Decay characteristics of some short-lived nuclides of low atomic number

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DECAY CHARACTERISTICS OF SOME SHORT-LIVED NUCLIDES OF LOW ATOMIC NUMBER

By
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July 1954

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Ames, Iowa
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ABSTRACT

The short-lived activities in Cl\(^{34}\), K\(^{38}\), Ca\(^{39}\), Li\(^8\), He\(^6\), and O\(^{15}\) formed by high energy bremmstrahlung irradiation have been examined, and some improved values for the half-lives of these nuclides are reported. A detailed description of the apparatus and procedure is included, together with a discussion of the weighted least mean square analysis used.

This report is based on a Master's thesis by Raymond Milton Kline submitted July, 1954 to Iowa State College, Ames, Iowa. This work was done under contract with the Atomic Energy Commission.
INTRODUCTION

Some of the nuclides which have half-lives in the neighborhood of a second have been previously studied. However, the theory of beta decay for certain of these nuclides (the case of mirror transitions in particular) has advanced to the point where it would be desirable to improve the accuracy of some of the original measurements. In addition, there are other new theoretical implications which half-life measurements can have. Until Stähelin observed the 1.58 second decay in Cl$^{34}$ (1), it was thought to be theoretically improbable that there were isomeric states in the light nuclides. This single discovery has been the basis for several important theoretical papers (2). There are even several undiscovered nuclides with atomic number less than 20 such as Mg$^{22}$, F$^{21}$, Si$^{28}$ and S$^{30}$, which should have half-lives between 0.5 seconds and 15 seconds. One may expect to produce the above isotopes by x-ray bombardment. For these reasons it was decided to try to build a piece of equipment which would accurately measure half-lives in the range from a few tenths of a second to a few minutes.

The purpose of this thesis is to describe the construction and testing of such a piece of equipment and to report the results obtained when this equipment was used in conjunction with the 70 Mev. synchrotron at Iowa State College to study x-ray induced activities. Of particular interest was the confirmation of the measurements made by Stähelin (1) on isomerism in the light nuclides.

REVIEW OF THE LITERATURE

There have been a great many methods used to measure half-lives in the range from a few minutes down to a few tenths of a second. One of the first was devised by Crane et al. (3) This consists of expanding a cloud chamber at several different time intervals after the production of the radioactive source. The number of tracks found in each expansion were then plotted as a function of time in order to obtain the decay curve. A more direct method due to Elliott and King (4) consists of automatically photographing a scaler's register and interpolation lights at predetermined times after the production of the radioactivity. White et al. (5) did practically the same thing by photographing an electroscope.

Two very popular methods in use at the present time will be termed the multiple scaler method and the oscilloscope method. In the multiple scaler method the decay curve is obtained by automatically allowing several scalers to count in sequences after the production of the radioactivity. With the oscilloscope method pulses from the decay of the radioactive nuclide are displayed on the screen of an oscilloscope and are then photographed. The resulting pictures are then analyzed to obtain the decay curve.
The multiple scaler method was first employed by Cassels and Lathan (6). The use of only one scaler in this method is impractical because of the difficulty in normalizing the various runs. This difficulty is overcome by employing two scalers. The first scaler is set to count during a fixed time interval following the production of the radioactivity and the second is moved from one position to a new position on the decay curve at the beginning of each run. In this way the data in each run can be normalized by using the counts in the fixed scaler. The next improvement in this method is to use a large number of scalers, each counting in sequence following the production of the radioactive material. This eliminates the possibility of error in normalization between the several runs, and it also greatly reduces the time required to take data.

Schelberg et al. (7) were the first to use the oscilloscope method. They simply displayed the pulses from a Geiger-Müller counter on the screen of an oscilloscope and photographed them along with a time marker. Later the data were analyzed to obtain the decay curve. Boley and Zaffarano (8) improved on this method by using "z-axis intensification" on the topmost part of the pulses coming from a scintillation counter. In this way they were able to obtain not only the decay curve but since the height of the top of the pulse above the baseline was a function of the energy of the detected particles, they were also able to obtain the energy distribution of the particles.

Recently a further improvement has been suggested by Morinaga and Zaffarano (9). In this method the pulses caused by the radioactive decay are displayed in the form of dots on the screen of an oscilloscope. The sweep is adjusted to include several half-lives of the decay. The oscilloscope is triggered each time a source of radioactivity is introduced to the counter. In this way the decay appears in the form of dots along the x-axis of the cathode-ray tube. A new source of radioactivity is produced and this sweep is triggered as often as is necessary to obtain good statistics. During this time the screen of the oscilloscope is photographed with a motion picture camera, the film moving in a direction parallel to the y axis. Since the decay appears as a function of time along a direction perpendicular to the length of the film, it is now possible to analyze the film quickly and accurately by using the automatic scanner recently developed by Hunt et al. (10). Since the difficulty in analyzing the data has always been the biggest disadvantage in using the oscilloscope method, this new method should be very useful.

Of course, there is a large number of other methods which have been used to measure short half-lives. The reader is referred to an article on the subject by Rowlands (11).
DESCRIPTION OF THE EQUIPMENT

The multiple scaler method was chosen for use in the experiments to be described in this thesis. This method seemed more desirable than any of the other methods because the data can be analyzed more quickly and may be checked as the run progresses.

The detector used in these experiments was a scintillation counter. It has the advantage over the simpler Geiger-Müller counter in that energy discrimination can be obtained. A NaI(Tl) crystal is often used in scintillation counters, especially in the study of gamma-rays. However, this presents a disadvantage when the crystal must be used near the synchrotron's x-ray beam because the iodine in the crystal has been found to have a fairly strong neutron capture activity, which is difficult to reduce sufficiently by shielding. For this reason, whenever it is desired to study gamma-rays one must move the detector away from the x-ray beam. This was not found to be necessary in the case of the detection of beta-rays, in which case an anthracene crystal may be used within one or two inches from the center of the beam if proper shielding is used. This makes the work much simpler because the sample may be bombarded and counted in the same position.

Figure 1 shows a block diagram of the entire equipment. At the proper time a relay closes in the timing circuit, allowing the x-ray beam to bombard the target. At the end of this bombardment time the synchrotron is turned off by the timing circuit and the sample is allowed to decay. The beta particles from the target impinge upon the scintillation crystal through a thin light-tight window. After passing into the crystal a beta particle gives up its energy to the crystal. The crystal in turn produces a light pulse whose amplitude is proportional to the amount of energy lost in the crystal by the beta particle, as will be demonstrated later. This light is directed to the photocathode of a photomultiplier tube. The photoelectrons produced are then amplified by the electron multiplication process and produce an electrical pulse at the anode of the photomultiplier tube. This pulse is then passed through a cathode follower in order to match the high impedance of the photomultiplier to the low impedance of the coaxial cable that runs from the synchrotron's magnet room to the control room. Upon reaching the control room the pulses are amplified by the pulse amplifier, Atomic Instrument Co. type 204-C. These pulses are then passed into an oscilloscope which is used as a monitor. The pulses also pass from the amplifier into a single channel discriminator. This discriminator may be set so as to allow only pulses above a certain energy to pass, (integral setting) or it may be set to allow only pulses in a certain narrow energy range to pass, (differential setting). Following the discriminator the pulses pass through a cathode follower to the timing circuit, which distributes the pulse to the scalers. Figure 2 is a picture of the pulse amplifier, discriminator, and oscilloscope, and Figure 3 is a picture of the timing equipment as well as some of the scalers.
Figure 1
Arrangement of the Entire Experimental Apparatus
Figure 2. Amplifier, Discriminator and Oscilloscope
Figure 3. Timing Equipment and Scalers
One cycle of the equipment proceeds as follows: The beam is turned on by the timing device for a predetermined length of time, which may be set in steps of one-half second from a minimum of one-half second to a maximum of 10.8 seconds. During bombardment the timing circuit provides an open circuit between the scintillation counter and the scalers. Following the bombardment, the timing device allows scalers 1 through 9 to count in sequence. In this way the decay of the sample is automatically recorded in the scalers. After the last scaler has finished counting, the equipment is automatically reset to the bombardment position. The sample is bombarded and the decay is recorded again, each scaler recording at the proper time. In this way the decay from many cycles may be added until good statistics are obtained. It should be pointed out that during all this time operation of the equipment has been entirely automatic.

