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Decay schemes of Cl-34 and Sc-47

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Decay schemes of Cl-34 and Sc-47

Abstract
Cl$^{34}$ was obtained from a gamma reaction upon ordinary chlorine and its radiation to determine the decay scheme. Three positron components were separated from the beta spectrum of Cl$^{34}$ obtained from an intermediate-image spectrometer. Coincidence beta spectra obtained with the use of the same instrument gave additional energy determinations for the two lower-energy beta groups as well as indicating the time relationships between the beta groups as well as indicating the time relationships between the beta ray and the gamma rays in Cl$^{34}$. The low energy gamma ray in Cl$^{34}$ was studied by means of its internal conversion peak. The 1.1 Mev and 2.1 Mev gamma rays were measured from photographs of the Cl$^{34}$ gamma scintillation pulse distribution. The decay scheme indicated from the results was completed with the use of beta log ft values and gamma lifetime values for the spin and parity assignments given to the nuclear energy levels.

Keywords
Ames Laboratory

Disciplines
Atomic, Molecular and Optical Physics | Physics

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DECAY SCHEMES OF Cl\textsuperscript{34} AND Sc\textsuperscript{41}

By
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E. N. Jensen

December 1954
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Technical Information Service Extension, Oak Ridge, Tenn.
F. H. Spedding, Director of Ames Laboratory.


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DECAY SCHEMES OF Cl$_{34}$ AND Sc$_{47}$*

by

R. T. Nichols and E. N. Jensen

ABSTRACT

Cl$_{34}$ was obtained from $^8$He, n reaction upon ordinary chlorine and its radiation to determine the decay scheme. Three positron components were separated from the beta spectrum of Cl$_{34}$ obtained from an intermediate-image spectrometer. Coincidence beta spectra obtained with the use of the same instrument gave additional energy determinations for the two lower-energy beta groups as well as indicating the time relationships between the beta groups as well as indicating the time relationships between the beta rays and the gamma rays in Cl$_{34}$. The low energy gamma ray in Cl$_{34}$ was studied by means of its internal conversion peak. The 1.1 Mev and 2.1 Mev gamma rays were measured from photographs of the Cl$_{34}$ gamma scintillation pulse distribution. The decay scheme indicated from the results was completed with the use of beta log ft values and gamma lifetime values for the spin and parity assignments given to the nuclear energy levels. This proposed decay scheme is shown in Fig. 26.

The Sc$_{47}$ sources were prepared from a $^7$He, p reaction upon ordinary titanium. The Sc$_{44}$ positron activity was eliminated from the beta spectrum of Sc$_{47}$ by use of a charge-discriminating baffle. Low counting rates and high source thickness lowered the precision of the measurements but two beta groups were separated by means of their Kurie plots with the energy of the lower-energy component agreeing very well with the energy measured for the same component as it was obtained from a coincidence beta spectrum. The energy difference between the two beta groups agreed well with the energy for the accompanying gamma ray which was measured by the scintillation spectrometer. A decay scheme was set up from these results using nuclear shell theory for orbital assignments. This proposed decay scheme as shown in Fig. 27 is self-consistent although it disagrees with the results of the three other investigators cited from recent literature.

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*This report is based on an M. S. thesis by R. T. Nichols submitted December, 1954, to Iowa State College, Ames, Iowa. This work was done under contract with the Atomic Energy Commission.
INTRODUCTION

Most of the gaps in information on a nuclide chart are associated with isotopes whose half-life is relatively short or intensity relatively weak compared to others formed in the same production processes. The Iowa State College Synchrotron can be used to obtain many of these isotopes but often the intensity is so low as to make it a major problem in investigation of their radiations. This is especially true of the more complex activities where coincidence measurements are necessary for determination of the decay scheme.

In order to examine some of the moderately short-lived low-intensity activities, an intermediate-image magnetic-lens spectrometer (1) has been constructed which has a transmission of 10 per cent at 5.5 per cent resolution. It is operated in conjunction with a scintillation spectrometer for electron-gamma coincidence spectra.

The information obtained from these instruments is used to determine decay schemes of various radioactive isotopes. These decay schemes indicate the energy and sequence of emission of the various radiation components and the energy, angular momentum and parity of the indicated energy levels of the nuclei. These results are used to check various nuclear theories and to contribute to the store of information which is needed for beta-ray systematics.

In this work, Cl34 and Sc47 were chosen for investigation. The subsequent discovery of the positron emitter, Sc44, in the scandium sources led to the installation of a charge-discriminating baffle to permit the separate investigation of the Sc47 beta spectrum.

A short presentation of beta decay theory is included in this paper for background information and some consideration of basic statistical theory which was used for least-squares evaluations is given in the form of an appendage.

REVIEW OF LITERATURE

A. Cl34

A number of investigators (2,3,4) gave early reports concerning a 33 minute positron activity in radioactive chlorine fractions. This activity was found to be from Cl34. From cloud chamber studies indicating a maximum energy of ~5.1 MeV, Zah-Wei (4) calculated an ft value of 107 which classified it as a first-forbidden transition.
The first magnetic-lens spectrometer measurements of Cl$_3^4$ were presented by Ruby and Richardson (5). Their Cl$_3^4$ sources were obtained from proton bombardments of both chlorine and sulfur targets. The beta spectrum revealed three positron groups which were separated by subtraction of Kurie plot lines. The maximum energies were evaluated as 4.55 ± 0.11 MeV, 2.58 ± 0.26 MeV and 1.3 ± 0.2 MeV. Two gamma rays were found and measured from inflection points on their Compton spectra. The corresponding energy values were reported to be 3.30 ± 0.14 MeV and 2.13 ± 0.12 MeV. They also discovered a 145 keV gamma ray from its internal conversion line. The ratio of positrons to conversion electrons was estimated as 17 to 1.

Subsequent reports from the same group (6,7) present revised measurements. Their latest values for the positron group energies are 4.50 ± 0.03 MeV, 2.43 ± 0.07 MeV and 1.33 ± 0.10 MeV.

The gamma rays from Cl$_3^4$ were examined by Ticho (8) with a scintillation spectrometer. His gamma energy measurements of 3.22 ± 0.03 MeV and 2.10 ± 0.03 MeV were in good agreement with the values of Ruby and Richardson. An additional gamma ray of 1.16 ± 0.03 MeV was also reported which corresponds to the energy difference of the two higher energy gamma rays. However, these energy values are not accurate enough to indicate the place of the 145 keV gamma ray in the decay scheme.

Stahelin et al. (9) have more recently reported that the 33 minute half-life is due to an isomeric state in Cl$_3^4$. The 145 keV gamma ray was identified with the transition from this metastable state to the ground state of Cl$_3^4$. The ground state was reported to decay by simple positron emission to the ground state of S$^3$. This would indicate that the other two positron groups are associated with transitions from 33 minute Cl$_3^{34m}$ (metastable state) to excited levels in S$^3$. The half-life of Cl$_3^{34m}$ was given as 1.45 second from which an ft value of 2650 was derived. This would indicate that the transition is of the "super-allowed" class. Since the daughter product has an even Z - even N nucleus with A = 4 n + 2 he concluded it to be an O+→O+ transition. From the internal conversion coefficient of the 145 keV gamma ray, which was evaluated as 0.13 ± 0.04, this radiation was calculated to be magnetic octupole and so a spin assignment of 3+ was given for the 33 minute metastable state.

The separation technique used in obtaining the 1.5 second Cl$_3^4$ activity is given in another paper (10). Carbon tetrachloride was formed with the 33 minute Cl$_3^{34m}$ and vaporized. The chemical bonds of decaying chlorine atoms were broken by a Szilard-Chalmers reaction. The resulting free Cl$_3^4$ was absorbed by a fine spray of water or by a Cu absorber and examined with a multi-channel scintillation pulse-height analyzer (11). The latter paper gives a revised value of 1.58 ± 0.05 seconds for the half-life of Cl$_3^4$. 
Peelman et al. (12) and Waffler et al. (13) give half-life values of 33.0 minutes and 33.2 minutes for Cl$^{32}$ but neither group indicate the accuracy of their values. Hints and Ramsey (14) report a value of 32.5 ± 0.5 minutes and the most recent paper of Richardson et al. (7) gives a value of 32.40 ± 0.04 minutes for the half-life.

B. Sc$^{47}$

A 3.4-day activity was found in scandium by Hibdon et al. (15) and identified in a later paper (16) as Sc$^{47}$. The activity was reported to decay by emission of negative beta particles and a gamma ray (17). Absorption measurements were used to obtain a value of 0.61 MeV for the maximum beta energy. The half-life was given as 3.43 ± 0.03 days.

Three reports have been published concerning Sc$^{47}$ since the initiation of this investigation. The results have not been in general agreement.

Cheng and Pool (18) prepared Sc$^{47}$ by deuteron bombardment of Ti$^{49}$-enriched titanium. Studies with a lens spectrometer indicated two beta groups with energies of 0.622 ± 0.005 MeV and 0.435 ± 0.008 MeV and a branching ratio of 34:66, respectively. A gamma ray of 0.185 ± 0.007 MeV was also reported. Coincidence absorption measurements indicated that this gamma ray was in coincidence with the lower energy beta group only. The conversion coefficient as estimated from spectral areas indicated that the gamma ray is of electric dipole or magnetic dipole origin. The magnetic dipole assignment was noted to be in better agreement with nuclear shell theory since the two beta groups are in the same order of forbiddenness. Log ft values of 5.2 and 6.0 for the lower and higher beta groups, respectively, indicated that both are allowed transitions. Their proposed decay scheme is shown in Fig. 1a. It was set up with the use of nuclear shell theory for orbital assignments.

