

3-D MODELING OF ULTRASONIC SCATTERING FROM
INTERGRANULAR STRESS CORROSION CRACKS

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INTRODUCTION

This paper describes the derivation of a three-dimensional model for scattering of elastic waves by an intergranular stress corrosion crack (IGSCC). The model is based on a geometrical abstraction of the IGSCC. The crack has a main stem and left and right branches. The transducer is on the upper surface and the crack breaks the lower surface of a flat plate. The beam of transverse wave motions radiated by the transducer insonifies the crack under an angle of 45° with the lower face of the plate. Figure 1 shows the plane of symmetry of the configuration. The current profile of the beam is Gaussian, but the use of other beam profiles is possible. The position of the center of the transducer can be varied in the plane of symmetry of the crack. This allows the simulation of sizing techniques which require probe motion. The backscattered field has been computed by using the Kirchhoff approximation for the crack-opening displacements of the individual components of the branched crack. The backscattering by the main stem and the branches has been analyzed and superimposed to provide the total backscattered field in the frequency domain. Numerical results are presented.

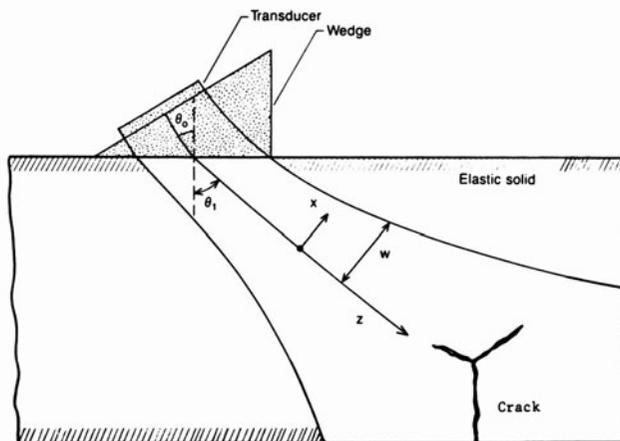


Fig. 1 Configuration in the plane of symmetry of the transducer and the crack.

Crack Model. The 3-D model presented herein is an extension of the 2-D model of Thompson et al, see [1]. The lengths and widths of the vertical stem and the branches, as well as the angular orientations of the two branches, were considered as variables. The mathematical crack was constructed by patching together three half-ellipses in the manner depicted in Fig. 2b.

Beam Model. In the description of the transducer field we follow the model proposed by Thompson and Lopes [2], which assumes that the beam profile is Gaussian in all cross-sectional planes. In this model the displacement field of a beam of transverse wave motion takes the form

$$u_i = A e^{-i\phi} d_i e^{ik_T z} e^{-r^2(1/w^2 - ik_T/2R_c)}, \quad (1)$$

where $|d| = 1$ and $d \perp y$, z is the direction of propagation, and $k_T = \omega/c_T$ is the wavenumber, c_T being the velocity of transverse waves. The third exponential of (1) defines the Gaussian nature of the beam, and the first is simply a phase factor which is premultiplied by an amplitude factor. The term $\exp(-i\omega t)$, where ω is the angular frequency, has been suppressed.

AULD'S FORMULA

To characterize the back-scattered field we will employ Auld's formula, as opposed to the usual representation integrals for stresses or displacements. There are two advantages to the use of Auld's formula. Firstly, it gives us a field quantity (here and henceforth called Auld's parameter) which can be measured (electrically) external to the elastic body. Secondly, the integration involved in Auld's formula is somewhat simpler, since the integrand is free of Green's functions. For the derivation of Auld's formula the reader is referred to [3]. For back-scattering by a flat crack lying in the x_1x_2 -plane the formula takes the form

