A production and pricing decision model for the Korean agricultural cooperatives

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A production and pricing decision model for the Korean agricultural cooperatives

Han, Saim Woo, Ph.D.
Iowa State University, 1992
A production and pricing decision model for the Korean agricultural cooperatives

by

Saim Woo Han

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

Major: Economics.

Approved:

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In Charge of Major Work //
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For the Major Department
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For the Graduate College

Iowa State University
Ames, Iowa

1992
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LIST OF SYMBOLS

A  The set of member patrons
B  The set of nonmember patrons
C  The set of outputs sold to and variable inputs and consumption goods purchased from the cooperative by the member patrons and nonmember patrons
D  The amount of debt employed by the cooperative
DS The total dividends on member stocks
ds The dividends on stock held by a member patron
fc The fixed costs of a member patron
FCC The fixed costs of a cooperative
NS The net savings of the cooperative
PVPR The present value of the allocated patronage refunds
r_i The expected rate of patronage refund on the ith product
X  The set of outputs produced by the members
X_C The subset of goods in set X that represent business through the cooperative
X_0 The subset of goods in set X that represent business outside the cooperative
Y  The set of variable inputs purchased by the members
Y_C The subset of goods in set Y that represent business through the cooperative
Y_0 The subset of goods in set Y that represent business outside the cooperative
Z  The set of consumption goods purchased by the member patrons and nonmember patrons
Z_C The subset of goods in set Z that represent business through the cooperative
\( V \)

- \( Z_0 \): The subset of goods in set \( Z \) that represent business outside the cooperative.
- \( G \): The set of the club goods which the cooperative provides to member patrons.
- \( F \): The set of fixed inputs of member.
- \( U \): The set of outputs produced by the cooperative and sold to buyers outside cooperative.
- \( V \): The set of variable inputs used by the cooperative and purchased by the cooperative from outside cooperative.
- \( W \): The set of consumption goods purchased by the cooperative from outside cooperative.
- \( F_C \): The set of cooperative's fixed inputs.
- \( \alpha \): The rate of reserve funds and the rate of transferred funds.
- \( \tau \): \( (1 - \alpha) \).
CHAPTER 1. INTRODUCTION

Introduction

Today, cooperatives exist in every country in the world and take many forms in relation to political system, economic activity and community organization. The actual incidence of cooperative activity also differs widely from country to country.

However, in general a cooperative is an organization of firms or consumers which is owned and controlled by those who use it and is operated for their mutual benefit as patrons. The established cooperative with proven purchasing and marketing techniques plays an important role in determining the economic success of a farm farmers' family (Williams). Therefore, a cooperative must be under good management to maximize its contribution to members' welfare.

Korean Cooperative

Formation

The Korean Agricultural Bank was first established as a joint-stock company in 1956 by the Financial Associations. But in 1957 it was reorganized into a special bank dealing exclusively in agricultural credit and banking. The
Agricultural Cooperative Law was approved by the National Assembly in 1957, and the agricultural cooperatives were founded in 1958 to engage in economic business other than banking business. A program for the reorganization of multi-purpose cooperatives was made on August 15, 1961, through merger of the agricultural bank and agricultural cooperatives.

The Korean agricultural cooperative was not established because farmers saw the need for its establishment. It was initiated as part of a government policy to promote the development of the rural economy.

The government, first of all, organized the National Agricultural Cooperative Federation (NACF). Afterwards, the organization was expanded vertically to include the primary cooperatives in the villages. The federation provided guidance and assistance for the establishment of primary cooperatives.

The agricultural cooperatives in Korea are organized vertically at two levels; primary cooperatives at the township level and the federation at the national level. They are horizontally classified into two categories; multipurpose cooperatives and special cooperatives. The multipurpose cooperatives are organized by farmers primarily engaged in production of grain such as rice and barley, while special cooperatives are established by the farmers who are mainly
engaged in fruit and vegetable farming. There are 143 multipurpose cooperatives and 41 special cooperatives in Korea.

Activities

An interesting feature of the Korean cooperatives is that all the activities concerning the farmers, i.e., marketing of members' output, provision of production inputs and consumption goods, processing, extension services, credit and banking, and even insurance are undertaken by the multi-purpose cooperatives. And in that sense, they are the real multi-purpose cooperative. The major businesses of the agricultural cooperatives are divided into economic business, credit and banking business, and insurance business.

Marketing Business

In Korea agricultural cooperatives are entrusted by the government with collecting rice, barely, corn, etc. under the government scheme for price stabilization. The most important among them is the collection of rice.

However, the proportion of the policy marketing business under which farm products are sold at the price designated by the government or the NACF, has declined steadily. At the same time the voluntary marketing business carried out by primary cooperatives in the competitive market has increased.
In Korea, as much as 68 percent of total value of farm products marketed by the primary level multi-purpose cooperatives was represented by food grains, 80 percent of which was rice. And 33 percent of the total marketing turnover in 1976 of the primary level cooperatives was on consignment basis and the remaining 67 percent on purchase basis. Under the consignment sales system, the cooperatives sell products consigned by their members and deduct incidental expenses from the sales prices as commissions, and then refund the remainder to their members. In the purchase sales, the purchasing prices are paid for farm products by the cooperatives, and the payments to farmers, are settled at the time of sale. It is believed that agricultural cooperatives in Korea succeeded in reducing the marketing margins and commissions to lower levels than those of private traders by modernizing marketing facilities and being content with small profit. Thus agricultural cooperatives not only contribute to higher prices received by farmers but also repress unfair trade on the part of the merchants (Saito). Therefore, it may be said that marketing activities of agricultural cooperatives has been relatively efficient. But this is said with qualification in that agricultural cooperatives handled only 29 percent of the total agricultural marketings in 1976. The agricultural cooperatives have not yet reached a stage wherein they command a dominant market share of agricultural products
that enables them to exert a very strong influence on price formation or take a price leadership role.

**Purchasing Business**

The prices and amounts of fertilizers used were determined by the government itself until 1982. Even the prices of fertilizers to farmers, transportation costs, storage charges and handling commission fees were annually determined by the government. The government notified the NACF of the prices and amount. In almost all cases, the prices to be paid by farmers were lower than the NACF purchasing prices. The differences were covered by government subsidies. NACF and primary cooperatives had generally functioned well in carrying out the government agency business for the fertilizer distribution. However, the government suffered snowballing deficits in its fertilizer management account. All the fund required for the procurement and management of fertilizer was provided by the government from the Bank of Korea, during the initial stage.

The government had been depending, however, upon NACF for financing the fertilizer account with increasing deposits received by mainly the cooperative banking branch offices in city areas. NACF and primary cooperatives have experienced partly free sale system with the special kinds of fertilizer, such as ammonium sulphate, since 1983 and increased their
portion from 20 percent in 1983 to 60 percent in 1986 even though the others still have been handled by the government. The government changed the fertilizer distribution system to the so-called free sales in 1988.

Farmers as cooperative members are both producers and consumers. The cooperative purchasing business for consumer goods may be regarded as a business in which farmers patronize their cooperatives as consumers. The multi-purpose cooperatives have carried out the purchasing business for consumer goods to serve farm consumers. The NACF introduced a nationwide cooperative chain-store system. At present, chain-stores are operated by expert managers, and daily necessities of the farmer are supplied not only through chain-stores, but also through the women's clubs organized through primary cooperatives.

Processing Business

Most primary cooperative associations have installed collection points for farm products to facilitate their transport. Almost all the multi-purpose cooperatives have their own warehouses, a fact which has contributed to the adjustment of demand and supply and price stabilization of farm products.

The cooperative processing business is divided into two groups; processing for supply to members and processing of
members' production for sale outside the cooperative, with value added. The amount of cooperative processing business is very small. It barely keeps itself in existence. However, agricultural cooperatives are in a very favorable position to participate in or further develop the processing businesses, since they have a nationwide sales network and can secure raw materials on more favorable conditions than private firms.

Banking Business and Insurance Business

The primary objective of the banking business is to provide enough cheap credit with comprehensive services to meet the financial needs of member farmers. The major funding resources of the banking business are deposits at the NACF's banking offices and borrowings from the government, the Bank of Korea, and other international or domestic financial organizations. The agricultural cooperatives also provide member farmers with provisions against unexpected accidents such as sickness, death and any loss and damage from unforeseen disasters with the handling of cooperative insurance. However, these are not just for the members, most of the commercial banking and insurance customers are nonmembers.
The government urged agricultural cooperatives to operate their business in line with the agricultural policy aimed at increasing food production. The Korean agricultural cooperatives adopted a multipurpose system to enhance the social and economic status of farmers. Agricultural cooperatives in Korea engage in various activities rather than specializing in a single type of business. There are advantages from employing a multipurpose system in a small country with small-scale farming. However in multipurpose cooperatives it is hard to devote skilled managerial efforts to each specialized business section. It is also difficult to train specialists, and to keep track of profits and losses by business activities and take appropriate actions. Therefore, the cooperatives need to develop the management skills and marketing methods for their businesses. After Korean farmers provided a sufficient amount of rice to Korean rice consumers, Korean farmers have tried to find other farm products which give them more chance to sell for or higher income from the market. At the same time they have shifted from production for family consumption to production for sale. Agricultural cooperatives in Korea are also presently changing from a government agency to organizations for the benefit of their members. The primary objective of cooperatives under NACF
(National Agricultural Cooperatives Federation) is to increase their own businesses rather than government policy businesses, and the presidents of the primary cooperatives are elected by the members. This cooperative movement away from being a quasi-governmental agency appears to be a welcome change in agricultural cooperatives in Korea. They can protect their members' interests and establish objectives that are consistent with members' goals.

The Differences Between Agricultural Cooperatives in the United States and Korea

1. In the United States, agricultural cooperatives are organized based on the commercial needs of members. Cooperatives and the members are closely related to each other in that agricultural cooperatives try to achieve the goals of their members. The members own and control the cooperatives. Unlike the cooperatives in the United States which have been established by farmers from the bottom up, the agricultural cooperatives in Korea were organized from the top-down. Therefore, the Korean agricultural cooperatives have very difficult positions to perform their businesses. They need to consider the government agricultural policy and members' interests.

2. In the United States, agricultural cooperatives are organized according to the special types of businesses, and
they have appropriately developed the management skills and decision methods for their businesses. On the other hand, agricultural cooperatives in Korea engage in various activities rather than in special types of businesses. A multi-purpose cooperative renders it hard to devote managerial endeavors to a specialized business section, therefore, it is difficult to train specialists and it is hard to keep track of profits and losses by business sections and thus to make appropriate decisions.

3. In the United States, the farm scale is much larger than the Korean farm scale. In Korea, the average farming acreage per farm household is about 1.17 ha which is equivalent to 2.89 acres.

4. In the United States, the percentage of cooperative members of all farmers is less than 40 percent. In Korea, more than 90 percent of farmers are members of cooperatives, even though participation in the cooperatives is not purely voluntary.

5. There are some legal differences between United States and Korea. In the United States, nonmember business can not be greater than half of total business, and all net profit is distributed to members with patronage refund even if a portion of the patronage refund is deferred. While, in Korea, nonmember business can not be greater than one third of total business, and at least 10 percent of net profit must be
reserved as legal reserve fund, at least 20 percent of the net saving also need to be reserved as a business reserve fund, and finally at least 20 percent of net saving must be transferred to next year. Therefore, only the rest of net saving can be distributed to members with dividend and patronage refund.

Problem Statement and Objectives

Korean agricultural cooperatives have several problems that have to be addressed. First of all, many members of Korean cooperatives have the perception that the cooperative is a governmental agency. Such misconception in the minds of the cooperative members on the real identity of cooperatives is bound to retard the growth and development of cooperative. Mendoza cited the Report of the 1966 ICA commission that in developing countries, the members of most cooperatives "are like the passengers of a train using it only when it becomes necessary to do so for their own individual purposes; the running of the train is not their business. This is what must inevitably happen when planning and organizing come from the top. The cooperative movement can not grow from the top downward for voluntary membership and democratic control are of the essence of the cooperative system (p. 25)." The more successful a cooperative is, the more likely are the members
to conceive the ambition of gaining independence from
government supervision and work to achieve this ambition. A
cooperative association is an organization of firms controlled
by those who use it and operated for their mutual benefit as
patrons. In the long run, cooperatives will be successful
only to the extent to which they become truly self-help
organizations. This responsibility calls for planning the
necessary supporting educational programs, developing a sense
of financial responsibility among members, and making a
deliberate plan to have government agencies and their
officials turn responsibility over to cooperatives just as
soon as they demonstrate ability to carry it.

Korean agricultural cooperatives need to select an
optimum labor allocation between credit business and the
noncredit business. The main sources of capital are now
customers' deposits in Korean agricultural cooperative. A
portion of profit from the credit business is transferred to
the noncredit business. Therefore, the credit business might
be important to Korean agricultural cooperatives, even though
there are arguments that the credit business of agricultural
cooperatives should be separated from the noncredit business.
A major component of the problem is the labor allocation
between the credit business and the noncredit business. The
management and employees of cooperatives may prefer to
concentrate on the credit business which is a more profitable
and less risky business than noncredit business. However, the cooperative should work for its members. Therefore, labor allocation should be made so the marginal member benefit with respect to labor in credit business equals the marginal member benefit with respect to labor in noncredit business.

Cooperatives also need to consider financial instruments including the rate of patronage refunds, the rate of the dividend, the level of reserve funds, and the level of debt. The member interest in patronage refunds can enhance members' involvement in the cooperative business; such interest has increased due to recent changes.

Korean cooperative's banking and insurance businesses are not just for the members, most of those businesses' customers are nonmembers. Cooperative's managers prefer banking and insurance businesses to marketing and purchasing businesses because marketing and purchasing businesses are riskier than banking and insurance businesses. However, the cooperatives are the organized for the members' benefit. Therefore, cooperative managers may be over emphasizing banking and insurance in which case the cooperatives need to pay increased attention to their marketing and purchasing businesses.

The multipurpose primary cooperatives adopt the purchase sales method more than the consignment sales method. The purchase sales method implies that the cooperatives buy farm products from their members and sell them to final consumers
or intermediary merchants. The purchase price of farm products is determined when farmers hand the products over to the cooperatives, and the payments to farmers made on delivery. The purchase sales method is applicable to food grains and other products which stand long storage and whose prices are comparatively stable. The supply of chemical fertilizers were monopolized by the NACF and its member cooperatives from 1962 to 1988. However, the NACF and its member cooperatives must operate in a free market in fertilizer production and marketing since 1988. Therefore, cooperatives decide the farm product prices which cooperatives pay to their members, and the farm inputs' prices and consumption goods' prices which members pay to cooperatives. These decisions are very important decisions which affect member income and utility. They affect the cooperatives' financial status.

The objective of this study is to introduce the idea that a primary agricultural cooperative is a decision unit in Korea. This study is about the managerial economic process, and is a conditional normative study: conditional upon the specified comparative objective function. I hope this study can give some idea about how Korean cooperatives ought to behave to provide maximum benefit to members.

This study assumes that the employees of cooperatives are optimally allocated between credit business and noncredit
business, although there is need for further research to examine whether this assumption holds. This study narrows down the objective to concentrate on marketing and purchasing decisions, which are to decide the prices and quantities for the short run, and leaves the financial problem to future research. It is assumed that the objective of a cooperative is maximization of members’ welfare. Under this assumption the cooperative model was constructed by modifying Royer’s production pricing model by adding the consumption goods. This study first constructs a cooperative model under the assumption of price certainty; then, develops a model under price uncertainty. This study is not intended to study welfare economics. Some previous studies (Carson, Anderson, Porter, and Maurice) do study welfare consequences of cooperatives.

