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Capital Controls and Exchange Rate Determination in a Ration Expectations Framework

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Capital Controls and Exchange Rate Determination in a Ration Expectations Framework

Abstract
Crucial to the comparison of fixed versus flexible exchange rates is the mechanism by which exchange rates are determined when governments do not directly support currencies. The monetary approach to exchange rate determination emphasized that an exchange rate is the relative price of two currencies; the equilibrium rate being that which equates desired and actual money stocks. Within this approach, both monetary and real phenomena affect the equilibrium exchange rate through their influence on the stock demand or supply for currencies. The fundamental idea of the monetary approach is that an asset pricing model must be used to examine exchange rate determination.

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Capital Controls and Exchange
Rate Determination in a
Ration Expectations Framework

by

Harvey E. Lapan and Walter Enders

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Crucial to the comparison of fixed versus flexible exchange rates is the mechanism by which exchange rates are determined when governments do not directly support currencies. The monetary approach to exchange rate determination emphasized that an exchange rate is the relative price of two currencies; the equilibrium rate being that which equates desired and actual money stocks. Within this approach, both monetary and real phenomena affect the equilibrium exchange rate through their influence on the stock demand or supply for currencies. The fundamental idea of the monetary approach is that an asset pricing model must be used to examine exchange rate determination.

The incorporation of rational expectations into the monetary approach has carried the concept of the exchange rate as a relative asset price to its logical extreme. In doing so, however, the predictions of the simple monetary models are greatly altered. If the exchange rate is the relative price of two assets, beliefs about the future value of the exchange rate will be the determinant of the current rate of exchange. The only mechanism by which real or monetary disturbances can alter the current exchange rate is for these disturbances to alter exchange rate expectations. For example, Kareken and Wallace (1977) show that in a rational expectations model with no uncertainty (i.e., perfect foresight), the exchange rate must be constant over time, regardless of changes in currency supplies or output levels. Moreover, they show that any constant exchange rate is an equilibrium time path. The links between asset supplies and productivity levels, which were so direct in the simple monetary approach, are totally absent in the Kareken and Wallace model. As long as all economic agents believe that the future value of the exchange rate will be constant, the current value of the exchange rate will not respond to changes in productivity or asset supplies.
Essential to the Kareken and Wallace result that current disturbances will not alter exchange expectations is the implicit assumption that currencies are perfect substitutes. They assume perfect capital mobility and no transactions demand for money. Money is held only as a store of value, and as capital is perfectly mobile - either currency can perform this function equally well. It is not surprising, then, that the relative price of perfect substitutes will be constant over time. While Kareken and Wallace argue that it is ad hoc to postulate country specific money demand functions, it is worthwhile to consider what is meant by "domestic currency" versus the "stateless" money of the Kareken and Wallace model. The fact that governments have the ability to declare a particular form of money as legal tender and to restrict the use of alternative forms of money serves to differentiate monies in the eyes of asset holders. Note that declaring a particular form of money as legal tender is not sufficient to cause national monies to be imperfect substitutes as currencies of various issues could circulate within a particular country. But when there are capital controls - restrictions on domestic holdings of foreign issued currency -- or even a probability of such controls, asset holders will not be indifferent to the composition of currencies within their portfolios. Thus, the Kareken and Wallace results concerning the indeterminacy and constancy of the exchange rate rely upon the assumption that capital controls will never be imposed, i.e., the probability of exchange controls is identically zero.

One purpose of this paper is to consider exchange rate determination in a rational expectations model in which there is a positive probability of capital controls in some future period. It is shown that exchange
rates are determinate and are responsive to both monetary and real disturbances in the manner suggested by the Monetary Approach. Further, as the probability of controls tends to zero, the exchange rate remains determinate. Only when capital controls are impossible (i.e., probability zero) do the Kareken and Wallace results hold.

After considering exchange rate determination, we examine the relative merits of fixed versus flexible exchange rate regimes. We follow the approach developed by Lapan and Enders (1979) and Helpman and Razin (1978) in which expected utility comparisons are the criterion used to select the best exchange regime. Both of these papers use an intergenerational model of the sort developed by Samuelson (1958). All asset and commodity demand functions are derived from microeconomic behavior, and all agents have rational expectations. Both papers consider only the small country case. Domestics hold only domestic currency in the Lapan and Enders paper, while currencies are "stateless" in the Helpman and Razin model. Here we consider the full range of currency substitutability by allowing the probability of currency controls to vary between zero and unity. As long as the probability of controls is not zero, expected utility under the two exchange regimes differ. With fixed rates, expected utility does not depend upon the degree of capital mobility, but expected utility with flexible rates depends upon the probability that controls will be imposed. We also show -- in a two-country model -- that controls can increase expected utility so that flexible exchange rates and an optimally selected probability of controls will be preferred to fixed exchange rates, or to flexible rates with no probability of controls.
The outline of our paper is as follows. In Section I we describe the behavior of individual agents in the context of the standard Samuelson consumption-loan model; we also derive the aggregate equilibrium conditions from individual optimizing behavior. In Section II we describe how the possibility of future capital controls affects the current exchange rate in an otherwise determinate model. Section III expands the analysis of Section II by assuming output in each country follows a Markov process; hence, we analyze how real factors influence exchange rate determination. Our principal results are that exchange rates are responsive to real phenomena, but are not very sensitive to the magnitude of the probability of controls, provided this probability is non-zero. Section IV uses the results of previous sections and shows that fixed rates are preferable to flexible rates if capital is not mobile. Section V compares the two regimes when the probability of controls is less than unity. It is demonstrated that the probability of controls affects expected utility under flexible rates and that at least one country will wish to choose a positive probability of capital controls. Thus, not only is it implausible that capital controls will never be imposed, it is also suboptimal. Conclusions and directions for further research are contained in the final section.

I) The Basic Model

We employ a standard consumption-loan model. It is assumed that:

i) there are two countries - the U.S. and Germany.

ii) each country produces a single (identical) tradeable good.

iii) individuals in each country live for two periods, working in the first, retired in the second.
iv) each generation in each country consists of the same number of people; in any time period, two generations are alive – the working, and the retired.

v) labor supply decisions are – for simplicity – exogenous; each individual is assumed to supply one unit of labor.

vi) output in each country is linear in labor supply.

vii) an individual’s utility depends upon (real) consumption in each period; further, all individuals have identical, homothetic preferences.

viii) commodities are not storeable.

ix) there are (given) outstanding stocks of each country ’s currency; these currencies are the (only) stores of value.

x) under fixed exchange rates, it is believed (accurately) that exchange rates will not change through time.

xi) under flexible exchange rates, the equilibrium exchange rate is determined by market forces (recognizing the probability of capital controls).

Consider a representative individual (i) of generation t; he is born at t, supplies one unit of labor in this period, consumes in t and (t+1). Let

(1) \( C_t^i \) – denote consumption in t by individual i of generation t.

(2) \( B_{t+1}^i \) – denote consumption in (t+1) by individual i of the \( t^{th} \) generation. \( \bar{C}_t^i, \bar{B}_{t+1}^i \) denote the corresponding variables for the
"foreign" country). The utility for this individual (i) is given by:

\[ (3) \quad U(C_t^i, B_t^i) \]

It is assumed that the individual chooses current consumption \( C_t^i \) after his output (income) and current prices are known, but before next period's prices are known.

We assume there are two currencies - dollars (D) and marks (M); the outstanding stocks of each currency are given for all time (denoted \( D, M \)). Under fixed exchange rates, the dollar/mark exchange rate, \( e \) is known for all time, and equal to \( e \); under flexible exchange rates, the current dollar/mark exchange rate is \( e_t \), as determined by market conditions. In general, today's exchange rate (\( e_t \)) will depend upon expectations concerning next period's exchange rate (\( e_{t+1} \)). We assume throughout that all expectations are rational in the sense that individual probability distributions concerning \( e_{t+1} \) coincide with the true distributions.

Under fixed exchange rates, the individual really has only one decision to make - how much to spend (save) today, as the asset composition of his savings is immaterial. On the other hand, under flexible exchange rates, two decisions are needed: (i) how much to consume (save) today; and (ii) how to allocate assets within the portfolio.

Let \( P_t \) denote the dollar price of commodities at \( t \); and let \( D_t^i(M_t^i) \) be the dollars (marks) held by individual \( i \) of generation \( t \). Then, if \( Q_t^i \) is individual output, the asset constraint is:

\[ (4) \quad P_t(Q_t^i - C_t^i) - D_t^i - e_t M_t^i \geq 0. \]

and next period's consumption is given by:

\[ (5) \quad D_t^i + e_{t+1} M_t^i - P_{t+1} B_{t+1}^i \geq 0; \]
or, using (4) and assuming non-satiation:

\[ P_{t+1}^i B_{t+1}^i = P_t^i (Q_t^i - C_t^i) + (e_{t+1} - e_t)M_t^i \]

At \( t \), \( P_t^i \), \( Q_t^i \), and \( e_t \) are known, as is the joint distribution of \( (P_{t+1}^i, e_{t+1}) \) -- denoted by \( g(P_{t+1}^i, e_{t+1}) \). Hence, at \( t \), the individual chooses \( C_t^i, M_t^i, D_t^i \) to maximize the expectation of (3), given (4), (6) and expectations:

\[
\text{Max } E \left[ U(C_t^i, \left\{ \frac{P_t^i (Q_t^i - C_t^i) + (e_{t+1} - e_t)M_t^i}{P_{t+1}^i} \right\}) \right],
\]

subject to (4). In (7), \( E[\cdot] \) denotes the expectation operator; the expectation runs over \( e_{t+1}, P_{t+1}^i \). The Lagrangean for the optimization is:

\[
L = E \left[ U(C_t^i, \left\{ \frac{P_t^i (Q_t^i - C_t^i) + (e_{t+1} - e_t)M_t^i}{P_{t+1}^i} \right\}) \right] + \lambda [P_t^i (Q_t^i - C_t^i) - e_t M_t^i]
\]

where \( M_t^i \in [0, \frac{P_t^i}{e_t} (Q_t^i - C_t^i)] \); note \( (e_{t+1} - e_t)M_t^i \) is the capital gains (losses) in dollars of mark holdings.

