An Overlapping Generational Model With Capital Accumulation in a Small Open Economy

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Abstract
In this paper we develop a portfolio balance model which can be used to make an equitable comparison of alternative exchange rate regimes. Our point of departure from standard portfolio models is that we derive all asset demand functions from microeconomic behavior. As Helpman and Razin (1979) and Lapan and Enders (1980) have observed, all asset demand and expenditure functions used in a comparison of exchange rate systems must be consistent with individual optimizing behavior. This seemingly innocuous observation implies that many of the macroeconomic comparisons of fixed and flexible exchange rates are not legitimate. Inherent in any portfolio balance model is the implication that asset demand functions depend upon the joint distribution of asset yields. As long as there are different risks in different exchange regimes, portfolio allocation rules will change when the exchange rate regime changes.

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An Overlapping Generational Model With Capital Accumulation in a Small Open Economy

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In this paper we develop a portfolio balance model which can be used to make an equitable comparison of alternative exchange rate regimes. Our point of departure from standard portfolio models is that we derive all asset demand functions from microeconomic behavior. As Helpman and Razin (1979) and Lapan and Enders (1980) have observed, all asset demand and expenditure functions used in a comparison of exchange rate systems must be consistent with individual optimizing behavior. This seemingly innocuous observation implies that many of the macroeconomic comparisons of fixed and flexible exchange rates are not legitimate. Inherent in any portfolio balance model is the implication that asset demand functions depend upon the joint distribution of asset yields. As long as there are different risks in different exchange regimes, portfolio allocation rules will change when the exchange rate regime changes. In general, one cannot postulate a set of asset demand equations and then hold various income, interest rate, and wealth elasticities constant when the exchange regime changes. Instead, the approach used in this paper is to derive the behavioral rules of individuals from the postulate of expected utility maximization. The individual demand and supply functions are aggregated in order to characterize macroeconomic equilibrium under fixed and flexible exchange rates. Once the properties of macroeconomic equilibrium are obtained, it is possible to determine which exchange regime yields the highest level of such variables as the capital stock, aggregate consumption, and expected utility.

A second reason for using a micro-theoretic approach is that the choice between exchange regimes can be made using a single criterion: the optimal exchange regime is the one which maximizes the expected utility of individuals. Intermediate goals such as price or consumption stability may not be in accord with individual preferences. To take a simple example, Oi
(1961) demonstrates that price variability may be desirable if the degree of risk aversion is sufficiently low. Thus, any finding indicating that a flexible exchange rate insulates an economy from external price variability has little normative content unless individual attitudes towards risk are known.

In this paper we extend the work on micro-theoretic models of exchange rates in several directions. Helpman and Razin (1979) argue that a consistent comparison of exchange regimes requires that agents face the same objective constraints regardless of the exchange regime. On this basis, it is claimed that it is inappropriate to compare a system of fixed rates (in which central banks hold foreign currency) to a flexible rate system in which there is complete currency immobility. Helpman and Razin (1979) and Kareken and Wallace (1977) demonstrate that in the absence of a country-specific demand for money, the presence of complete currency mobility renders a constant rate of exchange even if the central banks refrain from intervention in the foreign exchange market. As long as currencies are perfect substitutes, there will not be any change in the relative price of currencies. However, in Lapan and Enders (1980), we have shown that it is perfectly legitimate to compare a system of fixed rates to a system of flexible rates with complete currency immobility: flexible rates (without currency mobility) were shown to yield a higher level of expected utility than fixed rates when agents were sufficiently risk averse.

In this paper we expand upon the work of Helpman and Razin (1979) and Kareken and Wallace (1981) by allowing for country-specific demands for money. We believe that there is little to be gained by comparing a system of fixed rates to a system of flexible rates in which the exchange rate is constant. A virtue of these papers not found in Lapan and Enders (1980) is
the presence of interest-bearing assets. In Lapan and Enders, currency was
the only store of value; a flexible rate without currency movements implied
that no real saving was possible. In the model developed below, we allow
domestics to hold domestic currency and real physical capital. By introduc-
ing real capital into the analysis, we allow individuals to save when the
exchange rate is flexible. More importantly, we are able to examine the
effects of alternative exchange regimes on the capital accumulation process
without having to specify ad hoc asset demand equations. Using the micro-
theoretic approach, the demand for capital under each exchange regime will
be consistent with individual maximizing behavior.

The outline of our paper is as follows. In Section II, we develop the
model and obtain the individual behavioral rules that agents obey.

Section III obtains the macroeconomic equilibrium results for a system of
flexible rates. Results are obtained for the cases of perfect foresight and
rational expectations. In Section IV, we consider macroeconomic equilibrium
for the case of fixed rates and perfect foresight. It is this section which
contains our principal results concerning the comparison of alternative
exchange regimes. Unfortunately, we are not able to provide an analytic
solution for the case of fixed rates and rational expectations. In
Section V, we present a Monte Carlo study and compare fixed and flexible
rates when agents have rational expectations. Conclusions are presented in
Section VI.

II. The Model of Individual Behavior

In order to derive the microeconomic demand functions, we consider an
overlapping generations model of the type developed by Samuelson (1958).
The economy consists of agents who live for two periods, and there are N
members of each generation. In the first period of life, each individual supplies one unit of labor to a firm which combines the labor input with capital to produce output. Production functions are homogeneous of degree one in capital and labor and contain a multiplicative productivity term. While this productivity term changes from period to period, the return to labor is the realized marginal product of labor. Once outputs are realized and income is received, workers decide how to divide their income between consumption and saving. Wealth can be held in two forms: domestic currency and physical capital. For simplicity, we consider a one commodity/putty-clay framework in which capital lasts only one period. The portfolio size and composition decisions are made in such a way that agents maximize expected utility. At this point, the agent enters the second period of life as a capitalist. The individual now hires labor and pays the workers their realized marginal product. Since capital lasts only one period and there is no bequest motive, the agent consumes an amount equal to his/her real money holdings plus the difference between output and labor payments. After consumption in the second period of life, the individual leaves the economic system, and a new generation is born.

We assume that all agents have identical preferences and are identical in all essential ways. Each person has the utility function:

\[ U_t = \frac{(C_t D_{t+1})^\beta}{2^\beta}; \beta \neq 0, \beta < 1 \]

\[ = \ln C_t + \ln D_{t+1}; \beta = 0 \]

where \( U_t \) = utility of an agent born at period \( t \); \( C_t \) = consumption in \( t \) of an agent born at \( t \); \( D_{t+1} \) = consumption in \( t+1 \) of an agent born at \( t \); and \( \beta \) is a measure of risk aversion. Note that the willingness of an agent to undertake a gamble is positively related to the magnitude of \( \beta \).
Since money and capital are the only assets, the consumption in \( (t+1) \) of an agent born at \( t \) \((D_{t+1})\) is:

\[
D_{t+1} = M_{t+1}/P_{t+1} + R_{t+1}K_{t+1}
\]

where \( M_{t+1} \) = nominal money balances that the agent brings into \( t+1 \), \( K_{t+1} \) = real capital that the agent brings into \( t+1 \), \( P_{t+1} \) = price level in \( t+1 \), and \( R_{t+1} \) = real payment to capital in \( t+1 \).