Cork et al. (19) prepared radioactive calcium by neutron and deuteron bombardment of normal calcium. A 3.8-day negatron activity was ascribed to Ca$^{45}$ which would decay into Sc$^{47}$. The composite beta spectrum was resolved into four components by Kurie plots. These beta groups had maximum energies of 1.4 MeV, 0.64 MeV, 0.46 MeV and 0.26 MeV. The 0.26 MeV component was attributed to long-lived Ca$^{45}$, the 1.4 MeV and 0.46 MeV components were identified with 5.35 day Ca$^{47}$ and the 0.64 MeV beta group was ascribed to 3.4 day Sc$^{47}$ which would grow into equilibrium with its parent nuclide. Scintillation spectrometer measurements on a separated scandium fraction indicated that a 0.16 MeV gamma ray was associated with the Sc$^{47}$. Beta-gamma coincidence measurements were made using the Ca$^{47}$ source and counting coincidences with the 0.16 MeV gamma ray. The coincidence Feather absorption curve appeared to be of the same shape as a Feather absorption curve for the separated scandium isotope. Thus the gamma ray was concluded to be in cascade with the single reported beta group.
Marquez (20) examined $\text{Sc}^{47}$ which had been separated from $\text{Ca}^{47}$. He reported that two beta groups were observed in magnetic-lens spectrometer data. The energy maxima of the two groups were given as $0.280 \pm 0.003$ Mev and $0.490 \pm 0.005$ Mev. An internal conversion line was used to determine an energy value of $0.218 \pm 0.010$ kev for the gamma ray which was also present. The decay scheme evidenced by these values is in complete disagreement with the results of the other two investigators. The three indicated decay schemes are shown in Fig. 1. Marquez also gave a value of $3.44 \pm 0.05$ days for the $\text{Sc}^{47}$ half-life.

BETA-RAY SPECTROSCOPY AND NUCLEAR THEORY

A. Beta Decay Theory

Although much of the impetus for research with beta radiations has arisen from interest in the nature of the beta decay process, the fact that it is a nuclear process has meant that progress in such studies has gone hand-in-hand with progress in general nuclear theory. Some of the major steps towards the understanding of the beta decay process have come from other studies involved with the nature of the atomic nucleus. At the same time, studies of beta radiations have been fruitful avenues for exploring the little-known region inside the heart of the atom. Present day knowledge of nuclear force fields and energy configurations have drawn heavily upon the information furnished from such studies. There are still many aspects of nuclear physics that are unknown or little understood. Beta-ray spectroscopy continues to be a most promising source of new information from which to build or to test new nuclear theories that may unlock the secrets of the nuclear domain.

In beta-ray spectroscopy, the beta and accompanying gamma radiations of many radioisotopes are studied. Determinations may be made of half-life, energy distributions and time-relationships among the various radiation components. This information may lead to an estimation of the decay scheme of the radioactive isotope. The decay scheme shows the various energy states through which the nucleus passes in going from its initial to its final state. It often includes a description of the angular momenta and parities associated with these nuclear energy levels. These results may be used to test nuclear theories, both those which are involved in making the analysis and those which make predictions of nuclear energy level configurations. A brief discussion will be given here of beta-decay theory and the selection rules which it gives for determining decay schemes from spectroscopic data. A more detailed development of the theory is presented elsewhere (21, 22).

Nuclei are almost certainly composed of protons and neutrons. Although electrons are emitted in beta decay, various arguments may be presented
Fig. 1. Proposed decay schemes for $\text{Sc}^{47}$ from the three cited sources
(a) Cheng and Pool (18)
(b) Cork et al. (19)
(c) Marquez (20)
against the possibility of their existence in the nucleus (23). Therefore a nuclide of atomic number \( Z \) and atomic weight \( A \) must contain \( A - Z \) neutrons and in negative beta emission a neutron is transformed into a proton as the electron is created. There are other beta transformations in which positrons are emitted or orbital electrons captured by the nucleus and the number of protons in the nucleus is decreased by one.

The beta particle does not necessarily carry off all the energy lost by the nucleus as it changes states by beta decay. A gamma ray may be emitted to take some of the energy. Yet these two observable particles cannot account for all the energy loss of a decaying nucleus. Even in cases where there are no gamma rays, the beta energies are found to vary according to a bell-shaped distribution with energy values ranging from zero up to the maximum energy where the electron has all the energy lost by the nucleus. Thus it is evident that the observed particles do not carry away all the energy lost by the nucleus in most instances. This apparent disappearance of energy was accounted for by the Pauli neutrino hypothesis according to which an additional "unobserved" particle carries away part of the energy. The success of this hypothesis in explaining many other details of the beta process including nuclear recoil distributions (24), has been so great that it has almost universal acceptance today although anything resembling a "direct" observation of such a particle is very improbable. With a spin of \( \frac{1}{2} \hbar \) assigned to the neutrino it is possible to maintain conservation of angular momentum in the beta decay process. Measurements (25) indicate that the neutrino must have a rest mass of less than 0.25 kev and it is generally assumed to be zero.

The three types of beta transitions may be represented, in terms of the particles involved, by the following equations:

Negative beta emission:

\[
Z^A\rightarrow Z + A + \nu^- + e^+ + \gamma \tag{1A}
\]

Positive beta emission:

\[
Z^A\rightarrow Z + A + e^+ + \gamma \tag{1B}
\]

Orbital electron captures:

\[
Z^A + \nu^- \rightarrow Z + A + \gamma \tag{1C}
\]

where \( Z^A \) represents the nuclide with atomic number \( Z \) and atomic weight \( A \). The symbols, \( e^- \), \( e^+ \), \( \nu \), and \( \gamma \), represent an electron, a positron, a neutrino and an antineutrino, respectively. Although the neutrino and the
antineutrino are not distinguishable as are the electron and its anti-
particle (the positron), the reason for a distinction here will be evident
in the consideration of Fermi's beta decay theory.

Fermi utilized the neutrino hypothesis to put the theory of beta decay
into mathematical form. Fermi's assumption involved adding to the
Hamiltonian a perturbing energy term which may be represented schematically
as follows:

$$H = g_f (\bar{\psi}^* \psi, \phi)$$  \hspace{1cm} (2)

The "Fermi constant" $g_f$ is proportional to the strength of the interaction
and in principle may be evaluated from one case of a transition induced by
$H$ if the form of $H$ is known. $\bar{\psi}^*$ and $\phi$ represent the final and initial
states of the nucleon undergoing transformation, while $\psi$ and $\phi$ represent
the created and the absorbed small particles respectively. The emission of
an antiparticle is here viewed as the absorption of a particle from the
filled negative energy states of the vacuum. The whole process may be
viewed as one in which two particles are absorbed and two created. For
example, for negative beta decay the $\bar{\psi}^*$ and $\phi$ may describe the created
proton and electron while $\bar{\phi}^*$ and $\psi$ would describe the absorbed neutron
and (anti) neutrino.

In analogy to the Dirac theory for the emission of light (26), Fermi
constructed the interaction transition matrix in the form

$$\langle f | H | i \rangle = g_f \int \frac{\psi^* \phi^*}{H} (U_f^* K_n U_i) d^4r.$$  \hspace{1cm} (3)

Here $(\psi^* \phi^*)$ is some function of the electron and neutrino fields evaluated
at the position of the $n$th nucleon. It is assumed to be a linear function
in each for simplicity's sake. $K_n$ is some operator which transforms the
$n$th nucleon into a proton (27). $U_f$ and $U_i$ represent the initial and the
final states of the system and therefore contain $\bar{\psi}^*$ and $\phi^*$. The symbol $\star$
denotes the multiplicative operation appropriate to the forms of $(\psi \phi)$ and
$(U_f^* K_n U_i)$ in order to make $\langle f | H | i \rangle$, which is the expectation value of
the transition energy, an ordinary scalar as it must be.

The ambiguities that are left in Eq. 2 come from the fact that field
amplitudes at a point in space-time are not mere numbers. The corre-
sponding field quanta have spin $\frac{1}{2} \hbar$ and so have "internal" degrees of
freedom. This freedom is described by writing the field amplitudes as
four-vectors with components having the characteristics of the Dirac
spinor. The most general form for $(\psi \phi)$ then becomes some form of the
matrix elements represented as follows:

$$\langle \phi^* \psi \rangle = \langle c_k \bar{\psi}^* \phi \rangle$$  \hspace{1cm} (4)
where the subscripts denote the components of the four-vector wave functions and \( c_{kl} \) are the constants appropriate to each combination. Similarly the expression \( (u_f^* K_n u_i) \) may be written in terms of the possible four-vector as a matrix form as follows:

\[
(u_f^* K_n u_i) = \left[ (u_f^* K_n u_i)_{kl} \right] \]

(5)

where \( (K_n)_{kl} \) is the matrix operator element which performs the indicated transformation between the four-vector components of the nuclear states.

There are 16 independent ways by which such bilinear combinations as Eqs. 4 and 5 may be formed. This arbitrariness is reduced by the fact that the product of the two forms must be an ordinary scalar and therefore must be invariant to relativistic space-time transformations. This condition may be satisfied by five different forms for Eq. 4 which are described and symbolized as follows:

1. A scalar product of the four-vector components \( (\psi^* \phi)_{S} \)
2. A polar vector product of the four-vector components \( (\psi^* \phi)_{V} \)
3. An antisymmetric tensor function of the four-vector components analogous to the electromagnetic field tensor \( (\psi^* \phi)_{T} \)
4. An axial vector product of the four-vector components \( (\psi^* \phi)_{A} \)
5. A pseudo-scalar product analogous to the scalar product of an axial vector and a polar vector \( (\psi^* \phi)_{P} \).

With the bilinear combinations in Eq. 5 appropriately complementary, the transition matrix will be a scalar. Eq. 3 may now be written in terms of each possible interaction form as follows:

\[
\langle f | H | i \rangle_K = G_K \sum_n \int (\psi^* \phi)_{K} (u_f^* K_n u_i)_{kl} d\tau
\]

(6)

\( K = S, V, T, A, \) or \( P \).

