Survey of Promising Developments in Demand Analysis: Economics of Product Characteristics

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Abstract
I have subtitled this paper "Economics of Product Characteristics to identify the set of recent developments that will be covered and to inform you that not all recent developments will be dealt with. This paper will not cover the recent work in systems of consumer demand equations: e.g., the linear expenditure systems, additive-preference systems, Theil-Barten work, Houthakker and Taylor dynamic model. For discussions of these, see Barten (1977), Hassan, Johnson, and Green (1977), and Thill (1975, 1976). Another body of literature that I will not cover concerns probabilistic product-choice models, in which utility is a random function and the probability that a product is chosen depends upon the characteristics of the chosen product and of other products. Among the papers on this topic McFadden (1973, 1976) and Manski and Lerman (1977).

Disciplines
Economic Theory | Growth and Development | Income Distribution | Statistical Models

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George W. Ladd

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Another body of literature that I will not cover concerns probabilistic product-choice models, in which utility is a random function and the probability that a product is chosen depends upon the characteristics of the chosen product and of other products. Among the papers on this topic are McFadden (1973, 1976) and Manski and Lerman (1977).

PRODUCT CHARACTERISTIC

In most of our economics, the idea of "product", "good", or "service" is a basic or primitive concept. The economics of product characteristics takes "product characteristic" as the basic concept, and views a "product" as a collection of "characteristics." Product then becomes a derived concept. Different products result from combining the same characteristics in different proportions, or from combining different characteristics.

A product characteristic is an objective, universal property of a product, e.g., length of an automobile, amount of protein in a gallon of
skim milk, amount of net energy for maintenance in 100 pounds of no. 2 yellow corn. Starting with the idea of "characteristic" rather than with the idea of "product" permits us to distinguish between the objective properties of products and the subjective properties of consumer preferences for products: something that is not possible so long as "product" is the primitive concept. One result is that economics of product characteristics is more appropriate than is economics of products for problems involving product heterogeneity: e.g., product differentiation, product development, quality, grades and standards.

To anticipate a bit, one model to be presented later distinguishes product characteristic from consumption services obtained from the product characteristic. Is the preparation time saved by a cook by using convenience foods, or the cooking time and energy saved by using pre-cooked food a characteristic or service? I expect we can treat them as either one.

REVIEW OF MODELS

Waugh

The earliest study in this area that I am acquainted with is a study made by Waugh (1928, 1929) more than a half century ago. On the Boston wholesale market he collected information on wholesale prices and characteristics of individual lots of asparagus, tomatoes and cucumbers. For each lot of vegetables he computed the ratio of the price of that lot to the average price of all lots of that vegetable that were sold on the same day. He regressed this ratio on measures of product characteristics and converted the regression coefficients into season average prices of characteristics. From his analysis of prices of asparagus, for example, he concluded that
(a) each additional inch of green color per stalk added 34.45¢ to the price of one dozen standard bunches, (b) each additional stalk per bunch reduced price by 4.6¢, and (c) each additional percentage variation in size of stalk decreased price by 0.76¢ per one dozen standard bunches. He wrote, "There is a distinct tendency for market prices of many commodities to vary with certain physical characteristics which the consumer identifies with quality, and the relation of these characteristics to prices may in many cases be fairly accurately determined by statistical analysis. If this generalization is accepted as true, it opens up a field in the theory of prices which has been practically untouched" (1929, p. 87).

Waugh did not have a formal model but I like to start the review with his work because he was a pioneer in this area.

Theil and Houthakker Studies

This field of inquiry remained practically untouched for some time after Waugh opened it up. The next significant works in this area were published more than two decades after Waugh's work. Oddly enough they were published simultaneously in the same journal, by Theil (1952) and Houthakker (1952). Theil (1952) argued that a consumer's level of utility is determined by quantities of goods consumed and by their characteristics. Let

\[ p_i = \text{price of } i\text{-th product}; \quad i = 1, 2, \ldots, n \]

\[ q_i = \text{quantity of } i\text{-th product consumed} \]

\[ x_{ij} = \text{quantity of } j\text{-th characteristic provided by one unit of } i\text{-th product}; \quad j = 1, 2, \ldots, m \]
x_{ij} is a "consumption input-output coefficient." Also, define the vector of input-output coefficients \( x_i \)

\[
x_{i} = (x_{i1}, x_{i2}, \ldots, x_{im_i})
\]

Assume the value of \( p_i \) depends upon \( x_i \): \( p_i = p_i(x_i) \). Theil writes the consumer's utility function as

\[
U(q_1, q_2, \ldots, q_n, x_1, x_2, \ldots, x_m)
\]

This is maximized subject to

\[
\sum_{i=1}^{n} q_i p_i(x_i) - I = 0
\]

where

\( I = \) consumer's fixed income

The consumer's instruments for maximizing the value of \( U \) are the \( q_i \) and the \( x_{ij} \): the consumer can vary the quantity and the quality of each good.

First-order conditions are

1. \( \frac{3U}{3q_i} - \lambda p_i = 0 \)
2. \( \frac{3U}{3x_{ij}} - \lambda q_i \frac{3p_i}{3x_{ij}} = 0 \)

He investigated thoroughly only the case of proportionately moving prices of all qualities within each commodity, i.e., his budget constraint was

\[
\sum q_i p_i = I.
\]

In his empirical work with family budget data he regressed family purchases on family income and family size. To study demand for quality he regressed price paid on the same two independent variables.

Houthakker (1952) used a scalar to measure the quality of each good. Let \( x_i \) now be a scalar measure of quality of good \( i \). And write the price of good \( i \) as \( a_i + b_i x_i, b_i > 0, a_i + b_i x_i > 0 \). Now \( a_i \) is quantity price and \( b_i \) is quality price. Houthakker maximized \( U(q_1, q_2, \ldots, q_n) \),
subject to $\sum q_i (a_i + b_i x_i) = I$. He studied effect of variations in $a_i$, $b_i$, and $I$ on consumers. There are two notable differences in Theil's and Houthakker's treatments of quality: (a) the former measures it by a vector, the latter by a scalar, and (b) Theil treats it as a set of instruments subject to the consumer's choice (consumer varies $q_i$ and $x_{ij}$), whereas Houthakker treats $x_i$ as a parameter.