The amount of money and capital brought into period \( t+1 \) is the difference between labor income in \( t \) and consumption in \( t \). Since workers in \( t \) supply one unit of labor:

\[
P_t(W_t - C_t) = M_{t+1} + P_tK_{t+1}
\]

where \( P_t \) = price level in \( t \) and \( W_t \) = real wage in \( t \).

Given the constraints of equations 2) and 3), individuals born at time \( t \) choose \( C_t \), \( K_{t+1} \), and \( M_{t+1} \) to maximize their expected utility. We will consider two alternative expectational assumptions: perfect foresight and rational expectations.

Turning to the production side, we assume technology exhibits constant returns to scale; in particular, we let the output of firm \( i \) be given by:

\[
Q_t^i = \phi_t^i(K_t^i)^\alpha(L_t^i)^{1-\alpha}
\]

where \( \phi_t^i \), \( K_t^i \) and \( L_t^i \) represent the multiplicative productivity disturbance, capital and labor supply, respectively, for firm \( i \) at time \( t \). Furthermore, we assume that \( \phi_t^i \) is identical for all firms at time \( t \); thus, aggregate output at \( t \) can be represented by:\( 5/ \)

\[
Q_t = \phi_t(K_t)^\alpha(L_t)^{1-\alpha}
\]

where \( K_t \) and \( L_t \) represent the aggregate factor supplies at \( t \). When perfect foresight is present, we assume agents know the realizations of \( \phi_j \) for all
j; under rational expectations, agents at t are assumed to know the true distributions of \( \phi_j \), but not the realizations of \( \phi_j \) for \( j > t \).

Furthermore, it is assumed that the \( \phi_j \) are distributed independently, but identically, across time.

Assuming competition prevails in factor markets and that agents are paid their realized marginal products (thereby exhausting the total product):

1. \( W_t = (1-\alpha) \phi_t (k_t^\alpha) \); \( k_t = \frac{\bar{K}_t}{L_t} \)

2. \( R_t = \alpha \phi_t (k_t^\alpha)^{1-\alpha} \)

where \( \bar{k}_t \) is the aggregate capital-labor ratio at t. For simplicity, we normalize the labor supply of each agent to unity so that aggregate labor supply is N. Individual agents treat \( (k_t^\alpha) \) as exogenous to their actions; thus, agents at \( (t-1) \) know the true distribution (or value) of \( R_t \) but take it to be exogenous to individual actions. 5/ 

Returning to the individual maximization problem, the agent at t chooses \( (C_t, K_{t+1}^t, M_{t+1}^t) \) to maximize:

3. \( V = E \left\{ \left[ \frac{(C_t D_{t+1}^t)^{\beta}}{2 \beta} + \lambda (P_t (W_t - C_t) - M_{t+1} - P_t K_{t+1}^t) \right] | I_t \right\} \)

where \( D_{t+1}^t \) is given by equation 2, and \( I_t \) refers to the information set at t. Our notation in equation 7 stresses the fact that the information set may influence the optimal selection of \( C_t, M_{t+1} \), and/or \( K_{t+1}^t \). Below, we drop the conditional notation for convenience. While there will always be an interior solution for \( C_t \), and \( K_{t+1}^t \), there may be corner solutions for \( M_{t+1} \). As clearly shown by Cass, Okuno, and Zilcha (1980), models such as ours may yield solutions in which money is not held. The Kuhn-Tucker conditions yield:

4. \( V_{C_t} = \frac{C_t^{\beta-1}}{2} E(D_{t+1}^t) - \lambda P_t = 0 \)
b) \( V_{M_{t+1}} = \frac{C_t^b}{2} E(D_{t+1}^{b-1}/P_{t+1}) - \lambda \leq 0 \)

c) \( V_{M_{t+1}} - M_{t+1} = 0 \)

d) \( V_{K_{t+1}} = \frac{C_t^b}{2} E(D_{t+1}^{b-1}R_{t+1}) - \lambda P_t = 0 \)

e) \( V_\lambda = P_t (W_t - C_t) - M_{t+1} - P_t K_{t+1} = 0 \)

In order to determine \( C_t \), multiply 8d by \( K_{t+1} \) and add to 8c in order to obtain:

9) \( \frac{C_t^b}{2} E(D_{t+1}^{b-1}) = \lambda [M_{t+1} + P_t K_{t+1}] = \lambda [P_t (W_t - C_t)] \)

Using 8a and equation 9:

10) \( C_t = W_t/2 \)

Thus, consumption of the new generation will equal half of real wage income regardless of the information set. In order to determine capital and money holdings, however, it is necessary to specify how commodity and factor prices are determined. Before we turn to price determination, it will be helpful to obtain one further implication of the first order conditions. An interior solution requires that equations 8b and 8d both equal zero. Thus, an interior solution requires:

11) \( E[D_{t+1}^{b-1}(R_{t+1} - (P_t/P_{t+1}))] = 0 \)

As \( D_{t+1} > 0 \), in a world of perfect foresight in which both money and real capital are held, the real return on capital \( (R_{t+1}) \) must equal the real return on money holdings \( (P_t/P_{t+1}) \). No such presumption can be made in the case of rational expectations.
III. Market Clearing/Price Determination and Flexible Rates

If capital is not mobile internationally, a flexible exchange rate system is tantamount to a closed economic system. Market clearing requires that output be equal to consumption by members of the working generation \((NC_L)\), plus capital purchases \((NK_{t+1})\), plus consumption of the capitalists \((ND_t = NM_t/P_t + NR_tK_t)\):

\[
N\phi K_t^\alpha = \frac{NW_t}{2} + NK_{t+1} + \frac{NM_t}{P_t} + NR_tK_t.
\]

Competition and constant returns to scale means that \(NQ_t = NW_t\), so that equation 12 can be rewritten as:

\[
NW_t/2 = NM_t/P_t + NK_{t+1}.
\]

If the economy is closed or if the exchange rate is flexible, a passive monetary policy acts to fix the nominal value of the money supply at \(\overline{M}\) \((NM_t = \overline{M} \text{ for all } t)\). Given the magnitudes of the real variables in the economy, equation 13 determines the domestic price level. Ex ante, equation 13 determines the distribution of prices. It is important to note that equation 13 is in the information set of individuals; individuals use the information contained in this equation when making their portfolio composition decision. The simultaneous nature of the problem should be clear: given \(W_t\) and \(K_{t+1}\), equation 13 determines the current price level, but the distribution of prices, generated by equation 13, is used by agents in determining their holdings of capital.