The reader is referred to the simplified circuit diagram, Figure 4, and to the actual circuit diagram, Figure 5, for a more complete understanding of the timing equipment. The timing equipment obtains its accurate one cycle per second or one-half cycle per second time base from the output relay of a photoelectric cell circuit. The photocell's light beam is interrupted by the pendulum of a Seth Thomas Clock. The "clock method" of obtaining the time base, though slightly cumbersome, has been found to be highly accurate and practically trouble-free. The electrical pulses from the photoelectric cell's relay are then passed into a circuit containing two stepping relays arranged in series so that they act as a "frequency divider". This relay circuit can be adjusted so that it gives one output pulse for from 1 to 10.8 input pulses in steps of one pulse. The pulses from the frequency divider first pass through the normally open contacts of relay RL 1 or through the cycle stop switch and then actuate the stepping relay St 3. As it is actuated by pulses from the frequency divider, the upper deck of St 3 distributes the pulses from the scintillation counter to the various scalers. The lower deck of St 3 contains the control circuits. Consider the circuit labelled reset: This may be connected to any one of the 10 positions on the lower deck of the stepping relay at the operator's command. Assume it is connected to the eighth position as shown in the diagram. When the wiper of the stepping relay reaches the eighth position, the reset relay is actuated and stepping relay 3 is reset to the zero position. When the stepping relay reaches the zero position it opens the off normal contact. This deenergizes relay RL 1. When relay RL 1 is deenergized and if the cycle stop switch is closed, the circuit which turns the beam on is actuated. Also if the cycle stop switch is closed, pulses continue to pass from the frequency divider into the actuating coil of St 3. If the cycle stop switch is not closed the beam will not turn on and the wiper of St 3 will remain in position zero because no pulse can get to its stepping coil. Another thing happens if the cycle stop switch is open: Power energizes the coax relay, RL 11, for the background scaler, and the background is automatically recorded. If the cycle stop switch is closed observe that the first pulse
Figure 4
Simplified Circuit Diagram of the Timing Equipment
TO COIL RELAY IN INJECTOR GCT  
\( \text{X-ray beam is on when no current is flowing in this relay} \) \( \text{[Fig. 5]} \)

ABBREVIATIONS

**ST** • STEPPING RELAY

**SW** • SWITCH

**CONTACT** • RELAY

**SELECTOR** • SWITCH

**CHARGE** • RELAY

**CLOCK** • RELAY

**ID AC** • METER LIGHTS GCT

**ID DC** • METER LIGHTS GCT

**INDICATES THAT THIS WIRE CONNECTS SELECTOR SW 1 CONTACT 5**

**CLOCK & PHOTOCELL SW**

**TO BACKGROUND SCALER**

**FROM SCINTILLATION COUNTER**

**SCALERS 1-9**

**SELENIUM RECTIFIER**

**-0.1 SEC DELAY BEFORE CLOSING**

**250 MA SELENIUM RECTIFIERS**

**Fig. 5**
coming from the frequency divider steps the stepping relay St 3 to position 1. At this time the off normal contacts of St 3 are closed energizing relay RL 1. When relay RL 1 closes several things happen. First, another path is provided for the pulses from the frequency divider besides the path through the cycles stop switch. If the cycle stop switch is opened now the equipment will continue to operate until the reset position is reached. Also closing relay RL 1 allows the clock to start recording the time the scalers have counted and actuates a register which counts the number of cycles completed. In addition to this, closing of relay RL 1 actuates a delay circuit which after a delay preset by the operator a coax relay RL 12, closes allowing scaler 1 to count. There are two reasons for not allowing scaler 1 to count immediately when the stepping relay's wiper reaches position 1. First, there may be a very short unwanted background activity which one may eliminate with this delay. More important than this, a short delay is always necessary so that the equipment which turns off the x-ray beam has had time to operate. If this delay were not used, the first scaler might start counting before the beam were actually turned off, and since the x-rays cause a large number of pulses in a scintillation counter, an error would be caused in the counts received by scaler 1. Since scaler 1 does not count as long as the other scalers, a correction must be applied to the counts it has recorded. This point will be discussed more fully later.

Returning to the frequency divider circuit, the reason the frequency divider has an output consisting of two different time intervals is that it is often desirable to allow the last few scalers to count longer than the first, in order that long lived background activity may be measured more accurately, if it is present. The timing equipment is switched from one time base to another by relay RL 10. Notice that it may be connected to any one of the 10 positions of the lower deck of St 3; thus the operator may preset the equipment to change its time base at any one of the positions.

Having gained knowledge as to the basic circuit for the timing equipment the reader may turn to Figure 5, the actual circuit diagram. The resistor condenser combination across stepping coil 1 and across the several relay contacts are used to prevent the various contacts from arcing. Note that the stepping coils are called advancing coils in Figure 5.

TESTS OF THE EQUIPMENT

Most pieces of newly designed equipment contain an oversight of some kind. Usually its presence becomes obvious when the equipment is first operated. However, a piece of equipment designed to perform some measurement very accurately may contain a more concealed type of oversight which will be discovered only through an extensive series of tests.
In the case of the equipment used in these experiments, one might wonder if the pulses which provide the time base are always the same distance apart, if the delay in the first scaler is always the same, and if each scaler is truly counting for the same time. These are some things which could cause quite serious errors in the measurement and which might not be noticed if careful tests were not made. Actually two groups of tests were made. In the first, the equipment was tested as thoroughly as possible using pulses from a pulse generator. Then the half-lives of several fairly well known activities were measured. Finally, after the above test indicated that the equipment was operating properly, the measurements of interest were made. In order to be certain that the equipment continued to operate properly, some of the tests with the pulse generator were repeated at the beginning of each running period.

As a further precaution a circuit was installed in the timing equipment which allowed the accuracy of the timing of the equipment to be checked after each run. This check was made by noting the time recorded by the clock which measures the total counting time of all the scalers. This time was then compared with the time obtained by multiplying the number of cycles completed by the product of the number of scalers used times the length of time each counted. If the times obtained in these two ways varied by more than 0.75% the data were discarded. This was very seldom necessary during the more than 300 runs which have been made with this equipment.

The clock which provided the time base was also tested to see if it kept accurate time. This test was made by checking the clock with a Hamilton Chronometer which in turn was checked with radio station WWV. By adjusting the length of the pendulum it has been possible to get the clock to keep accurate time to within better than 5 seconds a day.

When the pulse generator was used as a testing device, its output was placed into the pulse amplifier. The discriminator was switched to the integral setting and adjusted so that its discrimination level was below the level of the pulses of the pulse generator. After timing equipment had been set to the desired basic time and delay, it was turned on and allowed to go through many cycles.

Table 1 below shows typical results obtained for one series of tests. In this case the repetition rate of the pulse generator was adjusted to approximately 1300 c.p.s. The counting time of each scaler was set for one second and the delay in the first scaler was set for 0.1 seconds. Note that scalers two through five received almost the same number of counts in each run, that the delay to the first scaler as calculated from the counts it received varied by about 3.5% between the three runs, and that the counting time of each scaler as calculated from the indication of the clock* and the number of cycles completed was very close to one second.

