PLATE MODES GENERATED BY EMATS FOR NDE OF PLANAR FLAWS*

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INTRODUCTION

The theory of SH-wave generation by electromagnetic-acoustic transducers (EMATs) and the representation of the waveform in propagating plate modes was developed in [1]. Since then several theoretical and experimental studies have been reported that deal with SH-wave scattering by normal flaws (cracks) and its use for sizing these cracks [2-10].

In this paper we have studied theoretically and experimentally SH-wave scattering by planar canted cracks in a plate. Generation of various plate modes by a transducer, their interaction with a canted crack, and then reception of the scattered signal by a receiver have been analyzed. Calculations have been based on a hybrid finite element and modal expansion technique. Predicted scattered waveforms due to canted cracks of different lengths are compared with experiments performed on slotted plates. This background narrows the experimental search for the best transducer geometry to obtain a simple signal most useful for flaw detection and sizing. The NDE parameter used in some recent studies has been the ratio of the amplitudes of signals back-scattered from and transmitted through a flawed region in a plate [9,10].

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Consider a canted crack in a plate as shown in Fig. 1. For the purpose of the present method of analysis, the plate is divided into three regions, \( R_1^- \), \( R_1^+ \), and \( R_2 \) as shown. It is assumed that the plate has the same homogeneous uniform properties in \( R_1^+ \) but may have different properties in \( R_2 \). Also, \( R_2 \) may have any number of cracks or inhomogeneities.

For SH motion with nonzero displacement \( w(x,y,t) \) in the \( z \)-direction only, the equation satisfied by \( w \) is,

\[
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + k_2^2 w = 0, \quad (x,y) \in R_1^+ \tag{1}
\]

where \( k_2 \) is the wavenumber and we have assumed that \( w(x,y,t) = w(x,y)e^{-i\omega t} \).

In \( R_1^+ \), \( w \) satisfies the boundary conditions

\[
\frac{\partial w}{\partial y} = 0 \text{ on } y = 0, \quad H
\]

Solution to equation (1) satisfying (2) admits an infinite set of propagating and nonpropagating waves of the form,

\[
w_n = \cos(n\pi y/H) \ e^{\pm ik_n x}, \quad n = 0, 1, 2, \ldots \tag{3}
\]

where \( k_n = \sqrt{k_2^2 - n^2\pi^2/H^2} \). It was shown in [1] that the waves generated by an EMAT placed on the surface \( y = 0 \) can be expressed in terms of the plate modes (3) as, (see [3])

\[
w(x,y) = \frac{ik_2 MD}{2\rho v_2 H} \frac{N_c I_e B_0}{\omega W} \sum_n \frac{\epsilon_n F}{k_n} \cos \frac{n\pi y}{H} \ e^{\pm ik_n x}, \tag{4}
\]

where:
- \( \rho \) = density of the plate,
- \( v_2 \) = shear wave speed in the plate,
- \( W \) = width of the transducer,
- \( I_e \) = eddy current induced in the plate,
- \( N_c \) = number of turns in the coil,
- \( B_0 \) = normal component of the applied magnetic field,
- \( \epsilon_n \) =
  \begin{cases} 
    1, & n = 0 \\
    2, & n > 0.
  \end{cases}
- \( F = \frac{\pi \sin(k_n D/4)}{2(k_n D/4)} \frac{\sin[(MD/4)(k_n-2\pi/D)]}{M \sin [(D/4)(k_n-2\pi/D)]} \exp(iM\pi/4) \)
- \( M \) = number of magnet pairs
- \( D \) = magnet spacing
Fig. 1 Cross section of transmitter and receiver (T and R) transducers on the surface of a plate containing a canted flaw. The plate is divided into three regions: $R_1^+$ = unflawed, $R_2$ = flawed. The $z$-coordinate is normal to this drawing.

Now for the laboratory configuration shown in Fig. 2 with the transmitter and receiver placed at $y = H$ at a distance $d$ apart, the normalized receiver signal as a function of time can be expressed approximately as

$$S(d,t) = \sum_{\ell=1}^{L} \sum_{n=0}^{N} F_{\ell n}(d,t) A_{\ell n}^T A_{\ell n}^R \sin(\omega_k t - k_{\ell n} d)$$

when there is no scatterer present. The definitions of various quantities above are:

- $\ell$ = frequency index,
- $n$ = mode index,
- $W_{\ell}$ = weight assigned to each frequency component,
- $N$ = highest order plate mode allowed to propagate,
- $F_{\ell n}(d,t)$ = envelope factor,
- $A_{\ell n}^T$ = normalized transmitter array factor,
- $A_{\ell n}^R$ = normalized receiver array factor,
- $k_{\ell n} = \sqrt{(\omega_k/\nu_2)^2 - n^2 \pi^2/H^2}$.