Any combination of these interaction forms is also possible so the general transition matrix will be the sum of the existing forms and may be written

\[
\langle f | H | i \rangle = \sum_K \langle f | H | i \rangle_K
\]

(7)

where the \( G \)'s will be zero for any non-existing interaction form.

This is the most definite form of the transition matrix that may be obtained from the original assumptions unless some further criteria are found to eliminate or relate the strengths of the interactions. No such criteria have been formulated which have not been eliminated by experimental evidence. The present day arguments for the interaction form are mostly phenomenological and inconclusive (22). Therefore it has been the practice to calculate the transition probability for each form and then to test with experimental data for the various possibilities.
From a given $\langle f | H | i \rangle$, it is a straightforward matter to obtain the rate of beta decay in terms of the energy or moments of the emitted particles. The transition probability per unit may be written as follows (28):

$$T(\varepsilon_o) = \frac{2\pi}{\hbar} |\langle f | H | i \rangle|^2 \frac{d\rho(\varepsilon_o)}{d\varepsilon_o}$$

(8)

where $\varepsilon_o$ is the energy of the transition and $\rho(\varepsilon_o)$ is the number of available states for the emitted particles in phase space.

The energy units for the following equations will be $m_0 c^2$ units and the momentum units, $m_0 c$ units where $m_0$ is the rest mass of the electron and $c$ is the velocity of light.

Considering an electron in the volume element $dV_e$ which has a momentum of magnitude between $\gamma$ and $\gamma + d\gamma$ in a direction within the solid angle $d\Omega_e$ and a neutrino in the volume element $dV_\nu$ with a momentum between $\xi$ and $\xi + d\xi$ within the solid angle $d\Omega_\nu$, the available element of volume in phase space is expressed in its natural units, $\hbar^6 = (2\pi \hbar)^6$, as follows:

$$\delta\rho(\varepsilon_o) = (2\pi \hbar)^{-6} \gamma^2 d\gamma \xi^2 d\xi \ d\Omega_e d\Omega_\nu dV_e dV_\nu.$$  

(9)

The transition probability for the particles being emitted within this range of values is

$$\delta T(\varepsilon_o) = (2\pi)^{-5} \hbar^{-7} |\langle f | H | i \rangle|^2 (d/d\varepsilon_o) \delta\rho(\varepsilon_o).$$  

(10)

$\delta\rho(\varepsilon_o)$ may be written in terms of $\varepsilon_o$ in order to perform the indicated differentiation. In $m_0 c$ units the momentum of the neutrino is

$$\xi = (\varepsilon_o - \varepsilon)$$  

(11)

where $\varepsilon$ is the energy of the electron. With this substitution the differentiation gives for Eq. 10

$$T(\varepsilon_o) = \left[ (2\pi)^{-5} \hbar^{-7} \right] |\langle f | H | i \rangle|^2 \gamma^2 d\gamma \left[ -(2(\varepsilon_o - \varepsilon)) \right] d\varepsilon$$

$$\times d\Omega_e d\Omega_\nu dV_e dV_\nu.$$  

(12)

By integrating over all the possible values for the other parameters the probability per unit time is obtained for the emission of an electron with a magnitude of momentum between $\gamma$ and $\gamma + d\gamma$:

$$P(\gamma) \ d\gamma = \left[ \frac{\nu^2}{2 \pi^3 \hbar^7} \right] |\langle f | H | i \rangle|^2 (\varepsilon_o - \varepsilon)^2 \gamma^2 d\gamma.$$  

(13)
where $V$ is the volume over which the integration is performed. When the electron and neutrino wave functions in $H$ are written as plane waves normalized over the volume of integration, a normalization factor proportional to $1/\sqrt{V}$ appears. Thus if this factor is left out in the handling of the wave functions the $V^2$ term drops out leaving

$$P(\eta)d\eta = \left[\frac{1}{2}\pi^{3/2}V^{3/2}\right]\langle f|H|i\rangle^2 \left(\cos^2 \theta \right)^2 d\eta$$

which is unchanged when the volume of integration is extended over all space.

Since the (anti) neutrino state which enters into $\langle f|H|i\rangle$ is practically unperturbed it may be written as a plane wave:

$$\phi = u_\nu \exp\left[\pm i\mathbf{S} \cdot \mathbf{r}/\hbar\right].$$

The beta-ray wave will be distorted by the Coulomb field but it may be Fourier-analyzed into plane waves as well. This brings into $\langle f|H|i\rangle$ the terms

$$\langle \phi^* \phi \rangle_K = (u_\nu^\ast u_\nu)_K \exp\left[-2\left(\mathbf{S} + \mathbf{F}\right) \cdot \mathbf{r}/\hbar\right].$$

These may be expanded into a power series in $r$ if the nuclear radius $R << \hbar/\mathbf{S} + \mathbf{F}$ is large because contributions to the integral vanish soon outside the nuclear region. In terms of this power series, Eq. 6 may be written:

$$\langle f|H|i\rangle = G_K \sum_{K} \int (U_K^* u_\nu^\ast)_K (U_K u_\nu)_K d^3r - \left(\mathbf{S} + \mathbf{F}\right) \int \mathbf{r} (u_\nu^\ast u_\nu)_K (U_K u_\nu)_K d^3r + \cdots.$$
and are known as Fermi selection rules. The rules for T and A interactions are the same and are called Gamow-Teller rules. These rules are compared in Table I. \( \Delta I \) represents the spin change between the initial and final states of the transition.

### Table I. Selection rules for beta-decay

<table>
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<th>Parity change?</th>
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<td></td>
<td>( \Delta I )</td>
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<td>0, 1 (no 0( \rightarrow )0)</td>
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<tr>
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<td>0, 1, no</td>
<td>0, 1, 2 (no 0( \rightarrow )0, 1( \leftrightarrow )0, ( \frac{1}{2})( \leftrightarrow )( \frac{3}{2}))</td>
</tr>
<tr>
<td>second-forbidden</td>
<td>( \pm 1, \pm 2 )</td>
<td>0( \rightarrow )0, ( \pm 2, \pm 3 )</td>
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In evaluating the momentum distribution for the emitted beta rays (Eq. 14) it is customary to calculate the transition matrix neglecting the Coulomb interaction on the electron and to calculate this Coulomb effect separately as another factor. The expression obtained comes from treating the electron as a wave and its transmission as an optical phenomenon. As the electron moves through the rapidly varying electrostatic potential near the center of the nucleus, it experiences a substantial reflection on its wave function. The reflected wave interferes destructively with the outgoing wave thus suppressing the emission of the particle. Quantum-mechanical calculation (29) shows that, other conditions being equal, the probability of emission of a beta ray of a given energy is proportional to the ratio:

\[
F_n (Z, \gamma) = \left| \frac{\psi (R)}{\psi (\text{free})} \right|^2
\]

where \( \psi \) is the wave function of the electron for the given energy evaluated at the nuclear radius \( R \) and outside the atom as indicated. The value of \( F_n \) depends primarily upon the electric charge of the daughter nuclide and upon the momentum of the electron which may be written in terms of its free momentum.

This separation of the Coulomb effect from the transition matrix leaves it approximately energy independent in most cases and proportional to \( (R / \alpha)^{2N} \) where \( N \) is the order of forbiddenness. The transition probability distribution (Eq. 14) may then be written in the form:

\[
P (\gamma) d\gamma = C_n F_n (Z, \gamma) \gamma^2 \left( \varepsilon_0 - \varepsilon \right)^2 d\gamma
\]
where \( C_N \) is \( \propto \left( \frac{R}{R_i} \right)^{2N} \sim (1/100)^N \). \( F_n (Z, \gamma) \) changes form according to additional selection rules. \( F(Z, \gamma) \) is called the "allowed" shape factor and is usually appropriate when \( |\Delta I| < N + 1 \). For this reason Eq. 19 may be conveniently written:

\[
P(\gamma) \, d\gamma = C_N \, a_n \, F(Z, \gamma) \, \gamma^2 \, (\varepsilon_0 - \varepsilon)^2 \, d\gamma
\]

where \( a_n \) is the additional shape factor including any energy dependence in the transition matrix factor outside the Coulomb effect. It is approximately equal to one for most transitions; however, it takes "unique" shapes when \( \Delta I = \pm (N+1) \). The following are several of the "unique" shape factors, \( a_n \) under the given conditions:

\[
\Delta I = \pm (N+1), \, n = N
\]

\[
a_1 \approx \gamma^2 + 5^2
\]  
\[
a_2 \approx \gamma^4 + (10/3) \, \gamma^2 \, 5^2
\]  
\[
a_3 \approx \gamma^6 + 7 \, \gamma^2 \, 5^2 \, (\gamma^2 + 5^2)
\]

In all cases \( \gamma^2 \, (\varepsilon_0 - \varepsilon)^2 \) is the dominant shape factor in Eq. 20, so this theoretical distribution obviously has a bell-shaped distribution such as has been experimentally observed.

The experimental determination of the momentum distribution for beta radiation may be obtained from a magnetic-lens spectrometer by plotting \( N/I \) versus \( I \) where \( N \) is the counting rate and \( I \) is the corresponding current setting. A single beta activity may emit a number of beta components. Then gamma rays will take off the balance of the energy for transitions which do not go to the ground state of the daughter nucleus. The resulting beta spectrum will be a composite of the corresponding bell-shaped distributions plus the narrow-peaked distributions due to internally converted electrons from the gamma transitions. For the sake of simplicity, the procedure for analyzing a spectrum with a single beta component will be discussed first and the extension to the more complex cases will follow.

The experimental momentum distribution of a simple beta emitter is shown in Fig. 2. The maximum energy \( \varepsilon_0 \), which represents the energy of the transition, is difficult to ascertain from such a plot because of the asymptotic approach of the curve to the momentum axis. Furthermore, the statistical scatter in these points increases since an average background has been subtracted and, as the counting rate falls to the background rate, the variation in background becomes prominent.