Optimizing over \( C_t^i \) and \( M_t^i \) yields:

\[
\begin{align*}
\text{Max } & E \left[ \left( \frac{P_t^i}{P_{t+1}^i} \right) U_2 \right] - \lambda P_t \leq 0 \quad \text{(9)} \\
\text{Max } & E \left[ \left( \frac{e_{t+1} - e_t}{P_{t+1}^i} \right) U_2 \right] - \lambda e_t \leq 0 \quad \text{(10)}
\end{align*}
\]

where \( U_1 \) (\( U_2 \)) reflects the marginal utility of current (future) consumption.

Throughout the paper we shall assume - as is consistent with homothetic preferences - that the utility function is homogeneous (of degree
no larger than one) in its arguments; so that all partial derivatives are homogeneous of the same (non-positive) degree in their arguments. This implies that portfolio composition is independent of wealth. Let:

\[(11) \quad c_t^i = c_t^i/Q_t^i = \text{average propensity for current consumption.}\]

\[(12) \quad W_t^i = P_t(Q_t^i - C_t^i) = \text{dollar wealth.}\]

\[(13) \quad m_t^i = (e_t^i \cdot M_t^i / W_t^i) = \text{fraction of current wealth held in marks.}\]

Then, given current prices \((P_t, e_t)\), output \((Q_t^i)\), and expectations, \((9)\) and \((10)\) uniquely determine the individual's consumption demand and demand for marks. Denote this solution by an * where, for convenience, the superscript \(i\) is dropped. Thus,

\[(14) \quad c_t^*(P_t, e_t; g(P_{t+1}, e_{t+1}))\]

\[m_t^*(P_t, e_t; g(P_{t+1}, e_{t+1}))\]

where the \(g(\) \(\) reflects the fact that expectations - as well as current prices - enter this solution.

By assumption, all individuals have identical preferences and expectations, and face identical prices; hence, the solutions \((14)\) are the same for all individuals. Assuming there are \(N\) people in each generation (in each country), and letting \(Q_t^i, Q_t^j\) denote total domestic (foreign) output, aggregate world demand of the new generation is given by:

\[(15) \quad c^*(Q_t^i, Q_t^j);\]

Finally, the outstanding stock of dollars (marks) is given by \(D(M)\); since no bequests are left, commodity market equilibrium yields:

\[(16) \quad (e_t^i \cdot M_t^i + D) = (1 - c^*) P_t(Q_t^i + Q_t^j).\]

Given the fraction of wealth people wish to hold in marks, asset market equilibrium becomes:
where, in (16) and (17'), \( c^* \), \( m^* \) depend on \( P^*, e^* \) and expectations. Hence, the equilibrium values of \( P_t \) and \( e_t \) are determined by \( D, M, \) and 

\[ (Q_t + \overline{Q}_t) \text{ given expectations of } P_{t+1}, e_{t+1}; \text{ by analogy, the equilibrium values of } P_{t+1} \text{ and } e_{t+1} \text{ are determined by } D, M \text{ (assumed constant) and } \\
(Q_{t+1} + \overline{Q}_{t+1}) \text{, given the expectations for } (t + 2). \]

Rational expectations require that the expectations held by individuals at \( t \) are identical with the true distributions of \( P_{t+1}, e_{t+1} \) as determined above.

Having set the stage, we now turn to an analysis of exchange rate determination. We consider first the case in which there are no exogenous disturbances; in section III we then assume output in each period is random.

II) Exchange Rate Determination and Capital Controls

In this section we assume there is no output uncertainty; hence 

\[ Q_t = Q \text{ and } \overline{Q}_t = \overline{Q} \text{ for all } t. \] 

Further, assuming no capital controls can occur at any \( t \), it is clear that the commodity and asset demands at 

\( (t + 1) \) are identical with those at \( t \). Hence, \( P_t = P_{t+1}, e_t = e_{t+1}. \)

But, from (10), for \( e_{t+1} = e_t, m^* \) is indeterminate; hence, any constant exchange rate yields an equilibrium time path. (Note that this exchange rate does not (necessarily) depend on real variables, or nominal asset supplies). This is the essence of the Kareken-Wallace argument.

Now, let us introduce governments into the model; the only function we permit them is the imposition of capital controls. By capital controls we mean the following: residents of each country are permitted to hold only domestic currency. Thus, U.S. residents currently holding marks
are obliged to swap them for either goods or dollars (at a market-determined rate), and German residents holding dollars are likewise obliged to swap these dollars for goods or marks. Furthermore, we assume that the imposition of controls is determined randomly and the public knows the probability distribution used by governments. We denote the probability that controls will be implemented in any period by \( \pi \) so that the probability of no controls is \( (1-\pi) \). This probability is time-independent – i.e., the probability of controls at \( (t+1) \) is independent of occurrences at \( t \), and is the same for all periods.

As is clear from the above description, the system outlined above is also stationary providing \( \pi \) does not change through time. Thus, call \( \hat{e}^*_t \) the equilibrium exchange rate at \( t \) if there are no controls, and \( \hat{e}_t \) the equilibrium exchange rate if controls are present at \( t \). Since the presence (or absence) of controls at \( t \) does not affect aggregate demand or expectations at \( (t+1) \), it follows that the equilibrium exchange rate at \( (t+1) \), \( \hat{e}^*_t+1 \) – if no controls are present – must be the same as that at \( t \): hence, \( \hat{e}^*_t+1 = \hat{e}^*_t \). By similar logic, \( \hat{e}^*_t+1 = \hat{e}^*_t \). Therefore, from (10), it is clear that \( \hat{e}^*_t \) must equal \( \hat{e}^*_t \). Suppose otherwise; if \( \hat{e}^*_t > \hat{e}^*_t = \hat{e}^*_t+1 \), then next period’s dollar value of the mark can be no higher, and may be lower (if controls are imposed), than its current value; hence, no one will hold marks (if capital mobility is allowed) and \( \hat{e}^*_t \) must fall (relative to \( \hat{e}^*_t \)). Similarly, if \( \hat{e}^*_t < \hat{e}^*_t \), no one would willingly hold dollars.

Thus, \( \hat{e}^*_t = \hat{e}^*_t \) for all \( t \), whether or not controls are present in any period; consequently, \( P \) – the dollar price of goods – is also time independent. As a result, the marginal (average) propensity to consume will be identical in the two countries. Finally, when capital controls are present, asset market equilibrium for each currency requires:
(18) \((1 - c^*) P \cdot Q = D\)
\((1 - c^*) P \cdot Q = eM\)

(i.e., demand for wealth in each country equals supply, since only domestic currency may be held).

From (18):

(19) \(e = \left( \frac{Q}{Q} \right) \left( \frac{D}{M} \right)\),

and the equilibrium exchange rate is determinate and responds conventionally to asset supplies and real economic variables.

To summarize the results of this section, we have seen that - if no controls are possible - the exchange rate is constant and indeterminate. In essence, since neither asset represents a claim on any particular set of goods, its price is simply determined by self-fulfilling expectations. However, we have also seen that this result is a singular one; the smallest probability of controls renders the exchange rate determinate, its value being determined in a way consistent with the Monetary Approach. Since it seems rather implausible to attach a zero probability to any event (particularly when governments are involved), the Kareken-Wallace indeterminacy result stands out as a theoretical curiosity.

The model we have used so far is a stationary one in which the equilibrium exchange rate was constant. In the next section we introduce output uncertainty in order to examine exchange rate determination when real factors may change over time.

III) Uncertainty, Capital Controls and Exchange Rate Determination

In general, the introduction of time-dependent variables into the model outlined in (I) will not alter the Kareken-Wallace conclusion that the equilibrium exchange rate is constant - but indeterminate (in the
absence of capital controls). For example, suppose $Q_t = 1$, but $\bar{Q}_t = \theta^t$, $\theta \neq 1$; since no uncertainty is present, a rational expectations solution requires $e_{t+1} = e_t$, as can be seen from (10). For, if $e_{t+1} > e_t$ (since the model is determinate, there is no probability distribution associated with $e_{t+1}$), then no one will hold $D$, whereas if $e_{t+1} < e_t$, no one will hold $M$. Thus, even if the two countries have divergent growth rates, the rational expectations solution requires the constancy - and indeterminacy - of the exchange rate.

Similarly, if each country's output is subject to random disturbances, it can be seen from (10) that $e_{t+1} = e_t$ for all $t$ remains a solution for the exchange rate path. Hence, even though the model is no longer determinate - i.e., $P_{t+1}$ is not known at $t$ even though its probability distribution is - the exchange rate path is determinate.

To this framework we now add the possibility that capital controls may be imposed at any period $t$ (the imposition of controls at $t$ does not imply controls will also exist at $(t+1)$), and ask how the possibility of controls at $(t+1)$ affects the equilibrium exchange rate at $t$. In section II we saw that this possibility rendered the exchange rate determinate - and constant; note, however, that the equilibrium exchange rate determined there depended upon the relative outputs of the two countries. Since we now take output to be random, it is clear that the equilibrium exchange rate (path) can not be a constant one, as relative outputs may change through time. Thus, the system is no longer autonomous, and the equilibrium exchange rate will vary over time (depending on the probability of controls and on the realized values of the random variables).
Assume that output in each country follows a Markov process:

\[ Q_{t+1} = \lambda_{t+1} \cdot Q_t; \quad E(\lambda_{t+1}) = 1, \quad \text{Var}(\lambda_{t+1}) > 0. \]

\[ \overline{Q}_{t+1} = \overline{\lambda}_{t+1} \cdot \overline{Q}_t; \quad E(\overline{\lambda}_{t+1}) = 1, \quad \text{Var}(\overline{\lambda}_{t+1}) > 0. \]

In (20), we assume \( \lambda_i \), \( \lambda_j \) are identically, but independently distributed; similarly, for \( \overline{\lambda}_i \), \( \overline{\lambda}_j \) \((i \neq j)\). By analogy to our previous discussion, since relative outputs vary over time, it is clear that – even if exchange controls are imposed next period – the exchange rate that will then prevail is not known \textit{ex ante}, but it will depend on \( \overline{Q}_{t+1}/Q_{t+1} \).