In order to solve the system, define:

\[
s_t = \frac{K_{t+1}/(W_t/2) - 2K_{t+1}/W_t}{W_t/2}.
\]

so that \(s_t\) is the percentage of savings allocated to real capital. Using equations 13 and 14:

\[
\overline{M} = P_t(W_tN)(1-s_t)/2
\]
The real wage is given by equation 5. Substitute equation 5 into equation 15:

16) \( p_{t}^{-1} = (1 - \alpha) \phi_t K_t^\alpha N^{1-\alpha}(1-s_t)/2M \)

Update equation 16 by one period and form the intertemporal price ratio:

17) \( \frac{p_t}{p_{t+1}} = \frac{\phi_{t+1} K_{t+1}^\alpha (1-s_{t+1})}{\phi_t K_t^\alpha (1-s_t)} \)

If both capital and money are held, equations 11 and 17 indicate:

18) \[ D [ \phi_{t+1} (\alpha_{t+1} N^{1-\alpha_{t+1}} - \frac{\phi_{t+1} K_{t+1}^\alpha (1-s_{t+1})}{\phi_t K_t^\alpha (1-s_t)}) ] = 0 \]

As long as \( 0 < \alpha < 1/3 \), a solution to equation 18 is:

19) a) \( s_{t+1} = s_t = \frac{2\alpha}{1-\alpha} \equiv s \)

b) \( K_{t+1} = \alpha Q_t K_t^\alpha N^{1-\alpha} \)

It is to be noted that \( s \) (the proportion of saving allocated to real capital) is the same under perfect foresight and rational expectations.

Further, the economy saves at a rate which satisfies the golden rule. The golden rule savings rate is such that the economy consumes all of its labor income and uses the remaining output for capital formation. Given that \( s = 2\alpha/(1-\alpha) \), equation 12 can be rewritten as:

12') \( \phi_t K_t^\alpha = C_t + sW_t/2 + D_t \)

12'') \( Q_t - \alpha \phi_t K_t^\alpha = C_t + D_t \)

The right-hand side of equation (12'') is aggregate consumption at \( t \) while the left-hand side is output minus the marginal product of capital. Thus, labor's share of output equals aggregate consumption in each period of time. One way to interpret this result is to consider the portfolio choice
problem in a world of perfect foresight. The rate of return on money \( \frac{P_t}{P_{t+1}} \) is given by equation 17. For a constant value of \( s \):

20) \( \frac{P_t}{P_{t+1}} = \frac{Q_{t+1}}{Q_t} \)

Equating the rate of return on money to the marginal product of capital \( \frac{\alpha Q_{t+1}}{K_{t+1}} \):

21) \( K_{t+1} = \alpha Q_t \)

Equation 21 demonstrates that capital accumulation \( K_{t+1} \) equals current payments to capital: capital accumulation is such that all labor income is consumed, and the remaining output is used for capital accumulation. While our ultimate aim is to compare fixed and flexible rates, some further discussion of the flexible rate regime is in order. Table 1 presents the solutions for the endogenous variables; the solutions are for the case of rational expectations as for the case of perfect foresight.

Three interesting observations arise from the table:

1. Current period values of the endogenous variables depend upon current and previous values of the exogenous productivity disturbances. The endogenous variables are serially correlated even if the exogenous productivity disturbances are serially uncorrelated. This result differs from models which do not allow for the presence of real capital. In models in which domestic money is the only asset (see Fischer (1977)), no saving is possible because the current account and the balance of payments are identical. Without real capital, flexible rates present any asset accumulations. In our model, with a constant proportion of current income being allocated to capital, the capital stock will tend to fluctuate directly with current output. Part of any generation's "above average" output can be transferred into capital which can be used to produce
Table 1

Reduced Form Solutions for a Flexible Exchange Regime

\[ K_{t+1} = aQ_t = \alpha^{(1-\alpha)\phi \pi} i_{t-1} \]

\[ Q_{t+1} = \alpha^{(1-\alpha)\phi \pi} i_{t+1-1} \]

\[ C_t = D_t = (1-\alpha)\phi \pi (1-\alpha)\phi \pi = (1-\alpha) Q_t \]

\[ D_{t+1} = (1-\alpha)\phi \pi i_{t+1-1} \]

\[ R_{t+1} = aQ_{t+1}/K_{t+1} = \phi^{(1-\alpha)\phi \pi (\alpha-1)} i_{t-1} \]

\[ P_t = \frac{\alpha}{(1-\alpha)(1-3\alpha)\phi \pi} i_{t-1} \]

\[ \frac{P_t}{P_{t+1}} = R_{t+1} \]

\[ U_t = (C_t/D_{t+1})^{2\beta} = (1/2\beta) \left[ \frac{\alpha}{(1-\alpha)\phi \pi} \right]^{2\beta} \phi^{\omega} \phi a^{(\alpha+1)\beta} \]

where: \( \omega \) is the multiplication operator for which the index (i) runs from zero to infinity.

Note that the realized values of the endogenous variables are presented in the table. Variables, such as expected utility, are found by taking the mathematical expectation of the entry in the table.
output for the second period of life. In doing so, output, capital, prices, and the other endogenous variables will tend to be serially correlated.

2. In any period $t$, consumption of the new generation and the old generation will be equal ($C^*_t = D^*_t$). Workers consume one half of labor income ($C^*_t = W^*_t/2$) and purchase $sW^*_t/2$ units of capital. The remaining output ($Q^*_t - W^*_t/2 - sW^*_t/2$) is the amount consumed by the older generation:

$$D^*_t = Q^*_t - (1-s)W^*_t/2 = Q^*_t(1-a)/2 = C^*_t$$

Even in the presence of uncertainty, individuals of overlapping generations consume equal amounts so that the exogenous output and endogenous price uncertainties are spread across overlapping generations. Examination of the solution for $U^*_t$ indicates that the utility of an agent born at $t$ is affected by previous values of the domestic output disturbances. Domestic disturbances in $t-j$ ($j > 0$) affect agents born in $t$ in that the amount of capital brought into $t$ (hence the marginal product of labor in $t$) will be positively related to past disturbances. This is to be contrasted with the case of fixed rates for which it will be shown that past values of the foreign (as opposed to domestic) disturbances affect $U^*_t$.

3. While the economy follows a golden rule path, the marginal product of capital need not equal unity (even if there is perfect foresight). Examination of Table 1 indicates that the marginal product of capital can be greater than unity, and a monetary equilibrium will still exist. The relevant comparison is between the rates of return on money and capital. Suppose $R^*_{t+1} = p^*_t/p^*_{t+1} > 1$. 
Additional capital holdings at the expense of money would lower $R_{t+1}$ but increase $P_t/P_{t+1}$ as $P_t$ would increase and $P_{t+1}$ would fall.

IV. Market Clearing/Price Determination and Fixed Rates

The system is quite different if the exchange rate is fixed. Notably, for a small economy with a fixed exchange rate, commodity prices are determined on world markets as opposed to internal markets. In this section, we will demonstrate that the differences in the commodity price determination mechanism lead to differences in resource allocation and utility levels under alternative exchange rate systems. The argument is rather straightforward: with a flexible exchange rate, commodity prices are negatively correlated with domestic output disturbances. If agents in period $t$ anticipate that output in $t+1$ will be greater than output in $t$, they will also anticipate that prices in $t+1$ will be lower than prices in $t$. Such is not the case when the exchange rate is fixed. Domestic disturbances in $t+1$ will have no effect on commodity prices in $t+1$. Price and output uncertainties are, then, independent when the monetary authorities fix the rate of exchange. Agents will respond differently to the different risks inherent in the two systems, and attitudes toward the alternative regimes will depend upon attitudes towards price uncertainty. (Clearly, then, there are differing risks inherent in the alternative exchange rate regime so that it is inappropriate to postulate asset demand functions which are invariant to exchange rate policy.)