The theory of beta decay may be utilized as follows to obtain an accurate value for \( \varepsilon_0 \). The momentum distribution should conform to the theoretical distribution in Eq. 20 so:

\[
(N/I) \, d\gamma = C \, a_n \, F(Z, \gamma) \, \gamma^2 \, (\varepsilon_0 - \varepsilon)^2 \, d\gamma
\]
where \( C \) is a constant proportional to the strength of the source and the transmission of the spectrometer. This equation may be rewritten to obtain

\[
\left[ \frac{N}{I \mathcal{E}_n^F} \right]^{1/2} = \sqrt{C(E_0 - E)}.
\]

From this equation one sees that a plot of \( \frac{N}{I \mathcal{E}_n^F} \) versus \( E \) should yield a straight line with an \( E \)– intercept at \( E = E_0 \). Such a plot is called a "Fermi" or a "Kurie" plot. The values for \( \gamma_{2F}(Z, \gamma) \) may be conveniently obtained from tables prepared by the National Bureau of Standards (30).

For a first plot of the data, \( a_n \) is assumed to be equal to one. Such a plot of the data in Fig. 2 is shown in Fig. 3. The energy intercept of a least-squares straight line fit yields a good determination of the energy of the transition.

In cases where a unique shape factor appears, the deviations from a straight line should be apparent on the first Kurie plot. The first Kurie plot of \( \gamma_{90} \) shows such a deviation (Fig. 4). The S-shaped appearance is removed with the division of each point by \( \sqrt{a_n} \), (Eq. 21A) as shown in the lower curve of Fig. 4.

The remaining nonlinearity is due to the approximation used for \( a_1 \) since Kurie plots made up with the more exact formula show no detectable deviation from linearity. From the selection rules for \( a_1 \), one may conclude that in the \( \gamma_{90} \rightarrow Z_{190} \) transition there is a spin change of \( \pm 2 \) and a parity change. For the calculation of \( a_1 \) the value of \( E_0 \) as obtained from the first Kurie plot is sufficiently accurate.

In the case of more complex beta spectra, the Kurie plot will curve upward where the counting rate is a sum of several beta components. The straight line portion above the energy of the other beta groups may be used to determine the Kurie plot of the highest energy component. The rest of the components' distribution may be calculated from the Kurie line. With this subtracted the remainder should represent the spectra of the other beta components. Subsequent Kurie plots and subtractions may be used to separate the rest of the beta groups and give a maximum energy determination for each of the groups. If the energy maxima of two groups are not sufficiently different or if a lower-energy component is relatively weak it may not be possible to see the presence of unique shapes. The resulting error in the Kurie plot may introduce errors in the apparent shape and in the energy determination for the lower energy groups. Thick sources which cause a piling up of electrons on the low energy side of the distribution may also introduce appreciable errors in the subtracted spectra of the lower-energy groups.
Fig. 2. Beta spectrum of P³²
Fig. 3. Kurie plot of P\textsuperscript{32}

\begin{equation}
\left( \frac{N}{I_{\eta^2F}} \right)^{1/2}
\end{equation}

\text{ENERGY IN MC}^2 \text{ UNITS}

1712 \pm 6 \text{ KEV}
Fig. 4. Kurie plots of $\gamma^{90}$ with and without "$a_1$" shape factor.
Another method of separating the beta spectra of various beta components is that of coincidence counting. A coincidence circuit linking the beta-ray spectrometer and a gamma-ray counter may be set to count only those beta particles that are in coincidence with certain gamma rays, thus eliminating the beta components which are not connected with the gamma transition.

In the case of unique distributions the Kurie plot gives an indication of the spin and parity change between the initial and final states of the beta transition. A more generally applicable indication of these changes may be obtained from the half-life and maximum energy values. In the case of beta transitions with allowed shape factors, the total transition probability per unit time:

$$T(\varepsilon_0) = \ln 2/ t = \int P(\eta) d\eta$$

$$= C_N \int \eta(\varepsilon_0) F(z,\eta) \eta^2 (\varepsilon_0 - \varepsilon) d\eta$$

(24)

where $t$ is the half-life of the beta component. By rearranging the terms another representation is obtained:

$$f(Z,\varepsilon_0) t = \ln 2/C_N \alpha(100)^N$$

(25)

where $f (Z,\varepsilon_0)$ is the value of the integral in Eq. 24. From this equation one sees that

$$\log f t = \text{constant} + 2N$$

(26)

where $ft$ represents the product shown in Eq. 25.

One can expect considerable spread in actual log ft values for various beta groups of the same order of forbiddenness because of the neglected factors in this treatment. Nevertheless, the experimental log ft values are found to cluster around certain values with sufficient regularity and spacing to indicate grouping according to degree of forbiddenness (31). With the exception of a few anomalies, the classifications from these groups are as expected from the nuclear configurations predicted by nuclear shell theory (32,33). Thus these log ft values indicate the spin and parity changes in the beta transitions as far as the selection rules may go in determining them. In cases where there are several radiation components originating from the same state, the half-life period for a particular beta group may be obtained from the half-life of the state divided by $p$ where $p$ is the relative number of the transitions going by way of that component. Moszkowski has constructed graphs from which a rapid calculation of log ft values may be made (34).
When the beta momentum distribution is not of the allowed shape, the integral in Eq. 24 is no longer correct due to the neglected $a_n$ factor. An additional correction factor of $(\epsilon_0^2-1)$ i.e., $\log (\epsilon_0^2-1)$ for, has been demonstrated to cluster the $|t|$ values for first-forbidden ($\Delta I=2$) unique shaped transitions (35).

B. Orbital Electron Capture

In orbital electron capture there is no observed beta radiation, for the neutrino is the only particle that is emitted in the process. The only observed radiations are the X-rays from the filling of the vacated orbitals by electrons from higher energy states and from the gamma rays that may complete the transition if the capture transition does not end at the ground state of the daughter product.

The neutrinos have the discrete momenta

$$\mathcal{S} = (\epsilon_0^2 - 1)$$

where $\epsilon_0$ is the energy of the transition as in the previous treatment and $\epsilon_B$ is the binding energy of the captured electron. The $l$ comes from the rest mass energy of the electron. Since there is only one emitted particle the number of available states in phase space

$$\rho(\epsilon_0) = \left(\frac{2\pi \hbar}{m}\right)^{3/2} \int \frac{d^3 \mathbf{r} \, d\Omega}{2\pi}$$

$$= \left[\frac{\pi^2/3}{\epsilon_0^4 + 1 - \epsilon_B^2}\right]^{3/2}$$

which has the derivative

$$\frac{d\rho(\epsilon_0)}{d\epsilon_0} = \left[\frac{\pi^2/3}{\epsilon_0^4 + 1 - \epsilon_B^2}\right]^{3/2}$$

so the transition probability per unit time from Eq. 8 becomes

$$T(\epsilon_0) = \left[\frac{2\pi \hbar^2}{m \mathcal{S}}\right] \left|\langle \psi| H| \psi\rangle\right|^2$$

The volume $V$ over which the integration has been performed drops out as before if the $1/\gamma V$ term is omitted from the wave function of the neutrino as it is normalized over all space. If, for direct comparison with Eq. 24, the same is done to the electron wave function, an additional factor comes in which is proportional to the probability of finding an orbital electron in the nucleus. By far the largest proportion of those captures are K-electrons from the innermost shell of the atom. The wave function of the K-electron is of the form

$$\mathcal{E} = N e^{-r/a}$$

(31)
where \( r_B \) is the Bohr radius and \( N \) is the normalization factor. The normalization factor is obtained by integrating over all space as follows:

\[
\int_{0}^{\infty} |\psi|^2 d\tau = 1 = N^2 \mathcal{N} r_B^3 \tag{32}
\]

to obtain the normalization factor

\[
N = 1/\sqrt{\mathcal{N} r_B^3}. \tag{33}
\]

This is approximately the factor which must be separated from \( \psi \) in \( \langle f | H | 1 \rangle \) to leave it the same as in Eq. 14 and the subsequent equations of sec A. Thus the transition probability is written:

\[
T(\varepsilon_o) = \left[ \frac{2}{\mathcal{N}} \mathcal{N}^2 \mathcal{N} r_B^3 \right] \left| \langle f | H | 1 \rangle \right|^2 (\varepsilon_o + 1 - \varepsilon_B)^2. \tag{34}
\]

The extra factor two comes from there being two K-electrons in the atom.

The binding energy \( \varepsilon_B \) for K-electrons in the non-relativistic approximation is given by

\[
\varepsilon_B = \frac{1}{2} \frac{(Z e^2/\hbar c)^2}{r_B}. \tag{35}
\]

The Bohr radius for these electrons is given by

\[
r_B = \frac{2 \hbar c^2}{Ze^2}. \tag{36}
\]

With these two substitutions, Eq. 34 becomes

\[
T(\varepsilon_o) = \left[ \frac{1}{4} \mathcal{N}^2 \mathcal{N}^2 \mathcal{N} r_B^3 \right] \left| \langle f | H | 1 \rangle \right|^2 (Z e^2/\hbar c)^3 \left[ \varepsilon_o + 1 - \frac{1}{2} (Z e^2/\hbar c)^2 \right]^2 \tag{37}
\]

which may be approximated as in Eq. 24 and 25

\[
T(\varepsilon_o) = \ln^2/t_K \sim C_N f_K \tag{38}
\]

where \( C_N \) is the same constant indicating the degree of forbiddenness and

\[
f_K = 2 \mathcal{N} (Z e^2/\hbar c)^3 \left[ \varepsilon_o + 1 - \frac{1}{2} (Z e^2/\hbar c)^2 \right]^2 \tag{39}
\]

These \( f_K \) values may also be conveniently obtained from Moszkowki's graphs (34). It should be emphasized that \( Z \) here is the nuclear charge of the parent nucleus.
Except in cases where the term \((\epsilon_0 + 1 - \epsilon_B)\) is especially small for the K-electrons the amount of L-electron capture will be small, but if this term for the L-electrons is much the larger of the two then L-electron capture probabilities must be included in \(T(\epsilon_0)\).

