A numerical study of blocking

Ren-Yow Tzeng

Iowa State University

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A numerical study of blocking

Tzeng, Ren-Yow, Ph.D.

Iowa State University, 1990
A numerical study of blocking

by

Ren-Yow Tzeng

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LIST OF SYMBOLS

a  radius of the earth
APE  available potential energy
Ag  eddy available potential energy
Az  zonal available potential energy

A_ctl  control run for Atlantic experiment
A_olr  Atlantic experiment with only remote forcing
A_res+olr  Atlantic experiment without transient forcing
A_res  Atlantic experiment with only local forcing
A_res+tns  Atlantic experiment without remote forcing
A_tns+olr  Atlantic experiment without local forcing
A_tns  Atlantic experiment with only transient forcing

Cp  specific heat of air at constant pressure

E  - 1/2(u'^2 + v'^2), kinetic energy
f  Coriolis parameter
Fr  friction terms
KE  kinetic energy

KE  eddy kinetic energy
KZ  zonal kinetic energy

KV  vertical diffusion coefficient
KT  horizontal thermodynamic diffusion coefficient
K_T  horizontal momentum diffusion coefficient

P  pressure
Ps  surface pressure

P_n  normalized associated Legendre polynomial at order m
degree n

P_ctl  control run for Pacific experiment
P_olr  Pacific experiment with only remote forcing
P_res+olr  Pacific experiment without transient forcing
P_res  Pacific experiment with only local forcing
P_res+tns  Pacific experiment without remote forcing
P_tns+olr  Pacific experiment without local forcing
Pacific experiment with only transient forcing

\( q \)  
\( \dot{q} \)  
\( R \)  
\( R_c \)  
\( R_\delta \)  
\( R_T \)  
\( R_q \)  
\( t \)  
\( T \)  
\( T_0 \)  
\( T_1 \)  
\( u \)  
\( v \)  
\( V \)  
\( \alpha_r \)  
\( \alpha_N \)  
\( \delta \)  
\( \zeta \)  
\( \kappa \)  
\( \lambda \)  
\( \mu \)  
\( \rho \)  
\( \sigma \)  
\( \delta \)  
\( \phi \)  
\( \Phi \)  
\( \chi \)  
\( \psi \)  
\( \omega \)  
\( \nabla^2 \)  

- \( q = \ln P_s \)
- diabatic heating
- gas constant
- residual term of vorticity equation
- residual term of divergence equation
- residual term of thermodynamic equation
- residual term of surface-pressure-tendency equation
- time
- temperature
- global-mean temperature
- \( T_1 = T - T_0 \)
- zonal wind velocity
- meridional wind velocity
- horizontal wind vector
- drag coefficient for Rayleigh friction
- Newtonian cooling constant
- horizontal divergence
- vertical component of horizontal vorticity
- \( \kappa = R/C_p \)
- longitude
- \( \mu = \sin \phi \)
- air density
- \( \sigma = P/P_g \), vertical coordinate
- vertical \( \sigma \)-velocity
- latitude
- geopotential height
- velocity potential
- streamfunction
- vertical \( p \)-velocity
- horizontal Laplacian operator
\( \overline{\cdot} \) time-average

\( \cdot \)' transient mode

\( \Omega \) time-smoothed quantity

\( \cdot \)' first time-split integration

\( \cdot \)'' second time-split integration

\( \cdot \)''' third time-split integration

\( \cdot \)\textsubscript{b} blocking composite

\( \cdot \)\textsubscript{n} nonblocking composite

\( \cdot \) vertical integration

\( [\cdot] \) global and vertical average
I. INTRODUCTION

A. Climatology of Northern Hemispheric Blocking

In synoptic terms, "blocking" is an interruption of the midlatitude westerly flow by a strong, long-lived, slow-moving, larger-scale anticyclone (or ridge) that extends through the entire depth of the troposphere and which is usually accompanied by a large-scale cyclone to the south. A blocking ridge appears as a closed anticyclone at lower levels and as a strong ridge in the westerlies in the upper troposphere. Over the blocking area, the upper level jet stream is split into two branches, one passing poleward and the other equatorward and giving a high-low dipole structure. Short-wavelength baroclinic disturbances tend to follow the jet stream in their poleward excursion; hence, these features temporarily "block" the normal eastward progression of the weather system in midlatitudes. This blocking feature forms a low-index pattern intensifying the north-south energy exchange and the activity of synoptic-scale baroclinic (cyclone) waves over this region.

A fully developed blocking pattern over the northeast Atlantic Ocean or northern Europe can produce highly anomalous weather in Europe, particularly in winter. When a blocking high settles over Scandinavia for a period of weeks in winter, it spreads extremely cold Arctic air westward across Europe along its southern flank and often generates intensive small-scale depressions with strong winds and heavy snow
falls. Similar severe winter conditions are often observed in North America when a blocking is locked over the northeastern Pacific Ocean (Tung and Lindzen, 1979). There is no doubt that a better understanding of blocking is a paramount objective of climatologists and of medium-range weather forecasters.

As shown in Figure 1, the geographic distributions of maximum blocking activity in the Northern Hemisphere are the North Pacific Ocean (PAC), the North Atlantic Ocean (ATL), and the northern Soviet Union (NSU) (Dole, 1982; Shukla and Mo, 1983). Over the PAC and ATL regions, the locations of blocking are consistent with the prominent Pacific-North Atlantic (PNA) and the Eastern Atlantic (EA) teleconnection patterns described, respectively, by Wallace and Gutzler (1981). These regions are also associated with storm tracks, where baroclinic wave activity is most intense (Blackmon et al., 1977).

These relatively preferred locations of blocking and their local structures do not change with the season, whereas the activity of blocking varies considerably (Shukla and Mo, 1983). It is unclear whether any trend or periodicity is involved, but winter and fall seem the most favorable seasons for blocking.

B. Definition of Blocking

Even though blocking is an old subject in the meteorological literature (e.g., Elliott and Smith, 1949; Rex, 1950a,b), there is no agreement on the definition of this term. Elliott and Smith used an
objective criterion based on the magnitude and persistence of the pressure's departure from normal to examine the blocking activity. Rex, on the other hand, identified the blocking events subjectively by visual inspection and then used semi-objective criteria to determine the exact dates of initiation and duration. Interestingly, these two definitions yielded contradictory results on the relative frequency of blocking in the Atlantic and Pacific sectors. Elliott and Smith showed that blocking was more frequent in the Pacific sector, whereas Rex indicated that it occurred more often in the Atlantic sector. More recent studies on blocking have defined blocks in various ways, all of which are objective and can easily be applied to computer searches through large data sets. Practically, there are two different approaches for defining a blocking episode: a deviation from regional mean and a temporal anomaly of the height field.

Hartmann and Ghan (1980) defined a ridging event as a deviation of the 500 mb height field from some regional zonal mean greater than some threshold value. If the ridge endured for more than six days and did not move more than 10° longitude in 12 hours or more than 30° longitude in its lifetime, then it was defined as a block. Otherwise, it was called a transient ridge. This definition of blocking, as a deviation from a regional mean, has also been used in other studies as well, such as those of Lejenäs and Økland (1983) and Mullen (1986, 1987).

On the other hand, Charney et al. (1981) considered temporal anomalies at each grid point along a latitudinal circle. They defined a block at a particular grid point as an anomaly greater than some
threshold value lasting longer than some specified time. Dole (1982) and Dole and Gordon (1983) applied this temporal-anomalies definition to the positive (e.g., blocking) and negative sign of a persistent anomaly. Both studies showed that the geographical distributions of persistent anomalies of both signs were quite similar. Shukla and Mo (1983) extended the work of Dole (1982) to document the geographical variation of blocking by season.

Recently, in an effort to cover every blocking episode, Chen and Tzeng (1990a) applied both spatial (Mullen, 1986) and temporal (Dole, 1982) anomalies criteria to nine winters (1978/79 to 1986/87) of the National Meteorological Center (NMC) analyzed 200 mb height field. The same approach to define a blocking episode was also adopted by Metz (1986) and Blackmon et al. (1986). Some statistics of the blockings can be referred to Metz and Blackmon et al.

Moreover, to obtain an overall view of the spatial structure of the selected blockings, Mullen (1987) defined the blocking composites over the Pacific and Atlantic regions as those with ridge located at about 135°W and 22.5°W, respectively. In this study, we apply the same approach to define the Pacific and Atlantic blocking composites. The details of these data are described in the "Initial and Boundary Data" section of Chapter II. Furthermore, the final analyses NMC data used in this study began from the 1978/79 (FGGE, the first GARP Global Experiment) winter. Before the 1978/79 winter NMC analyses did not include the divergent wind. The divergent circulation is very important to the study of blocking (Chen and Tzeng, 1990b).
G. Blocking Theories

Although the importance of blocking to the middle- and high-latitude weather systems has received much attention in the past four decades, the mechanisms for the initiation and maintenance of blocking are still not well understood. The difficulty may be attributed to the quality and quantity of the observational data, especially over the oceans and before the FGGE year (1979). Recently, using a stream-function budget analysis to investigate the formation of blocking, Chen and Tzeng (1990b) suggested that three blocking theories (constructive interference, transient eddy, and remote forcing) may be feasible to explain the formation of blocking. Beside these three theories, two more blocking theories are also reviewed in this chapter.

1. Constructive Interference Theory

Comparing the longitude of the initial splitting of the jet to the normal phase of the 500 mb planetary waves, Austin (1980) found that, between 50°N and 60°N, zonal wavenumbers one and two tend to interfere constructively in the sector 0°-40°E and wavenumbers two and three at about 140°W (Figure 1). These regions are consistent with Rex’s (1950a) results regarding the geographical distribution of blocking. Austin also indicated that, during blocking, the stationary planetary waves have normal phases but their amplitudes are much greater than normal. For example, when wavenumbers one and two become large in amplitude while still normal in phase, then there is blocking in the Atlantic
Figure 1. The longitude distribution of blocking activity in 11 winters (a) and the normal phases of the planetary waves at 500 mb (b) (after Eliassen, 1958)
sector. Similarly, Pacific blocking is formed by the amplification of quasi-stationary wavenumbers two and three. The relations between the amplitude of planetary waves and the locations of blocking suggested by Austin is:

<table>
<thead>
<tr>
<th>Wavenumber</th>
<th>Atlantic Block</th>
<th>Pacific Block</th>
<th>Double Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>One</td>
<td>large</td>
<td>small</td>
<td>large</td>
</tr>
<tr>
<td>Two</td>
<td>large</td>
<td>large</td>
<td>large</td>
</tr>
<tr>
<td>Three</td>
<td>small</td>
<td>large</td>
<td>large</td>
</tr>
</tbody>
</table>

As shown in Figure 1, however, the most preferable location of Atlantic blocking is in 0°-40°W sector instead of 0°-40°E sector as expected by Austin. Furthermore, this blocking is formed by the constructive interference of intensified wavenumbers one and three but not by wavenumbers one and two. This finding is consistent with an observational blocking composite study (Chen and Tzeng, 1990b) and will be investigated in more detail in this study.

Recently, Chen and Shukla (1983) adopted this notion to study the development of blocking by a spectral energetics analysis. They found from both observations and model simulation that zonal wavenumbers two and three become stationary when these waves propagate to the climatological locations of blocking ridges. The constructive interference of these two waves forms two persistent blocking ridges, one over the west coast of North America and the other over western Europe (Figure 2). Note that the amplitude of wavenumber one is also
Figure 2. Spectrally filtered Hovmöller diagrams of the height of various waves at 500 mb and 50°N for January-February 1977 (after Chen and Shukla, 1983). Thick lines in (d) denote blocking episodes.
intensified during the double-blocking episode. This matches Austin’s (1980) double-blocking criterion. Consequently, the blocking ridges decayed when wavenumber three and four started to move eastward. The amplitudes of these waves then decayed.

Applying Saltzman’s (1957) energetics scheme to both observed and simulated blocking episodes, Chen and Shukla (1983) indicated that the available potential energies (APE) of wavenumbers two and three ($A_2$ and $A_3$) are supplied by the zonal APE ($A_2$), a horizontal sensible heat transport. The kinetic energy of wavenumber two ($K_2$) is maintained by a baroclinic process, $C(A_2,K_2)$, a vertical sensible heat transport, which converts $A_2$ to $K_2$. $K_3$ is maintained by a barotropic process $C(K_2,K_3)$, the momentum transport from $K_2$ to $K_3$. In short, the development and maintenance of wavenumber two are due to the baroclinic process, whereas with wavenumber three they are due to both the baroclinic and barotropic processes. These baroclinic and barotropic processes are also observed in other blocking studies (Hansen and Chen, 1982; Dole, 1986).

Although the energetics studies can illustrate the maintenance process of the amplification of stationary planetary waves, what forcing causes this amplification and forms a block is still not clear. It is well known that the stationary planetary waves are maintained by quasi-stationary topographic and thermal forcings. These quasi-stationary forcings do not generate blocking. Therefore, there must be some other kind of locally enhanced intrinsic forcing to interact with these stationary planetary waves and to form a blocking anticyclone over its
preferable locations. As indicated by Blackmon et al. (1977), the split jet stream and, hence, the planetary-scale waves are enhanced locally. This enhanced split pattern is one of the major criteria defining a blocking (Rex, 1950a).

Regarding local forcing, the residual terms in the linear (in time) transient model are usually considered as a local forcing (e.g., Metz, 1986; Branstator, 1990). These residual terms consist of the flux of the perturbation vorticity, divergence, and temperature as well as the diabatic heating. The real physical processes of these local forcings are still not explained. The main purpose of this study, however, is to find a major mechanism that initiates a blocking and maintains it. It is beyond the scope of this study to investigate the physical characteristics of the residual forcings. In practice, we may input these forcings as time-averaged or parameterized forms.

In addition, observations have shown that transient eddies (cyclone waves) generally follow the splitting jet stream around the blocking anticyclone, and it has also been shown by many studies that transient eddies play a positive role in the maintenance of blocking (Hansen and Chen, 1982; Mullen, 1986; and others). The effect of these transient eddies on planetary waves, however, is usually not clearly isolated from the local forcing theory.

2. Transient Eddy Theory

The importance of mutual interaction between synoptic-scale eddies and blocking has been suggested earlier by Berggren et al. (1949),
Naraias (1964), Palmén and Newton (1969), and others. Berggren et al. noted that the persistence of blocking is associated with the absorption of anticyclonic and cyclonic eddies into the northern and southern halves of the blocking pattern, respectively and that this association often coincides with an overall westward translation of the pattern. Theoretical and observational studies have been conducted on this topic.

Investigating the 1976 summer drought in western Europe, Green (1977) suggested that eddy-transport processes might be important to the maintenance of blocking anticyclones. Austin (1980) used a quasi-geostrophic model to test Green's hypothesis. Her results indicated that anticyclonic eddy forcing one-quarter wavelength upstream of the ridge is needed to oppose the blocking's tendency to be advected downstream by the time-mean flow. Studying the predictability of atmospheric blocking, Bengtsson (1981) pointed out that interactions between smaller scale transient eddies and the quasi-stationary system are essential for the maintenance of the blocking pattern. Recently, Shutts (1983) conducted experiments with linear and nonlinear versions of barotropic channel models to examine the properties of eddies propagating in a split blocking flow. He found that anticyclonic forcing upstream of the blocking ridge was a natural consequence of the deformation experienced by eddies approaching the block. More recently, Mullen (1986), in a diagnostic study of blocking events simulated by the NCAR CCM, confirmed that anticyclonic eddy forcing tended to be one-quarter wavelength upstream of the blocking wave. All of the above theoretical studies clearly indicate the importance of transient eddies
to the maintenance of blocking, although Savijärvi (1977) reported no obvious relation between blocking and eddy-flux convergence of vorticity or potential vorticity.

In a spectral energetics study, Hansen and Chen (1982) found that an Atlantic block was developed by the nonlinear interaction between intense baroclinic cyclone-scale waves and barotropic ultralong waves. A Pacific block resulted from the baroclinic amplification of planetary waves. They also found that the intense transient baroclinic waves upstream of these two blockings were responsible for the initial development of the blocking in both instances. Hansen and Sutera (1984) indicated that synoptic-scale waves transport energy and enstrophy to planetary-scale waves during blocking periods but not during nonblocking periods. Using NMC analyses data, Illari (1984) calculated the vorticity and quasi-geostrophic potential vorticity budget over western Europe and the eastern Atlantic during July 1976. She found that blocking waves are maintained by time-averaged eddy transports of potential vorticity, which balance the block-dissipative effect of potential vorticity transport by mean flow.

Recently, Mullen (1987) calculated the quasi-geostrophic tendencies of the composite of the observed and model blocking events. He suggested that the net quasi-geostrophic geopotential tendencies due to transport by the synoptic-scale transient eddies exhibit a quadrature relation with the blocking pattern throughout the troposphere, with anticyclonic eddy-forcing being located about one-quarter wavelength upstream of the blocking anticyclone. The temperature tendencies caused
by transports of the synoptic-scale transient eddies tend to be out of phase with the temperature perturbations of the block. Another observational case study (Colucci, 1985), however, indicated that a rapid cyclogenesis often precedes the formation of a blocking ridge. The warm air east of the deepening surface cyclone is transported poleward from the subtropics into the region where the warm thermal anomalies associated with the block become established. In other words, it is possible to show that the transient (synoptic-scale) eddy is able to initiate and to maintain a blocking anticyclone, whereas the role of the synoptic-scale transient eddy during the mature phase of blocking may be different from that of the eddy during the initiation of blocking.

So far, we have seen that both theoretical and observational studies show a strong linkage between transient eddies and blocking. However, whether or not all planetary-scale blocking circulations are associated with antecedent synoptic-scale cyclones, and how frequently synoptic-scale cyclones are linked with future blocking systems have not yet been resolved. Colucci (1987) suggested that whether or not a 500 mb blocking structure will occur and the type of structure (cyclonic or anticyclonic vortex) which will follow an intense surface cyclone event may depend critically upon the amplitude of existing planetary waves and the phase of these waves related to surface cyclones and attendant 500 mb potential vorticity transports. This concurs with the conclusions of Hansen and Sutera (1984) that cyclones supply energy and enstrophy to planetary waves during blocking periods but not during nonblocking periods.
It has been shown that the effect of transient eddies on blocking is vital. From the point view of planetary-scale waves (or blocking), however, one may question to what extent the blocking flow field depends upon the existence of eddy forcing. Observational studies (Lau, 1979; Holopainen, 1978a; and Chen and Tzeng, 1990b) indicated that the climatological average winter eddy vorticity forcing is generally two or three times smaller than the mean vorticity-flux divergence. Shutts (1983) argued that the vertical integrated vorticity-flux divergence is dominated by the eddy contribution because the mean vorticity-flux divergence, although large at any particular level, has opposite signs in the upper and lower troposphere and these contributions strongly cancel. In contrast, using the NMC analyses' daily-mean data, Chen and Tzeng (1990b) analyzed the streamfunction budget formed by performing the inverse of Laplacian on the primitive vorticity equation. They found that the vortex stretching (or vorticity source) term, instead of eddy vorticity forcing, is the major effect to balance the advection of relative vorticity by the zonal flow. They concluded that vortex stretching (divergence) on the upstream of the ridge was needed to counterbalance the tendency induced by time-mean flow advection of vorticity associated with blocks. Furthermore, Blackmon et al. (1986) used the NCAR CCM and found that some blocking events were not preceded by an explosive cyclogensis; this finding suggests that the transient eddy may not be the only mechanism to enhance blocking flows.
3. Tropical (Remote) Forcing Theory

Chen and Wiin-Nielsen (1976) pointed out that APE developed by the global scale tropical heating, hence the north-south differential heating, is released by divergent circulations to maintain rotational circulations. Krishnamurti (1979) proposed a schematic theory that planetary-scale divergent circulations linked to three tropical rainfall centers may be the important energy source maintaining subtropical jet streams. Using ECMWF IIIb data (Chen et al., 1988) and different NCAR CCM1 experimental data (Tzeng, 1988) to investigate the maintenance of subtropical jet streams, both Chen et al. and Tzeng found that the planetary-scale divergent circulation is important in transporting tropical energy to maintain the jets, although it is not as great as the local ageostrophic effect (Holopainen, 1978b). Moreover, examining the maintenance of blocking by a streamfunction budget equation, Chen and Tzeng (1990b) found the same results. There can be no doubt, therefore, that tropical heating is important to the midlatitude planetary-scale waves.

It is well known that stationary planetary waves owe their existence to the nonuniform distribution of earth topography and thermal characters. What has not yet been determined, however, is which forcing is more important to the maintenance of planetary-scale waves. Nigam (1983) used a linearized GCM to examine the individual effect of tropical heating and orography on planetary-scale waves and found that the midlatitude stationary-wave's amplitude due to tropical heating is only about 50 m, whereas the orographically forced solution is about 300
m. Results of this and several other linear model studies (Lin, 1982; Jacqmin and Lindzen, 1985; and others) suggested that tropical heating is not an important forcing to midlatitude stationary waves.

Saltzman and Irsch (1972) and Alpert et al. (1983), however, pointed out that the conventional treatment of mean zonal air flow over orography was not adequate and should be avoided. Specifically, orography not only provides the external forcing to the planetary wave by lifting the boundary mean zonal flow upward or downward mechanically, but also interacts with the boundary eddies by forcing the flow to go around the mountains. Chen and Trenberth (1988a) used a linear balance model to examine the effect of the traditional orographic (wave-decoupled) forcing and the mountain interaction (wave-coupled) forcing. They suggested that the traditional orographic forcing model not only overestimated the magnitude of the orographic forcing (by a factor of two) but also gave rise to incorrect responses in phase. Comparing wave-coupled orographic forcing with thermal forcing, they found that both forcings were important in the troposphere with the thermal forcing somewhat dominant in the stratosphere. Because their model is a hemispheric model, to suppress reflection from the equatorial lateral boundary, they purposely reduced the heating rate over the tropics. Thermal forcing is more important when tropical heating is included in the model simulation. To what extent the tropical heating affects the planetary waves, however, is still an open question.

Namias (1964) proposed that blocking is directly related to anomalous heating. He suggested that midlatitude blocking actions can
be statistically correlated with anomalous sea-surface temperature in the midlatitudes and tropics oceans. Recently, Horel and Wallace (1981) have presented convincing observational evidence of a modest relation between interannual variability in tropical Pacific sea-surface temperature anomalies and the PNA teleconnection pattern. They interpreted extratropical circulation anomalies as a forced stationary Rossby wave response to tropical heating anomalies centered over the central equatorial Pacific (their Figure 11). Besides, modeling studies (e.g., Hoskins and Karoly, 1981; Simmons, 1982; and Branstator, 1983) have displayed a marked sensitivity in the extratropical response centered over the central North Pacific to forcing changes located in the southwestern Pacific. This linkage was also found in the Atlantic sector. Rowntree (1976) indicated that sea-surface temperature anomalies in the subtropical north Atlantic during wintertime may give rise to circulation anomalies at higher latitudes over the Atlantic sector and Europe.