Literature Review

There has been no previous research about Korean agricultural cooperatives as decision units. Korean multi-purpose agricultural cooperatives deal with consumption and production goods in their purchasing business. Therefore, the members patronize cooperatives as consumers and producers. There has also been no previous research about cooperatives which deal with consumption goods as well as farm inputs and
farm outputs. The cooperative is a special type of corporation which owned and controlled by its members. However, a reasonable assumption is that some degree of management control exists in most agricultural cooperative. Therefore, there is presumed to be a decision maker within the cooperative. Optimality is defined with reference to an assumed objective function. Theoretical models of cooperatives are idealizations and they represent their authors' views about cooperatives. A brief discussion of the previous research on cooperative theory will be presented.

Emelianoff, one of the earliest cooperative theorists, concluded that a cooperative was an organization of independent units which is coordinated, owned, and controlled by the same independent economic entities, and the membership must be homogeneous. Phillips viewed a cooperative as a vertical extension of its member firms. Phillips argued that cooperation is closely analogous with vertical integration, and employed the theory of the multi-plant firm to derive optimality conditions. Among the optimality conditions are the following; (1) for each participating firm, the marginal productivity of each resource allocated to the cooperative plant must be equal to the marginal productivity of that resource in the individual plants of that firm, and the marginal productivity of the last dollar must be equal in every use within each firm, (2) regarding the volume of
patronage, the cooperating firm equates the sum of the marginal costs in its individual plant or plants and the marginal cost in the joint plant with the marginal revenue facing the firm in the market where the product is sold. Emelianoff and Phillips viewed members as the sole decision agents in a cooperatives. Most early writers followed Emelianoff and Phillips in specifying that there was little or no role for management in cooperatives. Aresvik and Ohm expressed skepticism concerning Phillip's second optimality condition. Each member is likely to consider the average revenue obtainable from a marketing cooperative, and the average cost incurred by a farm supply or marketing cooperative, in determining his volume of patronage. One reason members will base their patronage decisions on average cost or revenue is that members are not informed about the joint plant's cost and revenue functions. Therefore, members may behave as price takers relative to the cooperative.

Trifon investigated Phillips' first optimality condition, and argued that it is inconsistent with proportionality, as well as unlikely to be realized in practice. Proportionality requires that each member's capital contribution be proportional to his patronage.

Stephen Enke was the earliest formal cooperative theorist and the only early writer to suggest an active role for management. He proposed a model of a 'consumer cooperative'
in which the objective pursued by the decision maker was to maximize the members' welfare, as measured by the sum of consumer surplus plus profits. He uses the term 'profits' to describe what most writers now term 'net savings'. Enke derives a criterion for optimal resource use: the general welfare requires that all business units follow a price policy which results in a scale of operation such that marginal cost is equal to the marginal demand price of the market. To achieve this, the cooperative business unit must produce a level of output such that average revenue equals marginal cost.

Clark presented a model in which the decision makers' objective was to minimize the cost of providing a good or service to members. Clark assumed that each member's physical patronage is a fixed amount. Not even in the short run is it reasonable to assume that supply and demand curves are perfectly inelastic, so this assumption is a major weakness of Clark's model. Clark assumed that the decision maker would achieve an optimal volume of business by regulating the number of members. This assumption may be inconsistent with the best interest of members.

Helmberger and Hoos developed a theoretical model of a single-product agricultural cooperative. In their model, the objective assumed for the cooperative is maximization of the net price paid to members. Equality of treatment among
members was achieved by paying the same net price to all members. The decision maker's problem can be broken into two parts. First, regardless of how much final product the cooperative produces, it is desirable to produce this amount of product at the lowest possible cost. Second, the quantity of output must be chosen to maximize the cooperative's surplus (net savings), which is equivalent to maximization of the net price paid to members. The necessary condition is that cooperative marginal revenue equals its marginal cost.

Hardie extended the Helmberger and Hoos model to deal with a multiproduct marketing cooperative. In order to cast the model into a linear programming framework, the cooperative's production function is assumed to be linear and homogeneous with discontinuous factor substitution. Hardie also assumed that the cooperative's average variable cost of producing each final product was known and constant, and that joint fixed costs were allocated. His multiproduct marketing cooperative had profit maximization as its objective. The price paid for a raw product was the shadow price associated with that product. The shadow price was the average monetary contribution of a raw product to the cooperative's total profit. He noted that separable programming methods could be used to apply his model to cases where demand curves for final products were downward sloping and average total costs were not constant.
Ladd also expanded the analysis of the Helmberger and Hoos model to a multiproduct cooperative. In Ladd's model of bargaining cooperative, the cooperative performed three services; (1) it sold a farm input to both members and non-members, (2) it provided an excludable public good to its members at no direct charge, (3) it bargained on behalf of its members for higher raw product prices. Ladd considered two alternative objectives which might be pursued by the decision maker; (1) maximize the price paid to members for their raw product, and (2) maximize the amount of raw product marketed through the cooperative. The first order conditions were different in the two cases, and different from those of a profit maximizing proprietary firm.

Royer's work in modeling cooperative associations incorporated many of the ideas presented by earlier researchers. A nonlinear programming model of a multiproduct cooperative was presented. Royer incorporated decision making at the level of the individual farmer. The cooperative did non-member business on a profit basis, and permitted members to patronize other firms. An individual farmer's profit is written:

\[ \pi = \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i - fc + ds + pvpr \]

where \( p_i \) = the price of the ith product
\( q_i \) = the quantity of the \( i \)th product
\( X \) = a set of farm outputs
\( Y \) = a set of farm inputs
\( fc \) = fixed costs
\( ds \) = dividends on stock
\(_pvpr\) = expected present value of patronage refunds

\[
_pvpr = \left( s + \frac{1 - s}{(1 + d)^\tau} \right) \sum_{i \in C} r_i^* q_i
\]

where \( s \) = the proportion of patronage refunds paid in cash
\( \tau \) = the revolving fund period
\( r_i^* \) = the expected per unit patronage refund on products sold to, or inputs purchased from cooperative
\( C \) = the set of outputs sold and variable inputs purchased from the cooperative by the member and nonmember patrons

By adding production and fixed factor usage constraints, a lagrangian function was formed. The necessary conditions for expected profit maximization are that, if the \( i \)th product is produced, it should be produced up to the point at which the marginal cost of producing it is equal to its effective price which is the cash price plus the discounted expected per unit patronage refund on the product. Under certain assumptions, the necessary conditions will also be sufficient for maximization of expected profit. Input demand and output
supply functions of the form:

\[ q_i = q_i(P_x, P_y, R_c^*, Q_c) \quad i \in X, Y \]

can be derived, where \( P_x, P_y, R_c^*, Q_c \) are vectors of a member's output prices, input prices, expected per-unit patronage refunds, and public goods provided by the cooperative respectively.

The cooperative decision maker's objective is to maximize the total profits of its members, written as:

\[ \Pi = \sum_{i \in X} p_i Q_{ic} - \sum_{i \in Y} p_i Q_{ic} - FC + DS + PVPR \]

where \( Q_{ic} \) = the quantity of the ith product purchased or sold by the member patrons

- \( FC \) = the total members' fixed cost
- \( DS \) = the total dividends on stock
- \( PVPR \) = the present value of all allocated patronage refunds

Three constraints - a production function, fixed factors usage, and the allowable amount of non-member business - were placed on the cooperative profit function. Royer derived first order conditions for the cooperative decision maker. The conditions are complex and I will discuss them in more detail in chapter 3. Royer recommends that the cooperative decision maker allocate joint fixed costs on an opportunity
cost basis. The opportunity cost of a fully employed factor is its shadow price.

Eversull employed Royer's model in an empirical study to show practical applications of the model. Hypothetical cooperatives were simulated to analyze the optimality conditions and provide insight into cooperative management practices. He assumed that producer demand and supply functions were linearly related to basis (futures prices - cash prices) values. He utilized quadratic programming to solve for optimal basis values that the cooperative should use. VanSickle also utilized Royer's work and estimated the supply and demand equations for several commodities using data from Iowa cooperatives. VanSickle studied the cooperative decision nexus between production, pricing, and financing and integrated these decisions into one model. A submodel using Royer's work was developed as the basis for the production and pricing decisions and a submodel which maximized total collective profits of all members was used for financial decisions. The study separated the cooperative decision nexus into three interdependent steps: (1) short-run determination of pricing and production practices, (2) long-run investment portfolio determination, (3) determining the long-run cooperative financial structure. The production and pricing decisions were solved by using an enhanced Royer's model whereas a cooperative financial model was developed to answer
long-run financial questions. VanSickle and Ladd used the theoretical model presented by VanSickle and derived optimal levels of qualified patronage refunds, stock dividends, revolving fund period, rate of cash patronage refund, and the amount of cooperative debt by maximizing cooperative profits. The level of price, production, and investment were assumed exogenous in the financial model of profit maximization. Their results supported the conclusion that cooperatives had in the past relied too heavily on deferred patronage refunds as a source of financing.

Fischer developed a model that consolidated the optimal production, pricing, and financing decision of the cooperative decision maker. Using an objective function similar to Royer's, Fischer assumed that a typical cooperative member would maximize his expected after-tax profit. A model of a single product farm supply cooperative was used to look at the cost of the cooperative's capital. Capital costs were determined for a risk neutral member on a pre-tax and after-tax basis, then for a risk averse member on an after-tax basis. Fischer found that the cooperative's cost of capital was a decreasing function of leverage and that most cooperative should utilize more debt capital.

Passe developed a cooperative model that allowed different types of members to be treated differently. He used Royer's model for his production and pricing decision model
and used a modification developed by VanSickle and Ladd following Jones's criticisms of his financial decision model. The Kuhn-Tucker conditions yielded by his model were similar to Royer's and VanSickle's. The computer model utilized three product lines with each being purchased or supplied by two distinct patron groups. A four stage procedure was used in the computer simulation model. The steps employed in the application procedure involved: (1) specifying the member and non-member supply and demand functions, (2) solving a pricing, production model to determine the price and quantity for each patron group, (3) solving a financial model through a two-step procedure to ascertain the optimal values of $\tau$ and $s$, and (4) updating the exogenous parameters of the pricing, production model with the new values for membership patronage, $\tau$, and $s$. This empirical analysis showed the applicability of theoretical model.

Carson derived basic pricing, production, and resource allocation rules for consumer and producer cooperatives. In order to do this, a somewhat general model of a firm was developed, capable of comprising cooperative, as well as the conventional owner-controlled firm of economic theory. He focused on cooperatives as sellers of private goods and buyers of inputs on the market. He considered cooperative as a welfare maximizing enterprise. Thus the objective of cooperative decision maker is the maximizing a welfare
which is increasing in each $U_k$, subject to constraints posed by its production function and the budget constraints, where $U_k$ is the utility function of the $k$-th member firm. He concluded that a generalized welfare maximizing firm maximizes member welfare by maximizing the sum of its total profit plus the value of members' consumer and factor surplus, which is similar to Enke's result. As a general rule, Carson's G-firm (generalized welfare-maximizing firm) will operate in the closed intervals between the profit maximizing and full welfare maximizing rates of output and inputs. The more consumer or producer members it has for given unrestricted demand and supply curves, the closer it will be to the latter.

Zusman analyzed the group choice process in a cooperative enterprise to derive its implications for the efficient allocation of the organization and the distribution of income among its members. The analysis dealt with a specific problem, the short-run allocation of the cost of operation among members of an agricultural marketing cooperative. He argued that a cost allocation rule is a rule that determines the part of total marketing cost allocated to each member as a function of the cost allocation criteria. In order to decide
the efficient choice of the cost allocation parameters, he adopted Carson's cooperative welfare function. The objective of a cooperative decision maker was maximization of cooperative welfare function subject to zero profit constraint.

Anderson, Porter, and Maurice extended Enke's work, but their work was based on a framework of utility maximization. They concluded that consumer-owned, consumer-managed firms, organized to maximize the utility of each member, reached the same long-run equilibrium position as the perfectly competitive firm, if the cooperatives could freely determine the number of members.

After reviewing the previously cited articles that consider member's utility as their objective criteria. I adopted Carson's cooperative welfare function for a cooperative decision maker's objective function, and modified Royer's pricing, production model in order to decide optimal values of cooperative instrument variables.

Following Chapters

Chapter 2 presents individual member's pricing and production decision including consumption goods under price certainty. In Chapter 3, a cooperative's model under certainty is presented, and first order conditions and their
economic interpretations are also presented. Chapter 4 presents simplification of the cooperative's model and interpretations. Chapter 5 considers these decisions under price uncertainty. Chapter 6 consists of a summary, conclusion, and suggestions for further research.
CHAPTER 2. DEVELOPMENT OF THE ECONOMIC MODEL

In Korea, an agricultural cooperative is a multipurpose cooperative. It includes in its functions that of cooperative credit business along with sales and purchasing businesses. A rural community in Korea is formed around this cooperative. More than 90 percent of farmers are members of primary agricultural cooperatives. The reason of the development of credit business in cooperative is to solve the financing problems of cooperatives and their members. However, in this model, we assume Korean agricultural cooperatives adopt the departmental accounting system and build the model for non-credit businesses, which are sales and purchasing businesses.

In this chapter, a model of a typical member patron and a model of a typical nonmember patron are presented. These models are sub-models of the cooperative model. After these, the cooperative model under certainty will be presented in Chapter 3. Figure 2.1 shows the relationships between the patrons and the cooperative. Figure 2.2 indicates the flows of products within the cooperative production process.

The cooperative purchases unprocessed products (set X) from member patrons and supplies them with variable inputs (set Y) which they use in production, and also supplies them and nonmember patrons with consumption goods (set Z). The
Figure 2.1 Model of the Cooperative
Figure 2.2 Flows of Products within Cooperative
cooperative determines the prices it pays for purchased unprocessed products and the prices it charges for the sale of variable inputs and consumption goods. Not all of the inputs which the cooperative supplies to its patrons are sold. Some of them are club goods (set G). Because it is assumed that member patrons enjoy the benefits of these club goods, the cooperative does not sell them. Instead, it provides them to member patrons free of charge and finances them from other business or capital. An example of a club good is a production technique that cooperative develops and provides to member patrons without any charge.

In addition to doing business with member and nonmember patrons, the cooperative deals with buyers and sellers outside the cooperative. The cooperative purchases variable inputs (set V) for use in processing the unprocessed products which it purchases from its member patrons and for use in producing the inputs which it supplies its member patrons. It also purchases the consumption goods (set W) to supply to its patrons. The cooperative also sells the processed products (set U).

In this model, a product of the member firms which is simply marketed by the cooperative could be included as a special case of an unprocessed product without the use of inputs. However, because it is assumed that the marketing of any product through the cooperative requires the use of some
inputs, there is technically no difference between a product processed by the cooperative and one marketed through it and no distinction is made between the two. Similarly, no distinction is made between a variable input supplied to member firms by the cooperative which is produced by the cooperative and one which is simply purchased by the cooperative and resold to member firms.

The cooperative distributes patronage refunds to only its member patrons and also pays members dividends on stock, but it is assumed for convenience that nonmembers do not hold stock in the cooperative. The members and nonmembers are not required to do business exclusively with the cooperatives. Instead, they purchase variable inputs and consumption goods from outside the cooperative as well as from the cooperative. They also can sell farm products outside the cooperative as well as to the cooperative.

Model of a Member Patron

In the sub-models of the typical member patron and the typical nonmember patron, it is assumed that each patron attempts to maximize his utility.

