REFLECTION OF BOUNDED ACOUSTIC BEAMS FROM A LAYERED SOLID

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INTRODUCTION

It is well known that when a bounded beam of acoustic waves is incident on a fluid-solid interface at certain critical angles, the reflected beam is significantly distorted and displaced due to the interference between specularly and nonspecularly reflected waves. Measurement and analysis of the reflected field can be used to estimate certain near surface elastic properties of the solid by means of several alternative non-destructive experimental arrangements [1,2]. In most of these experiments the interface generated leaky waves play a significant role. Thus a good understanding of the interface phenomena is a prerequisite to the design of experiments for their practical applications.

A detailed examination of the leaky waves and of their influence on the reflected field was carried out in [3] for a uniform half space model of the solid. The problem of the multilayered solid consisting of one or more isotropic elastic layers bonded to a homogeneous half space has been considered in a number of papers [4-6]. The multilayered plate with the bottom surface free of traction has also been considered in a more recent paper [7]. These and other recent studies have shown that in presence of layers, the interface phenomenon exhibits many features which, in general, can not be extrapolated from the simple half space [3] or thin layer approximations [4]. The primary reason for this is the presence of dispersive guided waves which can propagate along the layers with relatively low spatial attenuation.

In this paper we discuss the interface related wave phenomena in a solid consisting of a top layer (InSb) which is bonded to a homogeneous substrate (Si) by means of a relatively thin and low velocity bonding material (epoxy). We also consider the same problem for a plate obtained by removing the substrate as a reasonable model of the layered solid containing a large debond at the epoxy-Si interface. We present dispersion
curves for both models in a wide frequency range and show that the epoxy layer, however thin, has a strong influence on the phase velocity of the Rayleigh waves in the half space model. In addition, we calculate the $V(z)$ curve or the AMS for each model at a fixed frequency and show that the presence of the debond significantly alters their oscillatory character.

THEORY

Let a cartesian coordinate system be located at the fluid-solid interface with the $y$-axis directed into the solid (Fig.1). We assume, for simplicity that all field variables describing the wave motion in the medium are independent of $z$, so that the mathematical problem is two dimensional.

We first consider the guided wave problem in the solid in absence of the fluid. The general theory of Rayleigh wave propagation in a multilayered half space can be found in the standard seismological literature [8]. The theory of Lamb type wave propagation in multilayered plates appears to have received less attention in the literature except in the case of the uniform (i.e., single layered) plate which has been studied in great detail [9]. The dispersion or secular equation for a multilayered plate has been discussed in [7]. The general form of this equation for both the half space and the plate is

$$D(k, \omega) = 0$$  \hspace{1cm} (1)

where $k$ is the wavenumber of the guided waves, $\omega$ is the circular frequency and $D(k, \omega)$ is the dispersion function. For a homogeneous half

![Figure 1. Geometry of the problem.](image-url)
space or a plate, explicit expressions for \( D \) can be easily obtained. In presence of layers, it is difficult if not impossible to derive such closed form expressions. However, \( D \) can be expressed as an element of a 4x4 matrix which is itself the product of a number of other 4x4 matrices. The number of these matrices is equal to the number of the layers and the general method is based on the so called Thomson Haskell matrix method [8]. The elements of the individual matrices are certain functions of \( \omega \), \( k \) and the properties of the layers. Calculation of the wavenumber \( k \) or, equivalently, the phase velocity \( v_p = \omega/k \) requires a numerical evaluation of the matrix product. It is well known that a direct numerical evaluation of the matrix product becomes unstable at higher frequencies when evanescent waves are present within the layers. In the seismological literature, submatrix manipulation of the Thomson Haskell matrices and an alternative formulation called the reflectivity method have been used to avoid this so called precision problem. A discussion of this and other related issues can be found in [10].

We have used the submatrix (also called the delta matrix) manipulation in calculating the dispersion functions for both the half space and the plate models. We have developed computer codes for the calculation of the roots of the dispersion equations. The codes appear to be extremely efficient and yield all roots at a given frequency in double precision arithmetic. These codes have been used to calculate the phase velocities of the guided waves in the two models. The results of these calculations will be presented and discussed in the next section.

The next problem of interest is the calculation of the plane wave reflection coefficient from the interface as a function of the incident angle. It has been shown [3,4,6,7] that the general form of the reflection coefficient is

\[
R(k, \omega) = \frac{D(k, \omega) + i\omega q(k) G(k, \omega)}{D(k, \omega) - i\omega q(k) G(k, \omega)}
\]

where \( k \) is the horizontal wave number of the incident acoustic waves,

\[
q(k) = \frac{\omega \rho_f}{v_f} \quad (3)
\]
\[
v_f = (k_f^2 - k^2)
\]
\[
k_f = \frac{\omega}{\alpha_f}
\]

and \( \rho_f, \alpha_f \) are the density and acoustic wave speed in the fluid. The function \( G(k, \omega) \) is an element of the same product matrix from which the dispersion function \( D \) is derived. If the angle of incidence is \( \theta \) then

\[
k = k_i = k \sin \theta \quad (4)
\]

It can be seen from equation (2) that if the phase velocity of the guided waves is greater than \( \alpha_f \) and if

\[
\theta = \arcsin(\alpha_f/v_p) \quad (5)
\]

then the reflection coefficient becomes -1, resulting in a critical phenomenon. For a given frequency there may be several critical angles corresponding to the number of roots of the dispersion equation (1).