We have seen thus far that anomalous heating contributes positively to high latitude planetary waves. It should be emphasized that external forcing accounts for only a small fraction of the low-frequency variability of the atmospheric circulation in extratropical latitudes. Douglas et al. (1982) have pointed out that an anomalous midlatitude PNA pattern is observed in winter without anomalous tropical heating, suggesting that the origin of the pattern may indeed be confined to the midlatitudes. This result is also supported by the perpetual January simulations of the NCAR CCM (Blackmon et al., 1986).
4. Other Theories

Beside the three theories previously discussed, other hypotheses have been proposed to explain blocking. Examples are solitons/modons and multiple equilibria.

Solitons are exact solutions of a one-dimensional barotropic vorticity equation with weak dispersion and weak nonlinearity. They are localized in space and maintain temporal permanence. Examples of this application on soliton theory include models of Jupiter's Great Red Spot (Ingersoll, 1973) and models of tropical sea-surface temperature anomalies (Boyd, 1980). Studying the dynamics of oceanic Gulf-Stream rings, Stern (1975) derived an exact localized solution of the barotropic vorticity equation on beta plane. Stern called these solutions "modons". McWilliams (1980) recognized that the characters of modons, a long-lived and dipole structure vortex pair, resemble observed blocking and attempted a detailed analysis of a particular event in terms of modon theory. He found that the horizontal and vertical structure of the blocking pattern and its intensity are consistent with the modon dispersion relation, though questions remain concerning the consistency of the mean wind with modon requirements.

Recently, Tribbia (1984) and Verkley (1984) independently extended the beta-plane assumption to the spherical geometry of the earth, so a modon can be directly compared to the real atmospheric blocking patterns. However, many blocking events do not have the dipole structure of a simple modon and will require for their modeling either a soliton-like structure or a modon with rider. It will also be important
to examine the behavior of solitons and modons in more general non-uniform mean flows to understand the preferred geographical location of blocking. Furthermore, since solitons/modons are solutions of the barotropic vorticity equation, they are not consistent with the barotropic and baroclinic nature of blocking anticyclones revealed by the observational studies (Hansen and Chen, 1982; Dole, 1986). Hence, the solitons/modons theory may be inadequate to explain blocking.

The multiple equilibria theory was proposed by Charney and DeVore (1979) in a highly truncated barotropic channel model. They found from a case of topographical forcing that two stable equilibrium states of very different character may be produced by the same forcing: one is a "low-index" flow with a strong wave component and a relatively weaker zonal component which is locked close to linear resonance; the other is a "high-index" flow with a weak wave component and a relatively stronger zonal component which is much farther from linear resonance. They suggest that the phenomenon of blocking is a metastable equilibrium state of the low-index, near-resonant character. Efforts related to this theoretical study with observations have been described by a few authors (White, 1980; Dole and Gordon, 1983; Dole, 1986). However, they found no observational evidence to support this theory.

Recently, Sutera (1986) indicated that the probability density distribution of the amplitude of the zonal wavenumber 2-4 is bimodal, which is analogous to multiple equilibria. However, Hansen (1986) pointed out that blocking events can be observed in both of these two amplitude modés. In addition, as pointed out above, blocking energetics
involves both barotropic and baroclinic processes. Therefore, barotropic model is inadequate for modeling important aspects of the initial development of the blocking.

This review shows that no single theory can completely describe an entire blocking episode. It is implied in these theories that there must be persistent forcing to support the large amplitude of planetary waves (e.g., local forcing) and one or more of these forcings must work as a trigger to initiate the blocking when the planetary wave situation (including energy level and phase) reaches to some critical condition (e.g., Austin, 1980; Colucci, 1987).

D. Outline of this Study

A numerical model simulation will be used to examine the importance of these blocking hypotheses, since we can test these hypotheses individually or in combination. It has been stressed that a barotropic or a quasi-geostrophic baroclinic model is inadequate to simulate blocking (Bengtsson, 1981; Dole, 1986). Hence, the only alternative is primitive equation model. In addition, it was pointed out by Blackmon et al. (1986) that the blocking anticyclones produced in the NCAR CCM are quite realistic, thus we will use the NCAR CCM's equations and numerical structure to develop our model. Furthermore, using linearized NCAR CCM model equations, Branstator (1990) successfully studied the teleconnection pattern and SST anomalies. Branstator's model consists of standing mode, transient (anomaly) mode, and transient
eddy forcing including tropical heating. All of these characteristics can be fitted into our model requirement. Therefore, we will follow the same procedures as Branstator to derive our (anomaly) primitive equation and use the numerical structure and algorithms of the NCAR CCM to develop the anomaly (transient) primitive equation model.

The details of the numerical structure and algorithms of this model are described in Chapter II, which includes the horizontal spectral method, spectral transform method, vertical discrete finite difference scheme, centered time difference with a time filter function, and a semi-implicit time integration scheme. Three different forcings (local, transient, and remote forcings) and the experiments corresponding to different combination of these three forcings are presented at the end of Chapter II. The results of these experiments are discussed in Chapter III. Finally, a concise conclusion and some remarks for further work will be offered in the last chapter.
II. MODEL AND EXPERIMENTS

Atmospheric phenomena can be analyzed and understood by three approaches: observational diagnostic, numerical, and analytic studies. Diagnostic studies can provide implications of physical processes related to physical phenomena. To demonstrate or to prove this inference, however, we have to rely upon numerical and analytic studies. Because of the complexity surrounding the development of blocking, numerical experimentation seems a more flexible approach for exploring the mechanism causing blocking. Therefore, a numerical model was developed to investigate three mechanisms known to cause blocking. According to the considerations described in the previous section, the model needs to be a primitive equations (PE) model, especially with regards to tropical heating (divergence field). It also needs to express explicitly the standing and transient modes.

A. The Model Equations

1. The Primitive Equations

The primitive equations for a dry atmosphere using $\sigma$-coordinates ($\sigma = P/P_s$, Phillips, 1957) are the momentum equation

$$\frac{d\mathbf{u}}{dt} = -f \mathbf{k} \times \mathbf{u} - \nabla \Phi + RT \nabla \ln P_s + F,$$

(2.1)

the thermodynamic equation
The continuity equation (the surface-pressure tendency equation)

\[ \frac{dT}{dt} = \frac{R}{c_p} \left( \frac{\partial}{\partial \sigma} - \frac{\partial \Phi}{\partial \sigma} \right) - \nabla \cdot \mathbf{v} \]  

(2.2)

the hydrostatic equation

\[ \frac{d \ln P_o}{dt} = -\nabla \cdot \mathbf{v} - \frac{\partial \Phi}{\partial \sigma} \]  

(2.3)

and the equation of state

\[ P = \rho RT \]  

(2.4)

The notation is conventional and a complete list of symbols is included in the Table of Symbols.

The momentum equation (2.1) does not accurately calculate acceleration for the initial data, because the atmosphere is usually in approximately geostrophic balance and the acceleration is one order less than these geostrophic balance terms (the Coriolis and pressure-gradient forces). The observed winds are often 10-20% in error, and hence the estimated Coriolis force \((-f \mathbf{\varepsilon} \times \mathbf{v})\) may be 10-20% in error at the initial time. The order of error in the Coriolis force is the same as the order of error in acceleration. Therefore, the acceleration computed by using observed winds and geopotential fields will generally be 100% in error (Holton, 1979). To prevent this, the momentum equation is converted to the vertical component of relative vorticity \(\zeta\) and the horizontal divergent \(\delta\) equations (Bourke, 1974):
\[
\frac{\partial \zeta}{\partial t} = -\nabla \cdot (\zeta + f) \mathbf{V} + \mathbf{k} \cdot \nabla \times \left( RT_1 \nabla \ln P_e + \frac{\partial \mathbf{V}}{\partial a} + F_r \right),
\]

and

\[
\frac{\partial \delta}{\partial t} = \mathbf{k} \cdot \nabla \times (\zeta + f) \mathbf{V} - \nabla \cdot \left( RT_1 \nabla \ln P_e + \frac{\partial \mathbf{V}}{\partial a} + F_r \right)
- \nabla^2 \left( \Phi + RT_0 \ln P_e + \frac{1}{2} \mathbf{V} \cdot \mathbf{V} \right),
\]

where \( \zeta = \mathbf{k} \cdot \nabla \times \mathbf{V}, \delta = \nabla \cdot \mathbf{V}. \) To facilitate the incorporation of the semi-implicit time-integration scheme, the temperature has been divided into two parts,

\[
T_1(\lambda, \mu, \sigma, t) = T(\lambda, \mu, \sigma, t) - T_0(\sigma).
\]

The new set of primitive equations on spherical coordinates \((\lambda, \mu, r)\) can be written as

\[
\frac{\partial \zeta}{\partial t} = \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (N_a) - \frac{1}{a} \frac{\partial}{\partial \mu} (N_u) + F_{\zeta},
\]

\[
\frac{\partial \delta}{\partial t} = \frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (N_u) + \frac{1}{a} \frac{\partial}{\partial \mu} (N_a) + F_{\delta}
- \nabla^2 (\Phi + RT_0 \mathbf{q}),
\]

\[
\frac{\partial T_1}{\partial t} = -\frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} (u T_1) - \frac{1}{a} \frac{\partial}{\partial \mu} (v T_1) \cos \phi + F_{T_1}
+ T_1 \delta - \frac{\partial T}{\partial \sigma} + \frac{R}{C_p} T(\frac{\omega}{P}),
\]

\[
\frac{\partial q}{\partial t} = -\delta - \frac{\partial}{\partial \sigma} - \mathbf{V} \cdot \nabla q,
\]
\[ \frac{\partial \Phi}{\partial \sigma} = -RT , \] (2.13)

and

\[ P = \rho RT , \] (2.14)

where \( \mu = \sin \phi \), \( \phi \) - latitude, \( \lambda \) - longitude, \( a \) - radius of the earth.

\[ \eta_u = \left( \zeta + f \right) v - RT \frac{1}{a} \frac{\partial \sigma}{\partial \lambda} - \phi \frac{\partial u}{\partial \sigma} \cos \phi , \] (2.15)

\[ \eta_v = \left[ -(\zeta + f) u - RT \frac{1 - \mu^2}{a} \frac{\partial \sigma}{\partial \mu} - \phi \frac{\partial v}{\partial \sigma} \right] \cos \phi , \] (2.16)

\[ E = \frac{1}{2} (u^2 + v^2) , \] (2.17)

\[ q = \ln P_s , \] (2.18)

\[ \sigma = \sigma \int_{0}^{1} (\delta + \mathbf{v} \cdot \nabla q) \, d\sigma - \int_{0}^{\sigma} (\delta + \mathbf{v} \cdot \nabla q) \, d\sigma , \] (2.19)

and

\[ \frac{\omega}{P} = \mathbf{v} \cdot \nabla q - \frac{1}{\sigma} \int_{0}^{\sigma} (\delta + \mathbf{v} \cdot \nabla q) \, d\sigma . \] (2.20)

\( F_\delta \), \( F_\delta \), and \( F_\tau \) are the horizontal and vertical diffusion of vorticity, divergence and temperature, respectively. The spherical horizontal Laplacian operator is denoted by \( \nabla^2 \):

\[ \nabla^2 = \frac{1}{a^2 (1 - \mu^2)} \frac{\partial^2}{\partial \lambda^2} + \frac{1}{a^2} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial}{\partial \mu} \right] . \] (2.21)
2. The Linear Transient Primitive Equation

To explicitly examine the standing and transient modes, the primitive equations have to be split into these two modes. As indicated in the last chapter, the NCAR CCM can simulate a realistic blocking (Blackmon et al., 1986). Branstator's (1990) model, which is a linearized model from the NCAR CCM, can explicitly handle these two modes as well as different forcings. We, therefore, adopted Branstator's model for this study.

Consider a simplified vorticity equation

$$\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial x}.$$  \hspace{2cm} (2.22)

Decomposing the dependent variable $\zeta$ into a basic state component $\bar{\zeta}$ and a perturbation, $\zeta' = \zeta - \bar{\zeta}$, this equation becomes

$$\frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial \zeta'}{\partial t} = - \left( \bar{u} \frac{\partial \zeta}{\partial x} + \bar{u} \frac{\partial \zeta'}{\partial x} + u' \frac{\partial \bar{\zeta}}{\partial x} + u' \frac{\partial \zeta'}{\partial x} \right). \hspace{2cm} (2.23)$$

The time average of (2.23) is

$$\frac{\partial \bar{\zeta}}{\partial t} = - \left( \bar{u} \frac{\partial \bar{\zeta}}{\partial x} + \bar{u} \frac{\partial \zeta'}{\partial x} \right). \hspace{2cm} (2.24)$$

Subtracting (2.24) from (2.23), we obtain

$$\frac{\partial \zeta'}{\partial t} = - \left( \bar{u} \frac{\partial \zeta'}{\partial x} + u' \frac{\partial \bar{\zeta}}{\partial x} + u' \frac{\partial \zeta'}{\partial x} - u' \frac{\partial \zeta'}{\partial x} \right). \hspace{2cm} (2.25)$$

If we add a dissipative term into (2.25) and replace the time-mean product of perturbations by $R$ (the residual term), we arrive at a linear (transient) approximation of (2.22):
\[ \frac{\partial q'}{\partial t} = -\left( \frac{\partial q'}{\partial x} + u' \frac{\partial q'}{\partial x} + u' \frac{\partial q'}{\partial x} + \alpha' - R \right). \quad (2.26) \]

When this same procedure is applied to primitive equations (2.8)-(2.12), we are able to obtain the transient model

\[ \frac{\partial \zeta'}{\partial t} = -\frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (M_v + L_v) - \frac{1}{a} \frac{\partial}{\partial \mu} (M_u + L_u) \]
\[ - \alpha' \zeta' + K_h \nabla^2 \zeta' + K_v \frac{\partial^2 \zeta'}{\partial \sigma^2} + R \zeta', \]

\[ \frac{\partial \delta'}{\partial t} = -\frac{1}{a(1 - \mu^2)} \frac{\partial}{\partial \lambda} (M_u + L_u) + \frac{1}{a} \frac{\partial}{\partial \mu} (M_v + L_v) + R \delta, \]
\[ - \nabla^2 (\Phi' + R T_0 q' + E') - \alpha' \delta' + K_h \nabla^2 \delta' + K_v \frac{\partial^2 \delta'}{\partial \sigma^2}, \]

\[ \frac{\partial T_1'}{\partial t} = \frac{1}{a \cos \phi} \left( \frac{\partial}{\partial \lambda} (u T_1' + u T_1' + u T_1') - \frac{1}{a} \frac{\partial}{\partial \mu} (v T_1' + v T_1' + v T_1') \cos \phi \right) \]
\[ + \delta' T_1' + \delta' T_1' - \delta' \frac{\partial T_1'}{\partial \sigma} - \delta' \frac{\partial T_1'}{\partial \sigma} - \delta' \frac{\partial T_1'}{\partial \sigma} + \kappa T_1' \left( \frac{\omega}{P} \right)' \]
\[ + \kappa T_0' \left( \frac{\omega}{P} \right)' - \alpha' \frac{\partial T_1'}{\partial \sigma} + \kappa T_1' \left( \frac{\omega}{P} \right)' - \alpha' T_1' + K_v \nabla^2 T_1' + \frac{K_v}{\sigma^2} \nabla^2 \left( \frac{T_1'}{\sigma^2} \right) + R_T, \]

\[ \frac{\partial q'}{\partial t} = -\delta' - \nabla' \cdot \nabla q' - \nabla' \cdot \nabla q' + \Phi', \quad (2.29) \]

and

\[ \Phi' = - \int_0^1 \frac{R T_1'}{\sigma} \, d\sigma, \quad (2.30) \]

where

\[ M_u = \left[ \zeta' \nabla + (\zeta + f) \nabla - \frac{R}{a \cos \phi} \left( T_1 \frac{\partial u'}{\partial \lambda} - T_1' \frac{\partial \zeta'}{\partial \lambda} \right) - \delta \frac{\partial u'}{\partial \sigma} - \delta' \frac{\partial u'}{\partial \sigma} \right] \cos \phi, \]

\[ M_v = \left[ - \zeta' u' - (\zeta + f) u' - \frac{R}{a} \left( T_1 \frac{\partial q'}{\partial \phi} - T_1' \frac{\partial \zeta'}{\partial \phi} \right) - \delta \frac{\partial v'}{\partial \sigma} - \delta' \frac{\partial \zeta'}{\partial \sigma} \right] \cos \phi, \]
\[ L_u = \left( \zeta'v' - \frac{RT_1}{a \cos \phi} \frac{\partial \theta'}{\partial \lambda} - \theta' \frac{\partial u'}{\partial \sigma} \right) \cos \phi, \]

\[ L_v = \left( -\zeta'u' - \frac{RT_1}{a} \frac{\partial \theta'}{\partial \phi} - \theta' \frac{\partial v'}{\partial \sigma} \right) \cos \phi, \]

\[ \kappa = \frac{R}{C_p}, \] is the time average, \[ \overline{x} = \int_{t_1}^{t_2} x \, dt / \int_{t_1}^{t_2} dt, \]

and the vertical integration \[ \overline{x} = \int_0^1 x \, d\sigma, \overline{x^0} = \int_0^a x \, d\sigma. \]

The residual (anomalous perturbation covariance) terms, \( R_{\xi}, R_{\delta}, R_{\zeta}, \) and \( R_q \) are calculated from the time mean primitive equations.

\[ R_{\xi} = \frac{-1}{a(1-\mu^2)} \frac{\partial}{\partial \lambda} \left( \zeta'v' - \frac{RT_1}{a} \frac{\partial \theta'}{\partial \phi} - \theta' \frac{\partial u'}{\partial \sigma} \right) \cos \phi + \Phi_{\xi}, \] (2.32)

\[ = \frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \lambda} \left( -\zeta'U - \frac{RT_1}{a} \frac{\partial \theta}{\partial \phi} - \theta' \frac{\partial U}{\partial \sigma} \right) \cos \phi \]

\[ - \frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \mu} \left[ (\zeta + f) V - \frac{RT_1}{a} \frac{\partial \theta}{\partial \phi} - \theta' \frac{\partial U}{\partial \sigma} \right] \cos \phi, \]

\[ R_{\delta} = \frac{-1}{a(1-\mu^2)} \frac{\partial}{\partial \lambda} \left( \zeta'v' - \frac{RT_1}{a \cos \phi} \frac{\partial \theta'}{\partial \lambda} - \theta' \frac{\partial u'}{\partial \sigma} \right) \cos \phi + \Phi_{\delta}, \] (2.33)

\[ = \frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \mu} \left( -\zeta'u' - \frac{RT_1}{a} \frac{\partial \theta'}{\partial \phi} - \theta' \frac{\partial v'}{\partial \sigma} \right) \cos \phi \]

\[ - \frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \lambda} \left[ (\zeta + f) V - \frac{RT_1}{a \cos \phi} \frac{\partial \theta}{\partial \lambda} - \theta' \frac{\partial U}{\partial \sigma} \right] \cos \phi \]

\[ + \frac{1}{a(1-\mu^2)} \frac{\partial}{\partial \mu} \left[ (\zeta + f) \bar{U} - \frac{RT_1}{a} \frac{\partial \theta}{\partial \phi} - \theta' \frac{\partial V}{\partial \sigma} \right] \cos \phi \]

\[ - \nabla^2 \left[ \frac{1}{2} (\bar{U} \bar{U} + \bar{V} \bar{V}) + \Phi + RT_0 \bar{V} \right], \]
Equations (2.26)-(2.30) are the ones used in the model. The model actually forecasts the perturbation (anomalous) mode, which is a deviation from the standing (time-mean) mode. The numerical aspects of this transient primitive model are illustrated in the next two sections.

B. Numerical Algorithms

The vertical and time-derivative terms of the model are represented by finite-difference approximations, whereas horizontal structures are treated in terms of spherical harmonics functions (Platzman, 1960).

1. The Time Differences

The time differences are basically centered with the terms responsible for fast-moving gravity waves (e.g., divergence terms) treated semi-implicitly (Hoskins and Simmons, 1975). Compared with a fully explicit or centered scheme, the semi-implicit time-integration algorithm, which was first proposed by Robert (1966), (i) allows an
increase in time step from 600 sec to 3600 sec (for R15 resolution); (ii) introduces time-truncation error, which is negligible; and (iii) introduces a computational error of 3% (Bourke et al., 1977).

The vertical and horizontal diffusion terms are treated implicitly using a time-splitting procedure, which is vertical before the advection processes and horizontal after. In general, the prognostic equation for the generic variable Q has the form,

\[
\frac{\partial Q}{\partial t} = F_{QV}(Q) + \Gamma(Q) + F_{QH}(Q). \tag{2.36}
\]

vertical nonlinear horizontal diffusion terms and diffusion forcing

The \( F_{QV} \) and \( F_{QH} \) terms are time-split. The model is designed to provide for time splitting as follows:

\[
Q^{n+1} = Q^n + 2\Delta t F_{QV}^n(Q^{n+1}), \tag{2.37}
\]

\[
Q^{n+1} = Q^n + 2\Delta t \Gamma(Q^n, Q^n, Q^{n+1}, Q^{n+1}), \quad \text{and} \tag{2.38}
\]

\[
Q^{n+1} = Q^{n+1} + 2\Delta t F_{QH}^n(Q^{n+1}). \tag{2.39}
\]

The first step (2.37) represents the calculation of linearized vertical diffusion with coefficients independent of \( \lambda \) and \( \mu \). The superscript \( n \) on the operator \( F_{QV} \) implies that the variables in the coefficients of vertical diffusion are taken at time-level \( n \) and can vary in grid space, whereas the prognostic variable \( Q \) is diffused in \( Q^{n+1} \). The second step (2.44) includes all terms, except \( F_{QV} \) and \( F_{QH} \). The operator \( \Gamma \) uses variables \( Q^n, Q^{n+1}, \) and \( Q^n \) for the explicit part of the computation and
Q for the semi-implicit part. During the second step, the transform method (Orszag, 1970; and Eliasen et al., 1970) is applied to all nonlinear terms (e.g., vδ, vT, Tδ, ...). The spectral representation of these terms may be accomplished by (i) transformation of spectral fields, U_n, T_n, ..., to a two-dimensional latitude-longitude grid on the sphere, (ii) evaluation of the requisite products at each grid point, and (iii) inverse transformation. A detailed description of the transform method is given in the APPENDIX. The third step (2.39) is linear, horizontal diffusion, which is performed in spectral space. The operator \( F_{qH} \) includes only \( Q_{n+1} \), and any coefficients involved are independent of \( \lambda \) and \( \mu \).