The set of products produced by the member patrons is represented by X. The subset of products in X which are sold
to the cooperative is represented by $X_c$ while the subset of products which are sold to buyers outside the cooperative is represented by $X_o$. The set of variable factors of production purchased by the patron firms is represented by $Y$. The subset of variable inputs in $Y$ purchased from the cooperative is represented by $Y_c$ while the subset of variable inputs purchased from sellers outside the cooperative is represented by $Y_o$. Similarly, the set of consumption goods purchased by the patrons is represented by $Z$. The subset of the consumption goods in $Z$ which are purchased from the cooperative is represented by $Z_c$ while the subset of consumption good purchased from sellers outside the cooperative is represented by $Z_o$. Both the member and nonmember patrons are assumed to be price takers with respect to all of the prices they pay or receive. The set of fixed inputs of production which are available to the typical member patron is represented by $F$.

I assume that the patrons have perfect knowledge of the prices at the time they make their production and consumption decision. Therefore, they can immediately determine whether they sell their products to cooperative or outside cooperative. The typical member patron tries to maximize his utility subject to budget constraint and production constraint. His utility depends upon the amount of
consumption goods. Therefore, his problem is to solve:

$$\text{max } U = U(z_1, z_2, z_3, \ldots) \quad \text{s.t. } \pi - \sum_{i \in \mathbb{Z}} p_i q_i = 0$$

Note: $$\pi = \sum_{i \in \mathbb{X}} p_i q_i - \sum_{i \in \mathbb{Y}} p_i q_i - fc + ds + pr$$

$$pr = \sum_{i \in \mathbb{C}} r_i^* q_i$$

$$q_i = z_i \quad \text{for } i \in \mathbb{Z}$$

where $$p_i$$ and $$q_i$$ are respectively the price paid or received and the quantity of the $$i$$th product or factor or consumption good, $$fc$$ is the fixed cost of the member, $$ds$$ is the dividends on stock, and $$pr$$ is the patronage refund. The symbol $$\mathbb{C}$$ is the set of outputs sold to and set of variable inputs and consumption goods purchased from the cooperative by the member and nonmember patrons. $$r_i^*$$ is the rate of per-unit patronage refund for the $$i$$th good. The typical member firm can look at its own specific production function to determine optimal levels of production. The individual member's production function is assumed to be a single valued continuous function with continuous first and second order partial derivatives. This strictly concave production function for member is specified in the implicit form as:

$$\phi(q_x, q_y, q_f, Q_g) = 0$$
where \( q_x \) is a vector of the quantities of each of the products in set \( X \) produced by the firm, \( q_y \) is a vector of the quantities of each of the variable inputs in set \( Y \) used in production, and \( q_f \) is a vector of the quantities of each of the fixed inputs in set \( F \) used in production. The symbol \( Q_g \) represents a vector of the quantities of each of the set \( G \) of club goods provided by the cooperative.

The maximization is also subject to a set of constraints which come from the fact that production can utilize only the amounts of fixed inputs that are available. This availability constraints are given as:

\[
q_i \leq \bar{q}_i \quad i \in F
\]

where \( q_i \) is the stock of \( i \)th fixed factor available to the typical member. Then, we can set up the lagrangian function which represents a maximization of a member's utility subject to his budget constraint. This is given as:

\[
L = U(z_1, z_2, z_3, \ldots)
\]

\[
+ \lambda_1 \left[ \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i - \sum_{i \in Z} p_i q_i - fc + ds + \sum_{i \in C} r_i q_i \right]
\]

\[
+ \lambda_2 \left[ \phi(q_x, q_y, q_f, Q_g) \right]
\]

\[
+ \sum_{i \in F} \lambda_3 i (\bar{q}_i - q_i)
\]

(2.1)
where $\lambda_1$ is the lagrange multiplier corresponding to the budget constraint, $\lambda_2$ is the lagrange multiplier corresponding to the production function, $\lambda_n$ are the lagrange multiplier corresponding to the $i$th fixed input constraint. Corresponding to the lagrangian function is a set of Kuhn-Tucker conditions. These are necessary conditions for a global maximum. They are sufficient conditions for a global maximum if the objective function is concave, the constraints are concave, and the set of feasible solutions are bounded and nonempty. It is assumed that utility function is concave, and production function is also concave. The budget constraint and the fixed input constraints are linear and, therefore, can be considered as concave. Thus, if it is assumed that the set of feasible conditions are necessary and sufficient for a global maximum. The Kuhn-Tucker conditions for the problem are as follows:

(1) for all $i \in Z_c$

\[
\frac{\partial L}{\partial z_i} = \frac{\partial U}{\partial z_i} - \lambda_1(p_i - x^*_i) \leq 0
\]

\[
\frac{\partial L}{\partial z_i} z_i = 0
\]

\[z_i \geq 0\]
(2) for all \( i \in \mathbb{Z}_0 \)

\[
\frac{\partial L}{\partial z_i} = \frac{\partial U}{\partial z_i} - \lambda_1 p_i \leq 0
\]

\[
\frac{\partial L}{\partial z_i} z_i = 0
\]

\( z_i \geq 0 \)

(3) for all \( i \in X_c \)

\[
\frac{\partial L}{\partial q_i} = \lambda_1 (p_i + r_i^*) + \lambda_2 \frac{\partial \phi}{\partial q_i} \leq 0
\]

\[
\frac{\partial L}{\partial q_i} q_i = 0
\]

\( q_i \geq 0 \)

(4) for all \( i \in X_0 \)

\[
\frac{\partial L}{\partial q_i} = \lambda_1 p_i + \lambda_2 \frac{\partial \phi}{\partial q_i} \leq 0
\]

\[
\frac{\partial L}{\partial q_i} q_i = 0
\]

\( q_i \geq 0 \)

(5) for all \( i \in Y_c \)

\[
\frac{\partial L}{\partial q_i} = \lambda_1 (- p_i + r_i^*) + \lambda_2 \frac{\partial \phi}{\partial q_i} \leq 0
\]
\[ \frac{\partial L}{\partial q_i} a_i = 0 \]

\[ q_i \geq 0 \]

6) for all \( i \in Y_0 \)

\[ \frac{\partial L}{\partial q_i} = \lambda_1 (-p_i) + \lambda_2 \frac{\partial \phi}{\partial q_i} \leq 0 \]

\[ \frac{\partial L}{\partial q_i} q_i = 0 \]

\[ q_i \geq 0 \]

7) for all \( i \in F \)

\[ \frac{\partial L}{\partial q_i} = \lambda_2 \frac{\partial \phi}{\partial q_i} - \lambda_3 i \leq 0 \]

\[ \frac{\partial L}{\partial q_i} q_i = 0 \]

\[ q_i \geq 0 \]

8) for \( \lambda_1 \)

\[ \frac{\partial L}{\partial \lambda_1} = \sum_{i \in \mathcal{X}} p_i q_i - \sum_{i \in \mathcal{R}} p_i q_i - \sum_{i \in \mathcal{Z}} p_i q_i - f c + d s + \sum_{i \in \mathcal{C}} r_i q_i = 0 \]

\[ \frac{\partial L}{\partial \lambda_1} \lambda_1 = 0 \]

\[ \lambda_1 \geq 0 \]
(9) for \( \lambda_2 
\)
\[
\frac{\partial L}{\partial \lambda_2} = \phi(q_x, q_y, q_f, Q_o) = 0
\]

(10) for all \( \lambda_{ji} \), \( i \in F \)
\[
\frac{\partial L}{\partial \lambda_{3i}} = \bar{q}_i - q_i > 0
\]
\[
\frac{\partial L}{\partial \lambda_{3i}} \lambda_{3i} = 0
\]
\[
\lambda_{3i} > 0
\]

Given these first order conditions, if we assume the marginal value products of the fixed factors (set F) do not reach zero, the fixed factor will be exhausted in the production process. If we also impose the non-satiation assumption, the economic implications and results of the model can be more easily derived and understood.

The lagrange multiplier \( \lambda_i \) can be interpreted as the variation of the objective function due to change in the dividend and fixed cost. That is:

\[
\lambda_1 = \frac{\partial U}{\partial ds} \text{ or } \frac{\partial U}{\partial fc}
\]

So we may interpret \( \lambda_1 \) as marginal utility of money. If we set \( \lambda = \lambda_f/\lambda_1 \), then \( \lambda \) can be interpreted as the imputed value or shadow price of \( \phi \) (Royer, p.37), and \( \lambda_{ji} \) is interpreted as the
variation of objective function due to change in the level of
ith fixed input.

Then, condition 1 indicates that members consider not
only the price for buying consumption good from cooperative
but also the patronage refund caused by their patronizing the
cooperative. Condition 2 indicates that members consider only
the price for buying consumption good which they buy from
outside the cooperative. Therefore, the marginal rate of
substitution between good i (i ∈ Zc) and good j (j ∈ Z0) is
defined as:

$$\text{MRS}_{ij} = \frac{p_i - r_i^*}{p_j}$$

If we rewrite the condition 3 with

$$p_i + r_i^* = -\frac{\lambda_2}{\lambda_1} \frac{\partial \phi}{\partial q_i} \text{ for all } i \in X_c$$

$$= -\lambda \frac{\partial \phi}{\partial q_i} \text{ where } \lambda = \frac{\lambda_2}{\lambda_1}$$

Then, it indicates that the members will produce the
unprocessed product and sell to the cooperative at the point
where the effective price equals to marginal cost of producing
it (Royer, p. 38). The effective price is the price received
from the cooperative plus the patronage refund. Condition 4 is
similar to condition 3. The members will produce the
unprocessed product and sell outside the cooperative at the
point where the price received is equal to the marginal cost of producing it, i.e.,

\[ p_i = \frac{\lambda_2}{\lambda_1} \frac{\partial \phi}{\partial q_i} = \lambda \frac{\partial \phi}{\partial q_i} \text{ for all } i \in X_o \]

A similar conclusion may be reached about the demand for inputs (set Y). The member patron will demand inputs from the cooperative to the point where the effective price is equal to the marginal value product of using the inputs in the production, i.e.,

\[ p_i - r_i = \frac{\lambda_2}{\lambda_1} \frac{\partial \phi}{\partial q_i} = \lambda \frac{\partial \phi}{\partial q_i} \]

The member will purchase inputs outside the cooperative to the point where the price of the input equals the marginal value product of using the input in the production, i.e., from condition 6,

\[ p_i = \frac{\lambda_2}{\lambda_1} \frac{\partial \phi}{\partial q_i} = \lambda \frac{\partial \phi}{\partial q_i} \text{ for all } i \in Y_o \]

In condition 7, the level of fixed input usage is determined. If ith factor is used, its imputed value equals to its marginal value product. Condition 8 indicates the budget constraint, conditions 9, and 10 show production and fixed input constraints.
The typical nonmember patron tries to maximize his utility subject to the budget constraint. We assume that the cooperative does business with nonmembers only in the consumption goods business. The typical nonmember patron's utility function depends upon the amount of consumption goods. Therefore, his problem is to solve:

$$\max U = U(z_1, z_2, z_3, \ldots) \text{ subject to } I - \sum_{i \in Z} p_i q_i = 0$$

where $I$ is income, $p_i$ and $q_i$ are the price paid and the quantity of the $i$th consumption good. Then, the corresponding lagrangian function is:

$$L = U(z_1, z_2, z_3, \ldots) + \lambda (I - \sum_{i \in Z} p_i q_i)$$

and the corresponding Kuhn-Tucker conditions are:

$$(11) \text{ for all } i \in Z$$

$$\frac{\partial L}{\partial z_i} = \frac{\partial U}{\partial z_i} - \lambda p_i \leq 0$$

$$\frac{\partial L}{\partial z_i} z_i = 0$$

$$z_i \geq 0$$
(12) for \( \lambda 

\frac{\partial L}{\partial \lambda} = I - \sum_{i \in Z} p_i z_i = 0

Condition 11 indicates that nonmembers consider only the price in deciding whether to buy consumption goods from the cooperative or outside cooperative. Therefore, the marginal rate of substitution between good \( i \) and good \( j \), where \( i \in z_c \), \( j \in z_o \) is defined as:

\[ MRS_{ij} = \frac{P_i}{P_j} \]

Consumption Good Demand, Output Supply, and Input Demand Functions

Consumption-good demand functions, output supply functions, and input demand functions for the typical member and the typical nonmember patrons can be derived from the Kuhn-tucker conditions if the bordered Hessian matrix of the model is nonvanishing and negative definite. The results provide us output supply functions for products in set \( X_c \) and set \( X_0 \), input demand functions for inputs in set \( Y_c \) and set \( Y_0 \), and demand functions for consumption goods in set \( Z_c \) and \( Z_0 \). The general functions can be represented as:

for \( i \in X_c, X_0, Z_c, Z_0, Y_c, Y_0 \)
\[ q_i^* = q_i(p_x, p_y, p_z, r_c^*, ds, fc, Q_g) = q_i^*(.) \]

\textit{note; } \[ q_i^*(.) = z_i^*(.) \text{ for all } i \in \mathbb{Z} \]

where \( p_x \) is a vector of the prices of the products in set \( X \), \( p_y \) is a vector of the prices of the variable inputs in set \( Y \), \( p_z \) is a vector of the prices of the consumption goods in set \( Z \), \( r_c^* \) is a vector of the patronage refunds on the products in set \( C \). Then, a typical member patron \( n \)'s indirect utility function can be expressed as:

\[ V_{in} = V_{in}(z_i^*(.), z_z^*(.), . . . . , z_n^*(.)) \]

\[ = V_{in}(p_x, p_y, p_z, r_c^*, ds, fc, Q_g) \] \hspace{1cm} (2.2)

From the envelope theorem:

for all \( j \in Z_c, Y_c \)

\[ \frac{\partial V}{\partial p_j} = -\lambda_1 q_j \leq 0 \]

for all \( j \in X_c \)

\[ \frac{\partial V}{\partial p_j} = \lambda_1 q_j \geq 0 \]

for all \( j \in Z_c, X_c, Y_c \)

\[ \frac{\partial V}{\partial r_j^*} = \lambda_1 q_j \geq 0 \]
which imply that the prices of consumption goods and farm inputs offered by the cooperative have negative effects on members' utilities, the prices of farm products offered by the cooperative have positive effects on members' utilities, and finally the patronage refunds have positive effects on members' utilities.

By horizontally summing the individual output supply function for product $i$ in set $X$ across all member patrons, a supply function, relating the level of the output supplied by the member patrons to the parameters, can be determined. A demand function, relating the level of the $i$th variable input in set $Y$ demanded by the member patrons to the parameters, can be determined by horizontally summing the individual input functions for the input across all member patrons. In a similar manner, a demand function, relating the level of the $i$th consumption good in set $Z$ demanded by the member patrons and nonmember patrons to the parameters, can be determined by horizontally summing the individual consumption demand functions for the consumption good across all members. For all $i \in X_C, Y_C, Z_C$

$$q_i^{TM} = q_i^{TM}(p_x, p_y, p_z, r^*, f_c, d_s, Q_g)$$

where $q_i^{TM}$ is the total quantity supplied of the unprocessed products (set $X$) by the members or total quantity demanded of
the inputs (set $Y$) or consumption goods (set $Z$) by the member patrons. Using the Kuhn-Tucker conditions corresponding to the typical nonmember patron, consumption good demand functions for the nonmember patrons can be derived. The general functions can be represented as, for $i \in Z_c, Z_o$

$$q_i^* = q_i^*(p_z, I)$$

where $I$ is the income of nonmember patron.

