Tracer Advection Using Dynamic Grid Adaptation and MM5

John P. Iselin
Iowa State University

William J. Gutowski
Iowa State University, gutowski@iastate.edu

Joseph M. Prusa
Iowa State University

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Abstract
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Tracer Advection Using Dynamic Grid Adaptation and MM5

JOHN P. ISELIN
Department of Mechanical Engineering, and Department of Geological and Atmospheric Sciences, Iowa State University, Ames, Iowa

WILLIAM J. GUTOWSKI
Department of Geological and Atmospheric Sciences, and Department of Agronomy, Iowa State University, Ames, Iowa

JOSEPH M. PRUSA
Teraflux Corporation, Boca Raton, Florida, and Department of Mechanical Engineering, Iowa State University, Ames, Iowa

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ABSTRACT
A dynamic grid adaptation (DGA) technique is used to numerically simulate tracer transport at meso- and regional scales. A gridpoint redistribution scheme is designed to maximize heuristic characteristics of a "good" grid. The advective solver used in conjunction with the DGA is the multidimensional positive definite advection transport algorithm (MPDATA). The DGA results for regional tracer transport are compared against results generated using the leapfrog as well as MPDATA advection schemes with uniformly spaced, static grids. Wind fields for all tracer transport algorithms are provided by the fifth-generation Pennsylvania State University–NCAR Mesoscale Model (MM5). A mesoscale-sized test case with idealized initial condition and wind field clearly shows qualitatively and quantitatively the advantage of using the dynamic adaptive grid, which is a marked reduction in numerical error. These results are further corroborated by more realistic test cases that used NCEP–NCAR reanalysis data from 6–11 March 1992 to set initial and boundary conditions for (i) a mesoscale-sized, 24-h simulation with an idealized initial tracer field, and (ii) a regional, 5-day simulation with water vapor field initialized from the reanalysis data but then treated as a passive tracer. A result of interest is that MPDATA substantially outperforms the leapfrog method with fourth-order artificial dissipation (central to MM5) in all of our test cases. We conclude that with dynamic grid adaptation, results with approximately the same accuracy as a uniform grid may be obtained using only a quarter of the grid points of the uniform grid MPDATA simulations. Compared to results generated using the leapfrog method on a uniform grid, the DGA does even better.

1. Introduction

The atmosphere’s widely varying spatial and temporal scales and its nonlinearity pose a significant challenge to numerical modeling. One technique for addressing this issue is to utilize dynamic grid adaptation (DGA), which involves automatically adjusting the computational grid in response to flow changes. DGA can assume several forms (Iselin et al. 2002), including automatically moving nested grids, gridpoint insertion techniques that add and remove grid points according to resolution needs, and gridpoint redistribution schemes that adjust the coordinates of a constant number of grid points. Both gridpoint insertion and redistribution schemes have been used extensively in high-speed aerospace applications that have discontinuities in the flow (Hawken et al. 1991).

Atmospheric modeling has also used both gridpoint insertion and redistribution techniques. Dietachmayer and Droegemeier (1992) and Fiedler and Trapp (1993) used gridpoint redistribution to solve a two-dimensional frontogenesis problem and the evolution of a buoyant thermal, respectively, among other test simulations. Srivastava et al. (2001) used gridpoint redistribution in a finite-volume model to calculate the advection of an initially conical pollutant puff by a wind with constant angular velocity. Prusa and Smolarkiewicz (2003) used gridpoint redistribution in a global atmospheric model to solve the anelastic equations. Several atmospheric researchers have used gridpoint insertion and extraction coupled with a semi-Lagrangian scheme...
(Stevens and Bretherton 1996; Behrens 1996; and Behrens et al. 2000). Other forms of insertion and extraction have worked with the method-of-lines (Tomlin et al. 1997) and unstructured grids (Bacon and Coauthors 2000). Hubbard and Nikiforakis (2003) used a finite-volume method with nested grids in a global finite-volume model.

The motivation behind all of these techniques is to minimize numerical error globally by locally increasing resolution in regions where high numerical error would otherwise occur, thus avoiding the costs associated with globally increasing grid resolution. Srivastava et al. (2001) for example found that adaptive grid models were superior to uniform grids that had 4 times the nodes and required 3 times the computational time. Tomlin et al. (1997) found that, compared to uniform grids, adaptive grid models captured better not only key features of pollutant plumes, but also the average integrated concentrations. Hubbard and Nikiforakis (2003) found that adaptive grids yielded a fivefold reduction in computing time compared to a uniform grid with the same number of nodes.

The motivation in Iselin et al. (2002) and this work was to develop modeling techniques that would capture fine details in atmospheric moisture transport. Iselin et al. (2002) used DGA in combination with the multidimensional positive definite advection transport algorithm (MPDATA) scheme of Smolarkiewicz and Margolin (1998) to investigate the feasibility and characteristics of this combination for tracer transport. The grid redistribution scheme of Brackbill and Saltzman (1982) was used to automate the movement of the grid points. Both one- and two-dimensional models were developed and tested using idealized flow fields.