Lancaster

The study which seems to have attracted the most attention was Lancaster's book (1971). He formulated the consumer's utility-maximization problem as a nonlinear program. The consumer buys products in order to obtain characteristics. Products are wanted because of the utilities that they provide. The utilities that they provide depend upon their characteristics, and a consumer's utility depends upon the total amounts of product characteristics. Define

$$x_{0j} = \text{total amount of } j\text{-th characteristic provided to the consumer by the consumption of all products}.$$ 

And let $x_{ij}$ have the same meaning as before. In contrast with Theil, Lancaster assumes each $x_{ij}$ is a parameter to the consumer. Lancaster's program is

Maximize $U(x_{01}, x_{02}, \ldots, x_{0m})$

subject to $\sum_{i=1}^{n} x_{ij} q_i = x_{0j}; \ j = 1, 2, \ldots, m$

all $x_{0j} \geq 0$, all $q_i \geq 0$

One reason for the popularity of Lancaster's model is that he is able to reach a number of useful conclusions from a relatively simple analysis.
The program explicitly displays two of the assumptions that make his model fruitful:

a) linear consumption technology (LCT), explicit in the linear constraints, and

b) utility is independent of distribution of characteristics among products (IDC), explicit in the objective function where utility depends only on the total amount of a characteristic and not on the amount (or proportion) obtained from various sources. A third assumption that Lancaster made that does not show in the programming formulation is

c) every characteristic has nonnegative marginal utility (NNMU).

Two major contributions of Lancaster's model are the (objective) characteristics efficiency-frontier and the model's ability to distinguish between the (objective) efficiency substitution effect and the (subjective) private substitution effect. The existence of the characteristics efficiency frontier and the efficiency substitution effect makes it possible to judge a consumer's efficiency in satisfying his own tastes without knowing anything of his personal tastes.

Figure 1 presents a situation in which four different products -- A, B, C and D -- can be purchased, and each product provides two characteristics. The quantity of one characteristic is measured along the horizontal axis; the quantity of the other along the vertical axis. The four rays labelled OA, OB, OC and OD show the amounts of the two characteristics obtained from consuming various amounts of the products A, B, C and D, respectively. The lines $I_1^I_1$ and $I_2^I_2$ are one consumer's indifference curves. $I_2^I_2$ represents a higher level of utility than $I_1^I_1$. The lines $i_1^i_1$ and $i_2^i_2$ represent another consumer's indifference curves; $i_2^i_2$ represents a higher level of utility than $i_1^i_1$. 
Suppose that each consumer has the same specified amount of money to spend on the four products. Points A, B, C and D represent the amounts of the respective products that can be purchased if all money is spent on one product. In this situation, both consumers maximize utility by buying product C in the amount OC and buying none of the other products. Any consumer whose indifference curves slope downward to the right maximizes utility by buying OD of product D, OC of product C, or OB of product B. No utility-maximizing consumer will buy product A because point A yields less utility than points B, C or D. An efficient consumer, therefore, will be at some point on the line B C D. Any consumer operating at a point to the left of that line is inefficient. The line B C D is an efficiency-frontier. It is an efficiency-frontier for every consumer. The efficiency-frontier, therefore, is an objective concept and it can be constructed without knowledge of a consumer's preferences or utility function.

Now suppose that the price of product C rises until the maximum quantity of this product that can be purchased is represented by point F. The efficiency frontier is now the line B E D. An efficient consumer will not operate at any point to the left of that line. Although the consumer cannot obtain the amounts of Z₁ and Z₂ defined by point E by buying product C, because he does not have enough money, he can obtain these amounts of Z₁ and Z₂ by buying some of product B and some of product D. After the increase in the price of product C, the first consumer maximizes utility by operating at point B on indifference curve I₁₁; the other, by operating at point D on indifference curve I₁₁. The first consumer's movement from C to B can be graphically broken down into two steps: C to E and E to B. The other consumer's movement from C to D can be broken down into the steps C to E
and E to D. The step from C to E is common to both consumers. It represents the efficiency-substitution effect that is a part of every consumer's response to the increase in the price of C, regardless of the consumer's preferences or utility function. The first consumer's movement from E to B represents his private substitution-effect; the second consumer's movement from E to D represents his private substitution-effect. Unlike the efficiency-substitution effect, the private-substitution effect can differ for each consumer, and does depend upon each consumer's preferences.

The lines B C D and B E D are concave efficiency frontiers. The concave efficiency frontier consisting of linear segments is the basis of much of Lancaster's analysis.

The NNMU, IDC, and LCT assumptions of the Lancaster model have been questioned.

Hendler (1975) has shown that, if the NNMU assumption is violated, then the objective concave efficiency frontier does not exist, and it is impossible to judge a consumer's efficiency without knowing his own preferences. This is illustrated in Figure 2 in which the points A, B, D, E and F are the same as in Figure 1. Assume the marginal utility of characteristic two is negative. Then the indifference curves slope upward to the right. Lines $I_1I_1$ and $I_2I_2$ are different indifference curves for one consumer. $I_2I_2$ represents a higher level of utility and $I_1I_1$ a lower level. Line 11 represents an indifference curve for a different consumer. In Figure 1, where both marginal utilities were positive (and consequently indifference curves sloped downward to the right), it was not necessary to know a person's preferences or utility to know that he would not buy product A. In Figure 2, it is necessary to know a person's preferences.
The consumer depicted by $ii$ will buy $OB$ of product $B$. The consumer whose indifference curves are $I_{11}$ and $I_{22}$ will buy amount $OA$ of product $A$. The line $B E D$ is not an efficiency-frontier.

In Figure 2, the indifference curves slope upward to the right throughout their full length. This implies that the marginal utility for any amount of characteristic two is negative. Such a characteristic can be handled in the Lancaster framework by a simple change of sign on its quantity. It might be that marginal utility of characteristic two is positive up to some level and then turns negative. For example, increasing the aroma of pipe tobacco up to some point yields increasing satisfaction to the consumer. But increasing the aroma beyond that point yields reduced satisfaction as further increases become increasingly unpleasant and finally lead to an overpowering stench. In this situation, the smoker's indifference curves slope downward to the right for a ways and then turn upward.

Lucas (1975) and Hendler (1975) have questioned Lancaster's IDC assumption. Lancaster's analysis breaks down if this assumption is violated. To see the implications of this assumption and its violation, return to Figure 1. In that figure, the quantities $Z_{1E}$ of characteristic one and $Z_{2E}$ of characteristic two can be obtained in either of two ways: by consuming product $C$ alone in the quantity $OE$ or by consuming some of $B$ and some of $D$. Either way of obtaining $Z_{1E}$ and $Z_{2E}$ yields exactly the same utility to the consumer. It is, therefore, meaningful to draw one, and only one, indifference curve through $E$ for a consumer.

