The application of financial futures and options to home mortgages: An "optional" comparison of the ARM-FRM rate differential

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The application of financial futures and options to home mortgages: An “optional” comparison of the ARM-FRM rate differential

Maysami, Ramin Cooper, Ph.D.

Iowa State University, 1992
The application of financial futures and options to home mortgages: An "optional" comparison of the ARM-FRM rate differential

by

Ramin Maysami

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CHAPTER 1: INTRODUCTION

Today's home-buyer faces many different choices to satisfy his financing needs. The amount of the loan, as well as the length of the mortgage and mortgage rates play an important role in this decision making process.

The key to successful home buying is to get a favorable mortgage loan. Since the end of World War II, numerous changes have taken place in the mortgage markets. New and innovative financial techniques have provided for new challenges in managing the interest-rate risk for the lender, and also have allowed for shifting of such risk to the borrowers of mortgage funds. These alternative real estate financing tools keep mortgage markets continuously alive and mortgage lending flexible. However, vast developments of new techniques inevitably make mortgage markets more complex than before. For inexperienced home buyers, the various mortgage choices could be confusing. The "good old days" of going to the local savings and loan association and getting a "standard" real estate mortgage are gone. Today, successful real estate transactions depend largely upon having a clear understanding of basic financing tools.

Interest is the price of credit, loanable funds. The level of interest rates is determined by the interaction between the demand for and the supply of credit in the loanable funds theory framework. As the supply of loanable funds exceeds the demand, downward pressure would be exerted on interest rates. Excess of demand for loanable funds over supply would result in an increase in interest rates. When demand equals supply, the interest rate remains stable and is called the equilibrium interest rate. The equilibrium interest rate changes only if the supply (sources) of credit, the demand for credit (uses), or both change.

The sources of funds include personal savings by the households, and business savings in terms of undistributed corporate profits (retained earnings) and capital consumption allowances. Moreover, the Federal
Reserve System can increase the availability of credit through expansionary monetary policy including open market purchases of U.S. government securities, lowering the discount rate, and lowering the required reserve ratio. The government may also be a supplier of credit if government revenues exceed its spending. Finally, an inflow of foreign capital would increase the supply of credit.

The users of credit include households who borrow to finance the purchase of real estate and consumer durable goods, businesses that borrow for investment purposes, and government borrowing to finance the budget deficit.

Interest rates rise in periods of tight credit, if personal and business savings decline or when the federal reserve system follows a contractory monetary policy. An increase in the demand for real estate loans, consumer durable loans, or investment loans would also cause a rise in interest rates.

Moreover, interest rates are also affected by the rate of inflation. The inflation rate—a continuous increase in the general price level as measured by, say the Consumer Price Index or the Gross National Product (GNP) deflator—affects the purchasing power of the dollar. As prices increase, the purchasing power of money declines. To protect themselves against this decline, lenders of money will attempt to add an inflation premium to the pure rate of interest. Since forecasted inflation changes from time to time depending on expectations on various economic activities, lenders need to make adjustments in the inflation premium and hence change the stated nominal interest rates.

Mortgage rates are determined following the same line of reasoning, although in the real world several different mortgage rates are offered. Instead of a single market, we can think of several markets each using the interaction of supply and demand to come up with a particular mortgage rate.
A number of specific factors are also responsible for differences in mortgage rates. Among others, marketability, taxation, administrative costs and differences in risk are important.

Marketability affects mortgage rates. For example nonconforming loans—large loans exceeding the limits for loans that can be purchased by Fannie Mae and Freddie Mac—have higher rates than conforming loans, since they are more difficult to sell in the secondary market.

Mortgage rates are also affected by taxation laws. State and local government bonds—municipal bonds—are usually exempt from federal taxes on the earned interest, hence offer relatively low rates. When the funds thus generated are used to finance home purchases by individuals, the resulting mortgage rates are relatively low compared to conventional mortgages.

Mortgage loans with longer maturities pose a greater degree of risk to the lender due to the higher degree of uncertainty involved in predicting a more distant future. For example, the possibility of changes in interest rates is larger over a longer period of time. Lender of long-term funds are, thus, subjected to larger interest rate risk and generally require a higher rate of return. Finally, the efficiency of the lender will lower administrative costs, thus lowering mortgage rates.

The interest rate of a real estate loan affects the monthly payment of the borrower. As mortgage rates increase, the demand for real estate loans tend to decline as people are priced out of the market. Subsequently, the demand for real properties may also fall. Moreover, any rise in interest rates would cause a decline in the market value of mortgages, hence results in capital losses for investors who own mortgages. A decrease in mortgage rates on the other hand is the cause of capital gains in secondary mortgage markets as their value appreciates.

The future direction of market-driven interest rates is clearly uncertain. It is necessary for a lending institution to estimate future interest rates, and their uncertain effect on the volume and pricing of loans before any hedging attempts are made.
In addition to the economic uncertainties involved in interest rate changes, there are political uncertainties of when official, federal government-administered interest rate changes will be made. With the deregulation of the FHA rates, the VA rate is currently the only government-administered mortgage rate. Politically inspired interest rate actions tend to occur quickly or even lead the market to change the interest rates in a direction that favors the consumer/voter. With an increase in market rates, the politically managed rates may lag the market, creating huge discounts for borrowers of government administered mortgage funds.

**Statement of the Problem**

In the 1970s, the level and volatility of interest rates started to increase. As a result, savings and loans associations whose assets were primarily long-term fixed mortgages, while their deposits were mainly short-term, felt a financial squeeze. As rates on deposits increased, their profits declined.