In order to assess the applicability of DGA on more realistic atmospheric flows, this work extends Iselin et al. (2002) to model three-dimensional passive tracer advection. The flow fields used to drive the DGA tracer transport model were produced by the fifth-generation Pennsylvania State University–National Center for Atmospheric Research (Penn State–NCAR) Mesoscale Model (MM5) (Dudhia 1993). Although a detailed description of this model is not provided here, it should be noted that MM5 uses the leapfrog scheme with fourth-order dissipation to dampen dispersive errors. We have found that DGA can significantly reduce numerical error compared to a static grid with the same number of grid points. Alternatively, DGA needs only a quarter of the grid points in a uniform grid to produce comparable results. In order to put this work into context, a very brief review of the MPDATA scheme and the work in Iselin et al. (2002) follows.

The MPDATA scheme is a second-order, positive definite, iterative advection scheme. The donor-cell method is used to predict a field’s preliminary value. This predicted value is then used to construct an approximate truncation error, which is then subtracted from the preliminary field to construct a corrected, second-order approximation. Additional predictor and corrector steps may be used to further reduce the numerical error, although the scheme remains second-order accurate. Smolarkiewicz and Margolin (1998) discuss an extension (E3) that is third-order accurate only in the absence of velocity gradients. With velocity gradients, this extension is second-order accurate, but continues to be advantageous for reducing phase error. In this work all simulations used four MPDATA iterations and the E3 extension. Since the MPDATA scheme uses a conservative formulation it strictly conserves mass.

In Iselin et al. (2002), both one- and two-dimensional models were used to explore the basic behavior of the DGA model. The effects of the number of MPDATA iterations, the E3 extension, the number of grid points, as well as the Courant number, were initially investigated using a one-dimensional model. This model used a Gaussian profile advected by a constant velocity. A specified level of accuracy, DGA allowed the number of grid points to be halved. The two-dimensional model in Iselin et al. (2002) used a constant vortex to advect a cone-shaped initial tracer field for six complete revolutions. This case, DGA reduced the diffusion error by at least 25% (and by more than 96% in one case) compared to a uniform grid with the same number of points. Here the E3 extension did not greatly reduce all errors, but it did reduce the phase error by more than 50%. Uniformly spaced grid computations with over 5 times the number of grid points failed to reduce the diffusion error as significantly as DGA.

This work presents a three-dimensional advection model that uses realistic atmospheric flows and tracer fields. We compare simulations performed with the DGA/MPDATA combination, the MPDATA scheme on a uniformly spaced static grid, and MM5’s standard leapfrog advection routine, while keeping the advecting wind field the same. A set of subroutines called the dynamic grid advection component (DGAC) was developed to use the DGA/MPDATA combination on a grid that was specified independent of the MM5 grid. A second set of subroutines called the MM5 advection component (MM5AC) performed tracer advection using MM5’s numerics, but on a uniform grid specified independently of the MM5 grid. By holding the grid resolution of MM5 fixed while varying the resolution of the DGAC and MM5AC, we could more precisely isolate the effects of grid resolution and DGA on tracer advection.

In order to transfer fields such as tracer concentration, from MM5 to either the DGAC or MM5AC, the monotone interpolation scheme (MIS) of Smolark-
iewicz and Grell (1992) was used. Such interpolations are equivalent to solving a single advection problem and thus accounted for 39% of the CPU time in the DGAC. In a full DGA model for geophysical flows that solves an elliptic equation in addition to multiple advection problems, this interpolation overhead is less significant and vanishes if all fields are computed on the same adaptive grid. Full details of the implementation of the MIS into this model are provided in Iselin (1999).

The computational cost of calculating metric terms to account for the coordinate transformation is relatively small. The gridpoint redistribution scheme used in this study solved an elliptic set of equations at each time step that accounted for less than 2% of the CPU time in the DGAC. The computational cost of solving this system had significant initial cost with little relatively small subsequent CPU time at each time step. The initial grid distribution was determined from an evenly spaced grid and thus required many iterations to achieve convergence. Iterations for successive time steps, however, were initialized using the previous gridpoint distribution. In all cases this required fewer than 20 iterations to converge, and in most cases, less than 10. Detailed computational costs of this particular method can be found in Iselin et al. (2002). It is even possible that the use of DGA can reduce overall computational cost (Prusa and Smolarkiewicz 2003).

Although any number of numerical solvers and interpolation methods could have been chosen, the second-order accuracy combined with the monotone nature of both MPDATA and the MIS are especially advantageous, because the chosen grid redistribution scheme uses the curvature of the advected field as one of the criteria for redistributing grid points. Any dispersive ripples in the advected field from nonmonotonic schemes are picked up by the grid redistribution scheme as regions of high curvature, and thus grid points are inappropriately clustered around these ripples.

