Evaluation of the transient eddy current potential drop of a four point probe

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Abstract
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Keywords
nondestructive testing, eddy currents, electrical conductivity, QNDE

Disciplines
Materials Science and Engineering | Structures and Materials

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EVALUATION OF THE TRANSIENT EDDY CURRENT POTENTIAL DROP OF A FOUR POINT PROBE

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ABSTRACT. The transient electrical potential drop of a four point probe has been calculated for the case where a current pulse is injected into a conductive plate via two surface contact electrodes and the voltage measured between two other contact electrodes. The four contact points can be co-linear but this is not always case. For example, they can form a rectangle. Usually such probes carry direct current or alternating current and are used to measure electrical conductivity, crack dimensions or variations of conductivity and magnetic permeability with depth. However, the advantage of a current pulse excitation is that information on the variations of material properties with depth can be acquired rapidly and conveniently. What is needed is a means to infer material properties such as the conductivity variations with depth from the transient field measurements. Here, as an initial step in developing this analysis, we report on the evaluation of transient potential drop signals for four point probes on a homogeneous conductive plates.

Keywords: Transient Current Potential Drop, Nondestructive Evaluation
PACS: 41.20.Gx

INTRODUCTION

Four-point probes are used to determine the electrical conductivity of a sample by passing current between two contact electrodes on the surface while measuring the potential drop between two additional contact electrodes. Apart from the measurement of conductivity, the probes are also used to determine permeability [1], crack depth [2,3] and the effects of surface treatment such as case hardening [4,5]. Typically, the electrodes are sprung-loaded pins, Fig. 1, two of which carry the current, either alternating or direct. In order to determine variations of material properties with depth due to say stress, cold work, induction hardening etc., measurements of potential can be made over a range of frequencies and hence at different skin depths. However, a convenient and simple alternative, is to use a transient current potential drop (TCPD) technique.

The development of a model based inversion techniques for alternating current potential drop (ACPD) has been developed using the initial analysis of the field of a four point probe in a uniform conductor as a starting point [6]. The response of the probe in contact with a conductive plate is used to predict the ACPD as a function of frequency [7]. Here we build on the main results of this work by modifying the frequency domain equations to represent Laplace transforms of time domain signals. By taking the inverse Laplace transform of the expression
FIGURE 1. Evolution of the injected field of a four point probe for the case where the injected current is a step function in time. (a) Initially the field distribution is largely concentrated close to the surface. (b) Later it migrates deeper into the material. (c) In the long time limit, it reaches a steady state which is effectively the field produced by a direct current.

for the potential drop produced by an injected current pulse, we get predictions of TCPD between voltage electrodes on a homogeneous plate. This opens the possibility of analyzing the depth dependence of material properties by using a similar approach for predicting the TCPD for measurements on layered media. In developing here the homogeneous plate theory, we begin by giving a brief outline of the principles of potential drop measurement.

FORMULATION

In a four point probe, the contact points are commonly in line, Fig. 1, or they are located at the corners of a rectangle. However, the measured voltage can be determined for an arbitrary placement of the electrodes. In general we have current electrodes at say $C_1$ and $C_2$ and voltage electrode at $P_1$ and $P_2$, Fig. 2(a). The potential difference between two voltage electrodes is written

$$\Delta V = V_2 - V_1$$

(1)

and is dependent on the conductivity, injected current and pin positions. On a planar conductor of conductivity $\sigma$, the observed voltage can be expressed in the form

$$\Delta V = \frac{I_0}{2\pi\sigma} \left[ f(\rho_{22}) - f(\rho_{23}) - f(\rho_{12}) + f(\rho_{11}) \right]$$

(2)

where $\rho_{nm} = C_n - P_m$, $n, m = 1, 2$ is the distance between the contact point of the current electrode at $C_n$ and the contact point of the voltage electrode at $P_m$, Fig. 2(a). For DC injection, $f(\rho)$ and therefore $\Delta V$ is a real number. For AC, it is commonly represented as complex phasor and for transient current it is real function of time.

The simplest example is when DC is injected into a thick plate that can be modeled as a half-space conductor. In this case, the current spreads uniformly from the surface injection point, Fig 2(b), which means that the current density at a point a distance $R$ from the injection point is $I/(2\pi R)$ since the current is uniform over a hemisphere whose area is $2\pi R$. The electric potential in the conductor is therefore

$$\Phi(\vec{r}) = \frac{I}{2\pi\sigma |\vec{r} - \vec{r}_0|}$$

(3)

where $\vec{r}_0$ is the coordinate of the current contact point. The potential on the surface at the point $\vec{r}_1$ can be written

$$V(\rho) = \frac{I}{2\pi\sigma\rho}$$

(4)
\[ \Delta V = \frac{I_0}{2\pi \sigma a} \] (5)

which can be used for estimating conductivity from direct current measurements of potential drop.

