinal sound in the sample is $6.5 \times 10^5$ cm/s (the sound velocity of IN-100). These choices were made to simulate an experiment done in the bore geometry.

The results of the simulated experiment agree well with the ideas discussed above. If the flaw is near the focal point the estimated radius approaches its true value since in this case $B(\omega)$ is constant. For smaller values of $z_l$, $C(s)$ begins to filter out high frequency components, the estimated radius initially falls below its true value and then rises above. These features agree with the results of Addison and Elsley [17] who showed that the effect of decreasing the high frequency content of the scattering amplitude initially

![Bore Geometry Diagram](image)

Fig. 3.5. Shows effects on flaw sizing of focussing induced by a spherically curved sample surface; $z_l$ is flaw's depth.
caused the estimated radius to fall and then to rise (diffraction limited regime). These difficulties arise due to the near field null and hence are not very easily corrected. Significant errors in the sizing would also occur if the reference signal were evaluated in the near field. However, this effect can be straightforwardly corrected as long as the flaw is in the far field by multiplying the measured displacement amplitude by $i\omega D(2s_F)/C_s(s_F)$.

A cylindrically curved surface may also cause the incident beam to focus, although instead of a focal point there is a line of focus. We assume that the reference signal is measured on a separate planar sample and is in the far field, thus allowing the use of $D(s_R)$. The function corresponding to $C(s)$ is, however, more complicated and is defined as

$$C_y(s,\Delta s) = \sqrt{\frac{F}{F-z_1}} \frac{2\pi}{\omega} \int_\phi^\phi' \frac{d\phi}{2\pi} \left\{ \cos \left[ -\frac{\pi}{s+\Delta\cos 2\phi} \right] + i \sin \left[ -\frac{\pi}{s+\Delta\cos 2\phi} \right] \right\}$$

(3.13)

where

$$\bar{s} = s_0 + \frac{1}{2} s \left( 1 + \frac{F}{F-z_1} \right)$$

(3.14A)

$$\Delta s = \frac{1}{2} \frac{s_1 z_1}{F-z_1}$$

(3.14B)

The evaluation of this integral is difficult analytically and as yet results are limited to a far-field expansion in terms of $1/(s_1+s)$ [19]. Equation 3.13 must be treated with care numerically. Although for $z>F$ an accurate reliable computer code has been obtained, when $z<F$ the denominator inside the trigonometric functions will always vanish for some value of $\phi$, resulting in an infinitely rapid oscillation of the integrand. These oscillations lead to considerable numerical difficulties which have not been fully resolved.

For $z<F$ we carried out a numerical simulation to determine the effects of diffraction on the IBA. The transducer is at a distance $z_0=1$ inch from the curved surface whose radius of curvature was chosen to be $3\frac{1}{2}$ inches. The flaw lies on the axis of symmetry of the transducer which is oriented normally with respect to the sample's surface. In Fig. 3.6 we show the estimated radius for a $7\frac{1}{2}$ and a $15$ mil radius spherical "Born" flaw as a function of the flaw's
depth below the surface. For the 15 mil flaw the effects are slight and, in particular, the flaw is accurately sized at the focal line. Its value decreases about 10% as the flaw approaches the surface. For the smaller flaw the effects are somewhat more important. Accurate sizing is again obtained for the flaw located at the focal line, but as \( z_1 \to 0 \), the value of the estimated radius falls and then increases sharply as the flaw approaches the sample's surface. These results are very similar to those obtained for the case of a spherically curved surface. This suggests that the spherical analogy might be useful in understanding these results. However, if the flaw depth is greater than the depth of the focal line, neither analytic nor numerical results are available at present. This problem is currently being studied as well as the problem of sizing when the incident beam strikes the surface at an angle.

4. THE IBA, NEAR FIELD EFFECTS AND FOCUSED TRANSDUCERS

Important new effects occur in the use of the IBA when focused transducers are used in the sizing algorithm. One important consequence is that in some situations it may be possible to avoid
the difficulties associated with the near field nulls of a disk-shaped unfocussed transducer. This feature, as well as the improved ratio of signal-to-grain noise associated with focussed transducers, is an important advantage both in terms of flaw detection and for use in sizing via the IBA. In order to explain these effects we consider a planar sample containing a spherical flaw and immersed in a water bath. The flaw is assumed to lie on the axis of symmetry of the transducer which is oriented normal to the sample surface.

The physics of sizing in this case is analogous to that of the sample with a weakly spherically curved surface, which was discussed in the last section. However, the formalism is more easily expressed in a somewhat different manner. First, the corresponding on axis pressure can be written as

$$C_{FT} = \frac{F}{F - (z_o + \frac{v_1}{v_o} z_1)} C\left(\frac{\lambda_o}{a} \frac{F(z_o + \frac{v_1}{v_o} z_1)}{F - (z_o + \frac{v_1}{v_o} z_1)}\right). \quad (4.1)$$

Here $v_o$, $\lambda_o$, and $F$ are the velocity, the wavelength and the focal length in water. The reference signal transfer function, $D(s)$, Eq. 3.5B is evaluated by focussing the transducer on either the front or the back surface of the part. For this case the functional form of $s$ is given by

$$s_{FT} = \frac{2\lambda_o F}{a^2} = \frac{2\lambda_1 F_1}{a^2}. \quad (4.2)$$

Here $F_1$ and $\lambda_1$ are the focal length and the wavelength in the sample. The last equality follows since $\lambda_o F = \lambda_1 F_1$.

