CRACK MAPPING BY RAY METHODS

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ABSTRACT

Two methods to map crack edges by the use of arrival times of edge-diffracted signals have been briefly reviewed. They are a global triangulation method and a local crack-edge mapping technique. The local mapping technique, which generally has the greater accuracy, has been applied to both synthetic and experimental data. The effect of a uniform error in the data has been investigated. The experimental data were obtained at the Rockwell International Science Center for a doorknob specimen with a plane crack of elliptical shape at its center. Crack mappings of very satisfactory accuracy were achieved.

I. INTRODUCTION

In recent papers, [1]-[3], two analytical methods have been developed to map the edge of a crack by the use of data for diffraction of elastic waves by the crack-edge. The methods of Refs.[1]-[3] are based on elastodynamic ray theory and the geometrical theory of diffraction, and they require as input data the arrival times of diffracted ultrasonic signals. The first method maps flash points on the crack edge by a process of
triangulation with the source and receiver as given vertices of the triangle. By the use of arrival times at neighboring positions of the source and/or the receiver, the directions of signal propagation, which determine the triangle, can be computed. This inverse mapping is global in the sense that no a-priori knowledge of the location of the crack edge is necessary. The second method is a local edge mapping which determines planes relative to a known point close to the crack edge. Each plane contains a flash point. The envelope of the planes maps an approximation to the crack edge.

In Refs. [1]-[3] the material containing the crack was taken as a homogeneous, isotropic and linearly elastic solid. More recently, extensions to include anisotropy of the material have been given in Refs. [4]-[5].

Mathematical details and a fairly detailed error analysis can be found in Ref. [3]. References [1]-[3] also include applications of the methods to synthetic data. It is of particular interest that the local mapping technique allows for an iteration procedure whereby the result of a computation suggests an improved choice of the base point which in the subsequent iteration yields a better approximation to the crack edge.

Sections 2, 3 and 4 briefly review edge diffraction, the global triangulation method and the local crack-edge mapping technique. In Section 5 it is illustrated by a simple example that for the same accuracy of input data, the local mapping technique gives much better accuracy than the global triangulation method. An application of the local mapping technique on synthetic data is given in Section 6. Particular attention is given to the consequences of a uniform error in the input data. If the iteration procedure converges to a reasonable crack shape the uniform error will give rise to a translation of the crack and a rotation of the crack plane. The general shape of the crack will, however, be preserved. If the iterations do not converge to a reasonable shape the data should be adjusted in an adaptive manner by uniform increments or decrements until a crack shape emerges out of the iteration procedure.

As discussed in Section 7, the local crack-edge mapping has also been tested on experimental travel-time data for a simulated elliptical (yttria) crack in a spherical Ti-alloy specimen. The source transducer was placed at the North-Pole, and various configurations of receiver positions were considered. Very satisfactory crack-edge mappings were achieved by the use of the local mapping technique.
2. **Edge Diffraction**

The theory of elastodynamic edge diffraction is based on the result that two cones of diffracted rays are generated when a ray carrying a high-frequency elastic wave strikes the edge of a crack. The inner and outer cones consist of rays of longitudinal and transverse motion, respectively. For cracks in elastic solids, the three-dimensional theory of edge diffraction was discussed by Achenbach and Gautesen [6].

Figure 1 shows an incident ray of longitudinal motion and the corresponding cones of diffracted rays. The angle of the incident ray with the edge is $\phi_L$. Thus

$$\cos\phi_L = \mathbf{p} \cdot \mathbf{t} \quad (2.1)$$

where $\mathbf{p}$ defines the direction of propagation along the incident ray and $\mathbf{t}$ is a unit vector along the tangent to the edge, chosen in the direction with makes $\phi_L$ acute. The half-angles $\phi_\beta$ of the cones of diffracted rays of type $\beta (\beta = L, T)$ are given by $\phi_L$ and $\phi_T$, where

$$\cos\phi_T = \left( \frac{c_T}{c_L} \right) \cos\phi_L \quad (2.2)$$

In the direct problem, the position of the source and the receiver, which are defined by $x_S$ and $x_Q$, respectively, are known.
Also known is the location of the edge of the crack. The location of the flash point \( D \), with position vector \( \mathbf{x}_D \), must be determined. For L–L diffraction, it follows from (2.1) and (2.2) that \( \mathbf{x}_D \), satisfies the relation