Now suppose Lancaster's IDC assumption is violated. Specifically, suppose that the consumer's utility depends upon quantities of characteristics
one and two and upon the proportion of characteristic two obtained from product C. Suppose, e.g., that characteristics one and two are protein and vitamin B₁; that products B, C and D are milk, beef and pork; and that the consumer prefers that a high proportion of his protein come from beef. Then point E, which represents fixed amounts of the two characteristics, corresponds to two different levels of utility: one level of utility if products B and D are purchased, and a higher level of utility if the product C is purchased.

The three dimensional graph in Figure 3 may help to visualize the argument.¹ The amounts of characteristics one and two that are consumed are measured along the two horizontal axes labelled OZ₁ and OZ₂. The consumer's utility is measured along the vertical axis. The surface VPₜRZ₀₂R'V represents the consumer's utility if all of the consumer's protein (characteristic 2) is obtained from milk and pork (products B and C) and if no protein is obtained from beef (product D). The surface U'P'R Γ''Z₀'Z₂'R'''U represents the consumer's utility if all protein is obtained from product C and none is obtained from products B and D.

Lancaster's analysis is valid if each point in characteristics space represents a specific combination of characteristics and a unique level of utility, as in the first analysis of Figure 1. If the IDC assumption is violated, as in Figure 3, each point in two-dimensional characteristics space represents two (or more) levels of utility, each level being relevant for a different distribution of characteristics among products. Then the

¹/ I am indebted to Martin Zober for constructing this graph for me.
objective efficiency frontier in characteristics space no longer exists. To evaluate a consumer's efficiency one must know the relation of his utility to the distribution of characteristics consumed among the products purchased.

Lucas (1975) has questioned Lancaster's assumption of a linear consumption technology. Figures 1 and 2 reflect the LCT assumption in two ways. The LCT assumption means that increasing consumption of a product by \( P \) percent increases the amount of each characteristic obtained from that product by \( P \) percent. This is shown in the figures by drawing the rays \( A, B, C \) and \( D \) as straight lines through the origin. Under conditions of nonlinear technology, one or more of these rays would be a curved line of some sort. Figures 1 and 2 reflect the LCT assumption in a second way also. The amounts of \( Z_1 \) and \( Z_2 \) obtained by consuming \( B \) and \( D \) in the quantities \( aOB \) and \((1-a)OD \) where \( a \) varies from zero to one, are shown by the points on the straight line \( BED \). If the consumption technology were nonlinear the amounts of \( Z_1 \) and \( Z_2 \) obtained from \( aOB \) of \( B \) and \((1-a)OD \) of \( D \) (\( 0 \leq a \leq 1 \)) would not be given by a straight line connecting \( B \) and \( D \), but by some sort of curve connecting these two points, the exact nature of the curve depending upon the exact nature of the technology. If the consumption technology is nonlinear it is not possible to determine the characteristics efficiency frontier without detailed knowledge of the consumption technology.
Figure 1. An Illustration of Efficiency and Substitution Effects.
Figure 2. An Illustration of Negative Marginal Utility.
Figure 3. Violation of IDC Assumption.
The next model of economics of consumer goods characteristics that appeared was Ladd and Suvannunt's (1976). They did not assume LCT nor NNMU. In one model they assumed IDC; in another they did not. I will cover in some detail the model in which they assumed IDC and will mention the other. Their argument will be reported in some detail because two models to be subsequently discussed use similar arguments. They assumed n products and m common characteristics. Each of the common characteristics is provided by several products. Each product also supplies a unique characteristic. Total consumption of each characteristic then depends upon the quantities of products consumed and the consumption input-output coefficients, the $x_{ij}$:

$$x_{0j} = f_j(q_1, q_2, \ldots, q_n, x_{1j}, x_{2j}, \ldots, x_{nj})$$

$$x_{0m+1} = f_{m+1}(q_1, x_{im+1})$$

Their utility function is like Lancaster's

$$U(x_{01}, x_{02}, \ldots, x_{0m}, x_{0m+1}, x_{0m+2}, \ldots, x_{0m+n})$$

In their model as in Lancaster's, the magnitudes of the $x_{ij}$ are parameters to the consumer; their magnitudes are determined by producers. Also in both models, the consumer is viewed as selecting the combination of products that provides the combination of total amounts of product characteristics that maximizes utility, subject of course to the budget restriction $\sum p_i q_i = I$. Because the $q_i$ are the instruments for maximizing $U$, and $U$ is a function of the $x_{0j}$, and the $x_{0j}$ are functions of the $q_i$, it is necessary to use compound function (function of a function) rules to differentiate $U$:
\[ \frac{\partial U}{\partial q_i} = \sum_j \left( \frac{\partial U}{\partial x_{0j}} \cdot \frac{\partial x_{0j}}{\partial q_i} \right) \]

The first-order conditions are

\[ (3) \quad \sum_j \left( \frac{\partial U}{\partial x_{0j}} \cdot \frac{\partial x_{0j}}{\partial q_i} \right) + \left( \frac{\partial U}{\partial x_{0m+1}} \cdot \frac{\partial x_{0m+1}}{\partial q_i} \right) - \lambda p_i = 0 \]

\( \lambda \) is the marginal utility of income: \( \lambda = \frac{\partial U}{\partial I} \). Substituting this expression into (3) and solving for \( p_i \) yields

\[ (4) \quad p_i = \sum_j \left( \frac{\partial x_{0j}}{\partial q_i} \right) \left[ \frac{\partial U}{\partial x_{0j}} / \frac{\partial U}{\partial I} \right] + \left( \frac{\partial x_{0m+1}}{\partial q_i} \right) \left[ \frac{\partial U}{\partial x_{0m+1}} / \frac{\partial U}{\partial I} \right] \]

\( \frac{\partial x_{0j}}{\partial q_i} \) is the marginal yield of the \( j \)-th product characteristic by the \( i \)-th product. In the bracketed terms \( \frac{\partial U}{\partial x_{0j}} \) is the marginal utility of the \( j \)-th characteristic, and the ratio of the marginal utility of characteristic \( j \) to the marginal utility of income is the marginal rate of substitution between income and the \( j \)-th product characteristic. By the income constraint \( I = \text{total expenditure} \ E \). The bracketed term can be interpreted as the marginal rate of substitution between expenditure and characteristic \( j \), i.e., as the marginal implicit or imputed price for characteristic \( j \). Rewrite the bracketed term as \( \frac{\partial I}{\partial x_{0j}} = \frac{\partial E}{\partial x_{0j}} \) and assume one unit of each product provides one unit of its own unique characteristic. Then (4) becomes

\[ (5) \quad p_i = \sum_j \left( \frac{\partial x_{0j}}{\partial q_i} \right) \left( \frac{\partial E}{\partial x_{0j}} \right) + \frac{\partial E}{\partial x_{0m+1}} \]
Expressions (4) and (5) are hedonic price functions. They state that the price paid for a product equals the sum of the marginal yields of various characteristics provided by the product multiplied by the marginal implicit prices of the product's characteristics. The marginal money values of the characteristics of a purchased product exhaust the price of the product. This is one hypothesis that their model yields.

