1988

Photoelastic determination of stress intensity factors for sharp re-entrant corners in plates under extension

Mohammad Mahinfalab

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Photoelastic determination of stress intensity factors for sharp re-entrant corners in plates under extension

Mahinfalah, Mohammad, Ph.D.

Iowa State University, 1988
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Photoelastic determination of stress intensity factors for sharp re-entrant corners in plates under extension

by

Mohammad Mahinfalah

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

Department: Engineering Science and Mechanics
Major: Engineering Mechanics

Approved:

Signature was redacted for privacy.
In Charge of Major Work
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For the Major Department
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Iowa State University
Ames, Iowa
1988
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CHAPTER I.
INTRODUCTION

Fracture and effort to prevent fracture of materials such as metals, wood and glass costs US industry about $119 billion a year. This $119 billion cost, expressed in 1982 dollars, amounts to about 4 percent of the gross national product. It could be cut in half by using various prevention techniques and by current research [1].

It is no wonder that in the last quarter of the century, the theory of Linear Elastic Fracture Mechanics (LEFM) has been well developed and generally applied to the prediction of the load at which a structure containing a crack or flaw will fail. These cracks are usually a result of material defects created during material processing or of cyclic loading that causes crack initiation at stress raisers or notches which are present in most engineering components.

Sharp re-entrant corners and notches are introduced in design of components for ease of manufacturing and fabrication. Where they are called out, the first, automatic step is to introduce a fillet which changes the singular stress field to a conventional stress concentration problem. A repository of stress concentration factors are available to the designer or analyst to judge the severity of a re-entrant corner containing a fillet. If the geometry is unusual, standard numerical and
experimental techniques are available to determine the stress concentration factor.

There are many situations in which there is little control over the shape of the re-entrant corner and a fillet can not be introduced. For instance, in plates that have incomplete penetration of a butt weld it is common for re-entrant corners to be present [2]. These corners can not always be treated as a crack for fracture mechanics analysis because the stress field for a crack is different from that of a V-notch. A crack is a special case of a V-notch with the angle of the notch being zero degrees.

Whatever the reason, the presence of a sharp re-entrant notch or corner in a structure will result in stress singularities at the vertices of those notches or corners. Therefore, to be able to do any meaningful fracture mechanics analysis or to prevent future failure of a structure under design it is necessary to obtain the fracture parameters, called the stress intensity factors (SIF).

In order to analyze the stress field near a crack tip, it is necessary to define the type of crack displacements. There are three fundamental modes of crack extension as illustrated in Fig. 1 [3]. Mode I (opening mode) corresponds to normal separation of the crack walls under the action of tensile stresses; Mode II (sliding mode) corresponds to mutual shearing of the crack walls in a direction normal to the crack front; Mode III (tearing mode) corresponds to mutual shearing parallel to the crack front [3].
Fig. 1. The three basic modes of fracture: I opening mode; II, sliding mode; III, tearing mode [3]

In the vicinity of the crack tip, the distribution of the elastic-stress field in all machine/structural components subjected to either mode of loading are uniquely defined through the stress intensity factor $K$. This stress intensity factor has the character of a stress or load, is a linear function of the applied load, and a more complicated function of the crack and the specimen geometry. Consequently, the applied stress, the crack shape, size and orientation affects the stress intensity factors not the stress field distribution. The chief problem is then to obtain a solution for $K_I$, $K_{II}$ and $K_{III}$ (Mode I, Mode II, and Mode III stress intensity factors, respectively) as a function of part and crack geometry [4].
When the stress intensity factor at the crack tip reaches a critical value $K_C (K_{IC}, K_{IIIC}, K_{IIIIC}$ for Mode I, Mode II and Mode III, respectively) the unstable fracture occurs. $K_C$ represents the fracture toughness of the material and has units of Ksi in$^{1/2}$ (MN/m$^{3/2}$) for a crack. Therefore, $K_{IC}$ represents the inherent ability of a material to withstand a given stress field intensity at the tip of a crack and to resist progressive tensile crack extension under plane-strain condition.

In most cases it is difficult or impossible to obtain an analytical solution for flaws by the theory of elasticity. Hence, a great deal of attention is paid to numerical and experimental methods. Among the experimental methods the method of photoelasticity is the most developed. There has been numerous photoelastic studies of SIF for single cracks [4-7], multiple interactive cracks in isotropic materials [8] and single cracks in orthotropic materials [9]. However, there has not been any photoelastic studies of SIF in sharp re-entrant corners, and most work in this area has been limited to numerical analysis.

The primary objective of this dissertation is to use digital image analysis in combination with the photoelastic technique to obtain Mode I and Mode II stress intensity factors for sharp re-entrant corners of plates in tension. The special case of right angle re-entrant corner with stress free flanks will be considered for the present time, and it is the intent of the author to extend the research in the future for different angle notches.
CHAPTER II.
LITERATURE REVIEW

Brahtz [10] was one of the first investigators who examined the stress distribution and stress singularities at the roots of mathematically sharp notches. Later, Williams [11] independently obtained the stress singularities in angular corners with three different boundary conditions. In extension of his work [12] for special case of zero angle opening notch, Williams developed the equations for stress distribution near a crack. Since his work, there have been countless number of analytical, numerical and experimental studies of components with crack-like flaws.

It is not the intent of this investigator to review all the studies that have been done [13, 14, 15]. However, a few relevant cases will be reviewed in the following two sections.

Photoelastic Studies of Stress Intensity Factors in Cracks

Since the first use of photoelastic method by Post [16] to study the stress field near an edge crack, this technique has been used effectively in experimental determination of stress intensity factors and the far field stress effect, $\sigma_{ox}$, for components containing different orientations and combinations of cracks. Measurements of the fringe order $N$, angle $\theta$ and position $r$ for number of given points
(depending on the technique used) on a fringe loop(s) will be sufficient to
determine the stress intensity factors and $\sigma_{ox}$.

Sanford and Dally [17] explained in detail four different algorithms for extracting stress intensity factors by solving the non-linear "N-K" equation which relates the fringe order N to the stress intensity factors. These are: (1) a selected line approach in which two data points on the line $\theta = \pm \pi$ are selected to linearize and simplify the N-K relation; (2) the classical approach in which two data points at the apogees of two fringe loops (see Fig. 2), together with the condition that

$$\frac{\partial \tau_m}{\partial \theta} \bigg|_{(r_m, \theta_m)} = 0$$

at those apogees are used (this method is sensitive to data location); (3) a deterministic approach where three arbitrary points on the isochromatic fringe pattern of a loaded model are used; and (4) an overdeterministic approach where more than three data points (number of unknowns) are selected and the method of least squares [18] is used.

According to [17], all four methods gave satisfactory results (accurate to 0.1%) provided that exact input data was used. However, most of the time there will be some measurement errors in obtaining the necessary data. Therefore, the method that gives the least inaccuracy due to random measurement errors must be used. Sanford and Dally [19] showed that the error in determining stress intensity factors can be reduced by a factor of 3-4 with the use of least squares fitting procedure.
Seven different techniques for extracting stress intensity factors from stress-frozen photoelastic data in three dimensional problems were compared by Smith and Olaosebikan [20]. It was suggested that Sanford-Dally's iterative least squares method seems to give reliable estimates of stress intensity factors, converge rapidly and may be used to account for possible inaccuracy in crack tip location as described in ref. [18]. To obtain stress intensity factors in double edge cracked specimens made of PSM-1, Miskioglu et al. [21] combined Sanford-Dally's least squares method with Burger and Voloshin's [22] half fringe photoelasticity (HFP). HFP consist of a digital image analysis system with high optical resolution and can measure and convert the light intensity of a data point on a specimen into a fringe order. The stress intensity factors obtained by Miskioglu were within 4.0% of the existing theoretical and numerical values.
Kar [5] employed the method of ref. [21] to determine stress intensity factors in glass specimens with single edge cracks, 45° edge cracks and double edge cracks. For all models, twenty data points along five radial lines in an area limited by \(-140^\circ < \theta < 140^\circ\) were chosen. The SIF obtained from the data points were compared with the available numerical and experimental results. The average \(K_I\) and \(K_{II}\) values were within 5.2% for single edge cracks, 1.0% for double edge cracks, and 12.0% for 45° edge cracks of the existing values.

Mojtahed [15] used HFP combined with least squares method to obtain \(K_I\) and \(K_{II}\) and \(\sigma_{OX}\) for orthotropic materials. He showed that photoelasticity can be used effectively to determine Mode I stress intensity factor, when the crack is oriented along the weak axis. But for the case of a crack being parallel to the strong axis, there was a large discrepancy between the experimentally obtained \(K_I\) and \(\sigma_{OX}\) and the finite element results. Mojtahed suggested that this discrepancy was due to the existence of residual stresses and model’s sensitivity to crack growth, and it was reduced in one experiment by increasing the applied load.

An experimental study using photoelastic technique to obtain SIF for two dimensional inclined edge cracked models and three dimensional inclined semi-circular surface cracks that penetrated part-way through a thick plate was performed by [14]. Digital image processing system, fringe multiplication and fringe sharpening techniques were used to minimize the random experimental errors in measurements of \(r\) and \(\theta\). Iterative least squares method was used to solve an overdeterministic
system of equations. For accuracy evaluation of the experimental results, regenerated fringes using the fracture equation and coefficients estimated from the data sets were plotted and compared to the original fringe pattern. Then, the accepted values of the coefficients were those that gave the best fit to the original fringe pattern. The average values of $K_I$ and $K_{II}/K_I$ for two dimensional inclined edge cracked plates lie within the range of numerical solutions and previous experimental values.

Numerical Methods

Among the numerical methods, the methods of boundary collocation, Reciprocal Work Contour Integral and finite element have been used in determining the stress field and/or stress intensity factors for re-entrant notches. Gallagher [13] gives an excellent review of the numerical techniques used in this area.

Stress intensity factors in plates with V-notch were investigated by Gross [23] and Gross and Mendelson [24] using boundary collocation method. Later, stress intensity factors for notched plates were determined by Lin and Tong [25] using special hybrid elements to account for notch tip or corner singularities.

Carpenter [26] presented an overdeterministic collocation algorithm and used it to obtain the fracture mechanics parameters in a 90° notch and a plate with a single edge crack. More than one eigenvector was
considered and in this way stresses and displacements at the collocation points were better described than when only one eigenvector was used. The results for the 90° corner from the collocation method were within 4.1% of Reciprocal Work Contour Integral Method (RWCIM).

In a sequel to [26] Carpenter [27] extended his previous work to calculate the coefficients associated with eigenvectors for both the real and complex eigenvalues, and tested the technique for a problem of known solution. To determine the coefficients, stresses and/or displacements data generated from a finite element program at various node locations were used. Displacement data were taken directly from the nodal displacement values given by the program, and nodal stress values were the average of the stresses of the elements adjacent to the nodes. It should be noted that, the stresses from a displacement method finite element analysis (like the one used by [27]) contain greater errors than do displacements.

Several combinations of data from displacements and stresses were considered. In one part of the test, ref. [27] combined displacements with the stresses to determine the coefficients associated with the eigenvectors. It was shown that the determination of the coefficient associated with the first eigenvector was relatively insensitive to stress and displacement finite element idealization error and to the number of eigenvectors considered. However, this was not true for determination of the coefficients associated with the higher number eigenvectors because stresses from the displacement method finite element analysis contained too much error to allow the prediction of
these coefficients. In the other part, performance of the collocation procedure was examined by considering only the displacement data with various number of eigenvectors. The results showed that again, there was no problem in obtaining the coefficient of the first eigenvector. However, to obtain the correct coefficient value for the higher number eigenvector (i.e., #2), adequate number of eigenvectors (i.e., 7) combined with the displacement data must be used.

The RWCIM was developed by [28, 29, 30, 31]. This method is one of the most powerful approaches for using finite element results to obtain stress intensity factors for cracks [32]. This method uses a particular form of Betti's law [33] in the region enclosing the stress singularity in conjunction with a finite element solution. The technique of Stern et al. has the advantage that it does not require special elements, and can handle complex external boundaries and loading conditions.

The RWCIM was independently extended by Carpenter [32, 34], Sinclair and Mullan [35], Sinclair et al. [36] and Sinclair [37] to provide a means of obtaining stress intensity factors at sharp re-entrant corners. Two specific examples were examined by Carpenter [32]. The first example was for a configuration with $\alpha_2 = \pi/2$ and $\alpha_1 = \pi$ (see Fig. 3). This example was constructed to determine the effects of programming details. As seen in Fig. 4, as the number of segments increases, both predicted stress intensity factors converge to their exact values for $a_1 = 1$ in equation (45) of ref. [32].
Fig. 3. Polar stress components

Fig. 4. Convergence for example 1 [32]
The second example considered was a 90° corner in a lap joint as shown in Figs. 5a and 6a. In obtaining $a_1$, "the undetermined coefficient associated with the first eigenvector" the modulus of elasticity, $E$, of $30\times10^6$ psi and Poisson's ratio, $\nu$, of 0.3 were used. A coarse mesh idealization using 310 nodes and 264 elements (Fig. 6b), and a fine mesh using 3370 nodes and 3168 elements (Fig. 5b) were studied using the finite element program SUPERSAP [38]. For each mesh, three different contours, as shown in Figs. 5b and 6b, were used to obtain $a_1$. The results of both cases are presented and compared in Table 1 to the values obtained in ref. [26] from boundary collocation. The result indicated that the values obtained are insensitive to the choice of outer contour.