Interest rate risk to the mortgage banker is synonymous with the price risk. Price risk is the potential change in the value of the mortgage product because of future changes in its sale price. Changes in sale prices reflect the movement of long-term interest rates. The primary creators of price risk are interest-rate volatility and the uncertainty of the future direction, timing, and effect of interest rates.

A number of strategies were devised to reduce the interest rate risk exposure for such financial institutions. Included among them, are portfolio insurance, the buy-write strategy, the standby option, and the various types of adjustable rate mortgages. The goals of this study include the understanding of basic characteristics of the more popular mortgage types available in the market, and the examination of choice

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1 For a complete discussion of these and other hedging examples see, Kevin Commins, "The Eight Best Hedges," *Intermarket*, August 1986: 17-23
determinants for the borrowers of such mortgages. Since these mortgages were designed to provide greater flexibility for both borrowers and lenders in meeting their needs, the understanding of them may prove useful when the hedging demands of the mortgage investors are considered.

One of the more popular mortgage types for both the borrowers and lenders, for different reasons of course, has been the adjustable rate mortgage, ARM. The investors in mortgage funds have relied heavily on ARMs as a tool of controlling interest rate risk of their mortgage portfolios. The second goal of the current study is to present alternative hedging strategies utilizing financial futures and options. The structure of each market and the empirical results of hedging mortgage portfolios with futures and options will be examined. Finally, the above mentioned hedging activities will be used as comparison means to analyze the difference between the fixed and adjustable mortgage rates.

The study begins with an in-depth look at alternative financing tools. Chapter 1 examines the characteristics of each mortgage type, along with some advantages to each. An extensive literature review is conducted in Chapter 2 to explore the determinants of consumer demand for the two most popular choices—the fixed rate mortgage (FRM), and the adjustable rate mortgage. The significant borrower and mortgage characteristics that may affect a borrower's preferences are identified as, among others, the FRM-ARM rate differentials; the level of FRM rates; the relative variances of the inflation and mortgage rates; the covariances between additions to real wealth and the sources of change in the inflation and mortgage rates; the degree of risk aversion; income levels; availability of co-borrowers; and the expected housing tenure. Moreover, additional review of the literature is conducted in Chapter 2 in an attempt to understand the basics of mortgage pricing. The techniques that are commonly employed in pricing mortgages are the simulation or the Monte Carlo method and the option pricing principles.
The study continues with an analysis of alternative techniques available to lenders for hedging interest rate risk. Starting with the 1960s, borrowing short term and lending long term created serious financial difficulties for savings and loan associations as well as other mortgage lending institutions. One problem was the general upward trend in the levels of all interest rates which forced the S&Ls to pay higher rates of interest on their deposits while continuing to receive constant rates on their long term loans. If an S&L tried to liquidate its long-term loans by selling them, capital loans would result since it owned a long asset that earns below market rate of interest. Investors would not be willing to pay face value for such a loan. The S&L lost whether it chose to keep the loan or to sell it.

Until relatively recently, there was little that S&Ls could do to hedge the problem of a yield curve that shifts up or down. The introduction of variable-rate mortgages in 1981, however, allowed such institutions to better match the maturities of their assets and liabilities. On the other hand, they could not at the same time benefit from lower interest rates as when they owned FRMs. In other words, as market rates declined, the value of the S&Ls asset would not appreciate since mortgage rates were also adjusted downward. ARMs eliminated the possibilities of capital losses as well as the gains. The risk of owning a FRM is then the probability of rising market rates only. The existence of such one sided risks would seem to warrant the use of financial futures and options in the mortgage market.

Various types of financial futures and options are thus examined including Treasury bond futures in Chapter 4, and options on T-bonds and T-notes, options on interest rate indexes, and options on T-bonds futures contracts in Chapter 5. Moreover, the theory of hedging with financial futures is reviewed in Chapter 4, while Chapter 5 examines the Black options pricing theory.
Chapter 6 begins with position diagrams for the basic futures and options strategies. These diagrams are commonly used to study hedging outcomes, and will be employed here as well to analyze four hedging choices: short-futures, sell-calls, buy-puts, and the synthetic futures position of selling a call option and buying a put. The return distributions for the hedged and unhedged mortgage portfolios are then found mathematically and are used for the empirical study of this chapter. Mortgage and futures price series for the period 1983-1991, as well as option premia for the same period are employed to compare the mean and standard deviation of returns for the unhedged position to those of hedged portfolios. Chapter 6 concludes by the presentation and discussion of the results of the empirical study.

The second empirical study, performed in Chapter 7, is aimed at testing a hypothesis about the difference between the fixed and adjustable mortgage rates based on the cost of hedging FRM portfolios with put options on financial futures. A simulation study is performed by generating random changes in interest rates similar to those faced by the institutions holding only ARMs. The results of the hedging alternatives under this simulated interest rate environment are presented and discussed in Chapter 7.

The examination of the results suggests that put option hedging of the FRM provides for a situation similar to hedging the interest rate risk by offering ARMs. It is then proposed that since the same function, hedging interest rate risk of the FRM, may be performed by either of the two strategies, the cost difference between the fixed and adjustable mortgage rates should not be higher than the cost of hedging with put options on financial futures. The cost of hedging the portfolio with put options represents the "conversion cost" of turning the interest rate risk structure of the FRM portfolio similar to that of an ARM. The FRM-ARM rate differential should not exceed this cost. This hypothesis is tested in the last part of Chapter 7 by comparing the difference between the FRM series
and a simple ARM constructed as the sum of three-month Treasury bill rate and a margin of 1.5 or 2 percent to the cost of hedging the FRM with put options.