Equations 8a-8e remain the first-order conditions for expected utility maximization, and equation 10 yields the consumption rule. Normalizing the fixed rate of exchange to unity and ignoring transport costs, $P_t = P^*_t$ where
P* is the foreign price level in period t. If we assume an equilibrium in which both money and capital are held, equation 11 can be rewritten as:

11') \( E[D_t^B (R_t^{t+1} - (P^*_t/P_t^{t+1}))] = 0 \)

As was the case with flexible rates, to close the system it is necessary to specify how commodity prices are determined. We postulate that the structure of the large country replicates that of the small open economy in all respects save one: the distribution and/or realization of disturbances can differ across countries. The analysis in Section II can be used to solve for the endogenous variables in the large country. Letting \( \theta_t \) represent the productivity disturbance in the large country in period t, the intertemporal price ratio, identical in form to equation 20, is:

22) \( P_t^t / P_t^{t+1} = P_t^* / P_t^{t+1} = Q_t^* / Q_t^{t+1} = \theta_t + \frac{\theta_t}{t+1} \frac{(a-1)(a)^i}{v_{t+1}} \)

where (except for \( \theta_t \), we let starred variables represent the large country counterpart of small country variables.\(^7\) (Note that we assume \( a = a^* \), although this is not essential for our results.) With a fixed rate of exchange, individuals use equation 22 (as opposed to equation 13 or 20) in making their portfolio decisions. Having specified the price determination mechanism, it becomes possible to solve the system. In the case of rational expectations, however, it is not possible to obtain analytic solutions: we use Monte Carlo techniques to obtain rational expectation solutions in the next section of the paper. The remainder of this section considers the case of perfect foresight. We note at this point that the rational expectations and perfect foresight cases are qualitatively similar.

With perfect foresight, if both money and capital are to be held, equation 11') indicates that the marginal product of capital will be set equal to the intertemporal price ratio:
With a fixed rate of exchange, the capital stock in any period depends upon the domestic productivity term in that period and all present and previous foreign disturbances. This is to be contrasted to flexible rates where the capital stock depended upon the past and present values of the domestic productivity disturbances. With a fixed rate of exchange, the foreign productivity disturbances determine the intertemporal price ratio. The value of the domestic disturbance in \( t+1 \) is important in that it affects the marginal product of capital in \( t+1 \). As capital lasts only one period, other values of the domestic productivity terms will not affect the capital stock in \( t+1 \). Table 2 presents the solutions for the endogenous variables for the case of fixed exchange rates and perfect foresight. Notice that the domestic and foreign rates of return on capital are equal: in the large country, money and capital will be held if \( \frac{P^*}{P^*_t} = R^*_{t+1} \) while in the small country both assets will be held if \( R^* = \frac{P^*_t}{P^*_t} = \frac{P^*_t}{P^*_t} \). Thus, with a fixed exchange rate system, the marginal product of capital will be equalized across nations even though there are not any physical movements of capital across national boundaries. With a flexible rate system, there will be golden rule capital accumulation although the return to capital will differ across nations.

In comparing the values of the capital stock under the two exchange regimes, it is necessary to know the particular realizations of the foreign and domestic productivity terms. Even if such information were available, it would be of limited usefulness, since the amounts of capital held under
Table 2
Reduced Form Solutions for a Fixed Exchange Regime With Perfect Foresight

\[ K_{t+1} = \frac{1}{1-\alpha} \phi^{1-\alpha}_{t+1} \theta_{t+1}^{1-\alpha} \pi^{1-\alpha}_{t} a^{i}_{t} \]
\[ Q_{t+1} = \frac{1}{1-\alpha} \phi^{1-\alpha}_{t+1} \theta_{t+1}^{1-\alpha} \pi^{1-\alpha}_{t} a^{i+1}_{t} \]
\[ C_{t} = \left( \frac{1-\alpha}{2} \right) Q_{t} \neq D_{t} \]
\[ D_{t+1} = \left( \frac{p_{t}^{*}/p_{t+1}^{*}}{1-\alpha} \right) \left( \frac{1-\alpha}{2} \right) Q_{t} \]
\[ R_{t+1} = \frac{p_{t}^{*}/p_{t+1}^{*}}{1-\alpha} = R^{*}_{t+1} = \theta_{t+1}^{1-\alpha} \pi^{1-\alpha}_{t} (a-1)(a)^{i} \]
\[ U_{t} = \left( \frac{1}{2\beta} \right) \theta_{t+1}^{1-\alpha} \pi^{1-\alpha}_{t} (a-1)(a)^{i+1} \]
\[ U_{t} = (1/2\beta) \left[ \frac{1}{1-\alpha} \phi^{1-\alpha}_{t+1} \theta_{t+1}^{1-\alpha} \pi^{1-\alpha}_{t} (a-1)(a)^{i+1} \right]^{\beta} \]
each exchange regime would be fluctuating over time. Instead, it seems most
useful to compare the average or *ex ante* values of the capital stock under
the alternative exchange regimes. Specifically, we compare the expected
value of $K_{t+1}$ taken from Table 1 to the expected value of $K_{t+1}$ taken from
Table 2. In doing so, it is possible to determine whether, on average, the
value of the capital stock will be greater with a fixed rate or with a
flexible rate. We now demonstrate that, on average, the value of the
capital stock will be higher with a fixed rate if:

i) $\theta_t$ and $\phi_t$ are both identically and independently distributed (i.d.d.),
and

ii) $\theta_t$ and $\phi_t$ are uncorrelated.

From Tables 1 and 2:

\[ 25) \ E[K_{t+1}^{Fix} - K_{t+1}^{Flex}] = \alpha \frac{1}{1-\alpha} \left[ \frac{1}{1-\alpha} \theta_{t+1}^{1-\alpha} \alpha^t - \frac{\alpha}{\theta_{t+1}^{1-\alpha}} \right] \]

Since domestic and foreign productivity disturbances have the same
independent distributions, the right-hand side of equation 25) can be
written as:

\[ 26) \ \alpha \frac{1}{1-\alpha} \left[ \frac{1}{1-\alpha} \theta_{t+1}^{1-\alpha} \alpha^t \right] - \frac{1}{1-\alpha} \left[ \frac{\alpha}{\theta_{t+1}^{1-\alpha}} \right] = \frac{1}{1-\alpha} \left[ \frac{1}{1-\alpha} \theta_{t+1}^{1-\alpha} \alpha^t \right] - \frac{\alpha}{\theta_{t+1}^{1-\alpha}} \]

Using Jensen's inequality, $E\left[ \frac{1}{1-\alpha} \right] > 1$, so that the term in
expression 26) is positive. Thus, the expected value of the capital stock
will be greater with a fixed rate of exchange than with a flexible rate.
Furthermore, it can also be shown that aggregate consumption and the level of output can be expected to be greater with a fixed rate of exchange.