Sinclair et al. [36] following the ideas in an investigation by Stern and Soni [31] developed a set of path independent integrals, called $H$ integrals, for a stress-free notch. They examined a plate with a 90° re-entrant corner and the results obtained using path independent integrals are presented in the first half of Table 2. The values listed in the second half are of those obtained from the results in the first half and regularizing procedure of ref. [35]. These results indicate that the variation between the paths is of order of 20% for the direct coarse grid analysis and 0.1% for the fine grid, and it is much smaller for the regularizing procedure.

The conventional finite element displacement method was used by Walsh [39] to generate stiffness matrix of a special crack element
Fig. 5a. Fine grid idealization

Fig. 5b. Contour location

Fig. 5. Lap joint configuration

Fig. 6a. Coarse grid idealization

Fig. 6b. Contour location

Fig. 6. Lap joint configuration
### Table 1. Performance of the Reciprocal Work Contour Integral Method [32]

<table>
<thead>
<tr>
<th>Contour</th>
<th>Coarse Mesh</th>
<th>Fine Mesh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contour 1</td>
<td>2954</td>
<td>2662</td>
</tr>
<tr>
<td>Contour 2</td>
<td>2986</td>
<td>2678</td>
</tr>
<tr>
<td>Contour 3</td>
<td>2882</td>
<td>2729</td>
</tr>
<tr>
<td>Average</td>
<td>2941</td>
<td>2690</td>
</tr>
<tr>
<td>Collocation [26]</td>
<td>-</td>
<td>2580</td>
</tr>
<tr>
<td>% Difference Between Average and Collocation</td>
<td>14.0%</td>
<td>4.3%</td>
</tr>
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</table>

### Table 2. Numerical values of the dimensionless stress intensity factor for the 90° corner (exact value 1) [36]

<table>
<thead>
<tr>
<th>Grid</th>
<th>Direct calculation using on three paths</th>
<th>Calculation using regularizing procedure on three paths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_1$</td>
<td>$E_2$</td>
</tr>
<tr>
<td>Coarse</td>
<td>1.044</td>
<td>0.843</td>
</tr>
<tr>
<td>Medium</td>
<td>0.913</td>
<td>0.939</td>
</tr>
<tr>
<td>Fine</td>
<td>0.972</td>
<td>0.973</td>
</tr>
</tbody>
</table>
consisting of two regions. In the inner region, the stress and displacement distribution was defined by singular stress field associated with the crack tip. In the outer region, the conventional finite element mesh was constructed which satisfied the conditions of nodal compatibility and equilibrium on the interface between the two regions.

Walsh [40, 41] presented a numerical procedure for computing stress intensity factors for cracks and nonzero angle notches for both isotropic and orthotropic materials. His results for 5 different cases of Fig. 7 are presented in Table 3. Where in Table 3, $K_A$ and $K_B$ are symmetric and antisymmetric stress intensity factors respectively, and

$$S_A = 0.45552$$
$$S_B = 0.09147 \text{ for isotropic materials, and}$$
$$S_A = 0.45020$$
$$S_B = 0.10277 \text{ for the orthotropic material used in } [40].$$

To apply the finite element method to determination of stress intensity factors, Chen [42] formulated a stiffness matrix by using the principle of stationary complementary energy and singular elements at sharp corners. Stress intensity factors for rectangular cutouts of different shapes were calculated and Table 4 shows the results obtained for some typical stepped plates.
Fig. 7. Geometry of the re-entrant notch problems investigated by [40 and 41]
Table 3. Stress intensity factors for right angle notched specimens [41]

<table>
<thead>
<tr>
<th>Case (see Fig. 7)</th>
<th>L/W</th>
<th>S/W</th>
<th>T/W</th>
<th>Isotropic</th>
<th>Orthotropic</th>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>$\sigma_0^{VA}$</td>
<td>$\sigma_0^{VB}$</td>
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<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>0.5</td>
<td>0.44</td>
<td>0.22</td>
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<tr>
<td></td>
<td>4</td>
<td></td>
<td>0.5</td>
<td>0.33</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td>0.5</td>
<td>0.33</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0</td>
<td></td>
<td>0.54</td>
<td>0.18</td>
</tr>
<tr>
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<td>2</td>
<td>2.0</td>
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<td>0.53</td>
<td>0.19</td>
</tr>
<tr>
<td>2(a)</td>
<td>2</td>
<td>4</td>
<td>0.5</td>
<td>0.28</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>4 or 8</td>
<td></td>
<td>0.5</td>
<td>0.28</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.0</td>
<td></td>
<td>0.30</td>
<td>0.12</td>
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<td>4 or 8</td>
<td></td>
<td>0.5</td>
<td>0.30</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.0</td>
<td></td>
<td>0.31</td>
<td>0.11</td>
</tr>
<tr>
<td>2(b)</td>
<td>2,4 or 8</td>
<td>4</td>
<td>0.5</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>2,4 or 8</td>
<td>1.0</td>
<td></td>
<td>0.18</td>
<td>0.22</td>
</tr>
<tr>
<td>2(c)</td>
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<td>4</td>
<td>0.5</td>
<td>7.22</td>
<td>3.31</td>
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<tr>
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<td>4 or 8</td>
<td></td>
<td>0.5</td>
<td>7.21</td>
<td>3.31</td>
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<tr>
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<td>2</td>
<td>1.0</td>
<td></td>
<td>7.88</td>
<td>2.59</td>
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<tr>
<td>3(a)</td>
<td>8</td>
<td>1 or 4</td>
<td>0.5</td>
<td>0.63</td>
<td>0.33</td>
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<tr>
<td></td>
<td>1 or 4</td>
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<td>2.75</td>
<td>1.02</td>
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<tr>
<td>3(b)</td>
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<td>1 or 4</td>
<td>0.5</td>
<td>0.92</td>
<td>0.63</td>
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<tr>
<td></td>
<td>1 or 4</td>
<td>2.0</td>
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<td>4.86</td>
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<td>3(c)</td>
<td>8</td>
<td>1</td>
<td>0.5</td>
<td>12.83</td>
<td>7.15</td>
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<td>5.87</td>
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<td>14.59</td>
<td>7.21</td>
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<td>0.51</td>
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<td>1</td>
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<td>0.33</td>
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<td>1.37</td>
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<tr>
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<td>0.32</td>
<td>0.46</td>
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<td></td>
<td>0.25</td>
<td>0.44</td>
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<td>0.85</td>
<td>0.92</td>
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<td>2.0</td>
<td></td>
<td>0.61</td>
<td>0.84</td>
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Table 4. Stress intensity factors of some typical stepped plates [42]

<table>
<thead>
<tr>
<th>Geometries</th>
<th>$K_g (K_I)$</th>
<th>$K_a (K_{II})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x = 1$</td>
<td>0.929</td>
<td>1.437</td>
</tr>
<tr>
<td>$\sigma_x = 1$</td>
<td>1.466</td>
<td>1.494</td>
</tr>
<tr>
<td>$\sigma_x = 1$</td>
<td>1.506</td>
<td>1.568</td>
</tr>
<tr>
<td>$\sigma_x = 1$</td>
<td>2.245</td>
<td>1.627</td>
</tr>
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</table>
CHAPTER III.
THEORETICAL AND NUMERICAL PROCEDURES

In elastic analysis of components with notches or re-entrant corners, there exist a singular stress field at the vertices of the notches. In the section that follows, the equations for the stress field in the vicinity of a notch are developed following the line of procedure used by Williams [11]. These equations which will be in polar coordinates are the solution of the biharmonic equation and the traction free boundary condition for a re-entrant corner. In the last section, the numerical procedure for the overdeterministic least squares analysis of the photoelastic data for determination of stress intensity factors from a loaded model will be explained in detail.

Development of Stress Equations in the Neighborhood of a Notch

Williams [11] used the polar coordinates to derive the governing differential equation for the stress-free notch shown in Fig. 3. The origin of the coordinates lies at the notch tip and the X-axis bisects the notch opening angle.

For a homogeneous and isotropic material in state of plane stress or strain and no body forces, the stress function, \( \chi \), of the form

\[
\chi = r^{\lambda+1} f(\theta)
\]

(1)
with
\[ f(\Theta) = C_1 \cos(\lambda-1)\Theta + C_2 \sin(\lambda-1)\Theta + 
C_3 \cos(\lambda+1)\Theta + C_4 \sin(\lambda+1)\Theta \] (2)
satisfies the biharmonic equation, \( \nabla^2 (\nabla^2 \chi) = 0 \), obtained from equations of equilibrium and compatibility in absence of body forces. Subject to the boundary conditions
\[ \sigma_\theta = 0 \text{ and } \tau_{r\theta} = 0 \text{ at } \Theta = \pm\alpha \text{ (refer to Fig. 3)} \] (3)
where
\[ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2} \] (4)
and the polar stress components in terms of \( \chi \) are:
\[ \sigma_\theta = \frac{\partial^2 \chi}{\partial r^2} \] (5-a)
\[ \sigma_r = \frac{1}{r^2} \frac{\partial^2 \chi}{\partial \Theta^2} + \frac{1}{r} \frac{\partial \chi}{\partial r} \] (5-b),
\[ \tau_{r\theta} = -\frac{1}{r} \frac{\partial^2 \chi}{\partial r \partial \Theta} + \frac{1}{r^2} \frac{\partial \chi}{\partial \Theta} \] (5-c)
Through the use of equations (1), (3), and the biharmonic equation one can obtain the following fourth order differential equation
\[ \frac{d^4 f(\Theta)}{d\Theta^4} + 2(\lambda^2+1) \frac{d^2 f(\Theta)}{d\Theta^2} + (\lambda^2-1)^2 f(\Theta) = 0 \] (6)
subject to the condition:

\[
\frac{df(\theta)}{d\theta} = 0 \quad \text{at} \quad \theta = \pm \alpha
\]  

(7)

To solve for the unknown constants \( c_1 \) through \( c_4 \), substitution of eq. (2) into (7) after simple additions and subtractions will result in the following homogeneous eqs.

\[
\begin{bmatrix}
\cos(\lambda-1)\alpha & \cos(\lambda+1)\alpha \\
(\lambda-1)\sin(\lambda-1)\alpha & (\lambda+1)\sin(\lambda+1)\alpha
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_3
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(8-a)

\[
\begin{bmatrix}
\sin(\lambda-1)\alpha & \sin(\lambda+1)\alpha \\
(\lambda-1)\cos(\lambda-1)\alpha & (\lambda+1)\cos(\lambda+1)\alpha
\end{bmatrix}
\begin{bmatrix}
C_2 \\
C_4
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  

(8-b)

In order to obtain the nontrivial solution the determinants of equations (8) must be zero. This leads to the following characteristic equations for the eigenvalue \( \lambda \)'s.

\[
\lambda \sin 2\alpha = \pm \sin 2\lambda \alpha
\]  

(9)

From the second part of equation (8-a) and first part of equation (8-b), the constants are related to each other as:

\[
C_3 = \beta_1 C_1
\]  

(10-a)

and

\[
C_4 = \beta_2 C_2
\]  

(10-b)

where
\[ \beta_1 = -\frac{(\lambda-1)\sin(\lambda-1)\alpha}{(\lambda+1)\sin(\lambda+1)\alpha} \]

and

\[ \beta_2 = -\frac{\sin(\lambda-1)\alpha}{\sin(\lambda+1)\alpha} \]

A careful consideration of equations (9) reveals that zero is the only value of \( \lambda \) which satisfies both equations. The choice of \( \lambda = 0 \) results in stresses which identically satisfy the boundary condition of equation (3). Therefore for nonzero values of \( \lambda \), if one equation is satisfied the other one will not be [42].

To solve eqs. (9) divide both sides by \( 2\lambda \), then

\[ \frac{\sin2\alpha}{2\alpha} = \pm \frac{\sin2\lambda\alpha}{2\lambda\alpha} \]  \hspace{1cm} (11)

Fig. 8 [42 and 43] is the graphical solution of equation (11), where \( \lambda \) was plotted in the abscissa and quantities \( \sin2\alpha/2\alpha \) and \( \sin2\lambda\alpha/2\lambda\alpha \) as the ordinates. In Fig. 8, the solid curve is the plot of \( \lambda \) vs \( \sin2\lambda\alpha/2\lambda\alpha \) for the special case of a \( 90^\circ \) notch (i.e., \( \alpha = 135^\circ \)), and the horizontal dashed lines are the values of \( \sin2\alpha/2\alpha \) for \( \alpha = 3\pi/4 \). The intersections of the dashed lines and the solid curve will give the real roots (\( \lambda \)) of equation (11). The real and some of the complex roots of equation (11) are listed in Table 5 [42].

For \( \alpha = 180^\circ \) or a zero angle notch (i.e., sharp crack) and \( \alpha = 90^\circ \) or a \( 180^\circ \) angle notch (i.e., half space), there are infinite number of
Fig. 8. Graphical solution of real roots for 90° notch angle [42]
Table 5. The real and complex roots of characteristic equation for 90° notch angle [42]

<table>
<thead>
<tr>
<th>No.</th>
<th>Symmetrical</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tr>
<td></td>
<td>Real Part</td>
<td>Imaginary Part</td>
<td>Real Part</td>
<td>Imaginary part</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----</td>
<td>-------------</td>
<td>----------------</td>
<td>-----------</td>
<td>----------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.54448</td>
<td>0.0</td>
<td>0.90852</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.62925</td>
<td>±0.23125</td>
<td>2.30132</td>
<td>±0.31584</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.97184</td>
<td>±0.37393</td>
<td>3.64141</td>
<td>±0.41879</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.31037</td>
<td>±0.45549</td>
<td>4.97889</td>
<td>±0.48662</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5.64710</td>
<td>±0.51368</td>
<td>6.31507</td>
<td>±0.53763</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

real roots. However, in the case of an arbitrary notch angle between zero and 180°, there is only a finite number of real roots and an infinite number of complex roots. It is noted that the number of real roots decrease from infinity to two and then increase from two to infinity as the notch angle continues to increase from zero to 180° [42]. Moreover, there is at most one real symmetric and one real antisymmetric root with magnitudes less than one and these are the roots which result in singular stresses at the notch tip.