Section 2 presents model development, while sections 3 and 4 give results using idealized tracers and passive water vapor advection, respectively. Conclusions are given in section 5.

2. Model development

The DGAC’s advection equation used a form that was compatible with the other dynamic equations used in the nonhydrostatic version of MM5. This equation accounts for the map factors used in the projection and the transformation from a vertical scale based on length to terrain-following σ coordinates based on pressure.

The grids in the MM5, DGAC, and MM5AC discretized the same region but were inherently different because of potentially different numbers of grid points in each and the grid point movement within the DGAC. This necessitated a correspondence between the different grids and interpolation of wind fields from the MM5 to the DGAC and MM5AC using the MIS. Iselin (1999) provides details of how this correspondence was implemented.

a. Advection equation and algorithm

This section explains the effect of applying an advection equation that is compatible with MM5 to a dynamic grid and the subsequent use of MPDATA to solve it. Only enough detail is given to illustrate several key points. Full implementation details are available in Iselin (1999). The advection transport equation appropriate for atmospheric flows on a projected plane and using a terrain-following coordinate system is

\[ p^* \frac{\partial q}{\partial t} + m p^* \sum_{i=1}^{2} u_i \frac{\partial q}{\partial x_i} + p^* \frac{\partial q}{\partial \sigma} = 0, \]  

(1)

where \( q \) is an intensive property representing the concentration of an inert tracer, \( m \) is the map factor, \( u_i \)'s are the horizontal velocities in the corresponding \( x_i \) directions, \( \sigma \) is a terrain-following vertical coordinate defined as

\[ \sigma = (p_0 - p_i)(p_0 - p_i)^{-1} - (p_0 - p_i)^{-1}, \]  

(2)

and \( \dot{\sigma} \) is the vertical velocity

\[ \dot{\sigma} = -m \sigma u_i p_i^{-1} \frac{\partial p_i}{\partial x_i} - \rho_0 g \sigma p_i^{-1}. \]  

(3)

In (2) and (3), \( p_0 \) is the basic-state vertical pressure distribution, and \( p_i, p_0, \) and \( \rho_0 \) are the basic-state surface pressure, the basic-state pressure at model top, and the basic-state density, respectively, and are not functions of time. The acceleration due to gravity and the vertical velocity in the length-based vertical direction are \( g \) and \( w \), respectively.

A transformation from the physical domain with a potentially uneven and temporally changing grid to a computational domain with a static, orthogonal grid with unit spacing was performed on (1) in the same manner as in Iselin et al. (2002) to yield

\[ \frac{\partial (p^* q)}{\partial \tau} + \sum_{i=1}^{3} \frac{\partial (p^* u_i q)}{\partial \xi_i} = q \left[ \frac{\partial p^*}{\partial \tau} + \sum_{i=1}^{3} \frac{\partial (p^* u_i p^*)}{\partial \xi_i} \right], \]  

(4)

where \( \tau \) and \( \xi_i \) are the computational domain’s time and space coordinates, respectively. The relative velocities in the computational domain are

\[ \frac{\partial \xi_i}{\partial t} = m \sum_{j=1}^{2} u_j \frac{\partial \xi_j}{\partial x_j} + \frac{\partial \xi_i}{\partial \sigma}. \]  

(5)

Note that in the \( \sigma \) coordinate system \( p^* \) is constant with time but when the grid points are allowed to move it then becomes a function of computational time \( \tau \) and space \( \xi_\sigma \).
The MPDATA scheme uses pseudovelocities designed for use with the donor-cell scheme to approximate the truncation error of the previous iteration. Following Smolarkiewicz and Margolin (1998) and Iselin et al. (2002) the appropriate pseudoveLOCITY vector $\mathbf{U}^{(m+1)}$ at iteration level $m$ for the MPDATA scheme was developed. The initial pseudovelocity is the physical velocity at time step $n$. Full details appear in Iselin (1999). The MPDATA scheme then is simply an iterative application of the donor-cell scheme in three dimensions:

$$
q_i^{(m+1)} = q_i^{(m)} - \{ F[q_i^{(m)} q_{i+1,j,k}, \hat{U}_i^{(m)}] - F[q_i^{(m)} q_{i,j+1,k}, \hat{U}_i^{(m)}] \}
- F[q_i^{(m)} q_{i,j,k-1/2}, \hat{V}_{i+1/2,j,k}] - F[q_i^{(m)} q_{i,j,k+1/2}, \hat{V}_{i-1/2,j,k}] - F[q_i^{(m)} q_{i,j+1,k}, \hat{V}_{i+1/2,j,k}]
- F[q_i^{(m)} q_{i,j,k-1}, \hat{V}_{i-1/2,j,k}] - F[q_i^{(m)} q_{i,j+1,k}, \hat{V}_{i+1/2,j,k}] + R_{i,j,k}^{m+1}/\delta_{im},
$$