The total quantity demanded by all nonmember patrons from the cooperative of all consumption goods in set $Z_c$ will be the horizontal summation across all nonmembers for each good, i.e., for all $i \in Z_c$

$$q_i^{TN} = \sum_{i \in B} q_i^*(p_z, I)$$

Then, the total quantity demanded by all member patrons and nonmember patrons from the cooperative of all consumption goods in set $Z_c$ will be the horizontal summation across all members' and nonmembers' demand for each good, i.e., for all $i \in Z_c$

$$q_i^T = q_i^{TM} + q_i^{TN}$$

$$= q_i^T(p_x, p_y, p_z, z^*, fc, ds, Q_o)$$
where $q_i^J$ is the total quantity demanded of the $i$th consumption good by all member patrons and nonmember patrons.
CHAPTER 3. DEVELOPMENT OF COOPERATIVE MODEL UNDER CERTAINTY

The Cooperative's Model

The objective of the cooperative decision maker is the maximization of the welfare function whose elements are the individual members' indirect utility functions. The reason for this objective is that the purpose of the cooperative is to benefit the member patrons.

The cooperative determines the prices it will pay for purchased unprocessed products and the prices it charges for the sale of variable inputs and consumption goods to patrons in order to maximize the welfare function of cooperative. The cooperative also purchases variable inputs (set V) and consumption goods (set W) from sellers outside the cooperative and sells finished products (set U) to buyers outside cooperative. Therefore, the cooperative's decision maker also needs to decide the quantities of inputs and consumption goods to buy from outside sellers, and the quantity of output to sell to outside buyers.

Most Korean agricultural cooperatives save money from their net savings according to their articles. At least 10 percent of net savings must be set aside as a legal reserve until the fund reaches two times of sum of investment stock. At least 20 percent of the net savings must be saved as a
business reserve fund. And finally at least 20 percent of net savings must be transferred to next year's business. But, there is little difference between legal reserve and business reserve fund except that business reserve funds are spent first in case of loss. Therefore, at least 50 percent of net savings is reserved by cooperative without allocation to patrons. The unallocated reserve funds may avoid complex financing problems. However, use of unallocated reserve funds as a method of accumulating capital has problems. As the proportion of the unallocated reserve funds increase, members lose a sense of ownership and management may become more independent and less subject to member's control. It may reduce the member's welfare and efficiency of resource allocation. Later, we will discuss the effects of these restrictions. However, in this model, we first set up the model according to their articles, so we get constraints from these restrictions.

In this model, the cooperative decision makers decide the prices in the beginning of fiscal year. At the end of the fiscal year they decide the rates of the reserve funds, the rate of dividends on stock, and the rate of the patronage refunds. Therefore, at the beginning of the fiscal year, cooperative decision makers decide the prices based on the values of financial instruments which were decided during the last year in order to maximize the welfare function of the
The welfare function of the cooperative can be expressed as:

\[ W = W(V_1, V_2, \ldots, V_n) \]

where each \( V_i \) is defined by equation 2.2. This welfare function is increasing in each member's indirect utility function \( V_n \). It is maximized subject to constraints imposed by cooperative's restriction on distribution of net savings and cooperative's production.

### Determination and Distribution of Net Savings

The total net savings of the cooperative is determined by summing the net savings earned by the cooperative in all its transactions. It can be expressed as:

\[
NS = \sum_{i \in Y} p_i q_i + \sum_{i \in Y_c} p_i q_i^{TM} + \sum_{i \in X_c} p_i q_i^T - (\sum_{i \in Y} p_i q_i + \sum_{i \in W} p_i q_i + \sum_{i \in X_c} p_i q_i^{TM} + FCC + rD) = TCR - TCC
\]

where FCC, \( r \), and D are cooperative's fixed cost, interest rate, and the level of debt respectively. TCR the first line
of 3.1 and TCC the second line of 3.1 are total collective revenue of cooperative and total collective cost of cooperative. Total collective revenue is received by the cooperative for the actions it undertakes. Similarly, total collective cost is paid by the cooperative for the actions it undertakes. According to each cooperative's articles, at least 50 percent of net savings needs to be reserved by the cooperative. Let \( \alpha \) equal the reserve rate, and \( \tau = (1 - \alpha) \), then \( \tau \) is less than or equal to 0.5. The rest of net savings can be distributed to member patrons as dividends on stock and patronage refunds, i.e.,

\[
\tau NS = \sum_{n \in A} ds_n + \sum_{n \in A} pr_n
\]

\[
= DS + \sum_{i \in C} r_i q_i^T
\]

where \( A \) is the set of member patrons and \( DS \) is the total dividend on member stock.

Production Function

The technology of the cooperative is represented by a production function which, in its implicit form, is written as:

\[
\Phi (Q_u, Q_x, Q_v, Q_y, Q_w, Q_z, Q_g, Q_pc) = 0
\]
where $Q_u$ is a vector of the quantities of each of the products in set $U$, $Q_x$ is a vector of quantities of outputs in set $X_c$, $Q_v$ is a vector of the quantities of the variable inputs in set $V$, $Q_y$ is a vector of the quantities of each of the variable inputs in set $Y_c$, $Q_w$ is a vector of quantities of consumption goods in set $W$, $Q_z$ is a vector of quantities of the consumption goods in set $Z_c$, $Q_g$ is a vector of the quantities of each of the club goods in set $G$. $Q_{pc}$ is a vector of the quantities of each of the fixed inputs in set $F_c$. The assumptions made concerning this production function are similar to those made concerning the production function of the typical member patron. It is assumed that the production function possesses continuous first and second order partial derivatives. It is written in such a way that the partial derivatives with respect to the outputs are positive and the partial derivatives with respect to the inputs are negative.

Other Constraints

Other constraints come from full use of purchased goods from patrons and availabilities of fixed inputs. For all $i \in X_c$

$$q_{i}^{TM} - \sum_{i \in U, Y} q_{ij} = 0$$

The quantity of the $i$th product in set $X_c$, used in production
of the jth product by the cooperative is represented by \( q_{ij} \).

For all \( i \in F_c \)

\[
q_{i} - \sum_{j \in Y, z, \sigma} q_{ij} \geq 0
\]

The quantity of the ith fixed input in set \( F_c \) used in production of the jth product by the cooperative is represented by \( q_{ij} \).

In this model, since we build the model for noncredit business, we can ignore the limit upon nonmember business as a percentage of total business because nonmember business exists only in consumption goods' business, but the consumption good's business is less than 20 percent of the total noncredit business in 1988. Thus, the lagrangian function corresponding to the problem of the cooperative can be expressed as:

\[
L = W(V_1, V_2, \ldots, V_n) + \lambda_0 \left( \tau_{NS} - DS - \sum_{i \in C} x^*_i q_i^{TM} \right) + \lambda_1 \left( \Phi (Q_u, Q_x, Q_v, Q_y, Q_w, Q_z, Q_g, Q_{FC}) \right) + \sum_{i \in Y, u} \lambda_{2i} \left( q_i^{TM} - \sum_{j \in Y, u} q_{ij} \right) + \sum_{i \in FC} \lambda_{3i} \left( \overline{q}_i - \sum_{j \in Y, z, u, \sigma} q_{ij} \right)
\]
The instrument variables of the cooperative are the following prices and quantities:

- $p_i$ for $i \in Xc, Yc, Zc$
- $q_i$ for $i \in U, W, G$
- $q_{ij}$ for $i \in Xc$, and $j \in Y, U$
- $q_{ij}$ for $i \in Fc$, and $j \in Y, Z, U, G$
- $q_{ij}$ for $i \in V$, and $j \in U, Y, G$.

Now, if we drop the reserve fund restriction the lagrangian function representing the problem of the cooperative is obtained from the preceding lagrangian function by setting $\tau$ equal to unity in the first restriction.

Kuhn-Tucker Conditions and Interpretation

Kuhn-Tucker conditions are necessary conditions for a global maximum corresponding to the instruments and the lagrangian function. They are sufficient conditions for a global maximum if the objective function is concave, the constraints are concave, and the set of feasible solutions is bounded and nonempty. These properties are assumed to hold in this dissertation. The Kuhn-Tucker conditions of the lagrangian function with the reserve fund restriction are as follows. In obtaining these, the term NS in the first constraint is replaced by its definition from equation (3.1).
(13) for all \( j \in X_c \)

\[
\frac{\partial L}{\partial p_j} = \sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_j} \\
+ \lambda_0 \left\{ \tau \left( \sum_{i \in V_c} p_i \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} \right) \\
- q_j^{TM} - \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} - \sum_{i \in \mathcal{E}_c} r_i \frac{\partial q_i^{TM}}{\partial p_j} \right\} \\
+ \lambda_1 \sum_{i \in \mathcal{E}_c} \frac{\partial \Phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \\
+ \sum_{i \in \mathcal{E}_c} \lambda_{2i} \frac{\partial q_i^{TM}}{\partial p_j} \leq 0
\]

\[
\frac{\partial L}{\partial p_j} p_j = 0
\]

\( p_j \geq 0 \)

(14) for all \( j \in Y_c \)

\[
\frac{\partial L}{\partial p_j} = \sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_j} \\
+ \lambda_0 \left\{ \tau \left( q_j^{TM} + \sum_{i \in V_c} p_i \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} \right) \\
- \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^{TM}}{\partial p_j} - \sum_{i \in \mathcal{E}_c} r_i \frac{\partial q_i^{TM}}{\partial p_j} \right\} \\
+ \lambda_1 \sum_{i \in \mathcal{E}_c} \frac{\partial \Phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j}
\]
\[ \frac{\partial L}{\partial p_j} \] 

\[ p_j = 0 \]

\[ p_j \geq 0 \]

(15) for all \( j \in \mathcal{Z}_c \)

\[ \frac{\partial L}{\partial p_j} = \sum_{n \in A} \frac{\partial w}{\partial v_n} \frac{\partial v_n}{\partial p_j} \]

\[ + \lambda_0 \left[ \tau \left( \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} + q_j^T + \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} \right) \right. \]

\[ - \sum_{i \in \mathcal{E}_c} p_i \frac{\partial q_i^T}{\partial p_j} \]

\[ - \sum_{i \in \mathcal{E}_c} r_i \frac{\partial q_i^T}{\partial p_j} \]

\[ + \lambda_1 \sum_{i \in \mathcal{E}_c} \frac{\partial \phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \]

\[ + \sum_{i \in \mathcal{E}_c} \lambda_2 \frac{\partial q_i^T}{\partial p_j} \leq 0 \]

\[ \frac{\partial L}{\partial p_j} \] 

\[ p_j = 0 \]

\[ p_j \geq 0 \]

(16) for all \( j \in \mathcal{W} \)

\[ \frac{\partial L}{\partial q_j} = \lambda_0 \tau \left( - p_j - q_j \frac{\partial p_j}{\partial q_j} \right) + \lambda_1 \frac{\partial \phi}{\partial q_j} \leq 0 \]
\[ \frac{\partial L}{\partial q_j} q_j = 0 \]

\[ q_j \geq 0 \]

(17) for all \( j \in U \)

\[ \frac{\partial L}{\partial q_j} = \lambda_0 \tau (p_j + q_j \frac{\partial p_j}{\partial q_j}) + \lambda_1 \frac{\partial \phi}{\partial q_j} \leq 0 \]

\[ \frac{\partial L}{\partial q_j} q_j = 0 \]

\[ q_j \geq 0 \]

(18) for all \( j \in G \)

\[ \frac{\partial L}{\partial q_j} = \sum_{n \in A} \frac{\partial w}{\partial v_n} \frac{\partial v_n}{\partial q_j} \]

\[ + \lambda_0 [\tau \left( \sum_{i \in e_c} p_i \frac{\partial q_i^{TM}}{\partial q_j} - \sum_{i \in e_c} p_i \frac{\partial q_i^{TM}}{\partial q_j} \right) - \sum_{i \in e_c} r_i \frac{\partial q_i^{TM}}{\partial q_j}] \]

\[ + \lambda_1 \sum_{i \in e_c} \frac{\partial \Phi}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial q_j} \]

\[ + \sum_{i \in e_c} \lambda_2 \frac{\partial q_i^{TM}}{\partial q_j} \leq 0 \]

\[ \frac{\partial L}{\partial q_j} q_j = 0 \]

\[ q_j \geq 0 \]
(19) for all $i \in X_C$, and $j \in Y, U$

\[ \frac{\partial L}{\partial q_{ij}} = \lambda_1 \frac{\partial \Phi}{\partial q_{ij}} - \lambda_{2i} \leq 0 \]

\[ \frac{\partial L}{\partial q_{ij}} \frac{q_{ij}}{= 0} \]

\[ q_{ij} \geq 0 \]

(20) for all $i \in V$, and $j \in Y, U, G$

\[ \frac{\partial L}{\partial q_{ij}} = \lambda_0 \tau ( - p_i - q_i \frac{\partial P_i}{\partial q_{ij}} ) + \lambda_1 \frac{\partial \Phi}{\partial q_{ij}} \leq 0 \]

\[ \frac{\partial L}{\partial q_{ij}} \frac{q_{ij}}{= 0} \]

\[ q_{ij} \geq 0 \]

Note: $q_i = \sum_{j \in Y, U, G} q_{ij}$ for all $i \in V$

(21) for all $i \in F_C$, and $j \in Y, Z, U, G$

\[ \frac{\partial L}{\partial q_{ij}} = \lambda_1 \frac{\partial \Phi}{\partial q_{ij}} - \lambda_{3i} \leq 0 \]

\[ \frac{\partial L}{\partial q_{ij}} \frac{q_{ij}}{= 0} \]

\[ q_{ij} \geq 0 \]
Before we interpret the Kuhn-Tucker conditions, we need to interpret the lagrange multipliers. The lagrange multiplier $\lambda_0$ can be interpreted as the variation in welfare due to autonomous change in net savings, or

$$\lambda_0 = \frac{\partial W}{\partial (-\tau \text{FCC})} \quad \text{or} \quad \frac{\partial W}{\partial (-\tau \text{RD})}$$

where FCC, rD are the constant term in the constraint of net savings for distribution. That is marginal welfare of fixed
cost or marginal welfare of debt cost. We may interpret $\lambda_0$ as marginal welfare of the unit of cooperative's money income.

If cooperative's production constraint is solved for one output in set $U$, say $q_u^1$, then

$$q_u^1 = \Phi^1(Qu^1, Q_x, \ldots, Q_g)$$

where $Qu^1$ is set $U$ without the first product. Then, the third line of the cooperative's lagrangian function can be rewritten as:

$$\lambda_1 [q_u^1 - \Phi^1(Qu^1, Q_x, \ldots, Q_g)]$$

Therefore,

$$\lambda_1 = \frac{\partial W}{\partial q_u^1}$$

which is the contribution of $q_u^1$ to marginal welfare of members.