For a given value of λ, if $C_1$ and $C_3$ are not zero, then $C_2$ and $C_4$ must be zero and the stress function (1) becomes an even function of
\( \Theta \) which leads to symmetric stress distribution. The antisymmetric stress distribution is obtained when the stress function is an odd function of \( \Theta \), that is, the eigenvalue substituted in (9) results in nonzero \( C_2 \) and \( C_4 \), and zero \( C_1 \) and \( C_3 \).

In this dissertation, the eigenvalues which result in nonzero \( C_1 \) and \( C_3 \) will be denoted by \( \lambda^+ \), and the eigenvalues which result in nonzero \( C_2 \) and \( C_4 \) will be denoted by \( \lambda^- \). Therefore, the \( \lambda \) in (10-a) is \( \lambda^+ \) and the \( \lambda \) in (10-b) is \( \lambda^- \).

To obtain the equations for the stress distribution for an arbitrary angle notch all the symmetric and antisymmetric stress functions \( \chi \) for the eigenvalues must be added up. In summation notation the Airy's stress function will be of the form

\[
\chi = \sum_{i=1}^{\infty} C_{11} r^{1+\lambda} [\cos(\lambda^+-1)i + \beta_{11} \cos(\lambda^+1)i] + \\
\sum_{i=1}^{\infty} C_{21} r^{1+\lambda} [\sin(\lambda^-1)i + \beta_{21} \sin(\lambda^-1)i]
\]

When equation (12) is substituted into equations (5) that define the relationship between the stresses and the Airy's function, the stresses in a series form will be obtained.

For the sake of simplicity and the fact that singular stresses for most notch angles result from the first one or two eigenvalues, a two parameter approximation of stress equations would be considered from here on. Then, \( \chi \) can be written as:
\[ \chi = C_1 r^{1+\lambda^+} \left[ \cos(\lambda^+-1) \theta + \beta_1 \cos(\lambda^+ + 1) \theta \right] + \\
C_2 r^{1+\lambda^-} \left[ \sin(\lambda^- - 1) \theta + \beta_2 \sin(\lambda^- + 1) \theta \right] \]  
(13)

From the above definition of \( \chi \) and equations (5) one can obtain the polar stress components for a sharp re-entrant corner as:

\[\sigma_r = C_1 r^{\lambda^+-1} [f_1(\theta)] + C_2 r^{\lambda^- - 1} [f_2(\theta)] \]  
(14-a)

\[\sigma_\theta = C_1 r^{\lambda^+-1} \lambda^+ (1+\lambda^+) [f_3(\theta)] + C_2 r^{\lambda^- - 1} \lambda^- (1+\lambda^-) [f_4(\theta)] \]  
(14-b)

\[\tau_r \theta = C_1 r^{\lambda^+-1} \lambda^+ [f_5(\theta)] - C_2 r^{\lambda^- - 1} \lambda^- [f_6(\theta)] \]  
(14-c)

where

\[f_1(\theta) = [\lambda^+ (3-\lambda^+) \cos(\lambda^+-1) \theta - \lambda^+ (1+\lambda^+ \beta_1 \cos(\lambda^+ + 1) \theta]
\]

\[f_2(\theta) = [\lambda^- (3-\lambda^-) \sin(\lambda^- - 1) \theta - \lambda^- (1+\lambda^- \beta_2 \sin(\lambda^- + 1) \theta]
\]

\[f_3(\theta) = [\cos(\lambda^+-1) \theta + \beta_1 \cos(\lambda^+ + 1) \theta]
\]

\[f_4(\theta) = [\sin(\lambda^- - 1) \theta + \beta_2 \sin(\lambda^- + 1) \theta]
\]

\[f_5(\theta) = [(\lambda^+-1) \sin(\lambda^+ - 1) \theta + (\lambda^+ + 1) \beta_1 \sin(\lambda^+ + 1) \theta]
\]

\[f_6(\theta) = [(\lambda^- - 1) \cos(\lambda^- - 1) \theta + (\lambda^- + 1) \beta_2 \cos(\lambda^- + 1) \theta]
\]

As can be seen from equations (14), the degree of singularity of the stresses is in order of \( r^{\lambda - 1} \). It was shown by Williams [11] that the displacements are in order of \( r^\lambda \). Therefore, the possibility of \( \lambda \) being a negative value as the solution of equation (9) must be eliminated since such a negative value will result in unbounded displacements at the notch vertices.
To cancel the singularity, Modes I and II stress intensity factors are defined as follows:

\[ K_I = \lim_{r \to 0} (2\pi)^{1/2} r^{1-\lambda^+} \sigma_\theta \quad \text{at } \theta = 0 \quad (15) \]

\[ K_{II} = \lim_{r \to 0} (2\pi)^{1/2} r^{1-\lambda^-} \tau_{r\theta} \quad \text{at } \theta = 0 \quad (16) \]

From the above definition, the constants \( C_1 \) and \( C_2 \) in terms of \( K_I \) and \( K_{II} \) will be of the form

\[ C_1 = Q_1 K_I \quad (17) \]

\[ C_2 = -Q_2 K_{II} \quad (18) \]

where \( Q_1 \) and \( Q_2 \) are:

\[ Q_1 = \frac{1}{2(2\pi)^{1/2}(1+\beta_1)(\lambda^+)(1+\lambda^+)} \]

\[ Q_2 = \frac{1}{2(2\pi)^{1/2}(\lambda^-)[\lambda^- - 1 + \beta_2(\lambda^- + 1)]} \]

Then the final form of stresses in terms of \( K_I \) and \( K_{II} \) will be as:

\[ \sigma_r = K_I r^{\lambda^+ - 1} q^+ \alpha_1(\theta) - K_{II} r^{\lambda^- - 1} q^- \alpha_2(\theta) \quad (19) \]

\[ \sigma_\theta = K_I r^{\lambda^+ - 1}(1+\lambda^+) q^+ \alpha_3(\theta) - K_{II} r^{\lambda^- - 1}(1+\lambda^-) q^- \alpha_4(\theta) \quad (20) \]

\[ \tau_{r\theta} = K_I r^{\lambda^+ - 1} q^+ \alpha_5(\theta) + K_{II} r^{\lambda^- - 1} q^- \alpha_6(\theta) \quad (21) \]

with

\[ q^+ = \lambda^+(Q_1) \]

\[ q^- = \lambda^-(Q_2) \]
In the section that follows, a numerical procedure is developed to relate the photoelastic data and stress intensity factors to each other. This procedure takes the advantage of whole field photoelasticity and calculates the values of $K_I$ and $K_{II}$ using a least squares technique for any arbitrary angle notch.

Theory of Photoelasticity [44, 6, 45]

When certain polymeric transparent materials are subjected to stress, they become temporarily birefringent (double refractive). A beam of polarized light upon entering a stressed model made of birefringent materials splits into two components, each vibrating along a principal direction, and traveling at different speeds [44, p. 369]. As the light travels through the thickness, one of the components is progressively retarded relative to the other, and a phase shift (relative retardation) occurs between the two light components. This phase shift is proportional to the principal stress difference and the model thickness at each point.

After the two components emerge from the model, they enter and travel through an analyzer and an interference pattern will be produced. The directions of vibration of the two components give information about the principal stress direction and the relative phase shift determines the magnitude of the in-plane principal stress difference. The effect of principal stress direction can be eliminated with the use of a
circular polariscope (the kind used in this study). Fig. 9 shows a schematic diagram of a dark field circular polariscope.

In the case of two-dimensional state of stress and monochromatic light at normal incident to the plane of model, the stress-optic law can be written as [44]

\[ \Delta = \frac{2\pi hc}{\lambda} (\sigma_1 - \sigma_2) \]  \hspace{1cm} (22)

where

- \( \Delta \) = relative angular phase shift or relative retardation
- \( c \) = relative stress-optic coefficient in Brewsters (1 Brewster = 10^{-12} \text{ m}^2/\text{N})
- \( \sigma_1, \sigma_2 \) = in-plane principal stresses
- \( \lambda \) = wave length of the monochromatic light
- \( h \) = distance traveled by the light through the model for transmission polariscope \( h=t \), the thickness of the model.

Equation (22) can be written in the form:

\[ N = \frac{h(\sigma_1 - \sigma_2)}{f_\sigma} \]  \hspace{1cm} (23)

Here

- \( N = \Delta/2\pi \), is the fringe order, and
- \( f_\sigma = \lambda/c \), is material stress fringe constant.
Fig. 9. Schematic diagram of optical transformations in a circular polariscope. (Courtesy of W. F. Riley, Department of Engineering Science and Mechanics, Iowa State University.)
In a dark field circular polariscope setup, the intensity (I) of the light emerging from the analyzer is

\[ I = K \sin^2 \frac{\Delta}{2} \]

where K is a constant.

Extinction occurs when \( I = 0 \), i.e., when

\[ \frac{\Delta}{2} = n \pi \text{ for } n = 0, 1, 2, 3, \ldots \]

The resulting dark lines are called isochromatic and the corresponding integer numbers, i.e.,

\[ N = n = \frac{\Delta}{2\pi} \]

are called full fringe orders.

If the analyzer in Fig. 9 is rotated 90° such that it is parallel with the polarizer then this arrangement will result in one type of light field circular polariscope. The light intensity for a light field polariscope is:

\[ I = K \cos^2 \frac{\Delta}{2} \]

Extinction occurs when

\[ \frac{\Delta}{2} = \frac{2n+1}{2} \pi \text{ for } n = 0, 1, 2, 3, \ldots \]

or

\[ N = \frac{\Delta}{2\pi} = \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots \]
These are called "half-order" fringes.

From photographs of fringe patterns or "live" fringe patterns from both light and dark field arrangements, one can obtain a whole-field representation of the order of the fringes to the nearest 1/2 order.

In the following section, using equation (23), a relationship between the stress intensity factors and the fringe orders will be established. This relationship can be used to determine Mode I and Mode II stress intensity factors.

Numerical Analysis

To solve for the unknown stress intensity factors, $K_I$ and $K_{II}$ in equations (19)-(21), the overdeterministic least squares algorithm of Sanford [18] in conjunction with Newton-Raphson iterative method is used. This technique of using more data than the two needed to solve for the unknowns takes advantage of whole field photoelastic data and reduces the experimental error.

The maximum in-plane shear stress can be expressed as

$$
\tau_{\text{max}} = \frac{1}{2} (\sigma_1 - \sigma_2)
$$

and from equation (23)

$$
\sigma_1 - \sigma_2 = \frac{N f_e}{h}
$$
where \( h \) = model thickness

or \[ \tau_{\text{max}} = \frac{N_f}{2h} \] (24)

To establish the relationship between \( N \) and the stress intensity factors, the maximum in-plane shear stress is written in terms of polar stresses, i.e.,

\[ (\tau_{\text{max}})^2 = \left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2 + (\tau_r \theta)^2 \] (25)

or from (24) and (25)

\[ \frac{N_f}{h} \left(\frac{\sigma_r}{\sigma_\theta}\right)^2 = \left(\sigma_r - \sigma_\theta\right)^2 + 4(\tau_r \theta)^2 \] (26)

Define function \( G \) such that

\[ G(K_I, K_{II}) = \left(\sigma_r - \sigma_\theta\right)^2 + 4(\tau_r \theta)^2 - \left(\frac{\sigma_r}{\sigma_\theta}\right)^2 = 0 \] (27)

After substitution of stresses from equations (19)-(21), and simplification the function \( G \) can be written as

\[ G(K_I, K_{II}) = 4(K_I)^2 (0^+)^2 r^2 (\lambda^+ - 1) \left[ e_1(\theta) \right] + 4(K_{II})^2 (0^-)^2 r^2 (\lambda^- - 1) \left[ e_2(\theta) \right] + 8K_I K_{II} 0^+ 0^- r^{(\lambda^+ + \lambda^- - 2)} \left[ e_3(\theta) \right] - \frac{N_f}{h} \left(\frac{\sigma_r}{\sigma_\theta}\right)^2 \] (28)

where

\[ 0^+ = Q \lambda^+ \]
\[ q^- = a \lambda^- \]
\[ e_1(\theta) = (\lambda^+ - 1)^2 + (\beta_1^e)^2(1+\lambda^+)^2 + 2\beta_1(\lambda^+ - 1)(\lambda^+ + 1)\cos2\theta \]
\[ e_2(\theta) = (\lambda^- - 1)^2 + (\beta_2^e)^2(1+\lambda^-)^2 + 2\beta_2(\lambda^- - 1)(\lambda^- + 1)\cos2\theta \]
\[ e_3(\theta) = (\lambda^+ - 1)(\lambda^- - 1) + \beta_e(\lambda^+ + 1)(\lambda^- + 1)\sin(\lambda^+ - \lambda^-)\theta + \beta_1(\lambda^+ - 1)(\lambda^- - 1)\sin(\lambda^+ - \lambda^- + 2)\theta + \beta_2(\lambda^+ - 1)(\lambda^- + 1)\sin(\lambda^+ - \lambda^- - 2)\theta \]

