Rational Expectations, Uncertainty and Exchange Rate Determination

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Abstract
The theory of exchange rate determination has evolved considerably in recent years. Starting from a supposition that exchange rates were determined by Balance of Trade equilibrium, and hence-reflected "real" factors, the resurgence of the Monetary Theory has caused a sharp change in perceptions as to how exchange rates are determined. The Monetary Approach to the Balance of Payments focused attention upon the role of currencies as assets and hence viewed the exchange rate as a relative price of two assets. Thus, those factors that, determine the demand for each currency - as well as the supply - are seen to explain exchange rates. Since, in the context of the Monetary Approach the demands for currencies are generally transactions demands, and since transaction demands for each currency are generally assumed proportional to nominal domestic output (with, perhaps, the rate of interest also affecting transactions demand), the general conclusion emerges that the exchange rate between two currencies will depend upon the ratio of domestic output levels, as well as the ratio of currency supplies.

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Rational Expectations, Uncertainty and Exchange Rate Determination

by

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Rational Expectations, Uncertainty and Exchange Rate Determination

The theory of exchange rate determination has evolved considerably in recent years. Starting from a supposition that exchange rates were determined by Balance of Trade equilibrium, and hence reflected "real" factors, the resurgence of the Monetary Theory has caused a sharp change in perceptions as to how exchange rates are determined. The Monetary Approach to the Balance of Payments focused attention upon the role of currencies as assets and hence viewed the exchange rate as a relative price of two assets. Thus, those factors that determine the demand for each currency— as well as the supply—are seen to explain exchange rates. Since, in the context of the Monetary Approach the demands for currencies are generally transactions demands, and since transaction demands for each currency are generally assumed proportional to nominal domestic output (with, perhaps, the rate of interest also affecting transactions demand), the general conclusion emerges that the exchange rate between two currencies will depend upon the ratio of domestic output levels, as well as the ratio of currency supplies.

However, the rational expectations approach to exchange rate determination takes this argument one step further. Under this approach, it is required that the (expected) rate of return on the two assets are equal. To the extent that currencies are held only for transaction purposes, that domestic currencies must be held for domestic transactions, and that the use of a currency in this context provides the holder of the currency with some benefit, then the rational expectations approach cannot yield any real insight into exchange rate determination without first specifying how the transactions demands are determined, and what benefits are conferred by using a particular currency for a particular transaction. However, if the actual holding of currencies exceeds that needed
for transactions purposes, or if any currency can be used for transactions purposes, then one may view the holding of a particular currency as embodying speculative purposes. As such, if the two currencies have equal value as transaction media (including no value at the margin), then the rational expectations approach does, indeed, yield insights into the process of exchange rate determination.

It is this latter approach, followed by Kareken and Wallace (1977), which leads to their somewhat surprising - and unsettling - conclusion that the exchange rate between currencies must be constant - but is indeterminate (also, see Helpman and Razin (1979)). Once the assumptions are recognized, the conclusion seems apparent - since the two currencies have the same (perhaps zero) value as transaction media, the period rate of return on them must be identical; hence, the exchange rate must be constant. The fact that it is also indeterminate (ex ante) reflects the self-fulfilling expectations nature of the equilibrium.

While the approach taken by Kareken and Wallace is a valuable contribution in itself, the conclusion that exchange rates are insensitive to real or nominal factors is unsettling, and seemingly inconsistent with observed phenomenon. In the context of their model, differential growth rates of money supplies - or output levels - will not affect the time path of the exchange rate (the level, of course, being indeterminate).

It is the purpose of this paper to demonstrate that the constant exchange rate results are singular ones that are not robust to the introduction of various forms of uncertainty into the model. In the context of the consumption-loan model used by Kareken-Wallace, and to be employed here, uncertainty may enter the model in several ways - uncertainty as to the beliefs of future agents, uncertainty concerning future asset supplies or real output levels, and
uncertainty concerning government behavior. Since, for simplicity, we shall work with a model in which all agents are identical, we shall focus on the latter two types of uncertainty.

Suppose, for example, that money supplies are random. If, as predicted by the Monetary Approach, exchange rates are proportional to money supplies, then the exchange rate in any future period is also random. Hence, rational expectations does not explicitly require that the exchange rate is constant; rather, it implies that the expected utility derived from holding each currency must be equal. While a constant (indeterminate) exchange rate still represents an equilibrium solution, we show in Section II that there is also a nonstationary rational expectations solution that conforms to the predictions of the Monetary Approach.

The above still leaves something of a quandary in explaining exchange rate determination since multiple solutions exist. Again, the essence of the Kareken-Wallace approach is that – since there is no explicit transactions demand for a particular currency – the assets may be viewed as perfect substitutes; nothing intrinsically identifies the dollar with U.S. output or wealth, the mark with German output or wealth, etc. However, while agents may tend to view the assets as perfect substitutes, there is scope for government policy. For example, the institution of capital controls – banning domestics from holding foreign currency and foreigners from domestic currencies – will eliminate the perceived perfect substitutability of these currencies. Furthermore, while there may be no current controls, it seems unlikely that any agent will attach a zero probability to the event capital controls may be instituted in some future period. Thus, in Sections III and IV we analyze the implications of assuming agents assign a positive probability to the event controls are imposed. We show that this approach yields a unique (generally nonstationary) exchange
rate, and that this exchange rate is determined in accord with the implications of the monetary approach. Furthermore, we show that as the (perceived) probability of controls tends to zero, the exchange rate remains determinate. Hence, the Kareken-Wallace results can be seen as singular ones associated with the assumption agents hold the belief with probability one that governments will never impose controls.

The plan of this paper is as follows. In Section I we develop our basic consumption-loan model of individual behavior, and use it to derive agents' demand for currencies under rational expectations. We show that - in a world of perfect certainty - the Kareken-Wallace results provide the only (set of) solutions. In Section II we introduce (exogenous) uncertainty concerning currency supplies and show that - in addition to the Kareken-Wallace solutions - there is also a solution in which exchange rates are proportional to asset supplies. We also show how changes in the expected rate of growth - or in the variability - of asset supplies affects the exchange rate. In Section III we introduce endogenous uncertainty - i.e., the belief by agents that governments may impose capital controls. Assuming future currency supplies and output levels are known with certainty, we show how the (unique) exchange rate is determined and how differential real growth rates or differences in the growth rates of money supplies affects the exchange rate. Finally, in Section IV we reintroduce uncertainty concerning asset supplies and real output levels, and show how the combination of endogenous and exogenous uncertainty affects exchange rate determination. We conclude with suggestions for future research.

I) The Basic Model

In order to explain exchange rate determination, it is necessary to first specify the macro asset demands; it is the purpose of this section to show how the asset and commodity demands are derived from the optimizing behavior of
agents under uncertainty. In subsequent sections, these macro relationships are used to characterize the equilibrium exchange rate(s) under alternative sources of uncertainty.