Figure 2 shows the receiver signals at different values of $d$. The transducers were EMATs with 16 pairs of magnets (M) with a spacing of 3.9 mm (D). The frequency was 454 kHz (2.85 $\times$ 10$^6$ rad/s). The test material was A516 steel. Also shown in this figure are the signals computed according to Eq. (5) with $L = -3$, $\omega_1 = 2.639$, $\omega_2 = 2.827$, and $\omega_3 = 3.016$, all in $10^6$ rad/s. The weights assigned on the basis of experimental spectral analysis were $W_1 = W_3 = 0.6$, $W_2 = 1.0$, and the pulse envelope $F_{\ell n}$ was taken to be a triangle as shown in Fig. 3. The shear wave velocity in the plate was $\nu_2 = 3.17$ km/s. Qualitatively, there is good agreement between the computed and measured signals, although there are quantitative differences.
Signal characteristics of an SH-wave transmitted along an unflawed plate. This compares the experimental and calculated signal shape at several transducer separations. The rf carrier is omitted here and only the signal envelope is shown.

Fig. 3 Simple triangular envelope imposed here on the calculated signal. The true pulse shape is largely due to the duration of the gated rf tone burst (5 cycles here), and the transducer size and configuration. $t'$ is the arrival time of the pulse front, and is a function of the plate modes since they each travel down the plate at different speeds as determined by the angle of their wave fronts. $\Delta t$ is the pulse duration; the experimental value in this study was 20 $\mu$s.
Once the reflection and transmission coefficients are obtained [8] for a particular frequency, then the reflected and transmitted fields can be calculated at any point. In particular, for the experimental setup considered here (Fig. 1) we obtain the reflected or transmitted pulse at a point as

\[
S(d,d',t) = \sum_{m=0}^{L} \sum_{n=0}^{N} F_{k_{mn}}(d,d',t) A_{km} A_{ln} (-1)^{m+n} S_{kmn} \times \sin(w_{k} t - k_{km} d - k_{ln} d' - \alpha_{kmn})
\]

where the parameters are the same as in Eq. (5) except:

- \(m\) = incident mode index
- \(n\) = scattered mode index

\[
S_{kmn} = |R_{kmn}| e^{i\alpha_{kmn}} \quad \text{for the reflected field}
\]

\[
S_{kmn} = |T_{kmn}| e^{i\alpha_{kmn}} \quad \text{for the transmitted field}
\]

\(R\) and \(T\) are the reflection and transmission coefficients with phase shifts on scattering of \(\alpha_r\) and \(\alpha_t\), respectively.

The experimental conditions were the same as noted above except that now there was a sawcut canted at 30° to the surface normal to simulate the flaw shown in Fig. 1. On the test plate, \(b=0\) (the sawcut was on the bottom surface) while the calculation for the scattering coefficients assumed \(b/H=0.1\) (buried crack). To compare the theoretical predictions with experimental results, three different angular frequencies \(\omega_1, \omega_2, \omega_3\) as given before were chosen. For each of these frequencies there were five propagating modes (\(N = 4\)). Computed pulse shapes are compared with observed ones for the different flaw sizes in Figs. 4-7. At some receiver locations the agreement is reasonably good. However, at some other locations there is only qualitative agreement.

The discrepancies between the computed and observed pulse shapes may arise for several reasons:

1. Finite transducer size;
2. Actual flaws not buried, but surface-breaking;
3. Frequency content of the actual pulse continuous with a finite bandwidth;
4. Inhomogenous plate with non-flat surfaces;
5. Gated pulse shape.

Even with the simplifying assumptions made in the theoretical computations, there is a remarkable qualitative agreement with observations. Improving on items 2, 3, and 5 above would be especially important in increase this agreement.

As an aid to experimental NDE, these calculations will serve to guide the selection of transducer locations. With proper placement, it is possible to obtain a signal with maximum amplitude, simplest shape, and least sensitivity to minor changes in test geometry to increase probability of detection and sizing reliability. Flaw size, orientation, and location through the depth all influence mode interaction. This gives the possibility that detailed analysis of signal configuration will yield much information on flaw character.
Fig. 4  Reflected signal envelopes from a 30° canted flaw. D/H = 0.3.

Fig. 5  Transmitted signal envelopes from a 30° canted flaw. D/H = 0.3.
Fig. 6 Reflected signal envelopes from a 30° canted flaw. \( D/H = 0.5 \) for calculations and \( D/H = 0.6 \) for experiment.

Fig. 7 Transmitted signal envelopes from a 30° canted flaw. \( D/H = 0.5 \) for calculations and \( D/H = 0.6 \) for experiment.
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