Thus, asset holders find capital to be a more attractive store of value with a fixed rate of exchange. By way of explanation, suppose that all domestic and foreign disturbances had realized values of unity for all periods of time up to and including period \( t \). From Tables 1 and 2, the capital stocks for \( t+1 \) under the two exchange regimes would be:

\[
27) \quad K^\text{Fix}_{t+1} = \alpha^{\frac{1-\alpha}{\phi_{t+1}/\theta_{t+1}}} \frac{1}{1-\alpha}; \quad K^\text{Fex}_{t+1} = \alpha^{\frac{1}{1-\alpha}}
\]

With a fixed rate, the capital stock for \( t+1 \) is positively related to \( \phi_{t+1}/\theta_{t+1} \). A high realization of \( \phi_{t+1} \) would produce a proportionate increase in the marginal product of capital while a low realization of \( \theta_{t+1} \) would produce a proportionate decrease in the foreign (and domestic) price level. With flexible rates, however, the product of \( P_{t+1} \) and \( Q_{t+1} \) is invariant to the output disturbance in \( t+1 \) (\( K^\text{Flex}_{t+1} \) does not depend upon \( \phi_{t+1} \) because the value of the marginal product of capital would be invariant to \( \phi_{t+1} \)). Since domestic prices and domestic productivity disturbance are independent with fixed rates, the expected value of the capital stock will be higher with a fixed rate than a flexible rate. Assuming the same distribution for \( \phi_{t+1} \) and \( \theta_{t+1} \), Jensen's inequality indicates

\[
E\left(\left(\frac{\phi_{t+1}}{\theta_{t+1}}\right)^{\frac{1-\alpha}{1-\alpha}}\right) > 1.
\]

In other words, the expected value of the value of the marginal product of capital would be higher with a fixed rate exchange rate regime.

Individuals, however, will not base their preferences concerning the exchange regime on the criterion of the size of the capital stock. An individual will favor the exchange regime which yields the highest value of
utility. Depending upon the realizations of the domestic and foreign productivity terms, members of some generations might prefer fixed rates, while members of other generations prefer flexible rates. As above, we compare the average value of realized utility under fixed rates to the average value of realized utility under flexible rates. In doing so, we can determine whether members of a randomly selected generation would prefer fixed or flexible rates to prevail for all time periods. We continue to assume that \( \phi_t \) and \( \theta_t \) are i.i.d. and uncorrelated.

Let \( U_t^{\text{fix}} \) represent the utility that an agent born at \( t \) would receive if there were to be a fixed rate system and \( U_t^{\text{flex}} \) represent utility if there were to be a flexible rate. Define:

\[
J_t = U_t^{\text{fix}} - U_t^{\text{flex}} = \frac{1}{2^\beta} \left( \frac{1-\alpha}{2} \right)^{\alpha \frac{1}{1-\alpha}} 2^\beta \left\{ \theta^t_{t+1 \theta^{t-1} \phi_t^{\alpha+1} \phi_0^{\alpha+1}} \right\}^{2^\beta}
\]

\[
= \frac{1}{2^\beta} \left( \frac{1-\alpha}{2} \right)^{\alpha \frac{1}{1-\alpha}} X_t
\]

If the expected value of \( J_t \) is positive, members of a representative generation will choose to have a fixed rate of exchange. We proceed by showing that the expected value of \( X_t \) is positive so that the sign of the expected value of \( J_t \) is determined by the sign of \( \beta \). For the first step in our proof that \( E(X_t) \) is positive, note that the assumption that \( \phi_j \) is i.i.d. implies:
Thus, the expected value of $X_t$ is:

$$E(X_t) = E\left[\frac{\beta}{\alpha} \phi_{t+1} \theta_{t-1} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta - \frac{\beta}{\alpha} \phi_{t+1} \theta_{t-1} \phi_t \left(\alpha + \frac{1}{\alpha - 1}\right)^\beta - \frac{\beta}{\alpha} \phi_{t+1} \theta_{t-1} \phi_t \left(\alpha + \frac{1}{\alpha - 1}\right)^\beta \right]$$

So long as $\phi_j$ and $\theta_j$ come from identical and independent distributions:

$$E(X_t) \geq 0 \text{ as } E\left[\frac{\beta}{\alpha} \phi_{t+1} \theta_{t} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta \right] \geq E[\phi_t (\alpha + 1) \beta^\beta].$$

To complete our proof, note:

$$E\left[\frac{\beta}{\alpha} \phi_{t+1} \theta_{t} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta \right] < E\left[\frac{\beta}{\alpha} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta \right]$$

Lastly, since $\phi_t$ and $\phi_{t+1}$ are i.d.d.:

$$E\left[\frac{\beta}{\alpha} \phi_{t+1} \theta_{t} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta \right] = E\left[\frac{\beta}{\alpha} \phi_t \left(\alpha + \frac{2}{\alpha - 1}\right) \left(\frac{\alpha + 2}{\alpha - 1}\right)^\beta \right] > E\left[\phi_{t+1} (\alpha + 1) \beta^\beta \right]$$

Since $E(X_t) > 0$, it follows that the expected value of $J_t$ depends upon the sign of $\beta$ (recall that $\beta$ is a measure of the degree of risk aversion in an uncertain environment and a measure of the degree of homogeneity of the utility function). In order to provide an intuitive interpretation of our findings, substitute equation 3 into equation 2 in order to obtain:

$$D_{t+1} = \left[ P_t (W_t - C_t) - P_t K_{t+1} \right] / P_{t+1} + R_{t+1} K_{t+1}$$

Using equation 10, and assuming an interior solution: $D_{t+1} = P_t C_t / P_{t+1} = a^{P_t Q_t / 2P_{t+1}}$

It is easily shown that $C_t$ and $D_{t+1}$ have a higher correlation with a fixed rate of exchange than with a flexible rate. Domestic disturbances
simultaneously affect prices and output when the exchange rate is flexible: high values of the domestic productivity disturbance are associated with high values of $C_t$ and low values of $P_t/P_{t+1}$. This reduction in the intertemporal price ratio acts to reduce the correlation between $C_t$ and $D_{t+1}$: with flexible rates when $C_t$ is high, $P_t/P_{t+1}$ tends to be low. Whether the relatively high correlation between $C_t$ and $D_{t+1}$ present in a fixed rate system is deemed to be desirable depends upon the nature of the utility function. A positive correlation between intertemporal consumption bundles means that lifetime consumption $(C_t + D_{t+1})$ is highly variable: a plus for flexible rates, since the utility function is concave. It also means that the proportion in which goods are consumed tends to remain stable: a plus for fixed rates, since the marginal rate of substitution is diminishing. We consider this point in further detail when we consider rational expectations in the next section of the paper. The important point at this stage is that our results are in marked contrast to the work of Fischer (1977), Laffer (1973), and Mundell (1973), who argue that the choice of exchange regimes depends upon the source of disturbances. We have shown the choice between exchange regimes depends, in part, upon the nature of the utility function as opposed to intermediate goals such as price or consumption stability.