C. Gamma Transitions

Beta transitions do not necessarily terminate in the ground state of the daughter nucleus. When they do not, the nucleus decays from its excited state by a gamma transition to the ground state. There are two types of gamma transition processes. In one, the energy of the excited state is released with the emission of electromagnetic quanta which are called gamma rays. The alternative method of gamma decay comes with the emission of orbital electrons, which carry away the excess energy of the excited state and are called internal conversion electrons.

The probability for an electromagnetic radiation transition is proportional to the square of its transition matrix element as in Eq. 8. The matrix element in Dirac's radiation theory is given by

\[
\langle f \mid H \mid i \rangle = \frac{1}{\hbar} \int U_f^\dagger \hat{A} \cdot \hat{j} \, U_i \, d\tau \tag{40}
\]

where \(\hat{j}\) is the current operator and \(\hat{A}\) the vector potential of the emitted radiation. The other symbols have the same meaning as in the preceding section.

When the vector potential function \(\hat{A}\) is expanded as a power series in vector spherical harmonics, the terms with successively greater "polarity" \(\ell^\dagger\) \((\ell = 1, 2, \ldots)\) diminish rapidly in magnitude as in Eq. 17. The angular momentum carried by one quantum is \(\hbar \ell\) where \(\ell\) is the polarity of the transition matrix element by which it originates. Thus the angular momentum carried off by most of the quanta from a given transition is \(\hbar L\) where \(L\) is the polarity of the lowest order non-vanishing transition matrix element. Since angular momentum must be conserved

\[
\mathbf{I}_f = \mathbf{I}_i \tag{41}
\]

where \(\mathbf{I}_i\) and \(\mathbf{I}_f\) are the angular momentum quantum numbers of the initial and final states of the nucleus. Thus

\[
|\mathbf{I}_f - \mathbf{I}_i| \leq 1 \leq |\mathbf{I}_f + \mathbf{I}_i| \tag{42}
\]

The vector potential \(\hat{A}\) is directly proportional to the electric radiation field \(\vec{E}\), hence it has the same parity as \(\vec{E}\). The current operator \(\hat{j}\) has a parity of \(-1\) since it is an ordinary (polar) vector. Therefore the parity of \((\hat{j} \cdot \hat{A})\) is the opposite of \(\vec{E}\). Since the parity of the magnetic portion of the radiation field \(\vec{B}\) is opposite that of \(\vec{E}\) (36), the term \((\hat{j} \cdot \hat{A})\) has the same parity as \(\vec{B}\). The electric multipole fields have non-
vanishing radial components of $\vec{E}$ and the magnetic multipole fields have non-vanishing radial components of $\vec{H}$. So though electric and magnetic multipole radiation carry off the same angular momentum, they differ in parity as follows:

Parity of electric multipole $l$: $(-1)^l$  \hspace{1cm} (43)
Parity of magnetic multipole $l$: $(-1)^l$.

Since there does not exist any multipole radiation with $l = 0$ the relation in Eq. 41 implies that electromagnetic transitions between two states with angular momentum $I_r = I_i = 0$ are absolutely forbidden. The matrix element and with it the transition probability also vanishes unless

$II_i = II_f$ for "even" parity radiation
$II_i = -II_f$ for "odd" parity radiation  \hspace{1cm} (44)

where $II_i$ and $II_f$ denote the parity of the initial and final states of the transition.

Since for a given transition the predominant portion of the radiation will be of the lowest multipole order that is possible, Eqs. 43 and 44 indicate the selection rules from which to predict the multipole order of most of the radiation from a transition between a state with spin and parity $I_i, II_i$ and a state with spin and parity $I_f, II_f$. These selection rules are presented in Table II:

<table>
<thead>
<tr>
<th>Table II. Selection rules for the multipole order $L$ of a gamma transition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(A) $I_i \neq I_f$</strong></td>
</tr>
<tr>
<td>Parity-favored</td>
</tr>
<tr>
<td>$II_i II_f = (-1)</td>
</tr>
<tr>
<td>Parity-unfavored</td>
</tr>
<tr>
<td>$II_i II_f = (-1)</td>
</tr>
</tbody>
</table>

| **(B) $I_i = I_f \neq 0$** | Electric radiation | Magnetic radiation |
| Parity-favored | $L = 2$ | $L = 1$ |
| $II_i = II_f$ (no $I_i = I_f = \frac{1}{2}$) | | |
| Parity-unfavored | $L = 1$ | $L = 2$ |
| $II_i = - II_f$ (no $I_i = I_f = \frac{1}{2}$) | | |
It turns out that for the same value of 1, emission of electric radiation is much more probable than that of magnetic radiation. Hence in the parity-favored cases the radiation is practically pure electric multipole radiation. On the other hand, for parity-unfavored transitions where the multipole order of the magnetic radiation is less than that of the electric radiation this effect may turn out to offset the other and a mixture of the two radiations may be present. However, in most cases the radiation is predominantly magnetic in parity-unfavored transitions.

The $I_1 = I_f$ transitions form an exception to these rules. Here the $\Pi_1 \to \Pi_f$ transitions are practically pure electric dipole radiation whereas the $\Pi_1 = \Pi_f$ transitions are essentially magnetic dipole.

A more compact representation of the results of these selection rules in terms of the type of radiation predominant under various conditions of spin and parity change may be written as shown in Table III for convenience of application. The E and M represent electric and magnetic radiation, respectively, and the adjacent numeral represents the polarity of the radiation.

<table>
<thead>
<tr>
<th>Radiation</th>
<th>E1</th>
<th>M1</th>
<th>E2</th>
<th>M2</th>
<th>E3</th>
<th>M3</th>
<th>E4</th>
<th>etc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin change</td>
<td>0,1</td>
<td>0,1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>etc</td>
</tr>
<tr>
<td>Parity change</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>etc</td>
</tr>
</tbody>
</table>

Unless the energy difference between the two levels is especially low or the angular momentum difference especially high the transition rate may be so fast as to be experimentally immeasurable. If the half-life for the transition is measurable and there are no competing transitions that are much faster, the state is said to be a metastable state and the transition probability determined from the half-life gives a measure of the multipole characteristic of the gamma radiation. Weisskopf (37) has developed a theoretical formula which gives good agreement with experimental data. Montalbetti (38) has constructed a nomogram for convenient use of the formula. Goldhaber and Sunyar (39) present evidence that Weisskopf's formula gives too low a value for electric transitions of third order or higher and also give empirical equations for lifetime calculations.
The internal conversion process may be in competition with gamma-ray transitions since a nucleus in an excited state may perform the same change of states in either case.

Since the two processes are competing, the total transition probability is the sum of their individual transition probabilities. The ratio of internal conversion electrons to gamma quanta for a given gamma transition is called its internal conversion coefficient. This ratio is obviously the ratio of the transition probabilities for the respective processes. These probabilities may be calculated from the corresponding transition matrix elements. Many authors have tried various approximation methods for these calculations but their results do not check well with exact calculations. Exact calculations for K shell conversion coefficients have been published in tabular and graphical forms by Rose et al. (40). Experimental results such as obtained by Waggoner (41) and Petch (42) indicate that they are reliable for use in determining the multipole characteristics of radiation from experimental K shell conversion coefficients.

Electrons can also be ejected from the L shell. This process is usually much less probable than that of K conversion. However, the ratio of K shell to L shell conversion decreases with increasing multipole order. Thus where the ratio of K electron to L electrons are measurable, this K/L ratio may be used to determine the multipole characteristics of the transition. Graphs of the theoretical K/L ratio for different multipole orders have been published by Axel and Goodrich (43). Experimental curves have been made up from data from known gamma transitions and published by Goldhaber and Sunyar (39) for comparison with theoretical curves.

The energy of an internal conversion electron is equal to the energy of the corresponding gamma ray minus the binding energy which the electron possessed before conversion. Thus the K and the L conversion electrons are distinguishable by their energies. These energies may be measured in a magnetic-lens spectrometer to obtain the energy of the gamma transition.

D. Decay Schemes

The energies of the nuclear states involved in a radioisotope's decay may be obtained from the energies of its beta groups and its gamma rays or internal conversion electrons. The selection rules presented in the foregoing theoretical consideration give indication of the spin and parity changes between the nuclear states connected by the respective radiation components. Thus a decay scheme may be constructed indicating the nuclear energy level configurations of these states including the spin and parity of each level. This is only possible where the configuration of at least one of the levels is otherwise obtainable and then the ambiguities of the selection rules may have several alternatives for some of the spins and parities.
Nuclear shell theory also makes predictions concerning nuclear configurations. Radiation transition theory and nuclear shell theory have been used in conjunction to make fairly reliable determinations of decay schemes in many cases where neither would be sufficient alone. The two theoretical developments have made valuable contributions to each other by testing consistency of mutual predictions and where one has proved reliable it may make a positive check on the other. Selection rules for nuclear shell theory are given in review papers by Mayer et al. (32) and Nordheim (33) and compared to the results of beta decay theory.

INSTRUMENTATION

The intermediate-image beta-ray spectrometer used in these studies is described in detail elsewhere (1). A cross-sectional diagram of the instrument is shown in Fig. 5.

In the spectrometer, the turns-distribution in the three coils, $C_1$, $C_2$, and $C_3$, combines with the magnetic-iron shell and pole pieces to produce the magnetic field distribution indicated by the axial magnetic intensity shown in Fig. 6. Such a high gradient U-shaped field was found by Slatis and Siegbahn (44) to possess focusing properties which may be utilized to obtain much higher transmission than can be obtained with conventional types of magnetic-lens spectrometers at comparable resolutions. The focusing principle of the intermediate-image spectrometer is discussed in the previously-cited article (1). A transmission of ten per cent has been obtained in this spectrometer at a resolution of about six per cent.