A truncated Taylor series expansion of equation (28) about the unknown parameters, \( K_I \) and \( K_{II} \), will result in a linearized equation in the form of

\[ (G_k)_{i+1} = (G_k)_i + \left( \frac{\partial G_k}{\partial K_I} \right)_i \Delta K_I + \left( \frac{\partial G_k}{\partial K_{II}} \right)_i \Delta K_{II} \]  \hspace{1cm} (29)

where subscript \( i \) denotes the iteration step. From equation (29) if estimates of \( K_I \) and \( K_{II} \) combined with photoelastic data \( (r, \theta \text{ and } N) \) are given one can obtain the corrections \( \Delta K_I \) and \( \Delta K_{II} \) to those estimates. The corrections are added to the estimates and new estimates of \( K_I \) and \( K_{II} \) are obtained. This procedure is repeated until the desired result, \( (G_k)_{i+1} \) is obtained. Or

\[ \left( \frac{\partial G_k}{\partial K_I} \right)_i \Delta K_I + \left( \frac{\partial G_k}{\partial K_{II}} \right)_i \Delta K_{II} = - (G_k)_i \]

In matrix notation

\[ [G]_i = [B]_i (\Delta K)_i \]  \hspace{1cm} (30)
where

\[
[G] = \begin{pmatrix}
G_1 \\
G_2 \\
\vdots \\
G_k
\end{pmatrix}
\]

\[
[B] = \begin{pmatrix}
\frac{\alpha G_1}{\alpha K_I} & \frac{\alpha G_1}{\alpha K_{II}} \\
\frac{\alpha G_2}{\alpha K_I} & \frac{\alpha G_2}{\alpha K_{II}} \\
\vdots & \vdots \\
\frac{\alpha G_k}{\alpha K_I} & \frac{\alpha G_k}{\alpha K_{II}}
\end{pmatrix}
\]

\[
[\Delta K] = \begin{pmatrix}
\Delta K_I \\
\Delta K_{II}
\end{pmatrix}
\]

Multiplying both sides of eq. (30) from the left by transpose of \([B]\).

\[
[B]^T(G) = [B]^T[B](\Delta K)
\]

or

\[
(D) = (A)(\Delta K)
\]

Where

\[
(D) = [B]^T(G)
\]

\[
(A) = [B]^T[B]
\]
Finally,

\[ \Delta K = [A]^{-1}D \]  \hspace{1cm} (31)

where \([A]^{-1}\) is the inverse of \([A]\). The solution of the above equation gives the values of \(\Delta K_1\) and \(\Delta K_{II}\) which are the correction to the initial estimates of \(K_1\) and \(K_{II}\). Therefore, the new estimates \((i+1)\) of \(K_1\) and \(K_{II}\) would be given by

\[ (K_1)_{i+1} = (K_1)_i + \Delta K_1 \]  \hspace{1cm} (32-a)

\[ (K_{II})_{i+1} = (K_{II})_i + \Delta K_{II} \]  \hspace{1cm} (32-b)

This step is repeated until \(\Delta K_1\) and \(\Delta K_{II}\) become acceptably small. Or in other words, there is no difference within a specified error value between the final values of \(K_1\) and \(K_{II}\) and the one before.
CHAPTER IV.

EXPERIMENTAL TECHNIQUES

Model Material

The material used in constructing the model is a specially annealed polycarbonate called PSM-1. It is a ductile, highly transparent and completely free of time edge effect material. Its low photoelastic coefficient, \( f_\sigma \), makes this a suitable material for photoelastic studies [46]. PSM-1 is much easier to machine than most photoelastic materials, and unlike the other counterparts it does not have any problem with chipping or cracking in machining operations. Table 6 lists a few of PSM-1 material properties [46].

Table 6. Summary of the material properties for PSM-1

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity</td>
<td>( E )</td>
<td>GN/m(^2)</td>
<td>2.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>psi</td>
<td>(3.47 \times 10^5)</td>
</tr>
<tr>
<td>Photoelastic Fringe Constants</td>
<td>( f_\sigma )</td>
<td>KN/fringe-m</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lb/fringe-in.</td>
<td>40</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>( \nu )</td>
<td>-</td>
<td>0.38</td>
</tr>
</tbody>
</table>

*PSM-1 is a product of Photoelastic, Inc., Raleigh, NC 27611.*
The only disadvantage this material has is its sensitivity to localized heating from the machine tools in cutting operations. To take care of this problem, water and other aqueous coolants can be used during machining since PSM-1 is insensitive to moisture. To avoid occurrence of residual birefringence, beside the use of coolant, it is a good practice not to allow the work piece to remain in contact with a rotating tool.

Model Geometry

The geometry of the three models used in obtaining stress intensity factors in $90^\circ$ corners are shown in Figs. 10 and 11. All specimens were made from a single 1/4 inch thick plate of PSM-1. The model shown in Fig. 11 was chosen since theoretical and numerical values of stress intensity factors are available for comparison purposes. From each geometry in Fig. 10, three specimens with different dimensions as listed in Table 7 were made. To maintain the same sharpness of the corners from one specimen to another, the models were designed such that the same corners were present in all three specimens of each model. That is, one specimen was obtained from another by removing some material from certain sections without touching the corners. For example, specimen M2 was machined from M1 by reducing the width of specimen M1 by one inch, and M3 was obtained from M2 by reducing the width of specimen M2 by 0.5 inch.
Fig. 10. Geometry of models M and N
Fig. 11. Geometry of model W

Fig. 12. Loading frame used in this study
In designing the specimens' dimension, considerable information was gained from Table 4. For example, changes in dimension S of model M did not have any effect on the magnitude of stress intensity factors. Therefore, this dimension was kept the same for all three specimens (M1, M2, M3). Model M has two corners and model N has four corners. Each model was loaded and viewed under a monochromatic light polariscope, and the sharpest corner in each model was chosen for all data collection.

The specimens were loaded in tension using the loading frame shown in Fig. 12. The magnitude of the applied loads were read through a load cell with maximum range of 200 lbs and applied loads can be measured within 1.0% accuracy. The load cell and the loading frame are product of Photoelastic Division of Measurements Group Inc., Raleigh, North Carolina.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L (in.)</th>
<th>S (in.)</th>
<th>T (in.)</th>
<th>V (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>4.878</td>
<td>1.50</td>
<td>1.002</td>
<td>2.004</td>
</tr>
<tr>
<td>M2</td>
<td>4.878</td>
<td>1.50</td>
<td>1.002</td>
<td>1.002</td>
</tr>
<tr>
<td>M3</td>
<td>3.956</td>
<td>1.50</td>
<td>1.008</td>
<td>0.514</td>
</tr>
<tr>
<td>N1</td>
<td>2.000</td>
<td>4.65</td>
<td>0.958</td>
<td>0.479</td>
</tr>
<tr>
<td>N2</td>
<td>2.000</td>
<td>4.65</td>
<td>0.479</td>
<td>0.479</td>
</tr>
<tr>
<td>N3</td>
<td>2.000</td>
<td>4.65</td>
<td>0.234</td>
<td>0.479</td>
</tr>
</tbody>
</table>
Calibration for the Photoelastic Coefficient \([44, 47, 48]\)

To obtain the material fringe constant, \(f_\sigma\), the self-calibration property of the models \(M\) and \(N\) were used. These two models each contain at least two parallel sections under uniaxial tension where theoretical stresses can be easily calculated. These sections can be used to calculate the material fringe constant, and to check the uniformity of the applied stresses. This method of calibration eliminates the need for a separate calibration model such as a circular disk in diametral compression, and reduces the error due to time-temperature effects.

In the portions of the specimens where there is a uniform stress field, from equation (23) one can write

\[
\sigma_1 = \frac{Nf_\sigma}{h}
\]

and for \(\sigma_1 = P/Wh\)

\[
f_\sigma = \frac{P}{WN} \tag{33}
\]

where

\(f_\sigma = \text{material fringe constant}\)

\(P = \text{applied load}\)

\(N = \text{fringe order at the cross section of the uniform stress region}\)

\(W = \text{width of the specimen (section)}\)
Specimen N2 was loaded and at different load levels the fringe order in the two narrow regions of the specimen were recorded. Then a curve of P versus N was constructed and the slope of the line drawn through the points can be used in equation (33) to obtain the material fringe constant. Following the above outlined procedure and the slope from Fig. 13, the best estimate of material fringe constant for the PSM-1 plate used in this study was found to be 40.98 Psi/ fringe-in.

Stress Intensity Factor Extraction and Accuracy Evaluation

Several computer programs were developed to extract the stress intensity factors from the isochromatic fringe patterns of the loaded models. All the data acquisition and processing were done on an "EyeCom" digital image analysis system connected to an LSI-11 computer through an interface.

In the sections that follow, a brief description of the "EyeCom" system and the programs used will be given.

EyeCom System [5, 6, 49]

The image analysis system used in collecting data and regenerating the fringes is called "EyeCom III" system. This is an image processing machine which combines three types of man-machine communications, alphanumeric, graphic, and pictorial into a single entity to provide the tools necessary for efficient image processing. With the aid of an LSI-11 computer connected to the system the tasks of communication with
Fig. 13. Calibration curve for material fringe constant calculation using model N
the system, running of programs, saving and printing of needed information is possible. A schematic diagram is shown in Fig. 14. This system consists of picture digitizer and display monitor capable of both input and output functions for processing of pictures.

An analog picture scanned by the vidicon camera can be digitized and displayed either via a real time digitizer or from refresh memory. The image digitized in real time can be seen on the scanner display immediately. The scanned image is divided into 480 lines each consisting of 640 elements called pixels. Therefore, a full image is consisted of 307,200 pixels, and each digitized pixel will be assigned a gray level in the range of 0-256. When a pixel is digitized, its position (x, y), and its brightness (Z) are registered by the digitizer cursor and saved in the computer's memory for future use.

Calibration of the EyeCom System

The EyeCom system needs to be calibrated before it can be used for photoelastic analysis. That is, the relationship between the digitized Z-value and the fringe order N must be established. The built in program SETUP is used to achieve the calibration of EyeCom. It provides interactive adjustment of the EyeCom digitization parameters by determining proper values for the setup, zero and range registers. These registers control the video amplifier chain that provide the proper video signal for the digitizers as follows:

Setup - The digital value, 0 to 255, in this register sets the proper black level of the scanner video signal to produce linear
Fig. 14. Schematic diagram of the image processing system [22]
operation of the amplifier chain in the linear mode and correct logarithmic operation in the log mode.

Zero - The value in this register, 0 to 255, determines the video level corresponding to a digitized Z value of zero.

Range - The value in this register, 0 to 63, determines the range of video levels covering the span from a Z-value of zero to Z-value of 255.

The setup register is adjusted by first closing the lens which provides the scanner with a black picture. The computer program SETUP is then used to initialize the zero register and to setup the appropriate zero value and range. The brightest white level of the picture is adjusted by the f-stop of the lens on the scanner. Then the zero and range registers are set to digitize only that portion of the image gray scale of interest to the user.

The zero and range registers are then set with the correct values to place the darkest area at a Z-value of zero, and the brightest area at a Z-value of 255. Refresh memory picture is initialized and enabled for future use. The chart in Fig. 15 summarizes the whole procedure [6].

Program "MODATA" [50]

This program acquires the data from the stored image and stores it in a file to be used by other programs. The stored data are corner tip coordinates, fringe order, radial distance and angular position of
desired points from the corner tip. To reduce the error in locating the darkest point on the broad band isochromatic fringes, the cursor is pointed at or near the desired point. Then, an area of 5 by 5 pixel is searched and the darkest point in that area is picked automatically. For a selected point the user inputs the fringe order, N, and "MODATA" calculates radial distance, r, and angular location, θ, of the points.
In this program, a scaling factor between the model and the image is determined by pointing the cursor at two points in a grid and specifying the actual distance between them. The scaling factor together with the corner tip coordinates are used to give a mean of comparing real fringes with the regenerated ones.

Program "SIFACT" [50]

The stress intensity factors are obtained from the whole field photoelastic data collected by "MODATA". The program uses Newton-Raphson iterative technique combined with an overdeterministic least squares algorithm. The program is written for a two term approximation for a right angle notch, but by changing the values of two constants in the program, i.e., LP (lambda plus) and LM (lambda minus) it could be used for any arbitrary angle notch for which the photoelastic data are provided.

Program "BACKPL" [50]

This program regenerates fringes based on the stress intensity factors obtained from "SIFACT" for comparison with the original fringes. If the regenerated fringes match up with the original fringes then, one can conclude that the stress intensity factors obtained for a given data set are a good representation of the stress field. If the match up is not satisfactory a different data set is collected by using program "MODATA", and the procedure is repeated until a satisfactory data set is found.

The block diagram shown in Fig. 16 summarizes the steps used in
Fig. 16. Block diagram of steps used to obtain stress intensity factors.
analysis and determination of stress intensity factors in re-entrant corners. The above mentioned programs are in the Appendix. These and any other programs or subroutines used in this dissertation are all in the Program Bank of the Experimental Stress Lab, Department of Engineering Science and Mechanics, Iowa State University [50].
CHAPTER V.

RESULTS

Three different models were used to investigate stress intensity factors in 90° re-entrant corners. One was a rectangular plate with a 90° V-notch. Only one specimen from this model was made and its dimensions are given in Fig. 11, and three specimens from each of the other two models shown in Fig. 10 (with different ratios of T/V) were machined.

Before proceeding with the data collection, all specimens were loaded in tension. Then using a white light source polariscope, it was checked to make sure that the loading was uniform in the straight portions of the specimens. For the models with more than one 90° corner (refer Fig. 10), it was checked to see which corner was the sharpest and all the data collection for the specimens made from that model were on that corner.

The loaded models then were viewed in a dark field and light field circular polariscope. Fig. 17 shows typical fringe pattern in the vicinity of the corner tip for the three models used in this study.