\[
\mathbf{t} \cdot \left( |\mathbf{x}_D - \mathbf{x}_S| (\mathbf{x}_Q - \mathbf{x}_D) - |\mathbf{x}_Q - \mathbf{x}_D| (\mathbf{x}_D - \mathbf{x}_S) \right) = 0, \tag{2.3}
\]

where the unit vector \( \mathbf{t} \) is tangent to the edge at \( \mathbf{x} = \mathbf{x}_D \). When the vector \( \mathbf{x}_D \) has been determined, the total ray length may be computed as

\[
R = R_1 + R_L = |\mathbf{x}_D - \mathbf{x}_S| + |\mathbf{x}_Q - \mathbf{x}_D|. \tag{2.4}
\]

### 3. Global Triangulation Method

In the inverse problem, the position vectors \( \mathbf{x}_S \) and \( \mathbf{x}_Q \) are known. In addition, the time \( T(\mathbf{x}_S, \mathbf{x}_Q) \) which denotes the time span between emission at \( \mathbf{x}_S \) and reception of the diffracted signal at \( \mathbf{x}_Q \) is known. For diffraction of a longitudinal wave into a longitudinal wave we then also know

\[
R = R(\mathbf{x}_S, \mathbf{x}_Q) = c_L T(\mathbf{x}_S, \mathbf{x}_Q) \tag{3.1}
\]

where \( c_L \) is the speed of longitudinal waves. On the basis of ray theory, it then follows that \( \mathbf{x}_D \) is located on a spheroid \( E(\mathbf{x}) \), whose foci are at the points \( \mathbf{x}_S \) and \( \mathbf{x}_Q \), and whose major axis length is \( R \). To determine the actual position of the flash point, we need travel times for additional source and/or receiver positions. A large shift of the source and/or the receiver position will give rise to a quite different flash point. The shift of the flash point is, however, of second order when either or both the source and the observer are moved only slightly. This property will be used to determine the position of flash points.

For any point \( \mathbf{x} \) on the spheroid \( E(\mathbf{x}) \) we define the unit vectors

\[
\mathbf{p} = (\mathbf{x} - \mathbf{x}_S) / |\mathbf{x} - \mathbf{x}_S|, \quad \mathbf{q} = (\mathbf{x} - \mathbf{x}_Q) / |\mathbf{x} - \mathbf{x}_Q| \tag{3.2a,b}
\]

As shown in Ref.[3], a small shift of the observer in the direction of the unit vector \( \mathbf{v} \) results in the relation

\[
\mathbf{p} \cdot \left( \mathbf{v} \times \mathbf{q} \right) = 0
\]
Another shift of the observer in the direction $\mathbf{v}_1$ results in another equation of the form (3.3). Two of such equations together with the condition that $|q| = 1$ are sufficient to solve for $q$. A solution for $q$ and knowledge of the spheroid $E(x)$ provides the position of a flash point.

Similarly, the flash point can be determined by keeping the receiver fixed and moving the source in two nonparallel directions. Therefore, in terms of discrete measurements, the following combinations can be used in order to determine one flash point: (1) One source and three noncollinear receivers, (2) Two sources and two receivers, the four being noncollinear, and (3) Three non-collinear sources and one receiver.

The method of this section is a global triangulation method, i.e., it completes the triangle with vertices at the source, the receiver, and the flash point. In principle, the complete crack edge can be mapped by using data from one pair of source positions and $N$ pairs of receiver positions. Each pair of receiver positions provides a flash point. Three not necessarily adjacent flash points determine the plane of a flat crack. When we know the plane of the crack, we can determine the tangent at each flash point by the use of (2.1) or (2.3). In order that the flash points will not cluster too closely, and in order that a sizable segment of the crack edge will be mapped, the pairs of observation points should be at some distance from each other. Unfortunately, the method is rather sensitive to errors in the input data. It may, however, be expected that a single point close enough to the crack can be determined. This point can then be used as a base point for a local crack-edge mapping, which is discussed in the next section.

The computed position of a flash point will contain errors due to inaccuracies in the data and in the method of computation. The former are the inevitable result of errors in the measurement of the travel time of the first-arriving diffracted signal. The computational errors are incurred in using a finite difference approximation to the gradients $V_u(R)$ and $V_v(R)$. These errors have been illustrated in Refs. [2]-[3] by an application of the inverse method to synthetic diffraction data.