The trajectories of focused electrons are indicated in Fig. 5. Acceptable electrons must come off the source from within a certain interval of the azimuthal angle of the symmetrical ray ($\approx 45^\circ$). The width of this interval depends upon the width of the circular transmission slit in the central baffle, $B_d$. The high degree of curvature of the trajectories of focused electrons results in good energy dispersion and so a relatively wide slit may be used in the central baffle to obtain high transmission. This curvature also results in a low scatter background since scattered electrons are unlikely to get through such a baffle system.

The charge-discriminating baffle, $B_c$, was used to separate the positron radiation of $^{44}$Sc from the negative beta radiation of $^{47}$Sc. Fig. 7 gives an indication of the performance of this baffle. The 98 kev $^{75}$Se internal conversion line was used in this check. When the baffle was set to accept only electrons, the transmission was reduced only 26.3 per cent with negligible distortion of line shape and no detectable change in energy calibration. When the baffle was set to accept only positrons, the transmission was less than 0.04 per cent. When there is no need for charge discrimination, this baffle may be withdrawn against the resolving baffle, $B_r$, where it will not effect the transmitted beam.
Fig. 5. Cross-sectional diagram of intermediate-image spectrometer
Fig. 6. Magnetic field distribution for intermediate-image magnetic lens spectrometer
Sources are mounted as indicated in Fig. 8. Beta sources or internal conversion sources are deposited on an aluminum foil which is mounted on a Lucite holder. Photoconversion gamma-ray sources are deposited in the cavity of a brass holder as shown. The radiator foil is attached to a copper cap which is thick enough to stop all the electrons emitted by the source material. The source holders fit upon the end of the brass source holder tube by which they are inserted through the pole piece into the spectrometer chamber which is evacuated to a pressure of less than $5 \times 10^{-5}$ mm Hg.

A 25 kilowatt motor-generator provides the current for the spectrometer coils. The current is set by varying the field current of the generator with a current-control device. This current control contains a servo-mechanism by which the current may be kept at a setting well within one part in a thousand. The range of the current control is 0.5 ampere to 100 amperes. With this current range the spectrometer is capable of focusing electrons with energies of more than 5 Mev.

The linearity of the current-momentum relationship of a spectrometer containing iron must be checked experimentally. Calibrations may be obtained with known conversion lines and the current-momentum curve indicated by the calibration points may be used to determine energies. The linearity of the instrument is shown in Fig. 9. These data were obtained from the (Th B C C") conversion lines whose energies have been accurately determined ($45,46,47$). Within experimental error the current-momentum relationship was found to be linear in this energy range (up to 2.5 Mev).

The linearity of the instrument at higher energies is indicated by the agreement of energy values for higher energy beta groups with those of other investigators and by the linearity of the Kurie plots. The 4,47 Mev Cl$^{34}$ positron group reported in this paper gives evidence for the linearity of the spectrometer up to that energy. The small negative value for the energy intercept of the calibration line as indicated by the insert in Fig. 9 is due to residual magnetism in the surrounding iron.

The beta-particle detector is an anthracene crystal 3/32 inch thick and $\frac{3}{8}$ inch in diameter. It is mounted on the end of a Lucite light pipe by means of small spring clips as shown in Fig. 10. An optical contact between the anthracene and the Lucite is maintained with a silicone oil. Photons created in the crystal are transmitted by means of the light pipe to a 6292 DuMont photomultiplier tube outside the iron shell where it is well shielded from the magnetic field of the spectrometer.

The arrangement used on the source end of the spectrometer for coincidence measurements is shown in Fig. 11. The source material is deposited on an aluminum foil which is supported by an aluminum ring. A thallium-activated sodium iodide crystal one inch thick and one and one-eighth inch in diameter is used as the gamma-ray detector. It is preserved by layers of MgO and Mg(ClO$_4$)$_2$. Optical contact between the crystal and the
Fig. 7. Effect of charge discriminating baffle
Fig. 8. Source mountings
Fig. 9. Calibration curve of intermediate-image spectrometer
Fig. 10. Anthracene crystal and Lucite light pipe for counting beta rays
light pipe is made with silicone oil. A $\frac{3}{4}$-inch Lucite cap absorbs electron radiation from the source. An additional aluminum absorber of suitable thickness may be inserted between the source and the crystal for the elimination of more energetic beta particles.

A block diagram of the associated counting circuitry is given in Fig. 12. The pulse-height analyzer determines the energy of the gamma rays that are counted by the gamma scaler. The spectrometer current setting determines the energy of the beta rays that are counted by the beta scaler. The beta channel and the gamma channel may be operated independently or they may be operated in conjunction with the coincidence circuit to measure coincidences between radiation particles.

The pulse stretchers are 12 $\mu$sec univibrators which eliminate the counting of after-pulses from the photomultipliers. The resulting dead time is 12 $\mu$sec per count in the beta and gamma scalers.

Pulses are fed into the coincidence circuit from the beta and gamma channels behind the pulse stretchers and the gating circuit is used to prevent the counting of more than one coincidence per pair of beta and gamma particles. A pulse stretcher on the output side of the coincidence circuit is used so that a square wave is transmitted to the coincidence scaler for the duration of time when there are simultaneous square waves impressed upon the gating circuits by the pulse stretchers from the side channels. This brings the resolving time of the coincidence circuit down to that of the duration of the pulses leaving the beta amplifier and the pulse-height analyzer, although extra coincidences from after-pulses have been eliminated with the resulting dead time of 12 $\mu$sec per coincidence count.

The length of the pulses from the beta and gamma channels is energy dependent so the resolving time also varies with energy. A two $\mu$sec delay line is switched into the line of the beta pulses to directly determine the accidental coincidence rate at any time. The resolving time has been found to vary with durations up to 0.6 $\mu$sec occurring at high energies.

Gamma-ray measurements in this investigation were made with a scintillation spectrometer. The scintillation pulse-height distributions were obtained by photographing the pulses upon a synchroscope. The energy of a gamma ray may then be obtained with the measurement of its pulse height in the photograph. The integrating effect of longer exposures may be used to bring out pulse patterns that are otherwise invisible.

A block diagram of the spectrometer arrangement is shown in Fig. 13. The gamma-ray detector is much the same as described in the intermediate-image spectrometer coincidence circuitry. The pulse shaper converts the photomultiplier pulse into a flat-topped pulse about a microsecond wide with an amplitude proportional to the input pulse. The pulses are amplified.
Fig. 11. Arrangement of source and crystal of gamma counter for coincidence measurements
Fig. 12. Block diagram of intermediate-image spectrometer circuitry
and recorded upon photographic film with the use of the synchroscope.
The Sc47 gamma-ray pulse shown in Fig. 16b gives a good indication of
the shape of the resulting pulses.

Slow drifts in amplifier gains make it necessary to calibrate close
to the time of each gamma spectrum determination. A calibration source
with gamma rays of known energy is photographed directly before and after
the unknown to insure detection of significant changes in the relationship
between energy and pulse-height. Saturation effects may make this relation­
ship nonlinear especially in the higher-energy regions, so it is necessary
to use more than two calibration lines to obtain a calibration curve or
to bracket the unknown gamma ray with two known gamma rays having close
to the same energy as the unknown.

Either a recording densitometer or a calibrated eyepiece was used in
measuring pulse-heights from the photographic film.

PROCEDURE AND RESULTS

A. Cl34

Cl34 sources were examined in the intermediate-image spectrometer (Fig. 5).
Each source was studied for about one hour until its diminished intensity
made it necessary to obtain another source. This meant that only a portion
of the beta spectrum could be obtained with a single source. Therefore, the
spectral portions from each source were overlapped so that they could be fit

together by normalization. The apparent half-life of the source in the
spectrometer as indicated by the decay rate of spectral points was 31.3 min.
This indicated that sublimation of the source was taking place in the
spectrometer vacuum. Subsequent sources were made up with the use of a
bonding agent to prevent this escape of free Cl34.

The momentum spectrum for Cl34 is shown in Fig. 14. Three positron
groups were separated by Kurie plots. These Kurie plots are shown in Fig.
15. The Kurie lines were evaluated using the least-squares straight line
for the points indicated by the brackets. The straight portion of each
Kurie plot was long enough to indicate an allowed shape for each group
although the distortion due to source thickness made the results inconclusive.
The energy maxima evaluated by the lines were $4.47 \pm 0.02$ Mev, $2.49 \pm 0.05$ Mev
and $1.29 \pm 0.04$ Mev. The possible energy shifts due to accumulation of errors
from source thickness, etc., makes the statistical evaluation of errors for
the lower-energy beta groups meaningless. Therefore, these errors were estimated
as the probable magnitude of systematic errors.

The relative intensities of the three beta groups as measured from their
corresponding spectral areas are as follows:
Fig. 13. Block diagram of gamma ray spectrometer
Fig. 14. Beta spectrum of Cl$^{34}$
Fig. 15. Kurie plots of Cl$^{34}$ beta components
Fig. 16. Photograph of pulse-height distributions from gamma ray spectrometer

(a) Cl\textsuperscript{34} pulse-height distribution  
(b) Sc\textsuperscript{47} pulse-height distribution
Fig. 17. Calibration curve used in obtaining Cl$^{34}$ gamma measurements.
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1.29 Mev beta group------30%
2.49 Mev beta group------31%
4.47 Mev beta group------39%

One gamma ray produced an internal conversion line on the beta spectrum. Its energy was evaluated from this line as 139 ± 1 kev. There was no detection of the internal conversion lines of the three higher-energy gamma rays. The photoelectron peaks obtained with the use of a 65 mg/cm² Pb radiator were of insufficient intensity for measurement.