where $m$ refers to the iteration level. In (6) $F$ represents the flux of the advected quantity $q$. The $q_{i,j,k}$, $q_{i-1/2,j,k}$, and $q_{i+1,1,j,k}$ quantities refer to advected field quantities at the cell center and the left and right cell centers in the $\xi_1$ direction, respectively. Likewise, incrementing the $j$ and $k$ indices indicates similar neighboring cell centers in the $\xi_2$ and $\xi_3$ directions. The development of the pseudoveloCEITIES $\hat{U}_i^{(m)}, \hat{V}_i^{(m)}$, and $\hat{W}_i^{(m)}$ are derived in Iselin (1999) and are of the form

$$
\hat{U}_i^{(m+1)} = [\hat{U}_i^{(m)}] - \frac{\Delta \xi_1}{2 q_i^{(m)}} \frac{\partial q_i^{(m)}}{\partial \xi_1}
- \hat{U}_i^{(m)} \frac{\Delta \xi_2}{2 q_i^{(m)}} \frac{\partial q_i^{(m)}}{\partial \xi_2} - \hat{U}_i^{(m)} \frac{\Delta \xi_3}{2 q_i^{(m)}} \frac{\partial q_i^{(m)}}{\partial \xi_3},
$$

where the $\Delta \xi$’s represent the grid spacing in the computational domain, which for simplicity were chosen to be unity. The $\hat{V}_i^{(m)}$, $\hat{W}_i^{(m+1)}$, and $\hat{W}_i^{(m)}$ pseudoveloCEITIES were found by considering symmetric permutations of (7). Finite-difference forms for $\hat{U}_i^{(m+1)}, \hat{V}_i^{(m+1)}$, and $\hat{W}_i^{(m+1)}$ at the cell centers can be found in Iselin (1999). The $R = q [\partial^2 q / \partial r^2 + \partial (\rho \hat{u}_i) / \partial \xi] \partial \xi$ term in (6) is a divergence term that is nonzero only for the first MPDATA iteration as indicated by the Kronecker delta $\delta_{im}$.

b. Dynamic grid advection component

The DGAC is a set of subroutines developed to solve the advection equation on a nonuniform and temporally adaptive grid. Since the DGAC was specifically designed to interface with the MM5, an alignment of the DGAC and MM5 grids, a method of transferring the wind fields from MM5 to the DGAC, and a means of updating the boundary conditions of the DGAC were required.

The purpose of DGA is to minimize the truncation error by redistributing grid points. Minimizing this error requires a means of predicting the truncation error and moving the grid points. The two-dimensional grid redistribution scheme described in Iselin et al. (2002) was used to control the horizontal grid stretching in the DGAC. This scheme uses variational calculus to maximize global measures of grid smoothness and orthogonality while equally distributing a user-defined weight function that is heuristically related to the numerical error of the scheme. The larger the weight function the smaller is the spacing between the grid points. The details of this grid redistribution scheme can be found in Brackbill and Saltzman (1982), Iselin et al. (2002), and Iselin (1999). The chosen weight function was based on the horizontal gradient and curvature of the tracer field:

$$
\tilde{w}_{ij} = \sqrt{\frac{\partial q}{\partial x}^2 + \frac{\partial q}{\partial y}^2 + \left( \frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial y^2} \right)}.
$$

where $\tilde{w}_{ij}$ is the two-dimensional weight function, $q$ is the tracer concentration, and $L$ is a length scale here chosen to be the domain size. This basic definition was smoothed and rescaled as described in Iselin et al. (2002) to yield a final weight function $w_{ij}$, such that $1 \leq w_{ij} \leq 2$. This provides a normalized range of weight functions and ensures no region is void of grid points, which would happen if $w_{ij}$ were zero. The smoothness, orthogonality, and grid redistribution weight function compete in order to produce a grid that clusters grid points where numerical error is likely to be large while producing a grid with reasonable smoothness and or-thogonality traits.

The horizontal transformation is not a function of altitude. This allows vertical columns of grid points to remain vertical, following the basic hydrostatic character of the atmosphere. This choice has advantages of simplifying the transformation as well as in computing column-based statistics as would be used in moisture or radiation parameterizations. This choice requires a two-dimensional grid redistribution scheme to be derived from a three-dimensional flow field. Four different means of vertically contracting the features of the three-dimensional tracer field into a two-dimensional horizontal weight function were tested. The first two methods determine the weight function from a two-dimensional tracer field based on the average and maximum tracer value of a vertical column, respectively. The final two methods determined two-dimensional
weight functions on each \( \sigma \) level and contracted this three-dimensional weight field into two dimensions by taking the average and maximum weight values of a vertical column of weight values.