The model we employ is the standard Samuelson (1958) overlapping generational consumption loan model. There are two countries, the "home" country (U.S.), and the "foreign" country (Germany). Individuals in each country are assumed to live for two periods — working, and deriving income, in the first period of life, and consuming in each period. There are N people in each generation in each country; hence, at any time, t, (2N) people are alive in each country. Further, it is assumed that, within each country, all individuals of the same generation have the same income (received in the first period of life). Finally, we assume all individuals (of all generations) have identical preferences, and that all agents have rational expectations.

Each country is assumed to have its own currency; the supply of the "home" (U.S.) currency at time t, called dollars, is denoted by $D_t$, whereas that of the "foreign" currency, called marks, is denoted by $M_t$. These supplies may change over time; define $\lambda_t$, $\overline{\lambda}_t$:

1) $D_t = \lambda_t D_{t-1}$; $M_t = \overline{\lambda}_t M_{t-1}$

Thus, $(\lambda_t - 1)$, $(\overline{\lambda}_t - 1)$ are the period rates of growth of the two currencies.

It is further assumed that commodities are perishable, and that these two currencies are the only stores of value. Thus, $D_t - D_{t-1} = (\lambda_t - 1)D_{t-1}$ is viewed as the net transfer (or tax) at time t to U.S. residents; similarly, $(\overline{\lambda}_t - 1)M_{t-1}$ is the net transfer at t to German residents. For simplicity, it is assumed these transfers or taxes are given to (levied upon) the then working generation.1

We assume there is a single homogeneous commodity, produced in each country; aggregate U.S. (German) output at t is denoted by $Q_t$ ($\overline{Q}_t$). The
The dollar (mark) price of goods is denoted by $P_t$, and the dollar/mark exchange rate at time $t$ by $e_t$. Hence, commodity arbitrage implies:

2) $P_t = e_t P^*$

The income (in dollars) of a representative member of generation $t$ is given by:

3) $\frac{y_t}{N} = \frac{P_t Q_t + (\lambda - 1)D_{t-1}}{N}$; $\frac{\bar{y}_t}{N} = \frac{P_t \bar{Q}_t + e_t \lambda D_{t-1}}{N}$

where, throughout, the bar ($\bar{\phantom{a}}$) above variables refers to the foreign country.

As noted earlier, $((\lambda - 1)D_{t-1}/N)$ represents the net per capita transfer at $t$ to the young in the U.S.

Individual preferences are given by:

4) $U_t = \frac{(C^i_t)^{\alpha/2} \cdot (R^i_{t+1})^{\alpha/2}}{\alpha}$

where $C^i_t$ is consumption at $t$ by individual $i$ of generation $t$, and $R^i_{t+1}$ is consumption at $(t+1)$ by that same individual.

Governments do not fix exchange rates; hence, exchange rates are determined by supply and demand. If there are no capital controls, agents are free to hold either currency, and it is assumed that there is no explicit transactions demand for either currency. Assuming there are no capital controls, agents born at $t$ are assumed to determine current consumption ($C^i_t$), plus portfolio holdings ($D^i_t, M^i_t$) after current prices ($P_t, e_t$) are known, but before next period prices ($P_{t+1}, e_{t+1}$) are known. However, in making these decisions, agents are assumed to have rational expectations (i.e., the probability distributions they assign to $P_{t+1}, e_{t+1}$ are equal to the true distributions). Assuming they leave no bequests, their intertemporal budget constraint is:
5) \( P_t^i R_t^{i+1} \leq D_t^i + e_t^i M_{t+1}^i \)

6) \( D_t^i + e_t^i M_{t+1}^i \leq (y_t^i - P_t^i C_t^i) \)

where \((D_t^i, M_t^i)\) is the portfolio chosen at \(t\), \((y_t^i - P_t^i C_t^i)\) is nominal savings (in dollars) at \(t\), and \((D_t^i + e_t^i M_{t+1}^i)\) is the dollar value of the portfolio at \((t+1)\). Since nonsatiation and the Inada derivative conditions hold, (5) and (6) will be equalities. Hence:

7) \( R_t^i = \frac{(y_t^i - P_t^i C_t^i) + (e_t^i - e_t^i) M_{t+1}^i}{P_{t+1}} \)

Note that \((e_t^i - e_t^i) M_{t+1}^i\) represents the capital gain (in dollars) due to mark holdings. Barring short sales:

8) \( M_{t+1}^i \epsilon \left[ 0, \left(\frac{y_t^i - P_t^i C_t^i}{e_t^i}\right) \right] \)

Thus, at \(t\), expected utility for the young is given by:

9) \( \hat{U}_t = E[U_t] = \frac{1}{\alpha} \ln \left(\frac{C_t^i}{\alpha/2} \left(\frac{y_t^i - P_t^i C_t^i + (e_t^i - e_t^i) M_{t+1}^i}{P_{t+1}}\right)^{\alpha/2}\right), \quad \alpha < 1, \neq 0 \)

\(= E \left[ \ln C_t^i + \ln \left(\frac{y_t^i - P_t^i C_t^i + (e_t^i - e_t^i) M_{t+1}^i}{P_{t+1}}\right) \right] = 0. \)

The expectation in (9) runs over \((e_{t+1}, P_{t+1})\). Optimizing (9) yields:

10) \( \frac{\partial U_t}{\partial C_t^i} = \left(\frac{1}{\alpha/2} \right) E \left[ (R_{t+1}^i)^{\alpha/2} - \left(\frac{P_t^i}{P_{t+1}}\right) (C_t^i)^{\alpha/2-1} \right] = 0 \)

11) \( \frac{\partial U_t}{\partial M_t^i} = \left(\frac{1}{\alpha/2} \right) E \left[ (R_{t+1}^i)^{\alpha/2-1} \left(\frac{e_{t+1}^i - e_t^i}{P_{t+1}}\right) \right] > 0. \)

Note that (10), (11) hold for \(\alpha = 0\) as well. If (11) is positive at \(M_{t+1}^i = \left(\frac{y_t^i - P_t^i C_t^i}{e_t^i}\right)\), a corner solution results with \(D_t^i = 0\); similarly, if (11) is
negative at $M_t^i = 0$, then the optimal decision is $M_t^i = 0$; otherwise, an interior solution results.