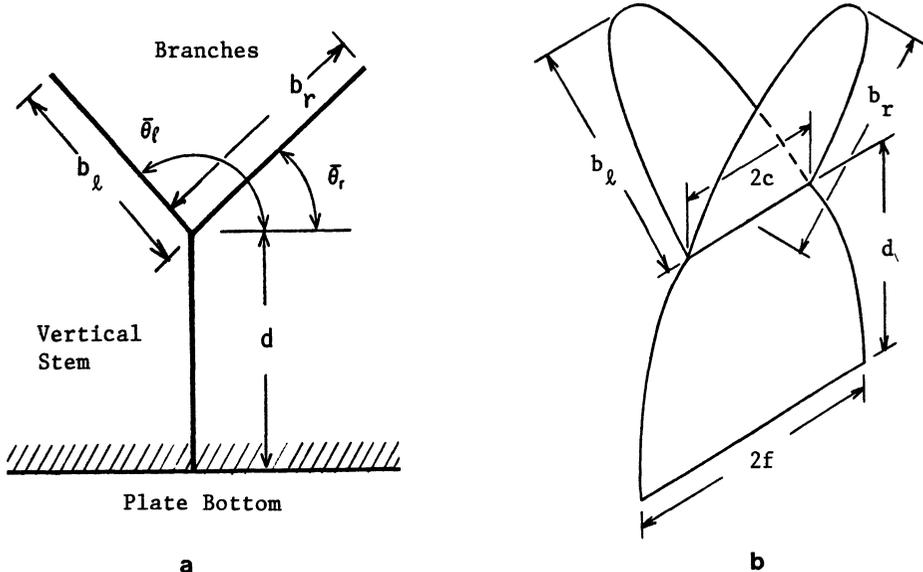


Fig. 2 Model of the IGSCC: (a) side view, (b) skew view.

$$\delta\Gamma = \frac{i\omega}{4P} \int_A \sigma_{3i}^{tr} \Delta u_i n_3 dA, \quad (2)$$

where $\delta\Gamma$ is Auld's parameter, and P is the electrical power incident on the transducer. Also, σ_{ij}^{tr} denotes the transducer stress field, i.e. the stress field that would exist in the absence of the crack, and Δu_i denotes the crack-opening displacement. The integral is carried out over the insonified surface with unit outward normal n_3 .

CALCULATION OF CRACK-OPENING DISPLACEMENT

The utility of (2) lies in our ability to first calculate Δu_i , the crack-opening displacement. To do so for the stated problem would involve the solution of a complicated singular integral equation over the crack surface, which for the geometry of our branched crack would necessitate the use of a numerical scheme, such as the boundary element method. Alternatively, we can make an intelligent estimate of the crack opening displacement with far less computational effort by making use of a few simple assumptions, each of which will be briefly discussed below.

Assumptions. Firstly, we will ignore the effect of the top surface of the plate. In doing so we can predict only those first-arrived signals that have interacted either solely with the crack or with the crack and the lower surface of the plate. Secondly, we will assume that the beam behaves locally like a plane wave. This assumption will greatly simplify both the evaluation of Auld's formula and the description of the beam after reflection from a traction-free surface. For example, the assumption implies that (for an angle of incidence of 45°) the incident beam is reflected as another beam of transverse motion, without mode conversion.

Lastly, and related to the previous assumption, in calculating Δu_i it will be assumed that the field on the insonified crack face is locally the same as on an insonified traction-free plane, while on the shadow side the field is assumed to vanish. This assumption is known as the Kirchhoff approximation, and amounts to taking as Δu_i the sum of the incident and reflected waves on an insonified traction-free plane.

Ray Cases. In order to apply the Kirchhoff approximation to the branched crack, we first consider the ways in which the segments of the crack can be insonified: either directly by the transducer, or after one reflection off the bottom surface of the plate. These paths of insonification are denoted as "ray paths". In considering the crack as consisting of three segments, i.e., two branches and the vertical stem, the crack's insonification consists of six different "ray cases". However, it may be assumed that the right branch is not insonified by wave motion reflected from the bottom surface of the plate, since this branch is shadowed by the left branch (see Figs. 1 and 2a). Hence, the six ray cases are further reduced to five. To each of these ray cases we apply the Kirchhoff approximation. For any one ray case we write the crack-opening displacement as

$$\Delta u_i = U_T(x_1, x_2) [d_i^T + R_L^T d_i^{rL} + R_T^T d_i^{rT}] e^{ik_T^T x_1}, \quad (3)$$

where we have affixed a local x_1, x_2 coordinate system to the plane

of the crack segment. The premultiplying function contains the amplitude, phase, and Gaussian nature of the beam. The three terms in square brackets represent the successive contributions from the incident T-wave, and the reflected L- and T-waves. Explicit expressions for the longitudinal and transverse reflection coefficients, R_L^T and R_T^T , can be found elsewhere, see for example [4].