Chapter 8 summarizes the results of the study and provides concluding remarks accordingly.
CHAPTER 2: MORTGAGE TYPES

The instruments of home financing have undergone a vast evolution. In the 1920s and earlier, the available mortgage instruments were of a short term nature, with relatively low loan to value ratio of 50-60 percent of the total home cost, and were offered at high mortgage rates.

The arrival of the Great Depression in the early 1930s resulted in the crash of the real estate market, and the federal government realized the need to stabilize the residential real estate market through legislation. The Home Owner's Loan Act of 1933 and the National Housing Act of 1934, established private home ownership as a national goal. The development of the long-term, fixed-rate, self-amortizing loan as a key tool in the revitalization of the American real estate market was led by the Federal Housing Administration. The "fixed rate mortgage" thus developed served as the predominant financing tool for nearly half a century.

A typical fixed rate mortgage assigns the borrower a certain fixed mortgage rate and hence a fixed monthly payment for the life of the loan. With each monthly payment, the loan balance declines and more of the payment will be applied to the principal rather than interest payment. At maturity, the loan balance will be approximately zero. The equal monthly payments of a FRM loan are calculated according to the following formula

\[
PMT = \frac{L(\frac{i}{12})}{1 - (1 + \frac{i}{12})^{-12N}}
\]

where \(L\) is the loan amount, \(i\) is the annual mortgage rate, and \(N\) is the term of the loan in years. For example, a $100,000 mortgage loan at 12% annual rate and a term of 30 years, requires a monthly payment of $1028.61. Total payments after 360 months will be $370,299.60, of which $270,299.60 is the interest portion.
The exact remaining portion of the loan after a number of payments have been made may be found using the following formula:

\[
BAL(k) = \frac{PMT}{i} \left[1 - \left(1 + \frac{i}{12}\right)^{-k} \right] - \frac{12N}{i} 
\]

where \( BAL(k) \) is the balance of the loan after the \( k \)th payment. In the previous example, the balance at the end of the 120 months is $93,417.

The monthly payment is composed of interest and principal. During the early years of the loan, only a small portion goes toward principal reduction. In other words, for many years the monthly payment is consumed by the interest expense. Since the interest portion is tax deductible, it is important to find the exact interest payment every month. This is accomplished by multiplying the balance at the beginning of the month by the monthly interest rate \( \frac{i}{12} \). For the example above, the first month's interest payment is $100,000 \times \frac{.12}{12} \), or $1000. Thus only $28.61 of the monthly payment will go towards principal reduction. The balance at the beginning of the second month will be $99,971.39 and the second month's interest cost will be $999.71.

It is important to realize however, that although the loan balance remains quite high for many years, no negative amortization occurs because each monthly payment does in effect lower the loan balance and eventually the loan will be exhausted.

The key advantage of a FRM is the fact that monthly payments remain the same until the loan is paid off. This "certainty" of the future events create peace of mind and a sense of ease for the borrower, hence has a high value for the borrower in the fixed rate mortgage, entailing a premium. Also, as the loan balance declines with every payment, the owner's equity in the property rises. Moreover, with declining loan balances, if the property appreciates in value, the equity of the home owner increases further. Finally, for assumable FRMs, if the mortgage rate is lower than
the current rates, sale of the property may be sped up and its value may increase.

The major disadvantage of a FRM is that borrowers may not benefit if interest rates drop. Due to the high cost of refinancing, obtaining a new FRM at the lower rate may not be economical. By the industry standards, a drop of at least 2 percent in rates is needed to make it financially wise to refinance a FRM.

The level and behavior of mortgage rates gradually changed until the second half of the 1970s, and in the latter years of that decade and early 80s, tremendous adverse effects were observed. In the 15 year period ending in 1979, mortgage rates doubled from 6 to 12 percent, and by 1981 they had almost tripled to 16 percent. With the high and volatile rates of the second half of the 1970s, and the mismatch between assets and liability durations of thrift institutions, a financial squeeze was placed on such firms.

Financial institutions involved in mortgage lending could finance their long-term fixed interest rate mortgage portfolios with their typical short-term deposits as long as long-term rates remained above short-term rates. In situations of rising interest rates, however, these institutions could face serious financial problems because they have to pay higher interest rates while receiving the same fixed rate on their mortgages. Moreover, any attempts to sell some of these assets would lead to capital losses due to lower value of mortgages resulting from higher interest rates (Marshal and Colwell, 1985).

The volatile interest rate climate resulted in a great deal of risk to both borrower and lender. It was common for mortgage rates to move by one percent point between the making of a loan application and the time the loan was closed. High and volatile interest rates combined with the rapid inflation in housing prices, made it all but impossible for effective real estate transactions subject to a reasonable amount of risk. A group of non-traditional mortgages, collectively called alternative mortgage
instruments (AMIs) were then created to cope with the above mentioned problems.

The variable rate mortgage (VRM) was the first flexible-rate mortgage to be authorized by the Federal Home Loan Bank Board in 1979. Under this contract, the mortgage interest rate can be adjusted periodically in response to changing market interest rates. This mortgage type eliminated the reluctance of mortgage funds lenders to commit their funds for a long period of time if they thought market rates were to rise in the future.