All four methods were tested on the full model. A simplified model, using only two \( \sigma \) levels, is shown in Fig. 1 to clearly illustrate characteristics of each contraction method. The simplified model was initialized with shifted but otherwise identical Gaussian profiles on each \( \sigma \) level (Fig. 1a). Figure 1b shows the values of the tracer on both levels along a midplane cross section at a constant \( x \) value (solid lines) and the resulting composite tracer fields from both average (dashed line) and maximum (dotted line) tracer values of a column. The composite weight function based on the average tracer values, shown in Fig. 1c, has a maximum at the center of the domain because of the high curvature of the average tracer value in this location. The resulting clustering of grid points at the domain center is inappropriate for either of the profiles considered separately. The actual high curvature in both of the individual profiles is lost in the averaging process and a false region of curvature is created at the center. An even more extreme case of creating a high false curvature is shown in Fig. 1d by using the maximum tracer value in a column. This technique interpreted the discontinuity in the slope at the intersection of the two profiles as extremely high curvature and assigned very high weight values inappropriately at this intersection. When a column-average weight value was used (Fig. 1e), the averaging process lost the actual maximum and created a false maximum at the center in much the same way as the averaging of the tracer field. An acceptable weight function shown in Fig. 1f was found by using the maximum weight value in a column. This method created a two-dimensional weight field that clearly resolves the high curvature in both of the tracer profiles and yet does not inappropriately combine features on different \( \sigma \) levels to create false areas of increased grid resolution. Based on these results this last method of calculating a two-dimensional weight function was chosen. This is expressed as

\[
w_{cij} = \max_{k=\sigma_l, \sigma_h} (w_{ij,k}),
\]

where \( w_{ij,k} \) is the three-dimensional weight function determined by calculating two-dimensional weight functions independently on each \( \sigma \) level; \( w_{ij} \) is the composite weight function at grid point \( i, j \); and \( \sigma_l \) and \( \sigma_h \) are upper and lower bounding \( \sigma \) levels. The \( \sigma_l \) and \( \sigma_h \) parameters permit a selective range of \( \sigma \) values to be used in the composite weight function.

c. MM5 advection component

A set of subroutines was developed to compute tracer advection on a uniform grid independent of the
MM5 grid. Denoted MM5AC, this module used the same leapfrog scheme as MM5. Consequently, the effect of changing the resolution of the MM5 advection scheme without changing the wind field could be studied. Therefore, as with the DGAC and the MM5, a correspondence between the MM5AC and the MM5 was required. As with the DGAC interpolations the MIS was used to transfer $p^*$ at the beginning of the simulation and the three velocity components ($u, v, \sigma$) at each time step from the MM5 grid to the MM5AC grid. To make a fair comparison, the horizontal diffusion coefficients based on the MM5AC grid spacing and simulation time step were calculated for the new grid. The boundary conditions of the MM5AC were implemented as in MM5, using a forcing frame. The MM5AC ran at the same or higher resolution than MM5, thus allowing interpolation of MM5 forcing frame directly to the MM5AC.

3. Test cases and results with idealized tracer fields

A series of experiments was conducted using MM5, DGAC, and MM5AC. The series tested the effectiveness of DGA when compared to the leapfrog and MPDATA schemes on a static, uniform grid while gradually introducing more realistic atmospheric conditions. Table 1 summarizes the test cases. The zonal and realistic advection, cases a–g, all used a 1620 km × 1620 km domain, a 30-km MM5 grid spacing, 24 vertical $\sigma$ levels for a simulation period of 24 h, and an initial idealized cylindrical tracer profile (Fig. 2), specified as

$$q(x, y) = \max \left( \frac{4}{243} \left[ 1 - ((x - 350)^2 + (y - 810)^2)^{1/2} \right], 0 \right).$$

(10)

Analytical expressions of the initial and lateral boundary conditions were developed for the zonal flow case (a) to test the ability of the schemes to translate the tracer pattern in a temporally invariant flow. The realistic advection cases (b–g) used MM5 initial and lateral boundary conditions derived from the 6 March 1992 National Centers for Environmental Prediction (NCEP)–NCAR reanalysis (Kalnay et al. 1996). The water vapor advection case (h) simulated a larger (5200 km × 3848 km) domain for a longer period of time (120 h) using 45-km MM5 grid spacing and 24 vertical levels. The water vapor field from 0000 UTC 6 March 1992 was chosen as the initial tracer field because an earlier study of observed atmospheric water vapor (Iselin and Gutowski 1997) showed substantial vertical and horizontal variability whose evolution could easily be diffused if not modeled accurately. The MM5 initial and lateral boundary conditions used the NCEP–NCAR reanalysis for 6–11 March 1992.

a. Zonal flow test case

The purpose of the zonal flow test case is to validate the MM5/DGAC and MM5/MM5AC systems with an example for which the analytical result is known. Detailed results of other idealized tests with varying numbers of grid points and Courant numbers can be found in Iselin et al. (2002). The initial and lateral boundary conditions for all prognostic fields were developed to produce a purely zonal flow with velocity decreasing