The gamma rays were measured with the scintillation spectrometer shown in Fig. 11. A photograph of the pulse-height distribution is shown in Fig. 16a. The 2.62 Mev Th B gamma ray and the 0.662 Mev Cs137 gamma ray were used for calibration purposes. The 2.62 Mev gamma ray produced prominent escape peaks thus giving a total of four points for a calibration curve. A graph of the pulse-height versus energy is shown in Fig. 17. The energies of the gamma rays in Cl34 were obtained from this calibration curve as shown. The 3.2 Mev gamma ray could not be evaluated because of saturation in the amplifiers. Values obtained for the other gamma rays were 2.08 ± 0.04 Mev and 1.17 ± 0.03 Mev.

A coincidence beta spectrum was taken using the intermediate-image spectrometer in conjunction with the scintillation spectrometer. The scintillation spectrometer was set to count all gamma rays above about 200 kev. The beta and gamma counting rates were taken at the same time as the coincidence counting rates to enable the detection of changes that might indicate improper performance of circuit.

The resulting coincidence beta spectrum is shown in Fig. 18. The highest-energy beta group has been eliminated and the lowest-energy group has been enhanced with respect to the intermediate-energy beta group (in comparison to their ratio of intensities in the total beta spectrum). Kurie plots from these data are shown in Fig. 19. Energy values from the energy-intercepts of the least-squares straight lines through the linear sections of the Kurie plots gave values of 2.57 ± 0.04 Mev and 1.33 ± 0.02 Mev.

The gamma-ray spectrometer was set to count only the 3.2 Mev gamma rays and a coincidence beta spectrum was obtained. The coincidence counting rate was too low for an accurate determination of the maximum energy of the resulting spectrum but it was at about one to one and a half Mev, indicating that only the lowest-energy beta group is in coincidence with that of 3.2 Mev gamma ray. Since a substantial portion of the counting rates of the three higher-energy gamma rays comes from accompanying Compton scattering, it was impossible to window with the pulse-height analyzer and thus get separate coincidence beta spectra for the two intermediate-energy rays.
Fig. 18. Coincidence beta spectrum of Cl\textsuperscript{34}
Fig. 19. Kurie plots of beta components in Cl$^{34}$ coincidence beta spectrum
The scintillation spectrometer was set to count only the 140 kev gamma rays and no coincidences were observed.

The half-life of Cl$^{37}$ was determined by putting a source in the spectrometer and following the intensity at one point of the spectrum. This method has the advantage of separating out counts due to any possible contaminants of different energy along with the low background count and the short resolving time of the spectrometer. The spectrometer was set for the peak counting rate of the spectrum and the activity followed for more than four half-lives. A log plot of the decay curve is shown in Fig. 20. The half-life as evaluated from this curve is $32.30 \pm 0.14$ min.

B. Sc$^{47}$

Scandium sources were examined in the intermediate-image spectrometer. The charge-discriminating baffle was used to eliminate positrons from 58-hr Sc$^{47m}$. The upper end of the resulting spectrum did not quite go down to the background counting rate, indicating the presence of a high-energy beta group of longer half-life. The relative counting rate of this activity compared to that of the 3.13-day Sc$^{47}$ activity was very low, so a linear subtraction was used to eliminate the effect of this contaminant. The resulting beta spectrum is shown in Fig. 21. The beta groups separated by the Kurie plots are shown in Fig. 22. The points at the upper end gave some indication of a weak conversion line evidently from a contaminant so they were not used in evaluating the Kurie line. Least-squares evaluations for the points indicated within the brackets gave values of $0.596 \pm 0.010$ Mev and $0.431 \pm 0.010$ Mev. The high error assignment for the low energy group was based upon the uncertainty in the Kurie subtraction, although the points indicated a much better precision. The relative intensities of the beta groups were determined from the spectral areas indicated in Fig. 21. The spectral curves were extended to zero by calculation from the Kurie lines of the corresponding groups. The relative activities of the lower-energy and the higher-energy groups were thus estimated to be 64% and 36%, respectively. There was no indication of an internal conversion line in the spectral data. The sharp drop of the experimental curve away from the theoretical curve is attributed to the great thickness of the source (50 mg/cm$^2$).

The gamma ray in Sc$^{47}$ was measured with the scintillation spectrometer. For the first measurement the pulse shapers were not in the circuit and a pulse-height analyzer was used in conjunction with a scaler to measure the energy. The resulting energy spectrum as shown in Fig. 23 indicated a value of 167 kev for the gamma energy.

Subsequent determinations of the Sc$^{47}$ gamma energy were made from photographs of the scintillation pulse distribution. Some of these measurements were taken with both the Sc$^{47}$ and the calibration sources counting simultaneously. This was done to check against possible shift of calibration with the change of sources. The values from these measurements ranged from...
Fig. 21. Beta spectrum of $^{47}\text{Sc}$
Fig. 22. Kurie plots of Sc$^{47}$
Fig. 23. Gamma spectrum of Sc$^{47}$ obtained from scintillation spectrometer.
Fig. 24. Calibration curve used in obtaining Sc$^{47}$ gamma ray measurement.
Fig. 25. Coincidence beta spectrum of Sc$^{47}$
165 kev to 169 kev with a mean value of $167 \pm 2$ kev. A typical graph of a calibration curve used in measuring the $\text{Sc}^{47}$ gamma ray is shown in Fig. 24. A photograph of the scintillation pulse distribution is shown in Fig. 16b.

A coincidence beta spectrum of $\text{Sc}^{47}$ was obtained with the results shown in Fig. 25. The curve through the points and extending down to zero momentum has been calculated from the least-squares line through the Kurie plot. The coincidence Kurie plot is shown in Fig. 22 along with the Kurie plots from the total beta spectrum. The higher-energy beta group has been eliminated and the maximum energy value for the lower-energy groups was evaluated as $0.430 \pm 0.005$ Mev.

**DISCUSSION**

A. $\text{Cl}^{34}$

The three positron groups in $\text{Cl}^{34}$ appeared to be well resolved by the Kurie plot separation. Maximum energy values for the three groups are estimated to be $4.47 \pm 0.02$ Mev, $2.54 \pm 0.03$ Mev and $1.32 \pm 0.02$ Mev. The values for the two lower-energy groups were obtained by weighting the values determined from the coincidence spectrum $2\times$ over those determined from the total beta spectrum where one additional Kurie subtraction was involved. These values are in fairly good agreement with those of Richardson et al. (7). The branching ratios were determined by the spectral areas of the individual components with the Kurie lines used to separate the groups and to determine the curves in the low-energy region where the detector efficiency falls off. The percentage of the activity in each of the 1.32 Mev, 2.54 Mev and 4.47 Mev groups was thus evaluated as 30 per cent, 31 per cent, and 39 per cent, respectively, compared to Ruby and Richardson's values of 26 per cent, 28 per cent and 46 per cent for the same groups.

The internal conversion peak from the low-energy gamma ray was well resolved. The energy value obtained was $139 \pm 1$ kev. The internal conversion lines of the three high-energy gamma rays did not appear in the beta spectrum. Photo conversion lines from these high-energy gamma rays were not evident in the spectrum obtained with the Pb radiator. The 1.1 Mev and 2.1 Mev lines may have been obscured by the fact that they are riding on Compton spectra from higher-energy gamma rays. Thicker radiators would be expected to increase the Compton count as well as the photo conversion count and also bring in uncertainty for the correction due to the thickness of the radiator.

Values for the energies of the 1.1 Mev and 2.1 Mev gamma rays as obtained from two independent determinations with the scintillation spectrometer are $1.17 \pm 0.03$ Mev and $2.08 \pm 0.04$ Mev. These are in good agreement with the results of Ticho (6). From the peak value and width of the 1.1 Mev gamma ray, it was assumed that the 2nd escape peak for the 2.1 Mev gamma ray was not interfering in the energy determination as it did in the case of similar 1-crystal detection for Ticho.
Since the 3.2 Mev gamma ray is of higher energy than any calibration source, it is difficult to get a reliable measurement of it with a scintillation spectrometer without some other means of calibration. An accurate measurement of this gamma ray would make it a valuable calibration source in the high energy region for use where Cl$^{34}$ is readily obtainable.

The half-life value for the Cl$^{34}$ activity was 32.30 ± 0.14-min which is lower than the values of Perlman and Waffler but in good agreement with the more recently reported values of Hintz and Ramsay (32.5 ± 0.5-min) and Richardson et al. (32.40 ± 0.04-min).

The energies for radiations from this 32.4-min Cl$^{34}$ activity are summarized in Table IV.

Table IV. Radiations from 32.4-min Cl$^{34}$ activity

<table>
<thead>
<tr>
<th>Beta groups</th>
<th>Maximum energy (Mev)</th>
<th>Gamma rays</th>
<th>Energy (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.32 ± 0.02</td>
<td>$\gamma_1$</td>
<td>0.139 ± 0.001</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>2.54 ± 0.03</td>
<td>$\gamma_2$</td>
<td>1.17 ± 0.03</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.47 ± 0.02</td>
<td>$\gamma_3$</td>
<td>2.08 ± 0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma_4$</td>
<td>3.22 (Ticho's value)</td>
</tr>
</tbody>
</table>

The energies may be summed as follows for an indication of the energy levels in the Cl$^{34}$ and $\bar{S}^{34}$ nuclei:

\[ \gamma_1 + \beta_3 = 4.61 \pm 0.02 \text{ Mev} \]
\[ \gamma_3 + \beta_2 = 4.62 \pm 0.04 \text{ Mev} \]
\[ \gamma_4 + \beta_1 = 4.54 \pm 0.04 \text{ Mev} \]
\[ \gamma_2 + \gamma_3 + \beta_1 = 4.57 \pm 0.03 \text{ Mev} \]

(using Ticho's value for $\gamma_4$).
Stahelin (11) showed that $\nu_1$ is associated with the transition from a metastable state to the ground state of Cl$^{34}$. These energy values then indicate that only $\nu_2$ is associated with the ground state of Cl$^{34}$ which Stahelin reported to decay with a half-life of $1.58 \pm 0.05$ sec. Kline has recently obtained a value of $1.53 \pm 0.02$ sec for Cl$^{34}$ (48).