The FHLBB guidelines on VRMs included many of the safeguards suggested by consumer groups. The mortgage rates were not to increase by more than .5 percent per year, with a maximum increase of 2.5 percent. The guideline also required the availability of FRMs and that a comparison between the terms of the two mortgage types be available to borrowers.

Renegotiable rate mortgages (RRM) were the second generation of adjustable loans authorized by FHLBB. Popular in Canada and also known as Canadian Rollover loans, RRMs are series of short term loans of three to five years, secured by a fixed rate mortgage that amortizes the loan principal fully over the life of the loan (typically 25-30 years). There is, however, a call provision requiring the payoff or renegotiation of the loan terms within the 3-5 year period. At that time rates would adjust to reflect the current market interest rates.

The main difference between VRMs and ARMs lie in the size and frequency of rate adjustments. FHLBB regulations allow the interest rate on a RRM loan to be adjusted no more than 1.5 percent every three to five years, and by no more than 5 percent over the life of the loan. This mortgage type then has looser rate caps, but allows less frequent adjustments. On the whole, both VRM and RRM, however, are consumer-oriented adjustable rate loans. Lenders, not surprisingly, felt that the regulations on both mortgage types did not allow them sufficient flexibility in offering mortgage loans.
A major reform in the real estate market, in April 1981, was the approval of Adjustable Mortgage Loans (AML) for federally chartered savings and loan associations by the FHLBB, and the comptroller of the currency’s authorization of Adjustable Rate Mortgages (ARM) for commercial banks shortly thereafter. The rules governing AMLs and ARMs are quite similar and much more relaxed compared to previous flexible rate mortgages, most consumer safeguards applicable to the original VRMs are removed.

The initial monthly payment of the ARM is calculated in the same way as the monthly payment of a FRM. The rate on ARMs, however, is adjusted periodically, hence, the monthly payments could increase or decrease as the general level of interest rates fluctuate. In some cases, the term of the loan as well as the balance increases.

The interest rate on an ARM is normally determined by adding 2-3 percent to an index rate chosen by, but not controlled by the lender, and verifiable by the borrower to determine ARM rates. The most widely used index rates include yield on Treasury securities, cost of funds to S&Ls, rates on certificates of deposits, and national average contract interest rates for major lenders on the purchase of previously occupied homes closed in the first five working days of each month.

Teaser rates, as the initial contract rates are commonly called, are generally lower than the index rate plus the margin. These rates are used to attract more borrowers by enabling them to qualify for loans. ARM rates will subsequently increase to a level equal to the index rate plus the margin after the first year.

Adjustment of these mortgage rates is limited to once every six months, and the adjustment cannot be more than 1 percent per six month period. There are no limits on the adjustment over the life of the loan. Written notices of any interest rate adjustments must be provided to the borrower.

In an environment of sharp increases in the index rate, most borrowers face "payment shocks" which possibly would result in foreclosure.
Most lenders agree to impose interest rate or payment caps. The interest rate caps limit interest rate changes for each adjustment period. Lifetime interest rate caps are absolute rates over which the mortgage rate cannot climb. The lender may specify an interest rate floor as well. The rate below which the mortgage rate cannot drop over the life of the loan.

When the new monthly payment subject to interest rate caps is smaller than the payment implied by the new contract rate, the new monthly payment may not be sufficient to cover interest on the loan. In these cases, the unpaid portion of the interest would be added to the loan balance, thus increasing the balance despite the payment. This process is called negative amortization. Since loans that are subject to negative amortization are not accepted for purchase in the secondary market by the Federal National Mortgage Association, most lenders set strict limits on this practice. For example, there may be requirements to adjust payments every five years to amortize the outstanding balance over the life of the loan.

Other features of ARMs are convertability and prepayment penalties. Convertability allows the conversion of the ARM to a FRM at predetermined dates. Prepayment penalties are zero if the loan is paid off at the time the interest rate is adjusted.\(^2\)

Compared to FRMs, ARMs have a lower initial rate, and are more affordable due to the lower initial monthly payment. Moreover, for people who relocate frequently, ARMs are more advisable since they allow the full benefit of the teaser rates. Also, ARMs allow borrowers to take advantage of lower rates by making lower monthly payments, or by applying some of the payment to the principal and lower the unpaid balance. Generally they are not subject to prepayment penalties if the loan is prepaid at the end of

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\(^2\) For further analysis of factors that should be considered in selecting a particular ARM, see A. G. Yohannes, "The Professional Desktop Guide to Real Estate Finance," Homewood, Illinois: Richard D. Irwin, Inc. 1990: Chapter 7.
the adjustment period. Finally, in many cases, points may be lower on ARMs compared to FRMs.

The major disadvantage of ARMs is the uncertainty of the future mortgage rate and the monthly payment. In the absence of interest rate caps, the increase in monthly payment could be quite substantial. Moreover, if negative amortization is allowed, an increase in rates may increase both the monthly payment and the loan balance, causing financial difficulties for the borrower.

Although today's real estate market is virtually dominated by the above mentioned ARMs and FRMs, there remain several other alternative mortgage types designed to suit particular homebuyers. The rest of this section is an explanation of such mortgages.

Graduated payment mortgages (GPM) were introduced by the Department of Housing and Urban Development in the mid-1970s. They are aimed at allowing families whose income were expected to grow, to qualify for mortgage loans.