Since at \( t \), \( \overline{Q}_t/Q_t \) is known, and since the distributions of \( \overline{\lambda}, \lambda \) are known, it follows that the exchange rate at \( t \) (if no controls exist) should depend upon \( \overline{Q}_t/Q_t \). In the remainder of this section, we show how the exchange rate depends upon relative productivity \( \overline{Q}_t/Q_t \), the probability of capital controls \( \pi \), individual attitudes towards risk, and the variability of the output disturbances.

Unfortunately, it is not possible to further characterize the equilibrium exchange rate without specifying the utility function and the distribution of the disturbances. In order to illustrate exchange rate determination, we provide some specific examples. First, we assume that the disturbances \( \lambda, \overline{\lambda} \) have identical – but independent – binomial distributions. For positive \( \varepsilon \) less than unity, we let \( \lambda \) and \( \overline{\lambda} \) take on values of \((1 - \varepsilon)\) and \((1 + \varepsilon)\) with probability 0.5. Define

\[ Y_{t+1} = \frac{\overline{Q}_{t+1}/Q_{t+1}}{\lambda_{t+1}/\lambda_t}; \quad \gamma_{t+1} = \begin{cases} \Delta \gamma_t \quad \text{with probability} \quad 0.25 \\ \Delta^{-1} \gamma_t \quad \text{with probability} \quad 0.5 \\ \Delta \gamma_t \quad \text{with probability} \quad 0.25 \end{cases} \]

Next, we assume the individual's preferences are given by:

\[ U(C, B) = \begin{cases} \frac{2}{\rho} \cdot (C - B)^{\rho/2} \quad ; \quad \rho \leq 1, \quad \rho \neq 0. \\ \text{ln}C + \text{ln}B; \quad \rho = 0. \end{cases} \]
Hence $U$ is homogeneous of degree $\rho$ (and hence homothetic); the degree of relative risk aversion is given by $(\rho - 1)$.

The specification given in (22) implies that the elasticity of substitution between consumption goods is 1; as can be seen from (9) and (10):

$$c^* = 1/2, \text{ or } C_t^i = (Q_t^i/2)$$

The individual consumes one-half of current output, selling the remaining half to provide for future consumption. The remaining question, then, is the consumer's portfolio decision (as determined by (10)). That decision depends upon current prices $(e_t, P_t)$ and expectations of future prices $(e_{t+1}, P_{t+1})$; these, in turn, depend upon the behavioral rules followed by individuals in $(t+1)$.

As argued earlier, the exchange rate at $(t+1)$ should depend on $\gamma_{t+1}$, $\pi$, and asset supplies $(D/M)$. Let $m^*(\gamma, \pi)$ denote the share of wealth the individual believes others will allocate to marks $(M)$; by rationality, this should reflect the true decision rules individuals will follow. Denote the exchange rate that will prevail if no capital controls prevail next period $(t+1)$ by $e_{t+1}^N$; from (17):

$$e_{t+1}^N = \left(\frac{D}{M}\right) \left(\frac{m^*(\gamma_{t+1}, \pi)}{(1 - m^*(\gamma_{t+1}, \pi))}\right)$$

Since $\gamma_{t+1}$ is unknown, it follows that next period's exchange rate - even if no controls are imposed - is unknown. From (16), the distribution of $P_{t+1}^N$ - assuming no controls, is

$$P_{t+1}^N = \frac{2D}{(1 - m^*(\gamma_{t+1}, \pi))Q_{t+1}(1 + \gamma_{t+1})}$$

If no controls are present at $(t)$, the behavioral rule $- m^*(\gamma_t, \pi)$ - will be the same as above, since $\pi$ is time-autonomous. Hence, $e_t^N, P_t^N$ are given by (24)-(25), with all time dimensions lagged one period.
If controls are present at \((t+1)\), then individuals born at \((t+1)\) can hold only domestic currencies; hence, the exchange rate \(e^c_{t+1}\) in the event of controls is:

\[
(26) \quad e^c_{t+1} = \gamma_{t+1}(D/M);
\]

and commodity prices - in dollars \((D)\) - are:

\[
(27) \quad P^c_{t+1} = \frac{2D}{Q_{t+1}} = \frac{2D}{\lambda_{t+1}Q_t}.
\]

Given these expectations for \((t+1)\), the individual chooses portfolio composition, assuming \(Q_t, P_t, e_t\) are known. From (10), for an interior solution, \(m^i_t\) is chosen so that:

\[
(28) \quad (1-\pi)E\left[U^*_2\left(\frac{e^N_{t+1} - e^c_{t}}{P^N_{t+1}}\right)\right] + \pi E\left[U^*_2\left(\frac{e^c_{t+1} - e^c_t}{P^c_{t+1}}\right)\right] = 0.
\]

Finally, since all agents are identical and rational, the decisions of this agent must be identical with those of his peers; hence, the decision rule \(m^i_t = m^*(\gamma, \pi)\). Substitution into (28), using (22)-(27) and (6) yields:

\[
(29) \quad (1-\pi)E\left[(\frac{\lambda + \lambda_0}{1+\gamma})^{\rho/2} \cdot (m^* (\gamma/\lambda) - m^*(\gamma))\right] + \pi E\left[(\frac{\lambda}{1+\gamma})^{\rho/2} \cdot (1 + \frac{\gamma}{\lambda})^{\rho/2-1}\left\{m^* (\lambda/\gamma, (1-m^*(\gamma))) - m^*(\gamma)\right\}\right] = 0.
\]

In (29), we drop the time subscript; \(\gamma\) is the current value of \(\lambda_0/\lambda\); hence, at \((t+1)\), \(\gamma_{t+1} = \gamma_{t+1}/\lambda\); the expectation runs over \((\lambda, \lambda')\).

Since the distribution of \(\lambda, \lambda'\) is discreet, so is the (ex ante) distribution for \(\gamma\); i.e., at any \(t\), \(\gamma\) can achieve the values \(\{\gamma^i\}\), for \(i\) in the range \((-t, t)\) where \(\gamma_0\) is the initial value of \(\gamma\). Thus, (29) yields a second order difference equation, which is solvable (at
least, by computer techniques). Let $\gamma_0 = 1$, and define $m_i(\pi)$ to be $m(\Delta^i, \pi)$ - i.e., the fraction of wealth held in marks when $\gamma = \Delta^i$. Then, from (29)

$$\text{(30) } (1-\pi) E\left[\left(\frac{\lambda + \lambda \gamma}{1 + \gamma}\right)^{\rho/2} \cdot m\left(\frac{\gamma \lambda}{\lambda}\right)\right] - m(\gamma) E\left[\left(\frac{\lambda + \lambda \gamma}{1 + \gamma}\right)^{\rho/2}\right] =$$

$$-\pi E\left[\left(\frac{\gamma \lambda}{\lambda + \gamma \lambda}\right)\left(\frac{\lambda + \lambda \gamma}{1 + \gamma}\right)^{\rho/2}\right]$$

For $\pi = 0$, $m\left(\frac{\gamma \lambda}{\lambda}\right) = m(\gamma)$, and the indeterminacy returns; for $\pi \neq 0$, we have a second order difference equation, and presumably two degrees of freedom. However, we shall show there is a unique solution consistent with the condition $m \in [0,1]$; for $\pi = 1$, it is clear the solution is unique.

For convenience, let $d_i = 1 - m_i$, i.e., $d_i$ is the fraction of wealth held in dollars when $\gamma = \Delta^i$. Then, for $\rho = 0$ (30) becomes:

$$\text{(31) } -(1-\pi)d_{i+1} + 2(1+\pi)d_i - (1-\pi)d_{i-1} = 4\pi E\left[\frac{\lambda}{\lambda + \Delta^i}\right]$$

$$= \pi\left[\frac{2}{1+\Delta^i} + \frac{1}{1+\Delta^{i-1}} + \frac{1}{1+\Delta^{i+1}}\right] \equiv \pi R_i$$

In general, for $\rho \neq 0$:

$$\text{(32) } -(1-\pi)d_{i+1}\left(\frac{1+\Delta^{i+1}}{1+\Delta^i}\right)^{\rho/2} + d_i\left\{(1+\pi)-(1-\pi)\left(1+\Delta^i\right)^{\rho/2}\right\} + \pi\left[\left(\frac{1}{1+\Delta^i}\right)^{\rho/2} + \frac{1}{1+\Delta^{i+1}}\left(\frac{\Delta + \Delta^i}{1+\Delta^i}\right)^{\rho/2}\right]$$

Since (31) is a second order, constant coefficient difference equation, it is possible to find the analytic solution; of course, there are two degrees of freedom in specifying this solution. Let $d(\gamma = 1) = d_0$, $d(\gamma = \Delta) = d_1$; then, it is readily shown that the solution to (31) is:
(33) \[ d_i = \left( \frac{\beta_2}{1-\beta_2} \right) \left( d_{i-1} - \beta_2 d_0 \right) \beta_1^i + \left( \frac{d_{0-\beta_2 d_1}^i}{1-\beta_2} \right) \beta_2^i - \frac{\sqrt{\pi}}{4} \sum_{j=1}^{i-1} \left[ (\beta_1^i - \beta_2^i) R_j \right], \quad i \geq 2 \]

\[ = \left( \frac{\beta_2}{1-\beta_2} \right) \left( d_{i-1} - \beta_2 d_0 \right) \beta_1^i + \left( \frac{d_{0-\beta_2 d_1}^i}{1-\beta_2} \right) \beta_2^i + \frac{\sqrt{\pi}}{4} \sum_{j=(i+1)}^{0} \left[ (\beta_1^i - \beta_2^i) R_j \right], \quad i < 0, \]

where

(34) \[ \beta_1 = \left( \frac{1+\sqrt{\pi}}{1-\sqrt{\pi}} \right); \quad \beta_2 = 1/\beta_1; \beta_1, \beta_2 \text{ are characteristic roots} \]

\[ R_i = \frac{2}{1+\Delta} + \frac{1}{1+\Delta} + \frac{1}{1+\Delta}; \quad R_1 + R_{-1} = 4 \]

From (33) and (34), after some manipulation:

(35) \[ d_i + d_{i-1} = 1 + \left( \beta_1^i + \beta_2^i \right) (d_0 - 1/2); \quad i > 0. \]

But \( d_i \in [0,1] \) for all \( i \) implies \( d_0 = 1/2 \); otherwise, \( d_i \) diverges.