**Field Representation for ACPD**

Assuming that the current is delivered to a horizontal plate via vertical wires aligned with the z direction, then the magnetic field has no z component. Consequently, the quasi-static electromagnetic field can be expressed solely in terms of a transverse magnetic (TM) potential
\[ \vec{H} = \nabla \times (\hat{z} \psi) . \] (6)

From Ampere’s law, the curl of the quasi-static magnetic field gives the injected current density, \( \vec{J} \), as
\[ \vec{J} = \nabla \times \nabla \times (\hat{z} \psi) = \nabla_i \frac{\partial \psi}{\partial z} - \hat{z} \nabla_i^2 \psi . \] (7)

Subscript \( t \) denotes the tangential components (x and y) with respect to the normal to the surface. From the first of the two terms on the right hand side of equation (7), it can be seen that the electric field in a tangential plane, \( (z \text{ = constant}) \) is the gradient of a scalar potential. Thus
\[ \vec{E}_t = -\nabla_i V \quad \text{where} \quad V = -\frac{1}{\sigma} \frac{\partial \psi}{\partial z} \bigg|_{z=0} . \]

Unlike the case of an electro-static field, the time-harmonic E-field is not the 3D gradient of a potential, therefore the use of the term potential drop when applies to an alternating current fields may seem inappropriate. However, because the field in a plane can expressed as a 2D gradient of a scalar, the alternating current at the surface flows from high to low potential and in this context one can refer to a surface potential drop.
FIGURE 3. Comparison of predictions of ACPD with experimental measurements on a titanium plate. The potential drop is divided by the drive current to give an impedance and the resistive part is shown.

The surface potential at distance \( \rho \) from an AC injection point on a plate can be written

\[
V(\rho, \omega) = \frac{I_0}{2\pi\sigma} f(\rho, \omega) \tag{8}
\]

where \( f(\rho, \omega) \) has the dimensions of reciprocal length. The analysis of plate measurements, carried out to determine \( f(\rho, \omega) \) and hence the potential drop from (2), is based on a few basic assumptions [8-11]:

- Connecting wires are represented as filaments normal to a plane surface.
- Current and voltage contacts are made at points.
- The plate is homogeneous isotropic and has linear material properties.

Subject these requirements, the exact solution for the ACPD on half-space conductor [6] implies that

\[
f(\rho, \omega) = \frac{\rho^k \rho}{\rho} + ik[E_1(-tk \rho) + \ln \rho], \tag{9}
\]

where \( E_1(z) \) is the exponential integral function. Equation (9) together with (2) gives a complex number representing amplitude and phase of the measured PD. Note that in the low frequency limit, equation (9) reduces to \( f(\rho, 0) = 1/\rho \) which is consistent with the DC result for a half-space conductor.

For a plate of finite thickness, the transverse electric potential can be found as an extension of the half-space result using the method of images. Instead of (9), a modified formula has been found in the form of an image series [1]. The image series represents a periodic field in an extended virtual image space which can be expressed in an alternative form as a Fourier series [7]. The image and the Fourier series have complementary convergence properties. This allows us to compute the solutions rapidly for thick plates using a few term in the image series. For thin plates, we need only a few terms in the Fourier series. A comparison of the computed values of ACPD with experimental results measured on a titanium plate over a range of frequencies, is show in Fig. 3 and 4. It can be seen that the agreement between theory and experiment is very good.
FIGURE 4. Comparison of predictions of ACPD with experimental measurements on a titanium plate. The potential drop is divided by the drive current to give an impedance. Here the reactive part is shown corrected for self induction of the pick-up circuit. Pick-up arises from flux linkage through a rectangular region between the pick-up pins bounded by the plate surface and the wire connections to the voltage electrodes.

TRANSIENT POTENTIAL DROP

Current Step

Use a tilde to denote the Laplace transform. Take the current to be a step function in time of magnitude $I_0$, which means it has a Laplace transform

$$\tilde{I}(s) = \frac{I_0}{s}.$$  \hfill (10)

Rewrite the potential due to a one-point injected current, equation (8), as

$$\tilde{V}(p, s) = \frac{I_0}{2\pi\sigma} \tilde{f}(p, s),$$  \hfill (11)

absorbing the $1/s$ factor into $\tilde{f}(p, s)$. Then the function that governs the potential at the surface of a half-space conductor is

$$\tilde{f}(p, s) = \frac{e^{-\sqrt{\mu\sigma sp}}}{sp} - \sqrt{\mu\sigma} / s \left[ E_1(\sqrt{\mu\sigma sp}) + \ln p \right].$$  \hfill (12)

This result can be obtained from a Laplace transform of the quasi-static diffusion equation for the transverse magnetic potential due to an injected current. Alternatively one can get it directly from (9) using the correspondence $-\omega \to s$ and $ik \to -\sqrt{\mu\sigma} s$, thereby circumventing a first principles derivation.