The lagrange multiplier $\lambda_{ii}$ ($i \in X_c$) can be interpreted as the variation in welfare from changes in the constraint that all the unprocessed product $i$ in set $X$ purchased from member patrons are transformed into final products. Finally the lagrange multiplier $\lambda_{ii}$ can be interpreted as the contribution of fixed factor $i$ in set $F_c$ to marginal welfare, that is the variation in welfare from changes in one of the cooperative's fixed factor.
In condition 13, the price of jth output sold to the cooperative by members is used as the instrument variable. The term

\[ \sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_j} = \sum_{n \in A} \frac{\partial W}{\partial V_n} \lambda_{1n} q_{jn} = \sum_{i \in \epsilon} \lambda_{1n} q_{jn} \]

may be interpreted as the marginal variation in welfare from the changes of indirect utility functions of members caused by a variation in the jth price, which is equivalent to the sum of weighted supply that members sell to the cooperative for j \( \in X_c \). The term

\[ \sum_{i \in \epsilon \in \epsilon} p_i q_{iM} \]

represents total collective revenue of cooperative's farm input business, since the revenue is received by the cooperative for providing farm inputs to the member patrons. Therefore, the term

\[ \sum_{i \in \epsilon \in \epsilon} p_i \frac{\partial q_{iM}}{\partial p_j} \]

can be interpreted as the marginal variation in total collective revenues (TCR) to the cooperative arising from shifts in factor demand induced by jth price. This effect was represented by Royer as:

\[ \sum_{i \in \epsilon \in \epsilon} \frac{\partial TCR}{\partial q_{iM}^{TM}} \frac{\partial q_{iM}^{TM}}{\partial p_j} \]
Similarly, the term

$$ - q_j^{TM} - \sum_{i \in X_c} p_i \frac{\partial q_i^{TM}}{\partial p_j} = - \sum_{i \in X_c} \frac{\partial TCC}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} $$

may be interpreted as the marginal variation in total collective costs (TCC) from the jth product and all other products in set $X_c$ arising from output shifts which are induced by a change in the jth price. The term

$$ \sum_{i \in X_c} p_i \frac{\partial q_i^{T}}{\partial p_j} = \sum_{i \in X_c} \frac{\partial TCR}{\partial q_i^{T}} \frac{\partial q_i^{T}}{\partial p_j} $$

can be interpreted as the marginal variation in total collective revenues arising from shifts in consumption goods demand induced by jth price. The term

$$ r_j q_j^{TM} + \sum_{i \in X_c} r_i \frac{\partial q_i^{TM}}{\partial p_j} = \sum_{i \in X_c} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} $$

can be interpreted as the variation in total member patronage refunds (TPR) from shifts in output supply, factor demand, and consumption good demand induced by a variation in the jth price. Therefore, the second and third lines of the condition can be interpreted as variation in welfare through the variation in total member patronage refunds and the change of net savings from variation of total collective revenue and total collective cost, all induced by $dp_j$.

The term

$$ \lambda_1 \left( \sum_{i \in X_c} \frac{\partial \Phi}{\partial q_i^{T}} \frac{\partial q_i^{T}}{\partial p_j} \right) $$
in the fourth line can be interpreted as the marginal variation in welfare from the production of the products in set \( X_c \), set \( Y_c \), and set \( Z_c \) arising from changes in the quantities demanded or supplied which are induced by \( d_p_j \). We can rewrite this term as:

\[
- \frac{\partial W}{\partial \Phi^1} \sum_{i \in C} \frac{\partial q_i^T}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j}
\]

The term

\[
\sum_{i \in C} \lambda_{2i} \frac{\partial q_i^{TM}}{\partial p_j}
\]

in the fifth line represents the variation in welfare that is caused by changes in cooperative's use of members' output due to changes in farm output which are generated by a change in price paid to producers. That is, the variation in welfare from the transformation of products in set \( X_c \) to final products arising from a change in the quantities supplied which are induced by \( d_p_j \).

If the cooperative offers a positive price for the \( j \)th product in set \( X_c \), the following equality must be satisfied for a maximum:

\[
\frac{\partial L}{\partial p_j} = \sum_{i \notin A} \frac{\partial W}{\partial v_n} \frac{\partial v_n}{\partial p_j} + \frac{\partial W}{\partial V_M} \left[ \tau \left( \sum_{i \notin Y_c} \frac{\partial M_i}{\partial TCR} \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \notin Z_c} \frac{\partial M_i}{\partial TCR} \frac{\partial q_i^{T}}{\partial p_j} \right) \right]
\]
where MI is cooperative's money income. This is equivalent to stating that, for a maximum, the sum of:

a) the marginal variation in welfare from the change of indirect utility functions of members induced by dp_j, which is equivalent to the sum of weighted supply that members sell to the cooperative for j ∈ X_c, plus

b) the marginal variation in welfare through the change of money income to the cooperative from the changes in total member patronage refunds, and total collective revenues, total collective costs due to change in the quantities supplied and demanded induced by dp_j, plus

c) the marginal variation in welfare arising from changes of cooperative's production induced by dp_j, plus

d) the marginal variation in welfare from the transformation of products in set X_c to final product arising from a change in the quantities supplied induced by dp_j must equal zero.
In condition 14, the price of jth input purchased by the member patrons from the cooperative is used as the instrument variable. The interpretation of this condition is similar to the interpretation of condition 13. The first line of the condition can be interpreted as the marginal variation in welfare from the change of indirect utility functions of members caused by a change in the jth price in set $Y_c$, which is equivalent to the sum of weighted demand that members buy from the cooperative for $j \in Y_c$. The second and third lines of the condition can be interpreted as the marginal variation in welfare through the change of total member patronage refunds and net saving from the change of total collective revenue and total collective cost from shifts in output supply, factor demand, and consumption good demand induced by a change in the jth price. The fourth line of the condition can be interpreted as the marginal variation in welfare from the production of the products in set $X_c$, set $Y_c$, and set $Z_c$ arising from changes in the quantities demanded or supplied which are induced by a change in the jth price in set $Y_c$. The fifth line of the condition represents the marginal variation in welfare that caused by changes in cooperative's use of member's output due to change of farm input demand which are generated by change in the price received from members.
If the cooperative offers a positive price for the jth product in set \( Y_c \), the following equality must be satisfied for a maximum:

\[
\frac{\partial L}{\partial p_j} = \sum_{i \in A} \frac{\partial W}{\partial v_n} \frac{\partial v_n}{\partial p_j} \\
+ \frac{\partial W}{\partial M_1} \left[ \tau \left( \sum_{i \in X_c} \frac{\partial M_1}{\partial TCR} \frac{\partial TCR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \in Y_c} \frac{\partial M_1}{\partial TCR} \frac{\partial TCR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right) \\
- \sum_{i \in X_c} \frac{\partial M_1}{\partial TCC} \frac{\partial TCC}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right) - \sum_{i \in Y_c} \frac{\partial M_1}{\partial TPR} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right] \\
+ \frac{\partial W}{\partial \Phi_1} \sum_{i \in E_c} \frac{\partial \Phi_1}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j} \\
+ \sum_{i \in X_c} \frac{\partial W}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} = 0
\]

This is equivalent to stating that, for a maximum, the sum of:

a) the marginal variation in welfare from the change of indirect utility functions of members induced by \( dp_j \), which is equivalent to the sum of weighted demand that members buy from the cooperative for \( j \epsilon Y_c \), plus

b) the marginal variation in welfare through the change of money income of the cooperative from the changes in total member patronage refunds and total collective revenues, and total collective costs due to change in the quantities supplied and demanded induced by \( dp_j \), plus
c) the marginal variation in welfare arising from changes of cooperative's production induced by dpj, plus
d) the marginal variation in welfare from the transformation of products in se Xc to final product arising from a change in the quantities supplied induced by dpj must equal zero.

In condition 15, the price of jth consumption good purchased from the cooperative by the member and nonmember patrons is used as the instrument variable. The first line of the condition can be interpreted as the marginal variation in welfare from the change of indirect utility function of members caused by a change in the jth price in set Zc, which is equivalent to the sum of weighted demand that members buy from the cooperative for j ∈ Zc. The second and third lines of the condition can be interpreted as the marginal variation in welfare through the change of total member patronage refunds and net savings from the change of total collective revenue and total collective cost from shifts in output supply, factor demand, and consumption good demand induced by a change in the jth price in set Zc. The fourth line of the condition can be interpreted as the marginal variation in welfare from production of the products in set Xc, set Yc, and set Zc arising from changes in the quantities demanded and supplied which are induced by a change in the jth price in set
The fifth line of the condition represents the marginal variation in welfare that caused by changes in cooperatives use of members' output due to changes of consumption good demands which are generated by change in the jth price paid to cooperative.

If the cooperative offers a positive price for the jth product in set \( Z_c \), the following equality must be satisfied for a maximum:

\[
\frac{\partial L}{\partial p_j} = \sum_{i \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_j} + \frac{\partial W}{\partial M^T} \left[ \tau \left( \sum_{i \in I_c} \frac{\partial M^T}{\partial TCR} \frac{\partial TCR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \in I_c} \frac{\partial M^T}{\partial TPR} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right) - \sum_{i \in I_c} \frac{\partial M^T}{\partial TCC} \frac{\partial TCC}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} \right] - \sum_{i \in I_c} \frac{\partial M^T}{\partial TPR} \frac{\partial TPR}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} + \frac{\partial W}{\partial \Phi^1} \sum_{i \in I_c} \frac{\partial \Phi^1}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} + \sum_{i \in I_c} \frac{\partial W}{\partial q_i^{TM}} \frac{\partial q_i^{TM}}{\partial p_j} = 0
\]

This is equivalent to stating that, for a maximum, the sum of:

a) the marginal variation in welfare from the change of indirect utility functions of members induced by \( dp_j \), which is equivalent to the sum of weighted demand that members buy from the cooperative for \( j \in Z_c \), plus
b) the marginal variation in welfare through the change of money income of the cooperative from the changes in total member patronage refunds and total collective revenues, and total collective costs due to change in the quantities supplied and demanded induced by \( dp_j \), plus

c) the marginal variation in welfare arising from changes of cooperative's production induced by \( dp_j \), plus

d) the marginal variation in welfare from the transformation of products in set \( X_c \) to final product arising from a change in the quantities supplied induced by \( dp_j \), must equal zero.

In condition 16, the term

\[ p_j + d_j \frac{dp_j}{dq_j} \]

is the marginal cost to the cooperative and the term

\[ \lambda_i \frac{\partial \Phi}{\partial q_j} \]

is the marginal variation in the welfare function due to change of the cooperative revenue through change in the amount of jth consumption good handled by the cooperative.

Therefore, if cooperative purchase jth consumption good in set \( W \), for a maximum, the marginal variation in the welfare due to change of cooperative's cost to purchase the consumption good induced by \( dq_j \), multiplied by \( \tau \), must equal the marginal
variation in the welfare due to change of cooperative's revenue induced by dq_j.

In condition 17, if cooperative produces a positive quantity of the jth product in set U, this condition must be satisfied as an equality. The term

\[ p_j + q_j \frac{\partial p_j}{\partial q_j} \]

is the marginal revenue to the cooperative and the term

\[ \lambda_i \frac{\partial \Phi}{\partial q_j} \]

is the marginal variation in the welfare due to change of the product cost through the change of cooperative's volume of production. Therefore, for a maximum, the marginal variation in the welfare due to change of cooperative's revenue from producing product j in set U induced by dq_j, multiplied by \( \tau \), must equal the marginal variation in the welfare due to change of cooperative's production cost induced by dq_j.

In condition 18, the first line of the condition can be interpreted as variation in welfare from the change of indirect utility functions of members caused by a variation in the quantity of the jth club good which cooperative provides to members. The club good does not enter members' direct utility functions but increases levels of outputs from each combination of inputs, and increases profits, hence increases
consumption expenditures. The second line of the condition can be interpreted as variation in welfare through the change of total member patronage refunds and net saving from the change of total collective revenue and total collective cost from shifts in output supply, factor demand, and consumption good demand induced by a variation in the jth club good which cooperative provided. The third line of the condition can be interpreted as the variation in welfare from the production of the products in set $X_c$ arising from changes in the quantities demanded or supplied which are induced by a variation in the jth club good. The fourth line of the condition represents the variation in welfare that is caused by changes in cooperative's use of member's output due to change of farm input demand which are generated by change in the jth club good. If the cooperative provides jth club good in set $G$, for a maximum, the sum of:

a) the marginal variation in welfare from the change of indirect utility functions of members induced by $dq_j$, plus

b) the marginal variation in welfare through the change of money income to the cooperative from the changes in total member patronage refunds and total collective revenues, and total collective costs due to change in the quantities supplied and demanded induced by $dq_j$, plus
c) the marginal variation in welfare from the production change of the products in set \( X_c \) arising from changes in the quantities supplied which are induced by \( dq_j \), plus
d) the marginal variation in welfare caused by changes in cooperative's use of members' outputs due to change of from input demanded induced by \( dq_j \) must equal zero.

In condition 19, the term

\[ \lambda_i \frac{\partial \Phi}{\partial q_{ij}} \]

is the variation in the welfare function of the cooperative from a change in the quantity of the \( i \)th factor of set \( X_c \) used to produce output \( j \) in the cooperative (\( dq_{ij} \)). \( \lambda_{ij} \) can be interpreted as the variation in the welfare function of the cooperative from a change in the amount of unprocessed product \( j \) used in the cooperative. Thus, condition 19 implies that, if \( q_{ij} \) is positive, the variation in the welfare function of the cooperative from \( dq_{ij} \) must be equal for all \( j \).

In condition 20, if the cooperative uses a positive quantity of the \( i \)th input in set \( V \) in the production of the \( j \)th product in set \( U, Y, G \), this condition is satisfied as an equality. The term

\[ p_j + q_j \frac{\partial p_j}{\partial q_j} \]
is the marginal input cost to the cooperative of using the ith variable input and the term

$$\lambda_i \frac{\partial \phi}{\partial q_j}$$

can be interpreted as the marginal variation in the welfare from the change in the quantity of input i used in the production of output j by the cooperative induced by dq_j. Therefore, for a maximum, the marginal variation in the welfare due to change of cooperative's cost from a change due to use jth input induced by dq_j multiplied by \( \tau \) must equal the marginal variation in the welfare due to change in amount of cooperative's jth product in set Y, U, G induced by dq_j.

The condition 21 can be interpreted as stating that cooperative's marginal value product of using ith fixed input in producing jth output equals the imputed value or shadow price of the ith fixed factor. This result implies that the cooperative's marginal value product of using ith fixed factor must be equal for all j. Condition 22 through 25 are just restatements of the constraints of the model. Condition 22 is constraint from the spending of net saving and 23 is the cooperative's production constraint. Condition 24 and 25 are the constraints from that all the unprocessed products in set
$X_c$ purchased from member patrons are transferred into final products and that all fixed factors of productions are fixed in short run.

The Impact of the Rate of Reserve Funds

In order to look at the impact of the rate of reserve funds, we can apply the envelope theorem. The envelope theorem concerns the rate of change of the optimal value of the objective function itself when a parameter changes.

$$\frac{dW^*}{d\tau} = \lambda_0 NS \geq 0$$

since $\frac{\partial W}{\partial \tau} = 0$

That is, $W$ is not function of $\tau$. Equality is satisfied when $NS$ equals zero. If $\lambda_0 > 0$, the result implies that a higher value of $\tau$ benefits the members. That is, lower $\alpha$ benefits members. It is possible that a higher value of $\alpha$ would provide larger future benefits to members at the expense of current benefits. But, in this study I deal with only current period's problem rather than dynamic multi-period's problem.
CHAPTER 4. SIMPLIFICATION OF THE MODEL

The optimality conditions to maximize the welfare function of the cooperative were presented in chapter 3. Further insight into those conditions is provided in this chapter by considering simplified models. First, we consider a single product consumer cooperative and then a single product marketing cooperative, after that we will consider single product supply cooperative.

Single Product Consumer Cooperative

This cooperative provides its patrons a single consumption good which cooperative produces. The cooperative does not market the products produced by the member patrons. And the cooperative does not supply its patrons with any inputs. In addition, the cooperative must purchase some of its inputs from sources outside the cooperative.

Given these assumptions, only Kuhn-Tucker condition 15, 16, 20 through 23 are relevant. Of those, the interpretations of all but 15 are similar to those of the general model in chapter 3. Condition 15 can be rewritten as:

$$
\sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_z} + \lambda_0 \left( \tau (q_z^T + p_z \frac{\partial q_z^T}{\partial p_z}) - r_z \frac{\partial q_z^M}{\partial p_z} \right) + \lambda_z \frac{\partial \Phi}{\partial q_z^T} \frac{\partial q_z^T}{\partial p_z} \leq 0
$$
The term
\[ \sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_z} \]
can be interpreted as the variation in welfare from the change of indirect utility functions of members caused by the change of the price of the consumption good.