---

**Table 1. DGAC test cases.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Simulation type</th>
<th>DGAC grid points</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Zonal flow</td>
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<td>Initial validation</td>
</tr>
<tr>
<td>b</td>
<td>Realistic advection</td>
<td>$53 \times 53$</td>
<td>Baseline advection of cylinder</td>
</tr>
<tr>
<td>c</td>
<td>Realistic advection</td>
<td>$105 \times 105$</td>
<td>High-resolution realistic advection</td>
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<tr>
<td>d</td>
<td>Realistic advection</td>
<td>$35 \times 35$</td>
<td>Effect of reduced grid points</td>
</tr>
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<td>Realistic advection</td>
<td>$26 \times 26$</td>
<td>Effect of reduced grid points</td>
</tr>
<tr>
<td>f</td>
<td>Realistic advection</td>
<td>$18 \times 18$</td>
<td>Effect of reduced grid points</td>
</tr>
<tr>
<td>g</td>
<td>Realistic advection</td>
<td>$12 \times 12$</td>
<td>Effect of reduced grid points</td>
</tr>
<tr>
<td>h</td>
<td>Water vapor advection</td>
<td>$100 \times 74$</td>
<td>Effect of realistic initial condition</td>
</tr>
</tbody>
</table>

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**Fig. 2.** Initial tracer condition for zonal and realistic advection test cases. The minimum contour (largest ring) is 0.25 with increments of 0.25. The initial profile is identical at all $\sigma$ levels.
slightly with latitude. The circulation used simplifying assumptions about the terrain, velocity field, thermodynamic parameters, and pressure distribution. The earth was assumed to be a flat (map factors $m = 1$) $\beta$ plane with no terrain ($p^*$ a constant). Since a purely zonal wind field was initially specified the integrated wind field was a steady state, longitudinally invariant zonal wind. The flow was assumed to be isothermal, with no water vapor and no radiation. Viscous diffusion was set to zero. The pressure perturbation ($p'$) was independent of the zonal direction. These assumptions produced a flow with $\dot{\sigma} = 0$, no flow divergence, and a pressure perturbation

$$p' = p_0 \left[ \exp \left( -\frac{c}{RT_{oo}} y \right) - 1 \right]. \tag{11}$$

where $c$ is a constant chosen such that the velocity at the middle latitude of the domain is $12.5 \, \text{m s}^{-1}$. $R$ is the ideal gas constant for air, $T_{oo} = 273 \, \text{K}$ is the basic-state temperature at the surface of the model, and $y$ is the distance in kilometers from the lowest latitude of the model. The chosen constants give a flow that advects a tracer profile across the domain in approximately $24 \, \text{h}$. Since the initial tracer field and velocity fields were independent of height with no vertical transport the tracer evolution on a given $\sigma$ level was identical to the evolution on all other levels. Thus the bounding $\sigma$ levels used in the grid redistribution weight function (9) were inconsequential.

Figure 3 shows the results and the associated error of this test case for the leapfrog MPDATA scheme on a static uniform grid and the dynamic grid adaptation scheme. Rms errors for each scheme were $6.56 \times 10^{-4}$, $3.28 \times 10^{-4}$, and $2.15 \times 10^{-4}$ for the leapfrog, MPDATA, and DGA schemes, respectively. A lagging phase error was found for each scheme and is very evident in the leapfrog solution. These phase errors for the leapfrog, MPDATA, and DGA schemes were 30, 15, and 8.6 km respectively.

b. Test cases with realistic advection

Realistic initial and lateral boundary conditions from the NCEP–NCAR reanalysis (Kalnay et al. 1996) for 6 March 1992 provide a more challenging test and make

Fig. 3. Results and error of the zonal flow test case after 20 h for each scheme: (a) leapfrog; (b) MPDATA on a static uniform grid; (c) dynamic grid adaptation; (d) error in leapfrog scheme; (e) error in MPDATA on a static uniform grid; and (f) error in dynamic grid adaptation scheme. The minimum contour level in (a)–(c) is 0.25 with 0.25 increments. The contours in (d)–(e) are 0.15. Negative quantities are shaded. The dots in (c) mark locations of the cell centers.
up cases b–g. Tracer characteristics in the lower half of the atmosphere determined the gridpoint redistribution by setting the bounding $\sigma$ levels in (9) to $\sigma_1 = 1.0$ and $\sigma_0 = 0.5$. These were chosen with an eye on future modeling of water vapor transport, which has much higher concentrations in the lower atmosphere. Figures 4a–c show the results of case b after 24 h on the $\sigma = 0.725$ level, which is typical of the lower-altitude results. The sharp gradients of the dynamic grid simulation suggest the expected result of reduced numerical error when compared to the static MPDATA scheme and even more so when compared to the MM5 leapfrog scheme.

