Optical interferometric techniques using single mode optical fiber waveguide embedded in composites and other layered materials have been used to measure one- and two-dimensional stress distributions and acoustic emission caused by applied point source loads. By interferometrically comparing the phases of coherent optical signals propagated through an embedded sample fiber and a bypass reference fiber, a signal proportional to the instantaneous strain integrated along the embedded length of the sample fiber has been detected. System calibration has been obtained by applying a one-dimensional dc strain field to a cantilever beam containing the fiber. Using this calibrated system, an array of fibers attached to a 15cm x 15cm x 0.3cm plate simply supported at the corners and subjected to point loading on the surface has been used to quantitatively determine the two-dimensional dc stress field in the plate. Finally, the calibrated ac response of the interferometer to acoustic emission events in a composite panel has been demonstrated. Potential applications are discussed.

INTRODUCTION

Embedded optical fibers offer significant advantages for the nondestructive evaluation of composite materials. First, they may
be embedded within the composite during manufacturing, thus allowing the NDE system to be "built in." Second, since the elastic properties of optical fiber waveguide are similar to those of composite fibers, composite mechanical properties are not degraded. Finally, optical fibers are a sensitive sensor of both dc internal residual stress and ac ultrasonic waves. The objective of this work has been to demonstrate the quantitative measurement of both stress and ultrasonic waves using fibers embedded in solid materials including composites.

During the past five years the sensitivity of the propagation of light through optical fiber waveguide to environmental conditions has been exploited to develop single mode fiber transducers which are sensitive to temperature, pressure, and rotation rate.\textsuperscript{1-3} In materials evaluation, similar quantitative techniques have been used to detect acoustic waves in liquids\textsuperscript{4-6} and solids\textsuperscript{7} as well as the pulsed ultrasonic waves associated with the energy release caused by acoustic emission events.\textsuperscript{8}

More recently, qualitative multimode fiber methods have been used to locate points of excessive continuous or impact loading on large material structures. First, multimode fibers have been applied with a primer coat of paint to areas of the commercial European jetliner AIRBUS that are sensitive to load-induced cracking.\textsuperscript{9} The fiber is thus in intimate contact with the surface and both displacement and stress continuity across the bond interface are assumed. If a crack begins to propagate within the structure, the fibers nearby will break, optical transmission through the fibers will stop, and the location of the break, and thus the crack, will be determined.

A second qualitative method has been developed which utilizes multimode fiber embedded between successive layers in a fiber matrix composite.\textsuperscript{10,11} Here, the fibers have been arranged in a mesh array in small laboratory samples and breaks in orthogonal fibers used to locate excessive stress near their intersection. Although neither of these multimode fiber techniques indicate the amplitude of the stress at the break location, they are inexpensive to apply and relatively simple to instrument.

Quantitative optical fiber techniques which may be applied to similar NDE problems are reported in this paper. In the following sections, the single mode interferometric sensor and previous measurements are first summarized. Next, the detection of a two-dimensional stress field in a flat plate is reported. Finally, the qualitative multimode and quantitative single mode fiber techniques are compared.

**SINGLE MODE FIBER INTERFEROMETRY**

The modified Mach-Zehnder interferometer shown in Figure 1 has been used to detect changes in the difference in the phase delay between two optical fiber paths. If the optical pathlengths are
equal to within the coherence length of the source, optically heterodyning the light from the output of the two fibers produces a stationary pattern of dark and light interference fringes. The relative phase change of light from one fiber with respect to the phase of the light in the other results in a displacement of the fringe pattern at the output. Spatial filtering of the light in the fringe pattern and optical detection of the transmitted light gives an electrical signal related to the pathlength difference $\Delta L$ between the two optical signals. Although this relationship is a complicated nonlinear function in general, for small pathlengths $\Delta L$ it is directly proportional to the electrical output of the detector. 12

To determine the sensitivity of the fiber, consider a fiber of length $L$, core diameter $D$, and core index of refraction $n$. If the laser light has a free space propagation constant $k_0$ and a single mode propagation constant $k$ inside the fiber, then the relative retardation of the light that propagates through both fibers is

$$\phi_s = \phi_r = kL.$$  \hspace{1cm} (1)

Deformation of the fiber due to an applied stress causes a phase
shift at the output
\[ \Delta \phi = \phi_s - \phi_r = k\Delta L + L\Delta k \] (2)
for small applied stresses.

The first term in (2) represents the optical phase modulation produced by a physical change in the length of the fiber due to strain. The second term is produced either by the strain-optic effect in the fiber material or by optical waveguide mode dispersion due to a change in fiber diameter. Thus, the second term may be written as a sum of two terms
\[ L(dk/dn)\Delta n + L(dk/dD)\Delta D. \] (3)
Equation (2) then becomes
\[ \Delta \phi = PkL(2\nu-1)/E + L(dk/dn)\Delta n + L(k/D)\Delta D, \] (4)
where \( P \) is pressure, \( E \) Young's modulus and \( \nu \) Poisson's ratio.