Regardless of the portfolio decision, it is readily seen that the optimal consumption rule is given by:

$$12) \quad C_t^i = (\frac{y_t^i}{2P_t^i}).$$

The utility function employed implies individuals possess constant relative risk aversion of degree $(1-\alpha)$; hence, the optimal portfolio rule may be expressed in terms of the per cent of wealth held in each currency. Define:

$$13) \quad m_t^i \equiv \left( \frac{2e_t M_t^i}{y_t^i} \right); \quad (1-m_t^i) \equiv \frac{2D_t^i}{y_t^i}; \quad \phi_t^i \equiv \left( \frac{m_t^i}{1-m_t^i} \right),$$

where $\phi_t^i$ is the ratio of (current) wealth held in marks to dollars. Using (12) and (13), (11) may be rewritten as:

$$14) \quad \frac{\partial U}{\partial M_t^i} = \left( \frac{y_t^i}{2} \right)^{\alpha-1} (P_t^i)^{-\alpha/2} E \left[ \left( 1 + \left( \frac{e_{t+1}}{e_t} - 1 \right) m_t^i \right)^{\alpha/2-1} \left( e_{t+1} - e_t \right) p_t^{\alpha/2} \right]$$

$$\quad = \left( \frac{y_t^i}{2} \right)^{\alpha-1} (P_t^i)^{-\alpha/2} (1-m_t^i)^{\alpha/2-1} e_t E \left[ \left( 1 + \left( \frac{e_{t+1}}{e_t} \phi_t^i \right) \right)^{\alpha/2-1} \left( \frac{e_{t+1}}{e_t} - 1 \right) p_t^{\alpha/2} \right]$$

where, for an interior solution, (14) holds with equality. Note that (12) and (14) hold for residents of either country, assuming no capital controls, since all agents have identical preferences and since there is no explicit transactions demand for either currency.

If, at time $t$, capital controls are imposed, then the new generation (of each country) is prohibited from acquiring foreign currency for use in the next period. Thus, they are forced to hold the domestic currency as the only store of value. However, the retired of the previous generation are allowed to dispose of their foreign currency holdings for goods or domestic currency (and
ultimately goods). Consequently, even if capital controls are imposed, commodity arbitrage will hold, and an equilibrium exchange rate is determined. From the previous analysis, it is clear that the consumption rule of the young is unaltered; however, their portfolio decisions are, of course, constrained. In particular, for U.S. residents, under capital controls, $m_t^c = 0$, $1 - m_t^c = 1$; whereas for German residents, $m_t^c = 1$, $1 - m_t^c = 0$ (the $c$ superscript stands for controls). We now turn to consider the macro equilibrium.

A) Macro Equilibrium - No Capital Controls

Since all agents have identical preferences, and since the retired spend all of their wealth, aggregate commodity demand, $(C_t^D + C_t^D)$ using (3) and (12), is given by:

$\text{15) } P_t (C_t^D + C_t^D) = (D_{t-1} + e_t M_{t-1}) + \frac{N_t}{2} (y_t + \bar{y}_t)$

$= [P_t (Q_t + \bar{Q}_t) + D_{t-1} (1 + \lambda_t) + e_t M_{t-1} (1 + \bar{\lambda}_t)]/2,$

where, of course, aggregate supply is $(Q_t + \bar{Q}_t)$. Since all agents of both countries allocate their wealth in the same fashion, asset market equilibrium is given by:

$\text{16) } M_t^D = m_t \left[ \frac{N_t}{2} (y_t + e_t \bar{y}_t) \right] = M_t$

Using (1), (3), (13), we have, for commodity and asset market equilibrium:

$\text{17) } P_t (Q_t + \bar{Q}_t) = D_{t-1} (1 + \lambda_t) + e_t M_{t-1} (1 + \bar{\lambda}_t) = D_t \left[ \frac{(1 + \lambda_t)}{\lambda_t} \right] + \phi_t \left( \frac{(1 + \bar{\lambda}_t)}{\bar{\lambda}_t} \right)$

$\text{18) } e_t M_t = m_t \left[ D_t + e_t M_t \right]; \quad e_t = \left( \frac{m_t}{1 - m_t} \right) \left( \frac{D_t}{M_t} \right) = \phi_t \left( \frac{D_t}{M_t} \right).$

Leading the time subscript forward one period gives the equilibrium values of $(P_{t+1}, e_{t+1})$, or - given information available at $t$, gives the probability distributions of $(P_{t+1}, e_{t+1})$. In choosing $\phi_t$, agents use the true probability distributions for $(P_{t+1}, e_{t+1})$. 
B) Macro Equilibrium - Capital Controls

If capital controls occur, then asset market equilibrium entails:

19) \( \left( \frac{N^y_t}{2} \right) = D_t; \quad P^c_t Q_t = D_t \left( 1 + \lambda_t \right) \)

20) \( \left( \frac{N^y_t}{2} \right) = e_t M_t; \quad \overline{P^c_t}_t = M_{t-1} \left( 1 + \lambda_t \right); \quad P^c_t = e_t \overline{P^c}_t \)

Hence:

21) \( e^c_t = \left( \frac{Q_t}{Q_t} \right) \left( \frac{D_{t-1}}{M_{t-1}} \right) \left( \frac{1 + \lambda_t}{1 + \lambda_t} \right) \)

where, as earlier, the superscript implies the presence of controls. Again, leading \( t \) forward by one period in (19) - (21) gives the equilibrium values at \( (t+1) \), and the distributions of \( (P^c_{t+1}, e^c_{t+1}) \) based on information available at \( t \). Under rational expectations, agents use these distributions (and the probability of controls) in choosing \( \phi_t \).

Using the analysis of this section, we now turn to address the issue of exchange rate determination under alternative forms of uncertainty.

II) Uncertain Asset Supplies and Exchange Rate Determination

Assume in this section that all agents assign a zero probability to the event capital controls may occur; the only sources of uncertainty arise from (potentially) random asset or output supplies. Individual portfolio decisions are given by (14); from (14) it is clear that if no uncertainty is present, then the only rational expectations solution is given by \( e_{t+1} = e_t \) for all \( t \). Hence, under certainty, we obtain the Kareken-Wallace solution that the exchange rate is constant over time - but indeterminate.

Even if asset or output supplies are random, it is clear that the Kareken-Wallace solution is still valid; if all agents believe the exchange rate will remain constant for all time, then it will remain constant. However, nothing
indicates how the level of the exchange rate is determined. In addition, the issue arises as to whether a nonconstant exchange rate solution exists under uncertainty.

From (17) and (18):

\[ e_{t+1}/e_t = \left( \frac{\phi_{t+1}}{\phi_t} \right) \left( \frac{\lambda_{t+1}}{\lambda_{t+1}} \right); \quad p_{t+1} = D_{t+1} \left[ \left( \frac{1+\lambda_{t+1}}{\lambda_{t+1}} \right) + \phi_{t+1} \left( \frac{1+\lambda_{t+1}}{\lambda_{t+1}} \right) \right] (q_{t+1} + q_{t+1})^{-1} \]

Substituting (22) into (14) yields:

\[ \frac{\delta \hat{U}}{\delta \hat{M}_t} = \left( \frac{y_t}{2} \right)^{\alpha-1} p_t^{-\alpha/2} (1-m_t)^{\alpha/2-1} e_t. \]

where the expectation runs over \((\lambda_{t+1}, \lambda_{t+1}, q_{t+1}, q_{t+1})\) (and \(\phi_{t+1}\) if it is nonconstant). Again, note that \(\phi_{t+1} = \phi_t \cdot \frac{\lambda_{t+1}}{\lambda_{t+1}}\), or \(e_{t+1} \equiv e_t\) is a solution to (23).