Furthermore, convergence of \( d_i \) as \( i \to \infty \) requires

(36) \[ d_i = \left( \frac{\beta_2}{2} \right) + \left( \frac{\pi}{1-\pi} \right) \bar{K}; \quad \bar{K} = \lim_{i \to \infty} \sum_{j=1}^{i-1} \left( \beta_1^{-j} R_j \right) \]

Since \( R_i \) decreases with \( i \) \( (i > 0) \), the limit exists. Hence, using (33)-(36):

(37) \[ d_i = \frac{\beta_1^i}{2} + \frac{\sqrt{\pi}}{4} \left( \beta_1^i - \beta_2^i \right) K_i + \frac{\sqrt{\pi}}{4} \beta_2^i \sum_{j=1}^{i-1} \left( \beta_1^{-j} - \beta_2^{-j} \right) R_j, \quad i \geq 2; \]

\[ K_i = \lim_{i \to \infty} \sum_{j=1}^{i} \beta_1^{-j} R_j \]

(38) \[ d_i + d_{i-1} = 1; \quad d_0 = 1/2; \quad d_1 = \left( \frac{\beta_2}{2} \right) + \left( \frac{\pi}{1-\pi} \right) \bar{K} \]

Consequently, for \( \pi > 0 \), there is a unique exchange rate solution; the actual exchange rate at any time depends on the state of the system \( \left( \hat{Q}_i/Q_1 \right) \) and the probability \( (\pi) \) of capital controls. Note that if capital controls are not present at a period, the equilibrium exchange rate is:
(39) \[ e^N(\gamma, \pi) = \left( \frac{1-d(\gamma)}{d(\gamma)} \right) \frac{D}{M} \]

whereas if capital controls are present at that period:

(40) \[ e^C(\gamma, \pi) = \gamma \left( \frac{D}{M} \right) \]

In Tables 1 and 2, we present the values of the proportionate demand for dollars \( d \) and the dollar price of marks \( e^N \) for various values of \( \gamma \), \( \pi \), and \( \Delta \) assuming that \( \rho = 0 \). Note that an increase in \( \gamma \) reflects an increase in German productivity relative to that of the U.S., and an increase in \( \Delta \) reflects greater variability of output disturbances. \(^{13}\) To ascertain the effects of output variability on the demand for dollars, consider -- for example -- a value of \( \gamma = (1.1)^2 \), so that Germany is 1.21 times as "productive" as the U.S., and a \( \pi \) of 0.1. For a \( \Delta = (1.1)^5 \), the demand for dollars (as a percent of wealth) would be .45262 by both U.S. and German residents. Increasing output variability to \( \Delta = 1.1 \) would change \( d \) to .45300, so that increasing output variability would increase the demand for the currency of the less productive country (the U.S. in this example, since \( \gamma > 1.0 \)). Examination of Table 2 indicates that the rate of exchange would change from 1.20935 \( \left( \frac{D}{M} \right) \) to 1.20750 \( \left( \frac{D}{M} \right) \): increasing output variability would act to depreciate the currency of the more productive country (Germany in this example).

Comparing these rates to that of the case in which capital is immobile -- the column entitled "Current Controls" -- shows that the value of currency of the more productive country is lower when capital is mobile than when there are capital controls [1.21 \( \left( \frac{D}{M} \right) \) for this example].

To determine the effects of an increase in German productivity relative to that of the U.S., holding output variability constant, let
### Table 1

'Proportionate Demand for Dollars When Rho = 0'

<table>
<thead>
<tr>
<th>DELTA = (1.1) ( \gamma )</th>
<th>1.0</th>
<th>.7</th>
<th>.5</th>
<th>.3</th>
<th>.1</th>
<th>.001</th>
<th>( 1 \times 10^{-6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (1.1)^5 )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( (1.1)^2 )</td>
<td>0.45250</td>
<td>0.45251</td>
<td>0.45252</td>
<td>0.45253</td>
<td>0.45262</td>
<td>0.46012</td>
<td>0.49622</td>
</tr>
<tr>
<td>( (1.1)^6 )</td>
<td>0.36084</td>
<td>0.36086</td>
<td>0.36088</td>
<td>0.36093</td>
<td>0.36117</td>
<td>0.38229</td>
<td>0.48868</td>
</tr>
<tr>
<td>( (1.1)^{10} )</td>
<td>0.27831</td>
<td>0.27833</td>
<td>0.27836</td>
<td>0.27843</td>
<td>0.27876</td>
<td>0.30988</td>
<td>0.48117</td>
</tr>
<tr>
<td>( (1.1)^{20} )</td>
<td>0.12945</td>
<td>0.12947</td>
<td>0.12950</td>
<td>0.12957</td>
<td>0.12988</td>
<td>.16653</td>
<td>0.46266</td>
</tr>
<tr>
<td>( (1.1)^{100} )</td>
<td>( 7.2599 \times 10^{-5} )</td>
<td>( 7.2617 \times 10^{-5} )</td>
<td>( 7.2640 \times 10^{-5} )</td>
<td>( 7.2695 \times 10^{-5} )</td>
<td>( 7.2972 \times 10^{-5} )</td>
<td>1.6001 ( \times 10^{-4} )</td>
<td>0.33613</td>
</tr>
<tr>
<td>( \gamma = 1.0 )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( (1.1)^2 )</td>
<td>0.45254</td>
<td>0.45256</td>
<td>0.45259</td>
<td>0.45266</td>
<td>0.45300</td>
<td>0.46871</td>
<td>0.49806</td>
</tr>
<tr>
<td>( (1.1)^6 )</td>
<td>0.36095</td>
<td>0.36101</td>
<td>0.36110</td>
<td>0.36129</td>
<td>0.36221</td>
<td>0.40717</td>
<td>0.49417</td>
</tr>
<tr>
<td>( (1.1)^{10} )</td>
<td>0.27846</td>
<td>0.27855</td>
<td>0.27866</td>
<td>0.27893</td>
<td>0.28023</td>
<td>0.34855</td>
<td>0.49030</td>
</tr>
<tr>
<td>( (1.1)^{20} )</td>
<td>0.12960</td>
<td>0.12968</td>
<td>0.12979</td>
<td>0.13004</td>
<td>0.13129</td>
<td>0.22351</td>
<td>0.48068</td>
</tr>
<tr>
<td>( (1.1)^{100} )</td>
<td>( 7.2723 \times 10^{-5} )</td>
<td>( 7.2794 \times 10^{-5} )</td>
<td>( 7.2888 \times 10^{-5} )</td>
<td>( 7.3110 \times 10^{-5} )</td>
<td>( 7.4241 \times 10^{-5} )</td>
<td>2.0858 ( \times 10^{-3} )</td>
<td>0.40966</td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( (1.1)^2 )</td>
<td>0.45270</td>
<td>0.45279</td>
<td>0.45291</td>
<td>0.45317</td>
<td>0.45435</td>
<td>0.47877</td>
<td>0.49901</td>
</tr>
<tr>
<td>( (1.1)^6 )</td>
<td>0.36139</td>
<td>0.36163</td>
<td>0.36195</td>
<td>0.36267</td>
<td>0.36595</td>
<td>0.43672</td>
<td>0.49704</td>
</tr>
<tr>
<td>( (1.1)^{10} )</td>
<td>0.27907</td>
<td>0.27941</td>
<td>0.27985</td>
<td>0.28087</td>
<td>0.28554</td>
<td>0.39587</td>
<td>0.49508</td>
</tr>
<tr>
<td>( (1.1)^{20} )</td>
<td>0.13016</td>
<td>0.13049</td>
<td>0.13092</td>
<td>0.13191</td>
<td>0.13673</td>
<td>0.30291</td>
<td>0.49017</td>
</tr>
<tr>
<td>( (1.1)^{100} )</td>
<td>( 7.3219 \times 10^{-5} )</td>
<td>( 7.3506 \times 10^{-5} )</td>
<td>( 7.3892 \times 10^{-5} )</td>
<td>( 7.4809 \times 10^{-5} )</td>
<td>( 7.9757 \times 10^{-5} )</td>
<td>2.5530 ( \times 10^{-2} )</td>
<td>0.45250</td>
</tr>
</tbody>
</table>

NOTES: 1) As \( d_1 + d_{-1} = 1 \), negative values of \( i \) are not presented in the table.

2) All calculations have been carried to sixteen significant digits, the numbers in the table reflect .5 rounding.
TABLE 2
'EXCHANGE RATES ($/M) WHEN RHO = 0'

<table>
<thead>
<tr>
<th>DELTA=(1.1)^</th>
<th>CURRENT CONTROLS</th>
<th>PROBABILITY OF CONTROLS NEXT PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>= (1.1)^2</td>
<td>1.21</td>
<td>1.20994</td>
</tr>
<tr>
<td>= (1.1)^6</td>
<td>1.77156</td>
<td>1.77128</td>
</tr>
<tr>
<td>= (1.1)^10</td>
<td>2.59374</td>
<td>2.59310</td>
</tr>
<tr>
<td>= (1.1)^20</td>
<td>6.72750</td>
<td>6.72472</td>
</tr>
<tr>
<td>= (1.1)^100</td>
<td>1.37806x10^4</td>
<td>1.37733x10^4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DELTA=(1.1)</th>
<th>CURRENT CONTROLS</th>
<th>PROBABILITY OF CONTROLS NEXT PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>= (1.1)^2</td>
<td>1.21</td>
<td>1.20994</td>
</tr>
<tr>
<td>= (1.1)^6</td>
<td>1.77156</td>
<td>1.77045</td>
</tr>
<tr>
<td>= (1.1)^10</td>
<td>2.59374</td>
<td>2.59114</td>
</tr>
<tr>
<td>= (1.1)^20</td>
<td>6.72750</td>
<td>6.71624</td>
</tr>
<tr>
<td>= (1.1)^100</td>
<td>1.37806x10^4</td>
<td>1.37498x10^4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DELTA=(1.1)^2</th>
<th>CURRENT CONTROLS</th>
<th>PROBABILITY OF CONTROLS NEXT PERIOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>= (1.1)^2</td>
<td>1.21</td>
<td>1.20896</td>
</tr>
<tr>
<td>= (1.1)^6</td>
<td>1.77156</td>
<td>1.76712</td>
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<tr>
<td>= (1.1)^10</td>
<td>2.59374</td>
<td>2.58338</td>
</tr>
<tr>
<td>= (1.1)^20</td>
<td>6.72750</td>
<td>6.68256</td>
</tr>
<tr>
<td>= (1.1)^100</td>
<td>1.37806x10^4</td>
<td>1.36567x10^4</td>
</tr>
</tbody>
</table>

**NOTES:**
1) To calculate exchange rates from the data in the table, multiply each entry by the ratio \( \overline{D/M} \).
The figures above will be the rates of exchange when \( D/M = \) unity.