If we assume each member patron's utility function has the same weight and marginal utility of income for each member is same, then
\[ \sum_{n \in A} \frac{\partial W}{\partial V_n} \frac{\partial V_n}{\partial p_z} = - \sum_{n \in A} \lambda_n q_{zn} = - \lambda q^M = - \delta q^M \]
where \( \delta = \lambda \)

If \( \delta = 1 \), then the variation in welfare from the changes in utility of members induced by \( dp_z \) equals the sum of members consumption of the good purchased from the cooperative.

The term
\[ q^T_z + p_z \frac{\partial q^T_z}{\partial p_z} \]
can be rewritten as:
\[ (p_z + q_z \frac{\partial p_z}{\partial q_z^T}) \frac{\partial q^T_z}{\partial p_z} \]
where
\[ p_z + q_z^T \frac{\partial p_z}{\partial q_z^T} \]
is the change of total collective revenue to the cooperative arising from the change of quantity demanded which is induced by the $dp_z$. Thus, the term

$$\lambda_0 \left( \tau (q_z^T + p_z \frac{\partial q_z^T}{\partial p_z}) - \tau^*_z \frac{\partial q_z^{TN}}{\partial p_z} \right)$$

can be interpreted as the variation in welfare through the change of net saving and total member patronage refunds from the change of total collective revenue to the cooperative from shifts in the consumption good demand induced by $dp_z$.

The term

$$\lambda_1 \frac{\partial \Phi}{\partial q^T_z} \frac{\partial q_z^T}{\partial p_z}$$

can be interpreted as the variation in welfare from production of product $Z$ arising from changes in the quantities demanded, which are induced by $dp_z$.

Thus, if the cooperative offers a positive price of the consumption good, for a maximum, the sum of:

a) the variation in welfare from the changes in indirect utility functions of members' induced by $dp_z$, plus

b) the variation in welfare through the change of money income of the cooperative from change in total member patronage refunds and total collective revenue due to change in the quantity demanded induced by $dp_z$, plus
the variation in welfare arising from changes of cooperative's production induced by \( dp_c \), must equal zero.

**Single Product Marketing Cooperative**

In this model, the cooperative markets one product produced by the member patrons. This product is used by the cooperative in the production of several outputs, each of which is sold outside the cooperative. The cooperative does not supply its patrons with any inputs or consumption goods; all of these must be purchased from sources outside the cooperative. In addition, the cooperative must purchase some of its inputs from sources outside the cooperative.

Given these assumptions, only Kuhn–Tucker condition 13, 17, 18, 20, and 21 through 25 are relevant. Of these the interpretations of all but 13 are similar to those of the general model in chapter 3. Condition 13 can be rewritten as:

\[
\frac{\partial L}{\partial p_x} = \sum_{n \in A} \frac{\partial w}{\partial v_n} \frac{\partial v_n}{\partial p_x} + \lambda_0 \left( \tau (-q_x^T - p_x \frac{\partial q_x^T}{\partial p_x}) - r^* \frac{\partial q_x^T}{\partial p_x} \right)
\]

\[+ \lambda_1 \frac{\partial \Phi}{\partial q_x^T} \frac{\partial q_x^T}{\partial p_x} + \lambda_2 \frac{\partial q_x^T}{\partial p_x} \leq 0\]

where \( X \) represents the product marketed by the cooperative.

In lagrange function (2.1), let
\[
\sum_{i \in \mathcal{X}} p_i q_i - \sum_{i \in \mathcal{X}} p_i q_i - F = M \quad \text{then}, \quad \frac{\partial U}{\partial M} > 0.
\]

Therefore, maximizing \( M \) (a member's profit) is identical to maximizing the member's utility. When each member patron's utility function has the same weight, and each member patron's utility function is identical, maximizing the welfare function is identical to maximizing sum of individual member's profit functions. Then, this simplified model is similar to Royer's.

If we ignore the dividends on stocks, the net savings' restriction can be rewritten by replacing \( \Sigma p r_n \) in \( \Sigma \pi_n \) by \( r NS \).

Then,

\[
\frac{\partial L}{\partial p_x} = (q_x^T + p_x \frac{\partial q_x^T}{\partial p_x}) - \sum_{i \in \mathcal{X}} p_i \frac{\partial q_i}{\partial p_x} + \tau (-q_x^T - p_x \frac{\partial q_x^T}{\partial p_x})
\]

\[
+ \lambda_1 \frac{\partial \Phi}{\partial q_x^T} \frac{\partial q_x^T}{\partial p_x} + \lambda_2 \frac{\partial q_x^T}{\partial p_x} \leq 0
\]

If we rewrite this condition as:

\[
(p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}) \frac{\partial q_x^T}{\partial p_x} - \sum_{i \in \mathcal{X}} p_i \frac{\partial q_i}{\partial p_x} - \tau (p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}) \frac{\partial q_x^T}{\partial p_x}
\]

\[
+ \lambda_1 \frac{\partial \Phi}{\partial q_x^T} \frac{\partial q_x^T}{\partial p_x} + \lambda_2 \frac{\partial q_x^T}{\partial p_x} \leq 0
\]

The term

\[
p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}
\]

represents the marginal revenue of the member patrons from
product X. Thus,

\[

teal\left( p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}\right) \frac{\partial q_x^T}{\partial p_x}
\]

represents the increase in total revenue of the member patrons arising from the output shift induced by the variation in the price which the cooperative offers for product X. The term

\[
\sum_{i \in \mathcal{I}} p_i \frac{\partial q_i}{\partial p_x}
\]

represents the increase in the total cost of the member patrons due to the shifts in factor use which are induced by \(dp_x\). The term

\[
p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}
\]

also represents the marginal factor cost to the cooperative. Therefore, the term

\[
- \tau \left( p_x + q_x^T \frac{\partial p_x}{\partial q_x^T}\right) \frac{\partial q_x^T}{\partial p_x}
\]

can be interpreted as the marginal variation in the profits of the member patrons due to change of net savings due to change of cooperative's factor cost from a change in the quantity of X used in production by the cooperative, induced by \(dp_x\). The term

\[
\lambda_1 \frac{\partial \Phi}{\partial q_x^T}
\]
is interpreted as the marginal variation in the profits of the member patrons arising from a change in the quantity of $x$ used in production by the cooperative. This is equivalent to the marginal revenue product to the cooperative from product X multiplied by $\tau$. Thus, the term

$$\lambda_1 \frac{\partial \Phi}{\partial q_x^T} \frac{\partial q_x^T}{\partial p_x}$$

can be interpreted as equivalent to the increase in the total revenue of the cooperative from use of product X arising from the change in the quantities supplied which are induced by $dp_x$, multiplied by $\tau$. The term

$$\lambda_2 \frac{\partial q_x^T}{\partial p_x}$$

is interpreted as the marginal variation in total revenue of the member patrons caused by changes in cooperatives' use of members' output due to changes of farm output which are generated by change in price paid to producers. That is, it is the marginal variation in total revenue of the member patrons from the transformation of products to final products arising from a change in the quantities supplied which are induced by $dp_x$.

Thus, for a maximum, the sum of:

a) the increase in the total revenue of the member patrons from the output shift induced by $dp_x$, plus
b) the increase in the total revenue of the cooperative from use of product X arising from changes in the quantity supplied which are induced by $dp_x$, multiplied by $\tau$, must equal the sum of:

c) the increase in the total cost of the member patrons due to the shifts in factor use which are induced by $dp_x$, plus

d) the increase in the total cost of the cooperative from product X arising from the changes in the quantities supplied which are induced by $dp_x$, multiplied by $\tau$, plus

e) the marginal variation in total revenue of the member patrons from the transformation of members' products to final products arising from a change in the quantities supplied which are induced by $dp_x$.

If the rate of allocation to reserve funds is zero, that is, $\tau = 1$, the marginal revenue to the member patrons and the marginal factor cost to the cooperative cancel. Then, for a maximum, the marginal increase in the cost of the member patrons from producing the product should equal its marginal revenue product to the cooperative.

The argument that the optimal quantity is that quantity at which the marginal cost of the product equals its marginal revenue product in the cooperative can be made in terms of producers' and consumers' surpluses. The producers' surplus can be defined as the difference between what the producers of
the product receive and what they would be willing to receive for a given quantity, a measure of the net benefit they derive from selling the product.

The proprietary firm, serving as a marketing agency, might be interested in maximizing the consumer's surplus alone. This would be accomplished by operating at the point at which the marginal revenue product curve intersects the marginal factor cost curve instead of where it intersects the marginal cost. However, the cooperative attempts to maximize the sum of the producers' and consumer's surpluses.

If the supply curve facing the cooperative is the marginal cost curve, the cooperative maximize the profits of its member patrons by setting a price equal to the marginal revenue product of the product. Unless the marginal revenue product is equal to the average revenue product, this price by itself will not result in all of the producer surplus being distributed to the member patrons. A price equal to the average revenue product would by itself result in all of the producer surplus being distributed to the member patrons, but it would not result in a maximum. The cooperative can set a price equal to the marginal revenue product and still distribute all of the producer surplus through the use of patronage refunds.
Single Product Supply Cooperative

In this model, the cooperative supplies member patrons with a single factor of production. This factor is used by the member patrons in their production of several outputs, each of which is sold outside the cooperative. The cooperative does not market any of the outputs for its patrons, and does not supply any consumption goods to its patrons. In addition, the cooperative must purchase from outside the cooperative inputs which it uses in the production of the factor.

Given this assumption, only Kuhn-Tucker condition 14, 20, 21 through 25 are relevant. Of these the interpretations of all but 14 are similar to those of the general model in chapter 3. Condition 14 can be rewritten:

\[
\frac{\partial L}{\partial p_y} = \sum_{n \in A} \frac{\partial \varphi_n}{\partial p_y} \frac{\partial v_n}{\partial p_y} + \lambda_0 \left\{ r \left( q_y + p_y \frac{\partial q_y^r}{\partial p_y} \right) - r^* \frac{\partial q_y^r}{\partial p_y} \right\}
\]

\[
+ \lambda \frac{\partial \phi}{\partial q_y^r} \frac{\partial q_y^r}{\partial p_y} \leq 0
\]

where \( y \) represents the factor supplied by the cooperative.

We can define a welfare function which gives the same weight to each member's utility function, and each member patron's utility function is identical. Then, maximizing the welfare function is identical to maximizing the sum of individual member's profit functions, which is similar to
single product marketing cooperative model. The net saving restriction can be rewritten by replacing $\Sigma r_{n}$ in $\Sigma p_{n}$ by $r_{NS}$. Thus, the condition is

$$
\sum_{i \in x_{n}} \frac{\partial q_{i}}{\partial p_{y}} - (p_{y} + a_{y} \frac{\partial p_{y}}{\partial q_{y}}) \frac{\partial q_{y}}{\partial p_{y}}
+ \tau (p_{y} + a_{y} \frac{\partial p_{y}}{\partial q_{y}}) \frac{\partial q_{y}}{\partial p_{y}} + \lambda_{1} \frac{\partial \phi}{\partial q_{y}} \frac{\partial q_{y}}{\partial p_{y}} \leq 0
$$

The term

$$
\sum_{i \in x_{n}} p_{i} \frac{\partial q_{i}}{\partial p_{y}}
$$

represents the increase in total revenue to the member patrons arising from the output shifts which are induced by the change in $p_{y}$. The term

$$
p_{y} + a_{y} \frac{\partial p_{y}}{\partial q_{y}}
$$

represents the marginal factor cost to the member patrons of factor $y$. Thus,

$$
(p_{y} + a_{y} \frac{\partial p_{y}}{\partial q_{y}}) \frac{\partial q_{y}}{\partial p_{y}}
$$

represents the increase in the total cost of the member patrons from shift in factor use induced by the change in the price the cooperative charges for $y$. The term

$$
p_{y} + a_{y} \frac{\partial p_{y}}{\partial q_{y}}
$$
also represents the marginal revenue to the cooperative. Therefore,

$$\tau (p_y + q_y \frac{\partial p_y}{\partial q_y}) \frac{\partial q_y}{\partial p_y}$$

can be interpreted as the marginal variation in the profits of the member patrons due to change of patronage refunds through the change of cooperative's net saving due to change of cooperative's total revenue from a change in the quantity of $Y$ demanded by the members, induced by $dp_y$.

The term

$$\lambda_1 \frac{\partial \Phi}{\partial q_y}$$

is interpreted as the marginal variation in the profits of the member patrons arising from a change in the quantity of $y$ produced by the cooperative. This is equivalent to the marginal cost to the cooperative of factor $y$ multiplied by $\tau$. Thus, the term

$$\lambda_1 \frac{\partial \Phi}{\partial q_y} \frac{\partial q_y}{\partial p_y}$$

can be interpreted as equivalent to the increase in the total cost to the cooperative of factor $y$ arising from changes in the quantities demanded which are induced by the change in $p_y$, multiplied by $\tau$. 
Thus, for a maximum, the sum of:

a) the increase in the total revenue of the member patrons from the output shifts induced by \( dp_y \), plus

b) the increase in the total revenue of the cooperative from factor \( y \) arising from changes in the quantities demanded which are induced by \( dp_y \), multiplied by \( r \),

must equal the sum of:

c) the increase in the total cost of the member patrons due to the shifts in factor use which are induced by \( dp_y \), plus

d) the increase in the total cost of the cooperative from factor \( y \) arising from the changes in the quantities demanded which are induced by \( dp_y \), multiplied by \( r \).

If the rate of allocation to reserve fund is zero, that is, \( r = 1 \), for a maximum, the cooperative's marginal cost of supplying the factor should equal the marginal increase in the revenue of the member patrons from using it.

As in the model of the single product marketing cooperative, the argument that the optimal quantity is that quantity at which the marginal value product of the factor equals its marginal cost can be made in terms of producer's and consumers' surpluses. With the exception that the producer is the cooperative and the consumers are the member patrons, the argument is identical to that used in the model of the single product marketing cooperative.
This result is identical to that found by Enke in his model of a consumer cooperative. He suggested that a consumer cooperative which took into account its consumers as owners as well as patrons should set the price it charges its members for a particular product equal to the marginal cost of producing it. A proprietary firm, serving as a supplier, would maximize its producer's surplus (profit) by operating at the point at which the marginal cost curve intersects the marginal revenue curve instead of the marginal value product curve or demand curve.

If the demand curve facing the cooperative is the marginal value product curve, the cooperative maximizes the profits of its member patrons by setting a price equal to the marginal cost of the factor. Unless the marginal cost is equal to the average cost, this price by itself results in all of the consumer surplus being distributed to the member patrons, but it would not result in a maximum. The cooperative can set a price equal to the marginal cost and still distribute all of the consumer surplus through the use of patronage refunds.
CHAPTER 5. PRODUCTION AND PRICING DECISION UNDER PRICE UNCERTAINTY

Introduction

Price uncertainty is created by uncertainties about future total supplies and demand for inputs and outputs which are determined by individual farm decisions. Mismatches of aggregate supply and demand similarly affect prices and create price uncertainty.

Marketing contracts are written agreements between cooperatives and their members stating the rights and duties of both parties regarding how products will be marketed or purchased for a specified period of time. Cooperative's forward contract provides the potential for improving macro coordination if a sufficient market share can be included and the problems of contingency contracting can be solved. Cooperatives have limited capacity to guarantee forward prices. A cooperative can not offer a guaranteed price because the price received by a member must depend on the performance of the cooperative.

However, they have potential to influence production plans through providing information to members, contracting with members, and to influence downstream market channel participants through collective bargaining, contracting, and promotion. A cooperative representing a large portion of
production could improve the match of aggregate production supply and demand, thus contributing to price stability and coordination.