We now consider the reflection of a bounded acoustic beam incident at a critical angle \( \theta \). Assuming that the incident beam has a Gaussian
profile of width $2b$ at the fluid solid interface, the incident and the reflected potentials in the fluid may be expressed in their fourier integral forms

$$
\phi_i(x,y,\omega) = \left(\frac{b_0}{2\sqrt{\pi}}\right) \int_{-\infty}^{\infty} \exp\{g(x,y,k,\omega)\} \, dk
$$

(7)

$$
\phi_r(x,y,\omega) = \left(\frac{b_0}{2\sqrt{\pi}}\right) \int_{-\infty}^{\infty} R(k,\omega) \exp\{g(x,-y,k,\omega)\} \, dk
$$

(8)

where

$$
g(x,y,k,\omega) = \left\{\left(k - k_b\right)b_0/2\right\}^2 + i(k - k)x + iv_y
$$

(9)

and

$$
b_0 = b \sec \theta
$$

Clearly, the calculation of the reflected field requires numerical evaluation of the integral (8). Although the significant contribution to the integral comes from a small interval of $k$ near $k_f$, the integrand is not very well behaved due to rapid variations in $R$ near the critical point and to the dense oscillations of of the sinusoidal. We have developed a special quadrature code [11] based on a modification of the original work of Clenshaw and Curtis [12]. The code appears to be extremely efficient in evaluating this type of integrals and is adaptive with excellent accuracy control features.

If the incident beam is convergent as in the case of the acoustic microscopy experiment, then it is decomposed into a finite number of narrow beams at the critical angles corresponding to the operating frequency of the microscope. The reflected waves from each of these beams are calculated separately as described above. In addition, the reflection of the normally incident beam is also considered as a special case. These reflected fields are then propagated through the buffer rod of the microscope and the resulting normal displacements are superposed to give the transducer response. The $V(z)$ curve is obtained by changing the focal distance $z$ of the lens from the interface. Details of these calculations can be found in [6]. Results for the two layered solid are presented in the next section.

RESULTS AND DISCUSSIONS

The properties of the two layered half space are given in Table 1. In the absence of any de-bonding the possible guided waves in the solid are dispersive Rayleigh waves which decay exponentially in the substrate. The calculated dispersion curves in the frequency range 0-100 MHz are presented in figure 2 for the model of Table 1 with a 5 m thick epoxy layer. It can be seen that there are three possible modes of propagation in this frequency range with cutoffs at 15 and 30 MHz. As expected the phase velocity of these higher modes is 6 mm/μsec (the shear wave velocity in the substrate) at the cutoffs and approaches the Rayleigh wave velocity in top layer (2.09 mm/μsec) at the high frequency limit. The phase velocity of the fundamental mode equals the Rayleigh velocity in the substrate (5.41 mm/μsec) at zero frequency and approaches 2.09 mm/μsec at high frequencies.
Table 1. Properties of Component Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness h (micron)</th>
<th>P-wave vel. α (km/sec)</th>
<th>S-wave vel. β (km/sec)</th>
<th>Rayleigh Wave vel. vR (km/sec)</th>
<th>Density ρ (gm/cc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>InSb</td>
<td>8</td>
<td>3.78</td>
<td>2.29</td>
<td>2.09</td>
<td>5.77</td>
</tr>
<tr>
<td>Epoxy</td>
<td>5</td>
<td>2.20</td>
<td>1.10</td>
<td>1.03</td>
<td>1.20</td>
</tr>
<tr>
<td>Silicon</td>
<td>∞</td>
<td>9.30</td>
<td>6.00</td>
<td>5.41</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Clearly, the low velocity epoxy layer has a strong influence on the velocity of the fundamental mode, which decreases rapidly to about 1.4 mm/μsec and then increases rather slowly toward its limiting value of 2.09 mm/μsec.

In order to further examine the influence of the bonding layer on the phase velocity of the fundamental mode, we calculated its values for different thicknesses of this layer. The results are shown in figure 3. It can be seen that as the thickness of the layer decreases, the decrease in the phase velocities becomes less rapid and that they approach the top curve (no epoxy layer). However, even the presence of a 1 m epoxy layer has a substantial influence on the fundamental mode.

![Figure 2. Phase velocity of the three Rayleigh modes in the half space model of Table 1.](image-url)
In Figure 4 we present the calculated phase velocity of the Lamb waves which can propagate in the two layered plate obtained by removing the substrate from the half space model. In this case the phase velocity of the fundamental mode is almost flat in the entire range of frequencies and all velocities approach 2.09 mm/µ sec at high frequencies as expected. It should be noted that in contrast to a homogeneous plate, the waves in the layered plate can not, in general be classified into symmetric and antisymmetric motions.

Finally the calculated V(z) curves at 60 MHz for the two layered solid in presence and in absence of the substrate are shown in figure 5. Only the total contribution from all the critically reflected beams are shown; their individual contributions have been presented in [7]. It can be seen that the oscillations in the curves in absence of debonding are of a significantly different nature than those in presence of debonding. For the half space model, two periodicities can be detected; the short period oscillations are due to leaky wave radiation from the fundamental mode Rayleigh waves, while the longer period ones are due to leakage from critically refracted P-waves propagating in the substrate. In the case of the plate, the oscillations are entirely due to the first higher mode of the Lamb waves; the fundamental mode does not produce any leaky waves in the fluid.
Figure 4. Phase velocity of Lamb waves in a two layer plate obtained by removing the Si substrate from the model in Table 1.

Figure 5. Computed $V(z)$ curves for the two layered model of Table 1 (solid) and the two layered plate in absence of the substrate (dashed) at 60 MHz.
ACKNOWLEDGEMENT

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REFERENCES