In the next section of the paper, we consider the case of rational expectations. While we cannot provide analytic solutions, we present a Monte Carlo study to ascertain expected utilities and expected capital holdings under alternative exchange systems. It will be shown that $\beta=0$ remains the dividing line in the choice between exchange regimes, although the expected value of the capital stock is not always higher when the exchange rate is fixed.
V. Rational Expectations Simulation

In Section IV we were not able to provide analytic results for fixed rates when agents had rational expectations. In this section, we simulate the rational expectations case for the alternative exchange regimes. The simulations allow us to compare the two regimes directly and to provide some indications of the magnitudes of the differences between the regimes. The results in this section reinforce the conclusion that very risk averse individuals prefer flexible rates while less risk-averse individuals prefer fixed rates. However, the results concerning the capital stock are somewhat different from those of Section IV. With rational expectations, the expected value of the capital stock under flexible rates can be greater than that of fixed rates when capital's share (α), risk aversion, and/or foreign output variability are sufficiently large.

In simulating the model, we assume that the productivity disturbances are drawn from a trinomial distribution. Specifically, we let \( \phi_L \) have the distribution:

\[
\phi_L = \begin{cases} 
1 - \Sigma & f(1-\Sigma) = 1/3 \\
1.0 & f(1) = 1/3 \\
1 + \Sigma & f(1+\Sigma) = 1/3
\end{cases} \text{ where: } 0 \leq \Sigma \leq 1
\]

For example, if we let \( \Sigma = .25 \), the domestic productivity disturbance can take on values of .75, 1.0, and 1.25 each with a probability of 1/3. We continue to assume that the domestic productivity disturbance is serially uncorrelated and independent of the foreign productivity disturbance. The foreign productivity disturbance is also assumed to be drawn from a symmetric trinomial distribution with a mean of unity and variance of \((2/3)(\Sigma^*)^2\) where \( \Sigma^* \) is the foreign counterpart of \( \Sigma \). Using the Monte Carlo technique, we performed the following steps:
1. For each of fifty-one time periods, we determined whether the domestic productivity disturbance would take on its low, mean, or high realization. Our procedure guaranteed that \( \phi \) would have a calculated mean of 1.0, variance of \( 2 \sigma^2 / 3 \), and would be serially uncorrelated. We repeated this process in order to determine realizations of the foreign productivity disturbance.

2. Values of \( a, \beta, \Sigma, \) and \( \Sigma^* \) were specified, all endogenous variables were initialized at their steady state levels, and the labor supply set equal to unity. At this point, it was possible to solve the system for fifty consecutive time periods under each exchange regime. The productivity disturbances remained invariant to the exchange regime. Averaging yielded the simulated values of variables such as the expected capital stock and expected utility under fixed and flexible exchange rates.

3. Step 2 was repeated in order to determine how the endogenous variables are affected by changes in \( a, \beta, \Sigma, \) and \( \Sigma^* \). At this stage, step 1 was not repeated so that the pattern of the realizations of the productivity disturbances (i.e., low, mean, or high) was invariant to the choice of \( a, \beta, \Sigma, \) or \( \Sigma^* \).

In Table 3, we present simulated values of the expected capital stock under flexible exchange rates for three values of \( a \) and three different assumptions concerning the variance of the productivity disturbances (the capital stock under flexible rates does not depend upon \( \beta \) or \( \Sigma^* \)). We have simulated over many sets of parameters, and the values chosen for the table were selected to illustrate the qualitative properties of all parameter sets.
Table 3

Capital Holdings with a Flexible Exchange Rate

<table>
<thead>
<tr>
<th>Σ</th>
<th>α = .1</th>
<th>α = .2</th>
<th>α = .3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.871</td>
<td>6.687</td>
<td>8.954</td>
</tr>
<tr>
<td>0.25</td>
<td>3.839</td>
<td>6.607</td>
<td>8.804</td>
</tr>
<tr>
<td>0.5</td>
<td>3.783</td>
<td>6.439</td>
<td>8.469</td>
</tr>
</tbody>
</table>

1/ Divide each entry in the table by 50.0 to find the expected value of the capital stock. The entries in the table are sums over fifty periods.
As can be seen from Table 3, the expected value of the capital stock with a flexible exchange rate is positively related to and negatively related to $\Sigma$. These results are in accord with our intuition and can be derived from Table 1. Alpha ($\alpha$) represents capital's share of output so that it is not surprising to find that increases in $\alpha$ act to increase the expected capital stock. In considering the effects of changes in $\Sigma$ on capital holdings, it is useful to consider the distinction between portfolio size and portfolio composition. We have already demonstrated that the proportion of income saved and the proportion of savings allocated to capital ($s$) were unaffected by the variance of the domestic productivity disturbance (see equations 10 and 19a). Thus, for a given level of current income, an increase in $\Sigma$ will have no effect on either portfolio composition or portfolio size. By way of explanation, consider an individual in the first period of life. With a flexible exchange rate, increased output variability will be reflected in increased price variability. For the individual currently determining portfolio composition, increased output uncertainty simultaneously increases uncertainty concerning the return from holding capital and the purchasing power of money balances during the second period of life. While portfolio composition is unaffected by changes in $\Sigma$, an increase in the variability of the domestic output disturbance reduces expected output and thus reduces expected saving (and capital formation).\textsuperscript{10}

The situation is quite different with a fixed rate of exchange: the expected value of the capital stock depends upon $\alpha$, $\beta$, $\Sigma$, and $\Sigma^*$. Table 4 shows how the expected value of the capital stock depends upon $\alpha$, $\beta$, and $\Sigma$ for a given variance of the foreign productivity disturbance. As was the case with a flexible rate, the expected value of the capital stock is positively related to and negatively related to $\Sigma$. However, the
Table 4

Capital Holdings with Fixed Rates: \( \gamma = .25 \)

<table>
<thead>
<tr>
<th>BETA ( \beta )</th>
<th>( \alpha = .1 )</th>
<th>( \alpha = .2 )</th>
<th>( \alpha = .3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>3.931</td>
<td>6.737</td>
<td>8.659*</td>
</tr>
<tr>
<td>( \gamma = .25 )</td>
<td>3.907</td>
<td>6.638</td>
<td>8.081*</td>
</tr>
<tr>
<td>( \gamma = .5 )</td>
<td>3.821</td>
<td>6.198*</td>
<td>6.927*</td>
</tr>
<tr>
<td>.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>3.967</td>
<td>6.779</td>
<td>8.679*</td>
</tr>
<tr>
<td>( \gamma = .25 )</td>
<td>3.931</td>
<td>6.630</td>
<td>8.047*</td>
</tr>
<tr>
<td>( \gamma = .5 )</td>
<td>3.800</td>
<td>6.079*</td>
<td>6.746*</td>
</tr>
<tr>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>4.003</td>
<td>6.820</td>
<td>8.700*</td>
</tr>
<tr>
<td>( \gamma = .25 )</td>
<td>3.954</td>
<td>6.622</td>
<td>8.014*</td>
</tr>
<tr>
<td>( \gamma = .5 )</td>
<td>3.780*</td>
<td>5.962*</td>
<td>6.568*</td>
</tr>
<tr>
<td>-1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>4.149</td>
<td>6.989</td>
<td>8.779*</td>
</tr>
<tr>
<td>( \gamma = .25 )</td>
<td>4.047</td>
<td>6.597*</td>
<td>7.865*</td>
</tr>
<tr>
<td>( \gamma = .5 )</td>
<td>3.700*</td>
<td>5.457*</td>
<td>5.851*</td>
</tr>
<tr>
<td>-2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \gamma = 0 )</td>
<td>4.296</td>
<td>7.156</td>
<td>8.855*</td>
</tr>
<tr>
<td>( \gamma = .25 )</td>
<td>4.135</td>
<td>6.578*</td>
<td>7.703*</td>
</tr>
<tr>
<td>( \gamma = .5 )</td>
<td>3.621*</td>
<td>5.073*</td>
<td>5.305*</td>
</tr>
</tbody>
</table>