TRANSDUCER STRESS FIELD

Since we have five crack opening displacements, Δu_1 , Auld's parameter $\delta\Gamma$ of (2) will consist of contributions from five separate integrals. Each of these integrals can, however, be further broken into two integrals, since the transducer stress field σ_{ij}^{tr} , which appears in (2), consists of two terms:

$$\sigma_{ij}^{tr} = (\sigma_{ij}^{tr})^d + (\sigma_{ij}^{tr})^r. \quad (4)$$

Here the d-superscript term denotes the field emanating directly from the transducer, and the r-superscript term denotes the field generated by one reflection off the lower surface of the plate. These two fields are calculated from their respective displacement fields, giving at $x_3 = 0$

$$(\sigma_{ij}^{tr})^{d,r} = ik_T \mu [d_{ip_j}^T + d_{jp_i}^T]^{d,r} U_T^{d,r}(x_1, x_2) e^{ik_T (p_1^T)^{d,r} x_1}, \quad (5)$$

where in carrying out the differentiation we have operated only on the plane-wave parts of the displacement fields.

EVALUATION OF THE INTEGRALS

In evaluating the integrals of (2), we make further use of the assumption that the incident beam may be locally approximated by a plane wave, by assigning the Gaussian function $U_T(x_1, x_2)$ of (3) and (5) its value at the centroid of the relevant crack segment. In doing so, the evaluation of the integrals of (2) is reduced to the evaluation of integrals of the form

$$I = ik_T \int_A e^{ik_T \gamma x_1} dx_1 dx_2, \quad (6)$$

where γ can take on any of the following forms, depending upon the ray case:

$$\gamma = \pm 2\sin\theta_T, \quad \pm (\cos\theta_T - \sin\theta_T), \quad \pm (\cos\theta_T + \sin\theta_T). \quad (7)$$

Here θ_T is the angle of incidence of the incoming beam of transverse motion measured from the outward normal of the particular crack segment.

The area integrals (6) over the half-elliptical branches can be converted to line integrals, as shown in Ref.[5]. The integration can subsequently be carried out rigorously using the result of [6]. For a branch of length b and width $2c$ (see Fig. 2.), there results

$$I_{\text{Branch}} = \frac{c}{\gamma} [2-\pi\{H_1(k_T \gamma b) - iJ_1(k_T \gamma b)\}] - \frac{2c}{\gamma}, \quad (8)$$

where H_1 and J_1 denote the first order Struve and Bessel functions, respectively, both of the first kind. Using the same approach, the evaluation of (6) over the truncated half-elliptical main stem can be approximated as the difference between integrations over two half-ellipses.

NUMERICAL RESULTS

Numerical results will now be given for a crack with the following configuration, see Fig. 2: $\theta_\ell = 120^\circ$, $\theta_r = 60^\circ$, $b_\ell = b_r = .15$ cm, $d = .25$ cm, $2f = 1.0$ cm, $2c = .96$ cm. The magnitude of a dimensionless version of Auld's parameter, $|\overline{\delta\Gamma}|$, is plotted versus the dimensionless shear wavenumber, $k_T d$. The plots are essentially frequency response functions of the branched crack.

In Fig. 3 we show the contributions to the response function of two of the five ray cases. In these two plots the beam centerline coincides with the crack mouth M, i.e. the beam is being directed towards the mouth of the crack. There is considerable difference in amplitudes in the two figures, since one ray case is much closer to specular reflection than the other.