Using the program "MODATA" the live image (fringe pattern) for each model was stored in the picture memory of the EyeCom III, and every stored image had a grid mesh for scaling purposes. On the stored image, the user in response to questions asked by "MODATA" points the cursor to
Fig. 17. Fringe patterns near the corners
c) Model N
specimen N2
\[ \sigma_o = 313.15 \text{ psi} \]

Fig. 17. (Continued)
two grid points (whose distance in inches have been entered before), and
the program calculates the scaling factor between the live and the
stored image on the screen of EyeCom. After the scaling factor have
been established, the user points the curser to the corner tip, to a
point on the "horizontal" (or left) free boundary, and to a point on the
"vertical" (or right) free boundary of the corner, and the program then
stores the coordinates of these points.

The coordinates of the corner tip and the point on the "horizontal"
free boundary are used to find the angle between the x-axis of the
"EyeCom" and the "horizontal" boundary of the corner or the notch which
in turn will allow one to find the axis (bisector of the notch) from
which the angle theta (θ) is measured.

The coordinates of the point on the "vertical" boundary was not
entered into any calculations. However together with the point on the
"horizontal" boundary and the corner tip, it was only used to draw a 90°
angle which outlined the boundary of the corner near the tip.

Once the coordinates of the points mentioned above together with a
scaling factor were specified and stored in a file, the program asks
the user to point the cursor at a desired data point on a fringe and hit
the return key. Then, the user enters the fringe order at that point in
response to a question asked. These last two steps will be repeated for
the number of data points desired by the user (given as input in the
beginning of the program) and the fringe number, radial distance from
the the corner tip and the angle Theta of the points will be stored in a
single file for future use. The location of the data points will be marked with a plus (+) sign on the screen of the "EyeCom".

The data file that contains \( r, \theta \) and fringe number is used in program "SIFACT" to calculate the stress intensity factors for the corner. The stress intensity factors are then inputed into the program "BACKPL" and the program regenerates the fringe pattern for the corner. If the regenerated fringe pattern matches or comes close to matching the actual fringes then the stress intensity factors are accepted as a representative values of the actual stress intensity factors for the specimen.

In the sections that follow each model is discussed in detail and the stress intensity factors obtained for each are presented.

Model V

The V-notch model was chosen as a means of checking the whole procedure of obtaining stress intensity factors in this study. The model is under Mode I loading and theoretical and numerical values of \( K_I \) have been obtained by different investigators [42, 51].

Using this model, effects of number of data points picked from the fringe patterns and load levels applied to obtain stress intensity factors were investigated. Tabulated values of \( K_I \) and \( K_{II} \) for different stress levels and number of data points for the V-notch specimen are given in Table 8. From the result presented in Table 8, one can conclude that as the stress level increases, as expected, the value of \( K_I \) increases and the value of \( K_{II} \) remains almost unchanged. The result
in Table 8 also indicates that the values of $K_I$ are independent of number of points used in the analysis as long as it is greater than 20. Theoretically, the values of $K_{II}$ must be zero but in this study that was not the case. However, $K_{II}$ did not increased as $K_I$ (or load) was increased. So, one can ignore this value of $K_{II}$.

Fig. 18 shows the data points and the regenerated fringes superimposed on the actual fringe patterns for the V-notch model at
Fig. 18. Data points and regenerated fringes superimposed on the actual fringes for model W
c) Case no. 20
Table 8

Fig. 18. (Continued)

d) Case no. 24
Table 8

Fig. 18. (Continued)
different stress levels. For the sake of clarity, the data points and the regenerated fringes (back plot) for the cases presented in Fig. 18 are shown in Fig. 19. The rectangular frame in which the back plot is drawn is actually a 0.25 by 0.25 inch square which is centered on the corner tip. The distortion of the square frame and the fringe loops as seen in some of the cases is caused by the misadjustment of the hardcopy unit used to copy the video images from the screen of the EyeCom III, and in no way this misadjustment affected the values of $K_I$ and $K_{II}$.

The average stress intensity factor at each stress level is plotted against the axial stress for the V-notch specimen in Fig. 20. Fig. 20 also shows a 95% confidence band on the lines fitted through the data points. The slope of the line fitted through the points in Fig. 20 is the normalized value of $K_I$. In Table 9, this normalized $K_I$ is compared with the theoretical and numerical values obtained by other investigators. From the values in Table 9, one can conclude that the method used in this study works well, and the difference between the experimental and the theoretical value is only 2%.

Table 9. Normalized Mode I stress intensity factors for a single-edge 90° notch plate

<table>
<thead>
<tr>
<th>Method</th>
<th>$K_I/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numerical Solution [42]</td>
<td></td>
</tr>
<tr>
<td>1 term solution</td>
<td>4.710</td>
</tr>
<tr>
<td>7 term solution</td>
<td>4.948</td>
</tr>
<tr>
<td>Theoretical Solution [51]</td>
<td>4.566</td>
</tr>
<tr>
<td>Experimental (this study)</td>
<td>4.661</td>
</tr>
</tbody>
</table>
Fig. 19. Data points and regenerated fringes for model W
c) Case no. 20  
Table 8

---

d) Case no. 24  
Table 8

---

Fig. 19. (Continued)
Fig. 20. Plot of average stress intensity factors for the V-notch specimen

\[ K_I = 12.6184 + 4.6612X \]

95% Confidence Band
Model M

Three specimens M1, M2 and M3 with different T/W ratios were machined from this model. It should be emphasized that the corner sharpness and the shape was the same for all three specimens, since for example specimen M2 was machined from specimen M1 by reducing the width by one inch and keeping T same as before. Then specimen M3 was obtained from specimen M2 in the same way. For both specimens M2 and M3, a new pin hole centered about the width of the specimen was drilled for loading purposes.

Mode I and Mode II stress intensity factors for the specimens at different load levels and number of data points were obtained. Tables 10-12 list the values of $K_I$ and $K_{II}$ for each specimen.

In Fig. 21, the data points and regenerated fringes are superimposed on the actual fringe patterns of the loaded specimens. The data points and only the regenerated fringes of the cases presented in Fig. 21 are shown again in Fig. 22. One can see in Figs. 21, and 22 that the match between the regenerated fringes and the actual fringe is real good especially near the corner.

The average values of the Mode I and Mode II stress intensity factors from Tables 10-12 at each stress levels with a 95% confidence band are plotted in Figs. 23-25. As before the slopes of the lines fitted through the points are the normalized stress intensity factors for the three specimens. These normalized values are presented in Table 13. From the result presented in Table 13, as the ratio of T/W
increases the normalized stress intensity factors increases also. This fact is also evident from case 3 (b) of Table 3, but unfortunately Walsh in references [40] and [41] never defined his $K_A$ ($K_I$) and $K_B$ ($K_{II}$), and so it is not possible to compare the result from this study with those in references [40] and [41].

Table 10. Modes I and II stress intensity factors for specimen M1

<table>
<thead>
<tr>
<th>No.</th>
<th>Load (lb)</th>
<th>No. of data points</th>
<th>Stress (Psi)</th>
<th>$K_I$</th>
<th>$K_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>25</td>
<td>53.33</td>
<td>151.9051</td>
<td>159.3886</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>25</td>
<td>53.33</td>
<td>146.7994</td>
<td>161.1025</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>30</td>
<td>66.67</td>
<td>172.7495</td>
<td>210.5351</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>30</td>
<td>66.67</td>
<td>182.3238</td>
<td>208.2290</td>
</tr>
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<td>50</td>
<td>25</td>
<td>66.67</td>
<td>178.4743</td>
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<td>66.67</td>
<td>179.2998</td>
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<td>60</td>
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<td>80.00</td>
<td>203.0695</td>
<td>238.9169</td>
</tr>
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<td>8</td>
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<td>30</td>
<td>80.00</td>
<td>221.0156</td>
<td>241.3755</td>
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<tr>
<td>9</td>
<td>60</td>
<td>30</td>
<td>80.00</td>
<td>192.0954</td>
<td>254.3865</td>
</tr>
<tr>
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<td>60</td>
<td>35</td>
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</tr>
<tr>
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<td>218.8621</td>
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</tr>
<tr>
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<td>210.0326</td>
<td>254.5570</td>
</tr>
<tr>
<td>13</td>
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<td>80.00</td>
<td>211.2440</td>
<td>251.9009</td>
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<tr>
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<td>270.9238</td>
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<tr>
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<td>80</td>
<td>40</td>
<td>106.67</td>
<td>264.3825</td>
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<td>40</td>
<td>106.67</td>
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<td>50</td>
<td>106.67</td>
<td>284.6257</td>
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</tr>
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<td>285.7025</td>
<td>298.6416</td>
</tr>
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</table>
Table 11. Modes I and II stress intensity factors for specimen H2

<table>
<thead>
<tr>
<th>No.</th>
<th>Load (lb)</th>
<th>No. of data points</th>
<th>Stress (Psi)</th>
<th>$K_I$</th>
<th>$K_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>40</td>
<td>43.91</td>
<td>175.3874</td>
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<td>22</td>
<td>40</td>
<td>43.91</td>
<td>191.5815</td>
<td>219.1303</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>30</td>
<td>59.88</td>
<td>253.5646</td>
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<td>250.0144</td>
<td>289.6076</td>
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<td>40</td>
<td>59.88</td>
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<td>40</td>
<td>59.88</td>
<td>245.4396</td>
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<td>30</td>
<td>40</td>
<td>59.88</td>
<td>270.0396</td>
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<tr>
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<td>30</td>
<td>45</td>
<td>59.88</td>
<td>251.4840</td>
<td>341.6810</td>
</tr>
<tr>
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<td>30</td>
<td>45</td>
<td>59.88</td>
<td>262.5169</td>
<td>312.3962</td>
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<td>79.84</td>
<td>324.9951</td>
<td>392.2993</td>
</tr>
<tr>
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<td>50</td>
<td>79.84</td>
<td>339.3569</td>
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Table 12. Mode I and II stress intensity factors for specimen M3

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a) Specimen M1
   case no. 3
   Table 10

b) Specimen M1
   case no. 1
   Table 10

Fig. 21. Data points and regenerated fringes superimposed on the actual fringes for model M
c) Specimen M2  
   case no. 13  
   Table 11


d) Specimen M3  
   case no. 19  
   Table 12

Fig. 21. (Continued)
Fig. 22. Data points and regenerated fringes for model M
c) Specimen H2  
   case no. 13  
   Table 11

d) Specimen M3  
   case no. 19  
   Table 12

Fig. 22. (Continued)
Fig. 23. Plot of the average stress intensity factors for specimen M1

\[ K_1 \quad Y = 15.1513 + 2.4621X \]

\[ K_{II} \quad Y = 15.0514 + 2.8574X \]

\[ \ldots \quad \text{95% confidence band} \]
Fig. 24. Plot of the average stress intensity factors for specimen M2
Fig. 25. Plot of the average stress intensity factors for specimen M3

- $K_I$: $y = 23.2908 + 6.8395x$
- $K_{II}$: $y = -4.3425 + 9.7770x$
- 95% Confidence Band
Table 13. Normalized stress intensity factors for model M

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<th>Specimen</th>
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<th>$K_{II}/\sigma$</th>
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Model N

Like model M, three specimens were made from this model. The machining of specimens were such that the same corners were present in each specimen. For example, specimen N2 was made from N1 by removing 0.5 inch of material from each side of the midsection of specimen N1. The narrow sections on each sides and the pin holes together with the four corners were unchanged from specimen to specimen.

The mixed mode stress intensity factors for the specimens at different stress levels are shown in Tables 14-16. The average intensity factors from Tables 14-16 for a given stress level with 95% confidence bands are plotted in Figs. 26-28. The bands on $K_{II}$ for specimen N1 include a wide region of Fig. 26. This is because the stress intensity factors were obtained for three load levels only.

The original fringes patterns and the regenerated fringes together with the data points are shown in Fig. 29. These pictures are the photocopies of the hard copy, and they show how well the back plots (regenerated fringes) match with the original fringes.
The slopes of the lines in Figs. 26-28 are the normalized stress intensity factors for model N. These values are shown in Table 17, and one can see that as the ratio of T/V is decreasing the normalized stress intensity factors are decreasing as well. The same trend between T/V and normalized stress intensity is seen in case 1 of Table 3, but unfortunately again there is no way that the actual values obtained here could be compared with those in Table 3.

From the results of experiments performed on models W, M, and N, one can conclude that values obtained for Mode I and Mode II stress intensity factors are representative of such class of problems. Since, first, the technique has been checked using model W and the result is well within 2% of the theoretical value of $K_I$, and second, the results for the other two models follow the same trend as the available numerical solution, then these must be representative stress intensity factors for models M and N.