GPMs have a fixed contract rate. However, the monthly payments start at a lower level than a comparable fixed rate mortgage, and rise during the graduation period, according to a graduation rate, to a level higher than that of a comparable fixed rate mortgage, and stay there.

In a variation of the scheme, the lender may establish an overall mortgage rate, and a corresponding initial interest rate to calculate monthly payments. The initial rate is then increased every year during the graduation period by a given percentage, thus increasing the monthly payments during that period.

The main problem with GPMs is that due to insufficiency of early payments to cover interest cost, the loan balance may increase, causing negative amortization. Some lending institutions have designed GPM with sufficiently high initial rates to cover the full interest expense, and thus eliminating this problem.
Graduated payment adjustable rate mortgages (GPARM) combine the features of the graduated payment mortgages with those of renegotiable mortgages.

As in a typical GPM, the initial monthly payments are smaller than the payments of a comparable level-payment mortgage and rise during the graduation period. At the end of the first adjustment period, however, the monthly payment could rise by more than the graduation rate, signifying higher market interest rates. By the end of the graduation period, any adjustment in the mortgage rate reflect changes in market rates.

Furthermore, every five years, payments may increase to the level that allows full amortization of the remaining loan balance over the remainder life of the loan. This adjustment is known as the "catch up" payment and is established at the time of approval along with the graduation rate, graduation period, any caps, and method of annual rate adjustments.

The main advantage of this type of loan to the borrower is the relative low initial monthly payment, thus qualifying the homebuyer for a larger loan. Moreover, if the note rate declines, the monthly payments could go down. And finally, from the lender's point of view, the interest rate risk has been largely shifted to the borrower.

The major disadvantage of GPARMS is the fact that the borrower is subject to the risk of interest rate fluctuations. Increase in future interest rates could cause increases in monthly payments, which already are subject to increase during the graduation period. Negative amortization is also a possibility.

Growing equity mortgages (GEM) allow loans with a higher loan-to-value ratio be made with fixed interest rates and terms of 25-30 years. Although the mortgage rate is fixed, monthly payments increase by a predetermined percentage per year for a specified period of time. This allows for a more rapid pay-off of the loan, since the increases in monthly payments are applied fully to the principal.
Since the monthly payments are determined based on the actual mortgage rate, the payments are enough to cover interest cost, and no negative amortization occurs. So as long as the income of borrower increases to provide for higher payments, GEMs reduce the life of the loan considerably, thus saving the borrower substantial amounts of interest expenses.

Advantages of GEMs include the certainty about payment amount, since the interest rate, the graduation rate, and the graduation period are all determined at the time of origination. Also, the borrower qualifies for the loan using the first year’s monthly payments despite higher payments in the subsequent years. Moreover, unlike a GPM, this type of loan does not generate negative amortization. Finally, from the lender’s point of view, the acceleration of equity build up for the home owner which possibly reduces the probability of default, and the shorter period of time for which their capital is tied up is very attractive.

The major disadvantage of the GEM is that if the borrower’s income does not grow as fast as the increase in the monthly payments, default will be a realistic possibility.

Reverse Annuity Mortgages (RAM) are loans which are more or less the reverse of fixed rate, level payment mortgages. Specifically, the borrower receives the loan in equal installments over a given period of time, and then makes a lump sum payment, including principal and interest, at the end of loan period.

This type of mortgage loan is designed to allow people who own their homes free, or have a very low outstanding balance on their mortgage to use the equity in their homes without selling or getting loans that require monthly payments.

The sum of the new loan, and any existing loans would be a set percentage of the appraised value of the home, typically 70 percent or less. The mortgage ends if the home is sold, the borrower dies, and in some other situations such as the nonpayment of taxes. In case of
homeowner death, the lender would collect from the estate of the borrower, although arrangements may be made to name the lending institute as the beneficiary in the borrowers life insurance contract.

The advantages of RAMs for homeowners with high equity is to receive a limited period annuity that is tax free, while living in their home, and taking advantage of its appreciation. Finally, as opposed to refinancing, and obtaining second mortgages, the homeowner need not make any payments until the term of the RAM ends.

The major disadvantage of this mortgage type is the accumulation of debt. If the owner outlives the mortgage, and there is not sufficient equity to guarantee another reverse annuity loan, the house must be sold as a means of repayment. For the lender, if accrual method of accounting is used, the interest that accrues may be taxable although it is not received until the maturity of RAM, since under this accounting method income is recognized when it is earned rather than when it is received.

Price level adjusted mortgages (PLAM), introduced in 1982, are designed to protect lenders against inflation by adjusting the mortgage balance for the changes in the purchasing power of money. If prices increase, the balances are revised upward, resulting in higher monthly payment.

In a typical PLAM, the lender requires a real rate of return equal to the nominal interest rate minus the rate of inflation. The monthly payments over the first year will be calculated using the real rate of return, and are typically low. If prices remain constant, the loan will amortize fully with the real interest rate as the prevailing mortgage rate. In cases of rising inflation, however, the loan balance will be revised upward by a factor of $1 + \text{the change in price level}$, thus preserving the real value of the loan balance. The new monthly payment is calculated using the adjusted balance, the real interest rate, and the remaining term of the loan.
The PLAM benefits the borrower with its relatively low payment in the first year, and the possibility of continuing such low payments if prices remain stable. Its advantage for the lender is the protection against inflation by maintaining a stable real rate of return on the loan. In times of hyperinflation, PLAM could increase the probability of default substantially. Due to higher monthly payments, the borrower may have a difficult time making the mortgage payment unless his/her income rises proportionally. Moreover, although the real rate of return on the loan is stable, the riskiness of the lender's portfolio is greatly increased due to the possibility of borrower default.