*The clock referred to is the one which measures the total counting time of all the scalers.
Table 1
Tests of the Timing Equipment

<table>
<thead>
<tr>
<th>Cycles completed</th>
<th>Scaler 1</th>
<th>Scaler 2</th>
<th>Scaler 3</th>
<th>Scaler 4</th>
<th>Scaler 5</th>
<th>Clock(^a) in sec.</th>
<th>Basic time calc. in sec.</th>
<th>% Var. of scaler most distant from mean of rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>127,567</td>
<td>142,663</td>
<td>142,442</td>
<td>142,554</td>
<td>142,556</td>
<td>552.5</td>
<td>0.1051</td>
<td>1.0045</td>
</tr>
<tr>
<td>110</td>
<td>128,113</td>
<td>142,668</td>
<td>142,459</td>
<td>142,631</td>
<td>142,577</td>
<td>552.8</td>
<td>0.1015</td>
<td>1.0051</td>
</tr>
<tr>
<td>110</td>
<td>127,918</td>
<td>142,662</td>
<td>142,501</td>
<td>142,590</td>
<td>142,562</td>
<td>552.7</td>
<td>0.1028</td>
<td>1.0049</td>
</tr>
</tbody>
</table>

\(^a\) The clock referred to is the one which measures the total counting time of all the scalers.
In addition to the tests made with a pulse generator, the equipment was also tested with a long lived radioactive source. In this case, the source was placed in the magnet room in the position normally occupied by the target. The pulses produced by this source were treated as if an ordinary run were being made except that the x-ray beam was not used. The discriminator was switched to "integral" and was set at a convenient level. The equipment was then allowed to cycle in the same way as when tests were made with the pulse generator. Of course, in the case of these runs, statistical variations appeared in the counts received by each scaler. Taking those into account, the counts in each scaler, as well as the calculated delay in the first scaler agreed with the tests made with the pulse generator. The pulse generator is more convenient for testing the equipment because no statistical variation enters into the results. For this reason the equipment was usually tested with the pulse generator.

Another thing which one might question in an experiment of this kind is whether or not the energy measurement equipment was linear. To prove that the equipment was linear two tests were made. First the pulses from a signal generator were passed through a linear step attenuator. The output from the attenuator was placed on the input to the photomultiplier's cathode follower and from this through the rest of the circuits until the output of the discriminator was reached. From here instead of going to the cathode follower and then to the timing equipment the pulses were passed into an oscilloscope. With the discriminator switched to "differential" the height of the incoming signal could then be measured very accurately because the differential discriminator would allow a signal to appear on the oscilloscope only when the discriminator was set to the voltage of the incoming pulse. The measurement was made for all ten positions of the step attenuator (input voltage from zero to two volts) and a graph of pulse height (as measured by the discriminator) versus the position of the step attenuator was plotted. This graph showed a straight line passing through all 10 points as well as through the zero discriminator setting for zero input voltage.

The other type of linearity test made was with gamma rays and conversion electrons of accurately known energies. This check was performed on both NaI(Tl) and the anthracene crystals; the exact method used on the two crystals differed slightly. Since NaI(Tl) gives a good photoelectric peak for gamma rays less than about 1.75 Mev, and a good pair peak for more energetic gamma rays, it was merely necessary to measure the position of the photo peak or pair peak (they have the same position if the positron is annihilated and both resulting gammas are captured in the crystal) with the discriminator set to differential.

In the case of anthracene one of the points on the linearity curve was obtained by measuring conversion electrons. This worked very well because
the source gave a narrow energy distribution around the known energy of the conversion electrons. However, there are not many suitable sources of conversion electrons arising from internally converted gamma-rays because they also usually contain intense ordinary continuous beta spectra, because they have low internal conversion coefficients, or because they have inconvenient half-lives. The Compton cross-section for gamma-rays in anthracene is fairly high. For this reason it was decided to use the Compton edges produced in anthracene by gamma-rays as other calibration points. This method is discussed in detail by Jordan and Bell (12). The sources used, together with the position of each Compton edge, are shown in Table 2. Anthracene does not have a large photoelectric cross-section because it does not contain any heavy elements as NaI(Tl) does. Since the Compton edge in a scintillation crystal is far from being a sharp "edge", the energy value obtained is much more uncertain than that which can be obtained with internally converted beta-rays in anthracene or with the method of detecting gamma rays in NaI(Tl). A typical graph obtained for anthracene of the number of counts per unit time versus the setting of the differential discriminator for the various sources is shown in Figure 6. Also shown in Figure 6 is a graph of the known energy of each Compton edge and that of the conversion electrons (indicated by the cross marks) versus the discriminator setting. The arrow on each curve indicates the estimated position of the Compton edge. The small peak on the high energy side of the ThC" and Po-Be curves appear at about the energy of the gamma rays of each source.

Table 2
Radioactive Sources Used for Calibration with the Anthracene Crystal

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy of gamma ray (in Mev)</th>
<th>Calculated energy of Compton edge (in Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Po-Be(^{a})</td>
<td>4.45</td>
<td>4.22</td>
</tr>
<tr>
<td>ThC&quot;</td>
<td>2.62</td>
<td>2.37</td>
</tr>
<tr>
<td>Zn(^{65})</td>
<td>1.12</td>
<td>0.92</td>
</tr>
<tr>
<td>Ca(^{137})</td>
<td>0.66</td>
<td>0.625 (Energy of conversion electrons)</td>
</tr>
</tbody>
</table>

\(^{a}\)Polonium decays by alpha emission. When mixed with beryllium these alpha particles often cause the reaction Be\(^9\)(\(\alpha\),n)\(^{12}\)C\(^{12}\) is sometimes left in the first excited state, which decays by the emission of a 4.45 Mev gamma-ray.
Figure 6

Linearity Check of the Equipment Using an Anthracene Crystal
At this point it will be profitable to discuss the scintillation crystals used in some detail. The anthracene crystal used for high energy beta particles was a clear crystal 13/4" thick mounted on a Dumont 6292 photomultiplier tube. It was prepared in a way described in the thesis of Phipps (13). The crystal used for the lower energy beta particles was 1" thick and mounted by the same technique on a R. C. A. 5819 tube. The above tubes had resolutions of 10% and 12% respectively at 625 kev. The NaI(Tl) crystal was prepared by Larco Nuclear Instrument Co. It was mounted on a R. C. A. 5819 tube using Dow Corning 10^6 centipoise silicone fluid to improve the light conduction between the crystal and the tube. This crystal had a resolution of 15% at 660 kev.

In addition to the tests mentioned above, a series of tests was performed on the "megavoltmeter", which measures maximum energy of the x-rays produced by the synchrotron. It was necessary to know the maximum energy of the synchrotron's x-rays in those experiments in which it was desired to eliminate the possibility of producing some contaminating reaction by using x-rays whose maximum energy was below the threshold of the undesired reaction.

The final energy of the electrons in the synchrotron, \( E \), and thus the maximum energy of the bremsstrahlung produced, is determined by the magnetic field, \( H \), at the orbit and by the frequency, \( f \), of the electric field applied across the R. F. cavity according to the equation \( E = \frac{300CH^*}{2\pi f} \), Eq 1,

where \( C \) is the velocity of light. In the case of the machine at Iowa State College the frequency is held constant at approximately 167 megacycles. This leaves only the magnetic field as the quantity which determines the final energy of the electrons.

The megavoltmeter measures the energy of the electrons by determining the magnetic flux, a quantity which is proportional to the magnetic field, at the time the electrons strike the target. This is done by taking the voltage developed by several turns around one leg of the magnet and allowing it to pass through a milliammeter. (The voltage developed by turns around the "flux bars" is actually subtracted from the voltage developed by turns around the leg of the magnet in order to get the true field at the orbit.) Now the voltage, \( V \), appearing at the terminals of the meter will be proportional to the change in flux at the orbit, \( V = k\frac{d\phi}{dt} \), Eq 2, where \( \phi \) is the flux, \( k \) is a constant, and \( t \) is time. Two thyratrons are used as switches in the meter circuit. The first thyratron turns the meter on at the time the flux goes through zero and the second turns it off at the time the radio-frequency oscillator is turned off. Now the average current, \( I \), through this time is

\[ I = \frac{1}{t} \int_{0}^{t} V \, dt \]

*This equation has \( E \) in electron volts, \( C \) in centimeters per second, \( H \) in emu., and \( f \) in cycles per second.
However \( \phi_1 = 0 \) and \( \phi_2 = \phi \), the final flux; therefore

\[
I \propto \int_0^1 \phi \, dk \phi / dt = k \phi . \tag{Eq 3}
\]

Thus the average current is proportional to the final flux, which is proportional to the final energy. (This assumes the kinetic energy of the electrons is much larger than the electron rest mass energy.) A more complete discussion of the megavoltmeter appears in an article by Westendorp (14).