1/ Divide each entry in the table by 50.0 to find the expected value of the capital stock. The entries in the table are sums over fifty periods.

* Indicates that the expected value of the capital stock with flexible rates is greater than that with fixed rates.
mechanism by which changes in $\Sigma$ affect the expected capital stock differs across exchange regimes. Recall that a fixed exchange rate means that the return on money holding ($P_t^*/P_{t+1}^* = P_t^*/P_{t+1}$) is determined in the world market while the return on capital is influenced by domestic and foreign disturbances. Thus, increases in the variance of the domestic productivity disturbance act to make money a relatively safer asset vis-a-vis physical capital. With a fixed rate of exchange, changes in $\Sigma$ will result in portfolio composition and portfolio size effects. Increases in the variance of the domestic productivity term induce risk-averse individuals to substitute money holdings for capital holdings within their portfolios. This portfolio-composition effect is in addition to the type of portfolio-size effect considered under flexible exchange rates.

In regard to the influence of the degree of risk aversion on expected capital holdings, we find: increases in the degree of risk aversion (decreases in $\beta$) act to increase the expected capital stock when $\Sigma$ is small and to decrease the expected capital stock when $\Sigma$ is large. With a fixed rate of exchange, a low value of $\Sigma$ means that (domestic) capital is the relatively safe asset. Extremely risk-averse individuals will hold more capital in their portfolios than less risk-averse individuals. Alternatively, when the variance of the domestic productivity term is large, money is the relatively safe asset. Extremely risk-averse individuals will hold less capital in their portfolios than less risk-averse individuals. For example, in Table 4, decreases in $\beta$ are associated with decreases in the expected capital stock when $\Sigma = .5$ and with increases in the expected capital stock when $\Sigma = 0$. 
While we have shown that an increase in domestic output variability decreases the expected capital stock, it would be incorrect to conclude that an increase in $Z^*$ would correspondingly increase the expected capital stock. Consider an agent in the first period of life at time $t$, and let the variance of the foreign output disturbance increase (of course, all agents know that $Z^*$ has changed, since we are considering rational expectations). In that particular period, equation 22) indicates that the expected return on money holdings has not changed since $p_t = p^*_t$ is in the individual's information set. From equation 22):

\[
E(P^t/P^*_t | P^*_t) = \pi_{t-1}^* E(\theta_{t+1})
\]

so that increasing the variance of the foreign productivity disturbances will not change the expected return on money holdings when $p^*_t$ is known. The increase in $Z^*$ does increase the variability of the return on money holdings so that the agent in the first period of life would be expected to hold more capital and less money. However, from an ex ante perspective (i.e., one in which $P^*_t$ is not given), the expected return on money does depend upon the distribution of the foreign productivity disturbance:

\[
\frac{\partial E(P^t/P^*_t | P^*_t)}{\partial Z^*} = \frac{\partial}{\partial Z^*} \left( \pi_{t+1}^* \pi_{t-1}^* \right) > 0
\]

where the expectations operator runs over all $\theta_i$. Since changes in the variability of the foreign productivity disturbance will increase the mean and variance of the return on money, there is no necessary relationship between $Z^*$ and the expected domestic capital stock.

Tables 3 and 4 can be used to compare the expected value of the capital stock under the alternative exchange regimes. In Table 4, a starred (*)
value indicates those instances in which the expected value of the capital stock was found to be greater with a flexible rate of exchange than with a fixed rate. These results are contrary to the case of perfect foresight for which it was found that average capital holdings with fixed rates were always greater than average capital holdings with flexible rates. From Table 4, it can be seen that flexible rates yield greater expected capital holdings than fixed rates when: capital's share ($\alpha$) tends to be large, individuals are strongly risk averse, and the variability of the domestic productivity term ($\Sigma$) is large. When $\Sigma$ is large, money holdings tend to be a safe asset vis-a-vis physical capital when the exchange rate is fixed. As mentioned earlier, highly risk-averse individuals will substitute money holdings for capital when $\Sigma$ increases and the exchange rate is fixed. Thus, it is not surprising to find that the expected value of the capital stock under fixed rates is less than that with flexible rates when $\Sigma$ is high and $\beta$ is low. In regard to $\alpha$, an increase in capital's share increases the expected capital stock under either exchange regime. However, there tends to be more portfolio diversification when the exchange rate is fixed as the return on money is determined in the world market and the return on capital is determined in the home and foreign market.

In Table 5 we present some of our simulations concerning expected utility under the alternative exchange regimes. While space considerations prevent a complete presentation of our results, all simulations indicated that $\beta=0$ was the dividing line between the choice of exchange regimes. Expected utility was always greater with a fixed rate when $\beta > 0$ and always greater with a flexible rate when $\beta < 0$. Moving from perfect foresight to rational expectations does not change our qualitative result that the choice between exchange regimes depends upon the degree of relative risk aversion.
Table 5

Expected Utility Under Alternative Exchange Regimes: 1/ $\Sigma = .25$

$\Sigma^* = .25$

\begin{align*}
\alpha &= .1 & \alpha &= .2 \\
\beta^{2/} & & U_{\text{Flex}} & & U_{\text{Fix}} & & U_{\text{Flex}} & & U_{\text{Fix}} \\
.5 & & 17.183 & & 17.616 & & 13.160 & & 13.495 \\
.25 & & 58.463 & & 58.906 & & 51.150 & & 51.586 \\
-1.0 & & -225.98 & & -240.42 & & -387.77 & & -402.45 \\
-2.0 & & -1111.5 & & -1409.1 & & -3302.0 & & -3836.8 \\
\end{align*}

$\Sigma^* = .5$

\begin{align*}
\alpha &= .1 & \alpha &= .2 \\
\beta^{2/} & & U_{\text{Flex}} & & U_{\text{Fix}} & & U_{\text{Flex}} & & U_{\text{Fix}} \\
.25 & & 58.463 & & 59.690 & & 51.150 & & 52.286 \\
-1.0 & & -225.98 & & -264.63 & & -387.77 & & -412.00 \\
-2.0 & & -1111.5 & & -1190.4 & & -3302.0 & & -4269.8 \\
\end{align*}

1/ Divide each entry in the table by 50.0 to find expected utility. The entries in the table are seen over fifty periods.