After completion of (2.37) to (2.39), a time filter is applied to the prognostic variables \( \zeta, \delta, T, \) and \( \ln p_s \). The time filter was originally designed by Robert (1966) (and later studied by Asselin, 1972) to stabilize the computational mode in the centered time-integration scheme. This time filter provides filtered values of the prognostic variable at time \( n \) after the values at time \( n + 1 \) are computed. The arbitrary variable \( Q \) has the form

\[
Q^n = Q^n + \alpha (Q^{n-1} - 2Q^n + Q^{n+1}),
\]

(2.40)

where \( n \) is the time index, \( \alpha \) is a small coefficient, typically 0.060, and underlining denotes time filtered data. In the model computation, the time filter is applied in two steps because all three time levels are never available simultaneously. At \( t = n \), the second half time-filter for \( t = n \) is

\[
Q^n = Q^n + \alpha Q^{n+1};
\]

(2.41)
and the first half time-filter for \( t = n + 1 \) is
\[
Q_{1}^{n+1} = Q^{n+1} + \alpha(Q^{n} - 2Q^{n+1}).
\]
where the subscript 1 denotes the first half time-filtered quantities.

The first half time filtered data \( Q^n \) is stored in a special buffer and is never used by the model except in the second half time-filter (2.41). The time filtered data, \( Q^n \), is now available for output to history files. At this point, a complete time step has been performed and the logic proceeds back to the starting point described immediately before (2.43)—the vertical diffusion step.

2. Vertical Finite Differences

The vertical discrete grid and the distribution of variables on this grid are shown in Figure 3.

a. Vertical Advection

Vertical advection of momentum and temperature at level \( k \) is approximated by
\[
\left( \frac{\partial Q}{\partial \sigma} \right)_k = \frac{1}{2} \left( \phi_{k+\frac{1}{2}} \frac{Q_{k+1} - Q_k}{\Delta \sigma_k} + \phi_{k-\frac{1}{2}} \frac{Q_k - Q_{k-1}}{\Delta \sigma_k} \right), \quad 1 \leq k \leq K,
\]
where \( \Delta \sigma_k = \sigma_{k+\frac{1}{2}} - \sigma_{k-\frac{1}{2}} \) and \( \phi_{\frac{1}{2}} = \phi_{k,\frac{1}{2}} = 0 \) (boundary condition).

b. Vertical Integrals

The vertical integrals in the \( \sigma \)-equation and in the surface-pressure-tendency equation are given by
\[
\phi_{k,\frac{1}{2}} = \sigma_{k,\frac{1}{2}} \sum_{j=1}^{K} \left( \delta_j + V_j \cdot \nabla q \right) \sigma_j - \sum_{j=1}^{K} \left( \delta_j + V_j \cdot \nabla q \right) \sigma_j, \quad 1 \leq k \leq K-1,
\]
<table>
<thead>
<tr>
<th>Vertical Index</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2, σ = 0</td>
<td>__________</td>
</tr>
<tr>
<td>1, Δσ₁, σ₁</td>
<td>__________</td>
</tr>
<tr>
<td>1 1/2</td>
<td>__________</td>
</tr>
<tr>
<td>2, Δσ₂, σ₂</td>
<td>__________</td>
</tr>
<tr>
<td>2 1/2</td>
<td>__________</td>
</tr>
<tr>
<td>3, Δσ₃, σ₃</td>
<td>__________</td>
</tr>
<tr>
<td>k - 1/2</td>
<td>__________</td>
</tr>
<tr>
<td>k, Δσₖ, σₖ</td>
<td>__________</td>
</tr>
<tr>
<td>k + 1/2</td>
<td>__________</td>
</tr>
<tr>
<td>K - 1</td>
<td>__________</td>
</tr>
<tr>
<td>K - 1/2</td>
<td>__________</td>
</tr>
<tr>
<td>K, Δσₖ, σₖ</td>
<td>__________</td>
</tr>
<tr>
<td>K + 1/2, σ = 1</td>
<td>__________</td>
</tr>
</tbody>
</table>

Figure 3. The vertical discrete grid and the distribution of variables on this grid (after Williamson et al., 1987)
and
\[
\frac{\partial \rho}{\partial t} = -\sum_{j=1}^{J} (\delta_j + v_j \cdot \nabla q) \Delta \sigma_j, \quad q = \ln P_s, \quad 1 \leq k \leq K. \tag{2.45}
\]

**c. Hydrostatic Equation** Integration of the hydrostatic equation to full-index levels gives

\[
\Phi_k = \Phi_s - R \int_{\sigma_1}^{\sigma_k} T \, d\ln \sigma, \tag{2.46}
\]

and

\[
\Phi'_k = -R \int_{\sigma_1}^{\sigma_k} T'_1 \, d\ln \sigma. \tag{2.47}
\]

This can be approximated by

\[
\Phi'_k = R \sum_{j=1}^{K} B_{kj} T'_{1j}. \tag{2.48}
\]

The matrix B is triangular (B_{kj} = 0 for j < k). T_1' is assumed to vary linearly with \ln \sigma between full-index levels and with extrapolation to the ground, assuming an isothermal first-half layer. This relation gives for the first level k = K,

\[
\Phi'_k = R T'_{1K} (-\ln \sigma_K), \tag{2.49}
\]

and for k < K,

\[
\Phi'_k = \Phi'_{k+1} + \frac{R}{2} \ln \frac{\sigma_{k+1}}{\sigma_k} (T'_{1,k+1} + T'_{1k}). \tag{2.50}
\]
The matrix \( B \) is thus given by

\[
\begin{bmatrix}
\frac{1}{2} \ln \frac{\sigma_2}{\sigma_1} & \frac{1}{2} \ln \frac{\sigma_2}{\sigma_1} + \ln \frac{\sigma_1}{\sigma_2} & B_{23} & \cdots & B_{K-1,K} \\
0 & \frac{1}{2} \ln \frac{\sigma_3}{\sigma_2} & \frac{1}{2} \ln \frac{\sigma_3}{\sigma_2} + \frac{1}{2} \ln \frac{\sigma_4}{\sigma_3} & \cdots \\
0 & 0 & \frac{1}{2} \ln \frac{\sigma_4}{\sigma_3} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \frac{1}{2} \ln \frac{\sigma_K}{\sigma_{K-1}} + \frac{1}{2} \ln \frac{\sigma_K}{\sigma_{K-1}} - \ln \sigma_K \\
0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\]  

\[(2.51)\]

d. **Energy Conversion Term**

The vertical integral associated with \( \frac{\omega}{P} \) in the conversion term of the thermodynamic equation can be written in a general form as

\[
(-\frac{\omega}{P})_k = [V_k \cdot \nabla q - \sum_{j=1}^{k} C_{kj} (\delta_j + V_j \cdot \nabla q)],
\]

where matrix \( C_{kj} = B_{jk} \frac{\Delta \sigma_j}{\Delta \sigma_k} \).

3. **The Horizontal Spectral Representation**

The concepts and notations of the truncated series of spherical harmonic functions described in this section are all standard. The reader may refer to other studies, e.g., Platzman (1960), Williamson et al. (1987), or any of several standard applied mathematics textbooks.

For an arbitrary variable \( Q \), it can be written in the truncated series form

\[
Q(\lambda, \mu) = \sum_{m=-M}^{M} \sum_{n=|m|}^{N} \Phi_n^m(\lambda, \mu) Y_n^m(\lambda, \mu).
\]

\[(2.53)\]
The base functions, $Y_n^m(\lambda, \mu) = P_n^m(\mu) e^{im\lambda}$, are the spherical harmonics, where $m$ is the order (the Fourier wavenumber included in the east-west representation) and $n$ is the degree of the associated Legendre functions included in the north-south representation. The normalized associated Legendre polynomials, $P_n^m$, for order $m$ degree $n$ has the form

$$P_n^m = \left[ \frac{2n + 1}{2} \frac{(n - m)!}{(n + m)!} \right]^{\frac{1}{2}} \left\{ \frac{1}{2^n n!} \right\} \frac{d^{n-m}}{d\mu^{n-m}} (\mu^2 - 1)^n,$$

and

$$\int_{-1}^{1} [P_n^m(\mu)]^2 d\mu = 1,$$

where $\mu = \sin\Phi = \cos\theta$. The $\theta$ is colatitude, $\phi$ the latitude.

The model is coded for a rhomboidal truncation illustrated in Figure 4. The coefficients of the spectral representation (2.60) are given by

$$Q_n^m = \int_{-1}^{1} \left[ \frac{1}{2\pi} \int_{0}^{2\pi} Q(\lambda, \mu) e^{-im\lambda} d\lambda \right] P_n^m(\mu) d\mu.$$  

(2.56)

The inner integral represents a Fourier transformation,

$$Q^m(\mu) = \frac{1}{2\pi} \int_{0}^{2\pi} Q(\lambda, \mu) e^{-im\lambda} d\lambda,$$

(2.57)

which is performed by a Fast Fourier Transform (FFT991) subroutine at the NCAR. The outer integral is performed via Gaussian quadrature:

$$Q_n^m = \sum_{j=1}^{J} Q^m(\mu_j) P_n^m(\mu_j) w_j,$$

(2.58)

where $\mu_j$ denotes Gaussian grid points in the meridional direction, $w_j$ the Gaussian weight at point $\mu_j$, and $J$ the number of Gaussian grid
Figure 4. The rhomboidal (OABC) and triangular (OAC) truncation parameters. $M$ is the zonal wave number, $N-M$ the meridional wave number.
points from pole to pole. Gaussian grid points ($\mu_j$) are given by the roots of the Legendre polynomial $P_j(\mu)$, and the corresponding weights are given by

$$w_j = \frac{2(1-\mu_j^2)}{(J P^0_{j-1}(\mu_j))^2}. \quad (2.59)$$

The weights themselves satisfy

$$\sum_{j=1}^{J} w_j = 2.0. \quad (2.60)$$

The Gaussian grid used for the north-south transformation is generally chosen to allow unaliased computations of quadratic terms only. For rhomboidal truncation, the number of Gaussian latitudes, $J$, must satisfy

$$J \geq \frac{(3N + 2M + 1)}{2}. \quad (2.61)$$

Furthermore, to allow exact Fourier transformation of quadratic terms, the number of points, $I$, in the east-west direction must satisfy

$$I \geq 3M + 1. \quad (2.62)$$

The actual values of $J$ and $I$ are not often set to the lower limit to allow use of more efficient transform programs. For R15 truncation, $I = 48$, and $J = 40$.

Finally, the spectral transform method is provided in the Appendix.
G. Details of Model Algorithm

In the previous sections, the model's numerical algorithms indicating both time-stepping procedures and spacial structure are described in general terms. In this section, we present the details of the formulas used by the model with an emphasis on the semi-implicit algorithm. Reference is made to the vertical finite differences, time filter, and the transformations between grid and spectral space described in the preceding sections.

1. Time-Split Vertical Diffusion

The first sub-time step of time integration includes the vertical diffusion of three prognostic variables \((\zeta', \delta', \text{ and } T_1')\), which are treated implicitly in time and spectral form in space. The time-split vertical diffusion equations of these three variables have the forms

\[
\frac{\partial \zeta'}{\partial t} = K_v \frac{\partial^2 \zeta'}{\partial \sigma^2} - F_{\zeta v}(\zeta'),
\]

(2.63)

\[
\frac{\partial \delta'}{\partial t} = K_v \frac{\partial^2 \delta'}{\partial \sigma^2} - F_{\delta v}(\delta'),
\]

(2.64)

and

\[
\frac{\partial T_1'}{\partial t} = K_v \frac{\partial^2}{\partial \sigma^2} \left( \frac{T_1'}{\sigma^2} \right) - F_{T v}(T_1'),
\]

(2.65)

where \(K_v\) is the vertical diffusion coefficient. The vertical distribution of \(K_v\) is listed in Table 1. \(K_v\) has an e-folding time about four days, the spin-down time scale in the planetary boundary
Table 1. The horizontal and vertical diffusion coefficients at $\sigma$-level\textsuperscript{a}

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\alpha_R$</th>
<th>$e^{-1}$ time of $\alpha_R$</th>
<th>$\alpha_N$</th>
<th>$e^{-1}$ time of $\alpha_N$</th>
<th>$K_H$</th>
<th>$K_V$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x10^-7)</td>
<td>(days)</td>
<td>(x10^-7)</td>
<td>(days)</td>
<td>(x10^5)</td>
<td>(x10^-9)</td>
</tr>
<tr>
<td>1</td>
<td>0.009</td>
<td>30</td>
<td>4</td>
<td>30</td>
<td>4.5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>0.074</td>
<td>30</td>
<td>4</td>
<td>24</td>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>0.189</td>
<td>5</td>
<td>24</td>
<td>15</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>0.336</td>
<td>5</td>
<td>24</td>
<td>12</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0.500</td>
<td>5</td>
<td>24</td>
<td>12</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>0.664</td>
<td>5</td>
<td>24</td>
<td>12</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>0.811</td>
<td>5</td>
<td>24</td>
<td>20</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>0.926</td>
<td>10</td>
<td>12</td>
<td>24</td>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>9</td>
<td>0.991</td>
<td>12</td>
<td>10</td>
<td>24</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Units of $\alpha_R$, $\alpha_N$, and $K_V$ are s\textsuperscript{-1}; and $K_H$ is m\textsuperscript{2}s\textsuperscript{-1}. 
layer (Holton, 1979), and about 20 days in the upper troposphere and stratosphere.

The finite-difference form of the vertical diffusion equation at level \( k \) can be written as

\[
Q_k^{n+1} - Q_k^n = \frac{2\Delta t K_v}{\Delta \sigma_k} \left( \frac{r_{k+1} Q_k^{n+1} - r_k Q_k^{n+1}}{\Delta \sigma_{k+\frac{1}{2}}} - \frac{r_k Q_k^{n+1} - r_{k-1} Q_k^{n+1}}{\Delta \sigma_{k-\frac{1}{2}}} \right)
\]

(2.66)

or

\[
Q_k^{n+1} = Q_k^n + x_k r_{k+1}^{n+1} - (x_k + y_k) r_k Q_k^{n+1} + y_k r_{k-1} Q_k^{n+1},
\]

where

\[
(2.67)
\]

The matrix form of (2.67) is \( \mathbf{A} Q^{n+1} = Q^n \),

where

\[
\mathbf{A} = \begin{bmatrix}
1 + x_1 r_1 & -x_1 r_2 & 0 & \cdots & 0 & 0 \\
-y_2 r_1 & 1 + r_2 (x_2 + y_2) & -x_2 r_3 & \cdots & 0 & 0 \\
0 & -y_3 r_2 & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 + r_{K-1} x_{K-1} \\
0 & 0 & 0 & \cdots & -y_K r_1 & 1 + y_K r_K
\end{bmatrix}
\]

(2.68)

Since the model actually forecasts vorticity, divergence, and temperature, that is \( \zeta^{n+1} \), \( \delta'^{n+1} \), and \( t_1^{n+1} \), respectively, the new
diffused quantities are not explicitly carried by the model. Rather, the net effects of the diffusion $F_{\xi_{\nu}}(\zeta^{n+1})$, $F_{\delta_{\nu}}(\delta^{n+1})$, $F_{\nu}(T^{n+1})$ are saved for addition to the nonlinear term $\Gamma(Q)$. Therefore, the vertical diffusion terms at level $k$ are written as

$$F_{\xi_{\nu}}(\zeta^{n+1}_k) = \frac{1}{2\Delta t} (\zeta^{n+1}_k - \zeta^{n-1}_k), \quad (2.69)$$

$$F_{\delta_{\nu}}(\delta^{n+1}_k) = \frac{1}{2\Delta t} (\delta^{n+1}_k - \delta^{n-1}_k), \quad (2.70)$$

and

$$F_{\nu}(T^{n+1}_k) = \frac{1}{2\Delta t} (T^{n+1}_k - T^{n-1}_k). \quad (2.71)$$

2. Semi-Implicit Time Integration

The semi-implicit time-difference scheme is applied to the terms responsible for fast moving gravity waves, e.g., the divergence term. Hence, the vorticity equation (2.26) in finite difference form is purely explicit:

$$\zeta^{n+1} = \zeta^{n-1} + 2\Delta t \left[ \frac{1}{\cos^2\phi} \frac{\partial}{\partial \lambda} (M_\nu + L_\nu) n - \frac{1}{a} \frac{\partial}{\partial \mu} (M_u + L_u) n \right]$$

$$+ 2\Delta t \left[ F_{\xi_{\nu}}(\zeta^{n+1}) + R_\xi \right]. \quad (2.72)$$

But the divergence equation (2.27), after substitution of the hydrostatic equation (2.30), contains implicit terms:

$$\delta^{n+1} = \delta^{n-1} + 2\Delta t \left[ \frac{1}{\cos^2\phi} \frac{\partial}{\partial \lambda} (M_\nu + L_\nu) n - \frac{1}{a} \frac{\partial}{\partial \mu} (M_u + L_u) n \right]$$

$$+ 2\Delta t \left[ F_{\delta_{\nu}}(\delta^{n+1}) + R_\delta \right] - 2\Delta t \nabla^2 E^n$$

$$- 2\Delta t \nabla^2 \left[ R \frac{1}{2} (T^{n+1}_1 + T^{n-1}_1) + \frac{RT_0}{2} (q^{n+1} + q^{n-1}) \right]. \quad (2.73)$$
The thermodynamic equation (2.28) is also semi-implicit. The terms in $\delta \frac{\partial T_0}{\partial \sigma}$ and $\kappa T_0 \left( \frac{\omega}{P} \right)'$ involving $\delta$ multiplied by a global mean temperature ($\overline{T_0}$) are treated implicitly, and all other terms are treated explicitly:

$$
T_{1}^{n+1} = T_{1}^{n-1} + 2\Delta t \left[ \frac{1}{\alpha \cos \phi} \frac{\partial}{\partial \lambda} \left( \overline{u''T'} + u'\overline{T'} + v'\overline{T'} \cos \phi \right) + \delta' T_{1}^n + \delta' T'_{1} \right. \\
- \frac{1}{a} \frac{\partial}{\partial \mu} \left( \overline{v''T'} + v'\overline{T'} + v'\overline{T'} \cos \phi \right) + \delta' T_{1}^n + \delta' T'_{1} \\
+ T_2 + F_{T_0}(T^{n+1}) + R'_T - 2\Delta t \sum_{j=1}^{K} \tau_{ij} T_{j}^n \right],
$$

where nonlinear terms

$$
(T2)_k = - \left( \delta \frac{\partial T_1'}{\partial \sigma} + \delta' \frac{\partial T_1'}{\partial \sigma} + \delta' \frac{\partial T_1'}{\partial \sigma} \right)^n + \overline{T_0} \left( Q_k - \sum_{j=1}^{K} C_{kj} Q_j \right)_k \\
+ \kappa \left[ \overline{T_1'} \left( \frac{\omega}{P} \right)' + T_1' \left( \frac{\omega}{P} \right)' \right]^n \overline{T_0} \left( \sigma_{j=1}^{K} Q_j \sigma_j - \sum_{j=1}^{K} C_{kj} \sigma_j \right)_k \\
- \frac{\partial \overline{T_0}}{\partial \sigma} \left( \sigma_{j=1}^{K} Q_j \sigma_j - \sum_{j=1}^{K} C_{kj} \sigma_j \right)_k,
$$

and

$$
\sum_{j=1}^{K} \tau_{ij} \delta_j^j = \kappa T_{0k} \sum_{j=1}^{K} C_{kj} \delta_j^j \\
+ \frac{T_{0k+1} - T_{0k}}{2\Delta \sigma} \left( \sigma_{j=1}^{K} \delta_j^j \sigma_j - \sum_{j=1}^{K} \delta_j^j \sigma_j \right) \\
+ \frac{T_{0k} - T_{0k-1}}{2\Delta \sigma} \left( \sigma_{j=1}^{K} \delta_j^j \sigma_j - \sum_{j=1}^{K} \delta_j^j \sigma_j \right),
$$

where

$$
\delta_j^j = \frac{1}{2} (\delta_{j}^{n+1} + \delta_{j}^{n-1}).
$$
The thermodynamic equation can now be written as the matrix,

\[ T^{n+1} = T^n - 2\Delta t Y + 2\Delta t \frac{1}{2} (\delta^{n+1} + \delta^{n-1}) \]

(2.78)

where \( Y \) includes all explicit terms at time level \( n \) in (2.74).

The divergence term in the surface-pressure-tendency equation is also treated implicitly.

\[ q^{n+1} = q^n - 2\Delta t \left[ \sum_{j=1}^{n} \xi \Delta \sigma_j + R_T + \frac{1}{2} (\delta^{n+1} + \delta^{n-1}) \Delta \sigma_j \right] \]

(2.79)

Therefore, equations (2.72), (2.73), (2.78), and (2.79) make up a complete set of semi-implicit time-integration equations set.

3. Solution of Semi-Implicit Equations

The vorticity equation is solved explicitly in spectral space:

\[ (\zeta^{n+1})^m_n = \sum_{j=1}^{n} \left[ \zeta^m_j(\mu_j) \right]^{n-1} + \sum_{j=1}^{n} \left[ \zeta^m_j(\mu_j) \right]^{n-1} \frac{P^m_n(\mu_j)}{\mu_j} \]

(2.80)

\[ + 2\Delta t \left[ \frac{M_L + L_L}{1 - \mu^2} \right] + \frac{H_n^m(\mu_j)}{a(1 - \mu^2)} \]

\[ + 2\Delta t \left[ F^m_T(\zeta^{n+1}(\mu_j)) + R^m_T(\mu_j) \right] \]

The divergence is implicated in equation (2.73) and is coupled with temperature and surface-pressure. By substituting \( T^{n+1} \) (2.78) and \( q^{n+1} \) (2.79) into (2.73) and by using the \( \nabla^2 \) operator, the divergence equation can be written as

\[ \delta^{n+1} = \delta^{n-1} + 2\Delta t \chi \]

\[ - 2\Delta t \nabla^2 \left[ R \left[ T^{n-1} + \frac{1}{2} \Delta t G(\delta^{n+1} + \delta^{n-1}) \right] \right] \]

(2.81)

\[ - 2\Delta t \nabla^2 \left[ R \frac{1}{\Delta t} \left[ q^{n-1} - \delta^{n+1} + \delta^{n-1} \right] \right] \]
where $X$ and $Z$ represent all explicit terms at time level $n$ in the divergence and surface-pressure equation, respectively. After some manipulation, (2.81) can be written as

$$\left[ 1 + \Delta t^2 R \frac{l(1+1)}{a^2} (BG + \overline{T}_0 I_d) \right] \delta^{m+1}$$

$$= 2 \Delta t x^n + \left[ 1 - \Delta t^2 R \frac{l(1+1)}{a^2} (BG + \overline{T}_0 I_d) \right] \delta^{m-1}$$

$$+ \Delta t R \frac{l(1+1)}{a^2} \left[ B (2 \underline{T}_{7}^{n-1} + 2 \Delta t Y^{7}) + \overline{T}_0 (2 \underline{Q}_{7}^{n-1} + 2 \Delta t E^{7}) \right]$$

or a matrix equation \( AD^{m+1} = F \),

where 1 is a unit vector of dimension $K$ and \( I_d = (\Delta \sigma_1, \Delta \sigma_2, \ldots, \Delta \sigma_K) \).