Table 14. Modes I and II stress intensity factors for specimen N1

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Table 16. Modes I and II stress intensity factors for specimen N3

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<td>417.54</td>
<td>211.4791</td>
<td>403.1554</td>
</tr>
<tr>
<td>11</td>
<td>100</td>
<td>25</td>
<td>417.54</td>
<td>204.5906</td>
<td>435.8032</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>25</td>
<td>417.54</td>
<td>229.8543</td>
<td>387.6093</td>
</tr>
<tr>
<td>13</td>
<td>100</td>
<td>25</td>
<td>417.54</td>
<td>214.5818</td>
<td>403.2960</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>25</td>
<td>417.54</td>
<td>234.3588</td>
<td>344.4563</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>25</td>
<td>417.54</td>
<td>212.7777</td>
<td>391.6839</td>
</tr>
<tr>
<td>16</td>
<td>150</td>
<td>40</td>
<td>626.30</td>
<td>333.4174</td>
<td>575.5357</td>
</tr>
<tr>
<td>17</td>
<td>150</td>
<td>40</td>
<td>626.30</td>
<td>342.5084</td>
<td>567.1274</td>
</tr>
<tr>
<td>18</td>
<td>150</td>
<td>40</td>
<td>626.30</td>
<td>329.0359</td>
<td>596.0101</td>
</tr>
</tbody>
</table>

Table 17. Normalized stress intensity factors for model N

<table>
<thead>
<tr>
<th>Specimen</th>
<th>T/W</th>
<th>$K_I/\sigma$</th>
<th>$K_{II}/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>2.0</td>
<td>0.7658</td>
<td>0.6760</td>
</tr>
<tr>
<td>N2</td>
<td>1.0</td>
<td>0.7141</td>
<td>0.7427</td>
</tr>
<tr>
<td>N3</td>
<td>0.5</td>
<td>0.5586</td>
<td>0.9708</td>
</tr>
</tbody>
</table>
Fig. 26. Plot of the average stress intensity factors for specimen N1

\[
X--X \quad K_I \quad Y = 5.5935 + 0.7658X
\]

\[
+---+ \quad K_{II} \quad Y = 26.7628 + 0.6760X
\]

--- 95% Confidence Band
Fig. 27. Plot of the average stress intensity factors for specimen N2.
Fig. 28. Plot of the average stress intensity factors for specimen N3

\[ K_I \quad Y = 13.9358 + 0.5585 \times X \]
\[ K_{II} \quad Y = 21.7632 + 0.9708 \times X \]

--- 95% confidence band
Fig. 29. The original and the regenerated fringes for model N
b) Specimen N2
   case no. 28
   Table 15

Fig. 29. (Continued)
c) Specimen N3
  case no. 1
  Table 16

Fig. 29. (Continued)
CHAPTER VI.
SUMMARY AND CONCLUSIONS

The experimental investigation undertaken used the method of photoelasticity and digital image analysis techniques to obtain Mode I and II stress intensity factors in plates with $90^\circ$ reentrant corners. A numerical procedure was developed to solve for stress intensity factors using the data obtained from the method of photoelasticity. To utilize the advantage of "whole-field" photoelasticity the overdeterministic least square method of Sanford [18] combined with Newton-Raphson iterative method was used to solve for $K_I$ and $K_{II}$ from "N-K" relationship, equation (28). Using this method more data points than the two needed for solving equation (28) were used, and therefore, random experimental errors in measurements of $r$ and $\theta$ were minimized.

Three computer programs were developed to collect data, to obtain stress intensity factors, and to generate the fringe patterns. Once the photoelastic data ($r$, $\theta$, and $N$) for number of points (usually more than 20) were obtained, they were used in a second program "SIFACT" to generate a set of overdeterministic equations. These equations were solved to obtain stress intensity factors. With a set of $K_I$ and $K_{II}$ values in hand, they were used in a third program "BACKPL" to regenerate and plot the fringe pattern for the set. If the regenerated fringes (superimposed on the actual fringes) matched or came close in matching the original fringes, the set of stress intensity factors were taken as
the accepted estimates of $K_I$ and $K_{II}$ for the stress field. Therefore, the accuracy of the collected data and $K_I$ and $K_{II}$ were checked by comparing the reconstructed fringe patterns with the experimentally obtained ones.

Seven specimens made from three different models (M, N, and W as shown in Figs. 10 and 11) were used to investigate the mixed mode stress intensity factors in reentrant corners. These specimens were all machined from a single 20 X 10 X 0.25 in. plate of PSM-1.

The normalized stress intensity factors for the three models are presented in Tables 9, 13 and 17. These stress intensity factors are the slopes of lines obtained from plots of average stress intensity factor versus axial stress. The V-notch model was chosen as a means of checking the whole procedure of obtaining stress intensity factors in this study. The model is under Mode I loading and theoretical and numerical values of $K_I$ have been obtained by different investigators [42, 48]. The "goodness of match" between the regenerated fringes and the original fringes of model W, as seen in Figs. 21 and 22, and the fact that the experimental values of the normalized $K_I$ for this model is within 2% of the theoretical value (see Table 9) are indications that the stress intensity factor extraction is correct. The normalized stress intensity factors obtained from experiments performed on models M and N follow the same trend as two different investigators [41, 42]. It was concluded that the Mode I and II stress intensity factors obtained in the study were indeed representative stress intensity factors for such re-entrant corners in models W, M, and N.
ACKNOWLEDGMENTS

I would like to express my sincere appreciation and thanks to Dr. Loren V. Zachary, my major professor for his advice, guidance and constant support throughout the course of my graduate study. Working under his supervision has been a joyful learning experience.

Sincere thanks are extended to Professors O. Buck, F. M. Graham, L. C. Peters and W. F. Riley for their advice and service as committee members.

I wish to thank Mr. Thomas J. Elliot for his technical assistance and his preparation of the specimens in such a short time.

Most of all, if it was not for my family's sacrifice and immeasurable support none of this was possible. So, I would like to thank all of my family, particularly my parents, my wife Fatemeh Aslroosta, and my children Ardalan and Yalda.

This dissertation is dedicated to my late grandfather, Mr. Saifallah Khan Mahinfalah.
REFERENCES


APPENDIX: PROGRAMS

Data Collection Program MODATA
This program collects photoelastic data for a reentrant corner which will be used to calculate the stress intensity factors for the corner.

```
COMMON /EYECON/ VIDEO(4), PICTUR, GRAPIC, CURSOR, RED, BLUE, GREEN, ALUAE, ALUDE, SHIFT, STAT, RAM(8)

COMMON /ALU/ ADD, ADDC, SUB, SUBC, AIN, ADEC, A, B, AINV, BINV, AND, OR, ORINV, XOR, XNOR, DICA, DICB, CLEAR, SET, OFLOW, FLAG

INTEGER ICOR(50, 2), ICORX(50), ICORY(50), PHD, XYCOR
REALA4 PH0T0(50, 3)
LOGICAL AIN, JNAME(16)
LOGICAL LNAME(16)
TYPEA, 'PROGRAM MODATA EYECON III'

TYPEA,''
C Start collecting data.

TYPEA,''
TYPEA,'At how many points do you want to collect data?...<50'
READ(5, A) N
K=0
KN=50
200  K=K+1
CALL DATCOL(N, K, PH0T0, ICORX, ICORY, KN)
TYPEA, 'PHOTO(1, 2)="', PHOTO(1, 2)
TYPEA, 'PHOTO(1, 3)="', PHOTO(1, 3)
TYPEA,''
```
TYPEA, 'Under what name do you want to store the data?'
READ(5,550) (LNAME(L), L=1,14)
OPEN(UNIT=2,NANE=LNAME,TYPB='NEW')
DO 10 I=1,N
  10 IF (PHOTO(I,2) .LT. 0.0) PHOTO(I,3)=0.0
C
  DUMMY=0.0
  DUMMY=DUMMY+1.0
  WRITE(3,250)N,DUMMY
  WRITE(3,250)N,DUMMY
250 FORMAT(14,ES1.5)
C
  DO 20 I=1,N
  20 WRITE(2,300) (PHOTO(I,II),II=1,3)
300 FORMAT(3F15.5)
C
  CLOSE(UNIT=2)
  WRITE(7,400)NAME
  WRITE(7,400)NAME
400 FORMAT(/,10X,' Data is stored in file =',14A1,//)
C
C Store the x and y coordinates of the data points in a file
C
17 TYPEA, 'What will be the name of the data file containing'
17 TYPEA, 'x and y coordinates of the data points?'
READ(5,550) (JNAME(I), I=1,14)
OPEN(UNIT=3,NANE=JNAME,TYPB='NEW',ERR=17)
DO 22 KJ=1,N
  ICOR(KJ,1)=ICORX(KJ)
  ICOR(KJ,2)=ICORY(KJ)
22 CONTINUE
DO 25 JJ=1,N
  ICOR(JJ)=ICOR(JJ)
25 CONTINUE
CLOSE(UNIT=3)
350 FORMAT(2I4)
C
26 TYPEA,'Do you want to collect another set of data? <Y> or <N>.'
READ(5,500,IER=26)ANS
IF (ANS .EQ. 'Y') GO TO 200
500 FORMAT(a)
550 FORMAT(14A1)
STOP
END

SUBROUTINE DATCOL(N,K,AA,ICORX,ICORY,KN):

This subroutine calculates the photoelastic data, fringe order, polar angle and the radial distance of N number of selected points.

COMMON /BYICON/ V1DB0(4),RICTU,8TABL,CUBSB0,RBD,BLUB,EBBN,
ALUAr,ALUB8,SHIfT,STAT,BANCS

COMMON /ALU/ ABD,ABBC,8UB,SUBC,AINC,ABBC,A,P,AINVI,DINVt,AND,
+ OB,ORBINT,XOB,XNOB,BICA,BICB,CLEAB,SET,DELOU,FLAG

COMMON /8B0M/XC,YC,XBN,YBN,XDV,YDV
COMMON /COR/IXTIP,ITYIP,IYDh,IXDN,IYDN,IXDv,IXDv
COMMON /H1SL/PI
INTEGERA2 ICORX(50),ICORY(50)
REALA4 AA(KN,3)
LOGICALA5,KNAME(16)

PI=4.0AATAN(1.0)
IF (K.GT.1) GO TO 50
TYPEA,'
Calculating the conversion factor from pixel to inches.

What is the actual distance between the two grid points?

READ(5,*) GRL
CALL SETUP
CALL DISPLAY (VIDEO) ! Show the live image from video camera.

Show the corner and the grid

Pause
CALL ACCUM
CALL DISPLAY (PICTUR)
CALL ERASE
CALL DISPLAY (GRAFIC)
CALL DISPLAY (CURSOR)

Type,''

Point the cursor to one of the grid points

Point the cursor to the other grid point and

Type ......<Return>......

Pause
CALL COORDS(I1, I2)
G1X = FLOAT(I1)
G1Y = FLOAT(I2)

Point the cursor to the other grid point and

Type ......<Return>....

Pause
CALL COORDS(J1, J2)
G2X = FLOAT(J1)
G2Y = FLOAT(J2)

C

PIXD = SQRT((GIX-G2X)AA2+(G1Y-G2Y)AA2)

Distance between the two grid points in pixel=',PIXD

FAC=GRL/PIXD! LENGTH SCALING FACTOR (IN/PIXEL).

C

Calculate the direction of the corner with the horizontal axis.

Type,''


POINT THE CURSOR TO THE CORNER TIP AND HIT <RETURN>.
PAUSE
CALL COORDS(IXC, IYC)
IXTIP=IXC
IYTIP=IYC
TYPEA,''
TYPEA,'CORNER TIP COORDINATES=', IXTIP, IYTIP
PAUSE
CALL COORDS(IXD, IYD)
IXDH=IXD
IYDH=IYD
TYPEA,'POINT THE CURSOR AT A POINT ON THE 'HORIZONTAL' FREE
BOUNDARY (LEFT BOUNDARY) OF THE CORNER AND HIT ...<RETURN>.'
PAUSE
CALL COORDS(JXD, JYD)
IXDV=JXD
IYDV=JYD
XC=FLOAT(IXTIP)
YC=FLOAT(IYTIP)
XDH=FLOAT(IXDH)
YDH=FLOAT(IYDH)
XDV=FLOAT(IXDV)
YDV=FLOAT(IYDV)

C
TGAMA=(YC-YDH)/(XC-XDH)
GAMA=ATAN(TGAMA)! GAMA IS POSITIVE WHEN CLOCK WISE.
651 TYPEA,'Type the name of a new file to store coordinates
of the corner tip and its direction w.r.t horizontal'
READ(5,550)(KNAME(K), K=1,14)
550 FORMAT(14A1)
OPEN(UNIT=4, NAME=KNAME, TYPE='NEW', ERR=651)
WRITE(4,100) IXTIP, IYTIP, IXDH, IYDH, IXDV, IYDV,
C SUBROUTINE POLAR calculates N, theta and r for a 90 degree corner.

SUBROUTINE POLAR(N, AA, GAM, FAC, KN, ICORX, ICORY)

COMMON /EYECOM/ VIDEO(4), PICTUR, GRIFIC, CURSOR, RED, BLUE, GREEN,
  + ALUAF, ALUDF, SHIFT, STAT, RAM, (8)

COMMON /ALU/ ADD, ADDC, SUB, SUBC, AINC, ADEC, A, D, AINV, DINVI, AND,
  + OR, ORINV, XOR, XNOR, DICA, DICB, CLEAR, SET, OFLOW, FLAG

COMMON /GEOM/XC, YC, XDH, YDH, XDV, YDV
COMMON /COR/IXTIP, IYTIP, IXTIP, IYDH, IXDV, IYDV
COMMON /MISL/PI
REAL AA(KN,3)
INTEGER IXI, IYI, ICORX(50), ICORY(50)
LOGICAL AMS
CALL DISPLAY (GRIFIC)

C Show the horizontal and vertical 'free boundary' of the corner
CALL SKIP(IXTIP, IYTIP)
CALL DRAW(IXHTIP, IYDH)
CALL SKIP(IXTIP, IYTIP)
CALL DRAW(IXDV, IYDV)