Figures 4d and 4e show results of simulations using the DGAC and MM5AC with double resolution. These tests, corresponding to case c, indicate convergence and help verify the accuracy of the DGA computations. Since the static and dynamic grid MPDATA solutions (Figs. 4b and 4c) appear to match more closely both higher-resolution simulations than does the lower-resolution leapfrog solution (Fig. 4a), it appears that the MPDATA scheme has less numerical error than the leapfrog scheme. This is consistent with the findings of Iselin et al. (2002) and the zonal flow test case in section 3a. Hence, the higher-resolution (15 km) MPDATA solution depicted in Fig. 4e will be used as our standard for assessing the other results.

Subtracting the standard solution (Fig. 4e) from the results in Figs. 4a–d gives an estimate of each scheme’s numerical error (Fig. 5). Figures 5b and 5e show leapfrog differences after horizontally translating solutions to minimize phase differences. The amount and direction of translation differed on each sigma level. The average distance shifted in the 30-km leapfrog simulation is 48 km. The average distance shifted in the 15-km resolution leapfrog simulation is 33 km.

Figure 6 shows the dynamic MPDATA realistic advection results and differences at the $\sigma = 0.125$ and $\sigma = 0.970$ levels. The significant vertical shear seen by comparing Figs. 6a and 6b highlights the need for a composite weight function to cluster points appropriately on different $\sigma$ levels. The invariance of the grid with height is evident by comparing the grids in Figs. 4c, 6a, and 6b. Although it appears that grid points have clustered inappropriately in regions with small gradients, comparison of Figs. 4c and 6b show that the “inappropriate” clustering in one $\sigma$ level is appropriate in others. This emphasizes the importance of a well-designed weight function that is capable of resolving multiple overlapping features that, in this particular case, consisted of the same field $q$ but at different heights. In models that use the DGA to resolve multiple fields, weight functions that resolve overlapping features in different fields are needed.

The differences in Fig. 6c shows that since the gridpoint redistribution did not depend upon the profile in

![Fig. 4. (a) The 30-km-resolution leapfrog scheme; (b) 30-km-resolution MPDATA on a static uniform grid; (c) MPDATA scheme with a dynamic grid using same number of grid points as in (a) and (b); (d) 15-km-resolution leapfrog scheme; and (e) 15-km-resolution MPDATA scheme on a static grid. All results are at $\sigma = 0.725$ after 24 h of simulation. The minimum contours are 0.25 with increments of 0.25.](image-url)
the upper half of the atmosphere the difference was significantly greater than at lower altitude. Nevertheless, acceptable results were still obtained at higher altitudes and were better than those of the leapfrog scheme (results not shown). At lower altitudes, the result in Fig. 6d shows that the sharp gradient depicted in Fig. 6b is justified and well resolved. In summary, Figs. 5 and 6 show that results of accuracy comparable to those generated by MPDATA on a static, uniform grid may be obtained using DGAC with only a quarter of the grid points. Compared to results generated using the leapfrog method on a uniform grid, the DGAC is even more advantageous.

Cases d–g permitted examination of the effect of repeated reduction in the number of grid points when using MPDATA with and without DGA. Using the MIS to interpolate the solutions to the high-resolution 15-km grid, an rms difference was calculated as

\[ E_{\text{rms}}(\sigma) = \left( \frac{1}{IJ} \sum_{i,j=1}^{I,J} (q_{ij} - \hat{q}_{ij})^2 \right)^{1/2} , \]  

where \( q_{ij} \) and \( \hat{q}_{ij} \) are the interpolated and the standard solution, respectively, at point \( i, j \) on the high-resolution grid with \( I \) and \( J \) grid points in their respective horizontal directions. Figure 7 shows that with the exception of case g, which has only 5% of the grid points as case b, the rms difference of each DGA simulation is substantially better than that using a static grid with the same number of grid points. This is especially true in the lower half of the atmosphere where the tracer field characteristics controlled the gridpoint redistribution. All of these curves tend to zero difference at the highest two \( \sigma \) levels because most of the tracer has advected out of the domain at these levels.

4. Passive water vapor advection case

The final simulations (case h) use the water vapor field from the NCEP–NCAR reanalysis at 0000 UTC on 6 March 1992 to prescribe the initial tracer field. The tracer field had no feedback with MM5 and thus did not participate in any evaporation, transpiration, or precipitation. The domain size, orientation, and initial tracer

![Fig. 5. Differences from the standard solution for realistic advection cases at \( \sigma = 0.725 \) after 24 h of simulation. All tracer fields were interpolated to a uniformly spaced 15-km grid for comparison. Contour levels are given at 0.2 increments. For clarity the zero contour is not included. Negative regions are shaded. (a) Differences in the 30-km leapfrog scheme; (b) differences in the 30-km leapfrog scheme, except the data has been horizontally shifted to minimize phase differences; (c) differences associated with the 30-km-resolution MPDATA solution on a static grid; (d) differences in the 15-km-resolution leapfrog solution; (e) differences in the 15-km-resolution leapfrog scheme, except the data has been horizontally shifted to minimize phase differences; and (f) differences associated with the dynamic MPDATA solution.](image-url)
field appear in Fig. 8. Because of the strong variation in the water vapor concentration with height the simulation used $\sigma_l = 0.870$ and $\sigma_h = 0.825$. The results after 44 and 120 h for both dynamic and static grid computations appear in Fig. 9.