Note that the notation of the degree of spectral coefficients in (2.82) has been changed to 1 to distinguish it from the time level $n$.

After an inverse transform of matrix $A$, the new time step of divergence $\delta^{m+1}$ is solved. With $\delta^{m+1}$ available, $T^{m+1}$ and $q^{m+1}$ are obtained from (2.78) and (2.79), respectively. Therefore, the semi-implicit sub-time step is completed at this point.

4. Time-Split Horizontal Diffusion

The horizontal diffusion (2.43) is computed implicitly by using the following form:

$$\zeta^{m+1} - \zeta^{m-1} + 2 \Delta t \left\{ - \alpha_R - K_\xi \left[ \frac{l(l+1)}{a^2} - \frac{2}{a^2} \right] \right\} \zeta^{m+1}, \quad (2.83)$$

$$\delta^{m+1} - \delta^{m-1} + 2 \Delta t \left\{ - \alpha_R - K_\delta \left[ \frac{l(l+1)}{a^2} - \frac{2}{a^2} \right] \right\} \delta^{m+1}, \quad (2.84)$$
and

$$T_1^{n+1} = T_1^{n+1} + 2\Delta t \left[-\alpha_T - K_T \frac{1(1 + 1)}{\alpha^2}\right] T_1^{n+1}. \quad (2.85)$$

The extra term, $\frac{2}{\alpha^2}$, in (2.83) and (2.84) is added to prevent damping of uniform rotations (Orszag, 1974; Bourke et al., 1977). The model's damping coefficients due to Rayleigh friction, $\alpha_R$, and Newtonian cooling, $\alpha_T$ (Table 1), are typical values used in many studies, e.g., Leovy (1964), Matsuno (1970), Hoskins and Karoly (1981). The simple $\alpha$ term is used to represent the effects of the boundary layer and of damping due to radiation and the influence of transient eddies (Lau, 1979). Relatively larger values of $\alpha_R$, however, are used at the top two levels to reduce possible energy reflection by the effectively "rigid lid" arising from applying the fixed upper boundary condition (Chen and Trenberth, 1988a).

The horizontal diffusion is a linear $\nabla^2$ form on the $\sigma$-surface throughout the entire model (e.g., Branstator, 1990). Boville (1984) indicated that the $\nabla^2$ form improves the NCAR CCM0 model simulation in the upper few levels of the model. The horizontal diffusion coefficients ($K_H$, Table 1) basically follow the NCAR CCM1 (Williamson et al., 1987), except those with relatively large values for the lowest two levels. The larger coefficients are to parameterize roughly the effects of surface friction (e.g., Charney and Eliassen, 1949; Alpert et al., 1983).

After the damping and diffusion coefficients are specified, the solutions of (2.83), (2.84), and (2.85) are
Therefore, an entire forecast time step has been completed at this point. The model then performs the time-filter processes described in (2.47) and (2.48) and goes back to the starting point at (2.43) to calculate the next time step.

D. Initial and Boundary Data

1. The Initial Data

Before the first time-step of model simulation, the initial data required by the model are

- standing mode data, only in grid space, \( \zeta, \bar{\zeta}, \bar{\xi}, \bar{\eta}, \bar{\nu}, \bar{\xi}, \bar{\sigma}, (\frac{\omega}{\rho}) \);
- two time levels of transient mode data, one in spectral and the other in grid space:
  - \( t = n - 1 \), in spectral space, \( \zeta^{m,n-1}, \delta^{m,n-1}, T^{m,n-1}, q^{m,n-1} \), and
  - \( t = n \), in grid space, \( \zeta', \delta', T', q', u', v', \theta', (\frac{\omega}{\rho})' \);
- anomalous residual forcing, in spectral space,
  \( R_{\zeta}^{m}, R_{\delta}^{m}, R_{T}^{m}, R_{q}^{m} \).
The perturbation mode is a composite over a period of many blocking episodes. The standing mode is a composite over the non-blocking period. The criterion for a blocking episode is defined by a combination of both Dole and Gordon (1983) and Mullen (1986). The definition of blocking according to Dole and Gordon is that for a band-average (50°N-60°N) of 200 mb height field, the anomaly (Δz) at any longitudinal point is equal to or greater than 100 m and for seven days or more. Mullen's definition states that for the same band-average (50°N-60°N), the value at a center longitude exceeding the mean of the entire sector (±45° longitudes from the center longitude) by more than 100 m continuously for at least seven days. In 11 winters (1976/77-1986/87), 34 cases satisfy either one of these two blocking criteria in the Pacific sector and 37 in the Atlantic sector (Figure 1).

Two sets of 18 cases are selected in this study for the Pacific and Atlantic blocking composites (Table 2). The criterion for the selected Pacific blocking cases is that their center be located around 135°W and for Atlantic cases, around 25°W. These two meridian locations represent the greatest number of blocking events in the Pacific and Atlantic sectors (Figure 1a), respectively. The nonblocking composite is a time-average over the period between these blocking episodes. There are a set of nonblocking composites for both Pacific and Atlantic cases.

In turn, the first time-step of the model is forward semi-implicit rather than centered semi-implicit, so only the variables at t = 0 are needed. The model performs this forward step by setting the variables
Table 2. The onset and duration (days) of selected blocking and nonblocking episodes for Pacific and Atlantic composite blocking cases

<table>
<thead>
<tr>
<th>Pacific blocking</th>
<th>Atlantic blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Blocking</strong></td>
<td><strong>Nonblocking</strong></td>
</tr>
<tr>
<td>onset (days)</td>
<td>onset (days)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>12/27/78</td>
</tr>
<tr>
<td>2</td>
<td>01/19/80</td>
</tr>
<tr>
<td>3</td>
<td>02/05/80</td>
</tr>
<tr>
<td>4</td>
<td>12/28/80</td>
</tr>
<tr>
<td>5</td>
<td>01/12/81</td>
</tr>
<tr>
<td>6</td>
<td>02/01/82</td>
</tr>
<tr>
<td>7</td>
<td>02/21/82</td>
</tr>
<tr>
<td>8</td>
<td>01/14/83</td>
</tr>
<tr>
<td>9</td>
<td>12/11/83</td>
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<tr>
<td>10</td>
<td>01/07/84</td>
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<td>11</td>
<td>01/26/84</td>
</tr>
<tr>
<td>12</td>
<td>12/12/84</td>
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<tr>
<td>13</td>
<td>01/14/85</td>
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<tr>
<td>14</td>
<td>12/07/85</td>
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<tr>
<td>15</td>
<td>01/04/86</td>
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<td>16</td>
<td>02/04/86</td>
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<td>12/04/86</td>
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<td>02/05/87</td>
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<td>02/21/86</td>
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<tr>
<td>20</td>
<td>12/15/86</td>
</tr>
<tr>
<td>21</td>
<td>01/23/87</td>
</tr>
</tbody>
</table>
at time $t = -\Delta t$ equal to those at $t = 0$ and by temporarily dividing $2\Delta t$
by 2 for this time step only. This is done essentially by a centered
semi-implicit but with a half of the regular time interval, so that no
additional code is needed for this step. After the first time-step, the
time interval is reset to its original value, and the model computation
is centered semi-implicit for the rest of the simulation.

2. The Boundary Data

The boundary condition is needed at $\sigma = 0$, the top of the model,
and at $\sigma = 1$, the surface of the earth (Figure 3). The boundary
conditions for the $\sigma$ vertical velocity $\sigma$ are $\sigma = 0$ at both $\sigma = 0$ and
$\sigma = 1$. The upper boundary condition is simply $\sigma = 0$. The lower
boundary condition, however, is more complicated than the upper one
because it also involves surface pressure, which is usually not reported
by the observational data and which needs to be estimated from the
hydrostatic relation by using height, temperature, and terrain
distribution. The topographic data are adopted from the NCAR CCM with
an RL15 truncation in spectral space. With topography available, the
initial time-step of surface pressure can be estimated through the
hydrostatic relation from the observational height field on the isobaric
surface. Following is the description of such a procedure:

The discrete form of the hydrostatic relation is

$$\Delta \ln P = -\frac{\Delta \Phi}{RT} \quad (2.89)$$
Using levels $s$ and $k$ (Figure 5), (2.89) can be written as

$$\ln \frac{P_s}{P_k} = -\frac{\Phi_s - \Phi_k}{RT^{k+\frac{1}{4}}},$$

or

$$P_k = P_s \exp\left(-\frac{\Phi_k - \Phi_s}{RT^{k+\frac{1}{4}}}\right).$$

(2.90a)

(2.90b)

Using level $k + 1$ and $s$,

$$P_{k+1} = P_s \exp\left(-\frac{\Phi_{k+1} - \Phi_s}{RT^{k+\frac{1}{4}}}\right).$$

(2.91)

(2.91) divided by (2.90) becomes

$$\frac{P_{k+1}}{P_k} = \exp\left(-\frac{\Phi_{k+1} - \Phi_k}{RT^{k+\frac{1}{4}}}\right).$$

(2.92)

Hence, we can find $T_{k+\frac{1}{2}}$ from (2.92):

$$T_{k+\frac{1}{4}} = \frac{\Phi_{k+\frac{1}{4}} - \Phi_k}{R \ln \left(\frac{P_{k+1}}{P_k}\right)}.$$

(2.93)

Substituting (2.93) into (2.91),

$$P_{k+1} = P_s \exp\left[-\left(\frac{\Phi_{k+1} - \Phi_s}{\Phi_{k+1} - \Phi_k}\right) \ln \frac{P_k}{P_{k+1}}\right].$$

(2.94)

Therefore,

$$P_s = P_{k+1} \exp\left(\frac{\Phi_{k+1} - \Phi_s}{\Phi_{k+1} - \Phi_k} \ln \frac{P_k}{P_{k+1}}\right)$$

- $$P_{k+1} \left(\frac{P_k}{P_{k+1}}\right)^{\frac{\Phi_{k+1} - \Phi_s}{\Phi_{k+1} - \Phi_k}}$$

- $$= P_k^{\Phi_{k+1} - \Phi_k} P_{k+1}^{\Phi_s - \Phi_k}. $$

(2.95)
Figure 5. The schematic diagram of the relationship among surface pressure, geopotential height, and temperature.
Finally, surface pressure can be estimated from the observed geopotential height field on the isobaric surface.

E. Experiments and Forcings

To investigate these three forcings: local, transient, and remote (tropical) forcings, we have to perform a series of numerical experiments with these prescribed forcings. The local forcing is represented by the residual terms of the model equations. The transient forcing is a time-mean of the synoptic-synoptic scale wave interactions, and tropical heating is represented by the outgoing longwave radiation (OLR) with vertical distribution parameters. Following is a detailed description of the procedures used to achieve these forcings.

1. Residual (Local) Forcing Anomaly

The model only simulates the anomaly (transient) part of the blocking flows. The term anomaly is defined as the difference between the blocking and nonblocking composite. Therefore, the standing mode of the model is the nonblocking composite, and a departure from this non-blocking composite is the transient (anomaly) mode. These anomaly flows are described below:

Again, the simplified time-mean vorticity equation has the form

\[
\frac{\partial \zeta}{\partial t} = - \nabla \cdot (\zeta + f) \mathbf{V} - \nabla \cdot (\zeta \nabla) + F_x = R + S + F_t \quad (2.96)
\]
where $R$ is the product of the time-mean mode, $S$ is the time-mean of the product of perturbations, and $F$ is the friction and other terms. Hence, the blocking composite flows can be expressed as

\[
\frac{\partial \overline{\zeta}_b}{\partial t} = 0 - \overline{R}_b + \overline{S}_b + \overline{F}_b,
\]

and the nonblocking composite flows are defined as

\[
\frac{\partial \overline{\zeta}_n}{\partial t} = 0 - \overline{R}_n + \overline{S}_n + \overline{F}_n,
\]

where $\overline{T} = \frac{1}{T} \int_0^T \eta \, dt$ and where subscripts $b$ and $n$ denote the blocking and nonblocking composites, respectively.

Assume that the blocking composite consists of the nonblocking composite (standing mode) and the difference between the blocking and the nonblocking composite. Thus, the blocking composite can be illustrated by

\[
\overline{\zeta}_b = \overline{\zeta}_n + \zeta'_b, \quad \overline{V}_b = \overline{V}_n + \overline{V}'_n,
\]

Substituting (2.99) into (2.97), we obtain

\[
\frac{\partial \overline{\zeta}_b}{\partial t} - \frac{\partial \overline{\zeta}_n}{\partial t} = \frac{\partial \zeta'_b}{\partial t} - \overline{R}_n + \overline{R}'_b + \overline{S}_n + \overline{S}'_b + \overline{F}_n + \overline{F}'_b.
\]

Subtracting (2.98) from (2.100), we arrive at a blocking anomaly (transient) vorticity equation:
\[ \frac{\partial \zeta'_b}{\partial t} = \mathbf{R}'_b + S'_b + F'_b - 0. \]  

(2.101)

The anomaly blocking residual term is thus

\[ \mathbf{R}'_b = - (S'_b + F'_b) - \mathbf{R}_b - \mathbf{R}_n. \]  

(2.102)

In the model simulation, the model starts at an arbitrary perturbation, say \( \zeta'_e \). Therefore, (2.101) becomes

\[ \frac{\partial \zeta'_e}{\partial t} = \mathbf{R}'_e + (S'_b + F'_b) - \mathbf{R}'_e - \mathbf{R}'_b, \]  

(2.103)

where \( \mathbf{R}'_e = - \nabla \cdot (\zeta'_e \mathbf{V}_e - \zeta'_n \mathbf{V}'_e - \zeta'_n \mathbf{V}'_e) \). Note that (2.103) is the same as the transient vorticity equation (2.26) illustrated in the first section of this chapter. With the prescribed forcing anomaly \( \mathbf{R}'_b \) available, steady blocking anomaly flows can be obtained when the model reaches its equilibrium state, i.e., \( \frac{\partial \zeta'_e}{\partial t} = 0 \); and hence \( \zeta'_e = \zeta'_b \), \( \mathbf{V}'_e = \mathbf{V}'_b \), and \( \mathbf{R}'_e = \mathbf{R}'_b \). Apply the same procedures to the divergence, thermodynamic, and surface-pressure-tendency equation (2.9)-(2.11), and the transient primitive equations (2.27)-(2.30) represent the blocking anomaly flows. This equations set is linear in time but nonlinear in space.

The residual forcing anomaly \( \mathbf{R}'_b \) of (2.102) is the difference between the residual forcing of the blocking and nonblocking composites. As stated in (2.38)-(2.41), these blocking residual forcings are directly calculated from the one-time-step blocking composite and nonblocking composite fields. Note that to reduce the error and uncertainty near the polar area caused by the boundary condition and the north-south derivatives near the poles, the residual forcings have been smoothed and reduced over these areas. Hence, the maximum/minimum
centers over these regions are moved somewhat equatorward. The spatial structures of these residual forcings are displayed on Figure 6.

The horizontal and vertical distributions of the vorticity residual forcing anomaly (R'\(\mathbf{f}\), Figures 6a and b) show a quarter wavelength out of phase with the observed streamfunction anomaly for both Pacific and Atlantic cases (Figures 9 and 22, respectively). Furthermore, these patterns are dominated by a long-wave regime. This is consistent with Metz's (1986) finding from a blocking stochastic forcing.

On the other hand, the horizontal distribution of thermodynamic residual forcing anomaly (R'\(T\), also including heating anomaly; Figures 6c and d) shows a chaotic pattern (a white noise). Its longwave mode (T8 truncated resolution, Figure 7), however, displays a very systematic pattern, basically representing the land-sea distribution and an emphasis over the blocking area. These smoothed R'\(T\) distributions are similar to the winter-mean residual heating calculated by Lau (1979) and Chen and Trenberth (1988b), especially over the areas of storm track. A significant difference between our blocking heating anomaly R'\(T\) and their winter-mean residual heating, however, is found in the blocking area. For example, a striking warming over the eastern and northern parts of the blocking center and a cooling over the western and southern parts of the blocking center are shown in Figures 6c and 6d for the Pacific and Atlantic cases, respectively, but these are not found in either Lau's or Chen and Trenberth's results.
Figure 6. The 200 and 850 mb horizontal distribution and vertical cross-section of anomaly residual forcing for both Pacific (a) $R'_{\zeta}$, (c) $R'_{\eta}$, (e) $R'_{\delta}$, and Atlantic (b) $R'_{\zeta}$, (d) $R'_{\eta}$, (f) $R'_{\delta}$ cases. The contour interval is 30 m$^2$s$^{-2}$ in (a), (b), (e), and (f); $5.0 \times 10^{-5}$ °Ks$^{-1}$ in (c) and (d). Positive areas are hatched.
Figure 6. (continued)
Figure 6. (continued)
Figure 7. The thermodynamic residual forcing ($R_T'$) with T8 truncated resolution at (a) 200 mb, (b) 850 mb for Pacific case and at (c) 200 mb and (d) 850 mb for Atlantic case. The contour interval is $0.2 \times 10^{-8}$ °Ks⁻¹. Positive areas are hatched.
Furthermore, the effect of local cooling over the North Pacific and Atlantic Ocean have been discussed by Namias (1964) and Douglas et al. (1982), respectively. The effect of a global distribution of anomaly heating on the formation of blocking has not been studied yet, however. Although Chen and Trenberth (1988b) used winter-mean diabatic heating to study the maintenance of the standing planetary-scale waves, their heating are very deficient over polar and tropical areas.

Concerning the residual term of the pressure-tendency equation, R'q (not shown), R'q displays a pattern very close to that of the R'T in the lower troposphere in both Pacific and Atlantic cases. This implies that the effect of R'T and R'q on the formation of a block is opposite because the cooling center should be accompanied by a high pressure tendency, and vice versa. The distribution of the anomaly divergence residual forcing (R'\delta, Figures 6e and 6f), however, is consistent with R'T, i.e., the cooling centers (negative R'T) are encompassed by the convergent areas (positive R'\delta) in the upper troposphere, and vice versa.

2. Transient Forcing Anomaly

Unlike residual forcing, transient forcing must be calculated at every time step, then we do a time-average for transient forcing over the same period as the blocking and nonblocking composite does. In the earlier literature, there is no consistent definition for transient eddies, and the results of these studies are somewhat different from each other (Mullen, 1987). Although Mullen suggested that a high-pass
filter in time which retains only the synoptic-scale transients (a cut-off period around 6-10 days) is sufficient for isolating synoptic-scale waves, Lejenás and Dóós (1987) pointed out from a case study that the transient planetary waves (free waves) interacting with the stationary planetary waves can also form a block. Instead of a high pass filter in time, Chen and Tzeng (1990a) suggested a nonlinear synoptic-synoptic scale wave interaction to represent transient forcing. A similar approach to this nonlinear interaction was also used by Metz (1986) in the study of transient cyclone-scale vorticity forcing for blocking highs. The advantages of this approach are that the synoptic-scale waves can be completely preserved (Fraedrich and Böttger, 1978) and that the long-long waves (zonal wave 1-4) and mean-long waves interactions can be excluded. No doubt, in the isolation of the blocking transient forcing, the filter in space is better than the filter in time.

Transient forcing, \( \nabla \cdot \left( \overline{\zeta' \mathbf{v}'} \right) \), used in this study is the divergence of synoptic-scale vorticity transported by synoptic-scale winds. The horizontal and vertical distributions of this forcing are displayed in Figure 8. The 200 mb horizontal distribution shows a maximum center at \((120°W, 55°N)\) for the Pacific blocking case and at \((10°W, 50°N)\) for the Atlantic blocking case. The locations of these maximum centers are somewhat closer to the observed blocking center than those of vorticity residual forcing (Figures 6a and 6b). As indicated by many studies (Lau, 1979; Chen and Tzeng, 1990b), however, the intensity of transient forcing is three to four times less than that of local forcing \(R'\zeta\). On the other hand, over the blocking area, the vertical cross-section at
Figure 8. The transient forcing at 200 and 850 mb and its cross-section at 55°N for both (a) Pacific and (b) Atlantic experiments. The contour interval is 10 m/s² in 200 mb and cross-section; 2.5 m/s² in 850 mb. Positive areas are hatched.
55°N exhibits a westward tilting with height in the middle and lower troposphere for the Pacific case, but this is not clearly so for the Atlantic case in the entire troposphere. All these differences may cause different influences on the blocking, and they will be discussed in the results section of this study. In addition, the maximum amplitude of transient forcing is located at 250 mb for both the Pacific and the Atlantic cases. Ultimately, as indicated by Metz (1986), these transient forcings are also dominated by the longwave regime.

3. Tropical (Remote) Forcing Anomaly

In the earlier heating anomaly studies, the heating anomaly is usually simplified by an idealized elliptic heating source (or sink), e.g., Hoskins and Karoly (1981) and Branstator (1985, 1990). To capture a realistic tropical heating, we use the anomalous OLR distribution, the difference between the OLR composites over the blocking and nonblocking composite periods (Figure 9). The OLR data are interpolated to selected levels. To obtain a vertical distribution of heating anomaly from these data, we adopted parameters similar to those of the normalized apparent heat source derived by Yanai et al. (1973). This vertical distribution has a maximum amplitude at 475 mb. The data in Figure 9a have been converted to the \( \sigma \)-coordinate with an assumption of 1000 mb surface pressure.