Transient Pick-up Voltage

The inverse Laplace transform of (9) can be obtained with the aid of the standard results listed in Table 1. In the case of the term containing the exponential integral, we have to deal with a function of the form

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FIGURE 5. Variation of the function \( f(\rho, t) \) with position at \( t = 1\mu sec \), \( t = 10\mu sec \), \( t = 100\mu sec \), and as \( t \to \infty \), solid curve.

\[
\tilde{f}(s) = \frac{1}{\sqrt{s}} E_i(k \sqrt{s})
\]  

(13)

whose inverse Laplace transform is not listed in commonly used tables [12, 13] but the required result can be easily derived as follows.

From the definition of the exponential integral

\[
E_i(z) = \int_1^\infty \frac{e^{-zu}}{u} \, du
\]  

(14)

we find that it is necessary to consider the inverse transform of

\[
\tilde{f}(s) = \frac{1}{\sqrt{s}} \int_1^\infty \frac{e^{-k \sqrt{s}u}}{u} \, du.
\]

Assuming that the inverse Laplace transform can be carried out before the \( u \)-integral, Table 1 item 3 gives the time dependence as

\[
F(t) = \frac{1}{\sqrt{\pi t}} \int_1^\infty \frac{\exp \left( -\frac{k^2u^2}{4t} \right)}{u} \, du
\]

With the change of variable \( u^2 = v \), \( du/u = dv/(2v) \) we recover a form that matches that of an exponential integral

\[
F(t) = \frac{1}{2\sqrt{\pi t}} \int_1^\infty \frac{\exp \left( -\frac{k^2v}{4t} \right)}{v} \, dv
\]

Hence from its definition, (14)

\[
F(t) = \frac{1}{2\sqrt{\pi t}} E_i \left( \frac{k^2}{4t} \right)
\]  

(15)

Thus we find from (15) and the results in Table 1, that the inverse Laplace transform of (12) is

\[
f(\rho, t) = \frac{1}{\rho} \text{erfc} \left( \sqrt{\frac{\mu \sigma}{4t}} \rho \right) - \sqrt{\frac{\mu \sigma}{4\pi t}} \left[ E_i \left( \frac{\mu \sigma}{4t} \rho^2 \right) + 2 \log \rho \right]
\]  

(16)
This shows that the decay of the transient signal with time is scaled by the product of the permeability and conductivity. In other words, if this product is increased, the decay time increases proportionally.

**Numerical Results**

Fig. 5 shows the variation of $f(\rho, t)$ with distance from an injection point for different values of time after a current of 1 Amp is started in the drive circuit. This function is proportional to the surface potential due to a single point injection of current. Initially, the potential is large since the current starts by being concentrated at the surface and later diffuses into the material. It is notable also that the initial potential decays relatively slowly with distance from the injection point, indicating that there is initially a remote field effect that decreases in time. Ultimately the potential tends toward the DC state in which the function $f(\rho, t)$ takes on its limiting form $1/\rho$ as shown by the solid curve in Fig. 5.

The variation of the potential drop with time, shown in Fig. 6, is computed using probe and material parameters given in Table 2. After a long time, the potential approaches the DC value.

**CONCLUSION**

If one chooses to have a DC current which is suddenly reduced to zero, then the initial potential will follow a similar curve to that shown in Fig. 6, except that there will be a constant voltage shift, such that the potential drop will tend to zero after a long time. In the case of a layered material, the initial shape of the curve will be indicative of near surface behavior exhibiting a faster initial transition for lower values of the conductivity-permeability product and a slower transition for a large value of the product. Similarly the rate at which

**TABLE 1.** Inverse Laplace transforms from [12].

<table>
<thead>
<tr>
<th>$\tilde{f}(s)$</th>
<th>$F(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\sqrt{s}}$</td>
<td>$\frac{1}{\sqrt{s}t}$</td>
</tr>
<tr>
<td>$\frac{1}{s}e^{-k\sqrt{s}}$</td>
<td>erfc($\frac{k}{\sqrt{s}t}$)</td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{s}}e^{-k\sqrt{s}}$</td>
<td>$\frac{1}{\sqrt{s}t}\exp(-\frac{k^2}{4t})$</td>
</tr>
</tbody>
</table>

**TABLE 2.** Parameters for the coil and test piece

<table>
<thead>
<tr>
<th>Four Point Probe</th>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{11}$</td>
<td>10 mm</td>
</tr>
<tr>
<td>$\rho_{21}$</td>
<td>20 mm</td>
</tr>
<tr>
<td>Conductivity</td>
<td>0.5 MS/m</td>
</tr>
</tbody>
</table>

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FIGURE 6. Potential drop of a four point probe excited by a unit step current in time. The probe parameters are given in Table 2.

the response tends towards the steady state limit will be indicative of the conductivity-permeability product of the substrate.

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