A second hypothesis provided by their model concerns demand for products. Remember that the effect of a change in product price $p_s$ upon the equilibrium level of $q_r$ is given by $\partial q_r / \partial p_s = -q_s \partial q_r / \partial I + S_{sr}$ where $S_{sr}$ is the substitution term. Now assume that the producer of product $u$ makes a small change in $x_{uv}$ while all other $x_{ij}$'s, all prices, and income remain constant. Differentiating the first-order conditions and manipulating the results yields (where $U_i = \partial U / \partial q_i$)

$$\frac{\partial q_r}{\partial x_{uv}} = -\left(\frac{1}{\lambda}\right) \sum_{i=1}^{n} \left(\frac{\partial U_i}{\partial x_{uv}}\right) S_{ir}$$

The effect of a change in $x_{uv}$ upon $q_r$ depends upon the effect of the change on marginal utilities and upon substitution terms. Thus, even though prices and income remain constant, purchases of product $r$ can vary if some producer varies at least one consumption input-output coefficient. This is their second hypothesis: demand depends upon product characteristics:

$$q_r = D_r(p_1, p_2, \ldots, p_n, I, x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, \ldots, x_{nm+n})$$

$$= D_r(\cdot)$$

Expression (6) is really a generalization of the Tintner-Ichimura relation for expressing effect of a parametric change (Basmann, 1956; Ichimura, 1950; Tintner, 1952). As is well known, the substitution terms
are invariant against monotonic transformations of the utility function. Basmann (1956) proves that the Tintner-Ichimura relation is also invariant against monotonic transformations of \( U \) and his proof shows that (6) is similarly invariant. Also, Basmann's proof of the linear dependence of the Tintner-Ichimura relations proves that changes in demand in response to a change in product composition are linearly dependent:

\[
\sum_{r=1}^{n} p_r \frac{\partial q_r}{\partial x_{uv}} = - \frac{1}{\lambda} \sum_{i=1}^{n} \left( \frac{\partial U_i}{\partial x_{uv}} \right) s_{ir} = 0
\]

From this expression and the fact that each \( p_r > 0 \) it follows that either: (a) no demand function is affected by changing \( x_{uv} \) or (b) demand for at least one product is increased and demand for at least one is decreased by a change in \( x_{uv} \). Not all demand functions affected by \( x_{uv} \) can be shifted in the same direction.

To take a simple case of (6) for illustrative purposes, suppose that \( \frac{\partial U_i}{\partial x_{uv}} > 0 \) but \( \frac{\partial U_i}{\partial x_{uv}} = 0 \) for \( i \neq u \), i.e., increasing the amount of characteristic \( v \) in product \( u \) increases the marginal utility of product \( u \) but leaves other marginal utilities unchanged. Then

\[
\frac{\partial q_r}{\partial x_{uv}} = \left( - \frac{1}{\lambda} \right) \left( \frac{\partial U_u}{\partial x_{uv}} \right) S_{ur}
\]

If products \( u \) and \( r \) are substitutes \( (S_{ur} > 0) \), increasing \( x_{uv} \) reduces \( q_r \). If the products are complements \( (S_{ur} < 0) \) increasing \( x_{uv} \) increases demand for product \( r \). Setting \( r = u \),

\[
\frac{\partial q_u}{\partial x_{uv}} = \left( - \frac{1}{\lambda} \right) \left( \frac{\partial U_u}{\partial x_{uv}} \right) S_{uu} > 0
\]

A third hypothesis that can be obtained from the Ladd and Suvannunt model concerns demand for product characteristics. Assume a LCT, then
Total sales of j-th characteristic depend upon product prices, income, and input-output coefficients:

\[ x_{0j} = d_j (\cdot) \]

By the price decomposition equation, each \( p_i \) can be expressed as a function of characteristics prices (say \( E_j = \partial E / \partial x_{0j} \)). Then total demand for j-th characteristic can be expressed

\[ x_{0j} = d'_j (E_1, E_2, \ldots, E_{m+n}, I, x_{11}, x_{12}, \ldots, x_{nm+n}) \]

The Ladd and Suvannunt model discussed thus far does make the IDC assumption but it can be easily modified so that it dispenses with this assumption. Define

\[ t_{ij} = \text{total quantity of } j \text{-th characteristic obtained from consumption of } i \text{-th product} \]

Then \( x_{0j} = \sum_i t_{ij} \). The arguments of the utility function are now the \( x_{0j} \)'s and the ratios \( x_{0j} / t_{ij} \)'s. But because the \( x_{0j} \)'s are functions of the \( t_{ij} \)'s, \( U \) can be expressed simply as a function of the \( t_{ij} \)'s. The resulting hedonic price equation is like (5) except that \( t_{ij} \) replaces \( x_{0j} \). Expression (6) is unchanged.

Ladd and Zober

The last model of characteristics of consumer goods to be summarized was presented by Ladd and Zober (1977). They avoided using the LCT, NNMU, and IDC assumptions. Some people have objected to the Lancaster model because they question the plausibility of a theory that assumes utility to
be a function of product characteristics. Ladd and Zober, therefore, stated
utility as a function of services rendered by products, and services
rendered depend upon product characteristics. They defined
\[ s_h = \text{amount of } h\text{-th consumption service that a consumer} \]
\[ \text{obtains from consumption of products; } h = 1, 2, \ldots, H. \]