If the flaw is located at the focal point of the transducer, the argument of $C$ in Eq. 4.1 becomes large since $F - (z_o + \frac{v_1}{v_o} z_1) \approx 0$. Thus, the flaw is in the far field and $C_{FT}$ becomes extremely simple,

$$C_{FT} = \frac{i\pi a^2}{\lambda_o(z_o + z_1 \frac{v_1}{v_o})}. \quad (4.3)$$

In general, $B(\omega)$ is given by Eq. 3.4 where $C(s_p)$ is now replaced by $C_{FT}$ evaluated at the appropriate arguments and $D$ is evaluated at
s_{FT}. Assume that the flaw is situated at the focal point so that the simple form $C_{FT}$ given in Eq. 4.3 can be used. We will consider the effects of diffraction on the IBA using a focussed transducer for two cases. In the first case we assume such a long focal length that the reference signal is evaluated in the far field ($s_{FT} \to \infty$). Then $B(\omega)$ becomes a constant independent of the frequency and no correction to the IBA is needed. The second case occurs when $D(s_{FT})$ is evaluated for the near field (a common case for commercially available focussed probes). Then we can write for $z_{1} \to F$

$$A(\omega) = U(\omega) \frac{D(\omega) (v_{o} z_{o} + v_{1} z_{1})^{2}}{i\omega a}$$

(4.4)

where we have used Eq. 3.2 with $C(s_{F})$ replaced by $C_{FT}$. The function $D(\omega)$ is smooth (and vanishes as $\omega \to \omega_{0}$) and so the correction is easily implemented and well-conditioned. This latter point is the main conclusion of this section. That is, for a focussed transducer, the near field effects can be cleanly corrected for. This stands in contrast to the case of the unfocussed, circular transducer where the occurrence of the near field null renders the correction for diffraction increasingly difficult for $s_{F} \leq 3/4$.

In experimental support of the preceding remarks, Fig. 4.1 illustrates the effects of using a focussed probe upon IBA sizing accuracy. A focussed transducer ($F = 10.7 \text{ cm in water}$) was used to illuminate an oblate spheroidal cavity ($200 \times 400 \mu\text{m semiaxes}$) in a titanium alloy disk for a range of distances $z$ between the transducer and sample surface. The reference signal was taken from the back surface of the sample ($z_{1} = 2.54 \text{ cm}$) with $z_{1} = 0.2 \text{ cm}$. In this case, the reference plane was not quite at the transducer focal point, which occurs at $z_{1} = 2.43 \text{ cm}$. Thus the curvature of the received wavefronts did not quite match that of the transducer, but this error is not believed to be too great. Also shown in the figure is a set of corresponding simulated data calculated using numerically exact results for the experimental flaw. Although the agreement between these two sets of results is not exact, the same qualitative trend is observed. Namely, a reduction in estimated radius occurs as the transducer is further removed from the sample surface. It should be noted here that due to the limited bandwidth available ($kR < 2.6$) these results are diffraction limited - the transducer was, in effect, "tuned" to a flaw of this size. Were greater bandwidth obtained, the degradation in sizing would be more severe, indicating the need for diffraction correction as discussed in Section 3 (e.g., Eq. 3.8).
Fig. 4.1. Estimated flaw radii using the uncorrected 1-D IBA with spherically focussed transducer. Radii are displayed as a function of the distance between the transducer and the front surface of the sample.
5. DISCUSSIONS AND CONCLUSIONS