Assuming output disturbances are not correlated with money supply, then the output uncertainty will not affect the equilibrium value of \(\phi_t\). Furthermore, since the remaining terms do not involve \(M_t, D_t\), it is clear that - if an interior solution to (23) exists, it will be a stationary one (i.e., \(\phi_{t+1} = \phi_t\)) provided \(\lambda_{t+1}\) and \(\lambda_{t+1}\) are not serially correlated. Hence, the equilibrium value of \(\phi\) (if one exists) is determined by:

\[ J(\phi) \equiv E[(\lambda+\phi\lambda)^{\alpha/2-1}(\lambda-\bar{\lambda})(\lambda(1+\lambda)+\phi(1+\lambda))^{-\alpha/2}] = 0 \]

In (24), we drop the time subscripts, because the distributions of \((\lambda, \lambda)\) are stationary. An interior solution will exist if, and only if, \(J(0) > 0 > \lim_{\phi \to \infty} J(\phi)\).

Whether or not such a solution exists depends on the distributions of \((\lambda, \bar{\lambda})\); and if one exists, the (unique) equilibrium value of \(\phi\) will also depend on
these distributions. Since $e_t = \phi(D_t/M_t)$, it follows that the (unique) nonstationary solution for the exchange rate will depend not only on relative asset supplies, but also on the distributions of these supplies.

An interior solution may fail to exist if either the (expected) rate of growth of one currency is considerably larger than that of the other, or if the supply of one asset is much more variable than that of the other. For example, suppose $\lambda = 1$, $E(\lambda) = 1$, $E[(\lambda-1)^2] > 0$. From (24), evaluated at $\phi = 0$:

25) $J(0) = E[(\lambda-1)(1+\lambda)^{-\alpha/2}] > 0$ as $\alpha > 0$.

Thus, for $\alpha > 0$, $\lambda = 1$, $E(\lambda) = 1$, the only solution is $\phi = 0$ (which is part of the Kareken-Wallace set). Note that the variability in dollar supply eliminates demand for the mark. Similarly, if $\lambda = 1$, $E(\lambda) > 1$, then:

26) $\lim_{\phi \to \infty} J(\phi) = E\left[\frac{(\lambda-1)}{\lambda^2 \alpha^2/2}\right] = \left(\frac{1}{2^{\alpha/2}}\right)[1 - E(1/\lambda)]$

Clearly, if $E(1/\lambda) < 1$, then no interior solution exists, and $\phi = \infty$ is the only solution (again part of the Kareken-Wallace set). Thus, corner solutions may result from large differences in variability or expected rates of growth of asset supply.

On the other hand, if $(\lambda, \lambda)$ are identically, but independently, distributed, then the symmetry of (24) makes it clear $\phi = 1$, $e_t = (D_t/M_t)$ represents a rational expectations solution. In general, then, one expects that the equilibrium $\phi$ (and hence $e_t$) depends on the relative means and variances of the two distributions. While an explicit solution to (24) cannot be found, a reasonable approximation, for small disturbances and small deviations of means (from each other), can be obtained. Let:

27) $\lambda = 1 + \beta_1 + \beta_2 \Delta; \ E(\Delta) = E(\Delta\lambda) = E(\Delta^2) = 0; \ E(\Delta^2) = E(\Delta^2) = V^2$

$\overline{\lambda} = 1 + \beta_3 \Delta; \ \beta_3 - parameters$
Thus, \( E(\lambda) = 1 + \beta_1 \), and the variances of \( (\lambda, \overline{\lambda}) \) are given by \( (\beta_2^2 \nu^2, \beta_3^2 \nu^2) \) respectively. Using (27) in (24) define:

\[
28) \quad J^*(\phi, \beta) = E[(\overline{\lambda} + \phi \lambda)^{\alpha/2 - 1} (\lambda - \overline{\lambda}) (\lambda(1+\lambda) + \phi(1+\lambda))^{\alpha/2}]
\]

Given the distributions of \( \Delta, \overline{\Delta}, J^* \) depends on \( (\phi, \beta) \); an interior solution entails \( J^*(\phi, \beta) = 0 \).

For small \( \beta \), (28) may be approximated by a Taylor series expansion, retaining the first three terms:

\[
29) \quad J^*(\phi, \beta) = J^*(\phi, 0) + \frac{3}{2} \sum_{i=1}^{n} \frac{\partial J^*}{\partial \beta_i} (\phi, 0) \beta_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 J^*}{\partial \beta_i \partial \beta_j} (\phi, 0) \beta_i \beta_j
\]

where \( J^* \), and its respective partial derivatives, are evaluated at \( \beta = 0 \). From (27) and (28), \( J^*(\phi, 0) = 0 \). After performing the necessary calculations, one obtains:

\[
30) \quad J^*(\phi, \beta) = \frac{1}{2(\alpha/2+1)} \left[ 2 \beta_1 (1+\phi) + \nu^2 (\beta_2^2 (\beta_3^2 + 2) - \beta_2^2 (\alpha + 2 \phi)) \right]
\]

where, in (30), it has been assumed \( \beta_1^2 \) is small (relative to \( \beta_1^2, \nu^2 \beta_2^2, \text{or} \nu^2 \beta_3^2 \)).

Note that, for \( \beta_1 = 0, \beta_2 = \beta_3, \phi = 1 \), as found earlier. Similarly, if \( \beta_2 = \beta_3 = 0 \), \( \beta_1 > 0 \) no interior solution exists (\( \phi = \alpha \)). Also, if \( \beta_1 = 0, \beta_3 = 0, \beta_2 > 0 \), then \( \phi = 0 \) for \( \alpha > 0 \). Thus, while (30) may not be exact, its properties conform with those previously discussed. From (30):

\[
31) \quad \phi = 0 \text{ if } 2 \beta_1 + \nu^2 (2 \beta_3^2 - \frac{\alpha}{2} \beta_2^2) \leq 0
\]

\[
\phi = \infty \text{ if } 2 \beta_1 + \nu^2 (\frac{\alpha}{2} \beta_3^2 - 2 \beta_2^2) \geq 0
\]

\[
0 < \phi = \frac{2 \beta_1 + \nu^2 (2 \beta_3^2 - \frac{\alpha}{2} \beta_2^2)}{-2 \beta_1 + \nu^2 (2 \beta_2^2 - \frac{\alpha}{2} \beta_3^2)} \quad \text{otherwise.}
\]

Thus, a corner solution will occur if either \( \beta_1 \) is very different from zero, or else if \( \alpha > 0 \), and the differences in variances of money supply are large.