2) All calculations have been carried to sixteen significant digits. The numbers in the table reflect \( .5 \) rounding.
If German productivity increases from \((1.1)^2\) to \((1.1)^6\) times that of the U.S., the proportion of dollars in portfolios falls from .45300 to .36221. The main results illustrated in Tables 1 and 2 can be summarized as follows:

i) For \(i > 0\) (i.e., \(\gamma > 1\)) an increase in \(\gamma\) acts to reduce the demand for dollars. Thus, the exchange rate is responsive to real factors in the conventional way; a relative increase in foreign productivity (increase in \(\gamma\)) will lead to a depreciation of the domestic currency.

ii) Perhaps the most interesting result is the relative insensitivity of \(d\) and \(e\) to \(\pi\); for \(\pi = 0\), we have seen that \(d(\gamma)\) and \(e(\gamma)\) is constant and indeterminate. For any \(\pi > 0\), \(d(\gamma)\) is determinate and responsive to productivity; hence, \(e(\gamma)\) depends on both productivity and asset supplies. However, note that, given \(\pi > 0\), the value of \(d(\gamma)\) is relatively insensitive to \(\pi\); for example, when \(\delta = 1.1\) and \(i = 6\), changing the probability of controls from .1 to unity only changes the proportion of dollars in domestic and foreign portfolios in the third decimal place. Hence, the important issue seems to be not what is the probability of capital controls, but whether this probability is zero or positive.

iii) As can be seen from (37), \(\lim d(\gamma) = (1/2)\), since \(\beta_1, \beta_2 \to 1\) and \(\pi \to 0\):

\[
\left( \lim_{i \to \infty} \sum_{j=1}^{i} R_{ij} \right)
\]

converges. Consequently, \(\lim e(\gamma) = (\bar{D}/\bar{M})\); this can be seen in Table 2. Thus, while the limiting solution to (37) corresponds to a constant exchange rate, it is a determinate one that depends only on asset supplies.
iv) For $\gamma > 1$, increases in $\pi$ lead to decreases in $d(\gamma)$ and hence to a depreciation of the domestic currency (i.e., of the currency of the less productive country). Since $d_1 + d_{-1} = 1$, for $\gamma < 1$, increases in $\pi$ lead to an appreciation of the domestic (more productive) country's currency.

v) For all $\pi$, the (capital mobile) exchange rate

$$e(\gamma, \pi) < e^c(\gamma) = \gamma \left( \frac{D}{M} \right) \text{ as } \gamma \leq 1.$$ 

Hence, capital mobility leads to an overvaluation of the less productive country's currency relative to the exchange rate that would prevail if holdings of foreign assets were banned. As we shall see in section V, this creates incentives for the less productive country to (occasionally) impose capital controls (choose $\pi > 0$).

vi) An increase in the variability of the disturbances to output ($\Delta$), ceteris paribus, leads to an appreciation of the less productive country's currency; i.e., the greater the degree of uncertainty, the more "overvalued" that currency becomes relative to the capital immobile exchange rate. This is apparent from section II, since, for no variability ($\Delta = 1$), any $\pi > 0$ yields the same exchange rate as would current capital controls; the greater risk leads to more portfolio diversification.

Returning to (32), the case where $\rho \neq 0$, we notice we also have a second order difference equation; however, since it is not a constant coefficient one, we have not been able to obtain an analytic solution. However, a computer simulation of (32) yields the characteristics of the portfolio demands and equilibrium exchange rates as a function of $\rho$, $\gamma$, ...
These results are presented in Tables 3 and 4. As can be seen from the tables, the characteristics of these solutions are comparable to those discussed for the prior case; even for $p = 1$ (constant marginal utility of income), the same qualitative results hold. Also, as can be seen from the tables, the larger is $p$ (in algebraic value), the less "overvalued" is the less productive country's currency. That is, an increase in $p$ (decrease in risk aversion) leads to an increase in demand for the currency of the more productive country and hence to an appreciation of that country's exchange rate. This result is not surprising, since it implies - in essence - that individuals demand less of a risk premium in order to hold that currency.

That concludes our analysis of the effect of (possible) exchange controls on the equilibrium exchange rate. It is particularly significant that the existence of the possibility of controls - rather than their magnitude - is of fundamental importance. It is also important to remember that - under uncertainty - the less productive country's currency will be "overvalued". So far, we have offered no motivation as to why controls might be employed (though perhaps motivation is unnecessary in addressing government behavior); now we turn to that issue. In section IV, we compare the expected utility of individuals under fixed and flexible exchange rates, considering the case in which no controls ($\pi = 0$) are possible and the case in which no capital mobility ($\pi = 1$) is permitted. Section V then addresses the whole remaining range in which controls occur randomly with a known probability ($0 < \pi < 1$).

IV) Expected Utility Under Fixed and Flexible Exchange Rate

We wish to compare expected utility for individuals under fixed and flexible exchange rates. Before considering how the probability of
TABLE 3

'PROPORTIONATE DEMAND FOR DOLLARS: VARIOUS RHO'

PROBABILITY OF CONTROLS NEXT PERIOD: DELTA = (1.1)

<table>
<thead>
<tr>
<th>RHO = 1.0</th>
<th>0.9999</th>
<th>0.7</th>
<th>0.5</th>
<th>0.3</th>
<th>0.1</th>
<th>0.001</th>
<th>10^-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ = 1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>= (1.1)^2</td>
<td>0.45252</td>
<td>0.45253</td>
<td>0.45254</td>
<td>0.45258</td>
<td>0.45275</td>
<td>0.46089</td>
<td>0.46968</td>
</tr>
<tr>
<td>= (1.1)^6</td>
<td>0.36088</td>
<td>0.36091</td>
<td>0.36095</td>
<td>0.36105</td>
<td>0.36151</td>
<td>0.38460</td>
<td>0.41013</td>
</tr>
<tr>
<td>= (1.1)^10</td>
<td>0.27836</td>
<td>0.27841</td>
<td>0.27846</td>
<td>0.26860</td>
<td>0.27925</td>
<td>0.31367</td>
<td>0.35363</td>
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<tr>
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<td>0.12950</td>
<td>0.12954</td>
<td>0.12960</td>
<td>0.12972</td>
<td>0.13035</td>
<td>0.17315</td>
<td>0.23407</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>RHO = 0.5</th>
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<th>0.5</th>
<th>0.3</th>
<th>0.1</th>
<th>0.001</th>
<th>10^-6</th>
</tr>
</thead>
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<td>γ = 1.0</td>
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<td>0.45253</td>
<td>0.45255</td>
<td>0.45257</td>
<td>0.45262</td>
<td>0.45288</td>
<td>0.46493</td>
<td>0.48175</td>
</tr>
<tr>
<td>= (1.1)^6</td>
<td>0.36092</td>
<td>0.36096</td>
<td>0.36103</td>
<td>0.36117</td>
<td>0.36186</td>
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<td>0.27876</td>
<td>0.27974</td>
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<td>0.12969</td>
<td>0.12988</td>
<td>0.13082</td>
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<table>
<thead>
<tr>
<th>RHO = -0.5</th>
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<th>0.7</th>
<th>0.5</th>
<th>0.3</th>
<th>0.1</th>
<th>0.001</th>
<th>10^-6</th>
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<tbody>
<tr>
<td>γ = 1.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>= (1.1)^2</td>
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<td>0.36256</td>
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<td>= (1.1)^10</td>
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<td>0.13020</td>
<td>0.13176</td>
<td>0.24858</td>
<td>0.49834</td>
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</tbody>
</table>
### TABLE 4

'*EXCHANGE RATES ($/M): VARIOUS RHO'*

**PROBABILITY OF CONTROLS NEXT PERIOD: DELTA = 1.1**

<table>
<thead>
<tr>
<th>RHO = 1.0</th>
<th>.999</th>
<th>.7</th>
<th>.5</th>
<th>.3</th>
<th>.1</th>
<th>.001</th>
<th>10^-6</th>
</tr>
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<tbody>
<tr>
<td>γ = 1.0</td>
<td>1.0</td>
<td>1.0</td>
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<td>1.0</td>
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<tr>
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<td>1.2099</td>
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<td>1.2097</td>
<td>1.2096</td>
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<td>2.5894</td>
<td>2.5811</td>
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<tr>
<td>RHO = 0.5</td>
<td>γ = 1.0</td>
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<td>1.0</td>
<td>1.0</td>
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</tr>
<tr>
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<td>1.2098</td>
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<td>1.2096</td>
<td>1.2094</td>
<td>1.2081</td>
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</tr>
<tr>
<td>RHO = -.5</td>
<td>γ = 1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>1.2097</td>
<td>1.2095</td>
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<td>1.2089</td>
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<td>2.5623</td>
<td>1.7442</td>
<td>1.0033</td>
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</table>
capital controls affects expected utility in a flexible exchange rate system, we first briefly discuss the two pure cases in which (i) there is no chance of capital controls being imposed, or (ii) holdings of foreign currency by private individuals are banned (no capital mobility).