4.1 Local Crack-Edge Mapping

The global triangulation method can be used to determine a point on or near the crack edge. In this section, it is shown that such a point can subsequently be used as a base point.
Relative to the base point other points on the crack edge can be determined by a local crack-edge mapping. The local mapping is also based on travel times from source to receiver via a flash point. However, instead of using single travel times to produce single flash points, we now use a set of travel times to simultaneously produce a set of flash points. The local crack-edge mapping should have greater accuracy than the global triangulation method.

As before, we have a source \( S \) at \( x_S \) and a receiver \( Q \) at \( x_Q \).

The travel time \( \tau \) for the diffracted signal is assumed known. With respect to the base point \( B \), whose position is defined by \( x_B \), we define the unit vectors \( p_B \) and \( q_B \), and the distances \( R_{SB}, R_{QB}, \) and \( R_B \) as follows

\[
\begin{align*}
  x_B &= x_S + R_{SB} p_B = x_Q + R_Q q_B \quad (4.1) \\
  R_B &= R_{SB} + R_{QB} \quad (4.2)
\end{align*}
\]

The geometry is shown in Figure 2.

![Figure 2](https://via.placeholder.com/150)

**Fig. 2.** Base point \( B \) near crack edge for local mapping technique.

Now we consider

\[
c_L \tau - R_B = R - R_B = R_1 - R_{SB} + R_L - R_{QB} \quad (4.3)
\]

where \( R_1 \) and \( R_L \) are shown in Fig. 2. Since

\[
R_1 - R_{SB} \simeq (x_D - x_B) \cdot p_B \quad (4.4)
\]
\[ R_L - R_{QB} = (x_D - x_B) \cdot q_B, \tag{4.5} \]
equation (4.3) yields
\[ (x_D - x_B) \cdot (p_B + q_B) = c_L \tau - r_B. \tag{4.6} \]
Equation (4.6) represents a plane which approximates the surface \( E \) near the flash point \( D \).

To map the edge of a crack, we consider \( n \) source positions \( S_i, i = 1, n \) and \( m \) observer positions \( Q_j, j = 1, m \). For each source observer pair \((S_i, Q_j)\), there is an associated travel time, which provides an input to equation (4.6). The base point defined by \( x_B \) remains the same for all cases. The planes that are obtained in this manner pass very close to the crack edge, in fact they envelope a curve which is an approximation to a segment of the crack edge. The inversion thus reduces to finding the congruence of all the planes. Since our data are finite, we will obtain a polygon of points which approximates a segment of the crack edge.

5. \textit{Comparisons of Accuracy}

It is to be expected that for the same accuracy of data (i.e., travel times \( \tau(x_S, x_Q) \)), the local crack-edge mapping will yield better results than the global triangulation method. To illustrate the differences in accuracy we consider the simple configuration shown in Fig. 3. A plane crack has its center at the center of a sphere of radius \( b \). Shown in Fig. 3 is a plane of symmetry of the crack. The source \( S \) is placed at \( \phi = 0 \), and the observer \( Q \) may be moved over the surface of the sphere, but in the plane of symmetry shown in Fig. 3. The position of the relevant flash point \( D \) is defined by \( \phi = \frac{1}{2} \pi, r = a \).

The configuration for the global triangulation method is shown in Fig. 3a. It is easily checked that the vectors \( q \) and \( V \) in Eq.(3.3) are defined by \( q = [-\sin(\phi - \chi), -\cos(\phi - \chi)] \) and \( V = (\cos \phi, -\sin \phi) \). Also, if \( r_1 = DQ \), we have
\[ \nabla_V(R) = \nabla_V(r_1) = \frac{1}{b} \frac{1}{d \theta} \frac{c_L}{b} \frac{\Delta \tau}{\Delta \theta} \tag{5.1} \]
Substituting \( q, V \) and \( \nabla_V(R) \) into Eq.(3.3), and using \( \Delta \theta = h/b \), where \( h \) is the magnitude of the shift of the observer over the surface of the sphere, we obtain
\[ \sin \chi = -c_L \Delta \tau / h \tag{5.2} \]
The angle $\chi$ defines the direction of the vector $q$. Since in this simple example the plane of the crack is known, the position of the flash point $D$ has been determined. In fact, since $\sin \chi = (a/b) \cos \phi$, we find

$$a = -(b/h)(c_L/cos \phi) \Delta \tau$$

(5.3)

In the sequel we will consider an experimental configuration for a sphere of radius $b = 27.94$ mm for a material with $c_L = 6.2$ mm/$\mu$sec.