50 TYPEA,""
*START COLLECTING DATA*****

DO 200 K=1,N
IP=K
TYPEA,'DATA POINT 4',IP
TYPEA,'('
TYPEA,'MOVE THE CURSOR TO A POINT ON A FRINGE AND
1 TYPE ......<RETURN>.'
PAUSE
CALL COORDS(IX1, IY1)
C SEARCH AN AREA OF 5 BY 5 PIXEL AND FIND THE DARKEST POINT
IZMIN=256
DO 55 II=IX1-2,IX1+2
  DO 55 JJ=IY1-2,IY1+2
    IZ=INTENS(IJ)
    IF (IZ.GE. IZMIN) GO TO 55
    IZMIN=IZ
    IXMIN=II
    IYMIN=JJ
55 CONTINUE
ICORX(IP)=IXMIN
ICORY(IP)=IYMIN
C RECORD THE DATA POINT ON THE SCREEN.
C
CALL SKIP(IXMIN, IYMIN-3)
CALL DRAW(IXMIN, IYMIN+3)
CALL SKIP(IXMIN-3, IYMIN)
CALL DRAW(IXMIN+3, IYMIN)
X1=FLOAT(IXMIN)
Y1=FLOAT(IYMIN)
R=SQR((X1-XC)**2+(Y1-YC)**2)
AA(IP,2)=R*AARC1 RADIAL DISTANCE IN INCHES.
TYPEA,'R=',AA(IP,2)
TYPEA,'.'
TVPEA,'INPUT THE FRINGE ORDER AT THE POINT.'
READ(5,A) AA(IP,3)
TVPEA,'ER0 = ',AA(IP,3)
C CALCULATING THE ANGLE THETA FOR THE POINT w.r.t. THE AXIS OF CORNER
IF (Y1.LE.YC .AND. X1.GT.XC) GOTO 7
IF (Y1.GT.YC .AND. X1.GT.XC) GOTO 17
IF (X1.LT.XC .AND. Y1.GT.YC) GOTO 37
IF (X1.LT.XC .AND. Y1.LE.YC .AND. GAN.GT.0.0) GOTO 47
IF (X1.LE.XC .AND. Y1.LT.YC .AND. GAN.LT.0.0) GOTO 57
C AAAAAAAAAA THETA IN REGION #1 AAAAAAAAAAA
7  TB1=(YC-Y1)/(X1-XC)
   DD=ATAN(TB1)
   THETA=PI/4.0+DD+GAN
   GO TO 77
C AAAAAAAAAA THETA IN REGION #2 AAAAAAAAAAA
17  TB2=(Y1-YC)/(X1-XC)
    DD=ATAN(TB2)
    THETA=PI/4.0-DD+GAN
    GO TO 77
27  THETA=-PI/4.0+GAN
    GO TO 77
C AAAAAAAAAA THETA IN REGION #3 AAAAAAAAAAA
37  TB3=(XC-X1)/(Y1-YC)
    DD=ATAN(TB3)
    THETA=GAN-(PI/4.0)+DD
    GO TO 77
C AAAAAAAAAA THETA IN REGION #4 AAAAAAAAA
C THIS CASE WILL EXIST ONLY IF GANA IS POSITIVE (C.W)
47  TB4=(YC-Y1)/(XC-X1)
    DD=ATAN(TB4)
    THETA=GAN-(0.75*PI+DD)
    GO TO 77
C THIS CASE WILL EXIST ONLY IF GAMMA IS NEGATIVE (C.C.W)

57  GAM=-GAM
    TB4=(XC-X1)/(YC-Y1)
    DD=ATAN(TB4)
    THEIA=(0.75*PI+DD)-GAM
77  THEIA=THEIAA(180./PI)
    TYPEA,'THEIA=',THEIA
    AA(IP,1)=THEIA
200 CONTINUE
    TYPEA,'
    TYPEA,'DATA COLLECTION IS COMPLETED.'
66  TYPEA,'DO YOU WANT TO CHANGE THE DATA POINTS..?'
    TYPEA,'AAAA <Y> OR <N>...AAAA'
    READ(S,150)ANS
    IF (ANS.EQ.'Y') GO TO 50
    IF (ANS.NE.'Y' .AND. ANS.NE.'N') GO TO 66
150 FORMAT(A)
    RETURN
    END
Data Analysis Program SIFACT
DATA ANALYSIS PROGRAM SIFACT (STRESS INTENSITY FACTORS)

This program uses the photoelastic data collected by program NODATA, and utilizes iterative least squares technique to calculate stress intensity factors for a corner.

COMMON /VAR/QE, DGWK1, DGWK2
COMMON /CONST/PI, LP, LM, ALF, ER, MIT, TH, FSIG
COMMON /SIF/GK1, GK2, FK1, FK2
LOGICAL ANS, LNAME(16), KNAME(16)
REAL A4, RA4(50), DG(50,2), FK(2), GK(2), X(2)
REAL A4, RA4(50), EN(50), IEITA(50)
REAL A4, EK1, EK2, ER, TH, FSIG, LP, LM, ALF, PI
INTEGER MIT

PI=4.0*ATAN(1.0)

TYPEA,'Input model thickness & material fringe value'
READ(5,A)TH, FSIG

TYPEA,'What are the values of the allowable error and'
TYPEA,'maximum number of iterations.
READ(5,A)ER, MIT

TYPEA,'Enter the initial values of K1 and K2'
READ(5,A)GK1, GK2

PI=4.0*ATAN(1.0)

IFK=0

TYPEA,'Which data file do you want to use to obtain SIFS?'
READ(5,550)(LNAME(L), L=1,14)
OPEN(UNIT=2, NAME=LNAME, TYPE='OLD')

IFK=IFK+1
JJI(IFK)=JJ
C

TYPEA,

READ(2,AN,DUMMY
DO 120 I=1,N
READ(2,A)IBEA(I),RA(I),FN(I)
IBEA(I)=IBEA(I)/180.0
TYPEA,*THETA,RA,IN=*,IBEA(I),RA(I),FN(I)
120 CONTINUE
CLOSE(UNIT=2)
FK1=0.0
FK2=0.0

C Call the subroutine CORSIF to calculate SIF
C
CALL CORSIF(N,RA,IN,THETA)
WRITE(7,10)FK1,FK2.
10 FORMAT('K1=',F16.8,'K2=',F16.8)

C Store the data in a file for back plotting
C
TYPEA,'WHAT IS THE FILE NAME FOR STORING SIFS?
READ(5,550)(KHAME(K),K=1,14)
OPEN(UNIT=8,NAM=KHAME,TYPE='NEW')
WRITE(8,30)FK1,FK2,TH,FSIG
30 FORMAT(4F16.8)
CLOSE(UNIT=8)
550 FORMAT(14A1)
999 STOP
END
Subroutine CORSIF accepts the values of theta, fringe order A C and radial distance of N number of points and outputs the A C stress intensity factors.

SUBROUTINE CORSIF(N,RA,PN,TETA)
COMMON /VAR/GE,BGWI,BGWK2
COMMON /CONST/PI,LP,LM,ALP,ER,NI,TH,FSIQ
COMMON /SIF/GK1,GK2,FK1,FK2
REALA4 RA(N),PN(N),TETA(N)
REALA4 C(2,2),A(50,2),W(2,50),CS(2,2),DBB(2),D(2)
REALA4 G(50),BG(50,2),FK(2),GK(2),X(2)
REALA4 GI(50),DG1(50),DG2(50)
REALA4 FK1,FK2,ER,IN,FSIQ,LP,LM,ALP,PI

C

13 B1=0
WRITE(7,15)N
15 FORMAT(2 ITERATION NO. = ',I3)
GK1=GK1+FK1
GK2=GK2+FK2

DO 25 I=1,N
R=RA(I)
PN=PN(I)
TET=TETA(I)

CALL GandBG(R,PN,TET,GK1,GK2)
G1(I)=GE
DG1(I)=DGWI
DG2(I)=DGWK2
E1=E1+G1(I)*AQ1(I)
25 CONTINUE
DO 27 KJ=1,N
Q(KJ)=G1(KJ)
DG(KJ,1)=DG1(KJ)
DG(KJ,2)=DG2(KJ)
27 CONTINUE
C Generate transpose of [DG]
DO 35 I=1,2
DO 35 J=1,N
W(I,J)=DG(J,I)
35 CONTINUE
C
C Generate a square matrix
C
DO 40 I=1,2
DO 40 J=1,2
C(I,J)=0.0
DO 40 K=1,N
40 C(I,J)=C(I,J)+DG(K,I)*DG(K,J)
C
C Calculations for \([V]^{-1}=[D]\)
C
DO 45 I=1,2
D(I)=0.0
DO 45 K=1,N
D(I)=D(I)+W(I,K)*DG(K)
45 CONTINUE
C
C Change matrices as \([D]=[DSS], [C]=[C] \) and \([CS]=[C]\)
C
DO 55 I=1,2
DSS(I)=D(I)
DO 55 J=1,2
C(I,J)=C(I,J)
CS(I,J)=C(I,J)
55 CONTINUE
CALL GAUSS(C,D,X,2,IERROR)
TYPE*,''
TYPE*,ERROR =ERROR
TYPE*,X(1)=X(1)
TYPE*,X(2)=X(2)

CALL RESCOR(CS,DSS,X,2,20)

DO 65 K=1,2
FK(K)=GK(K)+X(K)

65 CONTINUE
GK1=0.0
GK2=0.0
FK1=EK(1)
EK2=EK(2)
WRITE(7,130)FK1,FK2
130 FORMAT('ESTIMATE OF K1 =F16.8,ESTIMATE OF K2 =F16.8')
      X1=ABS(X(1))
      X2=ABS(X(2))
C Check to make sure that delta K1 and K2 are small enough

IF (X1.LE.0 .AND. X2.LE.0) GO TO 590
M=M+1
TYPOA,'MIT=',MIT
IF (M.LE.MIT) GO TO 13
TYPOA,'No convergence within the specified NO. of iterations'
590 WRITE(7,A),'RELATIVE ERROR=',E)
RETURN
END

C THIS SUBROUTINE CALCULATES FUNCTION G AND ITS DERIVATIVES A
C W.R.T. K1 AND K2
SUBROUTINE GabDB(B,PN,ME)
COMMON /DATA/B,PN,ME,PK1,PK2
COMMON /VAR/EB,EBK1,EBK2
COMMON /CONST/PI,LP,LN,ALF,ER,MIT,IN,FSIG
COMMON /SIE/EB,EBK1,EBK2
REAL4 LP,LN,LP1,LN1,LP2,LN2
REAL4 ER,IN,FSIG,ALF,PI,FK1,FK2

C Calculation of constants associated with K1 squared,
C K2 squared and K1 X K2. These constants are calculated for
C a 90 degree re-entrant corner and for any other angled-corner
C the values of LP, LN and ALF must be replaced with the proper
C values.
C
C Where
C LP = Lambda plus, LN = Lambda minus and
C ALF = Angle alfa measured from the bisector of the notch angle.
LP = 0.54448
LM = 0.90852
ALF = 2.0API/4.0
LP1 = LP - 1
LP2 = LP + 1
LM1 = LM - 1
LM2 = LM + 1
SLP1 = SIN(LP1ALF)
SLP2 = SIN(LP2ALF)
SLM1 = SIN(LM1ALF)
SLM2 = SIN(LM2ALF)

C
BET1 = -(LP1ASLP1)/(LP2ASLP2)
BET2 = -(SLM1)/(SLM2)

C
OP = 1.0/((SQRT(2API))A(1+BET1)ALP2)
QH = 1.0/((SQRT(2API))A(LM1+BET2ALM2))

C
C Constants associated with K1 squared
C
A11 = 4.0ALP1ALP1
A12 = BET1ABET1ALP2ALP2
A13 = 4.0ABET1ALP1ALP2

C
C Constants associated with K2 squared
C
A21 = 4.0ALM1ALM1
A22 = BET2ABET2ALM2ALM2
A23 = 4.0ABET2ALM1ALM2

C
C Constants associated with K1 x K2
C
B1 = LP + LM - 2
B2 = LP - LM
B3 = LP + LM
B4=LP-LH-2
B5=LP-LH+2
B6=LP+LH+2
AN1=4.0ALP1ALM1
AN2=(5.0/2.0)ALP1ALM2
AN3=ALP2ALP1ALM2
AN4=ALP1ALP2ALM1
AN5=(3.0/5.0)ALM2

C Calculate the functions which K1 squared is multiplied by
C
F11=1.0+3.0A((SIN(LP2ATBT))AA2)
F12=2.0A((COS(TET))AA2)
F13=(SIN(TET))AA2
F14=(COS(LP2ATBT))AA2
H11=A11+A12AF11+A12A(F12-F13-F14)

C Calculate the functions which K2 squared is multiplied by
C
F21=1.0+3A((COS(LP2ATBT))AA2)
F22=(SIN(LP2ATBT))AA2
H22=A21+A22AF21+A23A(F12-F13-F22)

C Calculate the functions which K1 X K2 is multiplied by
C
F31=(AN1+AN2)A(SIN(B2ATBT))
F32=(AN3+AN4)A(SIN(B3ATET))
F33=3.0AAN3A(SIN(B4ATET))
F34=3.0AAN4A(SIN(B5ATET))
F35=AN5A(SIN(B6ATET))
H33=F31+F32+F33+F34+F35

C
C Calculate the function GE

GE1 = (PMAESIG/UI) AA2.O
PR1 = 2.0ALP1
PR2 = 2.0ALM1
GE2 = G1AGK1AQPAQA(QAAPR1)
GE3 = G2AGK2AGMAQA(EAAPR2)
GE4 = 2.0APAQAAGK1AGK2A(AAAD1)
GE = GE2AH11 + GE3AN22 + GE4AH33 - GE

C Calculate the partials of GE w.r.t K1 and K2

DGE2 = (2.0AGE2)/GK1
DGE4A = GE4/GK1
DGUK1 = DGE2AH11 + DGE4AH33

DGE3 = (2.0AGE3)/GK2
DGE4B = GE4/GK2
DGUK2 = DGE3AN22 + DGE4BAH33
RETURN
END

C THIS LIBRARY CONTAINS A FEW OF DR. NISKIOGLU'S
C SUBROUTINES.