Assuming an interior solution exists, we obtain from (31):

\[
32) \quad \frac{\partial \phi}{\partial \beta_1} = \nu^2 (4 - \alpha) (\beta_2^2 + \beta_3^2) (G^{-1})^{-1} > 0; \quad G = \nu^2 (2 \beta_2^2 - \frac{\alpha}{2} \beta_3^2) - 2 \beta_1 > 0
\]
Consider first a change in $\beta_1$. An increase in the expected rate of growth of dollar supplies will cause current mark demand to rise; which in turn leads to a current depreciation of the dollar (although asset supplies have not changed yet), and an increase in the dollar prices of goods. This result is very similar to those found in which explicit transactions demands for currencies are assumed; in those cases, the anticipation of dollar supply increases in the future reduce dollar demands since, essentially, the cost of holding dollars has risen. This, in turn, leads to a current depreciation of the dollar and an increase in dollar commodity prices (see, for example, Fischer [1979]). It is important to note, however, that we do not postulate a transactions demand for dollars, nor do we have an explicit opportunity cost for holding dollars (and our model is stochastic, rather than deterministic). Nevertheless, our results replicate those obtained in which an explicit transactions demand is used. Of course, the increase in the expected rate of growth of dollars will lead to a decline in the future (expected) value of the dollar for two reasons—the likely increase in future dollar supplies, and the shift in asset demand away from the dollar.

Next, consider how a change in the variance of the rate of growth of dollar supply affects the current exchange rate. For $\beta_1 = 0$ (E($\lambda$) = E($\lambda$)), $\frac{\partial \phi}{\partial \beta_2} < 0$ as $\alpha < (-4)$; also, the larger is $\beta_1$ (ceteris paribus), the more likely it is $\frac{\partial \phi}{\partial \beta_2} < 0$. While it may seem paradoxical that an increase in the variability of dollar supplies will increase the demand for dollars (if agents are not too
risk averse), it really is simply an application of Jensen's inequality - and hence another example that price variability can benefit (some)-individuals. To see this, consider an agent with a given basket of currencies at \( t \), while the supply of dollars at \( (t+1) \) is random. An increase in dollar supply at \( (t+1) \) will cause dollar prices to rise, the dollar to depreciate against the mark, and the real income of the retired will fall; a decrease in dollar supplies will have the opposite effect (unless only marks are held). It is clear that the individual could (partially) hedge this risk by increasing his holdings of marks; however, it is also well-known that price variability can be beneficial, particularly if agents are not too risk averse. Thus, if agents are not "very" risk averse \( (\alpha > -4) \), the increased variability of real income associated with dollar holdings appears desirable and - rather than hedging - the individuals desire to magnify the risk by increasing current dollar holdings, consequently causing the dollar to appreciate. However, for very risk averse agents \( (\alpha > -4) \), the hedging motive dominates, and the increased variability of dollar supply causes a depreciation of the dollar.

This concludes our analysis of this section. We have shown that if currencies are in random supply, then - in addition to the Kareken-Wallace self-fulfilling expectations solution - there is an alternative solution whose properties are similar to those obtained from the Monetary Approach to the Balance of Payments, in which explicit transactions demands are postulated. However, we have seen only nominal - and not real - factors influence this equilibrium. Thus far, there is little to indicate which exchange rate solution is more robust; and, if the Kareken-Wallace results are accepted, there is nothing to indicate what the equilibrium exchange rate will be. It is the purpose of the next two sections to eliminate that impasse.
III) Government Controls and Exchange Rate Determination

In this section we wish to concentrate upon the role played by probabilistic capital controls; toward that end, we assume the agents know with certainty the future time path of asset and output supplies. In the next section we will reintroduce uncertainty concerning those variables.

While it is not the purpose of this paper to discuss optimal governmental foreign exchange policy, it is worth pointing out that there is some (nonpolitical) rationale for such controls. Assuming no exogenous uncertainty and no capital controls, the only equilibrium exchange rate will be constant over time. Suppose, for example, German output grows more rapidly than U.S. output. Given the constant exchange rate, Germany will be perpetually exporting commodities and importing paper money. Under these circumstances, the imposition of capital controls would prove beneficial to German residents. Similarly, if the supply of dollars increases more rapidly than that of marks, then - given the constant exchange rate - the U.S. will run a perpetual trade deficit, exporting dollars (and inflation), in return for commodities. Once again, the German government has an incentive to impose such controls. Thus, there would seem to be a strong rationale for considering the impact of such (probabilistic) controls on exchange rates.

Assume, then, that in any period, \( t \), there is a probability (\( \pi \)) that controls will be imposed, and hence (1-\( \pi \)) is the probability there will be no controls. For simplicity, we assume \( \pi \) is time-independent; i.e., the ex ante probability of controls in any period is stationary and independent of the history or current state of the economy. If controls occur at \( (t+1) \), the price level and exchange rate are given by \( (19) - (21) \); if no controls occur, then \( (17) - (18) \) determine the price level and exchange rate for that period (with \( t \) replaced by \( (t+1) \), of course).
The agent must determine his optimal portfolio rule, given full knowledge of the time path of assets and output supplies, and his (rational) expectations concerning the likelihood of controls next period and of the behavior of agents next period. If controls are currently in force, the agent has no decision to make; if not, his decision is determined by (14), where, as earlier, the expectation operator runs over \((e_{t+1}^c, p_{t+1}^c)\). Recognizing the only source of uncertainty is due to the possibility of controls, and using (14), the agent's portfolio decision is given by:

\[
(1-\pi) \left[ \left( \frac{e_{t+1}}{e_t} \right)^{\frac{\alpha}{2}} - 1 \right] \left( \frac{e_{t+1}}{e_t} - 1 \right) p_{t+1}^{\alpha/2} \right] + \pi \left[ \left( 1 + \frac{e_{t+1}^c}{e_t} \right)^{\frac{\alpha}{2}} - 1 \right] \left( \frac{e_{t+1}^c}{e_t} - 1 \right) p_{t+1}^{c,\alpha/2} = 0
\]

where \((e_{t+1}, p_{t+1})\) are \((t+1)\) prices in the absence of controls, and \((e_{t+1}^c, p_{t+1}^c)\) are \((t+1)\) prices with controls present. Substituting (17), (18), (19) and (21) in (35), and simplifying yields:

\[
(1-\pi) \left[ (1+\gamma_{t+1})^{\alpha/2} \cdot \left( \frac{\lambda_{t+1}}{\lambda_t} + \phi_{t+1} \lambda_{t+1} \right)^{\alpha/2} - 1 \right] \left( (1+\lambda_{t+1})^{\phi_{t+1} \lambda_{t+1}} - 1 \right) p_{t+1}^{\alpha/2} = 0
\]

where \(\gamma_{t+1} \equiv (\bar{Q}_{t+1}/Q_{t+1})\).