Measurements have been made by Von Tersch (15) of the value of the magnetic field as a function of the secondary current flowing through the magnet at several intervals of time after the magnet current has passed through zero. Selecting a convenient time and holding this value constant, several of Von Tersch's values for the field as a function of the secondary current were substituted into Eq 1 to obtain the energy the megavoltmeter should indicate for these several values of secondary current. The R. F. gate duration was then adjusted to be of the same length in time as used above in the calculations. Various values of the magnet secondary current were then used and the megavoltmeter was adjusted to indicate the energy as given by the calculation.

After the above method had been used to calibrate the megavoltmeter, the calibration was checked by measuring the threshold for the reaction \( \text{Cu}^{63}(\gamma, n)\text{Cu}^{62} \) and the energy at which the break appeared in the activation ratio between \( ^{0.15}(\gamma, n)^{0.15} \) and \( \text{Cu}^{63}(\gamma, n)\text{Cu}^{62} \). The copper threshold was found at 9.6 Mev and the break in the activation ratio was found at 20.9 Mev. Robinson et al. (16) have recently reported the copper threshold at 10.7 Mev and the break in the activation ratio at 21.9 Mev. From these data it appears that our meter reads about one Mev low. This was taken into account in all the experiments performed.

**ANALYSIS OF THE DATA**

The following is a description of the method used for analysis of the data from the scalers in order to obtain an accurate value for the half-life of the decay.
First, a correction was made to the counts as recorded by the first scaler because the counting interval for the first scaler was not as long as that for the other scalers. The correction formula is derived in Appendix 1.

After the constant background had been subtracted, the data were examined to see if any decaying background was present, as indicated by the background recorder, or as indicated by the counts in the last few scalers. Ordinarily a least squares plot would be made of the decaying background. However, in the experiments described in this thesis, the half-life of the background was, in each case, more than 400 times that of the activity being measured. For this reason the long lived background was approximated by the level of the background indicated by two scalers operating in time intervals at least 7 half-lives after the initial recorded activity. Only in the data from Cl$^{34}$ and K$^{38}$ was any long lived background present; thus the above approximation was necessary only in these two cases. Finally a least squares plot was made of the activity being studied. (The least squares adjustment of the data is discussed in Appendix 3).

The data from both the first and last groups of scalers were plotted at those points on the time scale where the counting rate was equal to the average counting rate. The equation which gives the position, in time, of the average counting rate is derived in Appendix 2.

RESULTS

The measurements performed in this thesis may be divided into two groups. In the first group are the rather well known activities of Li$^{6}$, He$^{6}$, and O$^{15}$. The main purpose in measuring these activities was to demonstrate that the equipment would measure half-lives accurately at both extremes of its working range. These three nuclides are particularly well suited for this because their half-lives are fairly well known, they are easily produced in large amounts by x-ray bombardment, and very little background is produced with them.

In the second group there are also three activities, those of Cl$^{34}$, K$^{38}$ and Ca$^{39}$. Cl$^{34}$ and K$^{38}$ are of interest because of the recent observation of the presence of isomeric states in them (1). The interest in Ca$^{39}$ stems from the fact that the ft value for the decay as reported by Hunt (17) does not agree well with the ft values of the other mirror nuclei. (Hunt reported the following ft values for some of the mirror nuclei: Mg$^{23}$, 3600; Si$^{27}$, 4000; S$^{31}$, 5150; Ca$^{39}$, 7250.)

The values for the half-lives are quoted with their probable internal and external errors. The definition of these errors is given in Appendix 3.
Lithium 8

Li\(^8\) was produced by the reaction Be\(^9\) (\(\bar{\nu},p\)) Li\(^8\). The beryllium was in the form of a pure metal block. It was bombarded at 65 Mev for one second per cycle and each scaler was allowed to count for one second per cycle. Before and after the run the long lived background was measured with the background scaler. In this experiment the discriminator was set so that only those beta-rays whose energy was above 3.5 Mev were counted. In this way there was no chance of interference from the He\(^6\) activity, produced by the reaction Be\(^9\) (\(\bar{\nu},2pn\)) He\(^6\). He\(^6\) has a half-life of almost the same value as Li\(^8\) but its beta-ray endpoint is 3.5 Mev (18) as compared to 13. Mev for Li\(^8\) (19).

Two acceptable measurements of the half-life of Li\(^8\) were made. These two measurements were made over three months apart, and the half-lives obtained fall well within the statistical errors of each run.

Figure 7 shows a typical decay curve obtained for Li\(^8\). Note that the Li\(^8\) activity was followed for nearly 10 half-lives, that there was no decaying background present, and that the constant background was very weak. The results of the measurement of the half-life of Li\(^8\) by this and other workers is listed in Table 3.

Table 3
Comparison of the Values for the Half-Life of Li\(^8\)

<table>
<thead>
<tr>
<th>Reported here</th>
<th>Reported in the literature</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.840±0.004 int. error</td>
<td>(0.825±0.02)(^b)</td>
<td>(20)</td>
</tr>
<tr>
<td>±0.002 ext. error</td>
<td>0.825±0.02</td>
<td>(21)</td>
</tr>
<tr>
<td>0.89</td>
<td>±0.01</td>
<td>(22)</td>
</tr>
<tr>
<td>0.89±0.02</td>
<td>0.85±0.016</td>
<td>(23)</td>
</tr>
<tr>
<td>±0.007 ext. error</td>
<td>0.88±0.03</td>
<td>(24)</td>
</tr>
<tr>
<td>0.88±0.02</td>
<td>0.87±0.02</td>
<td>(26)</td>
</tr>
</tbody>
</table>

\(^a\) Average of the above two values with the associated error calculated according to Eq 36 in Appendix 3.

\(^b\) Most likely value as determined by Ajzenberg and Lauritsen (20). (These authors did not state the criteria they used to get this result.)
Figure 7
The Decay of Li\(^8\)
(The half-life equals 0.840 $\pm$ 0.004 second)
(The points shown represent the total activity minus the constant background of 17 counts)
The present value for the half-life of Li$^8$ falls between the values reported by previous workers. It agrees very well with the value of Rall and McNeil and of Sheline.

Oxygen 15

$^{15}O$ was produced by the reaction $^16O(\alpha,n)^{15}O$. The oxygen target was obtained in the form of pure boric acid powder and was pressed into a cylindrical block. In this experiment the maximum energy of the x-rays from the synchrotron was held at 25 Mev to avoid interference from $^{14}O$ produced by the reaction $^16O(\alpha,2n)^{14}O$. The threshold for this reaction was calculated to be 25.8 Mev*. The integrals discriminator was adjusted so that all beta particles with energy greater than 150 kev were counted. The equipment was adjusted so that the scalers counted for 100 seconds each.

After the run the long lived background was followed with the background recorder for 13 minutes. No long lived background was found to be present. Figure 8 shows a plot of the results obtained in this experiment. Note that the activity was followed for more than 6 half-lives. Table 4 shows that the half-life obtained in this experiment agrees well with those reported in the literature.

Table 4
Comparison of the Values for the Half-Life of $^{15}O$
(All values are in seconds)

<table>
<thead>
<tr>
<th>Reported here</th>
<th>Reported in the literature</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>123.4±1.3 int. error</td>
<td>(118.0±0.6)$^a$</td>
<td>(20)</td>
</tr>
<tr>
<td>+0.05 ext. error</td>
<td>118.0±0.7</td>
<td>(27)</td>
</tr>
<tr>
<td></td>
<td>126. ±2</td>
<td>(28)</td>
</tr>
<tr>
<td></td>
<td>126. ±5</td>
<td>(29)</td>
</tr>
<tr>
<td></td>
<td>127. ±2</td>
<td>(30)</td>
</tr>
</tbody>
</table>

$^a$Most likely value as determined by Ajzenberg and Lauritsen (20). (These authors did not state the criteria they used to get this result.)