Moreover, because the OLR directly corresponds to temperature, it is impossible to distinguish clouds from low surface temperatures (e.g., snow cover) over latitudes higher than 45° and over Tibetan Plateau by
Figure 9. The structure of remote forcing: (a) its normalized vertical profile and its horizontal distribution for both (b) Pacific and (c) Atlantic cases. The contour interval is $2.0 \times 10^{-5} \, ^\circ{\text{Ks}}^{-1}$. Positive areas are hatched.
using these data. Hence, the OLR data are used only from 45°S to 45°N. Beyond this region, the data are replaced by divergence anomaly with an adjustment factor, since the centers of OLR anomaly are consistent with the centers of divergence anomaly (Chen and Tzeng, 1990b). After parameterization adjustment, the tropical heating anomaly has a maximum heating rate of about 1.5°C/day for the Pacific case and about 0.98°C/day for the Atlantic case. Note that because there is clear sky over a blocking center, a cooling center is located inside this area. In addition, a warming center upstream (east side) of blocking can be referred to retrogression or maintenance of blocking. Warming induces an upper level divergence and hence provides the vorticity source to maintain the block (Chen and Tzeng, 1990b).

The remote (OLR) forcing anomaly shows a significant positive anomaly over the maritime continent (from Indonesia to New Guinea) and a strong cooling over the tropical western hemisphere. Since this tropical heating anomaly is a difference between the blocking and non-blocking composites, this strong heating anomaly over the tropics may have a significant impact on the formation or maintenance of the blocking. These postulates will be tested by the model simulation.

To examine the effect of these three forcings, all seven possible combinations out of these forcings are studied. These seven experiments are:

- control run (with all three forcings),
- only residual (local) forcing,
- no residual forcing,
• only transient forcing,
• no transient forcing,
• only remote forcing, and
• no remote forcing.

Two sets of parallel experiments are simulated for the Pacific and the Atlantic blockings, separately.

The results of these experiments are presented in the next chapter. We shall concentrate on the effect of these three forcings on the formation of blocking. Finally a comparison between the simulated Pacific and Atlantic blockings and another comparison between the blocking simulated by higher (R15 truncated) and lower (T8 truncated) resolution models are also provided.
III. RESULTS

Before the results of the model simulations are presented, a brief review of the observed anomalous Pacific blocking pattern is outlined as follows (Figure 10):

- a striking dipole structure of streamfunction anomaly (positive $\Psi'$ to the north and negative to the south) is displayed in the entire troposphere;
- the blocking anticyclone tilts slightly westward with height;
- the temperature anomaly ($T'$) shows a quarter wavelength lag of streamfunction (a strong baroclinic effect) in the upper troposphere and a smaller lag in the lower troposphere; and
- the velocity potential anomaly ($\chi'$) presents a strong divergence (convergence) upstream (downstream) of blocking center in the upper troposphere. The sign of $\chi'$ is opposite in the lower troposphere. Note that the negative velocity potential corresponds to the divergent center.

The basic characteristics of Atlantic blocking are similar to those of Pacific blocking. The only significant difference between the two blockings is that the $\chi'$ of Atlantic blocking has a weaker and smaller divergence (convergence) upstream (downstream) of the blocking center than does that of Pacific blocking.

Although height field is conventionally used to present the blocking pattern, many recent studies, e.g., Dole (1986) and Chen and Tzeng (1990a, b) prefer to use streamfunction ($\Psi$) to describe blocking.
Figure 10. The observed streamfunction anomaly ($\Psi'$) at (a) 200 mb, (b) 850 mb, and (c) vertical cross-section at 55°N; temperature ($T'$) at (d) 200 mb, (e) 850 mb, and (f) vertical cross-section at 55°N; and velocity potential ($\chi'$) at (g) 200 mb and (h) 850 mb for Pacific blocking composite. The contour interval is $4.0 \times 10^4$ m$^2$s$^{-1}$ in (a), $2.0 \times 10^4$ m$^2$s$^{-1}$ in (b) and (c), $0.5^\circ$C in (d), $2.0^\circ$C in (e), $1.0^\circ$C in (f), and $0.5 \times 10^4$ m$^2$s$^{-1}$ in (g) and (h). Positive areas are hatched.
Figure 10. (continued)
because the height field analyses do not provide a good indication of the meridional component of energy propagation, whereas streamfunction analyses do (Hoskins et al., 1977)—especially over the subtropical and tropical areas. Moreover, the temperature field (T) with the streamfunction field (Ψ) can provide a good indication of the sensible heat transport. The spatial relation between temperature and streamfunction can manifest the baroclinicity of a weather system. Finally, velocity potential (χ) directly responds to the (tropical) heating (Chen and Wiin-Nielsen, 1976). Therefore, the velocity potential is important to indicate the response of blocking to the tropical heating and the local enhancement of the split westerlies in the blocking area. In the discussion of the model results, we shall use these three quantities (Ψ, T, and χ) to examine the performance of the model in the blocking simulation. Then, we shall concentrate on streamfunction and velocity potential to compare the effect of these three forcings on the formation of blocking.

A. Pacific Blocking Experiments

In this section, we discuss the blocking simulation of a control run. The comparison between control run and other experiments is used to illustrate various physical effects on the initiation and development of blocking.
1. Control Run

Although the control run includes all three forcings, the weightings of all forcings are not the same. The vorticity-forcing anomaly (including the residual and transient part) is increased by a factor of two, but the surface-pressure-tendency forcing anomaly ($R'_q$) is reduced by 50%. The former is because the model simulation always underestimates the intensity of streamfunction anomaly ($\psi'$) compared to the observations. The reasonable range of the maximum simulated $\psi'$ is defined as an average over the maximum $\psi'$ of selected blocking episodes, with one standard deviation ($\sigma$) error. The average of maximum $\psi'$ over 18 selected Pacific blocking cases (Table 2) is $3.478 \times 10^7$ m$^2$s$^{-1}$ at 200 mb. Their $\sigma$ is $0.7927 \times 10^7$ m$^2$s$^{-1}$. After the vorticity forcing anomaly is doubled, the intensity of maximum simulated $\psi'$ ($2.755 \times 10^7$ m$^2$s$^{-1}$) falls into this range ($3.4787 \times 10^7 \pm 0.7927 \times 10^7$ m$^2$s$^{-1}$). The latter adjustment ($0.5R'_q$) can be attributed to the reduction of error over the steep slope of the mountain in $\sigma$-coordinate (Kasahara, 1974) and to the minimization of the error of divergence around the mountain and polar areas. Furthermore, as pointed out in the previous chapter, the effect of the residual pressure tendency ($R'_q$) is almost opposite that of residual heating ($R'_T$). Therefore, the magnitude of $R'_q$ is reduced to avoid suppressing the effect of thermal forcing too much.

Furthermore, the combined thermal forcing anomaly ($R'_T$ and $R'_{\text{OLR}}$) has a maximum heating rate of about 2.5°C/day; which is approximately the latent heat release given by an extra 10 mm precipitation per day over the central North Pacific Ocean.
To assess the performance of the model, some selected globally averaged statistics, e.g., the energy level and the equilibrium state of the model, are computed.

Following Chen and Branstator (1989) and Williamson et al. (1987), the mass-weighted global mean temperature ([T]) throughout the entire depth of the model (T, in Figure 11) is used to objectively assess the equilibrium state of the model's atmosphere. To estimate the model's energy level and the exchange between kinetic and available potential energies during model simulation, two energy variables—global mean available potential energy ([APE]) and global mean kinetic energy ([KE]), are also calculated by the same procedures. The mathematical expression of these quantities in the \(\sigma\)-coordinate system can be found in Williamson et al.'s report.

The control run (see [T], Figure 11) took about 22 days to reach equilibrium. This spin-up time is about the same as that (20 days) in the cold spin-up study with the NCAR CCM0B by Chen and Branstator (1989). On the other hand, the [APE] and [KE] reach equilibrium much earlier than does [T]. The [KE] takes about 17 days to reach equilibrium; then it starts to oscillate. The [APE] takes an even shorter time (about 16 days) to reach equilibrium. The time difference between [KE] and [APE] can be attributed to the response of rotational flows to the thermal forcing. It is interesting to note that the time evolution of [APE] shows two peaks, one at day 3 and the other at day 20. A detailed examination related to this fluctuation will be discussed in the next two sections.
Figure 11. The time variations of the mass-weighted global and vertical mean temperature \([T]\) (dash and dot line), available potential energy \([\text{APE}]\) (solid line), and kinetic energy \([\text{KE}]\) (dash line) for the Pacific control run. The base value of temperature is 248°K. The unit of \([\text{APE}]\) and \([\text{KE}]\) is \(10^5 \text{ Jm}^{-2}\).
a. Energetic and spectral analyses  

Regarding the energetics of the simulated blocking, Figure 12 illustrates the time variations of the zonal and eddy APE ($A_Z$, $A_E$) and KE ($K_Z$, $K_E$), which are vertically averaged over a 22.5°N to 82.5°N zonal band. These four energy variables show a very good agreement with the observational energetic studies of blocking by Lejenäs (1977) and by Chen and Shukla (1983) (Figures 12c-e). Note that during the onset of blocking, both $A_E$ and $K_E$ increase, whereas $A_Z$ and $K_Z$ decrease. The energy levels of the model's $A_Z$ and $K_Z$ are comparable to those of the observations. The energy levels of $A_E$ and $K_E$ of the control run, however, are equivalent to those of the observed composite blocking flows, while they are smaller than those of another observational case study (Chen and Shukla, 1983). The values of our simulated $A_E$ and $K_E$ are about half those of Chen and Shukla's because of two possible reasons. First, our model simulation used the composite standing mode and the composite forcing, which are the time average of many blocking cases, but Chen and Shukla studied only a single blocking event. Moreover, the eddy activity has been smoothed out during the composite procedure. Second, the model resolution is coarser than that of the observational data used by Chen and Shukla, and therefore the intensity of shortwave eddies is significantly reduced in the model.

Concerning the constructive interference of planetary-scale waves, the spectrally filtered Hovmöller diagrams of 500 mb total streamfunction (standing mode + simulated streamfunction anomaly, $\bar{\Psi} + \Psi'$) at 55°N are presented in Figure 13. The ridge of simulated wavenumber one
Figure 12. The time variations of energies: (a) $A_Z$ and $A_E$, (b) $K_Z$ and $K_E$ for Pacific control run. (c) the same as (a) and (b), except after Lejenës (1977). Day $t = t_o$ is the onset of blocking. (d) and (e) the same as (a) and (b), respectively, except after Chen and Shukla (1983). The blocking onset is Jan. 30. The unit is $10^9$ J m$^{-1}$.
Figure 12. (continued)
Figure 13. The spectrally filtered Hovmöller diagrams of various waves of streamfunction ($\psi$) at 500 mb over 55°N for the Pacific control run. The contour interval is 2.5 x 10^6 m$s^{-1}$ in (a)-(d), and (f), 5.0 x 10^6 m$s^{-1}$ in (e), and (g)-(i). Positive areas are hatched.
is first located around 45°W in the beginning of model simulation and then migrates to around 70°W when the intensity of model blocking anticyclone reaches its first maximum. This westward shifting of the wavenumber-one ridge is consistent with Austin's (1980) findings. She indicated that Pacific blocking is formed when wavenumber one is weak over its climatological location (around 0° longitude). After the first intensity peak, this simulated ridge is quasi-stationary located around 90°W. This result is similar to the anomaly run of Chen and Shukla (1983). But compared with the ridge of observed wavenumber one at 0° longitude (Figure 2), this ridge is too far westward.

The ridge of wavenumbers two and three are quasi-stationary in the control run, and their locations are very close to those of the observations. The ridge of wavenumber 2 is located at 135°W, and wavenumber 3 at 140°W. In addition, the constructive interference between wavenumbers 2 and 3 in the control run forms a Pacific blocking, and the large amplitude of wavenumbers 1 and 3 forms an Atlantic blocking. These results are consistent with Austin's (1980) and Chen and Shukla's (1983) findings.

Note that the Hovmöller diagram of longwaves (wavenumber 1-4) shows three pairs of ridge-trough patterns during the model simulation. The most intense ridge is located over the western coast of North America, and the weakest ridge over Central Asia. This finding suggests that the model is capable of simulating blocking at these three preferable locations.
Furthermore, the simulated streamfunction anomaly ($\psi'$) possesses a 16 days fluctuation cycle for both Pacific and Atlantic blockings (Figure 13e and g). This duration of a blocking episode is consistent with that of the observations.

Finally, from the time evolution of the globally averaged temperature ($\langle T \rangle$, Figure 11) and from the Hovmöller diagram (Figure 13g), we found that simulated blocking starts to oscillate with a period of blocking lifecycle (10-17 days) after the model reaches its equilibrium. Therefore, it is sufficient to run the model for 25 days.

**b. Structure of simulated block**

Figure 14 presents the 200 mb streamfunction anomaly ($\psi'$) fields of various simulation times. During this 25-day simulation, the 200 mb blocking anticyclone anomaly shows one and a half blocking cycles (Figure 13g). The first maximum is reached at day 12; then its intensity decreases, but the blocking does not decay. The blocking reaches its minimum intensity at day 17; it then revives and reaches its second maximum intensity at day 24. The intensity of the second cycle is greater than that of the first. During the second cycle, the maximum intensity of $\psi'$ in the blocking center is $2.775 \times 10^7$ m$^2$s$^{-1}$. This value is within the range ($3.478 \times 10^7 \pm 0.7929 \times 10^7$ m$^2$s$^{-1}$) decided from the selected Pacific blocking composite cases. This continuous two-blocking event is frequently observed in the real atmosphere when the blocking episode is relatively long, or when two blocks occur consecutively at the same location (Chen and Tzeng, 1990a).
Figure 14. The 200 mb streamfunction anomaly ($\Psi'$) at day 1, 3, 12, 20, and 24 for Pacific control run. The contour interval is $2.0 \times 10^6$ m$^3$s$^{-1}$ in (a) and (b), and $4.0 \times 10^6$ m$^3$s$^{-1}$ in (c)-(e). Positive areas are hatched.
On the other hand, the blocking anticyclone is first formed over southwestern Canada in response to the (vorticity residual) forcing. It then migrates to the western coast of Alaska during its developing stage (Figure 14c). This finding supports Hansen's (1981) finding that the retrogression of a midtropospheric ridge across northern Canada seems to stimulate the initial development of high latitude Pacific blocking. Consequently, as the blocking anticyclone revives, the location of the blocking center shifts back to the western coast of Canada (Figure 14d). Subsequently, it retrogresses westward again during its second cycle.

Over the western North Atlantic Ocean, an Atlantic block is also observed in the model simulation after day 3. It reaches its maximum intensity at about day 6 (Figures 13e and 14). The formation of this Atlantic blocking by the anomalous Pacific blocking composite forcing could result from the composite forcing itself because in the 18 selected Pacific blocking cases (Table 2), more than half have double blocks or a strong ridge over the Atlantic Ocean. Hence, the signal of the Atlantic blocking forcing is probably contaminated in the Pacific blocking forcing anomaly, though the signal of Atlantic blocking forcing anomaly is relatively weak.

With regard to the overall view of simulated blocking, Figure 15 displays time-averaged (days 2-16) streamfunction anomaly (Ψ') and velocity potential (χ'). The simulated blocking anticyclone (Figures 15a-c) is in good agreement with the observation (Figure 10), especially at 200 mb. The only difference in 200 mb Ψ' between the observation and the control run is that the center of the simulated blocking is located
Figure 15. The time-averaged (days 2-16) streamfunction anomaly ($\Psi'$) at (a) 200 mb, (b) 850 mb, (c) cross-section at 55°N; and velocity potential ($\phi'$) at (d) 200 mb and (e) 850 mb for Pacific control run. The contour interval is 2.0 in (a) and (c), 4.0 in (b), and 1.0 in (d) and (e). The unit is $10^5 \text{ m}^2\text{s}^{-1}$. Positive areas are hatched.
about 5° latitude farther south than that of the observation. As mentioned in the section of "Experiments and Forcings," this southward shifting of model blocking can be derived from the imposed smoothing and reduction factors of the forcing around polar areas.

The time-averaged (days 2-16) vertical cross-section of simulated streamfunction over 55°N (Figure 15c) clearly shows that the simulated blocking has a westward tilt with height. The slope of westward tilting increases with time until the 200 mb blocking anticyclone revives at its second cycle (not shown). This westward tilting indicates that the development of Pacific blocking is significantly influenced by baroclinic processes and by the possibility of vertical energy transport. This baroclinic character of model blocking is in accordance with that of the observational (Dole, 1986) and energetics (Hansen and Chen, 1982) studies. In addition, the degree of the westward tilt of the simulated blocking anticyclone (Figure 15c) is greater than that of observations (Figure 10c) possibly because the observational blocking composite is equivalent to the mature stage of blocking (Mullen, 1987), whereas days 2-16 of model simulation are still in the developing stage of model blocking. Furthermore, our model simulation and Dole's (1986) study indicate that the westward tilting with height is significant during the developing stage of blocking, but not during the mature stage.

The velocity potential anomaly ($x'$) pattern over the blocking area (Figures 15d and e) is very close to that of the observation (Figures 10g and h), and shows a strong divergence (convergence) upstream
(downstream) of the blocking center in the upper troposphere. The opposite structure of the divergence field appears in the lower troposphere. To accompany the blocking center, however, the simulated divergence centers are also shifted a little southward compared to the observation. Regarding planetary-scale velocity potential, the simulated 200 mb $\chi'$ is in very good agreement with the observations (Figure 10), i.e., there is a positive $\chi'$ (convergence) over the western hemisphere and, negative $\chi'$ (divergence) over the eastern hemisphere. Interestingly, the major divergence center over the maritime continent and the major convergent center over the central Pacific Ocean (east of the dateline) at 200 mb are well simulated by the control run. Moreover, the simulated 850 $\chi'$ is consistent with the observation (Figure 10h).

Regarding the temperature ($T'$) distribution, Figure 16 presents the 200 mb simulated temperature at days 1, 3, and 12. Note that an asterisk marks each panel to indicate the center of the blocking anticyclone at that time step. The simulated 200 mb $T'$ at day 1 (Figure 16a) shows a significant positive anomaly located at the southwest of the blocking center. To the east and southeast sides of the blocking center, there is a negative temperature anomaly. At days 1 and 3, the positive $T'$ is strikingly elongated in N-S direction. This elongation implies that a northward positive sensible heat transport to the west of the blocking center is very remarkable during this period, i.e., the initial stage of blocking, whereas the northward positive sensible heat transport is not very noticeable at the mature stage of blocking (day
Figure 16. The temperature anomaly (T') of (a) day 1, (b) day 3, and (c) day 12 at 200 mb for Pacific control run. The contour interval is 0.5°C in (a) and (b), 1.0°C in (c). Positive areas are hatched.
12. Figure 16c). On the other hand, a significant southward negative sensible heat transport is found to the east of the blocking center during the mature stage. This north- and southward sensible heat transport is consistent with the observations (e.g., Palmén and Newton, 1969). Moreover, the location of the blocking centers is a quarter wavelength out of phase with the $T'$ maximum. This indicates that a strong baroclinic contribution is involved in the developing stage of the blocking, as pointed out by Hansen and Chen (1982) and Dole (1986).

In short, we have found that the control run can simulate Pacific blocking fairly well, not only in terms of spectral structure and energetics but also in terms of the time evolution of spatial structure. Comparing different forcing experiments to control run, we shall examine the effect of these three forcings (residual, transient, and remote) on the formation of the blocking anticyclone.

2. Residual Forcing Experiments

In this section, the effect of residual (local) forcing on the formation of blocking is studied. First, we investigate two special cases: experiment 1) with only local forcing ($P_{\text{res}}$) and 2) without local forcing ($P_{\text{tns+olr}}$). We shall then add either transient or remote forcings to local forcing ($P_{\text{res+tns}}$ and $P_{\text{res+olr}}$, respectively) to examine the effects of these two forcings on blocking simulation.
a. Experiment with only residual forcing

The results of this experiment ($P_{res}$, Figure 17) are similar to those of control run (Figure 15) in terms of both time evolution and spatial structure. The only noticeable differences are:

i) a lesser intensity (92%) of the blocking anticyclone in the $P_{res}$ experiment,

ii) a minor phase shift (about 5° longitude eastward) of the blocking center (Table 3), and

iii) a poorly simulated divergence field over the tropics (Figures 17d and e).

The slight under-simulation of $\Psi'$ and $\chi'$ and the slight dislocation of the blocking center in this ($P_{res}$) experiment suggest that residual forcing alone is not sufficient to stimulate blocking.

Furthermore, residual forcing can simulate blocking better in the upper troposphere (92%) than in the lower troposphere (79%) (Table 3). This finding can be directly related to forcing itself, since residual forcing is greater in the upper levels than in the lower levels, especially $R'_r$ (Figure 6a). This finding can also be viewed in terms of the characteristics of planetary-scale waves during the Pacific blocking episode. Chen and Shukla (1983) indicated that Pacific blocking is due to the baroclinic and barotropic amplification of the ultralong wave. Because the amplitude of ultralong waves is more pronounced in the upper troposphere, the response of these ultralong waves to the blocking forcing would be more sensitive in the upper troposphere than in the lower troposphere. Additionally, using stochastic forcing with synoptic-synoptic and synoptic-planetary scale wave-interactions to simulate Pacific blocking, Metz (1986) suspected that the failure of his simulation might have been due to the physical characteristics of...
Figure 17. Same as Figure 15, except for the Pacific blocking experiment with only local forcing ($F_{res}$). The contour interval is 2.0 in (a) and (c), 4.0 in (b), and 0.5 in (d) and (e). The unit is 10$^6$ m$^2$s$^{-1}$.
Table 3. Time-averaged (days 2-16) location of simulated Pacific blocking center and the intensity of streamfunction (percentage)

<table>
<thead>
<tr>
<th>Ψ, a</th>
<th>Intensity (%)</th>
<th>Location</th>
<th>Pattern</th>
<th>Forcing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pctl</td>
<td>Ub 2.010 100</td>
<td>145°W, 55°N</td>
<td>strong anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>Lc 1.335 100</td>
<td>140°W, 50°N</td>
<td>anticyclone</td>
<td>transient</td>
</tr>
<tr>
<td>Pres</td>
<td>U 1.934 92</td>
<td>150°W, 55°N</td>
<td>strong anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>L 1.051 79</td>
<td>140°W, 50°N</td>
<td>anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td>Ptns</td>
<td>U 0.506 24</td>
<td>125°W, 55°N</td>
<td>weak anticyclone</td>
<td>transient</td>
</tr>
<tr>
<td></td>
<td>L 0.319 24</td>
<td>120°W, 55°N</td>
<td>anticyclone</td>
<td>transient</td>
</tr>
<tr>
<td>Polr</td>
<td>U 0.201 10</td>
<td>165°W, 65°N</td>
<td>weak ridge</td>
<td>remote</td>
</tr>
<tr>
<td></td>
<td>L 0.240 18</td>
<td>140°W, 60°N</td>
<td>ridge</td>
<td>remote</td>
</tr>
<tr>
<td>Ptns+olr</td>
<td>U 0.503 24</td>
<td>135°W, 55°N</td>
<td>weak anticyclone</td>
<td>transient</td>
</tr>
<tr>
<td></td>
<td>L 0.566 42</td>
<td>130°W, 60°N</td>
<td>anticyclone</td>
<td>remote</td>
</tr>
<tr>
<td>Res+olr</td>
<td>U 1.857 88</td>
<td>145°W, 55°N</td>
<td>strong anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>L 1.220 91</td>
<td>135°W, 50°N</td>
<td>anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td>Res+tns</td>
<td>U 2.047 97</td>
<td>145°W, 55°N</td>
<td>strong anticyclone</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>L 1.190 89</td>
<td>140°W, 50°N</td>
<td>anticyclone</td>
<td>transient</td>
</tr>
</tbody>
</table>

aUnit of Ψ': $x10^7$ m$^2$s$^{-1}$.
bU: 200 mb.
cL: 850 mb.
Pacific blocking, as indicated by Chen and Shukla. Comparing Metz's (transient) stochastic forcing with our residual forcing, we found that the ultralong-ultralong and zonal-ultralong waves interactions are included in the residual forcing, but not in Metz's stochastic forcing. Therefore, Pacific blocking obviously results from the amplification of ultralong waves.