Clearly significant differences are evident when comparing the leapfrog scheme to the two MPDATA solutions. The leapfrog scheme gives smoother gradients and substantially less detail. In particular, the leapfrog solution does not show filamentary structures evident in the other two solutions, especially in the 44-h figures. Comparing the dynamic MPDATA solution after 44 h to the static MPDATA solutions, the most noticeable difference is resolution of a bimodal filament structure captured by the dynamic MPDATA scheme near the Texas–Mexican border that extends southeast into the Gulf of Mexico. Additionally, the dynamic MPDATA solution resolved the filament of high tracer concentration that starts in the Gulf of Mexico and continues too the Georgia–Alabama border. It is narrower, steeper, and more continuous than in the static MPDATA result. Although the low resolution of the NCEP reanalysis data precludes any meaningful comparison with the computed results, the work of Iselin and Gutowski (1997) and Behrens et al. (2000) suggest that these filamentary structures may be physically realistic.

After 120 h, there is a large concentration in the leapfrog solution in the Bay of Campeche in the Gulf of Mexico that does not appear in the MPDATA simulations. Examination of the tracer fields at the southern boundary revealed weak fluxes and approximate balance between eastern Gulf of Mexico tracer inflow and western Gulf of Mexico outflow during the last 24 h of simulation time. Thus it is unlikely that the concentration anomaly was due to advection into the computational domain. A more likely cause is the nudging used in the MM5AC forcing frame. By contrast, both the static uniformly spaced MPDATA scheme and the dynamic grid MPDATA scheme used extrapolation for outflow conditions and the true boundary condition for inflow conditions.

5. Conclusions

The use of dynamic grid adaptation (DGA), in conjunction with the multidimensional positive definite ad-
vection transport algorithm (MPDATA), was found to greatly enhance the accuracy of tracer transport simulations at meso- and regional scales. Dramatic reductions in numerical dissipation were observed when compared to results generated using only the standard leapfrog method typically employed within MM5. This enhancement in accuracy is due in part to the use of MPDATA alone—which even on a uniform grid consistently produced sharper results than did the leapfrog simulations.

These results were typically observed throughout a suite of numerical experiments designed to test the dynamic grid algorithm in successively more realistic scenarios. In the final test series, a set of regional, 5-day simulations were conducted using the water vapor concentration at 0000 UTC on 6 March 1992 as the tracer field initial condition. NCEP–NCAR reanalysis data were used during the course of the experiment to provide necessary boundary conditions as well as to provide other necessary field initializations. The dynamic grid computations developed tracer filamentary structures that are highly reminiscent of typical regional-satellite IR images. In computational results using MPDATA on a uniform grid, these filamentary structures tended to be broader, while in the computational results generated by the leapfrog scheme they were essentially unresolved.

We have also found that it is possible to define the grid adaptation primarily as a two-dimensional horizontal mapping and still provide reasonable levels of adaptation at multiple atmospheric levels—even in the presence of significant shear. This is accomplished through the weight function used to redistribute the grid points. Restricting the mapping to be only a function of the horizontal coordinates offers the advantage of a significant enhancement in computational efficiency. It also maintains the verticality of atmospheric columns in computational space that simplify precipitation and radiative transfer calculations.

The only cases in which DGA did not significantly outperform a uniformly spaced grid occurred when only a very few grid points (12 in each horizontal direction) were used. However, as the number of grid points increased, the DGA results rapidly improved and soon surpassed the quality of uniform grid results using either MPDATA or the leapfrog advection algorithms. Ultimately, it was found that DGA results of accuracy comparable to those generated using MPDATA on a uniform grid could be obtained using only 25% as many grid points as for the uniform grid. This result is consistent with the findings in Iselin et al. (2002), where in a two-dimensional test case, it was found that only 20% as many grid points sufficed. Compared to the leapfrog algorithm, the DGA algorithm fares even better.

Successful implementation of DGA hinges strongly upon the choice of the weight function in the grid redistribution scheme. In the present study, design of the weight function was relatively straightforward as only the tracer field was of interest. In the general case, the

![Fig. 7. Rms difference as a function of elevation and the number of grid points for static and dynamic MPDATA schemes. All tracer fields were interpolated to a uniformly spaced 15-km grid before rms differences were calculated.](image1)

![Fig. 8. Water vapor field at $\sigma = 0.825$ at 0000 UTC 6 Mar 1992. Legend units are g kg$^{-1}$.](image2)
proper “targeting” of the weight function may require a substantial research effort before DGA methods can become widespread in meteorological models. However, the results here are very encouraging and suggest the possibility that significantly improved flow fields could be simulated by using the DGA technique on all of the transported quantities including mass, momentum, and energy.
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