Write the services production function as
\[ s_h = S_h(t_{11}, t_{12}, \ldots, t_{1n}, t_{21}, \ldots, t_{mn}) \]
And, of course,
\[ t_{ij} = f_{ij}(q_1, q_2, \ldots, q_n, x_{ij}) \]
The utility function is
\[ U(s_1, s_2, \ldots, s_H) \]
This is maximized subject to
\[ \sum_i p_i q_i \leq 1, \text{ all } q_i \geq 0 \]
Three results they obtain are
\[(9) \quad p_i = \sum_j \left( \frac{\partial t_{ij}}{\partial q_i} \right) \left( \frac{\partial E}{\partial t_{ij}} \right), \text{ if } q_i > 0 \]
This expression says almost exactly the same thing as equation (5), if \( q_i > 0 \).
("Almost" because one contains \( \frac{\partial E}{\partial x_{0j}} \) and the other contains \( \frac{\partial E}{\partial t_{ij}} \).)
\[(10) \quad p_i \geq \sum_j \left( \frac{\partial t_{ij}}{\partial q_i} \right) \left( \frac{\partial E}{\partial t_{ij}} \right) \text{ if } q_i = 0 \]
\[(11) \quad q_i = 0 \text{ if } p_i > \sum_j \left( \frac{\partial t_{ij}}{\partial q_i} \right) \left( \frac{\partial E}{\partial t_{ij}} \right) \]
These three expressions provide some insight that the previous model does
not. If the price of a product exceeds the marginal money value of the
product's characteristics, the product "is not worth what it costs" and
it is not purchased. The consumer purchases only products whose marginal
money values equal their prices. The Ladd and Zober model also yields
hypotheses concerning demand for products and demand for product characteristics. It also yields a new hypothesis:

\[ p_i = \sum_{h} \left( \frac{\partial s_h}{\partial q_i} \right) \left( \frac{\partial E}{\partial s_h} \right), \text{ if } q_i > 0 \]

\( \frac{\partial s_h}{\partial q_i} \) is the marginal contribution of product i to service h. \( \frac{\partial E}{\partial s_h} \) is the marginal implicit price of service h. If product i is purchased, its price equals the total of the money values of its contributions to various consumption services.

Ladd and Martin

It is curious that we have at least six, perhaps more, models concerning consumer goods' characteristics but models. Treatments of producer goods' characteristics are rare: the only one I know is one by Ladd and Martin (1976). They present a neoclassical model and linear programming models. Define

- \( v_{ih} \) = quantity of i-th input used in production of h-th product
- \( r_i \) = price paid for i-th input
- \( p_h \) = price received for h-th product
- \( q_h \) = output of h-th product
- \( x_{jih} \) = quantity of j-th productive characteristic provided by one unit of i-th input used in production of h-th product
- \( x_{j,h} \) = total quantity of j-th characteristic used in producing product h

For example \( q_h \) might be number of hundred weight of choice beef produced; \( x_{jih} \) amount of protein in 100 pounds of soybean oil meal; and \( x_{j,h} \) total amount of protein used in production of choice beef. The values of the \( x_{jih} \) are parameters whose values are not controlled by the user, and each
production function is independent of other production functions. Write
the production function for product h as

\[ q_h = F_h(x_{1,h}, x_{2,h}, \ldots, x_{m,h}) \]

The value of each \( x_{j,h} \) can be expressed as

\[ x_{j,h} = x_{jh}(v_{1h}, v_{2h}, \ldots, v_{nh}, x_{j1h}, x_{j2h}, \ldots, x_{jnh}) \]

The firm's profit function is

\[
\pi = \sum_h p_h F_h(x_{1,h}, x_{2,h}, \ldots, x_{m,h}) \\
- \sum_h \sum_i r_i v_{ih}
\]

Applying the rule for differentiating a compound function to (12), and
manipulating the results yields

\[
 r_i = p_h \sum_j \left( \frac{\partial F_h}{\partial x_{j,h}} \right) \left( \frac{\partial x_{j,h}}{\partial v_{ih}} \right)
\]

The term \( \frac{\partial x_{j,h}}{\partial v_{ih}} \) is the marginal yield of characteristic j to production
of product h from input i. The term \( \frac{\partial F_h}{\partial x_{j,h}} \) is the marginal physical
product from using characteristic j in producing output h; \( p_h \frac{\partial F_h}{\partial x_{j,h}} \) is
the value of this marginal physical product. It is interpreted as the
marginal implicit price paid for characteristic j used in product h.

Expression (11) is a hedonic price function for an input. Ladd and Martin
also presented the demand function for use of input i in output h: \( v_{ih} \)
depends upon output prices, input prices, and input-output coefficients \( x_{jih} \).
They did not derive a demand function for characteristics, but one can be
obtained from their model.

Ladd and Martin also presented two linear programs of different
blending problems and used the shadow prices to determine implicit prices
of characteristics. One of their programs was seriously faulty. Westgren
and Schrader (1977) corrected their error.

None of the models presented here, and no combination of them,
constitutes a theory of prices of characteristics. Such a theory must deal
with supply as well as demand side of the market. The models presented
here complement existing theories. Suppose that a combination of floods
and drought in the United States reduces the domestic corn crop at the same
time that foreign demand increases. Existing theory tells how the resulting
shifts in supply and foreign demand affect domestic corn prices. Equation
(13) only asserts that whatever the price of a grade of corn may have
been before the changes, its price equalled the total of the money values
of its characteristics for each firm that used it. And it states that the
same equality holds after the change in supply and foreign demand.

Three final observations are in order. (a) Even though Lancaster paid
little attention to the dual of his program, the dual provides a hedonic
price function that is like (5) but is linear:

\[ p_i = \sum_j x_{ij} \frac{\partial E}{\partial x_{0j}} \]

(b) The earlier models -- Theil's and Houthakker's -- assumed the existence
of a hedonic price function. The four later works summarized -- Lancaster's,
Ladd and Suvannunt's, Ladd and Zober's, and Ladd and Martin's -- derived the
existence of a hedonic price function. (c) Models in which \( x_{ij} \) is an
instrument -- Theil's, e.g., -- do not yield hypotheses on relation of
product demand to product characteristics.
REVIEW OF EMPIRICAL WORK

This summary of empirical work will cover only agricultural products.

Hedonic Price Functions

I have already mentioned the hedonic price functions that Waugh (1928, 1929) estimated for asparagus, tomatoes, and cucumbers. Clarke and Bressler (1938) found that prices of crates of strawberries were related to average size, condition, uniformity, color, and variety. The premium for a given size declined as supply of that size increased. Perregaux et al. (1938) found that from 76 to 97 percent of variance in prices of lots of eggs at auctions was related to weight, grade, and color. Premiums for large eggs declined as supply of large eggs rose. In a study of retail prices of 31 different meat, dairy and poultry products, Ladd and Suvannunt (1976) found that these prices were significantly related to amounts of food energy, protein, carbohydrates, phosphorous, iron, potassium, riboflavin, and ascorbic acid provided per pound. Some of these studies found a linear equation

\[ P_i = \sum_{j} X_{ij} E_j \]

to be appropriate. Some found that adding squared terms as in

\[ P_i = \sum_{j} \alpha_i X_{ij} + \sum_{i} \beta_i X_{ij}^2 \]

yielded superior results. Most of the studies of nonagricultural products have used logarithmic or semi-logarithmic equations.