*All threshold calculations were based on semiempirical mass values given by Metropolis and Reitwiesner (42). The same thresholds calculated using semiempirical values given by Barkas (43) give constantly higher values.
Figure 8
The Decay of $^{15}$O
(The half-life equals 123±1. seconds)
(The points shown represent the total activity minus the constant background of 784 counts)
Helium 6

He$^6$ was produced by the reaction Li$^7$ ($\alpha$,p) He$^6$. The lithium was bombarded in the form of a pure lithium block. Since lithium metal oxidizes rapidly when exposed to air, it was packaged in a rubber-hydrochloride bag. The bag had been previously tested and was found to give negligible activity. This experiment was performed with 65 Mev x-rays, and the integral discriminator was adjusted so that only those events above 0.625 Mev were counted. The equipment was adjusted so that the bombardment time, as well as the counting time, of each scaler was one second.

Two runs were made for the decay of He$^6$. Figure 9 is a graph of the decay of He$^6$ from one of the two runs. Note that the decay was followed for more than 9 half-lives and that no long-lived background was present. Table 5 shows the half-lives obtained in these two experiments as well as those reported in the literature. There is internal consistency in the values reported in the two separate experiments. The half-life as obtained here is in agreement with the values reported by others.

Calcium 39

Ca$^{39}$ was produced by the reaction Ca$^{40}$ ($\alpha$,n) Ca$^{39}$. The calcium used was in the form of small lumps of calcium metal and was packed in a rubber-hydrochloride bag to prevent it from reacting with the air. The following impurities were present: Mg less than 0.1%; and Fe, N, Mn, and Al less than 0.01%.

Table 5

Comparison of the Values for the Half-Life of He$^6$

<table>
<thead>
<tr>
<th>Reported here</th>
<th>Reported in the literature</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.803±0.005 int. error</td>
<td>(0.828±0.01)$^b$</td>
<td>(20)</td>
</tr>
<tr>
<td>±0.005 ext. error</td>
<td>0.85 ±0.05</td>
<td>(31)</td>
</tr>
<tr>
<td>0.823±0.013</td>
<td></td>
<td>(32)</td>
</tr>
<tr>
<td>0.796±0.005 int. error</td>
<td>0.823±0.004</td>
<td>(33)</td>
</tr>
<tr>
<td>±0.004 ext. error</td>
<td>0.825±0.022</td>
<td>(13)</td>
</tr>
<tr>
<td>0.825±0.06</td>
<td></td>
<td>(34)</td>
</tr>
<tr>
<td>0.799±0.003±a ext. error</td>
<td>0.87 ±0.06</td>
<td>(6)</td>
</tr>
<tr>
<td>±0.0032 ext. error</td>
<td>0.86 ±0.03</td>
<td>(24)</td>
</tr>
</tbody>
</table>

$^a$Average of the above two values with the associated error calculated according to Eq 11 in Appendix 3.

$^b$Most likely value as determined by Ajzenberg and Lauritsen (20). This value was obtained from a weighted mean of the reported half-lives.
The Decay of He$^6$

(The half-life equals 0.796 ± 0.005 second)

(The points represent the total activity minus the constant background of 700 counts)
In this experiment the maximum energy of the x-rays was held at 19.5 Mev, which is below both the threshold of the reaction $\text{Ca}^{40} \left( \gamma, 2n \right) \text{Ca}^{38}$, as well as below the threshold of the reaction $\text{Ca}^{40} \left( \gamma, \text{pn} \right) \text{K}^{38}$. The calculated threshold for the reaction $\text{Ca}^{40} \left( \gamma, 2n \right) \text{Ca}^{38}$ is 26.6 Mev, the threshold of the reaction $\text{Ca}^{40} \left( \gamma, \text{pn} \right) \text{K}^{38}$ is 19.8 Mev and the threshold for the reaction $\text{Ca}^{40} \left( \gamma, d \right) \text{K}^{38}$ is 17.6 Mev. Although the $\text{Ca}^{39}$ activity appeared to be present below the $\text{Ca}^{40} \left( \gamma, d \right) \text{K}^{38}$ threshold, the activity was too weak to make an accurate determination of the half-life at this point. The reason for this is probably that at 17.6 Mev the cross-section is too low for the reaction $\text{Ca}^{40} \left( \gamma, d \right) \text{K}^{38}$ to produce large amounts of $\text{Ca}^{39}$. There is the possibility that a mixture of the $\text{K}^{38}$ isomer was produced in measurable amounts in this experiment along with the $\text{Ca}^{39}$, but this does not seem to be likely because 19.5 Mev is too close to the threshold for the production of the reaction $\text{Ca}^{40} \left( \gamma, d \right) \text{K}^{38}$.

In this experiment the integral discriminator was adjusted to 150 kev. The timing equipment was adjusted so that the sample was bombarded for 2 seconds and each scaler was adjusted to count for 1 second.

After this run the constant background was followed for approximately 12 minutes. Figure 10 is a graph of the decay of $\text{Ca}^{39}$. Note that the activity was followed for nearly 9 half-lives and that there was no long-lived background present. Table 6 shows the results for this experiment, as well as those reported in the literature. Note that the half-life is obtained by this experiment is lower than values reported in the literature.

Table 6

Comparison of the Values for the Half-Life of $\text{Ca}^{39}$

(All values are in seconds)

<table>
<thead>
<tr>
<th>Reported here</th>
<th>Reported in the literature</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.901 \pm 0.013$ int. error</td>
<td>$1.06 \pm 0.03$</td>
<td>(35)</td>
</tr>
<tr>
<td>$\pm 0.009$ ext. error</td>
<td>$1.1 \pm 0.2$</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>$1.00 \pm 0.03$</td>
<td>(36)</td>
</tr>
<tr>
<td></td>
<td>$1.00 \pm 0.05$</td>
<td>(37)</td>
</tr>
</tbody>
</table>
Figure 10
The Decay of Ca³⁹
(The half-life equals 0.901±0.013 second)
(The points shown represent the total activity minus the constant background of 686 counts)
Chlorine 34

The fact that there is a 1.58 second isomer in Cl$^{34}$ has been known for less than a year (1). Since the present equipment is ideally suited for the study of this reaction, and since there is at present a good deal of interest in the isomeric states of light nuclei, it was decided to check the assignment of the 1.58 second activity to an isomeric state rather than to the well known 2.3 second activity of Cl$^{33}$. 

In this experiment Cl$^{34}$ was produced by the reaction Cl$^{35}$ ($\gamma$,n) Cl$^{34}$. The source of Cl$^{35}$ was pure NH$_4$Cl powder pressed into the form of a cylindrical block. Of course no background activity was expected from the hydrogen, and the only nitrogen activity expected was the 10 minute N$^{13}$ produced by the reaction N$^{14}$ ($\gamma$,n) N$^{13}$. 

The threshold of the reaction Cl$^{35}$ ($\gamma$,2n) Cl$^{33}$ was calculated to be 23.8 Mev. The maximum energy of the x-rays in this experiment was kept below 22.0 Mev, and the integral discriminator was adjusted to 150 kev. When a short lived activity appeared, there was no other alternative but to attribute it to an isomer in Cl$^{34}$. The half-life obtained was 1.533 seconds with an internal error of 0.025 second and an external error of 0.005 second. This is in good agreement with the only other reported value of 1.58±0.05 seconds by Stahelin (1). A typical decay curve is shown in Figure 11. Note that the activity was followed for more than 7 half-lives.