As already noted, if the probability of capital controls is zero, fixed and flexible exchange rates are (essentially) identical. In particular, the exchange rates will be constant under either regime, so that the actual consumption path of individuals will be identical under the two regimes (though, of course, the consumption path will depend on the realized values of the random variables). Thus, in this rational expectations framework, there is no difference between the two regimes. On the other hand, if capital is immobile, then the two regimes are not identical, and hence will yield different expected utility. Under either regime, first period consumption for domestic (foreign) citizens is:

\[ C_t = \left( \frac{Q_t}{2} \right); \overline{C}_t = \left( \frac{\overline{Q}_t}{2} \right) \]

Under flexible exchange rates with no capital mobility, the Balance of Trade must be zero; in essence, individuals cannot transfer purchasing power intertemporally (as a group). Thus, second period consumption, for a person born at \( t \), is:

\[ B^*_t = Q_{t+1}/2; \overline{B}_t = \left( \frac{\overline{Q}_{t+1}}{2} \right). \]

Hence, utility under flexible exchange rates (denote by \( U^* \)) is:

\[ U^* = \frac{2}{\rho} \left[ \left( \frac{Q_t}{2} \right)^{\rho/2} \cdot \left( \frac{Q_{t+1}}{2} \right)^{\rho/2} \right]; \overline{U}^* = \left( \frac{2}{\rho} \right) \left[ \left( \frac{\overline{Q}_t}{2} \right)^{\rho/2} \left( \frac{\overline{Q}_{t+1}}{2} \right)^{\rho/2} \right] \]

where the bar denotes the foreign country.

Under fixed exchange rates, second period consumption is given by:

\[ B_t = \frac{P_t \cdot (Q_t/2)}{(P_{t+1})} ; \overline{B}_t = \frac{P_t (\overline{Q}_t/2)}{P_{t+1}} \]
which, using (16) reduces to:

\[(45) \quad B_t = \left( \frac{Q_{t+1}}{2} \right) \left( \frac{1+\gamma_{t+1}}{1+\gamma_t} \right); \quad \overline{B}_t = \left( \frac{Q_{t+1}}{2} \right) \left( \frac{1+\gamma_{t+1}}{1+\gamma_t} \right) \]

Letting \( U' \) denote utility under fixed exchange rates:

\[(46) \quad U' = \frac{2}{\rho} \left[ \left( \frac{Q_t}{2} \right)^{\rho/2} \left( \frac{Q_{t+1}}{2} \right)^{\rho/2} \left( \frac{1+\gamma_{t+1}}{1+\gamma_t} \right)^{\rho/2} \right]; \]

\[\overline{U}_t' = \frac{2}{\rho} \left[ \left( \frac{\overline{Q}_t}{2} \right)^{\rho/2} \left( \frac{Q_{t+1}}{2} \right)^{\rho/2} \left( \frac{1+\gamma_{t+1}}{1+\gamma_t} \right)^{\rho/2} \right] \]

Comparing (46) and (43):

\[(47) \quad E[U' - U^*] = \frac{2}{\rho} \left( \frac{Q_t}{2} \right)^{\rho} E \left[ \left( \frac{\lambda+\overline{\lambda} \gamma_t}{1+\gamma_t} \right)^{\rho/2} - \left( \lambda \right)^{\rho/2} \right] \]

where, in (47), the expectation is over \( \lambda, \overline{\lambda} \), given \( Q_t, \gamma_t \). But, assuming \( \lambda \) and \( \overline{\lambda} \) are identically, but independently distributed, it is readily shown that:

\[(48) \quad E \left[ \left( \frac{\lambda+\overline{\lambda} \gamma_t}{1+\gamma_t} \right)^{\rho/2} - \left( \lambda \right)^{\rho/2} \right] > 0, \quad 0 < \gamma < \infty. \]

Thus, for any non-zero, finite \( \gamma \), the fixed exchange rate regime yields a higher expected utility than is obtainable under a flexible exchange rate, no capital mobility regime. By symmetry, the same results hold for residents of the foreign country.

In conclusion, we see that fixed and flexible exchange rates yield identical expected utility if there is capital mobility, but that in the absence of private capital mobility, fixed exchange rates dominate flexible exchange rates. Consequently, residents of both countries will want either a fixed exchange rate regime or, if flexible exchange rates are permitted, will oppose absolute bans on foreign currency holdings. This, however, does not imply that they will be opposed to some bans on
capital mobility. In particular, we show in the next section that some probability of capital controls will increase expected utility for residents of at least one country.

V) Probabilistic Controls and Expected Utility

While an absolute ban on capital controls is undesirable, it is possible that the random imposition of controls may raise expected utility, particularly for residents of the less productive country. The reason for this stems from the fact that, as noted earlier, the less productive country's currency tends to be overvalued. Hence, if at \( t \), controls are imposed, but no controls are present at \( (t+1) \), then the residents of the less productive country are likely to experience capital gains on their domestic currency holdings, whereas the residents of the more productive country will experience capital losses. Consequently, the use of controls at \( t \) - and their absence at \( (t+1) \) - will cause a redistribution of consumption towards the (older) residents of the less productive country, away from those of the other country. Therefore, the probability of controls - particularly if this probability is small - may increase expected utility.

Consider the consumption vector for an individual born at \( t \); this consumption vector will depend upon the realized levels of output, as well as upon whether controls actually occur or not. As earlier, we assume the probability of controls in any period is \( \pi \), and that this probability is independent of the state of the economy. The first period consumption for this individual (of either country) is given by:

\[
(49) \quad C_t = \frac{Q_t}{2}; \quad \bar{C}_t = \frac{\bar{Q}_t}{2}
\]

If no controls are present at \( t \), residents of each country will hold both dollar (d) and mark (m) assets; total wealth holdings of domestic
residents (in dollar units) will be \( (P_t Q_t / 2) \), of which a fraction \( d(\gamma, \pi) \) will be held in dollars. Total foreign wealth holdings (in dollar units) will be \( (P_t \bar{Q}_t / 2) \), of which the same fraction \( d(\gamma, \pi) \) will be held in dollars. Thus, next period's exchange rate will not alter the distribution of wealth - or purchasing power - between these two groups. At \((t+1)\), aggregate consumption of the new generation will be \( [(Q_{t+1} + \bar{Q}_{t+1})/2] \), leaving a comparable amount for older residents to divide between themselves, according to the share of wealth they possess. Consequently, if no controls are present at \( t \), consumption in \((t+1)\) of the \( t^{th} \) generation will be:

\[
B_t = \left( \frac{Q_{t+1} + \bar{Q}_{t+1}}{2} \right) \left( \frac{1}{1+\gamma_t} \right) ; \quad \bar{B}_t = \left( \frac{Q_{t+1} + \bar{Q}_{t+1}}{2} \right) \left( \frac{\gamma_t}{1+\gamma_t} \right)
\]

Note that this is identical with the fixed exchange rate consumption vector; the probability of this event is simply \( (1-\pi) \), since it is independent of the use of controls at \((t+1)\).

On the other hand, if controls occur at \( t \) and \((t+1)\), then - for members of the \( t^{th} \) generation - this is identical to a flexible exchange regime with no capital mobility. The probability of this event is \( \pi^2 \), and the \((t+1)\) consumption for members of generation \( t \) is:

\[
B_t = \left( \frac{Q_{t+1}}{2} \right) ; \quad \bar{B}_t = \left( \frac{\bar{Q}_{t+1}}{2} \right)
\]

Finally, consider the case in which controls occur at \( t \), but not at \((t+1)\) - the corresponding probability being \( \pi(1-\pi) \). This is the case in which wealth can be redistributed between the older generations of each country. All older domestic residents hold dollars, while foreign citizens hold marks; hence, \((t+1)\) consumption for the older generation is:

\[
B_t = \frac{\bar{D}}{P_{t+1}} ; \quad \bar{B}_{t+1} = \left( \frac{e_{t+1} \bar{M}}{P_{t+1}} \right)
\]
Using (16) and (17*) yields:

\[(53) \quad B_t = d(\gamma_{t+1}, \pi) \left( \frac{Q_{t+1} + Q_{t+1}}{2} \right) ; \quad \overline{B}_t = (1-d(\gamma_{t+1}, \pi)) \left( \frac{Q_{t+1} + Q_{t+1}}{2} \right)\]

These consumption rules are summarized in Table 5.

Expected utility for domestic residents under flexible exchange rates - with the probability \( \pi \) of controls is

\[(54) \quad U^* = (1-\pi) E \left[ U \left( \frac{Q_t}{2} , \frac{Q_{t+1} + Q_{t+1}}{2(1+\gamma_t)} \right) \right] + \pi^2 E \left[ U \left( \frac{Q_t}{2} , \frac{Q_{t+1}}{2} \right) \right] + \pi(1-\pi) E \left[ U \left( \frac{Q_t}{2} , \frac{(Q_{t+1} + Q_{t+1}) d(\gamma_{t+1}, \pi)}{2} \right) \right]\]

Hence, the difference between expected utility under flexible and fixed exchange rates is given by:

\[(55) \quad J = U^* - U' = \pi \left\{ \pi E \left[ U \left( \frac{Q_t}{2} , \frac{Q_{t+1} + Q_{t+1}}{2(1+\gamma_t)} \right) \right] + (1-\pi) E \left[ U \left( \frac{Q_t}{2} , \frac{(Q_{t+1} + Q_{t+1}) d(\gamma_{t+1})}{2} \right) \right] \right. \]

\[\left. - E \left[ U \left( \frac{Q_t}{2} , \frac{Q_{t+1}}{2(1+\gamma_t)} \right) \right] \right\} \]

Similarly, for foreign residents:

\[(56) \quad \overline{J} = \overline{U}^* - \overline{U}' = \pi \left\{ \pi E \left[ U \left( \frac{Q_t}{2} , \frac{(Q_{t+1} + Q_{t+1}) \gamma_t}{2(1+\gamma_t)} \right) \right] + (1-\pi) E \left[ U \left( \frac{Q_t}{2} , \frac{(Q_{t+1} + Q_{t+1})(1-d(\gamma_{t+1}))}{2} \right) \right] \right. \]

\[\left. - E \left[ U \left( \frac{Q_t}{2} , \frac{(Q_{t+1} + Q_{t+1}) \gamma_t}{2} \right) \right] \right\} \]