In this paper we have discussed two classes of problems which arise in the sizing of defects in aircraft jet engine components. The problems discussed were: 1) the adaptation of current 1-D inversion algorithms to the case of cracks; and 2) the effects of focusing, diffraction and refraction on sizing estimates. Flaw characterization in these geometries is also affected by the finite aperture available for sonification and by the presence of surfaces in close proximity to the flaw. These latter features of the problem have been treated by Kogan and Rose [3] and by Hsu et al. [4] in these proceedings. Hsu et al. [4] showed experimentally that some nearly ellipsoidal inclusions in the volume of a part can be inverted (size, shape and orientation) using a number of 1-D Born shots distributed over a fairly restricted aperture (a 50° half-angle cone). They also showed that near surface, spherical inclusions in thermoplastic can be sized using the currently implemented form of the 1-D IBA without modification. Further, they were able to obtain the size, shape and orientation of a 2-1, nearly prolate spheroidal inclusion which was located one major diameter below the surface of the sample. Again, the unmodified form of the 1-D IBA was used. Kogan and Rose [3] examined the effects of a finite aperture on the full 3-D IBA. Their conclusions were that the discontinuity at the flaw surface is maintained in the reconstruction over the "lighted" area of the flaw. Systematic distortions of the flaw were found in the lateral dimension. Finally, non-analytic, unphysical features appeared in the reconstruction. Overall these two papers are extremely encouraging. First, it was found that almost ellipsoidal flaws can be reconstructed solely from a series of independently treated pulse-echo records. This avoids the three dimensional Fourier transform needed in the full 3-D IBA and greatly reduces the numerical processing. Secondly, it was found that the near surface flaws studied could be straightforwardly inverted using the current form of the 1-D IBA. No special modification was necessitated by the presence of the surface. Both of these experimental results (especially that for the near surface flaw) would benefit from theoretical study. The effects of the aperture on the 3-D IBA was shown to leave the discontinuity at the flaw's surface unaltered over the "lighted" region. This is very encouraging since it indicates that the needed information is to be found in the reconstruction. Effort is needed now to remove the effects of blurring and streaking in the reconstruction induced by the finite aperture.

In this paper we suggested two new algorithms which use intermediate frequencies to size cracks. These algorithms are based upon the 1-D IBA and upon the inversion method suggested by the Kirchhoff approximation. They were tested using theoretically generated "numerically exact" scattering amplitudes for a flat penny shaped crack. The results of these tests were quite encouraging for both
algorithms. The 1-D IBA gave excellent results and appeared to be relatively insensitive to noise and the appearance of a Rayleigh wave contribution in the impulse response function. The 1-D inverse Kirchhoff approximate, IKA, had higher resolution than the IBA. That is, for a given maximum bandwidth, the 1-D IKA defined the size of the crack more sharply. On the other hand, the 1-D IKA seemed to be more sensitive to noise and to the Rayleigh wave contribution. Both algorithms yield encouraging results and more study, especially experimental, is needed.

The presence of curved surfaces in jet engine parts give rise to focusing effects. We have treated the effects of focusing within the scalar wave diffraction theory of Thompson and Gray [6]. Additionally, the effects of diffraction was noted for non-focusing geometries.

If the geometry of the transducer and the sample surface are both planar, then we found the following results which apply in considerably more general cases. First, if the flaw is in the near field, the inversion algorithm behaves as if it were bandlimited with a maximum usable wavevector defined by \( kR = \frac{4\pi zR}{a} \). Due to the appearance of zeros at this and higher frequencies, the effects of flaw illumination in the near field are not easily corrected. Secondly, if the reference signal is in the near field, the inversion algorithm will seriously undersize the flaw. However, this problem may be straightforwardly corrected using Eq. 3.8. An application of this latter result is found in the use of the front surface echo to deconvolve the flaw signal. It was found experimentally some time ago that the front surface echo often provided an inadequate reference signal. We have explained these problems and corrected them by introducing the diffraction corrections of Section 3.

The effects of spherically and cylindrically focusing surfaces were also studied. As long as the reference was in the far-field, good inversion results were obtained when the flaw was sufficiently near the focus. The relative effects of near and far-field were explained straightforwardly in terms of the diffraction correction \( C(s_F) \) and \( D(s_R) \). Further work is required to study the effects of refraction where the beam is non-normally incident on the surface.

We also studied the case of a focused transducer and a planar sample oriented normal to the axis of the transducer. For this geometry it was found that all near field effects could be approximately corrected in a well-conditioned manner. This fact, together with the superior signal-to-noise offered by the focused transducer, suggests that the focused transducer should be routinely used in quantitative NDE inspections.
In conclusion, this paper studied the effects of the practical geometry of an aircraft jet engine on flaw sizing. The effects of cracks, refraction, diffraction and focussing were discussed.

REFERENCES

3. V. G. Kogan and J. H. Rose, these proceedings.
4. D. K. Hsu, J. H. Rose, and D. O. Thompson, these proceedings.
6. R. B. Thompson and T. A. Gray, these proceedings.
11. A. N. Norris, these proceedings.
16. J. L. Opsal, these proceedings.
DISCUSSION

R.C. Addison (Rockwell International Science Center): In regard to the experimental data with the spherically-focused transducer, could you describe how you acquired the reference for that one?

J.H. Rose (Ames Laboratory): Yes. The transducer was focused on the back surface of the piece that contained the flaw to within about 5%. We focused slightly in front because we didn't have a long enough focal length, but we used the back surface echo.

R.C. Addison: You moved the transducer--

J.H. Rose: Right up to the surface.

R.C. Addison: In your conclusions, you said that you could compensate for that through the D function?

J.H. Rose: Yes.

R.C. Addison: But you didn't do that in that experiment?

J.H. Rose: No. The purpose of that was to show what happened if you didn't compensate.

R.C. Addison: You think you could compensate?

J.H. Rose: Oh, absolutely.