Further, the vertical cross-section of the streamfunction ($\psi'$) over 55°N (Figure 17c) shows a similar distribution to that of the control run in terms of both intensity (90%) and baroclinic characteristics (tilted westward with height). This similarity suggests that the major contribution to the intensity and baroclinicity of Pacific blocking is associated with residual (local) forcing.

The intensity of the velocity potential ($\chi'$) in this experiment is about three-fourths that of the control run over the blocking area and about only half that over the tropics (Figures 17c and d). This finding clearly indicates that the residual forcing in the simulation of divergence circulation over the tropics is not conclusive and that undoubtedly some other forcing is needed to improve this deficit.

b. Experiment without residual forcing

The intensity of the blocking anticyclone simulated by this ($P_{tns+olr}$) experiment is about one-third that of the control run (Table 3) even though the location and pattern of the blocking are similar to those of the control run. The vertical cross-section of $\psi'$ (Figure 18c) shows that the blocking anticyclone does not tilt westward with height, especially in the upper
Figure 18. Same as Figure 15, except for the Pacific blocking experiment without local forcing ($P_{\text{tms+olr}}$). The contour interval is $1.0 \times 10^4$ m$^2$s$^{-1}$ in (a)-(c), and $0.5 \times 10^4$ m$^2$s$^{-1}$ in (d) and (e)
troposphere. This finding implies that the local baroclinic character of Pacific blocking is caused by residual forcing. This experiment confirms, therefore, that residual forcing anomalies play a crucial role in blocking simulation.

On the other hand, velocity potential ($\chi'$, Figures 18d and e) displays a pattern similar to that of the control run ($P_{ctl}$) over the tropics, where, as mentioned already, local forcing alone cannot properly simulate the divergence field. This finding may be related to the tropical (remote) forcing. Further discussion will take place in the section on remote forcing.

c. Other two-forcing experiments

In the previous two sections, we discovered that local forcing can stimulate a blocking very similar to that of the observation although it still under-simulates the intensities of $\Psi'$ and $\chi'$. We, therefore, added one of the other two forcings (either transient or remote) to the local forcing to study its effect on blocking simulation.

We found that when transient forcing is added to local forcing ($P_{res+tns}$), the intensity of $\Psi'$ is increased (by about 10%) and the location of the blocking is closer to the control run than it is in the local forcing experiment (Table 3). Moreover, the velocity potential is also improved over the blocking area, although not in the tropics (Figure 21). This improvement suggests that transient forcing can modify the location and intensity of the blocking anticyclone to some extent, although this effect is still minor. Over the tropics, velocity
potential is not affected by transient forcing. Remote forcing can thus also be important to blocking simulation, in regards to divergent circulation.

The experiment with both local and remote forcings ($P_{\text{res+olr}}$) shows that the remote forcing intensifies the $\tilde{\psi}$ in the lower troposphere but not in the upper troposphere (Table 3). This intensification can be related to the location of the blocking center simulated by the remote forcing. Furthermore, the velocity potential simulated by this experiment has a strong resemblance to that of the control run (Figures 15d and e). Hence, during the developing stage of blocking, remote forcing intensifies the planetary-scale divergence circulation. A detailed discussion of remote forcing is found in the section on that subject.

Finally, we can conclude that residual forcing is the major forcing initiating and maintaining a block. Transient and remote forcings modify the location and intensity of blocking. The effects of these two forcings on blocking are investigated in more detail in the next sections.

3. Transient Forcing Experiments

The arrangement of this section is similar to that of the previous section. The experiment with only transient forcing ($P_{\text{tns}}$) is the theme experiment of this section. The experiment without transient forcing ($P_{\text{res+olr}}$) is used as a contrast experiment. The other two-forcing experiments ($P_{\text{tns+olr}}$ and $P_{\text{res+tns}}$) are also reviewed.
Since Green (1977) proposed his hypothesis that the blocking anticyclone might be maintained by the transient forcing, many studies have dealt with this subject diagnostically and numerically. Many of these studies have indicated, however, that transient forcing contributes positively to the maintenance of the blocking anticyclone, but that its magnitude is not adequate to form a block (Lau, 1979; Austin, 1980; Chen and Tzeng, 1990b). The results of the experiment with only transient forcing (P\textsubscript{tns}, Figure 19) confirm the preceding studies, which indicate that the pattern of the blocking anticyclone simulated by transient forcing is consistent with that of the control run although the intensity of the blocking is only one quarter that in F\textsubscript{ctl} (Table 3). Similar results were obtained by Metz (1986), and by Egger and Schilling (1984).

Moreover, the location of the blocking center of this P\textsubscript{tns} experiment is about 20° longitude east of the location of the control run (Table 3). Compared with that of a lower resolution (T8 truncated) experiment, which will be explained in more detail in the end of this Chapter, this eastward shifting of the blocking center is observed only in the R15 truncated resolution. This finding implies that the shifting is dominated by the shortwave regime. The same result was obtained by Metz (1987) in a barotropic model with high-frequency stochastic forcing. Therefore, the effect of the transient forcing is dominated by the shortwave regime. In other words, a nonlinear interaction between the shortwaves with very close wavelengths is the major mechanism intensifying the longwave regime by transient forcing. From current and
Figure 19. Same as Figure 15 (a)-(c), respectively, except for the experiment with transient forcing only ($P_{\text{trans}}$). The contour interval is $1.0 \times 10^6 \text{ m}^2\text{s}^{-1}$.
previous studies, however, we found that amplification of planetary-scale waves by the resonance of shortwave transient forcing is too small to form a blocking.

In turn, the results of the experiment without transient forcing ($P_{\text{res+olr}}$) show that the blocking anticyclone is almost the same as that of the control run, with only about 10% less intensity than the control run (Table 3 and Figure 20). It can concluded from these two experiments that transient forcing is not as crucial for the formation and maintenance of a blocking as many researchers expect (e.g., Shutts, 1983; Illari, 1984; and Colucci, 1985, 1987). However, transient forcing may still affect the shape and location of a blocking.

Finally, the experiment with both transient and local forcings ($P_{\text{res+tns}}$) and the experiment with both transient and remote forcings ($P_{\text{tns+olr}}$) are reviewed here. When local forcing is added to transient forcing, the simulated $\psi'$ is improved dramatically. The intensity and location of blocking are almost the same as those of the control run (Table 3). Therefore, this also confirms that local forcing is the major forcing to initiate and maintain the blocking.

The remote forcing in the experiment with transient forcing ($P_{\text{tns+olr}}$), on the other hand, can slightly enhance the blocking in the lower atmosphere but not in the upper troposphere. Velocity potential (Figure 18), however, is improved over the entire troposphere, especially over the tropics and the blocking area. The intensity and pattern of $\chi'$ over the tropics are very similar to those of the control run (Figure 15). These findings suggest that the effect of remote
Figure 20. Same as Figure 19, except for the experiment without transient forcing ($P_{res+olr}$). The contour interval is $2 \times 10^4$ m$^2$s$^{-1}$ in (a) and (c), and $4 \times 10^4$ m$^2$s$^{-1}$ in (b).
forcing is likely to provide a favorable environment for the formation of a block, as indicated by Chen and Tzeng (1990b).

4. Remote Forcing Experiments

The streamfunction anomaly ($\Psi'$) of the experiment with only tropical heating ($P_{o1r}$) shows a blocking ridge of greater intensity in the lower troposphere than in the upper troposphere over the west of coast of North America (Figure 21). The intensity of this simulated blocking ridge is about 18% that of the control run (Table 3) at 850 mb, but only 10% the intensity of the control run at 200 mb. Other significant features in response to the effect of remote forcing are a deep depression over Southeastern Asia and anticyclones over the central North Pacific Ocean and the southwestern North Atlantic Ocean, at 850 mb (Figure 20b). Comparing these features with the distribution of remote forcing (Figure 9), we found that these patterns of streamfunction anomaly are decided by the direct response of the model to heating. In other words, a deep depression responds to strong latent heat released over the maritime continent (from Indonesia to New-Guinea) and, highs correspond to the clear-sky radiation cooling over the oceans.

Blackmon et al. (1977) pointed out that the intensified subtropical oceanic highs are one precursor of the local enhancement of the split of the jet streams, since these highs are connected to the intensified thermally indirect circulation downstream of the jet stream. We can, therefore, conclude that the effect of tropical forcing supports Rex's (1950a) first blocking criterion, which states that the basic
Figure 21. Same as Figure 15, except for the experiment with only remote forcing ($F_{polr}$). The contour interval is $1.0 \times 10^4 \text{ m}^2\text{s}^{-1}$ in (b) and $0.5 \times 10^4 \text{ m}^2\text{s}^{-1}$ in (a) and (c)-(e).
westerly current should be split into two branches. Furthermore, in
response to this split westerly, the center of the anticyclone simulated
by this experiment (P_{olr}) is located about 20° longitude upstream of the
blocking center of the control run. This fact suggests that the
retrogression of Pacific blocking is forced by remote forcing.
Furthermore, from the daily synoptic charts, Pacific blocking is usually
retrograded. Therefore, the effect of the remote forcing is also
important to simulation of Pacific blocking.

Unlike streamfunction, the divergence field is vigorous,
especially over the tropics (Figures 2Id and e). The intensity of
velocity potential (\chi') is about 80% that of the control run over the
tropics. In spite of relative weak intensities, the divergence
(convergence) upstream (downstream) of the block is still clearly shown
over the blocking area (Figure 2Id). The intensified planetary-scale
divergent circulation is one of the major agents modulating the
midlatitude general circulation endorsed by many studies such as those
of Chen and Wiin-Nielsen (1976), Chen et al. (1988), and Yen (1990).
Recently, Chen and Tzeng (1990b) pointed out that the vorticity source
term, -(f + f)^*V, is the major means of counterbalancing the advection
of vorticity in a streamfunction-budget analysis, particularly over the
Pacific Ocean. As suggested by Austin (1980), Shutts (1983), and
Colucci (1987), therefore, the intensified divergence upstream of
blocking is the major source to provide a small enough background flow
for the blocking formation.

In contrast, the results of experiments without remote forcing
(Pres+tns, Figure 22) exhibit a blocking pattern that is very consistent with that of the control run. The location of the blocking center and the intensity of $\psi'$ (97%) are almost the same as those of the control run (Table 3). The only difference between Pres+tns and P_ctl is found in the velocity potential ($\chi'$, Figure 22b). Over the blocking area, the pattern of the velocity potential in Pres+tns experiment is about the same as that of the control run, but its intensity is about 15% less than that of the control run. Moreover, the intensity of velocity potential over the tropics is about one-third that of the control run. This finding, however, is consistent with that of the previous experiment (Polr), which showed that planetary-scale divergent circulation is dominated by remote forcing.

Therefore, according to the results of these two experiments, remote forcing does not directly affect the intensity of the blocking anticyclone, but provides a preferable location for the formation of a blocking. It may also force the blocking retrogressively.

5. Summary of Pacific Blocking Experiments

The control run showed that the model can simulate a very realistic Pacific blocking. Therefore, we can use the control run as a reference to examine the three forcings. We conclude from our experiments that residual forcing is the major forcing forming and maintaining Pacific blocking. This forcing amplifies the ultralong waves baroclinically or barotropically (e.g., Hansen and Chen 1982; Chen and Shukla, 1983) and hence induces a constructive interference for
Figure 22. Same as Figure 15. except for the experiment without remote forcing (Pres+tns)
initiating and maintaining blocking. Further, the results show that the Pacific residual forcing can not only simulate Pacific block but also simulate a very realistic double blocks, one over Pacific sector and the other over Atlantic sector.

It is still not clear what physical process forms these residual forcing anomalies. Metz (1987) pointed out that stochastic forcing is dominated by a low frequency mode and a large-scale pattern. His result is consistent with our residual vorticity and divergent forcings. Metz's stochastic forcing, however, retains only the divergence of the vorticity flux by the interactions of synoptic-synoptic and synoptic-planetary scale waves. In our model, residual forcing consists of not only the vorticity but also the divergent, thermodynamic, and surface-pressure-tendency residual terms. Note that computational error and friction terms are also included in the residual terms. The reason causing these blocking residual forcings is beyond the scope of this study.

Transient forcing, in turn, can enhance the blocking pattern and location. As supported, however, by many other studies (Lau, 1979; Chen and Tzeng, 1990b), and by these numerical experiments, the intensity of the blocking simulated by this forcing is too small even though the resonance of the shortwaves may prevail in this experiment. It is impossible to form a full intensity of a blocking by transient forcing alone.

Finally, results of the remote forcing experiments suggest that this forcing does not directly intensify the simulated blocking.
Instead, tropical heating strikingly amplifies divergent circulation, which provides a background adequate (a slow, split westerly over the blocking area) for the commencement of a block. This background is also important in the duration of a blocking episode. Trenberth and Mo (1985), for example, indicated that the duration of blocking events in the SH is shorter than that in the NH. They attributed this fact to the presence of the generally stronger westerlies throughout the troposphere at the middle to high latitudes of the SH. Moreover, remote forcing shifts the location of the Pacific blocking center westward. Such a shift suggests that this forcing is the major mechanism retrogressing the blocking. We, also, found that the effect of remote forcing is more important than that of transient forcing for Pacific blocking.

B. Atlantic Blocking Experiments

Before we discuss the simulated Atlantic blocking, characteristics of the observed Atlantic blocking anomaly (Figure 23) are summarized as follows:

- like Pacific blocking, a striking dipole structure of streamfunction anomaly ($\Psi'$) exists in the troposphere;
- the positive $\Psi'$ extends zonally from Alaska to the Baltic Sea. This suggests that the Atlantic blocking anomaly not only amplifies the North Atlantic ridge but also enhances the North Pacific ridge and fills the North American trough;
- the blocking anticyclone tilts westward with height;
Figure 23. Same as Figure 10, except for the observed anomaly
Atlantic blocking composite
Figure 23. (continued)
the temperature anomaly \( (T') \) shows a quarter wavelength lag of 
\( \psi' \) (a strong baroclinicity); and

the velocity potential anomaly \( (\chi') \) displays a pattern similar to that of Pacific block, but the intensity and size of divergence-convergence pair are smaller than those observed in the Pacific block (Figure 10).

The reasons for using \( \psi, \chi, \) and \( T \) to represent the blocking characteristics are explained at the beginning of this chapter. We shall use these three quantities to investigate the performance of the control run in the simulation of Atlantic blocking. Comparing the control run with other experiments, we shall examine various physical effects of these three forcings on the initiation and development of Atlantic blocking.

1. Control Run

Based upon the explanation in the Pacific control run, those three forcings (local, transient, and remote) are also weighted by different factors in the Atlantic control run. These factors are the same as those used in the Pacific blocking experiments, i.e., the vorticity residual forcing \( (R_{\zeta}') \) is increased by 100\%, and the surface-pressure-tendency residual forcing \( (R_{q}') \) is reduced 50\%. Recall that the former adjustment \( (2 \times R_{\zeta}') \) is due to the under-simulation of \( \psi' \) by the model. After \( R_{\zeta}' \) is doubled, the simulated maximum intensity of \( \psi' \) at the blocking center \( (3.872 \times 10^7 \, \text{m}^2\text{s}^{-1}) \) satisfies the criterion \( (3.656 \times 10^7 \pm 0.870 \times 10^7 \, \text{m}^2\text{s}^{-1}) \) defined by the average of the maximum \( \psi' \) over those 18
selected Atlantic blocking cases (Table 2) and their standard deviation. During model simulation, we calculated the mass-weighted global and vertical average temperature ([T]), the available potential energy ([APE]), and the kinetic energy ([KE]) to estimate the equilibrium stage and the energy level of the model. Figure 24 displays the time evolution of these three quantities. The [T] curve indicates that the control run of the Atlantic blocking reaches equilibrium on day 24, about two days later than that of Pacific blocking. This spin-up time is also slightly longer than the cold spin-up study (about 20 days) by Chen and Branstator (1989). Comparing their results with those of Manabe and Wetherald's (1975), Chen and Branstator suggested that the NCAR CCM08 spins up much faster, which perhaps because of the differences in the vertical diffusion parameterizations used in the two models. This suggestion, however, may not apply in our study, since the vertical diffusion parameters are exactly the same in both the Pacific and Atlantic blocking experiments. The only difference between these two experiments is their prescribed forcings. Therefore, the difference in spin-up time between the Pacific and Atlantic blocking control runs may be caused by the prescribed forcings and by response of model to the forcings.

The energy level of [APE] and [KE], on the other hand, are consistent with Chen's (1982) result. As with the Pacific control run, the time series of [APE] exhibits two peaks during these 25 days of simulation. Moreover, the linear trend of [KE] and [APE] in the Atlantic control run (Figure 24) is about the same as that in the
Figure 24. Same as Figure 11 except for the Atlantic control run
Pacific control run (Figure 11). i.e., $[KE]$ increases as $[APE]$ decreases.

**a. Energetic and spectral analyses**  The time variations of various energy components ($A_Z$, $A_E$, $K_Z$, and $K_E$) of the Atlantic control run ($A_{ctl}$) are presented in Figure 25. The variations are consistent with those of the Pacific control run ($P_{ctl}$) and of the earlier blocking energetics studies (Lejenäs, 1977; and Chen and Shukla, 1983) (Figure 12) in which $A_Z$ and $K_E$ increased (decreased) when $A_Z$ and $K_Z$ decreased (increased) during the developing (decaying) stage of blocking. The energy levels of these energy components are similar to the energy levels of those in the $P_{ctl}$ experiment, whereas the difference between the maximum and minimum of every energy component is smaller in the Atlantic control run than in the Pacific control run.

Figure 26 displays the Hovmöller diagrams of the individual zonal wavenumbers 1 to 4 and the wave groups 1-15, 5-15, 1-4, 2+3, and 1+3 of 500 mb streamfunction over 55°N. Unlike the Pacific case, the amplitude of wavenumber 1 increases during the developing stage of the Atlantic blocking and then decreases gradually after the mature stage. Its ridge is quasi-stationary located at about 45°W. Wavenumber two, however, has a quite different character from that of other longwaves. The amplitude of wavenumber 2 decreases during the developing stage of the Atlantic blocking. Before this blocking reaches its maximum intensity at day 12, wavenumber 2 diminishes to its minimum and shifts away from its climatological location. After the mature stage of the Atlantic
Figure 25. Same as Figure 12 (a) and (b), respectively, except for the Atlantic control run.
Figure 26. Same as Figure 13 except for the Atlantic control run
blocking, wavenumber two starts to intensify over its climatological location. The fluctuation of wavenumber 2 during the Atlantic blocking is similar to that of wavenumber 1 during Pacific blocking. These fluctuations are consistent with Austin's (1980) results.

The result of the comparison made between the maximum simulated $\psi'$ of the Atlantic blocking ($3.872 \times 10^7 \text{ m}^2\text{s}^{-1}$) with that of the Pacific blocking ($2.755 \times 10^7 \text{ m}^2\text{s}^{-1}$) is consistent with the result of Austin (1980), who found that Atlantic blocking is stronger than Pacific blocking. Austin indicated that this difference in strength is caused by wavenumber 1's usually being of larger amplitude in the wintertime circulation and by its ridge normally being located over the eastern North Atlantic Ocean. Besides the location of the simulated wavenumber 1, the intensity also confirmed Austin's criterion for the formation of Pacific and Atlantic blockings. Atlantic blocking is formed when wavenumber 1 is strengthened over its climatological location (Figure 26a), and Pacific blocking is formed when wavenumber 1 is weakened (Figure 13a).

As pointed out in the Introduction, however, the most preferable location of Atlantic blocking is at $0^\circ-40^\circ$W (e.g., Rex, 1950a; Dole, 1986; Mullen, 1987; and Chen and Tzeng, 1990a) rather than at $0^\circ-40^\circ$E as indicated by Austin (1980). Furthermore, the preferable Atlantic blocking region ($0^\circ-40^\circ$W) corresponds to the climatological locations of wavenumbers one and three. Therefore, Chen and Tzeng (1990b) suggested that Atlantic blocking is constructed by intensified wavenumbers 1 and 3. Our model simulation, confirming Chen and Tzeng's finding, indicates
that constructive interference of the amplified wavenumbers 1 and 3 (Figure 26i) forms Atlantic blocking. Note that the Hovmöller diagram of wave group 1+3 exhibits a very similar pattern and intensity to wave groups 1-4 and 1-15.

Moreover, wavenumbers 2 and 3 construct a block-like ridge over the west coast of North America (the Pacific block, Figure 26h). Hence, the forcing of Atlantic blocking can also form double blocks. This finding is consistent with observations stressed in the beginning of this section that the Atlantic blocking composite forcing still amplifies the North Pacific ridge. Interestingly, a weak ridge over the Center Asia is clearly simulated by the Atlantic control run.

b. Structure of simulated block The horizontal structures of the simulated streamfunction ($$\Psi'$$) at different time instants at 200 mb are presented in Figure 27. Note that, as displayed in Figure 26, the horizontal structure of the wave group 1+3 (not shown) is very similar to that of the total (wave group 1-15, Figure 27). The 200 mb $$\Psi'$$ shows a double-ridge pattern, one ridge of which is over the south coast of Iceland, and the other over the central North America. The former ridge is, in fact, Atlantic blocking. The latter weakens the North American trough and intensifies the North Pacific ridge as emphasized in the observed Atlantic blocking anomaly (Figure 23).