An IOF (investor owned firm) offering fixed prices either on a spot or forward contract market may assume considerable risk due to uncertain future price movements. In a cooperative, members assume this risk and the price of the raw product is more like an internal transfer price than a transaction across a market. Thus, farmers have the opportunity of benefitting from future favorable price movement because of patronage refunds.

A long established tenet of cooperative theory is that a monopoly held by an open-membership marketing cooperative can not exploit consumers. The reason is the cooperative conveys the benefits of its high selling prices to its producers via some combination of high firm prices and patronage refunds. The producers respond to the high returns by increasing output, and consumer prices fall to where they would be with cooperatives.

There is a widespread notion throughout agriculture that the only way to achieve group bargaining market power is to control supply; and likewise, there is a mistaken notion that unless you do control supply you do not have bargaining power. Both notions are wrong, since market power comes in various shapes and sizes (Cooperative Bargaining pp.130 - 131). If
bargaining power is so successful as to raise farm prices sharply there will be a tendency to increase output. The effect to increase price would be self-defeating unless supply were limited. Can and will farmers impose strong production restraints upon themselves? The cooperative association is not a horizontal integration of its members' profit centers, and members act independently except as they have agreed to own firms jointly or have negotiated to act collectively. The cooperative association has the potential to affect horizontal coordination, as in the case of a bargaining cooperative, but market power requires a mechanism of collective action to control the purchase or production decisions of independent members. But members remain independent. It is important to distinguish farmer collective action through cooperatives to achieve improved macro coordination from collective action designed to extract monopoly advantage.

Farmer patrons receive many benefits when their cooperatives use contracts. Members like contacts especially because they assure a market for their products. Contracts also give producers an important advantage in securing credit, because loan officers recognize that a contract reduces risk and provides a market at reasonable prices. A cooperative could act as the farmers agent, thus reducing search costs and uncertainty. Cooperatives may reduce concentration in the markets of a farm commodity subsector by their entry into the
subsector. Even the threat of entry may change behavior of existing firms in concentrated markets. Further, enforced contracts help to solve the free-rider problem.

Cooperatives themselves can also improve their business by using contracts. First, they can reduce or eliminate cost associated with annual solicitation of patronage. Also, cooperatives can offer forward contracts with quantity, quality, and schedule guarantees; and they can acquire supplies and capital at more favorable rates and prices because of image of reliability and the stability created by producer contracts. Contracts enable cooperatives to plan for the best combination of resources to minimize costs.

Contracts are not, however, appropriate for all situations; and even when they are appropriate, cooperatives need to make efforts to minimize their potential weaknesses. For example, some potential members may be hesitant to make the type of binding commitment that the contract requires. One of the greatest drawbacks for farmers who sign contracts, particularly for those producing bulk commodities such as grain and oilseeds, is the loss of freedom to use other marketing alternatives. In addition, a cooperative's management may become careless because it may take the nearly guaranteed volume for granted. However, cooperatives must keep in mind that contracts are no substitute for efficiency.
No contract will hold members together indefinitely unless it produces expected benefits.

Risk and uncertainty influence the efficiency of resource use in agriculture and the decision making processes of farmers. Farmers may reduce price uncertainty by selecting enterprises with a low expected price variability. Forward contracting prior to harvest also introduces some flexibility in marketing. A cooperative can offer forward contracts. A cooperative's forward contract provides the potential for improving macro-coordination. However, cooperatives have limited capacity to guarantee forward prices. In order to continue the cooperative's business, cooperatives also need to consider the market price uncertainty, even though they may have better information about the demand than individual members.

The objective of this chapter is to provide the decision rules to the cooperative's managers under price uncertainty when they offer forward contracts to their members.

Development of the Model

The objectives of cooperatives are to increase members' profits and/or decrease the members' risk. For this study we assume that the cooperative purchases unprocessed products (set X) from member patrons under forward contract, and
supplies them with variable inputs (set \( Y \)) which they use in production. The cooperative determines the forward prices it will pay for purchased unprocessed products and the prices it charges for the sale of variable inputs. The cooperative also purchases variable inputs (set \( V \)) from sellers outside the cooperative and sells finished products (set \( U \)) to buyers outside the cooperative. We assume the prices of the variable inputs that the cooperative purchases from sellers outside are not random, but the prices of the finished products which the cooperative sells to buyers outside the cooperative are random. We assume there is no patronage refund for the variable inputs which members use in production and purchase from cooperatives, because input price is known when the member purchases the input from the cooperative. The cooperative can provide these variable inputs at the cooperative's cost.

**Individual Member Decision**

Assume each individual member farm firm wishes to maximize its utility, which is

\[
U = E(\bar{\pi}) - \frac{\lambda}{2} \text{var}(\bar{\pi})
\]

In this study, cooperatives offer forward contract to members. Cooperatives can acquire supplies at more favorable prices because of the image of reliability created by producer
contracts. By having contracts, cooperatives can reduce procurement, assembly, and delivery costs. They can also carry out processing and other activities at minimum costs because their volume is known, and have potential advantage in gaining reliable information compared with a government agency or private firm if they were able to generate a sense of community among their members. In general, cooperatives also have better position than members to get information about the market. Therefore, members can reduce the risk from private treaty, pooling risk, and get advantages of cooperative's information, economies of scale, and various of cooperative's services. Cooperatives can provide the opportunity for the adjustment of supply throughout a year. Cooperatives store the products when the prices are low and sell the products when the prices improve. Such adjustment can be made by warehouse storage. Therefore, cooperatives contribute to stabilize the prices, hence reduce the variances of the prices. Thus, as long as the variance of per unit patronage refund is less than the variance of price for a particular product faced by members, members will sell all their production to cooperatives under contract. In this study, we assume members sell their entire production to cooperatives under contract. Thus, a member's profit defined as:
\[ \bar{\pi} = \sum_{i \in X} p_i^c q_i - \sum_{i \in Y} p_i q_i - F + \sum_{i \in X} f_i q_i \]

\[ = \sum_{i \in X} (p_i^c + f_i) q_i - \sum_{i \in Y} p_i q_i - F \]

where \( X \) is the set of outputs produced by the members and sell to the cooperative, \( Y \) is the set of variable inputs purchased by the members from cooperative, \( F \) is the fixed cost, and \( r_i \) is the per unit patronage refund; its expected value is assumed equal to zero. This patronage refund is similar to an unbiased basis in a futures market price. For simplicity, covariances of patronage refunds between items \( i \) and \( j \) are assumed equal to zero. Therefore,

\[ E(\bar{\pi}) = \sum_{i \in X} p_i^c q_i - \sum_{i \in Y} p_i q_i - F \]

\[ \text{var}(\bar{\pi}) = \sum_{i \in X} q_i^2 \sigma_{r_i}^2 \]

Then, an individual member's utility function is:

\[ U = \sum_{i \in X} p_i^c q_i - \sum_{i \in Y} p_i q_i - F - \frac{\lambda}{2} \sum_{i \in X} q_i^2 \sigma_{r_i}^2 \]

where \( \lambda \) is the member's level of risk aversion. An individual member's decision is to maximize his utility function subject to his production function and fixed inputs. The individual member's implicit production function is:

\[ \phi(q_x, q_y, q_f) = 0 \]
Then, we can set up the lagrange function which will be represented as a maximization of a member's utility function subject to his constraints. This is given as:

\[ L = \sum_{i \in x} p_i^c q_i - \sum_{i \in y} p_i q_i - F - \frac{\lambda}{2} \sum_{i \in x} q_i^2 + \mu_1 (\phi(q_x, q_y, q_\ell)) + \sum_{i \in F} \mu_2 (\bar{q}_i - q_i) \]

In this individual's problem, the decision variables are:

\[ q_i, \quad i \in X, Y. \]

By solving the first and second-order conditions for maximizing this function, we can get the individual member's supply functions of farm outputs, demand functions of variable inputs, and indirect utility function, which are functions of \( p_i^c, p_i, \lambda, F \), and variances of patronage refunds.

The Kuhn-Tucker conditions for the individual's problem are as follows:

(26) for all \( j \in X \)

\[ \frac{\partial L}{\partial q_j} = p_j^c - \lambda q_j q_j^2 + \mu_1 \frac{\partial \phi}{\partial q_j} \leq 0 \]

\[ \frac{\partial L}{\partial q_j} q_j = 0 \]

\[ q_j \geq 0 \]
(27) for all $j \in Y$

\[
\frac{\partial L}{\partial q_j} = -p_j + \mu_1 \frac{\partial \phi}{\partial q_j} \leq 0
\]

\[
\frac{\partial L}{\partial q_j} q_j = 0
\]

$q_j \geq 0$

(28) for all $j \in F$

\[
\frac{\partial L}{\partial q_j} = \mu_1 \frac{\partial \phi}{\partial q_j} - \mu_{2j} \leq 0
\]

\[
\frac{\partial L}{\partial q_j} q_j = 0
\]

$q_j \geq 0$

(29) for $\mu_1$

\[
\frac{\partial L}{\partial \mu_1} = \phi(q_x, q_y, q_z) = 0
\]

(30) for $\mu_{2j}, j \in F$

\[
\frac{\partial L}{\partial \mu_{2j}} = \overline{q}_j - q_j \geq 0
\]

\[
\frac{\partial L}{\partial \mu_{2j}} \mu_{2j} = 0
\]

$\mu_{2j} \geq 0$
Before interpreting the Kuhn-Tucker conditions, it is useful to interpret the lagrange multipliers. The lagrange multiplier, $\mu_i$, can be interpreted as the imputed value or shadow price of $\phi$ (Royer, 37). If $i \in X$,

$$-\mu_i \frac{\partial \phi}{\partial q_i}$$

can be interpreted as the member's marginal imputed cost of producing the ith product. On the other hand, if $i \in Y$,

$$\mu_i \frac{\partial \phi}{\partial q_i}$$

can be interpreted as the marginal value product of the ith input to the member. The lagrange multiplier $\mu_{2i}$ can be interpreted as the imputed value or shadow price of the ith fixed factor.

Then, condition 26 can be rewritten as:

$$p_j^c - \lambda q_j \sigma_{ij} \leq -\mu_i \frac{\partial \phi}{\partial q_j}$$

The term on the left-hand side of the inequality is the price the member patron receives from the sale of product $j$ in set $X$ to the cooperative minus risk premium. The term on the right-hand side is equal to the marginal cost of producing product $j$. Therefore, if the $j$th product is produced, for maximum utility, it should be produced up to the point at which the
marginal cost of producing it is equal to the price adjusted with risk premium.

Condition 27 corresponds to the use of the jth variable input of production, purchased from the cooperative. Condition 27 can be rewritten as:

\[ P_j \geq \mu_1 \frac{\partial \phi}{q_j} \]

The term on the left-hand side of the inequality is the price the member patron pays the cooperative for input j in set Y. The term on the right-hand side is equal to the marginal value product of input j. Therefore, if the jth input is used, for maximum utility, it should be used up to the point at which its marginal value product is equal to its price.

Condition 28 corresponds to the use of the fixed factor of production. Condition 29 is simply a restatement of the firm's production function. Condition 30 corresponds to the fixed factor constraints.

Output supply functions and input demand functions for the member patron can be derived from the conditions. Output supply functions and input demand functions are function of parameters, the forward prices of farm products, the prices of inputs, the level of risk aversion, and variances of patronage refunds. These functions can be represented as:

\[ q_i = q_i(p_x, p_y, \lambda, \sigma_r^2) \quad i \in X, Y \]
where $p_x^c$ is a vector of the forward prices of farm products in set $X$, $p_y$ is a vector of the prices of the variable inputs in set $Y$, $\lambda$ is the level of risk aversion, $\sigma_r^2$ is a vector of variances of patronage refunds.

**Cooperative's Decision**

The cooperative's decision makers need to decide the prices of inputs, forward prices of farm product, the quantity of output to sell to outside buyers, and the quantity of inputs to buy from outside sellers. The objectives of the cooperatives are to give benefits to their members, not just maximize their profits. According to the individual member's utility function he tries to increase his profit but also to reduce his risk. Individual member's risk is measured by variance of profit which depends on variance of patronage refund. The variances of patronage refunds depend on the variance of cooperative's net saving. Therefore, the cooperative tries to increase sum of members' expected profits and to reduce the variance of net saving. Then, we can set the cooperative's welfare function as:

$$W = \sum_{n \in N} \bar{\pi}_n - \frac{\lambda_c}{2} \text{var}(N\bar{S})$$

where $\bar{\pi}_n$ is member $n$'s expected profit, $\lambda_c$ is cooperative's
level of risk aversion, and \( \text{var}(NS) \) is the variance of cooperatives net saving.

The cooperative's decision maker tries to maximize the cooperative's welfare function subject to zero expected net saving, cooperative's production function and fixed input constraints. The cooperative's net saving is:

\[
NS = \sum_{i \in U} \tilde{p}_i q_i - \sum_{i \in V} p_i q_i - \sum_{i \notin X} \tilde{p}_i^C q_i^T + \sum_{i \in V} p_i q_i^T - FCC
\]

where \( U \) is the set of the outputs produced by cooperative and sold to outside buyers, \( V \) is the set of the variable inputs used by cooperative and purchased from outside sellers, \( FCC \) is cooperative's fixed input cost. In the net saving equation, the only random variables are prices of the outputs in set \( U \). For simplicity, covariances between product \( i \)'s patronage refund and product \( j \)'s patronage refund are assumed equal to zero. Cooperative's input prices are not random because input prices in set \( V \) are known when cooperative makes decisions about the prices of products in set \( X \) and \( Y \). Then,

\[
E(NS) = \sum_{i \in U} \tilde{p}_i q_i - \sum_{i \in V} p_i q_i - \sum_{i \notin X} \tilde{p}_i^C q_i^T + \sum_{i \in V} p_i q_i^T - FCC
\]

\[
\text{var}(NS) = \sum_{i \in U} q_i^2 \sigma_i^2
\]

where \( E(p) = \bar{p} \),

\[
\sigma_i^2 = \text{variance of price of product } i \text{ in set } U.
\]
The cooperative's production function is:

\[ \Phi (Q_u, Q_v, Q_x, Q_y, Q_{e'}, \sigma_{i}^{2}) = 0 \]

The reason \( \sigma_{i}^{2} \) is one of the elements of the production function is that cooperative uses its resources to reduce \( \sigma_{i}^{2} \). Then, we can set up the lagrange function as:

\[
L = \sum_{i \in x} p_i^c q_i^T - \sum_{i \in y} p_i q_i^T - F_T - \frac{\lambda}{2} \sum_{i \in u} q_i^T \sigma_i^2 \\
+ \mu_0 \left( \sum_{i \in u} p_i q_i - \sum_{i \in x} p_i q_i - \sum_{i \in x} p_i^c q_i^T + \sum_{i \in y} p_i q_i^T - FCC \right) \\
+ \mu_1 \Phi (Q_u, Q_v, Q_x, Q_y, Q_{e'}, \sigma_{i}^{2}) \\
+ \sum_{i \in x} \mu_{2i} (q_i^T - \sum_{i \in u, y} q_{ij}) \\
+ \sum_{i \in e} \mu_{3i} (\bar{q}_i - \sum_{j \in u, y} q_{ij})
\]

The decision variables for this cooperative's problem are:

- \( p_i^c \) \( i \in x \),
- \( p_i \) \( i \in y \),
- \( q_i \) \( i \in u \),
- \( \sigma_i^2 \) \( i \in u \),
- \( q_{ij} \) \( i \in v, j \in u, y \),
- \( q_{ij} \) \( i \in e, j \in u, y \).
This model is different from Royer's model. His model assumed certainty. This study deals with price uncertainty which cooperatives meet when they sell their products to outside buyers. For simplicity, we ignore the cooperative's consumption goods businesses and the rates of allocation to reserve funds are not considered.