Shared appreciation mortgages (SAM) are mortgage loans made at low interest rates—2 to 3 percent below the current comparable mortgage rates—and in return the lender receives 20 to 30 percent of the appreciation in the value of the home as "contingent interest" when the loan matures, when the property is sold, or at some other specified time. In this manner, when the general level of interest rates is high, inflation in the value of the property is in part substituted for higher mortgage rates.

In a typical SAM, the borrower has to put a substantial down payment, up to 40 percent of the price of the home, and may be subject to substantial prepayment penalties. The life of the loan is typically 10 years, although it amortizes over a much longer 20 to 30 year period. Existence of SAMs allows many otherwise rejected borrowers, to qualify for mortgage loans since it allows much lower monthly payments. The large down payment is appealing to lenders by lowering the default risk, and uncertainties of interest rate movements are lowered due to shorter terms of the loan.

The disadvantages of SAMs include the borrower's inability to pay the "contingent interest" without the sale of the home. If the desire to sell before the terms of the SAM exist, prepayment penalties could be very prohibitive. Moreover, additions to the property, and home improvements pose a problem for computing the portion of the increase in home value is
due to inflation and sharable by the lender. Finally, the lender may face trouble if the expected appreciation in home prices does not materialize.

Wraparound mortgages (WM) are those that include, or are wrapped around an existing first mortgage. The buyer purchases a property subject to an existing first mortgage and gives the seller a wraparound mortgage with a face amount larger than the balance on the existing loan. The difference between the face amount and the balance is financed by the seller, who continues to make payments on the first mortgage but retains the difference between the monthly payment on the wrap and the monthly payment on the first mortgage.

This mortgage is used when the interest rate on the existing mortgage is lower than current mortgage rates. The seller thus takes advantage of the difference in rates by taking back a wraparound mortgage from a buyer at an interest rate that is higher than that of the first mortgage but lower than that of the current mortgage rates. The face amount of the wraparound mortgage consists of the balance of the existing loan and the amount financed by the seller. Since the interest rate on the wraparound mortgage is higher than the rate on the existing mortgage, the seller earns extra interest on the existing mortgage.

The term of the wraparound mortgage is usually the same as that of the remaining term of the existing mortgage. The buyer or the wraparound borrower sends monthly payment to seller and gets title to the property. The seller however is still responsible for making the payment to the original lender.

The advantages of this mortgage type for the seller includes facilitating the sale of the property and earning some interest on the first mortgage. The buyer benefits by paying an interest rate that is generally below the current market rates.
CHAPTER 3: REVIEW OF MORTGAGE FINANCING LITERATURE

Chapter 3 is an examination of the mortgage financing literature. Such literature includes those exploring the characteristics of the borrowers of different mortgage types and those examining the mortgage pricing principals.

The Mortgage Choice


In 1980, Baesel and Biger developed the first theory about the determinants of borrowers' choice between fixed-rate and index-linked mortgages. As mentioned before, in a fixed-rate mortgage, both the interest rate and the monthly payment are fixed. However, an index-linked mortgage is a special kind of adjustable-rate mortgage, in which the interest rate is adjusted with the Consumer Price Index (CPI).

Baesel and Biger (BB) start their analysis with the effect of inflation on mortgage choice. It is argued that the profit gain or loss for borrowers and lenders due to inflation depend on three factors: mortgage contract rate, inflation rate, and expectations about future economy.

Both borrowers and lenders face uncertainty in real terms, and since during periods of rising inflation rate, interest rates are likely to increase, borrowers who have locked in lower interest rates will be better off. In a recession the opposite is true—interest rate generally fall, and borrowers of FRMs will be worse off. Unlike the fixed-rate mortgage, the index-linked and the variable rate mortgages allow for the mortgage rate to be adjusted in response to a higher rate of inflation. The index-linked mortgages are written with both principal and interest payments indexed. It requires the borrower to repay sufficient nominal dollars to insure the
lender a specified real return on his investment. The variable rate mortgages can be adjusted at specified times to offset inflationary effects. Thus, both index-linked and variable-rate mortgages eliminate the uncertainty and the risk of inflation for the lenders. However, since lenders are usually large financial institutions with ownership traded in capital markets, they can diversify their portfolios to pass the risks on to the market. Hence both fixed-rate and index-linked loans will exist.

Baesel and Biger make this clear:

Since the risk characteristics of a firm are determined by the risk characteristics of their assets, capital market theory suggests that the risks of the alternative mortgage contracts will pass through to the market. If the risks of alternative mortgage contracts are different, then the yields necessary to allow market equilibrium will also differ. In equilibrium, there will not be dominant assets on a risk-adjusted basis. The lender should be indifferent between the alternative loans as long as the contract rate is appropriate to the level of risk.3

Baesel and Biger, then, focus on the borrowers' preference for index-linked or fixed-rate mortgages, which is assumed to depend on the characteristics of their income stream. The analysis is conducted within a single period and under the following assumptions:

1. Mortgage lenders are specialized companies with common stock traded in an efficient capital market.
2. Lenders offer a fixed rate mortgage at rate "G1," and a index-linked mortgage at "G2 = G1 - Z," where "Z" is the constant representing the rate differential.
3. The borrowers are risk averse in the mean-variance sense and concerned only with their terminal real wealth "W."
4. Borrower utility is expected to increase with the expected real wealth (W) and decrease with the variance of real wealth.
5. The uncertain labor income for the period is denoted by "Y."