Let us consider a shift of the receiver of $h = 3$ mm, and let us assume that the maximum error of $\Delta \tau$ is $\varepsilon_{\tau} = 0.01$ $\mu$sec. The corresponding error for $a$ follows from Eq.(5.3) as

$$\varepsilon_a \sim 0.58/cos \phi \quad (mm)$$

(5.4)

The configuration for the local crack-edge mapping is shown in Fig. 3b. As base point we select the center of the sphere $O$. Referring to Eqs.(4.1)-(4.6) we know $R_B = SQ + OQ = c_L \tau_O$. We also know $\chi_D - \chi_B = (a,0)$, $p_B = (0,-1)$ and $q_B = (-sin \phi, -cos \phi)$. 

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Fig. 3. Configuration for global triangulation method (a) and local crack-edge mapping (b).
Substitution of these results in Eq.(4.6) yields
\[ a \sin \phi \equiv c_L (\tau_0 - \tau) \] (5.5)

where terms of order \( O(a^2/b) \) have been ignored. In this case an error of 0.01 \( \mu \text{sec} \) in \( \tau \) yields
\[ \varepsilon_a = c_L \frac{\varepsilon}{\sin \phi} = 0.062/\sin \phi \quad (\text{mm}) \] (5.6)

Except for \( \phi \) very small, a comparison of (5.4) and (5.6) clearly illustrates the superior accuracy of the local crack-edge mapping over the global triangulation method for the same error in travel time.

6. Local Crack-Edge Mapping with Synthetic data

In this section we consider a particular configuration of a crack, sources and receivers, and we use synthetically generated data to demonstrate the local crack-edge mapping. The iterative nature of the procedure and the effect of uniform errors in the data are discussed.

Fig. 4. Configuration of elliptical crack, and source S and observers Q.
The configuration is illustrated in Fig. 4. A flat elliptical crack in the plane \( z = 0 \), centered at the origin has major and minor axes of 1 and .5 in the x and y directions, respectively. Two sources and twenty receivers are located in the plane \( y = 5 \). The two sources are at \((0,5,-5)\) and \((0,5,-10)\), and the twenty receivers are equally spaced between \((-10,5,5)\) and \((10,5,5)\). Thus, the crack is oriented vertically with respect to the plane \( y = 5 \), and the sources and receivers are on opposite sides of the plane of the crack. Only the longitudinal-longitudinal diffracted signals are considered. The time is made non-dimensional by taking \( c_L = 1 \).

For each source-receiver pair the travel times along the rays diffracted from the "near" edge \( y > 0 \) are computed numerically. These travel times then serve as synthetic data for the local edge mapping.

The algorithm used is as follows: First we choose a base point \( \mathbf{X}_B \), then for each travel time we define a plane by eq. (4.6). Let \( P_1, P_2, \text{and} P_3 \) be three planes corresponding to one source, and let \( \bar{P}_1, \bar{P}_2 \) be two planes for the other source. Define the lines \( L_1 \) and \( L_2 \) as the intersections of \( P_1 \) with \( P_2 \) and \( P_2 \) with \( P_3 \) respectively, and the line \( \bar{L} \) as the intersection of \( \bar{P}_1 \) with \( \bar{P}_2 \). A tentative flash point occurs if \( \bar{L} \) intersects \( P_2 \) between \( L_1 \) and \( L_2 \). This is equivalent to finding the intersection of \( \bar{P}_2, \bar{P}_1 \) and \( \bar{P}_2 \). If this point lies on the same side of both \( P_1 \) and \( P_3 \) it is a tentative flash point. Note: a point lies on the same side of two planes if the normals from the point to the planes form an acute angle.

An initial base point is necessary to start the mapping. The points produced by the initial base point \((2,2,2)\) are shown in Fig. 5a along with the actual crack edge. The set of points in Fig. 5a does not define a smooth curve, let alone a flat crack edge. An attempt to improve the mapping is made by redefining the base point. This was done by selecting a point in the middle of the agglomeration of points. From Fig. 5a we selected the point \((1,-1,-1)\). The same procedure was used in all the examples in this paper. In practice, the selection was done by viewing the plot of points on-line at a video display terminal. Thus, to a certain extent the procedure is adaptive, requiring pattern recognition at each step of the iteration in order to choose a new base point. For our purposes, this was done most conveniently using the eye and a certain amount of judgment.
Computed flash points using synthetic data. Initial base point at (2,2,2) (a), and two iterations, (b) and (c).
Fig. 6. Computed flash points with a uniform error of +2 in the travel times; (a) base points at (0,0,0) and (b) one iteration.