A number of people have studied auction prices of lots of feeder cattle. One of the most recent was done by Menkhaus and Kearl (1976), who studied prices of lots of cattle sold at feeder sales in Worland, Wyoming in 1973 and 1974. Another recent study was made by the North Central
Regional Livestock Marketing Research Committee (1975). They studied prices of cattle sold at six auctions in Nebraska and Kansas in late 1972. They found: (a) Steer prices were higher than heifer prices. (b) Prices for Hereford and Angus X Hereford crosses were not significantly different. (c) Prices for Hereford and Angus X Hereford crosses were significantly higher than Angus prices. (d) Prices for Charolais differed little from prices for Angus. (e) Price per pound rose as lot size increased. (f) Price fell as weight increased, but the estimated relationship was not linear. For example, steer price fell by $2.81 per hundred pounds as weight rose from 450 to 550 pounds, but only fell by $1.36 per hundred pounds as weight rose from 650 to 750 pounds. The breeds arranged in order from lowest to highest price were dairy breeds, Shorthorn, Angus, Charolais, Okie No. 1, "other," Hereford X Angus crosses, and Hereford. Price was positively correlated with grade. A lot of cattle that came directly from a farm or ranch was classified as "fresh." A lot that was being resold after a recent purchase was classified as "trader." Trader cattle sold for slightly less than fresh cattle. Under-filled (shrunken-out) cattle sold for a $0.91 premium relative to normally filled cattle. Over-filled cattle sold for $0.77 less than normally filled cattle. Thin cattle brought a premium of $0.78 per hundred pounds over cattle of normal fleshiness. Fleshy or relatively fat cattle brought a slightly (non-significantly) smaller price than normal cattle. Presence or absence of horns had a negligible effect on prices. Prices did differ among auction locations. There was a statistically significant tendency for lots sold later in a sale to bring higher prices than lots sold earlier in a sale.
Neville et al. (1976) found prices of boars sold from the Georgia Swine Testing station to be related to final age, average daily gain, feed efficiency, backfat thickness, and size of litter in which boar was born. Hyslop (1970) found that a large proportion of variance in prices of hard red spring wheat in Minneapolis was accounted for by a linear combination of percentage dockage, protein content, test weight, percentage of damaged kernels, percentage of foreign material, percentage of shrunken and broken kernels, area of origin, destination, and mode of transport. Griliches (1958) found that 95 percent of the variance in national average prices per ton of mixed fertilizer was accounted for by a linear combination of amounts of N, P\textsubscript{2}O\textsubscript{5}, and K\textsubscript{2}O per ton. Fettig (1963) found U.S. farm tractor prices were related to type of engine and horsepower. Cowling and Rayner (1970) found a similar relation for prices of farm tractors in the United Kingdom.

Ladd and Zober (1977) distinguish between a product's characteristics and its services. Ohta and Griliches (1975) distinguish between a car's physical characteristics and its performance, and they regress automobile prices upon measures of performance. Characteristic input-output coefficients tend to be correlated. To eliminate this intercorrelation some people have applied factor analysis to the coefficients and estimated implicit prices for the factors. Ladd and Zober (1977, p. 98) show how the factors can be interpreted as measures of services and their coefficients as implicit prices of services.

**Constant Quality Price Index**

In some studies, people have estimated hedonic price functions as one step in the construction of a constant-quality (hedonic) price index. If
a product's quality is increased by imparting increased amounts of desired characteristics to the product, an index of prices paid for this product can rise while a constant-quality price index is falling. A constant-quality price index measures changes in prices that would have occurred if product characteristics had not changed.

Let \( p_{it} \) and \( p_{10} \) be prices of product at times \( t \) and \( 0 \) and let \( dx_{ij} \) be the change in \( x_{ij} \) between times \( t \) and \( 0 \) and write

\[
p_{it} - p_{10} = p_{it}' - p_{10}' + \sum_j (\partial p_i / \partial x_{ij}) \, dx_{ij}
\]

In the terms of Adelman and Griliches (1961) \( p_{it}' - p_{10}' \) is the polygenetic price change. It is the price change that would have occurred in the absence of quality variations. Time period 0 is the base period. Define \( p_{10} = p_{10}' \) so that we take the product existing at time 0 as the base for comparing price and quality changes. Multiplying both sides of this equation by \( q_{i0} \) and summing over \( i \) we obtain

\[
\sum_i q_{it} p_{i0} = \sum_i q_{i0} p_{it}' + \sum_j q_{i0} \sum_i (\partial p_i / \partial x_{ij}) \, dx_{ij}
\]

To obtain a price index divide both sides by \( \sum_i q_{i0} p_{i0} \)

\[
(14) \quad \frac{\sum_i q_{it} p_{i0}}{\sum_i q_{i0} p_{i0}} / \frac{\sum_i q_{i0} p_{10}}{\sum_i q_{i0} p_{10}} = \frac{\sum_i q_{i0} p_{it}'}{\sum_i q_{i0} p_{i0}} / \frac{\sum_i q_{i0} p_{10}}{\sum_i q_{i0} p_{10}} + \sum_j q_{i0} \sum_i (\partial p_i / \partial x_{ij}) \, dx_{ij} / \sum_i q_{i0} p_{i0}
\]

To interpret this equation, go back to equation (2) in Theil's model and derive some marginal rates of substitution. For two characteristics of the same product,

\[
(15) \quad (\partial U / \partial x_{ij}) / (\partial U / \partial x_{ik}) = (\partial p_i / \partial x_{ij}) / (\partial p_i / \partial x_{ik})
\]
Compare the right-hand-side of (15) with the numerator of the last term in (14). The relative weights of two characteristics of the same product are equal to the ratio of the marginal rates of substitution between the two characteristics. Further,

\[
\frac{\partial U/\partial x_{ij}}{\partial U/\partial x_{rs}} = q_i \frac{\partial p_i/\partial x_{ij}}{q_r} \frac{\partial p_r/\partial x_{rs}}
\]

The ratio of weights of characteristics of different products again equals marginal rate of substitution between characteristics. The left-hand-side of (14) is a conventional price index. The last term in (14) can be computed from hedonic price functions. Subtracting provides a constant-quality price index. For further discussion see Adelman and Griliches (1961) and Griliches (1971).

Fettig (1963) used his relation between tractor prices, horsepower, and engine type to construct a constant-quality price index for farm tractors. Rayner (1968) computed a constant-quality price index for farm tractors, and Rayner and Lingard (1971) constructed a constant-quality price index for British fertilizer.