In Fig. 4a we show the combined contributions of all five ray cases, giving us the total backscattered response function. A composite plot of the two ray cases corresponding to corner reflection looks essentially like Fig. 4a without the wiggles. The wiggles in Fig. 4a are due to scattering by the branches.

In Fig. 4b we show the effect of moving the transducer over the upper plate face, so that the center of the beam will be directed to points a distance of $\sqrt{2} \cdot x$ from the crack mouth M. This simulation of probe motion is essential to crack sizing techniques.

To conclude we mention the utility of the complex-valued frequency response functions, whose norms have been plotted in Fig. 4a,b. By multiplying the complete response functions by the Fourier transform of an incident time-domain waveform, and then taking the inverse Fourier transform of the product, there results the time-domain back-scattered pulses. Thus one can create synthetic experimental data with which to

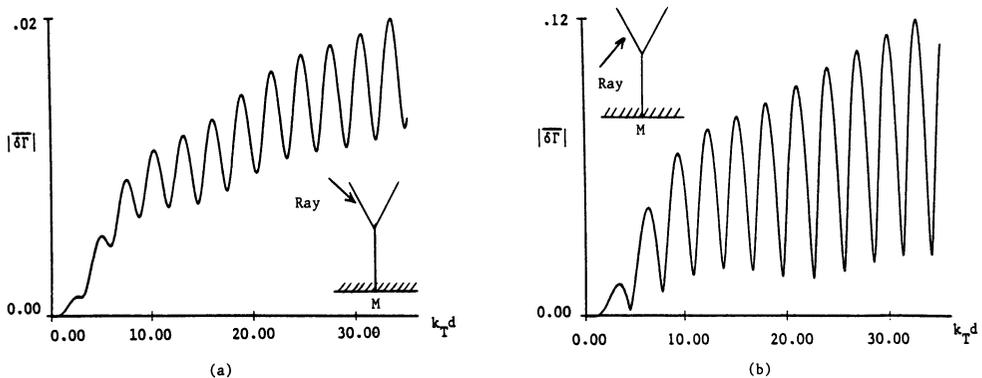


Fig. 3 $|\overline{\delta\Gamma}|$ versus $k_T d$ for (a) direct insonification of left branch, and (b) insonification of left branch by reflected wave.

compare to actual experimental back-scattered pulses. The details of this procedure have been carried out for the two-dimensional case in Ref.[1]. Furthermore, for an unknown crack geometry, measured data can be contrasted with model results in a method to infer the geometrical parameters of the crack.

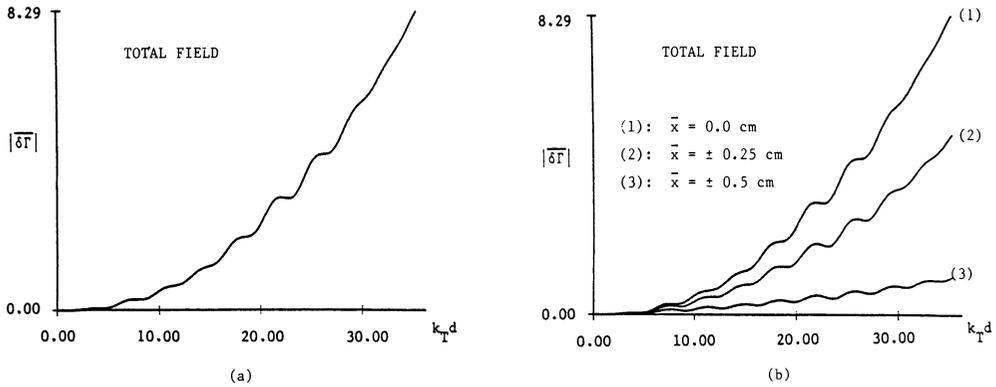


Fig. 4 $|\delta\Gamma|$ versus $k_T d$ for (a) total backscattered field, and (b) total backscattered field with simulated probe displacement.

ACKNOWLEDGMENT

This work was partially sponsored by the Electric Power Research Institute, under subcontract SC-85-089 from Ames Laboratory, and partially supported by the US Army Office of Research under Project DAAG29-84-K-0163.

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