SUBROUTINE GAUS (A, B, X, N, IERROR)
REAL A(N,N), B(N), X(N)
MM1 = N - 1
DO 5 I = 1, MM1

DO 3 J = I, N
IF (A(J,I) .EQ. 0.0) GO TO 3
DO 2 K = I, N
TEMP = A(I,K)

5 CONTINUE
3 CONTINUE
2 CONTINUE
1 CONTINUE
A(I,K) = A(J,K)
A(J,K) = TEMP
TEMP = B(I)
B(I) = B(J)
B(J) = TEMP
GO TO 4
3
CONTINUE
GO TO 8
C
C
IP1 = I + 1
DO 5 K = IP1, N
Q = -A(K,I)/A(I,I)
A(K,I) = 0.0
B(K) = Q*A(K,I) + B(K)
DO 5 J = IP1, N
A(K,J) = Q*A(K,J) + A(K,J)
IF (A(N,N) .EQ. 0.0) GO TO 8
C
C
X(N) = B(N)/A(N,N)
NP1 = N + 1
DO 7 K = 1, NM1
Q = 0.0
NHK = N - K
DO 6 J = 1, K
Q = Q + A(NHK, NP1 - J)*X(NP1 - J)
6
X(NHK) = (B(NHK) - Q)/A(NHK, NHK)
7
ERROR = 1
RETURN
8
ERROR = 2
RETURN
END
SUBROUTINE RESCOR(A,B,XC,N,M)
REAL A8 RES(20)
REAL A(N,N),SA(20,20),B(N),XC(N),SRES(20),E(20)
C
C
DO 1 I=1,N
DO 1 J=1,N
1 SA(I,J)=A(I,J)
CONTINUE
DO 2 I=1,N
DO 2 J=1,N
2 A(I,J)=SA(I,J)
DO 3 I=1,N
DO 3 J=1,N
3 A(I,J)=SA(I,J)
DO 4 I=1,N
RES(I)=B(I)
DO 4 J=1,N
4 RES(I)=RES(I)+A(I,J)*XC(J)
DO 5 I=1,N
5 SRES(I)=RES(I)
CALL GAUS(A,SRES,E,N,IERRO)
DO 6 I=1,N
6 XC(I)=XC(I)-E(I)
C
M=M-1
IF(M.GT.0) GO TO 2
RETURN
END
Back Plot Program BACKPL
PROGRAM BACKPL (BACK PLOT)

COMMON /EYECOM/ VIDEO(4), PICTUR, GRAPIC, CURSOR, RED, BLUE, GREEN, + ALUA, ALUBG, SHIFT, STAT, RAM(8)

COMMON /COR/ IXC, IYC, IXDH, IYDH, IXDV, IYDV
COMMON /RCOR/ XC, YC, XD, YD, XDV, YDV
COMMON /GEOH/ GRL, PIXD, FAC, GAMA, TH, FSIG, PI
COMMON /DAT/ FK1, FK2
LOGICAL A1 INAME(16), KNAME(16), JNAME(16)
LOGICAL A1 LNAME(16), ANS
REAL A4 TET
REAL A4 TETA(50), R(50), NF(50)

CALL ERASE
CALL DISPLAY (GRAPIC)
PI = 4.0 / ATAN(1.0)

C RECALL THE INFORMATION ON CORNER TIP, ITS DIRECTION AND SCALING FACTOR

10 TYPE*, 'ENTER THE FILE NAME IN WHICH YOU HAVE STORED
  THE COORDINATES OF THE CORNER TIP AND etc.'
READ(S, 105)(INAME(I), I = 1, 14)
OPEN(UNIT=4, NAME=INAME, TYPE= 'OLD', ERR=10)
READ(4, A) IXC, IYC, IXDH, IYDH, IXDV, IYDV
READ(4, A) XC, YC, XD, YD, XDV, YDV, GAMA
READ(4, A) GRL, PIXD, FAC
CLOSE(UNIT=4)

C RECALL THE VALUES OF K1, K2, MODEL THICKNESS AND MATERIAL
C FRINGE CONSTANT

20 TYPE*, 'ENTER THE FILE NAME CONTAINING SIF RESULTS'
  TYPE*
READ(S, 105)(KNAME(K), K = 1, 14)
OPEN(UNIT=8, NAME=KNAME, TYPE= 'OLD', ERR=20)
READ(8,*)FK1,FK2,TH,FSIG
CLOSE(UNIT=8)

C PROCEDURE FOR DRAWING THE CORNER
C
CALL SKIP(IXC,IYC)
CALL DRAW(IXDH,IYDH)
CALL SKIP(IXC,IYC)
CALL DRAW(IXDV,IYDV)

C PROCEDURE TO RECOVER THE INFORMATION OBTAINED BY DATA
C COLLECTION PROGRAM "HODATA". R, THETA, FRINGE ORDER
C
30 TYPEA
TYPEA,'WHAT IS THE FILE NAME ON WHICH YOU HAVE STORED
1 THE PHOTOELASTIC DATA?'
READ(5,105)(LNAME(L), L=1,14)
OPEN(UNIT=2,NANE=LNAME,TYPE='OLD',ERR=30)
READ(2,A,N,DUMMY)
DO 100 I=1,N
READ(2,A)TETA(I),R(I),NF(I)
100 CONTINUE

40 TYPEA,'WHAT IS THE FILE NAME ON WHICH YOU HAVE STORED'
TYPEA,'THE X AND Y COORDINATES OF THE DATA POINTS USED'
TYPEA,'TO OBTAIN K1 AND K2?'
TYPEA
READ(5,105)(JNAME(J), J=1,14)
OPEN(UNIT=3,NANE=JNAME,TYPE='OLD',ERR=40)
DO 200 I=1,N
READ(3,A)IXP,IYP
CALL SKIP(IXP,IYP-3)
CALL DRAW(IXP,IYP+3)
CALL SKIP(IXP-3,IYP)
CALL DRAW(IXP+3,IYP)
200 CONTINUE
C DRAW A WINDOW TO LIMIT THE BACK PLOTTING REGION

C

EBOX=.25/EAC
IX1=IXC+INT(EBOX+.5)
IX2=IXC-INT(EBOX+.5)
IY1=IYC+INT(EBOX+.5)
IY2=IYC-INT(EBOX+.5)
CALL SKIP(IX1,IY1)
CALL DRAW(IX2,IY1)
CALL SKIP(IX2,IY1)
CALL DRAW(IX1,IY2)
CALL SKIP(IX2,IY2)
CALL DRAW(IX1,IY2)
CALL DRAW(IX1,IY1)

C ROUTINE FOR BACK PLOTTING

C

MN=1
50 TYPEA
   TYPEA,'ENTER THE FRINGE ORDER YOU WANT TO PLOT'
   READ(5,A)ENU
   MN=MN+1

C CALCULATE THE COEFFICIENTS OF THE FUNCTION G(r)

C

D=(ENUSIG/IN)AA2.0
C IF(MN.GT.1) GO TO 51
00 60 I=-135,135,3
   IET=IAP(PI/100.0)
   CALL COEFIC(IET,A,B,C,LP,LM)
C
C FIND THE ROOT(S) R
C
RT=FNROOT(D,GR,2.,0.,.00001)
IF (RT.GE.2.9995) GO TO 60
C
C FOR EACH ROOT CALL PLOT
C
CALL PLOT(RT,TBT)
CONTINUE
60 CONTINUE
65 TYPE*, 'DO YOU WANT TO PLOT THE FRINGE LOOP FOR
& ANOTHER FRINGE NUMBER?
READ(5,67)ANS
67 FORMAT(A)
IF(ANS.EQ.'Y') GO TO 50
IF(ANS.NE.'Y'.AND.ANS.NE.'N') GO TO 65
RETURN
END

C FUNCTION GR(R)
REAL FUNCTION GR(R)
COMMON /COEF/LP,LM,A,B,C
REAL LP,LM
PA=2.0A(LP-1)
PB=2.0A(LM-1)
PC=LP+LM-2.0
GR=AA(RTAPA)+BA(RTAPB)+CA(RTAPC)
RETURN
END

C SUBROUTINE COEF(A,B,C,LP,LM)
C THIS SUBROUTINE CALCULATES COEFFICIENTS A, B, AND C
C OF THE BACK PLOTTING FUNCTION
C
C
COMMON /EYECOM/ VIDEO(4),PICTUR,GRAFIC,CURSOR,RED,BLUE, GREEN,
+ ALUAE,ALUBG,SHIFT,STAT,RAM(8)
Calculation of constants associated with $K_1$ squared.

These constants are calculated for a 90 degree re-entrant corner and for any other angled corner, the values of $L_P$, $L_M$ and $A_L$ must be replaced with the proper values.

Where

$L_P = \text{Lambda plus}$, $L_M = \text{Lambda minus}$ and

$A_L = \text{Angle alpha measured from the bisector of the notch angle.}$

**LP** = 0.54448  
**LM** = 0.90852  
**ALE** = $3.0 \times \pi / 4.0$

**LP1** = $L_P - 1$
**LP2** = $L_P * 1$
**LM1** = $L_M - 1$
**LM2** = $L_M * 1$

**SLP1** = $\sin(L_{P1} A_{L})$
**SLP2** = $\sin(L_P A_{L})$
**SLM1** = $\sin(L_{M1} A_{L})$
**SLM2** = $\sin(L_{M2} A_{L})$

**BET1** = $-(L_{P1} A_{L} SLP1) / (L_{P2} A_{L} SLP2)$
**BET2** = $-(SLM1) / (SLM2)$

**QP** = $1.0 / (\sqrt{2} \pi A_{L} (1 + BET1) A_{L} P2)$
**QM** = $1.0 / (\sqrt{2} \pi A_{L} (L_{M1} + BET2 A_{L} M2))$

**Constants associated with $K_1$ squared**

**A11** = $L_P ALP1$
**A12** = $BET1 A L_{P1} A L_{P2} A L_{P2}$
**A13** = $2.0 A B E T 1 A L P 1 A L P 2$
C Constants associated with K2 squared
C
A21=LM1ALM1
A22=BET1BET2ALM2
A23=2.0BET2ALM1ALM2
C Constants associated with K1 X K2
C
B1=LP-LM
B2=LP-LM+2.0
B3=LP-LM-2.0
B4=LP+LM-2.0
AM1=LP1ALM1
AM2=BET1BET2ALP2ALM2
AM3=BET2ALP1ALM2
AM4=BET1ALP2ALM1
C
C Calculate the functions which K1 squared is multiplied by
C
H11=A11+A12+A13A(C0S(2ATET))
C
C Calculate the functions which K2 squared is multiplied by
C
H22=A21+A22+A23A(C0S(2ATET))
C
C Calculate the functions which K1 X K2 is multiplied by
C
F31=(AM1+AM2)A(SIN(B1ATET))
F32=(AM4)A(SIN(B2ATET))
F33=AM3A(SIN(B3ATET))
H33=F31+F32+F33
C
A=4.0A(FK1AA2)A(QPAA2)AH11
B=4.0A(FK2AA2)A(QHAA2)AH22
C=8.0AQPAAFKA1FK2AH33
RETURN
END
C PRINT THE INFORMATION OBTAINED FROM PROGRAM "SIFACT"
C
WRITE(5,300)TH,FS1F,FK1,FK2
WRITE(6,300)TH,FS1G,FK1,FK2
300 FORMAT(*/,' THICKNESS OF THE MODEL (IN) = ',F13.4,/,  
& ' MATERIAL FRINGE VALUE (Lb/ft-IN) = ',F13.4,/,  
& ' MODE I STRESS INTENSITY FACTOR, K1 = ',F13.4,/,  
& ' MODE II STRESS INTENSITY FACTOR, K2 = ',F13.4,/)  
C
C CALL SUBROUTINE ARDPLT TO GENERATE THE RACK PLOT OF THE  
C FRINGE PATTERN OBTAINED FROM K1 AND K2
C
105 FORMAT(14A1)
CALL ARDPLT(N)
STOP
END
C
SUBROUTINE ARDPLT(N)
EXTERNAL GR
LOGICAL AI ANS
C
COMMON /EYECOM/ VIDEO(4),PICTUR,GRAFIC,CURSOR,RED,BLUE,GREEN,  
& ALUAE,ALUBG,SHIFT,STAT,RAM(8)
C
COMMON /COR/IXC,IYC,IXH,IYH,IXB,IYB
COMMON /RCOR/XC,YC,XDH,YDH,XBV,YDV
COMMON /GEOM/GR,L,PIXD,FAC,GAMA,TH,FS1G,PI
COMMON /DAT/FK1,FK2
COMMON /COEF/LP,LM,A,B,C
REALA4 TET
REALA4 TETA(50),R(50),NF(50)
REALA4 LP,LM
SUBROUTINE PLOT(RT,TET)
C
COMMON /EYECOM/ VIDEO(4),PICTUR,GRAFIC,CURSOR,RED,BLUE,GREEN,
ALUAF,ALUBG,SHIFT,STAT,RAM(8)
C
COMMON /COR/IXC,IXC,IXDH,IYDH,IYDH,IYDV
COMMON /RCOR/IXC,YC,IXDH,YDH,IYDV
COMMON /GEOM/RCL,P1X0,FAC,GAMA,TH,FSIG,PI
COMMON /DAT/FK1,FK2
C
C CALCULATE ANGLE BETA MEASURED FROM HORIZONTAL AXIS OF EYECOM
C POSITIVE CCW AND NEGATIVE CW
C
BET=TET-(PI/4.0+GAMA)
RI=RT/FAC
C
X AND Y COORDINATES OF THE POINTS FROM THE CORNER TIP
C
X=RIACOS(BET)
Y=RIASIN(BET)
C
X AND Y COORDINATES OF THE POINTS BASED ON EYECOM'S AXIS
C
IXP=IXC+INT(X+0.5)
IYP=IXC+INT(Y+0.5)
C
PLOT THE POINT ON THE SCREEN FOR A GIVEN THETA AND FRINGE ORDER
C
CALL SKIP(IXP,IYP)
CALL DRAW(IXP,IYP)
RETURN
END