(36) is a first-order nonlinear difference equation in \(\phi_t\); clearly, the solution will depend on \(\pi\). For \(\pi = 0\), the (set of) solutions are given by:

\[
\phi_{t+1} = \left( \frac{\lambda_{t+1}}{\lambda_t} + \phi_t \lambda_{t+1} \right); \ \text{or} \ \phi_t = a \cdot \frac{M_t}{D_t}
\]

Hence, for \(\pi = 0\), the Kareken–Wallace constant - and indeterminate - exchange rate is the only solution set.
However, for $\pi > 0$ it is clear that the Kareken-Wallace solution will not work unless
\[
\frac{\gamma_{t+2}}{\gamma_{t+1}} = \left(\frac{1+\lambda_{t+2}}{1+\lambda_{t+1}}\right)\left(\frac{1+\lambda_{t+1}}{1+\lambda_{t+1}}\right) = \frac{\lambda_{t+1}}{\lambda_{t+1}}
\]
and even in this case the exchange rate will be determinate (but constant). Thus, the constancy and indeterminacy of the exchange rate is a singular result that hinges upon the assumption all agents attach a probability of zero to the possibility of capital controls.

To get an idea of the properties of a solution for $\pi \neq 0$, suppose the growth rates of outputs and money supplies are constant:

37) $\lambda_{t+1} = \lambda_t = \lambda, \quad \lambda_{t+1} = \lambda_t = \lambda$ for all $t$; $\gamma_{t+1} = \phi^{t+1} \cdot \gamma_0$

where $\theta$ is the relative growth rates of real output, $\gamma_0 = \bar{Q}_0/Q_0$. For no (differential) real growth ($\theta = 1$), (36) becomes:

38) $(1-\pi)\left[(1+\gamma_0)^{\alpha/2}(\lambda+\phi\lambda)^{\alpha/2-1}((1+\lambda)\lambda+\phi(1+\lambda))^{-\alpha/2}\phi(\lambda-\lambda) + \pi(1+\lambda)
+ \gamma_0(1+\lambda)^{\alpha/2-1}(1+\lambda)^{-\alpha/2}(1+\lambda)^{-\alpha/2}(\gamma_0(1+\lambda) - \phi(1+\lambda))\right] = 0$

In (38), the time subscript on $\phi$ is omitted, since it is clear - with constant $\gamma$ - that the solution is time independent.

For $\lambda = \bar{\lambda}, \phi = \gamma_0$, and hence $e_t = \phi \frac{D_t}{M_t} = \left(\frac{\bar{Q}_0}{Q_0}\right)\left(\frac{D_0}{M_0}\right)$ is the unique (and constant) exchange rate, which is independent of $\pi$. Note that - unlike our results of Section II - real factors ($\gamma$) must influence the equilibrium exchange rate. The reason for this is clear - with no specific transactions demand for either currency and with no probability of controls, there is nothing that associates the demand for a currency with the output of the country that issued that currency. However, if $\pi > 0$, then in some future period the demand for dollars will be associated with U.S. output, and hence the current exchange rate must reflect that.
For \(\lambda \neq \bar{\lambda}\), the solution for \(\phi\) is stationary, but clearly the exchange rate (controlled or uncontrolled) will change over time. Also, if \(\lambda \neq \bar{\lambda}\), then \(\phi\) will depend on \(\pi\), the probability of controls. Let:

\[
39) \quad \phi = a \cdot \gamma_0 \left( \frac{1+\lambda}{1+\bar{\lambda}} \right)
\]

Substituting (39) in (38), and simplifying, yields:

\[
40) \quad a \cdot (1-\pi) \left[ (1+\gamma_0) \frac{\alpha}{2} \left( 1+a\gamma_0 \frac{\lambda}{\bar{\lambda}} \right) \right]^{\alpha/2} \left( 1+a\gamma_0 \frac{\lambda}{1+\lambda} \right)^{\alpha/2-1} \left( 1+\gamma_0 \left( \frac{1+\lambda}{1+\bar{\lambda}} \right) \right)^{1-\frac{\alpha}{2}} = \pi(a-1) = 0
\]

Hence, \(a > 1\) as \(\lambda > \bar{\lambda}\).

Suppose, for example, \(\lambda > \bar{\lambda}\). Then \(e_{t+1} = \phi \cdot \frac{D_{t+1}}{M_{t+1}} > e_t = \phi \frac{D_t}{M_t}\). If anyone is to be induced to hold dollars at \(t\) (assuming no controls then), it must be true that \(e_{t+1}^c < e_t\). Since \(e_{t+1}^c = \gamma_0 \left( \frac{1+\lambda}{1+\bar{\lambda}} \right)^{D_t} M_t\), it follows that this condition holds. Hence:

\[
41) \quad \lambda > \bar{\lambda}: \quad e_{t+1} = a \left[ \gamma_0 \left( \frac{1+\lambda}{1+\bar{\lambda}} \right) \right]^{D_{t+1}} M_{t+1} > e_t = a \left[ \gamma_0 \left( \frac{1+\lambda}{1+\bar{\lambda}} \right) \right]^{D_t} M_t > e_{t+1}^c
\]

for \(\lambda < \bar{\lambda}\), \(a < 1\), and the direction of the inequalities is reversed.

Thus, the exchange rate will be nonstationary, though it does not (necessarily) monotonically increase over time (\(\lambda > \bar{\lambda}\)) because of controls; however, the uncontrolled exchange rate \((e_t)\) and the controlled exchange rate \((e_t^c)\) each increase at the rate \((\lambda / \bar{\lambda})\). As is clear from (40), \(a\) depends on \(\pi\). Further, it is readily seen that \(\frac{\partial a}{\partial \pi} < 0\) \((\lambda > \bar{\lambda})\), and \(\frac{\partial a}{\partial \pi} > 0\) \((\lambda < \bar{\lambda})\). As \(\pi \to 1\), \(a \to 1\); however, as \(\pi \to 0\), \(a \to \infty\) for \(\lambda > \bar{\lambda}\) \((a \to 0\), \(\lambda < \bar{\lambda}\)). In the absence of supply uncertainty, an interior solution requires some likelihood of controls. It is
interesting to note that - for \( \pi = 0 \), the exchange rate is constant (and indeterminate) - whereas for \( \pi \) close to zero, the value of the more rapidly growing currency will be small. Hence, it would seem that - from the point of view of the country whose currency grows less rapidly - a small, but positive probability of controls is desirable.

From (40), it can be seen that for \( \lambda \) near \( \bar{\lambda} \), \( \frac{\partial a}{\partial \lambda} > 0 \), \( \lambda > \bar{\lambda} \) (\( \frac{\partial a}{\partial \lambda} < 0 \), \( \lambda < \bar{\lambda} \)); thus, the growth rate of a currency affects the equilibrium exchange rate, not just by affecting asset supplies, but also through shifting demand. In particular, for \( \lambda \geq \bar{\lambda} \), an increase in the rate of growth of the dollar will cause agents to shift away from dollars, causing an immediate depreciation of the currency. This is analogous to the results of Section II in which the asset supplies were uncertain.