For the log case \((p = 0)\) (55) and (56) reduce to:

\[(55') \quad J = \pi \left\{ E \left[ \ln(1+\gamma_t) \right] + (1-\pi) E \left[ \ln(d(\gamma_{t+1}, \pi)) \right] - \pi E \left[ \ln \left( 1+ \frac{\gamma_{t+1}}{\lambda} \right) \right] \right\} \]

\[(56') \quad \overline{J} = \pi \left\{ E \left[ \ln \left( \frac{1+\gamma_t}{\gamma_t} \right) \right] + (1-\pi) E \left[ \ln(1-d(\gamma_{t+1}, \pi)) \right] - \pi E \left[ \ln \left( 1+ \frac{\lambda}{\gamma_{t+1}} \right) \right] \right\} \]

At \( \pi = 0 \), \( J = \overline{J} = 0 \). From (55'):

\[(57') \quad \frac{\partial J}{\partial \pi} = \left\{ E \left[ \ln(1+\gamma_t) \right] + (1-\pi) E \left[ \ln(d(\gamma_{t+1}, \pi)) \right] - \pi E \left[ \ln \left( 1+ \frac{\gamma_{t+1}}{\lambda} \right) \right] \right\} \]

\[+ \pi \left\{ - E \left[ \ln(d(\gamma_{t+1}, \pi)) \right] + (1-\pi) E \left[ \frac{\partial d}{\partial \pi}(\gamma_{t+1}, \pi) \right] - \pi E \left[ \ln \left( 1+ \frac{\gamma_{t+1}}{\lambda} \right) \right] \right\} \]
<table>
<thead>
<tr>
<th>State at t</th>
<th>State at (t+1)</th>
<th>Probability</th>
<th>Domestic Citizens</th>
<th>Foreign Citizens</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Consumption</td>
<td></td>
</tr>
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<td>$Q_t/2$</td>
<td>$(\frac{Q_{t+1}+\bar{Q}_{t+1}}{2})(\frac{1}{1+\gamma_t})$</td>
<td>$\bar{Q}_t/2$</td>
</tr>
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<td>FLEXIBLE EXCHANGE RATES:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No controls</td>
<td>either</td>
<td>$(1-\pi)$</td>
<td>$(Q_t/2)$</td>
<td>$(\frac{Q_{t+1}+\bar{Q}_{t+1}}{2})(\frac{1}{1+\gamma_t})$</td>
</tr>
<tr>
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<td>Controls</td>
<td>$\pi^2$</td>
<td>$Q_t/2$</td>
<td>$Q_{t+1}/2$</td>
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<tr>
<td>Controls</td>
<td>No controls</td>
<td>$\pi (1-\pi)$</td>
<td>$Q_t/2$</td>
<td>$(\frac{Q_{t+1}+\bar{Q}<em>{t+1}}{2}) d(\gamma</em>{t+1})$</td>
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<td></td>
<td>$(\frac{Q_{t+1}+\bar{Q}<em>{t+1}}{2})(1-d(\gamma</em>{t+1})$</td>
</tr>
</tbody>
</table>
Now, as \( \pi \to 0 \), \( d(\gamma_{t+1}, \gamma_t) \to 1/2 \); and \( (\partial d/\partial \gamma) \) approaches a finite limit.\(^{19}\)

Hence,

\[
\lim_{\pi \to 0} \frac{\partial J}{\partial \pi} = [E(ln(1+\gamma_t)) - ln2]
\]

At \( t \), if \( \gamma_t > 1 \), this limit is positive. Ex ante, if the initial value of \( \gamma(\gamma_0) \) is \( \Delta^i \), the distribution of \( \gamma_t \) is given by:

\[
(58) \quad \gamma_t = \Delta^{i+s} \quad \text{with probability:} \quad g(s) = \frac{2^{-2t}(t!)^2}{s! (k!(t-k)!(k+|s|)!(t-k-|s|)!)} \sum_{k=0}^{t-|s|} \frac{1}{(k!) (t-k)! (k+|s|)! (t-k-|s|)!}
\]

where \( s \in [-t,t] \). Clearly, \( g(s) = g(-s) \). Hence, ex ante:

\[
(59) \quad \lim_{\pi \to 0} \frac{\partial J}{\partial \pi} = \sum_{s=-t}^{t} (\ln(1+\Delta^{i+s}) \cdot g(s)) - \ln2; \quad \sum_{-t}^{t} g(s) = 1
\]

By symmetry:

\[
(59') \quad \lim_{\pi \to 0} \frac{\partial J}{\partial \pi} = \ln(1+\Delta^i)g(o) + \sum_{s=1}^{t} [\ln(1+\Delta^{i+s}) + \ln(1+\Delta^{i-s})]g(s) - \ln2;
\]

and:

\[
(60) \quad \ln(1+\Delta^{i+s}) + \ln(1+\Delta^{i-s}) > 2\ln(1+\Delta^i), \quad \Delta \neq 1, \quad s \neq 0. \quad \text{Hence:}
\]

\[
(61) \quad \lim_{\pi \to 0} \frac{\partial J}{\partial \pi} = \left\{ \ln(1+\Delta^i)[g(o) + 2\sum_{s=1}^{t} g(s)] - \ln2 \right\}
\]

\[= \ln(1+\Delta^i) - \ln2 > 0, \quad i > 0.\]

Thus, from an ex ante perspective, if \( \gamma_0 > 1 \), then increasing \( \pi \) from zero can increase expected utility for any future generation of the domestic country; by symmetry, if \( \gamma_0 \leq 1 \), \( \lim_{\pi \to 0} \left( \frac{\partial J}{\partial \pi} \right) > 0 \), and - again from an ex ante perspective - some \( \pi > 0 \) will yield a higher expected utility for any future generation of the foreign country.

Economically, this says that if the countries currently have equal outputs, then both countries will choose some \( \pi > 0 \) in order to increase expected utility of future generations. It also follows from the above
analysis that governments may wish to rethink - or reshape - their policy towards controls depending upon the current state of the world economy. As can be seen from the prior analysis, the larger is the current $y$, the more desirable controls become for the (domestic) less productive country, whereas the less desirable they become for the (foreign) more productive country. Hence, even if they agree upon a particular system (value of $\pi$) ex ante, they will wish to change their minds at some later date; and conflicts may develop between the countries concerning the desirability - or legitimacy - of employing controls.

While the case $\rho \neq 0$ is considerably more complicated, a similar analysis shows that the expected utility of the current domestic generation can be increased by increasing $\pi$ above zero if $\gamma > 1$; for $\gamma = 1$, $\frac{\partial J}{\partial \pi} = 0$ at $\pi = 0$. In other words, if the government is concerned only with the expected utility of the current generation (for example, due to electoral considerations), then the less productive country will have an incentive to choose a positive probability for imposing controls. However, unlike the log case, it is not possible to infer from this that the ex ante expected utility of any future generation will also be increased for some positive $\pi$.

Since we were not able to derive analytic results, we once again turned to a simulation. Specifically, we have calculated the differences between the expected utility under flexible and fixed exchange rates (using the true probability distribution) as a function of $\pi$. The magnitude and sign of this difference will depend not only on $\pi$, but also on: i) the degree of risk aversion ($\rho$); ii) the relative productivity of the economies ($\gamma$); iii) the planning horizon of policymakers -
the number of future generations policy makers consider; and iv) the variability of the output disturbances ($\Lambda$). To ascertain the effects of risk aversion on the desirability of capital controls, consider the case in which countries are equally productive ($\gamma=1$), policymakers are willing to look twenty periods into the future, and $\Lambda=1.1$. Simulation results for this case are given in Table 6. As the numbers in the table indicate, for any degree of risk aversion there exists a flexible exchange rate system with a positive probability of capital controls that policy makers (looking twenty generations into the future) in both countries will prefer to either fixed exchange rates or flexible rates with no controls. Further, as risk aversion increases, the desirability of capital controls also increases, i.e., higher optimal values of $\Pi$ are associated with greater degrees of risk aversion.

The situation changes if both countries are not equally productive. As shown in Table 7, the less productive country will always desire a positive probability of capital controls while the more productive country will favor no controls. The figures in Table 7 are for $\rho=0$, $\Lambda=1.1$, and policy makers looking twenty periods into the future. In all cases, the more productive country will prefer a fixed rate to a flexible rate with controls. However, the less productive country will always favor a positive probability of capital controls: as productivity declines, the greater the optimal probability of controls. It is remarkable that if Germany is $(1.1)^5$ --- or approximately 61% --- more productive than the U.S., the U.S. will prefer a 30% probability of capital controls to no controls or a fixed exchange rate.

Table 8 indicates that the longer the planning horizon of policy makers, the greater the beneficial effects of capital controls for the
TABLE 6
EXPECTED UTILITY DIFFERENCE BETWEEN FLEXIBLE AND FIXED RATES
DELTA = 1.1, γ = 1.0, TWENTY PERIOD FUTURE

<table>
<thead>
<tr>
<th>Probability of Controls</th>
<th>Degrees of Risk Aversion: ( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.5</td>
</tr>
<tr>
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<td>0.0</td>
</tr>
<tr>
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<td>1.4819 \times 10^{-6}</td>
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<td>2.5819 \times 10^{-6}</td>
</tr>
<tr>
<td>.01</td>
<td>2.0394 \times 10^{-6}</td>
</tr>
<tr>
<td>.05</td>
<td>-8.5758 \times 10^{-6}</td>
</tr>
<tr>
<td>.1</td>
<td>-2.3298 \times 10^{-5}</td>
</tr>
<tr>
<td>.2</td>
<td>-5.3116 \times 10^{-5}</td>
</tr>
<tr>
<td>Probability of Controls</td>
<td>(1.1)^{-5}</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001</td>
<td>-1.2712 x 10^{-5}</td>
</tr>
<tr>
<td>0.001</td>
<td>-6.0158 x 10^{-5}</td>
</tr>
<tr>
<td>0.005</td>
<td>-1.1727 x 10^{-4}</td>
</tr>
<tr>
<td>0.01</td>
<td>-1.4028 x 10^{-4}</td>
</tr>
<tr>
<td>0.05</td>
<td>-1.8384 x 10^{-4}</td>
</tr>
<tr>
<td>0.1</td>
<td>-2.0680 x 10^{-4}</td>
</tr>
<tr>
<td>0.2</td>
<td>-2.4474 x 10^{-4}</td>
</tr>
<tr>
<td>0.3</td>
<td>-2.8065 x 10^{-4}</td>
</tr>
<tr>
<td>0.5</td>
<td>-3.5142 x 10^{-4}</td>
</tr>
</tbody>
</table>
TABLE 8
EXPECTED UTILITY DIFFERENCE BETWEEN FLEXIBLE AND FIXED RATES:
RHO = 0; DELTA = 1.1, Y = 1.0