Hocker evaluated Equation (4) theoretically to determine the fiber sensitivity to pressure changes and found the sensitivity to static pressure changes to be low (154 kPa-m/fringe for a He-Ne laser source) compared to other methods. For an ITT-110 single mode optical fiber the phase shift per unit pressure per unit length is approximately
\[ \Delta \phi/PL = 4.09 \times 10^{-5} \text{ rad/Pa-m}. \] (5)

SINGLE SAMPLE FIBER MEASUREMENTS

The fiber sensitivity indicated in (5) has been verified by measuring the fringe shift produced when the sample fiber is attached along the length of the top of a cantilever beam and an increasing downward load is gradually applied to the free end. If the reference fiber is attached to the bottom of the beam, the sample fiber is in relative tension, the reference fiber is in relative compression, and an output signal proportional to strain along the beam is produced. Reported theoretical and experimental data for this type of dc pressure calibration differ by less than eight percent. Variations in pressure due to both continuous and pulsed acoustic and ultrasonic waves in liquids and solids have similarly been measured using systems having a single sample fiber. The same single fiber technique has also been applied to the detection of acoustic emission events in graphite epoxy composite panels.

MULTIPLE SAMPLE FIBER MEASUREMENTS: TWO-DIMENSIONAL STRAIN

Measurements of ac or dc strain using a single sample fiber as
the strain sensitive element are complicated because the strain effects are integrated over the entire interaction length of the fiber with the strain field. If instead, several fibers in a known geometrical arrangement interact with the field, the individual fiber signals may be processed simultaneously to yield field components in several coordinate directions. To demonstrate this, six single mode optical fibers, each nominally 0.5m long, were attached to a 15.24 cm x 15.24 cm plexiglass plate 0.32 cm thick as shown in Figure 2. The plate was simply supported at each corner and a point source displacement was applied at each location indicated by the labeled circles. As in the cantilever beam calibration experiment, such a displacement compresses the attached fibers causing an incremental change in the length of the fiber. If one embedded fiber is used as the sample arm and a bypass fiber as the reference arm of the interferometer in Figure 1, the compression causes a phase shift in the output proportional to the strain along the sample fiber. By sampling each fiber of the matrix, the normal strain throughout the plate may be determined. At each circle on the plate, a 0.04 cm displacement was applied and the output response of each fiber recorded on an x-y plotter connected to the output of the optical detector. Examination of each individual fiber for the same displacement yielded a two-dimensional matrix of the dc strain in the plate.

Figure 2. Rectangular plate with attached fiber grid.
The equation for the deflection of the plate is
\[ \nabla^4 w = 12(1-v^2)P/Et^3, \]  
(6)
where \( w \) is displacement, \( E \) Young's modulus, \( t \) thickness, and \( v \) Poisson's ratio. Once the displacement is known at each node, the normal strains \( \varepsilon_x \) and \( \varepsilon_y \) may be calculated using (6) to be
\[ \varepsilon_x = -z \frac{\partial^2 w}{\partial x^2}, \text{ and} \]
(7)
\[ \varepsilon_y = -z \frac{\partial^2 w}{\partial y^2}. \]  
(8)

The solutions to (7) and (8) for a corner supported plate are best obtained using a finite element approach with the proper boundary conditions for each node in the analysis. For the applied stress at A, the strain over the entire plate was calculated using a quarter plate section and the properties of symmetry were applied to obtain a total solution. By sectioning the quarter plate as shown in Figure 3 nodes 1-5 lie along fiber 2 in Figure 2.

Figure 3. Plate partitions for finite element analysis.
Similarly, nodes 16-20 lie along fiber 3, nodes 3, 7, 11, 15, and 24 along fiber 4, and 1, 6, 11, 16, and 21 along fiber 5. By symmetry, the stress along fiber 1 is the same as that along fiber 3 and fiber 6 is the same as that along fiber 4. The average strain was calculated by multiplying the incremental length of each section by the strain for the node corresponding to that section. For example, the strain along fiber 2 is given by:

$$\varepsilon_x = 4(1.27\varepsilon_1 + 2.54\varepsilon_2 + 1.91\varepsilon_3 + 1.27\varepsilon_4),$$  \hspace{1cm} (9)

where the factor of 4 arises due to the double length of the fiber and the quarter plate symmetry. The number of shifted fringes is then

$$N = \frac{\varepsilon}{\lambda}. \hspace{1cm} (10)$$

For the asymmetrical loading of points B-D, the entire plate must be used in the analysis to obtain a correct solution. The experimental data are compared to scaled and normalized values for the symmetric case of a load at point A in Table 1. Errors between the modeled and measured data are due to singular boundary conditions at the nodes, the size of the cells used in the numerical analysis, and measurement error.

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**RESULTS**

Quantitative single mode fiber interferometric methods may be used to measure both dc and ac one- and two-dimensional strain fields in materials which can support the fibers. Unlike the qualitative multimode fiber NDE methods which depend upon the breakage of fiber to indicate excessive strain, the detection capability of this technique is not irreversibly destroyed during the measurement process. A combination of the simple multimode system as a gross failure-predicting device and the single mode system as a quantitative strain field detection system is suggested for the in-use monitoring
of composite materials used in critical structural applications.

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REFERENCES