Returning to (36), it is clear that if outputs grow at different rates, the solution for \( \phi \) is nonstationary. Assume \( \lambda = \bar{\lambda} \), and \( \gamma_{t+1} = \theta_{t+1} \gamma_{0} \); then (36) becomes:

\[
(1-\pi)\gamma_{t+1} - \gamma_{t} = \left( \frac{(1-\pi)}{1+\gamma_{0} \theta_{t+1}} \right) ; \quad \gamma_{t} = \left( \frac{1}{1+\phi_{t}} \right) = (1-\phi_{t}) .
\]

For \( \pi = 0 \), \( \gamma_{t} \) is constant, but indeterminate; however, for any \( \pi > 0 \), a unique solution exists; it is given by:

\[
43) \quad \gamma_{t} = a_{0}(1-\pi)^{-t} + \pi \sum_{i=0}^{\infty} \frac{(1-\pi)^{i}}{(1+\gamma_{0} \theta_{t+1}+1)} , \quad t > 0 .
\]

Further, since \( \gamma_{t} \in (0, 1) \), and since the second term of (43) is bounded above by 1, it is clear \( a_{0} = 0 \), i.e., the boundary conditions imply the homogeneous solution vanishes. Hence:

\[
44) \quad \gamma_{t} = \pi \sum_{i=0}^{\infty} \left\{ \frac{(1-\pi)^{i}}{(1+\gamma_{0} \theta_{t+1}+1)} \right\} ; \quad \phi_{t} = \left( \frac{1-\gamma_{t}}{\gamma_{t}} \right) ; \quad e_{t} = \phi_{t} \left( \frac{D}{M} \right)
\]

Thus, \( \phi_{t+1} \geq \phi_{t} \) as \( \theta < 1 \); for \( \theta > 1 \) \( (\theta < 1) \), \( e_{t} \), \( e_{t} \) both increase (decrease) over time. Further, since;
and since \((1+\gamma_0\theta^{t+1})y_t < 1\) as \(\theta > 1\), it follows that:

\[
\begin{align*}
45) \quad e_{t+1}^c &= \gamma_0\theta^{t+1} \frac{D}{M} ; \quad e_t = \left(\frac{1-y_t}{y_t}\right) \frac{D}{M} ,
\end{align*}
\]

as required by an interior solution. As for the case of monetary growth,

\[
\frac{\partial \phi_t}{\partial \pi} < 0 \quad \text{as } \theta > 1; \quad \text{as } \pi \to 0, \phi_t \to \infty, \theta > 1 (\phi_t \to 0, \theta < 1). \quad \text{Thus, the limiting solution, as } \pi \to 0, \text{ is that one currency is valueless.}
\]

Finally, if we write (44) as:

\[
\begin{align*}
46) \quad e_{t+1} > e_t > e_{t+1}^c ; \quad \theta < 1: \quad e_{t+1} < e_t < e_{t+1}^c
\end{align*}
\]

it is clear, given \(\gamma_t\), that an increase in \(\theta\) decreases \(y_t\) \((\theta > 1)\); hence, for any given current state \((\gamma_t)\), the more rapid is the German growth rate relative to the U.S., the lower will be the current value of the dollar, as agents anticipate the more rapid depreciation of the currency.

Thus far, we have demonstrated two separate reasons why the constant, indeterminate exchange rate may not be a (the only) solution. If asset supplies are uncertain, the Kareken-Wallace solution still holds, but an alternative solution will (may) exist. Furthermore, if agents believe there is any positive probability that foreign exchange controls will be used at some future date, then the Kareken-Wallace solution fails; for this case, an alternative (unique) exchange rate exists that conforms to the predictions of the Monetary Approach concerning the impact of real and nominal factors on the exchange rate. In our final section we integrate these two separate sources of uncertainty.

IV) Controls and Asset Supply Uncertainty

The preceding sections have indicated how exchange rates may be determinate in the presence of alternative sources of uncertainty, and have shown that the
Kareken-Wallace stationary exchange rate may not be an equilibrium solution (or the only equilibrium solution). In our final section, we combine the two sources of uncertainty—asset supplies are random, and controls are deemed possible. For simplicity, we assume real output levels do not change over time.\(^{12}\)

As earlier, the agents' asset demands are determined by (14). If no controls are present next period, then that period's exchange rate and price level is given by (17, 18); if controls are present, they are given by (19) – (21). Hence, the equilibrium exchange rate at \(t\) — given expectations and assuming agents attach a probability \(\pi\) to controls next period — is given by (35), recognizing \(e_{t+1}, P_{t+1}\) are random (due to asset supply uncertainty):

\[
(1-\pi)E\left[\left(\frac{e_{t+1}}{e_t}\right)^{\phi_t} \left(\frac{P_{t+1}}{P_t}\right)^{-\alpha/2} \left(\frac{e_{t+1}}{e_t} - 1\right) \left(\frac{P_{t+1}}{P_t}\right)^{-\alpha/2}\right] = 0.
\]

Assuming \(Q_t/Q_t = \gamma_0\) for all \(t\), and using (17) – (21), we have:

\[
(1-\pi)[(\lambda_{t+1} + \phi_{t+1})^{-\alpha/2} - 1] + \piE[(1+\lambda_{t+1})^{-\alpha/2} - 1] = 0.
\]

where (48) is identical to (36) with the exception of uncertainty concerning \(\lambda_{t+1}, \lambda_{t+1}\). As seen earlier, if \(\pi = 0\), then multiple solutions exist, including the Kareken-Wallace solution \(\phi_t = \frac{M_t}{D_t}\), as well as one in which \(\phi\) is stationary. However, for any \(\pi > 0\), it is clear the Kareken-Wallace solution cannot hold. Assuming that \((\lambda_{t+1}, \lambda_{t+1})\) have stationary distributions, it is
apparent from (48) - due to the absence of a state variable - that the unique solution for \( \phi \) is stationary.

Define \( \phi^* \) as the stationary solution (for \( \phi \)) to (48) for \( \tau = 0 \); as seen in Section III, such a solution will exist if the distributions of \((\lambda, \bar{\lambda})\) are similar. Also, the value of \( \phi^* \) will depend upon the moments of the distributions, as discussed in Section III. It is clear, however, that \( \phi^* \) is independent of \( \gamma_0 \) - i.e., it is not influenced by real factors.

Further, define \( \phi^c \) as the solution to (48) for \( \tau = 1 \); i.e., \( \phi \) solves:

\[
49) \quad E\left[ \left( (1+\lambda) + \gamma_0 (1+\lambda) \right)^{\alpha/2-1} (\gamma_0 (1+\lambda) - \phi^c (1+\lambda)) (1+\lambda)^{-\alpha/2} (1+\bar{\lambda})^{-\alpha/2} \right] = 0
\]

In (49), the time subscripts are omitted due to the stationarity of \((\lambda, \bar{\lambda})\). Clearly, a solution for \( \phi^c \) always exists, and it is apparent that this solution depends on real factors \((\gamma)\), as well as the distributions of \((\lambda, \bar{\lambda})\). For example, if \((\lambda, \bar{\lambda})\) are identically - but independently distributed, then

\[
\text{Max} [\gamma_0, 1] \geq \phi^c \geq \text{Min} [\gamma_0, 1].
\]

Thus, for \( \gamma_0 > 1 \), \( \gamma_0 > \phi^c > 1 \), and clearly

\[
\frac{\partial \phi^c}{\partial \gamma_0} > 0.
\]

Since (48) is monotonic in \( \phi \), it follows that, for any \( \tau \in (0, 1) \), the solution \( \phi \in (\phi^*, \phi^c) \). Thus, for example, if \( \phi^c > \phi^* \), then as \( \tau \) decreases from 1 to 0, \( \phi \) decreases from \( \phi^c \) to \( \phi^* \). Note that, as \( \tau \to 0 \), a unique solution exists \((\phi^*)\), even though for \( \tau = 0 \), multiple equilibria occur.