Furthermore, the observed dipole structure of $$\Psi'$$ (an anticyclone to the north and a cyclone to the south) over the blocking area is strikingly simulated by the Atlantic control run (Figure 27). This
Figure 27. 200 mb streamfunction anomaly (Ψ') at day 3, 6, 12, and 22 for the Atlantic control run. The contour interval is $2.0 \times 10^6 \text{ m}^2\text{s}^{-1}$ in (a) and $4.0 \times 10^6 \text{ m}^2\text{s}^{-1}$ in (b)-(d). Positive areas are hatched.
dipole structure of $\Psi'$ indicates that the split of westerlies is locally enhanced during the blocking lifetime. The mechanism of the enhanced split pattern will be studied further in terms of divergent circulation (velocity potential, $\chi'$).

As indicated by the Hovmöller diagrams (Figure 26), the intensity of the Atlantic blocking anticyclone increases with time during its developing stage. The blocking was first initiated over the southeast of Greenland to the west coast of North Europe (Figure 27a); it then migrated gradually to the south of Greenland at day 12 (Figure 27c). At this time, the blocking reached its maximum amplitude ($3.872 \times 10^7 \text{ m}^2\text{s}^{-1}$), which is within the criterion ($3.656 \times 10^7 \pm 0.870 \times 10^7 \text{ m}^2\text{s}^{-1}$) obtained from the 18 selected blocking cases for the Atlantic blocking composite (Table 2). After the mature stage of the block, its intensity decreased and it retrograded northwestward. Finally, the blocking anticyclone merged into the polar circle, and thus a life cycle of the Atlantic blocking was complete.

Chen and Wiin-Nielsen (1976) indicated that the APE generated by the tropical heating is released by the divergent circulation (velocity potential) to support the rotational circulation. Velocity potential ($\chi'$) is, therefore, used to indicate the response of the model to (tropical) heating and to thermally direct (or indirect) circulation (e.g., the ageostrophic effect).

The velocity potential anomaly ($\chi'$) of the Atlantic control run (Figure 28) shows a very similar pattern to that of the observations (Figure 23g and h), especially over the blocking area; that is a
Figure 28. Time-averaged (days 2-16) velocity potential ($\chi'$) at (a) 200 mb and (b) 850 mb, for the Atlantic control run. The contour interval is $0.5 \times 10^6$ m$^2$s$^{-1}$. Positive areas are hatched.
negative $\chi'$ (divergence) is located on the upstream side of the blocking center and a positive $\chi'$ (convergence) on the downstream side in the upper troposphere. This pattern suggests that the split of the westerlies over the blocking area is enhanced locally by intensified divergent circulation (the ageostrophic effect) during the Atlantic blocking, since the intensity of split flows is affected by thermally indirect circulation (Blackmon et al., 1977).

In view of the temperature anomaly ($T'$, Figure 29), the 200 mb temperature time-evolution displays a significant northward sensible heat transport from southwest of the Atlantic blocking to upstream of blocking. The cold air downstream of the blocking center shows a significant southward sensible heat transport (Figure 29b). Both this northward warm advection upstream of blocking and this southward cold advection downstream of blocking are consistent with the observations (Figure 23d). Furthermore, the horizontal structures of $T'$ and $\Psi'$ (Figure 29) indicate that $T'$ has a quarter wavelength phase lag of $\Psi'$. Simulated Atlantic blocking, therefore, possesses a strong baroclinic effect, as shown in the observations (Figure 23).

Finally, we have found that the Atlantic blocking control run can simulate a very realistic Atlantic blocking in terms of energetics, and vertical and horizontal structures. We shall, therefore, use the control run as a reference to examine the effect of these three forcings (local, transient, and remote) on the formation of Atlantic blocking.
Figure 29. The 200 mb temperature anomaly ($T'$) at days 3, 6, and 12, for the Atlantic control run. The contour interval is 0.5°C in (a) and (b) and 1.0°C in (c). Positive areas are hatched.
2. Residual Forcing Experiments

Like the Pacific blocking experiments, residual forcing is also studied by comparing of the experiments with and without residual forcing ($A_{res}$ and $A_{tns+olr}$, respectively) with the control run ($A_{ctl}$). The two-forcing experiments ($A_{res+tns}$ and $A_{res+olr}$) are then compared with the $A_{res}$ experiment to determine the effect of the other two forcings on the formation of the Atlantic blocking. The intensity, location, and pattern of simulated streamfunction ($\Psi'$) by different experiments are listed in Table 4 for the Atlantic blocking case.

a. Experiment with only residual forcing

As with the results of the Pacific blocking experiments, the results of this experiment ($A_{res}$) are very close to those of the control run. The intensity of streamfunction anomaly ($\Psi'$) of $A_{res}$ is about 85% (95%) of $A_{ctl}$ at 200 mb (850 mb) (Table 4). Note that the $\Psi'$ of $A_{res}$ is simulated better in the lower troposphere (95%) than in the upper troposphere (85%). This finding is different from the results of the Pacific residual forcing ($P_{res}$) experiment and can be attributed to the characteristics of the blocking itself. Hansen and Chen (1982) showed that Atlantic blocking is forced by the nonlinear interaction between the intensified cyclone-scale and ultralong waves. Since the midlatitude (polar) fronts are usually more intense in the lower troposphere (below 500 mb) than in the upper troposphere (Palmén and Newton, 1969), the nonlinear interaction between synoptic-scale waves and the ultralong waves would be more significant in the lower troposphere than in the
Table 4. Time-averaged (days 2-16) location of simulated Atlantic blocking center and the intensity of streamfunction (percentage)

<table>
<thead>
<tr>
<th></th>
<th>Intensity (%)</th>
<th>Location</th>
<th>Pattern</th>
<th>Forcing</th>
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</thead>
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<tr>
<td>Act1</td>
<td>Ub 2.851</td>
<td>35°W, 65°N</td>
<td>strong</td>
<td>residual</td>
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<tr>
<td></td>
<td>Lc 1.798</td>
<td>10°W, 65°N</td>
<td>anticyclone</td>
<td>transient</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>remote</td>
</tr>
<tr>
<td>Ares</td>
<td>U 2.416</td>
<td>35°W, 65°N</td>
<td>strong</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>L 1.706</td>
<td>3°E, 65°N</td>
<td>anticyclone</td>
<td></td>
</tr>
<tr>
<td>Atns</td>
<td>U 0.656</td>
<td>5°E, 50°N</td>
<td>weak</td>
<td>transient</td>
</tr>
<tr>
<td></td>
<td>L 0.396</td>
<td>0°W, 45°N</td>
<td>anticyclone</td>
<td></td>
</tr>
<tr>
<td>Aolr</td>
<td>U 0.143</td>
<td>35°W, 55°N</td>
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<td>remote</td>
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<td></td>
<td>L 0.107</td>
<td>35°W, 65°N</td>
<td>ridge</td>
<td></td>
</tr>
<tr>
<td>Atns+olr</td>
<td>U 0.756</td>
<td>10°W, 50°N</td>
<td>weak</td>
<td>transient</td>
</tr>
<tr>
<td></td>
<td>L 0.505</td>
<td>3°E, 45°N</td>
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<td>remote</td>
</tr>
<tr>
<td>Ares+olr</td>
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<tr>
<td></td>
<td>L 1.749</td>
<td>5°W, 65°N</td>
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<tr>
<td>Ares+tns</td>
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<td>30°W, 60°N</td>
<td>strong</td>
<td>residual</td>
</tr>
<tr>
<td></td>
<td>L 1.696</td>
<td>5°W, 65°N</td>
<td>anticyclone</td>
<td>transient</td>
</tr>
</tbody>
</table>

*aUnit of $\psi'$: $10^7$ m$^2$s$^-1$.
*bU: 200 mb.
*cL: 850 mb.
higher troposphere. However, this interaction is directly related to the intensified synoptic-scale waves (the transient forcing). We shall investigate this phenomenon in more detail in the next section.

Furthermore, the intensity and location of divergent circulation ($x'$) in this ($A_{res}$) experiment (Figures 30a and b) are almost the same as those of the control run ($A_{ctl}$) over the Atlantic blocking area (Figure 28). In other words, a divergence (convergence) upstream (downstream) of the blocking center (asterisked) is also significantly shown in Figure 30a and b. The split of the westerlies is, therefore, intensified by local forcing (thermally indirect circulation), through divergence circulation. On the other hand, the only noticeable difference between these two experiments is that the divergence over the tropics is much weaker in this ($A_{res}$) experiment than in the control run. This difference could have been resulted from the remote forcing (tropical heating), since the maximum heating and cooling centers of remote forcing are located over this region (Figure 9). A further examination will be performed in the experiment of the remote forcing.

b. Experiment without residual forcing

The result of the experiment without residual forcing ($A_{tns+olr}$) shows that the intensity of $\psi'$ is about one-fourth that of the control run and that the location of the blocking is about 25° longitude east that of the control run (Table 4). Obviously, residual forcing plays the most vital role in simulation of the blocking anticyclone.
Figure 30. Time-averaged (days 2-16) velocity potential anomaly ($\chi'$) for experiment with only residual forcing ($A_{res}$) at (a) 200 mb, (b) 850 mb; and experiment without residual forcing ($A_{trans+olr}$) at (c) 200 mb and (d) 850 mb for Atlantic case. The contour interval is $0.5 \times 10^6$ m$^2$s$^{-1}$. Positive areas are hatched.
Regarding the location change of the blocking center in this experiment, we might suspect that the migration of blocking during its lifecycle is forced either by transient or by remote forcing. On this topic, a detailed discussion will be performed in the next two sections.

Furthermore, the intensity of divergent circulation ($\chi'$, Figures 30c and d) is very close to that of the control run ($A_{\text{ctl}}$) (70% of intensity) over the tropics, whereas its intensity over the blocking area is too weak (25%). This finding, however, is consistent with that of the previous experiments ($A_{\text{res}}$), which indicated that the divergent circulation over the Atlantic blocking area is dominated by residual forcing and that the divergent circulation over the tropics is decided largely by tropical heating (Figure 30).

c. Other two-forcing experiments  
After the comparison between experiments with and without local forcings, we shall also examine the effect of the other two forcings (transient and remote) on local forcing in the blocking simulation.

The experiment with local and transient forcings ($A_{\text{res+tns}}$) shows that the intensity of $\Psi'$ is amplified (10%) in the upper troposphere and the location of the blocking center is closer than that of the control run than that of the single forcing ($A_{\text{res}}$) experiment. Therefore, the transient forcing can slightly amplify the blocking anticyclone and modify the location of the blocking center. The effect of our transient forcing on the Atlantic blocking, however, is somewhat different from the result of Metz's study (1986), which indicated that the Atlantic
blocking is due to the transient forcing. This difference will be
examined in more detail in the next section.

Finally, when we add remote forcing to local forcing ($A_{res+olr}$),
we find that the intensity and location of the blocking anticyclone
change very little (2%). Hence, residual forcing is still a dominating
forcing.

3. Transient Forcing Experiments

From the previous section (Residual Forcing Experiments), we
suspected that the nonlinear interaction of intense synoptic-synoptic
scale waves, i.e., transient forcing, may be important to the formation
of the Atlantic blocking. When we added transient forcing to residual
forcing ($A_{res+tns}$), however, the results showed that the effect of
transient forcing was unimportant. Therefore, it is necessary to
investigate transient forcing further to discover the reason for this
difference.

In this section, we shall first examine the difference between the
experiments with and without transient forcing ($A_{tns}$ and $A_{res+olr}$,
respectively) to determine the importance of this forcing in the
blocking formation. We then shall discuss the difference between the
results of $A_{tns}$ and $A_{res}$ experiments and to find out a possible
mechanism causing this difference.

Like the Pacific experiment ($P_{tns}$), the experiment with only
transient forcing ($A_{tns}$) shows that the intensity of the simulated
blocking anticyclone is about quarter that of the control run and that
the blocking center shifts eastward 20° longitude (Table 4 and Figure 31). Moreover, the experiment without transient forcing ($A_{res+olr}$) indicates that the intensity of $\psi'$ is about the same as in the residual forcing experiment ($A_{res}$) (about 90% of control run) and that the blocking center shifts slightly westward compared with the $A_{res}$ experiment. Thus, this finding is similar to the Pacific experiments in which transient forcing was not found to be important in the simulation of Atlantic blocking. Nevertheless, reconciling the results of the residual forcing experiment and those of the transient forcing experiment presents a great challenge.

Metz (1986) showed that, in a barotropical model with a stochastic forcing (function of time), Atlantic blocking can be well simulated by the model in terms of both location and intensity (two-thirds of the observations) of the blocking. He concluded that the time-mean of the transient forcing (like our transient forcing) does not support the blocking pattern, so that the model blocking activity is due only to the transient (in time) forcing. The first part of Metz's conclusion is consistent with the result of our model simulation ($A_{cns}$). The second part of his findings, however, is not consistent with ours. We have shown from the residual forcing experiment ($A_{res}$) that the time-mean of the local forcing anomaly can produce very realistic blocking in terms of both intensity (90%) and pattern for the Atlantic and the Pacific sectors. The important point is therefore to find a proper forcing, as is done in this study, rather than to find the type (time-mean or transient) of forcing.
Figure 31. Time-averaged (days 2-16) streamfunction anomaly ($\Psi'$) for experiment with only transient forcing ($\Delta_{\text{trans}}$) at (a) 200 mb, (b) 850 mb, and (c) cross-section at 55°N; and experiment without transient forcing ($\Delta_{\text{res+olr}}$) at (d) 200 mb, (e) 850 mb and (c) cross-section at 55°N for Atlantic case. The contour interval is $1.0 \times 10^6 \text{ m}^2\text{s}^{-1}$ in (a)-(c), $4.0 \times 10^6 \text{ m}^2\text{s}^{-1}$ in (d), and $2.0 \times 10^6 \text{ m}^2\text{s}^{-1}$ in (e). Positive areas are hatched.
Finally, we suggest that the time-mean transient forcing's inability to simulate Atlantic blocking is the result of feedback effects between synoptic-scale waves and ultralong waves, since this forcing does not represent the feedback effect. Residual forcing, however, includes all types of nonlinear interactions. Therefore, the response to the feedback effect by the ultralong waves and the reinforcement between synoptic-scale waves and ultralong waves are captured by the residual forcing, although this forcing is still in a time-mean form. This finding is consistent with Egger and Schilling's (1984) result that, as long as the statistical characteristics of forcing are correctly presented, the model will produce the correct climate. A further investigation of the feedback effect is accomplished by means of a comparison of blockings simulated by two different model resolutions, which is presented in the last section of this chapter.

4. Remote Forcing Experiments

From the diagnostic study (Chen and Tzeng, 1990b) and the Pacific blocking experiments, we found that the effect of tropical heating (through divergent circulation) is to amplify the vorticity source (divergence term) upstream of the blocking center and, hence, to split and to slow down the strong westerly over the blocking area. The observed Atlantic blocking anomaly indicates that the size and intensity of the divergent-convergent pair over the Atlantic blocking area are much smaller than those over the Pacific blocking area. To find out whether or not these differences in divergence field are affected by
tropical heating, we conducted two experiments: one with and one without tropical (remote) forcing ($A_{olr}$ and $A_{res+tns}$, respectively).

The results of the experiment with only tropical heating ($A_{olr}$) display a pattern similar to that of $P_{olr}$ experiment, since the OLR distributions of these two (Pacific and Atlantic) cases also resemble each other, especially over the tropics. On the other hand, the simulated $\psi'$ shows a very weak ridge (about 5% of the intensity of the control run) over Western Europe, and its location is about 5° longitude west of the blocking center of the control run. We can still see, however, that the divergence (convergence) center is located on the upstream (downstream) of the 200 mb blocking center in spite of its very weak intensity (Figure 32). Therefore, tropical heating still provides a positive effect on the formation and maintenance of Atlantic blocking by providing vorticity source through the divergent circulation (Chen and Tzeng, 1990b). Note that the subtropical oceanic high (Figure 32a) is also intensified in this experiment. This oceanic high is connected to the split of the subtropical jet streams through a thermally indirect circulation (corresponding to divergence circulation) (Blackmon et al., 1977). Hence, like the $P_{olr}$ experiment, the split of the jet stream is also enhanced to fulfill Rex's (1950a) first blocking criterion.

The experiment without remote forcing ($A_{res+tns}$) presents a strong resemblance to the control run ($A_{ctl}$) in both intensity (95%) and distribution. The percentage of the intensity of this no remote forcing experiment ($A_{res+tns}$) is larger than other experiments. Therefore, we can conclude that the effect of remote forcing is less important to
Figure 32. The streamfunction anomaly ($\psi'$) and divergent circulation ($\chi'$) at 200 and 850 mb for the experiment with only tropical forcing ($A_{o1r}$), for the Atlantic cases. The contour interval is $1.0 \times 10^4$ m$^2$s$^{-1}$ in (b) and $0.5 \times 10^4$ m$^2$s$^{-1}$ in (a) and (c)-(e). Positive areas are hatched.
Atlantic blocking than to Pacific blocking.

5. Summary of Atlantic Blocking Experiments

The control run of the Atlantic blocking experiment can also simulate the Atlantic blocking to a fairly good extent in terms of intensity, energetics, time evolution, and horizontal structure. The location of the blocking center, however, shifts slightly southward. This shift may be caused by the smoothed forcing over the polar area, as mentioned in the previous chapter.

As with the Pacific blocking experiments, the results of the Atlantic blocking experiments show that the residual (local) forcing is the major forcing to form the Atlantic blocking in that this forcing can simulate the blocking better in the lower troposphere. This finding may be supported by the results of energetics studies (Hansen and Chen, 1982), which indicates that the Atlantic block is forced by the nonlinear interaction of intense baroclinic cyclone waves with barotropic ultralong waves, since the baroclinic cyclone waves are more vigorous in the lower troposphere.

Time-mean transient forcing is unimportant for the formation of blocking over the Atlantic and Pacific sectors, possibly because transient forcing is a time-mean form, which does not include the feedback effect between synoptic-scale waves and ultralong waves. This feedback effect, however, can be captured by the residual forcing because residual forcing consists of all types of wave-wave interactions. In the next section, examination of the feedback effect
is provided by means of a comparison between two different model resolutions (R15 and T8).

The experiments with and without tropical heating show results similar to those of the Pacific experiments. In other words, tropical (remote) heating directly intensifies the divergent circulation over the tropics and upstream of the blocking center, as indicated by Chen and Wiin-Nielsen (1976). The amplified divergence upstream (downstream) of the blocking (jet stream) then locally enhances the defluence (split) of the strong westerly and hence reduces the westerly over the blocking area. These procedures involve two effects: First, the divergence center upstream of the blocking center provides a vorticity source, \(-(f + \zeta)\nabla \cdot \mathbf{v}\), counterbalancing the advection of the vorticity by the strong westerlies (Chen and Tzeng, 1990b). Second, the intensified divergent circulation downstream of the jet stream can amplify the thermally indirect circulation and hence increase the split of the westerlies (Blackmon et al., 1977). Both of these effects slow down the strong westerlies and as suggested by Shutts (1983) and others, provide a preferred location for the formation of blocking. On the other hand, tropical heating can shift the location of the blocking center westward during the blocking simulation and force the retrogression of the simulated blocking. These characteristics are consistent with observations.

In short, the blocking is formed and maintained primarily by residual (local) forcing. These procedures are possibly proceeded by the feedback effect between the intense synoptic-scale waves and the
ultralong waves. The transient and tropical (remote) forcings work as agents to modify the blocking location and to provide a favorable background for the blocking, respectively.

C. A Comparison of R15 and T8 Truncated Resolutions

It is controversial whether or not a very low resolution model can simulate a block. Charney and DeVore (1979) indicated that a highly truncated barotropic channel model can simulate blocking. Bengtsson (1981) and Tibaldi and Ji (1983), however, pointed out that the model resolution seems to play a major role in the success of the blocking simulation. But, as stressed in the beginning of this study, model resolution is not a major topic of this investigation. Nevertheless, to test the performance of this newly developed linear transient primitive equation model and to save computer resources, we reduced the model's horizontal resolution from R15 to T8. Consequently, we were able to reduce the memory size and computation time of the model by about 85% and to test this low-resolution model on our SUN workstation. Although this testing was a sidetrack of this study, the results are very promising.

We have pointed out in the previous section that transient forcing (time-mean of the synoptic-synoptic scale waves' nonlinear interactions) cannot simulate the Atlantic blocking properly. Moreover, we proposed that this inability is due to the feedback effect between the synoptic-scale waves and the ultralong waves. The comparison between higher and
lower resolution model simulations provides us with a feasible way estimating the strength of the feedback effect. Because the T8 truncated resolution retains only longwaves (wavenumbers \( \leq 8 \)), the nonlinear interaction of shorter waves (wavenumber \( > 8 \)) is naturally excluded from this model. The model with R15 truncated resolution, however, includes more shortwave components, especially the synoptic-scale waves. Therefore, the interaction (or feedback effect) between synoptic-scale waves and ultralong waves can be simulated significantly by the R15 resolution model. The difference between R15 and T8 resolution model simulations, therefore, is a good indicator for estimating the strength of the feedback effect.

Regarding the intensities of blockings over the Pacific and Atlantic regions, the results from the R15 truncated resolution (Figure 10 and 26) are different from those from the T8 resolution (Figure 33). The maximum simulated streamfunction of both Pacific and Atlantic blockings, from both resolutions, are as follows:

<table>
<thead>
<tr>
<th></th>
<th>Pacific blocking</th>
<th>Atlantic blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>T8</td>
<td>(3.060 \times 10^7 \text{ m}^2\text{s}^{-1} )</td>
<td>(2.250 \times 10^7 \text{ m}^2\text{s}^{-1} )</td>
</tr>
<tr>
<td>R15</td>
<td>(2.755 \times 10^7 \text{ m}^2\text{s}^{-1} )</td>
<td>(3.872 \times 10^7 \text{ m}^2\text{s}^{-1} )</td>
</tr>
<tr>
<td>difference</td>
<td>-10%</td>
<td>42%</td>
</tr>
</tbody>
</table>

Obviously, the shortwaves (particularly the synoptic-scale wave) regime plays the ultimate role in the simulation of Atlantic blocking. This not only increases the intensity of Atlantic blocking by 42%, but
Figure 33. Time-averaged (days 2-16) streamfunction anomaly ($\Psi'$) at (a) 200 mb and (b) 850 mb for the Pacific T8 control run; and at (c) 200 mb, and (d) 850 mb for the Atlantic T8 control run. The contour interval is $2.0 \times 10^6$ m$^2$s$^{-1}$. Positive areas are hatched.
it also simulates better blocking location and pattern. The feedback effect is therefore important to the Atlantic blocking simulation and can be achieved by the following procedures: First, the ultralong waves are intensified by residual forcing (heating or vorticity source), which then enhances the activity of the synoptic-scale waves; second, the nonlinear interaction between the intense synoptic-scale waves and the ultralong waves amplifies the ultralong waves (or the blocking), as indicated by Hansen and Chen (1982). Then, this procedure is repeated. Hence, the synoptic-scale waves and the ultralong waves reinforce each other and form a significant Atlantic blocking.