The Kuhn-Tucker conditions for the cooperative's problem are as follows:

(31) for all $j \in X$

$$\frac{\partial L}{\partial p_j^c} = (1 - \mu_0) \left( q_j^T + \sum_{i \in x} p_i^c \frac{\partial q_i^T}{\partial p_j^c} \right) - (1 - \mu_0) \sum_{i \in r} p_i \frac{\partial q_i^T}{\partial p_j^c}$$

$$+ \sum_{i \in x, r} \mu_i \frac{\partial \Phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j^c}$$

$$+ \sum_{i \in x} \mu_{2i} \frac{\partial q_i^T}{\partial p_j^c} \leq 0$$

$$\frac{\partial L}{\partial p_j^c} p_j^c = 0$$

$$p_j^c \geq 0$$

(32) for all $j \in Y$

$$\frac{\partial L}{\partial p_j} = (1 - \mu_0) \sum_{i \in x} p_i^c \frac{\partial q_i^T}{\partial p_j} - (1 - \mu_0) \left( q_j^T + \sum_{i \in y} p_i \frac{\partial q_i^T}{\partial p_j} \right)$$

$$+ \sum_{i \in x, y} \mu_i \frac{\partial \Phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j}$$
\[ + \sum_{l \in \mathcal{E}} \mu_{2l} \frac{\partial q_i^T}{\partial p_j} \leq 0 \]

\[ \frac{\partial L}{\partial p_j} p_j = 0 \]

\[ p_j \geq 0 \]

(33) for all \( j \in U \)

\[ \frac{\partial L}{\partial q_j} = -\lambda_c q_j q_j^2 \]

\[ + \mu_0 \left( P_j + q_j \frac{\partial p_j}{\partial q_j} \right) \]

\[ + \mu_1 \frac{\partial \Phi}{\partial q_j} \leq 0 \]

\[ \frac{\partial L}{\partial q_j} q_j = 0 \]

\[ q_j \geq 0 \]

(34) for all \( j \in U \)

\[ \frac{\partial L}{\partial \sigma_{j2}^2} = \left( 1 - \mu_0 \right) \left( \sum_{l \in \mathcal{E}} \sum_{l \in \mathcal{E}} p_i \frac{\partial q_i^T}{\partial \sigma_{r_j}^2} - \sum_{l \in \mathcal{E}} \sum_{l \in \mathcal{E}} p_i \frac{\partial q_i^T}{\partial \sigma_{r_i}^2} \right) \]

\[ + \mu_1 \sum_{l \in \mathcal{E}} \sum_{l \in \mathcal{E}, \gamma} \frac{\partial \Phi}{\partial q_{l \gamma}} \frac{\partial q_{l \gamma}}{\partial \sigma_{r_i}^2} \frac{\partial \sigma_{r_i}^2}{\partial \sigma_{j2}^2} \]

\[ - \lambda_c q_j^2 + \mu_1 \frac{\partial \Phi}{\partial \sigma_{j2}^2} \leq 0 \]
\[ \frac{\partial L}{\partial \sigma_j} \sigma_j^2 = 0 \]
\[ \sigma_j^2 \geq 0 \]

(35) for all \( i \in V, j \in U, Y \)
\[ \frac{\partial L}{\partial q_{ij}} = \mu_0 \left( -p_i - q_i \frac{\partial \Phi}{\partial q_i} \right) + \mu_1 \frac{\partial \Phi}{\partial q_{ij}} \leq 0 \]
\[ \frac{\partial L}{\partial q_{ij}} q_{ij} = 0 \]
\[ q_{ij} \geq 0 \]

(36) for all \( i \in F_C, j \in U, Y \)
\[ \frac{\partial L}{\partial q_{ij}} = \mu_1 \frac{\partial \Phi}{\partial q_{ij}} - \mu_{31} \leq 0 \]
\[ \frac{\partial L}{\partial q_{ij}} q_{ij} = 0 \]
\[ q_{ij} \geq 0 \]

(37) for \( \mu_0 \)
\[ \frac{\partial L}{\partial \mu_0} = \sum_{i \in U} p_i q_i - \sum_{i \in Y} p_i q_i - \sum_{i \in X} p_i q_i^T + \sum_{i \in Y} p_i q_i^T - \text{FCC} = 0 \]

(38) for \( \mu_1 \)
\[ \frac{\partial L}{\partial \mu_1} = \Phi(q_X, q_Y, q_U, q_V, q_{F_C}, \sigma_j^2) = 0 \]
Before we interpret the Kuhn-Tucker conditions, we need to interpret the lagrange multipliers. Most of the lagrange multipliers in this case have the same meaning as in the cooperative's model under certainty. The lagrange multiplier \( \mu_0 \) can be interpreted as the variation in total private profits (TPP) due to autonomous change in cooperative's expected net savings, or

\[
\mu_0 = \frac{\partial W}{\partial (-FCC)} = \frac{\partial TPP}{\partial (-FCC)}
\]

where FCC is the constant term in the constraint of expected net savings. Thus, \( \mu_0 \) is equivalent to marginal variation in total private profits of the unit of cooperative's money income.
In condition 31, the forward price of jth output sold to the cooperative by members is used as the instrument variable. Expression 31 and 32 are deterministic, variances of refunds do not appear in them. The term

\[
q_j^T + \sum_{i\in X} p_i^c \frac{\partial q_i^T}{\partial p_j^c}
\]

is marginal variation in total private revenue (TPR) from the jth product and all other products in set X arising from output shifts which are induced by \(dp_j^c\). This effect can be represented by

\[
\sum_{i\in X} \frac{\partial TPR}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j^c}.
\]

This term is also marginal variation in total collective revenue to the cooperative induced by \(dp_j^c\), which reduces the net savings. Therefore, the term

\[
(1 - \mu_0) \left( q_j^T + \sum_{i\in X} p_i^c \frac{\partial q_i^T}{\partial p_j^c} \right)
\]

is marginal variation in total private revenue after adjusting for the effects on the cooperative's net saving.

Similarly, the term

\[
\sum_{i\in \tau} p_i \frac{\partial q_i^T}{\partial p_j^T}
\]

can be interpreted as marginal variation in total private cost (TPC) arising from shifts in factor use which are induced by
dp_j^c. This effect can be represented by

\[ \sum_{i \in Y} \frac{\partial TPC}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j^c} \]

This term is also marginal variation in total collective revenue to the cooperative induced by dp_j^c, which increases the cooperative's net saving. Therefore, the term

\[ (1 - \mu_0) \sum_{i \in Y} P_i \frac{\partial q_i^T}{\partial p_j^c} \]

is marginal variation in total private cost after adjusting for the effects on the cooperative's net saving. The term

\[ \sum_{i \in X, Y} \mu_1 \frac{\partial \Phi}{\partial q_i^T} \frac{\partial q_i^T}{\partial p_j^c} \]

is interpreted by Royer (p. 90) as the variation in the profits of the member patrons from the production of the products in set X, and/ or purchases of products in set Y arising from changes in the quantities demanded or supplied which are induced by dp_j^c.

The term

\[ \sum_{i \in X} \mu_2 \frac{\partial q_i^T}{\partial p_j^c} \]

represents the variation in the profits of the members from the changes in use of products in set X to final product arising from a change in the quantities supplied, which are induced by dp_j^c.
Therefore, if $p_j^c > 0$, this condition is equivalent to stating that, for a maximum, the sum of:

a) the variation in total private profits arising from input and output shifts, after adjusting for the effects on the cooperative's net saving, induced by $dp_j^c$, plus

b) the variation in the profits of the members arising from changes in the cooperative's production induced by $dp_j^c$, plus

c) the variation in the profits of the members from the transformation of products in set $X$ to final products arising from a change in the quantities supplied induced by $dp_j^c$, must equal zero.

Condition 32 can be interpreted the same as condition 31 except that condition 32 measures the influences of the variation of a input price.

In condition 33, the quantity of the jth product in set $U$ is used as the instrument variable. In contrast with 31 and 32, condition 33 contains measures of price variability. The term

$$- \lambda_c q_j \sigma_j^2$$

can be interpreted as the cooperative's risk premium. The term

$$\overline{P}_j + q_j \frac{\delta \overline{P}_j}{\delta q_j}$$
is change in expected net savings. Therefore,

$$\mu_0 \left( \bar{p}_j + q_j \frac{\partial \bar{p}_j}{\partial q_j} \right)$$

can be interpreted as the marginal variation in the expected profits of the members due to the change in the cooperative's expected net savings. The term

$$\mu_1 \frac{\partial \Phi}{\partial q_j}$$

can be interpreted as the marginal variation in the expected profits of the members arising from a change in the quantity of the output produced by the cooperative (d$q_j$, $j \in U$).

Thus, if the cooperative produces a positive quantity of the $j$th product in set $U$, the condition 33 is equivalent to stating that, for a maximum, the marginal variation in the expected profits of the members by the change in the expected net savings minus the cooperative's risk premium must equal the marginal variation in the profits of the members arising from a change in the quantity of the output produced by the cooperative.

In condition 34, the variance of the price of output $j$ which cooperative produces, is an instrument variable. It reflects the cooperative's use of resources in searching for, and providing to members, information on markets, in bargaining and contracting and in other activities that reduce
variances of prices of outputs. The term

$$(i - \mu_0) \left[ \sum_{i \in X} \sum_{i \in X} P_j \frac{\partial q_i^T}{\partial \sigma_{r_1}} - \sum_{i \in X} \sum_{i \in X} P_j \frac{\partial q_i^2}{\partial \sigma_{r_1}^2} \right] \frac{\partial \sigma_{r_1}^2}{\partial \sigma_j^2}$$

is marginal variation in the members expected profits due to changes of production in set X and changes of input demand through the changes of patronage refunds induced by $d\sigma_j^2$, after adjusting for the effects on the cooperative's net saving.

The term

$$\mu_1 \sum_{i \in X} \sum_{i \in X, Y} \frac{\partial \Phi}{\partial \sigma_i} \frac{\partial \sigma_i^2}{\partial \sigma_{r_1}^2} \frac{\partial \sigma_{r_1}^2}{\partial \sigma_j^2}$$

is the variation in the expected profits of the members from the changes of cooperative's production arising from changes in the quantities supplied and/or demanded due to changes of patronage refunds which is induced by $d\sigma_j^2$. The term $\lambda_0 \sigma_j^2$ indicates the amount of cooperative's risk premium. The term

$$\mu_1 \frac{\partial \Phi}{\partial \sigma_j^2}$$

can be interpreted as the marginal variation in the expected profits of the members arising from a change in cooperative production from a change in the variance of price of the output j.

Therefore, if the cooperative use its resources to reduce the variance of its output j's price, the condition 34 is
equivalent to stating that, for a maximum, the sum of:

a) the marginal variation in the profits of members after adjusting for the effects on the cooperative's net saving due to changes of output supplies and input demands through the changes of the variances of patronage refunds induced by $\delta\sigma_j^2$, plus

b) the marginal variation in the profits of the members from the changes of cooperative's production arising from changes in the quantities supplied in set X and/or demanded in set Y due to changes of the variances of patronage refunds induced by $\delta\sigma_j^2$, must equal to the sum of:

C) change of cooperative's risk premium, plus

d) the marginal variation in the expected profits of the members arising from a change in cooperative production due to change in the variance of price of the output $j$.

The interpretations of condition 35 and 36 are same as the interpretation in general model except that 35 and 36 use the profits of the members instead of cooperative's welfare. Condition 38 is simply a restatement of the cooperative's production, condition 37, 39, and 40 are the restatements of the constraints of the model.

Risk and uncertainty influence the efficiency of resource use in agriculture and the decision-making process of farmers
and cooperative's decision makers. However, the Kuhn-Tucker conditions for their problems are very complex. In addition, there is a great amount of information which is necessary to evaluate them. This suggests that the cooperative decision maker's task of maximizing the expected profits of the cooperative's member patrons is a difficult one. In fact, it is doubtful that a cooperative of any complexity will be able to fully attain the objective of maximizing the expected profits of its member patrons.

Nevertheless, the optimality conditions presented here should be of value to the cooperative which is attempting to maximize its member patrons' expected profits even if it is not entirely successful in doing so.
CHAPTER VI. SUMMARY AND CONCLUSIONS

Summary

Some of the problems associated with Korean agricultural cooperative were discussed, and the relevant literature was reviewed to see how well it provided solutions. The purposes of this study were to introduce the idea that a farm cooperative is a decision unit in Korea, and to develop a short-run model of the cooperative which could be used to analyze its problems.

In particular, an attempt was made to develop under certainty a normative-prescriptive model of a diversified multi-product marketing, supply inputs and consumption goods cooperative which served both member and nonmember patrons, and also develop a model of a multi-product marketing and supply cooperative under uncertainty.

Development of the model under certainty began with the construction of a sub-model of a typical multi-product member patron. The typical member patron was assumed to maximize its utility functions. The typical member patron purchased its consumption goods and some of its inputs from the cooperative and marketed some of its outputs through the cooperative. The production of the member patron was augmented by the provision of club goods by the cooperatives. A similar sub-model of a
typical nonmember patron was also developed. The primary differences between this sub-model and that of the typical member patron were that the nonmember patron purchased only some of its consumption goods from the cooperative and did not receive patronage refunds.

From the optimality conditions determined for these two sub-models, individual output supply, input demand, and consumption good demand functions were derived for the patrons. By horizontally summing these individual supply and demand functions across all member and across all nonmember patrons, aggregate functions were determined.

The cooperative decision-maker was assumed to maximize the welfare function whose elements were the individual members' indirect utility functions. The decision-maker determined the optimal prices for the products it marketed and supplied and optimal level of club goods it provided.

The optimality conditions for the general model of the cooperative were analyzed. Simplified models, including that of a single-product consumer cooperative, that of a single-product marketing cooperative, and that of a single-product supply cooperative were analyzed.

Finally, a model of the cooperative was developed to consider the effects of price uncertainty on the objective function. The cooperative decision-maker was assumed to maximize its welfare function which is to increase the sum of
individual members' expected profits, and decrease the member's risk. The optimality conditions for this model were analyzed.

Conclusions

The principal conclusion determined in this study is that the task of the cooperative decision-maker is a difficult one. The optimality conditions derived for the cooperative in this study are complex. Not only are the optimality conditions which were derived for the cooperative in this study complex, but there is a great amount of information which is necessary to evaluate them. It is doubtful that a cooperative will be able to fully attain the objective of maximizing the cooperative's welfare function.

However, it is likely that the cooperative decision-maker will have some idea of how his decisions will affect the members' benefits. Then, many of the results of this study should be useful. The optimality conditions presented here can be of value to the cooperative which is attempting to maximize the welfare function even if it is not entirely successful in doing so. The model developed in this study was not intended to be a positive one but a normative-prescriptive one which would provide rules of behavior by which
cooperatives might strive to optimize their members' benefits which are measured by the cooperative's welfare function.

Further Research

There is no general agreement on what the objective or objectives of the cooperative are or should be. It would be interesting to develop and analyze the model with different objective functions. Candidates might include maximization of sum of the members' money income, where cooperative's consumption business tries to minimize members' expenditures on a fixed set of consumption goods.

It would also be interesting to extend this model to enable us to handle the cooperative's financial problems. This would require developing a dynamic multi-period model, since this year's business affects next year's business through the reserve and transferred funds.

Finally, it would be interesting to try to develop a programming model of the cooperative and see how cooperative differ from proprietary corporations in the pricing of products. This would require a survey of cooperative and of proprietary corporations to see if the pricing mechanism differs between the two types of corporations.
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