Fig. 7. Same as Fig. 6 except the uniform error is -2.
In Fig. 5b are shown the results of using the base point (1,-1,-1). Now a definite curve in space is observed. One more iteration produces a set of points indistinguishable from those in Fig. 5c, which were generated with the base point (0,0,0).

It should be taken into consideration when considering field data, that significant errors in the measured travel times may occur. Even after smoothing, relatively large uniform errors may still persist. In fact, the random errors observed may be much smaller in magnitude than the uniform errors. Therefore, we now consider the effect of uniform errors on the same synthetic data as before. First, a constant value of 2 was added to all the delay times. The results of the local edge mapping with initial base point (0,0,0) and one iteration are shown in Fig. 6. Further iteration converged to Fig. 6b which depicts a curve similar to the crack edge but centered at (0,-1.2,1.8) and rotated in the y-z plane.

In Fig. 7a, a uniform error of -2 is considered. However in this case, iteration did not produce convergence. Instead, the points became more unlike sections of a closed convex curve as can be seen from Fig. 7b. Similar synthetic experiments with different uniform errors gave either of two results: the iterations converge to a closed convex curve which is similar to the crack edge but shifted in space and rotated, or, the iterations diverge to produce a meaningless set of points occupying a large volume. If we were presented with the latter situation, we would immediately suspect erroneous data. The remedy would be to add or subtract uniform delays to the data until a consistent pattern emerges.

7. Inversion of Experimental Data

The experiments were carried out at the Rockwell International Science Center on a spherical specimen (trailer hitch) which contains an elliptical (yttria) crack at its center. The material of the sphere is Ti-alloy. Similar experiments have been discussed in Ref.[7]. Since the material is somewhat anisotropic, the velocity of longitudinal waves depends on the polar angle. Representative values are given in Table 1:

<table>
<thead>
<tr>
<th>polar angle ( \phi ) (degrees)</th>
<th>Sample #37 ( c_L ) (mm/( \mu )sec)</th>
<th>Sample #39 ( c_L ) (mm/( \mu )sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55.5</td>
<td>6.193</td>
<td>6.175</td>
</tr>
<tr>
<td>73.5</td>
<td>6.219</td>
<td>6.203</td>
</tr>
<tr>
<td>90</td>
<td>6.232</td>
<td>6.215</td>
</tr>
</tbody>
</table>
Fig. 8. Geometry of Ti-alloy sphere with source point S. The elliptical crack is in the xy-plane.

The radius of the sphere is 27.94 mm. The yttria crack is almost elliptical with major and minor axes of 2.57 and 1.41 mm, respectively. The geometrical configuration is shown in Fig. 8. The crack is located in the xy-plane, with its center at the center of the sphere.

For all measurements the source-transducer was placed at the North Pole (ϕ=0). For each specific set of data the position of the receiver transducer was at a fixed value of the polar angle φ, but the azimuthal angle ψ was varied incrementally. In order to obtain accurate measurements from the sample, a precision goniometer was constructed. This device allows fine control of the position of the center of the spherical sample and provides a good contact of the transducer with the specimen. The experiments were made in pitch-catch using 5 mhz broadband longitudinal transducers with the transmitting transducer hard bonded to the sample. The bonding material was Devcon 5 minute epoxy.

Travel time data were obtained for the following configurations:

A. \( \phi = 85^\circ, \psi = j \times 10^\circ \) (j=integer, 1 - 36)
B. \( \phi_1 = 70^\circ, \psi = j \times 10^\circ \) (j=0, integer, 1 - 18)
   \( \phi_2 = 55^\circ, \psi = j \times 10^\circ \) (j = integer, 18-36)
C. $\phi_1 = 85^\circ, \psi = j \times 10^\circ$ (j = 0, integer, 1-18)

$\phi_2 = 55^\circ, \psi = j \times 10^\circ$ (j = 0, integer, 1-18)