On the other hand, the pattern and location of Pacific blocking are not affected by the shortwaves, whereas the its intensity is reduced by about 10% in the R15 simulation. Pacific blocking, therefore, is intensified by the ultralong waves. The short waves just consume the blocking kinetic energy.

The results of Atlantic and Pacific blockings are in accord with Hansen and Chen's (1982) and Chen and Shukla's (1983) results. In other words, Atlantic blocking is forced by the feedback effect between intense synoptic-scale waves and ultralong waves. Pacific blocking results from the amplification of planetary-scale waves.

In addition, the location and pattern of Pacific blocking are about the same in the R15 and T8 resolution simulations. The model with R15 truncated resolution, however, can simulate an Atlantic blocking much closer to the observations than the model with the T8 resolution, in terms of the location, pattern, and intensity. We may therefore also
conclude that a higher resolution model is necessary to simulate a realistic blocking, especially for Atlantic blocking.

Finally, although Metz (1986) pointed out that the model’s blocking activity is due to transient (in time) forcing, our R15 truncated resolution model with residual forcing can still simulate a very realistic Atlantic blocking, as shown in the previous sections. Therefore, we conclude from our model simulations and from another blocking study (Egger and Schilling, 1984) that, with respect to a simulation of the long-term climate, it is not necessary to represent every detail of the cyclone-scale fluxes. The model will produce the correct climate if the statistical characteristics of forcing are correctly presented.
IV. CONCLUSIONS AND REMARKS

A. Conclusions

Chen and Tzeng (1990b) suggested that three possible forcings (local, transient, and remote) initiate blocking development. To demonstrate the validity of this suggestion, we used a numerical study. We constructed a linearized (transient) nine-level primitive equation model to investigate the formation of blocking. A control run, which includes all three forcings with different weighting factors, was performed to examine the capability of the model to simulate blocking. Comparing the experiments using different forcing combinations with the control run, we were able to study the effect of these three forcings on the formation of blocking.

Both the Pacific and Atlantic control runs can simulate very realistic blockings over their respective sectors. Significant blocking features simulated by both control runs are summarized below:

- The duration of simulated blocking is about two weeks;
- A dipole structure of $\Psi'$ is strikingly exhibited in the entire troposphere;
- The blocking anticyclone is tilted slightly westward with height. This tilting is stronger during the initial stage of blocking than during the mature stage (Dole, 1986);
- The maximum intensity of the simulated $\Psi'$ at the blocking center is within one standard deviation error range from the mean.
of maximum $\Psi'$ over 18 selected blocking cases ($\bar{x} \pm \sigma$);

- The global- and area-mean energy levels of various simulated energy quantities ([APE], [KE], $A_Z$, $A_E$, $K_Z$, and $K_E$) are consistent with those of the observations (e.g., Lejenās, 1977, and Chen and Shukla, 1983);

- A northward positive sensible heat transport upstream of the blocking center is simulated by the model during the developing stage of blocking (Colucci, 1987); and a southward negative sensitive heat transport downstream of the blocking center is noticeable during the mature stage of blocking;

- Divergence (convergence) upstream (downstream) of the 200 mb blocking center is simulated by the model. This result is consistent with observations and shows that both the size and intensity of this convergence-divergence pair are smaller in the Atlantic than in the Pacific sector; and

- The constructive interference of ultralong waves (wavenumbers 1-4) is in accordance with observations (Austin, 1980 and Chen and Shukla, 1983), i.e., the intensified wavenumbers 2 and 3 form Pacific blocking and wavenumbers 1 and 3 form Atlantic blocking.

Moreover, both control runs can simulate two strong ridges, one over the Atlantic sector and the other over the Pacific sector, as well as a weak ridge over Central Asia. The simulation of these ridges indicate that the model can not only simulate double blocks but can also simulate the
Central Asian block, which is missed by the NCAR GCM (Malone et al., 1984 and Blackmon et al., 1986).

Six experiments (three single-forcing and three double-forcing) are performed in the Pacific blocking simulation. Comparing the results of these experiments with those of the control run, we can investigate the effect of the three forcings on the formation of blocking. The same procedures are applied to the Atlantic blocking simulation, except that Atlantic blocking forcings are used.

The results of these experiments show that residual forcing is the major forcing to initiate and maintain blocking for both the Pacific and Atlantic blockings. The response of Pacific blocking to residual forcing, however, is different from that of Atlantic blocking. Pacific blocking is forced by the amplification of the ultralong waves (barotropically or baroclinically, e.g., Chen and Shukla, 1983), whereas Atlantic blocking resulted from nonlinear interaction (a feedback effect) between the intense synoptic-scale waves and the ultralong waves (Hansen and Chen, 1982). The rationale for these conclusions are presented next.

For Pacific blocking, three aspects are offered to depict its mechanism of formation. First, residual forcing can simulate Pacific blocking better in the upper troposphere than in the lower troposphere. The reason for this may be that residual forcing is greater in the upper levels, especially vorticity residual forcing ($R_f'$), and that planetary-scale waves are more significant in the upper troposphere than in the lower troposphere. Hence, the response of planetary-scale waves to
forcing is considerably stronger in the upper than in the lower levels. Second, comparing our residual forcing with Metz's (1986) stochastic forcing, we found that the difference between these two forcings can be attributed to the planetary-scale wave regime and to the blocking anomaly heating ($R'_T$), which are not included in Metz's forcing. Therefore, our residual forcing can simulate a very realistic Pacific blocking, whereas Metz's forcing cannot. Metz attributed his failure in Pacific blocking simulation to an inadequate forcing. Our results support his postulation. Finally, in a comparison of model simulations from two different horizontal resolutions (R15 and T8 truncated), we found that the lower resolution (T8) model using large-scale forcing is sufficient to simulate Pacific blocking.

Atlantic blocking, on the other hand, results from the feedback effect between synoptic-scale waves and ultralong waves. This result can be substantiated by our model simulations in two ways. First, residual forcing can simulate Atlantic blocking better in the lower than in the upper troposphere, possibly because the polar fronts (cyclone waves) are more intense and active in the lower (below 500 mb) than in the upper troposphere. Thus, the interaction between intense synoptic-scale waves and ultralong waves is more important in the lower than in the upper troposphere.

Second, Metz (1986) indicated that the time-mean of transient forcing does not support a blocking pattern. Therefore, the model blocking activity is due only to transient forcing. His first finding is consistent with that of our experiment with time-mean transient
forcing. His second finding, however, may not always be consistent with ours, since our time-mean residual forcing can also simulate a realistic Atlantic block. We proposed that this difference can be caused by the feedback effect in the forcing and in the model simulation. Time-mean transient forcing includes only nonlinear interaction of synoptic-scale waves, whereas the residual forcing takes all types of wave-wave interactions into account. Therefore, the time-mean transient forcing does not include the feedback effect between intense synoptic-scale waves and ultralong waves, whereas the residual forcing does. Moreover, comparison between the results from the R15 truncated model and the T8 truncated model shows that the R15 truncated model can simulate the Atlantic blocking better than can the T8 truncated model, in terms of both intensity and location. The R15 model possesses more shortwave components than does the T8 model, especially for the synoptic-scale waves. Consequently, the interaction between synoptic-scale waves and planetary-scale waves are more significant in the R15 model. Therefore, the feedback effect (reinforcement) between planetary-scale waves and synoptic-scale waves is more vigorous in the R15 model.

Transient forcing, in turn, can generate good blocking patterns in both the Pacific and Atlantic experiments, but the intensity of these patterns is too weak (only one-quarter that of the control run) to form a block. In the residual forcing experiment, we showed that the time-mean transient forcing is inadequate to represent the nonlinear interaction between the synoptic- and the planetary-scale waves, especially in terms of feedback effect. It is, therefore, very
important to find a correct forcing for the model simulation. Transient forcing can still modify the pattern and location of blocking and can slightly amplify (by about 10%) the intensity of Pacific and Atlantic blocks.

The importance of anomalous tropical heating to the middle and high latitude large-scale anomaly patterns has been stressed by many studies. For example, Horel and Wallace (1981) suggested that an anomalous warm sea-surface temperature (enhanced rainfall) in the equatorial Pacific (over the dateline) may enhance the PNA teleconnection pattern. Namias (1964) also pointed out that midlatitude blocking actions can be statistically correlated with anomalous sea-surface temperatures in the midlatitude and tropical oceans. All of these studies attempted to relate heating anomaly to the midlatitude persistent anomaly pattern. In contrast, our model simulation indicates that remote (tropical) forcing does not amplify the intensity of the blocking directly. Rather, it intensifies divergent circulations both globally and locally and hence modulates the midlatitude westerlies (Chen and Wiin-Nielsen, 1976) to become a favorable environment for blocking formation. An amplified divergent circulation enhances the split of the westerlies on the downstream side of the subtropical jet stream (the upstream side of the blocking center) and as such can suppress the westerlies over these areas. The effect of this remote (tropical) forcing anomaly is to provide a sufficiently weak westerly over the preference locations to allow blocking to form. Many studies, e.g., Shutts (1983), Colucci (1987) have supported this finding.
Another effect of remote forcing is to retrogress the blocking anticyclone northwestward.

Finally, we may conclude from these simulations that residual forcing is the primary forcing for the initiation and maintenance of both the Pacific and Atlantic blocks. The mechanisms of these two blocks are different, however. Pacific blocking was forced by the amplification of planetary-scale waves (barotropically or baroclinically) (e.g., Chen and Shukla, 1983). Atlantic blocking resulted from a nonlinear interaction between intense synoptic-scale waves and ultralong waves and from the feedback effect (e.g., Hansen and Chen, 1982). Furthermore, the time-mean of the transient forcing is inadequate to represent the feedback effect. This transient forcing, however, can still modify the pattern and location of the blocking. Finally, remote forcing does not directly amplify the intensity of blocking, but it does provide a favorable environment for blocking formation (e.g., it splits and suppresses the strong westerlies and supports the available potential energy).

B. Remarks

Although our linear nine-level transient PE model can simulate realistic blocking over the Pacific and Atlantic sectors, the comparison between the R15 and T8 truncated resolution models showed that a higher resolution model is necessary to simulate a realistic blocking, especially for Atlantic blocking. This is consistent with the model
study by Bengtsson (1981). Bengtsson indicated that the actual evolution of blocking simulated by a low resolution model differs substantially from observations and consequently the practical value of this model is reduced and restricts its use for predicting blocking.

Furthermore, the model resolution affects the distribution and altitude of earth orography. Blackmon et al. (1987) indicated that removing mountains greatly reduced the amplitude of the stationary wavenumbers 2 and 3 in the upper troposphere. The spectral (constructive interference) analyses (Austin, 1980; Chen and Shukla, 1983) showed that these two waves are the major factors responsible for the formation and maintenance of blocking over the Pacific and Atlantic sectors.

The lower model resolution creates a higher truncation error in both the zonal Fourier components and the meridional Legendre coefficients. The major effect of zonal wave truncation can be found from the distribution of orography. A well known problem is that earth orography with the R15 resolution has about two hundred meters of negative altitude over the ocean near the coast. Another problem is the location of the mountains. For example, the Rocky mountains shifts westward about 10° longitudes in the R15 resolution. On the other hand, the meridional truncation of the Legendre coefficients is serious in the polar area, especially for a white noise field such as divergence. This problem causes major errors in the north-south derivatives of spectral coefficients. Finally, although these deficiencies are intrinsic to the model, they can be improved by increasing the model resolution.
After the higher resolution model is considered, another crucial factor affecting model climate that should be considered is physical parameterization. Bengtsson (1981) indicated that the high growth rate of ultralong waves seems to be a problem mainly for models with oversimplified physical parameterizations. Comparing the spin-up time between two different models, Chen and Branstator (1989) suspected that the difference in spin-up time is perhaps the result of the differences in vertical diffusion parameterizations used in the two models. Thus, it is important to consider physical parameterizations in a detailed manner.

Moreover, the standing mode is a weak nonlinear mode, and as such is convenient for modeling. That is why we chose the linear transient PE model. The results of our blocking experiments also suggest that these three forcings are still more or less linear in space during the beginning of the model simulation. The weak nonlinear assumption, however, is not always true for every forcing under every condition. Geisler et al. (1985) pointed out that the amplitude of the model response is a highly nonlinear function of the amplitude of the sea-surface temperature anomaly. Shutts (1983) found from a nonlinear barotropic vorticity equation that dipole blocking patterns can be created simply by introducing an eddy generator into a sufficiently weak, uniform westerly flow. This background flow does not have to be a split jet stream as is used in the linear model. Therefore, a complete primitive equation model is necessary if we are to simulate blocking in more detail by including nonlinear effects.
Finally, we are still not sure what causes blocking residual forcing anomalies. From the equations, we know that this forcing includes all types of nonlinear interactions (e.g., short-short, short-long, long-long, and long-zonal mean waves interactions), anomaly diabatic heating, diffusion, dissipation, and even computational and observational errors. It is beyond the scope of this study to investigate the characteristics of diffusion, dissipation, and other error terms. From the model simulation, however, we found that this forcing is dominated by nonlinear interactions, especially the feedback effect, and by heating anomaly around the blocking. Moreover, the nonlinear interaction terms are dominated by the planetary-scale wave regime (or low-frequency mode) (Metz, 1986). The residual heating anomaly ($R'_T$) shows a significant heating anomaly on the west and north side of the blocking center and a cooling anomaly on the east and south side. This blocking heating anomaly is different from the winter mean anomaly heating, which is important over storm track (e.g., Lau, 1979 and Chen and Trenberth, 1988b). All these features are characteristics of blocking forcing. We still are uncertain, however, how this forcing is formed.

C. Recommendations for Further Studies

As indicated in the previous section, the linear transient model has some deficiencies (resolution, physical parameterization, and nonlinearity). On the other hand, Blackmon et al. (1986) showed that the
NCAR GCM is able to generate realistic blocking episodes over the wintertime North Atlantic and Pacific oceans. Thus, it would be worthwhile to use a complete general circulation model (e.g., NCAR's CCM) to test these blocking hypotheses again.

Furthermore, the wintertime blocking episodes are always accompanied by a locally enhanced split westerly on the downstream side of subtropical jet streams (Rex, 1950a,b). The intensity of the split flow (difluence) is related to the intensity of the jet streams (Blackmon et al., 1977). Recently, Dole (1986) found that the most systematic precursors for the Pacific anomalies are related to the variations in jet intensity and structure over eastern Asia and the southwestern North Pacific Ocean. It could be interesting to investigate further the relation between the subtropical jet stream and blocking.

The jet stream possesses a significant annual cycle. During the summer season, many blocking episodes are also observed, but the midlatitude westerly pattern is dramatically changed. It would also be interesting to analyze the effect of summertime midlatitude westerlies on blocking and the effect of these three forcings on summertime blocking.

Moreover, Shukla and Mo (1983) indicated that the geographically preferable blocking locations remain nearly the same throughout the year. Seasonal changes in blocking activity, however, are still in dispute. Shukla and Mo attributed this to the uncertainty of the definition of the blocking. Recently, by synthesizing Charney et al.'s
(1981) definition of spatial anomaly and Dole's (1982) of temporal anomaly, a degree of agreement has been reached in the definition of blocking. Hence, it is worth reexamining the annual variation of the blocking activity under this new blocking definition.

From this model simulation and another observational study (Chen and Tzeng, 1990b), we found that the divergence circulation is important in the initiation of blocking. But the 30-50 day oscillation is also dominated by the planetary-scale divergent circulation. The relation between these two phenomena, however, is another interesting topic.

Finally, although Pacific and Atlantic blockings are both dominated by residual forcing, the response of the ultralong waves to the forcing is different over these two areas. Austin (1980) pointed out that the Pacific blocking is formed by the intensified wavenumbers 2 and 3, whereas the Atlantic blocking is intensified by wavenumbers 1 and 3. The energetics study (Hansen and Chen, 1982; Chen and Shukla, 1983) have indicated that these ultralong waves are amplified by different mechanisms (barotropic and baroclinic). Another question that should be addressed is what causes the enhancement of these ultralong waves and the differences between the influences of ultralong waves for Pacific and Atlantic blockings.
V. REFERENCES


VI. ACKNOWLEDGEMENTS

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A. Transformation to Spectral Space

Before we solve the model (2.26)-(2.29), we need to transform every term in the equations into spectral space. There are three types of terms in the equations set, such as the undifferentiated term $Q$, the longitudinally differentiated term $Q_\lambda$, and the meridionally differentiated term $Q_\mu$.

Transformation of the undifferentiated term is obtained by straightforward application of (2.56)-(2.58)

$$Q_n^m = \sum_{j-1}^j Q^m(\mu_j) w_j,$$

(A.1)

where $Q^m(\mu_j)$ is the Fourier coefficient of $Q$ with wavenumber $m$ at the Gaussian latitude line $\mu_j$. The longitudinally differentiated term is handled by integration by parts using the cyclic boundary conditions,

$$\left( \frac{\partial Q}{\partial \lambda} \right)^m = \frac{1}{2\pi} \int_0^{2\pi} \frac{\partial Q}{\partial \lambda} e^{-im\lambda} d\lambda,$$

(A.2)

so that the Fourier transform is performed first, then the differentiation is carried out in spectral space. The transformation to spherical harmonic space then follows (A.1),

$$\left( \frac{1}{a(1-\mu^2)} \frac{\partial Q}{\partial \lambda} \right)_n^m = \sum_{j-1}^j i m Q^m(\mu_j) \frac{P_n^m(\mu_j)}{a(1-\mu_j^2)} w_j.$$

(A.3)
The latitudinally differentiated term is handled by integration by parts using zero boundary conditions at the poles,

\[
\left( \frac{1 - \mu^2}{a(1 - \mu^2)} \frac{\partial \mathcal{Q}}{\partial \mu} \right)_n^m = \int_{-1}^{1} \frac{1}{a(1 - \mu^2)} (1 - \mu^2) \frac{\partial}{\partial \mu} \mathcal{Q}^m P_n^m d\mu \\
= -\int_{-1}^{1} \frac{1}{a(1 - \mu^2)} \mathcal{Q}^m (1 - \mu^2) \frac{dP_n^m}{d\mu} d\mu.
\]

(A.4)

Defining the derivative of the associated Legendre polynomial by

\[
H_n^m = (1 - \mu^2) \frac{dP_n^m}{d\mu} = -n \varepsilon_n^m P_{n+1}^m + (n+1) \varepsilon_n^m P_n^m,
\]

(A.5)

where \( \varepsilon_n^m = \sqrt{\frac{n^2 - m^2}{4n^2 - 1}} \), (A.4) can be written as

\[
\left( \frac{1}{a(1 - \mu^2)} (1 - \mu^2) \frac{\partial \mathcal{Q}}{\partial \mu} \right)_n^m = -\sum_{j=1}^{l} \mathcal{Q}^m(\mu_j) \frac{H_n^m(\mu_j)}{a(1 - \mu^2)} w_j.
\]

(A.6)

Similarly, the \( \nabla^2 \) operator in the divergence equation can be converted to spectral space by sequential integration by parts and then applying the relationship,

\[
\nabla^2 P_n^m(\mu) e^{im\lambda} = -\frac{n(n+1)}{a^2} P_n^m(\mu) e^{im\lambda},
\]

(A.7)

to each spherical harmonic function individually,

\[
(\nabla^2 \mathcal{Q})_n^m = \sum_{j=1}^{j} -\frac{n(n+1)}{a^2} \mathcal{Q}^m(\mu_j) P_n^m(\mu_j) w_j.
\]

(A.8)
B. Transformation from Spectral to Grid Space

To perform the transform method on the nonlinear terms, we must convert the spectral data back to the grid space. From (2.54), the transformation of any variable $Q$ is given by

$$Q(\lambda, \mu) = \sum_{m=-M}^{N} \left[ \sum_{n=-N}^{N} Q_n^m P_n^m(\mu) \right] \theta^{im\lambda}. \quad (A.9)$$

The inner sum is done essentially as a vector product over $n$, and the outer is again performed by an FFT subroutine.

The advection and gradient of surface-pressure, $\mathbf{v} \cdot \nabla q$ and $\nabla q$ ($q = \ln P_s$), are needed on the grid space,

$$\mathbf{v} \cdot \nabla q = \frac{u}{\cos \phi} \frac{\partial q}{\partial \lambda} + \frac{v}{\cos \phi} (1 - \mu^2) \frac{\partial q}{\partial \mu}. \quad (A.10)$$

These required derivations are given by

$$\frac{\partial q}{\partial \lambda} = \sum_{m=-M}^{N} \left[ \sum_{n=-N}^{N} q_n^m P_n^m(\mu) \right] \theta^{im\lambda}, \quad (A.11)$$

which involve the same operations as (A.9). The other variables needed on the grid are the horizontal wind $u$ and $v$. These can be computed directly from the relative vorticity and divergence coefficients using the relations

$$\zeta_n^m = -\frac{n(n+1)}{a^2} \psi_n^m, \quad (A.12)$$

$$\delta_n^m = -\frac{n(n+1)}{a^2} \chi_n^m, \quad (A.13)$$
and

\[ u = \frac{1}{a} \frac{\partial \chi}{\partial \lambda} - \frac{1}{a} \frac{\mu^2}{\partial \mu} \]

\[ - \sum_{m=-M}^{M} \sum_{n=\text{lm}} \left[ \frac{im}{n(n+1)} \delta_{n,m}^m (\mu) - \frac{1}{n(n+1)} \zeta_{n,m}^m (\mu) \right] e^{im\lambda}, \tag{A.14} \]

\[ v = \frac{1}{a} \frac{\partial \chi}{\partial \lambda} + \frac{1}{a} \frac{\mu^2}{\partial \mu} \]

\[ - \sum_{m=-M}^{M} \sum_{n=\text{lm}} \left[ \frac{im}{n(n+1)} \zeta_{n,m}^m (\mu) + \frac{1}{n(n+1)} \delta_{n,m}^m (\mu) \right] e^{im\lambda}. \tag{A.15} \]