Further, from the prior analysis, it is clear that the current exchange rate will reflect not only current factors (real output supplies and asset supplies), but also expectational factors. From our prior analysis, it is clear that an increase in the expected rate of growth of dollars will cause an immediate depreciation of the currency, as in the case when a specific transactions demand is posited. Finally, note that the presence of uncertainty concerning asset supplies and concerning exchange controls tends to cause the currency of
the less productive country to be overvalued. In the case where \((\lambda, \overline{\lambda})\) are identically distributed, then \(\phi^* = 1 < \phi^c < \gamma_0\), for \(\gamma > 1\). Hence, for any \(\pi \leq 1\), the current exchange rate (in the absence of controls) is \(e_t = \phi \frac{D_t}{M_t} < \gamma_0 \frac{D_t}{M_t}\), whereas were controls enacted at \(t\): \(e_t^c = \gamma_0 \left( \frac{D_{t-1} + D_t}{M_{t-1} + M_t} \right)\).

VI) Conclusion

Using an overlapping generations model, we have shown how exchange rates are determined in a world of uncertainty, assuming agents have rational expectations. We have demonstrated that when asset supplies are random then, in addition to the self-fulfilling constant exchange rate solution, an alternative solution in which the exchange rate reflects current asset supplies also exists. Moreover, we have shown that the properties of this latter solution are similar to those obtained using an explicit transactions demand for money in that changes in the expected growth rate of a currency will affect the current exchange rate.

However, neither the Kareken-Wallace solution, nor this alternative solution, reflect real factors in that agents of both countries maintain the same portfolio. In addition, nothing indicates how to choose between these alternative solutions. However, once agents believe there is any positive probability of controls, the indeterminacy of the exchange rate vanishes, and the unique exchange rate will be responsive to both monetary and real factors. Furthermore, we have shown that the self-fulfilling expectations solution is a singular one, in that, for any positive probability of controls, it does not represent a solution, and as the probability tends to zero a unique (nonstationary exchange rate) solution exists. We believe that it would be interesting to extend the model to consider alternative stores of value, such as real capital. Further, insights into the exchange rate determination process would be gained by considering an activist monetary policy and an endogenous probability of controls.
1 Little in the following analysis would be altered if these changes in money supply were viewed as being caused by government purchases or sales of commodities.

2 This is readily seen from the Kuhn Tucker conditions when optimizing

\[
E \left[ \left( \frac{C_t}{\alpha} \right)^{a/2} \left( \frac{e_t^{1} + M_t^{1} - e_t^{1}}{P_t+1} \right)^{a/2} \right] = \frac{\alpha}{\lambda} \left( y_t^{1} - P_t C_t^{1} - D_t^{1} - e_t^{1} M_t^{1} \right). \]

However, the treatment in the text is more succinct.

3 If \( a < 0 \), an interior solution will exist for this case.

4 By Jensen's inequality, \( E[1/\lambda] > \frac{1}{E(\lambda)} \); hence, \( E(\lambda) > 1 \) is a necessary, but not sufficient, condition for this result.

5 Computer simulations for binomial distributions indicate the approximation is quite good.

6 If the increase (decrease) in the expected rate of growth of dollar supplies is viewed as transitory, a depreciation (appreciation) of the dollar and rise (fall) in dollar prices of goods will still occur, but not by as much as if the change is viewed as a permanent one. Hence, the efficacy of a (an announced) policy change depends on both the short-run and long-run beliefs of agents.

7 For brevity, we restrict ourselves to considering \( \frac{\partial \phi}{\partial \beta_3^2} \); note, however, that the results for changes in \( \beta_3^2 \) are symmetric. Since \( \beta_1 = E[\lambda] - E[\bar{\lambda}] \), we have:

\[
\frac{\partial \phi}{\partial \beta_3^2} = -V^2(4-\alpha)[E(\lambda-\bar{\lambda})+V^2 \beta_3^2 (1+\frac{\alpha}{4})]G^2 \quad \text{and} \quad \frac{\partial \phi}{\partial \beta_1^2} = -V^2(4-\alpha)[E(\lambda-\bar{\lambda})+V^2 \beta_2^2 (1+\frac{\alpha}{4})]G^2
\]

8 For an interior solution, \( [\beta_1 + V^2 \beta_2^2 (1+\frac{\alpha}{4}) - \frac{\alpha}{4} V^2 (\beta_3^2 + \beta_2^2)] > 0 \); thus, for \( \alpha > 0 \) and an interior solution, \( \frac{\partial \phi}{\partial \beta_2^2} < 0 \) regardless of \( \beta_1 \).
For the simple closed economy, \( \frac{D_t (1+\lambda_{t+1})}{Q_{t+1}} \); hence, for the retired at \( (t+1) \): \( R_{t+1} = \frac{D_t}{P^c_{t+1}} = \left( \frac{Q_{t+1}}{1+\lambda_{t+1}} \right) \). Let \( E(\lambda_{t+1}) = 1 \); then: \( E[R_{t+1}] = \left( Q_{t+1} \right) \)

\( E\left[ \frac{1}{1+\lambda_{t+1}} \right] > \frac{Q_{t+1}}{2} \), and the money supply variability raises expected consumption.

For expected utility in this case, \( \frac{E[(R_{t+1})^{\alpha/2}]}{\alpha} = \frac{(Q_{t+1})^{\alpha/2}}{\alpha} E\left[ \left( \frac{1}{1+\lambda_{t+1}} \right)^{\alpha/2} \right] \) \( \alpha > -2 \). Thus, the money supply variability can be viewed as a benefit by a retired individual who is not too risk averse.

While in this simplistic model, capital controls in all periods would be desirable from the German perspective, we do not mean to imply we favor such a policy. There are strong reasons - in more sophisticated models - for not always imposing such controls; among them are included the role of exogenous uncertainty and the risk-sharing afforded by temporary trade deficits or surpluses. However, even under uncertainty, if growth rates are divergent, some probability of such controls would be desirable. For details, see Lapan and Enders [1980b].

It is clear that, in the presence of possible controls \( \pi > 0 \), an interior solution must exist since, if controls occur, each currency will have a non-zero, finite value.

For details on the case in which output supplies are random, see Lapan and Enders [1980b].
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