The arrival times of the edge-diffracted signal were estimated by visual inspection of the waveforms plotted versus time. The same point of the waveform was consistently selected as defining the arrival time. Any random errors were effectively removed by regularizing the data, i.e., by plotting the data versus time and fitting a least-square-fit curve through the data points. It is, however, conceivable that the point which was selected for the arrival time is not totally accurate. Since the same point of the waveform was selected consistently, this would imply that all data would contain the same uniform error. It would seem to be very difficult to totally avoid such a uniform error. Best judgment should be used to keep the uniform error as small as possible, and the error should then be dealt with as discussed in Section 6. With a uniform error, it may be expected that the final result of the mapping procedure will be a crack of approximately the right
shape, but rotated and displaced somewhat. A comparison of the raw experimental data (minus a uniform delay time) with the smoothed data is illustrated in Fig. 9. This figure spans the first and second arrival times for the data case C with $\phi = 85^\circ$.

The general method as described in section 6 has been applied to the data of configuration A defined above. Since the data is not in the same form as in section 6, some modifications of the algorithm were required. For each receiver position at a given azimuthal angle, there are two flash points on the crack edge, corresponding to the first and second arriving pulses. For a given base point, each arrival time defines a plane, which we denote a near flash-point plane and far flash-point plane, respectively. The exact procedure of section 6 was then followed with the set of near (far) flash point planes substituted for the planes corresponding to the first (second) source. The results after two iterations are shown in Fig. 10.

An alternative inversion procedure, which we call the method of opposite flash points, has also been used with the same experimental data. In this procedure, we assume that the near-flash point of a receiver at the azimuthal angle $\psi$ is the same as the far-flash point for a receiver at azimuthal angle $\psi + \pi$. A tentative flash point is found as the common intersection of the neighboring near-flash point planes at angle $\psi$ with one far-flash point plane at $\psi + \pi$. In Fig. 11 the results of applying this procedure to the A configuration data are shown. We observe that many more points are generated by this method, giving a better definition of the crack edge. Of course this method is approximate in that it makes an a-priori assumption. The assumption of opposite flash points is equivalent to the assumption that the crack is almost parallel to the $xy$-plane. However, as justification we note that the solution in Fig. 11 is almost parallel to the plane $z = -.5$, implying self-consistency.

The same method has been applied to the data from configuration B and C with the results shown in Figs. 12 and 13, respectively. In Fig. 12 the iteration is demonstrated, with convergence after only two iterations. The two separate solutions in Fig. 13 show the end result of iteration for two different values of the base time. The base time is that value which is added to the data in Fig. 9 in order to give the total signal delay time. By changing the base time, or adding a uniform error, it is apparent that the solution is only slightly changed in shape, although there is a gross translation of the points.
Fig. 10. The general method applied to the data of configuration A.

Fig. 11. The method of opposite flash points applied to the same data.
Fig. 12. Configuration B data, using the opposite flash points method.

Fig. 13. Configuration C data, with two values of the base time $T_0$. 
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References

DISCUSSION

B.R. Tittmann (Rockwell International Science Center): Would you tell the audience how big that uniform error was?

J.D. Achenbach (Northwestern University): Yes, it was half a micro-second.

B.R. Tittmann: The other point I want to make is the fact that during the machining, the diffusion bonding and machining of the part, there are lots of changes that take place which are quite new, so we don't really know very precisely where the crack is, both in terms of the location vis-a-vis the shape of the trailer hitch as well as its position laterally with respect to the edges of the block. I think in all fairness to the experimentalists, it's a very good result.

D.O. Thompson (Ames Laboratory): Can you say anything about an optimal viewing situation and its effect upon the accuracy of the reconstruction, i.e., the limited aperture, from the number of data points that you have?

J.D. Achenbach: We have not explored that to the fullest extent that we would like. This configuration is favorable because the transducer is right above the crack, and you are taking a measurement from the side. As you can see, we took many points. I don't think it's necessary to take as many points as we took if you just are interested in kind of a gross overall picture of the crack. Now, in the way of optimization, those things hadn't really been explored.

B.R. Tittmann: You have essentially a full 180° aperture, do you not, as you interrogate these?

J.D. Achenbach: That is not necessary. Those were the data that we obtained. We obtained data for a 180° aperture and at two different levels, but we would be very happy if we could get data for the kind of picture that I showed first for the synthetic data where we simply have data along a line. If somebody could supply us with data for a source point here and for a number of observers along the line, that would be, in principle, all right for what we are trying to do. It works with synthetic data, as I showed. We would like to try it with experimental data to try out this method for that kind of configuration.