In this experiment the first five scalers were allowed to count for 2 seconds each while the last four were allowed to count for 5 seconds each. This was done to try to pick up the decay of the long lived background. However the level of the long lived background was so low that its statistical error blotted out any evidence of decay. The background recorder was of great use in this case in that it indicated the nature of the long lived background present. The result was that the long lived background appeared to be a compound activity of approximately a 10 minute half-life and a 30 minute half-life. The 10 minute activity is probably due to N$^{13}$ and the 30 minute activity is due to the well known Cl$^{34}$. No background activity with half-life shorter than 10 minutes appeared to be present. Since a 10 minute activity known within the statistical accuracy shown in Figure 11 can result only in a small correction, it was merely treated as a constant background whose level was determined by average of the last two scalers.

Potassium 38

The fact that an isomer exists in K$^{38}$, as is the case with Cl$^{34}$, has been known less than a year. The K$^{38}$ activity was produced by the reaction K$^{39}$ ($\gamma$,n) K$^{38}$. The potassium used was in the form of pure KI powder which had been pressed into a cylindrical block. The maximum energy of the x-rays
Figure 11
The Decay of the C134 Isomer
(The half-life equals 1.53 ± 0.02 seconds)
Total activity minus the constant background of 562
○ Total activity minus both the constant and the long-lived background
was kept at 22.0 Mev, which is well below the calculated threshold of 24 Mev for the reaction $K^{39}(\gamma, \alpha)K^{37}$. The $K^{38}$ activity was studied by using the discriminator to allow only those beta particles which had energies greater than 0.150 Mev to be counted. The equipment was adjusted so that the first five scalers counted for 1 second each and the last four for 4 seconds each.

The fact that a strong activity existed below the $K^{38}$ threshold leads one to the conclusion that $K^{38}$ has an isomeric state. The half-life obtained was 0.935 second with a 0.025 second internal error and a 0.006 second external error, which is in good agreement with the only other reported value of $0.95\pm0.03$ second by Stahelin (1). A typical decay curve is shown in Figure 12. Note that the decay was followed for more than 6 half-lives.

The background recorder indicated that there was a long lived background present whose half-life was approximately 7 minutes. This was probably due to the well known 7.7 minute activity in $K^{38}$. In this case, as in the case of $Cl^{34}$, the statistical uncertainty in the level of the background, coupled with the fact that the half-life of the background was over 400 times as long as the measured activity made it unnecessary to subtract decaying background. For this reason the long lived background was treated as constant background whose level was determined by the average of the counts in the last two scalers.

In connection with the $K^{38}$ decay it should be mentioned that $K^{37}$ as reported by Boley (6) and Langmuir (38) is probably this $K^{38}$ isomer. If this is true, $K^{37}$ has probably never been observed.

The half-life of $K^{37}$ will probably be about 1 second. Any attempt to produce $K^{37}$ by bombarding $K^{39}$ with x-rays will result in the production of large amounts of $K^{38}$ isomer mixed with the $K^{37}$. For this reason the observation of the $K^{37}$ activity without the interference of the 0.95 second $K^{38}$ activity will be very difficult.

Theoretically there is reason to believe that less than 1% of the decay of $K^{37}$ will be accompanied by gamma-rays. This makes the measurement by detecting gamma-rays difficult. The method of detecting $K^{37}$ by observing the beta-rays whose energies are greater than the endpoint of the $K^{38}$ isomer is made difficult because the beta-ray endpoint for $K^{37}$ is expected to be about the same as that of the $K^{38}$.

DISCUSSION OF THE RESULTS

It will be noted that the half-lives obtained here are sometimes lower than those reported by other people. The reason for this is not easy to understand. It may be that some people using G.M. counters have not made dead time corrections properly, as Rall and McNeill (21) have reported.
Figure 12
The Decay of the $K^{38}$ Isomer

(The half-life equals $0.94\pm0.03$ second)
- Total activity minus the constant background of 2130
- Total activity minus both the constant and the long-lived background
The pulses from a scintillation counter, even when the relatively slow NaI(Tl) crystals are used, are at most one micro-second wide; thus the effective dead time is approximately one micro-second. For this reason no dead time corrections are necessary when a scintillation counter is used, except for extremely high counting rates, and none were made in these experiments.

It was fortunate in the measurements reported here that the constant background was usually low and that when there was decaying background it was so long that it could be treated as constant. The error which poor subtraction of the background activity can produce in the measured half-life cannot be over emphasized. In order to obtain good background subtraction in these experiments not only was a background recorder used continuously, but the counting time of the last few scalers was often lengthened in order to get more points on the background. It should also be mentioned that the buildup of long lived background over a period of many cycles will cause an error in the half-life if the final level of the long lived background is subtracted, instead of some value between that of the long lived background present at the beginning of the run and that at the end.

When evaluating the reliability of the results reported here one should recall that in all six measurements the decay was followed for at least 6 half-lives and in three cases it was followed for 9 or more half-lives. In addition, most possible contaminating activities were eliminated by using x-rays whose maximum energy was below the threshold of the unwanted activity. These conditions have not always been observed in the measurements reported in the literature. Reference to the decay curves shown in the foregoing sections indicates that there is no particular deviation from exponential decay beyond statistical variations, for the last few points on any of the curves. This is at least a rough indication that approximately the correct background has been subtracted.

A comparison of the external error to the internal of all the values reported in these experiments shows that the external error is usually of the same size or a little smaller than the internal error. The significance of this is discussed in Appendix 3.

One criticism of the multiple scaler method of measuring half-lives is that because scalers are expensive, one is always limited as to the number of points one may put on the decay curve. This disadvantage could be partially overcome by using a fast digital printer such as the Berkeley Digital Recorder made by Beckman Instrument Company. This recorder can handle data as fast as one print-out every 0.8 second; thus it would be very useful in the range of half-lives from a few tenths of a second up. However, this printer would sacrifice one good feature of the multiple scaler method, that of automatic summing the data from each cycle. This could be an important disadvantage should several hundred cycles be necessary to obtain good statistics.
The fact that the equipment described in this thesis automatically sums the data from the several runs and the fact that the incoming data are always ready for examination makes this equipment particularly useful for rapid survey of decay characteristics. For such work this method seems superior to the oscilloscope method and all other methods presently available.

During the process of using a new piece of equipment improvements often come to mind, and many of these thought practical were added to this equipment. However there is one improvement worthy of consideration which has not yet been added. This consists of removing the delay from the first scaler and placing it between the bombardment period and the first scaler. The reason for this would be two-fold. First, it would eliminate the calculation necessary to correct the number displayed on the first scaler. Secondly, it would make the equipment more accurate because there is a slight variation in the delay as presently used. If the delay were between the bombardment time and the first scaler, any variation in the delay would not affect the value of the half-life obtained.

Besides the fact that there are many short lived nuclides whose half-lives should be determined more accurately, there is at least one entirely different problem on which this equipment could be used, which is that of measuring photo-nuclear cross-sections for short-lived nuclides. The dose control equipment necessary for this work has already been added to the short half-life measurement equipment.

SUMMARY AND CONCLUSIONS

The present data confirm Stähelin's conclusions that Cl$^{34}$ and K$^{38}$ contain isomeric states. The half-lives found for Cl$^{34}$ and K$^{38}$ are in good agreement with Stähelin's values (1). The essential results of this work are presented in Table 7. Both electrical tests of the accuracy of the half-life measurement equipment, and the measurement of the half-lives of Li$^6$, He$^6$ and O$^{15}$ indicate that this equipment should yield accurate values for half-lives. New values for the half-life of Li$^6$, O$^{15}$, He$^6$ and Ca$^{39}$ have been found. The new value for the half-life of Ca$^{39}$ does not change the ft value reported by Hunt (17); therefore the ft value of Ca$^{39}$ is still in disagreement with the other ft values reported by him for the mirror transitions.
Table 7  
Summary of Half-Lives Measured

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>Half-life (in sec.)</th>
<th>Max. energy of x-rays (in Mev)</th>
<th>Possible interfering reaction</th>
<th>Threshold for interfering reaction (in Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li$^8$</td>
<td>0.841±0.004</td>
<td>65</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>He$^6$</td>
<td>0.799±0.003</td>
<td>65</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>O$^{15}$</td>
<td>123.4±1.3</td>
<td>25.0</td>
<td>O$^{16}$ (p,2n) O$^{14}$</td>
<td>26.8</td>
</tr>
<tr>
<td>Ca$^{39}$</td>
<td>0.90±0.01</td>
<td>19.5</td>
<td>Ca$^{40}$ (d,d) K$^{38}$</td>
<td>17.6</td>
</tr>
<tr>
<td>Cl$^{34}$</td>
<td>1.53±0.02</td>
<td>22.0</td>
<td>Cl$^{35}$ (p,2n) Cl$^{33}$</td>
<td>23.8</td>
</tr>
<tr>
<td>K$^{38}$</td>
<td>0.935±0.025</td>
<td>22.0</td>
<td>K$^{39}$ (p,2n) K$^{37}$</td>
<td>24.0</td>
</tr>
</tbody>
</table>

*a* The first two nuclides listed were formed by (p,p) reactions. The others were formed by (d,n) reactions.