<table>
<thead>
<tr>
<th>Probability of Controls</th>
<th>Periods of Planning Horizon</th>
<th>1</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.001</td>
<td>4.4958 \times 10^{-8}</td>
<td>4.9994 \times 10^{-7}</td>
<td>9.9988 \times 10^{-7}</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>7.6384 \times 10^{-8}</td>
<td>2.9528 \times 10^{-6}</td>
<td>6.1040 \times 10^{-6}</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>-1.3725 \times 10^{-6}</td>
<td>5.1513 \times 10^{-6}</td>
<td>1.2245 \times 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>-3.8634 \times 10^{-6}</td>
<td>4.1925 \times 10^{-6}</td>
<td>1.2910 \times 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>-2.6110 \times 10^{-5}</td>
<td>-1.59821 \times 10^{-5}</td>
<td>-5.1256 \times 10^{-6}</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 9
EXPECTED UTILITY DIFFERENCE BETWEEN FLEXIBLE AND FIXED RATES:
RHO = 0.0, Y = 1.0, TWENTY PERIOD FUTURE

<table>
<thead>
<tr>
<th>Probability of Controls</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.1</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0.0001</td>
<td>9.9989 \times 10^{-7}</td>
</tr>
<tr>
<td>0.001</td>
<td>6.1040 \times 10^{-6}</td>
</tr>
<tr>
<td>0.005</td>
<td>1.2245 \times 10^{-5}</td>
</tr>
<tr>
<td>0.01</td>
<td>1.2916 \times 10^{-5}</td>
</tr>
<tr>
<td>0.05</td>
<td>-5.1256 \times 10^{-6}</td>
</tr>
<tr>
<td>.1</td>
<td>-3.3187 \times 10^{-5}</td>
</tr>
<tr>
<td>.2</td>
<td>-9.0751 \times 10^{-5}</td>
</tr>
</tbody>
</table>
less productive country. Note, in particular, that even if only the current working generation is considered [i.e., the length of the planning period is one], some positive probability of capital controls will be desirable even if countries are equally productive. Table 9 indicates that the greater the variability of the output disturbance, the greater the desirability of capital controls.

In closing this section, let us briefly summarize our results. We have shown - rather surprisingly, we think - that probabilistic capital controls will always appear desirable to at least one country. This, in turn, completes the motivation for our earlier sections in which we showed that the existence of such controls creates a determinate exchange rate. Furthermore, we have shown that the attitude of any country towards controls will depend upon its economic situation vis-a-vis the rest of the world; the weak countries will (rationally) prefer controls, the stronger ones oppose them. This, inevitably, poses conflicts between these governments, ones we have not pursued here. Finally, a country's attitude towards controls may change over time as it finds its economic position changing. Previously strong (productive) countries that had opposed controls may find their position shifting as their economic position deteriorates. There is nothing irrational in a change in such attitudes due to changing economic conditions.

VI) Conclusion

Our principal result in this paper has been to show that a determinate exchange rate path exists even if agents have rational expectations. The Kareken-Wallace conclusion that the exchange rate is constant - but indeterminate - is implausible and seems inconsistent with observed behavior. We have demonstrated that even the smallest probability of
controls renders the exchange rate determinate and responsive in traditional ways to asset supplies and relative productivity.

As a by-product of this analysis, we have also shown that such controls are not irrational, but may well increase expected utility for at least one country. Hence, it is not appropriate to argue that capital controls interfere with the behavior of private individuals and therefore lower their expected utility.

The paper raises as many questions as it answers. Further investigation could entail extending the analysis to a multi-country, or multi-commodity framework. In addition, since conflicts arise between the desires of the governments for controls, it would seem worthwhile to attempt to model this conflict. Moreover, since - from the perspective of each country - the optimal probability for any period will depend on relative productivities, it would be desirable to attempt to model this phenomenon, under the (rational expectations) assumption that economic agents are aware of the policy rule used by governments. In short, we believe that the approach presented here provides fruitful ideas for further research.
*Respectively, the authors are Professor and Associate Professor of Economics at Iowa State University. This material is based upon work supported by the National Science Foundation under Grant Soc - 7907066. We would like to thank the National Science Foundation for this support.

1 The relationship between currency substitution and exchange rates was first rigorously analyzed by Boyer (1972) and Girton and Roper (1976).

2 We defer discussion of the institutional setting in which capital controls are imposed until Section I. It should be noted that the qualitative nature of our results will hold as long as governments probabilistically impose policies that discriminate between national monies.

3 While somewhat awkward, this notation allows us to avoid two time subscripts - one for the generation, the other for the period in which consumption takes place.

4 The motivation for why they may wish to exercise these controls will be taken up in Sections IV and V; for now, we analyze the positive aspect of these controls - i.e., how they would affect exchange rates.

5 Laws banning (or restricting) domestic holdings of foreign currency are not uncommon; those banning foreigners from holding domestic currency are less common, but do exist. The latter ban could be enforced, for example, by issuing new currency in place of the old to domestic citizens only. Of course, as with any law, enforcement is not guaranteed, but that is not our problem here.

6 It would be a bit more desirable, perhaps, to assume the ban applies only to domestic holding of foreign assets, and to assume each government may (independently) impose such controls. This alternative would make little difference in this section; however, the analysis for the subsequent section would be greatly complicated. However, the qualitative conclusions that the possibility of such controls makes the exchange rate determinate, and dependent on asset supplies and real variables (in conformity with standard models) would be unaltered.

7 If each government can impose controls, then \((1-\pi)\) is the probability neither government imposes controls.

8 We have not been able to prove that this is the only solution for all utility functions, but we believe that to be the case.

9 Clearly, other forms exist for homogeneous utility functions; however, this specification has the simplicity that \(c^*\) - the MPC - is constant. Without this assumption, we have not been able to find analytic solutions.

10 Obviously, the rational expectations solution must be an interior one if \(\pi > 0\), since, in the event of controls, \(0 < e_{t+1}^C < \infty\); hence, \(0 < e_t^N < \infty\), implying interior solutions.
For \( \pi = 1 \), \( d_1 = \frac{1}{4} \left[ \frac{2}{1+\Delta^i} + \frac{1}{1+\Delta^{i-1}} + \frac{1}{1+\Delta^{i+1}} \right] \), so

\[
e(\gamma = \Delta^i) = \left( \frac{D}{M} \right) \left[ \frac{\Delta^i(\Delta+1)^2+\Delta^{2i}(3\Delta^2+2\Delta+3)+4\Delta^{3i+1}}{4\Delta^i+3\Delta^{i+1}+3\Delta^{i+2}+\Delta^{2i}(\Delta+1)^2} \right] \leq \Delta^i \cdot \left( \frac{D}{M} \right) \text{ as } i > 0.\]

Thus, even for \( \pi = 1 \), the less productive country's currency will be overvalued relative to the exchange rate that would prevail if no capital movements were possible this period.

For \( \pi = 0 \), (31) yields \( d_1 = a_0 + a_1 \pi \), since \( \beta_1 = \beta_2 = 1 \); again, \( d_1 \in [0,1] \rightarrow a_1 = 0 \); hence, for \( \pi = 0 \), \( d_1 = a_0 \), but is indeterminate - once again, the case of self-fulfilling expectations.

Since (31) provides the solution \( d_1 \), it gives the demand for dollars when \( \gamma = \Delta^i \). Consider an alternative economic system where \( \Delta' = \Delta^2 \); then the corresponding solution for dollar demands \( d_1 \) corresponds to a \( \gamma = (\Delta')^i = \Delta^{2i} \). To ask how variability of the disturbances affects the fraction of wealth held in dollars is to ask how - for the same \( \gamma \) - an increase in \( \Delta \) affects \( d \). Hence, if we compare \( d_{21}(\gamma = \Delta^{2i}) \) to \( d_{1i}(\gamma = (\Delta')^i = \Delta^{2i}) \), then we can ascertain how the increase in \( \Delta \), holding \( \gamma \) constant, affects portfolio demand.

The authors have calculated all values to twelve significant digits; however, for space consideration, we present no more than six significant digits. Further, the results are presented only for \( \gamma > 1(\gamma = \Delta^i, i > 0) \), since \( d(1/\gamma) = 1 - d(\gamma) \); \( d_1 + d_{-1} = 1 \).

Note that since it is really price uncertainty - not income or wealth uncertainty - that is involved here, it is not surprising that "risk neutrality" - as defined by constant marginal utility of income - yields qualitatively similar results to the case of risk aversion. Even with risk neutrality, we still have diminishing marginal utility for each commodity.

As noted by Helpman and Razin, this creates a fundamental asymmetry in comparing the two regimes, since, under fixed rates, the central bank will be holding foreign currency, whereas under flexible exchange rates, no agent holds foreign currency.

Identical results hold for the log case in which \( \rho = 0 \).

Note that as \( \gamma \rightarrow \infty \), the fixed exchange rate regime corresponds to the case in which the home country is infinitesimal; in this case, fixed and flexible yield identical expected utility. As \( \gamma \rightarrow 0 \), the foreign country becomes infinitesimal, so that, from the perspective of the home country, fixed and flexible exchange rates are logically equivalent.
The proof is omitted to save space; details will be supplied by the authors upon request.

Of course, if the \( \pi \) chosen depends on \( \gamma \), the analysis presented here does not reflect rational expectations by agents, since they have not internalized this. Nevertheless, policy decisions - or changes - are not always capable of being predicted ex ante. The main point is simply that the desire to change \( \pi \) arises because of the actual realizations of output.
REFERENCES