The coincidence data gave no measurable coincidence counting rate between the 140 kev gamma ray and the beta groups. This is as expected if the transition is from a metastable state to a 1.5-sec ground state of Cl$^{34}$.

The coincidence data show that the 3.2 Mev gamma ray is in coincidence with the low-energy beta group only. In the coincidence spectrum obtained from coincidences with all gamma radiations, both the 1.3 Mev and the 2.5 Mev beta groups appear, but the increase in intensity of the 1.3 Mev group relative to that of the 2.5 Mev group is 85%, which indicates that 45% of the counts in the lower-energy spectrum are associated with gamma rays which are not in coincidence with the 2.5 Mev group. This difference is several times greater than that which may be due to the counting of more 3.2 Mev radiation than when the 1.3 Mev coincidence beta spectrum was obtained. These results confirm the energy level scheme which is implied by simply considering the energies of the radiation components and taking into account the metastable state in Cl$^{34}$.

The proposed decay scheme of Cl$^{34}$ and Cl$^{34m}$ is shown in Fig. 26.

The 0+$\rightarrow$0+ assignment to the ground state of $\lambda^{34}$ is based upon the fact that it is an even-even nucleus (32). The log ft value of 3.8 for the 4.47 Mev beta group classifies it as a "super-allowed" transition indicating an $\nu \rightarrow 0+$ transition according to Stahelin.

The branching ratios of the three beta groups may be used to obtain a mean lifetime value of 7400 sec for the 140 kev gamma transition from a metastable state. The mean lifetime for a 140 kev M3 transition in $\lambda^{34}$ as determined theoretically with the use of Montalbelli's nomogram (34) is 500 sec. Using Rose's tables (48) to correct for the additional internal conversion transitions, the value for the case of an M3 transition is predicted to be about 430 sec. The next longer lifetime possibility is an E4 transition which is predicted to be about $2.5 \times 10^7$ sec. Thus the experimental lifetime of the transition definitely indicates that it is M3. This leads to an assignment of 3+$\rightarrow$0+ to the metastable state in Cl$^{34}$ (Table III, p.22) which agrees with Stahelin's assignment from measured internal conversion values (11). The internal conversion spectrum could not be determined from the beta spectrum in Fig. 14 since the gamma peak is near the low energy cut-off of the beta detector which was set high to reduce background. Ruby and Richardson's value of 17:1 for the ratio of beta-rays to conversion electrons plus the branching ratios determined in this experiment give a value of 0.15 for the K-shell conversion coefficient of Cl$^{34}$ which compares well with the theoretical value of 0.17 obtained by extrapolation from Rose's Tables.
Fig. 26. Proposed decay scheme of Cl$^{34}$
The log ft value for the 2.5 Mev beta group as calculated from the branching ratios is 6.3. This classifies it as a first-forbidden transition with a spin change of zero and a parity change or possibly an allowed 1-forbidden transition which has a spin change of one and no parity change. The log ft values for the first suggested possibility are found to cluster around 6.5 while the log ft values for 1-forbidden transitions scatter from about 5.0 to 9.0.

The first alternative would give a 3- assignment to the 2.08 Mev level in $^3$He. The gamma transition to the ground state would then be E3. The nomogram gives a value of 0.5 $\mu$sec for this transition. Goldhaber and Sunyar (39) present experimental evidence that Montalbelli's nomogram (Weisskopf's formula) gives low values for lifetimes of E3, E4 and E5 transitions. They give an empirical formula for $\Delta I = 3$ transitions as follows:

$$T \text{ (mean lifetime)} = 17.5 - 7 \log E \text{ (kev)}$$

from which the expected lifetime of 2.1 Mev E3 transition is calculated to be 2.5 $\mu$sec. Since the resolving time of the coincidence spectrometer is 0.5 $\mu$sec a first-forbidden transition is considered unlikely.

If the transition is an 1-forbidden one, the 2.08 Mev excited state must be 4+ or 2+ and the radiation associated with the transition E4 or E2, respectively. The possibility of E4 radiation is eliminated since such a transition would be slower than an E3 transition. Therefore the state is given a 2+ assignment and 2.54 Mev beta transitions is classified as 1-forbidden.

A log ft value of 5.0 is obtained for the 1.32 Mev beta group. This classifies it as a normal-allowed transition giving possibilities of 2+, 3+ or 4+ configurations for the 3.25 Mev level in $^3$He. A 4+-0+ transition would be too slow to produce coincidences in the coincidence circuit. The branching ratio for the 1.17 Mev and 3.25 Mev gamma rays may be calculated from their predicted lifetimes. If the second excited state is 3+ this ratio would figure to be 5000:1. Therefore the 3.25 Mev excited state is assumed to be 2+ which would indicate a branching ratio of 3:1. Such a branching ratio is acceptable since the scintillation spectrometer data indicated that the two gamma rays are in the same order of intensity.

This completes the spin and parity assignments for the proposed decay scheme of $^3$He. The assigned configurations in $^3$He are in agreement with the observation of Glaubman (49) that in even-even nuclei low lying states have even parity with even spin states and odd parity with odd spin states. Talmi (50) shows that such a rule may be explained in terms of either j-j coupling or LS coupling configurations. The 2+ assignments for the excited states are also in agreement with the observations of Scharff-Goldhaber (51) that the first excited state in most even-even nuclei is 2+ and either 2+ or 4+ for the second excited state as predicted by the j-j coupling model (52).
Although the source of Sc\textsuperscript{47} was very thick (~58 mg/cm\textsuperscript{2}), the straight lines obtained for the lower-energy group indicated a sufficient interval of undistorted spectral data for the purpose of energy measurements. The low counting rate and the lack of sufficient points on the portion of the Kurie plot above the lower-energy beta group made interpretation of the hump at the upper end of the Kurie plot uncertain. The shape of the hump suggested the presence of a weak conversion line from a contaminant. Therefore, these points were not included in the calculations. The success of this procedure in giving a subtracted Kurie line that agrees very well with that of the coincidence spectrum and the agreement in energy difference of the resultant beta energies with the energy of the gamma ray is considered as confirmation of the validity of this procedure.

The value of 167 kev obtained for the Sc\textsuperscript{47} gamma ray with the scintillation spectrometer (Fig. 23) was at first questioned because of the value of 185 kev obtained by Cheng and Pool from a conversion line in their spectrometer data. Repeated measurements by the photographic method were made. Some pictures were taken of the simultaneous reception of radiations from a calibration source as well as the Sc\textsuperscript{47}. A value of 167 ± 2 kev was obtained with the use of five independent measurements.

The absence of a conversion peak from the same gamma transition may indicate that the polarity of the radiation is low and may also be partially due to the absorption in the source which apparently sets in at about 180 - 200 kev as indicated in Fig. 21 by the deviation of the experimental curve from that calculated from the Kurie line.

The branching ratio of the lower-energy group to the higher-energy group as indicated by the extrapolated spectral areas is 64% to 36% which compares well with the branching ratio of the two components found by Cheng and Pool.

The coincidence data shows that only the lower-energy beta group is in coincidence with the gamma ray. The energy determination from the coincidence spectrum was weighted over that obtained by Kurie plot subtraction with a resulting value given for the lower-energy beta group of 0.430 ± 0.005 Mev.

The maximum energy for the other beta group was determined from one spectrum as 0.596 ± 0.010 Mev. The significant portion of the Kurie line of this group was too short to give an indication of shape factors other than that implied by the success of the allowed Kurie plot subtraction in obtaining a straight line for the other component with the expected energy.

The discrepancy of these results with those of Cork et al. may be partially due to the coincidental presence of beta groups of about 150 kev in both Ca\textsuperscript{47} and the Sc\textsuperscript{47}. The agreement of the Feather absorption curve
obtained for the separated Sc\textsuperscript{47} fraction with that obtained from coincidence with the 160 kev gamma rays using the original Ca\textsuperscript{47} + Sc\textsuperscript{47} source may conceivably be explained by the counting of some gamma rays from Ca\textsuperscript{47} which would bring in some counts from the higher-energy beta groups in Ca\textsuperscript{47}.

The disagreement of these results with those of Marquez is unexplained.

The discrepancy between these results and those of Cheng and Pool is particularly troublesome because of the agreement on other points. A lower-energy beta group is measured with about the same energy in both cases. The gamma ray and the other beta group are both given about 20 kev higher energy assignments from their work, the gamma ray determination having been made both from the conversion peak and a scintillation spectrum. At the same time the branching ratios found in both cases are approximately the same and the three components in either case fit together to form a logical decay scheme. Further investigation is desired in order to resolve the apparent conflict of facts in these two papers.

The proposed decay scheme for Sc\textsuperscript{47} as determined from the results of this investigation is shown in Fig. 27. The calculated energy changes for the alternative transition paths to the ground state of Ti\textsuperscript{47} agree well within the probable error. Klinkenberg's convenient table of nuclear shell structure (53) was used for orbital assignment of the ground states of the two nuclides. According to these tables Ti\textsuperscript{47} is predicted to have a f7/2 ground state. Sc\textsuperscript{47} is expected to have the same orbital configuration as Sc\textsuperscript{45} which is also classified with the f7/2 group. The log ft value of 5.2 for the lower-energy group indicates that it is an allowed transition. This leads to an assignment of f5/2 to the 167 kev state in Ti\textsuperscript{47}. The gamma transition would then be magnetic dipole which has a K-shell conversion coefficient of about 4 \times 10^{-3}. This would partially indicate the reason for the absence of a detectable conversion line in the beta spectrum of Sc\textsuperscript{47}. A beta spectrum from a thinner source might reveal such a conversion line.
Fig. 27. Proposed decay scheme of Sc$^{47}$
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