LITERATURE CITED


34. W. J. Knox, Phys. Rev. 74, 1192 (1948).
APPENDICES

Appendix 1. Derivation of the Correction to the First Scaler

The first scaler counts during time $t_1$ to $t_2$ and it is desired to multiply the counts received during this time by some ratio so that the counts obtained would be the same as if the scaler had counted from 0 to $t_2$.

The equation for radioactive decay is:

$$N = N_0 e^{-\lambda t} \quad \text{Eq 5}$$

where $N$ is the number of radioactive atoms present at time $t$, $N_0$ is the number present at time zero, and $\lambda$ is the decay constant. The number of counts received by a scaler counting from $t_1$ to $t_2$ would be:

$$N_1 - N_2 = N_0 (e^{-\lambda t_2} - e^{-\lambda t_1}). \quad \text{Eq 6}$$

The number of counts received by a scaler counting from 0 to $t_2$ would be:

$$N_0 - N_2 = N_0 (1 - e^{-\lambda t_2}) \quad \text{Eq 7}$$

therefore if the number of counts received by a scaler counting from $t_1$ to $t_2$ is multiplied by the ratio

$$R = \frac{N_0 - N_2}{N_1 - N_2} = \frac{e^{\lambda t_2} - 1}{e^{\lambda (t_2 - t_1)} - 1} \quad \text{Eq 8}$$

it will be the same as if the scaler had counted from 0 to $t_2$, provided there is no background.

In the case when background is present the counts, $N$, as recorded by the scaler, must have the background, $N_B$, subtracted in the amount:

$$\left(\frac{t_2 - t_1}{t_2}\right)N_B.$$
The counts in the scaler due to the decaying activity then are:

\[ N - \left( \frac{t_2 - t_1}{t_2} \right) N_B \]

The general formula for the correction to a scaler then becomes:

\[ N' = \left[ N - \left( \frac{t_2 - t_1}{t_2} \right) N_B \right] \frac{e^{\lambda t_2} - 1}{e^{\lambda (t_2 - t_1)}} \]  

Eq 9

where \( N' \) is the counts the scaler would have received if it had counted from 0 to \( t_2 \) with the background subtracted.

Appendix 2. Determination of the Position in a Time Interval Where the Counting Rate is Equal to the Average Counting Rate

A scaler has recorded for time \( t \) the decays from a radioactive substance. It is desired to know the position \( x_t \) at which the counting rate is equal to the average counting rate during the interval.

Now

\[ \frac{dN}{dt} = -\lambda N \]  

Eq 10

where \( N \) is the number of radioactive atoms present at any time, \( t \) is the time, and \( \lambda \) is the decay constant. If the terms of Eq 10 are integrated, it becomes

\[ N = N_0 e^{-\lambda t} \]  

Eq 11

where \( N_0 \) is the number of radioactive atoms present at time \( t = 0 \). Now \( \frac{dN}{dt} \) is the counting rate. Substitute Eq 11 into Eq 10.

\[ \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} \]  

Eq 12
When \( t \) is set equal to \( x_t \), \( \frac{dN}{dt} \) is the average counting rate during the interval. With this substitution Eq 12 becomes

\[
\frac{dN}{dt} = -\lambda N_0 e^{-\lambda x_t}.
\]

Eq 13

The average counting rate \( A \) during the interval \( t \) also is:

\[
A = -\left[ \frac{\text{total counts during } t}{t} \right] = -\frac{N_0 (1 - e^{-\lambda t})}{t}.
\]

Eq 14

Equate Eq 13 and Eq 14 and solve for \( x \):

\[
x = \frac{1}{\lambda t} \ln \left[ \frac{\lambda t}{1 - e^{-\lambda t}} \right].
\]

Eq 15

Appendix 3. The Least Squares Adjustment of the Data

Since the method of least squares is a rather extensive subject only a brief outline of this method will be presented here for convenience. Reference should be made to Birge (40) and Worthing and Geffner (41) for a more rigorous and detailed treatment.

If one has a set of points \((x,y)\) which represent a set of physical measurements the points will not in general lie on a straight line. This is true even in the case of radioactive decay where theory says that the logarithm of the number of decays per unit time plotted against the time should be a straight line. In the case of radioactive decay the points may deviate from a straight line simply because of statistical variations, because the decay is compound of two or more simple decays, or because of some error made in the measurements.

It is desired to have a method which will analytically determine a "best straight line" through the points \((x,y)\). In addition to this it is desired that the method be able to give the probable error in the measurement. Finally, it is desired that the method be able to give some indication as to whether or not the data can actually be fitted by a straight line within the precision of the measurements. The method of least squares is useful for accomplishing these things. The actual proof that this method gives the most probable straight line through a set of points has been given by Cohen (39).
When analyzing radioactive decay data, it is desired to fit the data by the straight line

$$\ln N = \ln N_0 - \lambda t$$  \hspace{1cm} \text{Eq 16}$$

where $N$ is the number of counts received by a scaler per unit time at time $t$ with the background subtracted, and $\lambda$ is the decay constant. $\ln N_0$ is the intercept on the ordinate axis. Let

$$Y_0 = \ln N, \quad a = \ln N_0, \quad \text{and} \quad b = -\lambda.$$  \hspace{1cm} \text{Eqs 17}$$

Eq 16 then becomes

$$Y_0 = a + b t.$$  \hspace{1cm} \text{Eq 18}$$

If $Y_i$ has been determined for several values of $t$ and if it is desired to put a straight line through these points, the least square method requires that the quantity

$$S = \sum \omega_i (Y_i - Y_0)^2$$  \hspace{1cm} \text{Eq 19}$$

be a minimum, where $Y_i$ is the natural logarithm of $N_i$, $N_i$ is the number of counts received by the $i$th scaler with the background subtracted, and $\omega_i$ is the weighting factor. The weighting factor is necessary because each of the points $N_i$ is not known with the same precision.

Worthing and Geffner (40) obtain the following relation between the weighting factor, $\omega$, and the standard deviation, $\sigma$:

$$\omega = \frac{1}{\sigma^2}.$$  \hspace{1cm} \text{Eq 20}$$

Substitute Eq 18 into Eq 19,

$$S = \sum \omega_i (Y_i - a - b t_i)^2.$$  \hspace{1cm} \text{Eq 21}$$
When Eq 21 is applied it is assumed that the points \((Y_i, t_1)\) deviate from a straight line simply because of statistical variations in \(Y_i\) and that there is no error at all in \(t_1\). The various tests made of the timing equipment indicate that \(t_1\) contains very little error.

In the experiments performed for this thesis

\[
N_r = n_r - B_c - B_1
\]

Eq 22

where \(n_r\) is the gross counts received by a scaler, \(B_c\) is the constant background, and \(B_1\) is the long-lived background. In most cases \(B_c\) and \(B_1\) were obtained by counting for a different time interval than that for which \(n_r\) was counted. For this reason

\[
B_c = \frac{b_c}{f_c} \quad \text{and} \quad B_1 = \frac{b_1}{f_1}
\]

Eq 23

where \(b_c\) and \(b_1\) are the gross background counts, and \(f_c\) and \(f_1\) are the factors which correct \(b_c\) and \(b_1\) to the proper time interval.