Dhrymes (1971) identifies two problems that have not been satisfactorily dealt with in the hedonic price index literature: (a) implicit prices vary among manufacturers, e.g., implicit prices for automobile horsepower are different for Ford cars than for General Motors cars, and (b) implicit prices vary over time. Even so, in study of real farm income, costs, returns or parity it might be worthwhile to compute constant-quality price indexes for fertilizer, tractors, buildings, seed, breeding animals and feeder livestock. Also, what do you suppose would be the behavior of a constant-quality food price index over the past quarter century? How much of the food price inflation in the CPI is due to the higher cost of living
with the food marketing services of 25 years ago and how much is due to the cost of living higher because of additional services incorporated into food products?

Product Demand

Some studies that relate product demand to product characteristics will be summarized next.

Harrington and Gislason (1956) studied effects of appearance on retail sales of fruits. They found that sales of peaches were affected by percentage of peaches with extra color; sales of apricots were affected by percentage having extra color, percentage having moderate and severe defects, and percentage of hard apricots; and sales of cherries were affected by percentage of ripe cherries. Naumann et al. (1959) investigated retail sales of shank portion smoked hams, rib end loin roasts, and center-cut ham slices to determine effect of leanness of a cut on market-share of the cut.

Johnson (1976) studied rail transportation of grain from Michigan grain elevators. The quality variables or product characteristics that he considered were truck equipment delay (average days of delay in delivery of motor trucks), rail equipment delay (average days of delay in delivery), damage by truck (average value of damage in truck transit per $1,000 value), damage by railroad (average value of damage in rail transit per $1,000 value), and average railroad speed. He concluded that the quantity of railroad services demanded by an elevator is influenced by the delay in delivery of rail cars, damage by railroad shipment, and promotional efforts by trucking firms, as well as by characteristics of the elevator. He also concluded that the ratio of rail shipments to truck shipments by an
elevator is related to truck equipment delay and railroad speed as well as to characteristics of the elevator.

Let us refer again to the product demand equation no. (7). Assume we have a set of such equations and they are linear, and we can solve for the vector of product prices

\[
p_r = D_r(q_1, q_2, \ldots, q_n, I, x_{11}, x_{12}, \ldots, x_{m+n})
\]

If supply is predetermined and markets are cleared, then the demand equations can be estimated in the form (16). This is closely akin to what Hyslop (1970, pp. 17-21) did. Instead of having wheat prices as dependent variables, however, he had premiums. He had four dependent variables: premiums on 16 and 15 percent protein spring wheat, premium on 13 percent protein spring wheat, and premium on 13 percent protein hard winter wheat. His independent variables were: average protein contents of current year's crops of hard red spring wheat and hard red winter wheat, hard red spring wheat production and carry-in stocks, hard red winter wheat production and carry-in stocks, and domestic consumption of hard red wheat as percentage of consumption of all wheat except durum. In their study, Bale and Ryan (1977) used price ratios rather than price premiums. Their independent variables were supply and quality variables. They cite four other studies that have examined demand for wheat by class.

These studies follow the hypotheses presented earlier in relating product demand to product characteristics. A number of studies have found differences in demand functions for different products, but have not related these demand differences to differences in product characteristics. Duewer (ca. 1974) studied demand for retail cuts of pork. He estimated price-quantity relationships for each of eight cuts of pork at each
of three types of outlets, for a total of 24 relationships, and found
different income, own-price, and cross-price elasticities. Other examples
are provided by Raunikar et al. in their treatments of different cuts of
beef and pork (1965, pp. 21, 23) and different sizes and grades of eggs
(1965, p. 33), and by Purcell et al. in their treatment of processed and
fresh peaches (1972, p. 10).

Abaelu and Manderscheid (1968) estimated separate green coffee demand
functions for different varieties of coffee. They found that price and
income flexibilities varied by variety of coffee. Matsumoto and French
(1971) studied demand for brussels sprouts. They had six brussels sprouts
products (three different sizes of sprouts, and two different containers)
and estimated a separate demand function for each of the six products.

One implication of the product-demand hypotheses is that a brand's
or firm's share of market is related to the brand's or firm's product
characteristics. Cowling and Cubbin (1971) presented one procedure for
using this implication. Briefly stated, their model was

\[
p_{it} = f(V_{it}) + u_{it}
\]

\[
q_{it} = g(u_{it}, q_t, A_{it}, q_{it-1}, e_{it})
\]

where

- \( p_{it} \) = price of brand \( i \) at time \( t \)
- \( V_{it} \) = vector of characteristics of brand \( i \) at time \( t \)
- \( u_{it} \) = disturbance term
- \( q_{it} \) = sales of \( i \)-th brand in time \( t \)
- \( q_t \) = total sales of all brands in time \( t \)
- \( A_{it} \) = share of industry's total advertising expenditures by maker
  of brand \( i \) in time \( t \)
- \( e_{it} \) = disturbance term
The first expression relates brand price to brand characteristics. The disturbance term is positive for a "high-priced" product, that is for a product whose market price exceeds the estimated average monetary value of its characteristics, and is negative for a "low-priced" product. The second expression relates brand sales to this disturbance, advertising expenditure share, total sales of all brands, and lagged sales of brand i. Results obtained by Cowling and Rayner (1970) in their study of United Kingdom tractor market were consistent with the hypothesis that market share is affected by product characteristics.

Hedonic price functions and product demand functions can be used in studying questions of product improvement, product development, new product design: using hedonic price functions or demand functions to determine optimum amount of each characteristic in a product. This issue is briefly discussed in Ladd and Zober (1977, pp. 95-96) and in Ladd and Martin (1976, pp. 29-30).

The topic of the preceding section concerns adjusting price indexes for temporal changes in product characteristics that determine quality. The existence of such changes creates a problem for demand estimation. Temporal changes in characteristics mean temporal changes in product. Thus, for example, data for 1972 and 1978 provide observations on two different auto demand functions because 1972 and 1978 autos were not the same product. When we are dealing with a changing product, time series data from year \( y = 1 \) to year \( y = Y \) does not provide a sample of \( Y \) observations on one demand function; it provides one observation on each of \( Y \) demand functions. Lacking a theory that tells how changes in product quality affect parameters in structural equations, it is appropriate to try an
estimation procedure on time series data that recognizes the existence of variation in parameters: perhaps a random coefficients model.