6. The purchasing power of money is expected to remain constant, \( E(P') = 1 \), where, \( P \) represents one plus the rate of change in the Consumer Price Index (CPI). Thus, the real wealth of the borrower at the end of the period is computed as follows:

\[
W = Y*P' - [h G1* P' + (M-h) G2]
\]

Where, "M" is the total amount of the loan; "h" is the amount of the mortgage loan given a fixed rate, and \( 0 \leq h \leq M \). The impact of \( h \) on expected terminal real wealth is a constant:

\[
dE(W)/dh = -Z
\]

Considering the inflation risk, fixed-rate mortgages are offered at a higher interest rate as compared with the initial interest rate in an index-linked mortgage, i.e. \( Z > 0 \). Thus, \( h \) has a fixed negative effect on the expected terminal real wealth. However, the impact of \( h \) on the variance of terminal real wealth depends on borrower's labor income and \( P' \). The effects of the expected terminal real wealth of the borrower, \( E(W) \), and its variance, \( Var(W) \), on the choice between alternative mortgages are then examined under several cases.

Case I. Nominal labor income is index-linked and certain

Under this condition, the real labor income, denoted by \( I \), is certain. Therefore, the terminal real wealth is

\[
W = I - [h (G2 + Z)' + (M-h) G2]
\]

The variance of the terminal real wealth is:

\[
Var(W) = h^2 (G2+Z)^2 Var(P')
\]

The impact of \( h \) on the variance of real terminal wealth is found by differentiating the variance with regard to \( h \):

\[
dVar(W)/dh = 2h (G2+Z)^2 Var(P') > 0
\]

Equation 3.5 shows a positive effect of \( h \) on the variance of real terminal wealth \( [Var(W)] \). Thus, under this condition, expected real wealth decreases with \( h \), but the variance of real wealth increases with \( h \). Since
borrowers are assumed to prefer expected wealth and to dislike variance of wealth, index-linked mortgages will be chosen.

Case II  
Real labor income is statistically independent of $P'$

The real end-of-period wealth is

$$W = YP' - [h(GZ+Z)P' + (M-h)G2] \tag{3.6}$$

with variance:

$$\text{Var}(W) = \text{Var}(YP') + h^2(GZ)\text{Var}(P') \tag{3.7}$$

and

$$\frac{d\text{Var}(W)}{dh} = 2h(GZ)^2 \text{Var}(P') > 0 \tag{3.8}$$

The result is the same as in Case I, indicating that $h$ negatively relates to expected terminal wealth but relates positively to the variance of the real wealth. Both effects lead to a borrower's choice of index-linked mortgage.

Case III  
Real labor income is stochastic but not independent of $P''$

In this case,

$$\text{Var}(W) = \text{Var}(YP') + h^2(GZ)\text{Var}(P') - 2h(GZ)\text{Cov}(YP',P') \tag{3.9}$$

and

$$\frac{d\text{Var}(W)}{dh} = 2h(GZ)^2 \text{Var}(P') - 2(GZ)\text{Cov}(YP',P') \tag{3.10}$$

The sign of the covariance term is the key for mortgage decision. If a borrower expects his real labor income to rise with inflation, the covariance term will be negative. As a result, the variance of the terminal real wealth will increase with $h$, indicating a borrower's preference for index-linked mortgage. On the contrary, if the real labor income is expected to fall as a result of inflation, the covariance will be positive. The variance of terminal real wealth, in this case, might rise or fall with $h$ depending on the relative signs of the components of the derivative. In the case that $\text{Var}(W)$ decreases with $h$, the borrower's mortgage choice depends on the trade-off between expected real wealth ($E(W)$) and the inflation risk ($\text{Var}(W)$). This suggests that both fixed-rate and index-linked mortgages will exist.

Case IV  
Labor income is fixed in nominal terms

Under this condition, the terminal real wealth is
W = YP' - [h(G2+Z)P' + (M-h)G2]  \tag{3.11}

with Variance:

\[ \text{Var}(W) = \left[YP' - h(G2+Z)\right] \text{Var}(P') \tag{3.12} \]

Thus,

\[ \frac{d\text{Var}(W)}{dh} = -2(G2+Z) (YP'' - h(G2+Z) \text{Var}(P')) < 0 \tag{3.13} \]

h has a negative impact on \( \text{Var}(W) \), showing that the variance of real wealth will decline as the amount of fixed-rate mortgages increases. Since \( Z > 0 \), the expected real wealth (\( E(W) \)) will also decline. As a result, the borrower's choice of mortgage will again depend on the trade-off between these two factors.

In general, Bæsel and Biger suggest that a borrower's choice between a fixed-rate and index-linked mortgage depends on the relationship between future income and the inflation variable represented by the rate differential of the two mortgages (\( Z \)). Assuming a borrower prefers expected wealth and dislike variance of wealth, if the borrower's future income is either fixed or stochastically independent of inflation, he will choose an index-linked mortgage. As shown, Bæsel and Biger's model is particularly concerned with the borrower's cash flow.