2/ Values for $\beta=0$ are not reported. Simulations for $\beta$ around zero indicate that fixed and flexible rates yield the same expected utility for $\beta=0$. 
Further, this is precisely the result found in our previous paper [Lapan and Enders (1980)] in which money was the only store of value, and domestic and foreign disturbances were present. The key to explaining the choice between exchange regimes does not hinge on the presence (or absence) of physical capital, but rather on the nature of risk sharing present in the alternative exchange rate regimes. Under either exchange regime, the return on capital depends upon realizations of the domestic productivity disturbance. The return on money depends upon domestic disturbances with a flexible exchange rate and upon foreign disturbances with a fixed exchange rate. In this regard, our results support those of Fischer, Laffer, and Mundell in that there is more international risk sharing with a fixed rate than with a flexible rate. However, as we have demonstrated, it would be incorrect to conclude that agents will prefer fixed to flexible rates. The international risk sharing with fixed rates is desirable because it increases the covariance between consumption in each period of life. Suppose, for example, that wage income is above its mean value so that $C_t$ and saving are above their mean values. Since fixed rates act to stabilize the value of one's savings, $C_t$ and $D_{t+1}$ tend to be highly correlated when the exchange rate is fixed. On the other hand, the presence of international risk sharing acts to increase the variance of lifetime consumption ($C_t + D_{t+1}$); when $C_t$ is above (below) average, $D_{t+1}$ will tend to be above (below) average. This relatively high variance of lifetime consumption detracts from the relative desirability of a fixed rate system for risk-averse agents. To put matters another way, there is more intergenerational risk sharing with flexible rates than with fixed rates. With a flexible rate, consumption of the working generation at $t$ is given by $(1-a)Q /2$, and capital purchased by the
workers is $aQ_t$ (see Table 1). Thus, the amount consumed by the older generation at $t$ is also $(1-a)Q_t/2$. Members of the younger and older generations consume equal amounts and share equally in output risk when the exchange rate is flexible. Whether international risk sharing is preferable to intergenerational risk sharing depends upon the degree of risk aversion.

Before concluding, it is interesting to note that Table 5 indicates that expected utility with a fixed rate may increase when $\Sigma^*$ increases. The reason is rather straightforward once it is remembered that price uncertainty may be desirable [see Oi (1961)]. With a fixed rate, an increase in $\Sigma^*$ increases foreign and domestic price variability. When the degree of risk aversion is sufficiently low ($\beta > 0$), increased price variability can be deemed desirable.

VI. Conclusions

We have taken a step in the direction of obtaining a portfolio balance model which can be used for a fair comparison of alternative exchange rate regimes. Rather than postulate asset demand functions, the demands for money and capital were derived from expected utility maximizing behavior. This is particularly important, because the asset demand functions are not invariant to the exchange regime. Portfolio diversification occurs because of the presence of risk, and there are different risks inherent in each exchange regime. Specifically, the real return on money holding is determined on the world market with a fixed exchange rate and on the domestic market when the exchange rate is flexible. The differing risk structures present in the alternative exchange regimes mean that the asset demand functions which prevail under fixed rates will not, in general, prevail under flexible exchange rates. Furthermore, the asset demand functions will depend upon
the information set of individuals. In this regard, we have considered asset holdings under conditions of perfect foresight and rational expectations.

We have also considered the issue of the "choice between exchange regimes" from a utility maximizing perspective. Using an overlapping generations model, we have found that an individual's consumption during the first period of life has a higher correlation with consumption during the second period of life when the exchange rate is fixed. However, the fact that fixed rates act to stabilize an individual's total lifetime consumption has no normative significance. Instead, attitudes towards intertemporal consumption variability must be considered in choosing the optimal exchange regime. Agents who are highly risk averse will prefer flexible exchange rates, while agents who are mildly risk averse will prefer fixed rates.
Footnotes

1. As examples of what we mean by standard portfolio models, see Branson (1977), Girton and Henderson (1976), and McKinnon (1969).

2. This point is made in both the Helpman and Razin (1979) and Lapan and Enders (1980) papers.

3. Fischer (1977), Laffer (1973) and Mundell (1973) argue that fixed rates are preferable to flexible rates in the presence of internal real disturbances and that flexible rates are preferable in the presence of external or internal nominal disturbances. The criterion used for determining the optimal exchange regime was consumption stability. A similar result was found in Enders and Lapan (1979), which extended the Fischer paper to consider nontraded goods and capital mobility.

4. As is well known, there is some ambiguity in measuring risk aversion in a two-good model. Forming the indirect utility function, the measure of relative risk aversion used by Pratt (1964) would be $1-2\beta$ for $\beta < 1/2$. Using the principle of decreasing risk aversion to concentration developed by Leland (1968), the degree of relative risk aversion is $1-\beta$. In either case, risk aversion increases as $\beta$ decreases.

5. Assuming constant returns to scale for each firm, if the $\phi^f_t$ are distributed independently across firms, then all uncertainty can be eliminated by having an "infinite" number of firms, each producing at an "infinitesimal" level of output. Thus, we assume $\phi^f_t$ is identical for all firms at time $t$.

6. Because productive functions are homogeneous of degree one and the $\phi^f_t$ are identical for all firms in period $t$, there is little to be gained by distinguishing between output of the firm and aggregate output. In the text, we use the same notation to represent individual and aggregate variables whenever the meaning is unambiguous.

7. Notice that when the exchange rate is flexible, the rate of exchange depends upon the current and past realization of domestic and foreign productivity terms (the exchange rate in $t$ is given by $P_t/P^f_t$). This is in contrast to Mussa (1977) in which it is shown that the current exchange rate depends upon future disturbances. In our model, the presence of real capital produces serial correlation in the endogenous variables.

8. Introducing physical capital movements is rather quite simple. With real capital movements: $R_t+1 = R^f_{t+1}$. Thus, our results for fixed rates would be essentially unaltered. With a flexible rate, $R_t+1 = R^f_{t+1}$ requires that $P_t/P_{t+1} = P^f_t/P^f_{t+1}$. Introducing the commodity arbitrage, equations $F_t = e_t P_t$ and $F_{t+1} = e_{t+1} P^f_{t+1}$ immediately demonstrate that $e_t = e_{t+1}$. Thus, the flexible rate system would be identical to our case of fixed rates. Below, we show that flexible rates without capital movements may be preferable to fixed rates (with
or without capital movements) and, thus, flexible rates with real capital movements. Our results for the rational expectations case would be altered, however, due to unanticipated differences in the realizations of the domestic and foreign productivity terms.

9. Note that the concept of the golden rule is somewhat ambiguous in an economy which can run a current account surplus or deficit, since current consumption need not equal current output. While there is not golden rule capital accumulation with fixed rates, the usual significance of golden rule saving is not applicable.

10. The application of Jensen's inequality to the reduced form solution for output ($Q_{t+1}$ in Table 1) shows that expected output is negatively related to the variance of the domestic output disturbance.
References