Now

\[
Y_r = \ln N_r = \ln (n_r - B_c - B_1).
\]

Eq 24

Worthing and Geffner (40) give the standard deviation, \(\sigma\), in a quantity \(U\) which is a function of \(X_1, X_2, X_3, \ldots\) as:

\[
\sigma_U^2 = \left( \frac{\partial U}{\partial \bar{X}_1} \sigma_{\bar{X}_1} \right)^2 + \left( \frac{\partial U}{\partial \bar{X}_2} \sigma_{\bar{X}_2} \right)^2 + \ldots \quad \text{Eq 25}
\]

The standard deviation for \(Y_r\) becomes:

\[
\sigma_{Y_r}^2 = \left( \frac{\partial Y_r}{\partial \bar{n}_r} \right)^2 \sigma_{\bar{n}_r}^2 + \left( \frac{\partial Y_r}{\partial \bar{B}_c} \right)^2 \sigma_{\bar{B}_c}^2 + \left( \frac{\partial Y_r}{\partial \bar{B}_1} \right)^2 \sigma_{\bar{B}_1}^2 \quad \text{Eq 26}
\]

Now

\[
\sigma_{\bar{n}_r}^2 = \bar{n}_r \quad \text{therefore} \quad \sigma_{\bar{B}_c}^2 = \left( \frac{\partial \bar{B}_c}{\partial \bar{b}_c} \right)^2 \sigma_{\bar{b}_c}^2 = \left( \frac{1}{f_c} \right)^2 \sigma_{\bar{b}_c}^2 = \frac{\bar{B}_c}{f_c}.
\]
and \( \sigma^2_{b_1} = \left( \frac{\partial B_1}{\partial b_1} \sigma_{b_1} \right)^2 = \left( \frac{1}{f_1} \right)^2 b_1 = + \frac{B_1}{f_1} \). Eqs 27

Eq 26 then becomes:

\[ \sigma^2_Y = \frac{n - B_0 + B_1}{(n - B_0 - B_1)^2} \]

therefore

\[ w = \frac{1}{\sigma^2_Y} = \frac{(n - B_0 - B_1)^2}{n - B_0 + B_1 + \frac{B_1}{f_1}} \]

Eq 29

The weighting factor must be modified slightly in the case of the data obtained from the first scaler because of the delay. By analysis similar to the above the weighting factor for the data from the first scaler is:

\[ w = \frac{[n - R(B_0 + B_1)]^2}{n + R(B_0 + B_1)} \]

Eq 30

where \( R \) is the ratio of the time the first scaler counted to the time it would have counted had there been no delay in the first scaler.

Since Eq 21 must be minimized with respect to both \( a \) and \( b \), the partial derivative of \( S \) is taken with respect to \( a \) and set equal to zero, and then the same thing is done for \( b \).

\[ \frac{\partial S}{\partial a} = -2 \sum w_n (Y_n - a - b t) = 0. \] Eq 31

\[ \frac{\partial S}{\partial b} = -2 \sum w_n t_n (Y_n - a - b t) = 0. \] Eq 32

Solutions of Eq 31 and Eq 32 for "a" and "b" are
\[
a = \frac{\sum w_i t_i ^2 \sum w_i y_i - \sum w_i t_i \sum w_i y_i t_i}{D} \quad \text{Eq 33}
\]
\[
b = \frac{\sum w_i \sum w_i y_i t_i - \sum w_i y_i \sum w_i t_i}{D} \quad \text{Eq 34}
\]

where
\[
D = \sum w_i t_i ^2 \sum w_i - \left( \sum w_i t_i \right)^2 \quad \text{Eq 35}
\]

Eq 34 above gives the slope of a straight line fitted to the decay data \((Y_i, t_i)\), hence the implied half-life.

Next it is desired to obtain the probable error in the slope of the straight line fitted by the method of least squares. There are two different ways of expressing the probable error in the slope. One of the ways is called the external error. The external error is based in the deviation of the measurement \(Y_i\) from the least squares line. On the other hand the internal error is a measure of how well the least squares line would fit the data if the points \(Y_i\) lie off the line simply because of statistical variations.

From the above definition it is evident that the significance of the external error being larger than the internal error is that the data do not fit the least squares line as well as they should if only statistical variations were present in the data. This may mean that the background was not subtracted accurately, that an error occurred in the independent variable \(t\), or that some other error occurred. On the other hand the significance of the external error being smaller than the internal error is not so evident. In some cases it may simply mean that the statistical errors assigned to the data may have been too conservative. A more complete discussion of the significance of the ratio of the external error to the internal is given in an article by Birge (40) and in a book by Worthing and Geffner (41).

The probable error in any quantity \(U\) which is a function of \(X_1, X_2, X_3\), is given by:
\[
P_U ^2 = \left( \frac{\partial U}{\partial X_1} P_{X_1} \right)^2 + \left( \frac{\partial U}{\partial X_2} P_{X_2} \right)^2 + \ldots \quad \text{Eq 36}
\]
where \( P_y \) is the probable error in \( U \) and \( P_{X_1}, P_{X_2}, \ldots \) are the probable errors in \( X_1, X_2, \ldots \).

Make the following substitutions in Eq 34:

\[
B' = \frac{\sum \omega \tau}{D} \quad \text{and} \quad B = \frac{\sum \omega \tau}{D}.
\] Eqs 37

Eq 34 then becomes:

\[
b = B' \sum \omega \tau Y \omega - B \sum \omega \tau Y \omega = \sum \omega \tau (B' \tau - B).
\] Eq 38

Now apply Eq 36 to Eq 38.

\[
P_{b^2} = \sum \left( \frac{\partial \sum \omega \tau Y \omega (B' \tau - B) P_y^2}{\partial \omega \tau} \right) \right) = \sum \omega \tau (B' \tau - B) P_y^2.
\] Eq 39

Up to this point the derivation of the probable error is the same no matter whether the external or the internal error is wanted. The difference between the two types of errors comes in the interpretation of \( P_{Y_1} \). For the internal error

\[
P_{Y_1} = \frac{F}{\sigma Y_1^2},
\] Eq 40

where \( F = 0.6745 \ldots \) and \( \sigma Y_1^2 \) is the standard deviation, given by Eq 28. Since the weighting factor has been defined as

\[
\omega \tau = \frac{1}{\sigma Y_1^2},
\] Eq 41

\[
P_{Y_1}^2 = \frac{F^2}{\omega \tau}.
\] Eq 42

Substitute Eq 42 into Eq 39.

\[
P_{b_{int}} = \sum \omega \tau (B' \tau - B) \frac{F^2}{\omega \tau}.
\] Eq 43
Now
\[ B' = \sum \frac{w_x}{D} \quad \text{and} \quad B = \sum \frac{w_x t_x}{D}, \]
Eqs 44

Substitute these into Eq 43 and simplify.
\[ P_{b_{\text{int}}}^2 = \frac{F \sum w_x [\sum w_x t_x^2 \sum w_x (\sum w_x)^2]}{D^2} = \frac{E^2 (\sum w_x) D}{D^2}, \]
Eq 45

Therefore
\[ P_{b_{\text{int}}} = F \sqrt{\sum w_x (B' t_x - B)^2} = F \sqrt{\sum w_x} = F \sqrt{B'}. \]
Eq 46

It was previously stated that the external error is a measure of the deviation of the points \( Y_i \) from the least squares line. Now the quantity
\[ (Y_i - a - b t_i) \]
gives the deviation of \( Y_i \) from the least squares line. Using this quantity, Birge (40) derives an external probable error given by the relation:
\[ P_{b_{\text{ext}}} = F \sqrt{\frac{B' \sum w_x (Y_i - a - b t_i)^2}{n - 2}}, \]
Eq 47

where \( n \) is the number of points \((Y_i, t_i)\).