The earlier statement that "data for 1972 and 1978 provide observations on two different auto demand functions" probably over-simplifies the situation. If the data are average annual car price and total annual car sales, each year provides an observation on some sort of annual weighted average car demand. What are the characteristics of a weighted average car? I am sure that they are quite different from the characteristics of the cars whose prices are used to compute the average price, and I suspect that the demand for a weighted average product cannot be used to determine demand for product characteristics.

Many time series studies of product demand find significant time trends. Is there any possibility of determining what part of a trend is due to changes in consumer preferences and what part is due to change in characteristics of the product? Can we determine what portion of a trend in a supply equation is due to changes in characteristics of inputs (e.g., average daily gain of feeder cattle) and how much is due to changes in characteristics of output (e.g., backfat of slaughter hogs)?

Demand For Characteristics

It is natural for agricultural economists to think of using the hypotheses on demand for characteristics to study demand for nutrients.

Adrian and Daniel (1976) related household consumption of protein, carbohydrate, fat, vitamin A, calcium, iron, thiamine, and vitamin C to family income, size, place of residence, race, education of homemaker, and stage of household in the family life cycle. All F ratios were highly significant. Their demand equations for nutrients did not contain
implicit prices of nutrients or prices of foods containing these nutrients.

Price et al. (1978) studied intake of 10 nutrients by 728 school-aged children. They used multiple regression to relate intake of each nutrient to socioeconomic variables, including participation in food stamp plan, national school lunch plan, and breakfast programs. Pinstrup-Anderson and his colleagues have made two interesting applications in studying impacts of public policy on nutrition. Pinstrup-Anderson et al. (1976) developed complete matrices of price elasticities of product demand for five income strata in Cali, Colombia. They developed market elasticities as weighted averages of the strata elasticities. They assumed a 10 percent horizontal (rightward) shift in the supply curve for each of 22 food commodities, keeping supply curves for the other 21 commodities unchanged, and determined the impact of each of these shifts on per capita calorie and per capita protein consumption in each of the five income strata. The results enabled them to suggest which commodities should receive high priorities in production research to achieve goals of improved calorie nutrition and improved protein nutrition. In another study of Cali, Colombia, Pinstrup-Anderson and Caicedo (1978) reported income elasticities of demand for these 22 food products for each of five income strata and used these to compute income elasticities of demand for calories and protein. They considered various methods of changing the existing distribution of income and investigated the effect of each one on per capita calorie and protein consumption in
each income strata. They did this under the assumption of fixed food supply and under the assumption that food supplies rise to meet the increased demands at unchanged prices.

Two studies on farm tractor demand estimated demands for tractor horsepower. Fox (1966) found that demand for tractor horsepower in the United States was related to tractor prices and stock of horsepower on farms, among other variables. Rayner and Cowling (1968) found United States and United Kingdom demands for tractor horsepower were related to prices and stocks of tractor horsepower.

Economics of product characteristics has other applications to nutrition in addition to study of demand for nutrients. Various people have used linear programs to determine minimum cost diets for humans. See, for example, Smith (1959, 1963). The variables whose values are determined by the solutions to these programs are the amounts of various food items (e.g., enriched white flour, nonfat dry milk, lima beans, ground beef) to be used. The duals to such problems provide shadow prices of nutritional elements. Three observations in relation to these programs are in order: (a) They provide "minimum-ingredient-cost diets" and may or may not provide "minimum-cost diets." The only costs they include are costs of items used in the diet. They do not include costs for the energy and the cook's time used in preparation. (b) Knowledge of these solutions is like having a list of the ingredients required for a recipe, but not having instructions for mixing, blending, warming, cooking, and serving. These solutions still leave the cook the sizeable challenge of finding or creating recipes for preparation of the ingredients. The magnitude of the challenge is increased by the nature of the list of foods contained in the
solutions to most such problems. (c) The solutions provide a bland, monotonous diet unless specific constraints are incorporated into the problem to assure some variety and improved palatability.

It is possible to formulate linear programs that do not have these three features. To do so, define each activity to be the number of servings of a certain dish or recipe (e.g., the number of hamburger sandwiches, the number of bowls of vegetable soup, the number of servings of tuna casserole). The cost of one unit of an activity can then be defined in either of three ways: (a) as the cost of the ingredients (e.g., the cost of the ground beef, of the bun, and of the condiments used to make one hamburger sandwich), or (b) as the cost of the ingredients plus the cost of utilities used in mixing and cooking one serving, or (c) as the cost of ingredients plus cost of utilities used in preparation plus a charge for the cook's time. Hall (1977) used this approach in her linear programming study of menu-planning. In some problems, she minimized food cost. In one problem, she minimized cost of food and energy used in cooking. Shadow prices of her programs do provide implicit prices of nutrients. Her model can be used to investigate the effect of variations in food prices on minimum-cost menu, effect of including costs of energy used in preparation on menu, and effect of including cook's time on minimum cost menu. It could also be used to study added costs, or savings, from using convenience foods. Use of linear programming provides an alternative to statistical estimation as a way of estimating implicit prices (shadow prices) of characteristics and hedonic price functions.
Other Applications

Some applications have already been mentioned: constant-quality price indexes; product demand, including studies of market-share or brand and product design; demand for product characteristics, e.g., nutrients or services in convenience foods; menu-planning; and measuring and pricing consumption services. A number of other applications are not closely related to the topic of this symposium but will be listed:

- a) Definition of product quality,
- b) Evaluation of grades and standards,
- c) Blending problems,
- d) Component pricing of milk and soybeans,
- e) Advertising readership,
- f) Role of nonprice terms in allocation of bank credit,
- g) Estimation of production functions,
- h) Microeconomics of technical change,
- i) Job safety, and
- j) Spatial equilibrium.

These are discussed in Ladd and Martin (1977) or in an Iowa Experiment Station bulletin (Ladd 1978).

In spatial or temporal equilibrium studies carried out by agricultural economists they have assumed product homogeneity. In a study of the Australian sugar cane processing industry Ryland and Guise (1975) considered a cartel-type processing industry producing a standard final product from a raw material whose quality varies seasonally and geographically. Quality was simply the sugar content of sugar cane. An activity analysis model was developed to determine optimum period of production
at a chain of sugar cane processing plants and to determine optimum regional flows of sugar cane and raw sugar. The results suggested that taking explicit account of raw material variability leads to higher net revenues than is obtained by assuming homogeneous raw material. Would consideration of regional differences in product quality improve any of our spatial equilibrium studies in this country? Would consideration of national differences in quality improve our studies of international trade flows? I understand that Italy prefers Argentinian corn because its use as poultry feed imparts a desired color to the eggs.
REFERENCES