In 1982, Meir Statman expanded Bæsel and Biger's study on consumers' mortgage choice by taking account of the effect of the home value as well as their cash flow. In Statman's view, a borrower's terminal wealth should consist not only of his cash flow but also of his equity in the house, because mortgages are not necessarily paid out of labor income. When interest rate rises, mortgage payments must increase to avoid negative amortization. Instead of taking additional mortgage payments out of his labor income, borrower may also borrow against his equity to meet the increased payments, i.e., securing a second mortgage. Thus, given the assumption of no participation in capital markets, "the terminal wealth of the borrower is composed of the difference between real labor income and real mortgage payments, plus the net (of mortgage) terminal real value of
The Hypothesis is the same as specified by BB: borrowers are risk-averse in the mean-variance sense and concerned only with their terminal real wealth, which now includes value of the house (net of the mortgage obligation), $XP^i$.\(^5\)

$$W = YP^i - (hG1P^i + (M-h)G2) + XP^i$$ \hspace{1cm} 3.14

where,

- $W$ is the terminal real wealth;
- $Y$ is the (uncertain) labor income during the period;
- $P$ is one plus the rate of change in the consumer price index, CPI, (for simplicity, assuming $E(P^i) = 1$);
- $M$ is the total amount of mortgage;
- $h$ is the amount of mortgage loan taken with a fixed rate $0<h<M$;
- $X$ is the net terminal nominal value of the house;
- $G1$ is the mortgage rate of FRM, fixed-rate mortgages;
- $G2$ is the mortgage rate of index-linked mortgages (assuming $G1-G2 = Z$, where $Z$ is a constant).

Statman then analyzes the borrower's utility function to explore the determinants of the borrower's choice on mortgages. According to Statman, the borrower's utility function is assumed to be in the form of

$$U(W) = E(W) - \frac{1}{2L} \cdot \text{Var}(W)$$ \hspace{1cm} 3.15

where $L$ is a measure of risk aversion assumed to be positive.

Using equation 3.1, this utility function can be transformed as follows:

$$U(W) = E(YP^i) - h(G2+Z)E(P^i) - (M-h)G2 + E(XP^i) - \frac{1}{2L} \cdot \text{Var}(YP^i) + h(G2+Z)^2 \cdot \text{Var}(P^i) - 2h(G2+Z) + \text{Cov}(YP^i,P^i) + 2 \cdot \text{Cov}(YP^i,XP^i) - 2h(G2+Z) \cdot \text{Cov}(XP^i,P^i).$$ \hspace{1cm} 3.16


\(^5\) Recall BB's model: $V = WP^i - (hG1P^i + (M-h)G2)$. 
To maximize borrower’s utility, the first order conditions are taken for the optimal amount to be financed by the fixed rate mortgage:

\[
\frac{dU(W)}{dh} = -\frac{Z}{L(G2+Z)^2} Var(P^i) + \frac{1}{L(G2+Z)} \cdot Cov(XP^i, P^i) + Cov(XP^i, P^i) \cdot Cov(YP^i, P^i) \]

3.17

Setting the derivative equal to zero, the amount of fixed rate mortgages is:

\[
h = \frac{-Z}{L(G2+Z)^2} Var(P^i) + \frac{1}{L(G2+Z)} Var(P^i) \cdot Cov(XP^i, P^i) + Cov(XP^i, P^i) \]

3.18

Since 0<h<M, if the optimal level of h is below or above M, it will take the values of zero or M, respectively. The significant difference of this model and BB’s model is the inclusion of the term Cov(XP^i, P^i). The borrowers’ preference behaviors were analyzed for the same four cases mentioned earlier.

Cases I and II Nominal income is index-linked and certain or is stochastically independent of P^i

The implication of these conditions is that the covariance between YP^i and P^i equals zero: Cov(XP^i, P^i) = 0. If the term Cov(XP^i, P^i) as in BB’s model,

\[
h = \frac{Z}{L(G2+Z)^2} Var(P^i) < 0.
\]

Assuming Z>0, and 0<h<M, the optimal value of h is zero, which means to maximize utility, the borrowers choose only index-linked mortgages. However, if Cov(XP^i, P^i) is included in the model and is larger than Z/L(G2+Z), then the value of h will be greater than zero, meaning that some or all of the mortgages will be at fixed rate. Thus, for cases I and II, if borrower expects the real value of the house to decline with inflation, his preference for fixed-rate mortgage rises.

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6 Second-order conditions for maximum are satisfied, because

\[
\frac{d^2 U(V)}{dh^2} = -L(G2+Z)^2 Var(P^i) < 0.
\]
Case III Real labor income is stochastic but not independent of $P'''$

Under this condition, $\text{Cov}(YP', P')$ does not equal to zero. A negative (positive) sign indicates that a borrower expects his/her real labor income to rise (fall) with inflation. When the term $\text{Cov}(XP', P')$ is neglected, a negative sign of $\text{Cov}(YP', P')$ leads to an optimal $h$ value of zero, and borrower’s preference of index-linked mortgages. On the other hand, if $\text{Cov}(YP', P')$ is positive and greater than $Z/L(G2+Z)$, $h$ will exceed zero, indicating the presence of demand for fixed-rate mortgages. If the term $\text{Cov}(XP', P')$ is not zero and sufficiently large, a fixed-rate will be chosen even if $\text{Cov}(YP', P')$ is negative.

Case IV Labor income is fixed in nominal terms

In this case, $\text{Cov}(YP', P')$ becomes $C \text{Var}(P')$ where $C$ is the nominal labor income. Disregarding $\text{Cov}(YP', P')$ term, if $C \text{Var}(P')$ is larger than $Z/L(G2+Z)$, the $h$ value will be greater than zero, implying that at least some fixed-rate mortgages will be taken. However, when the term $\text{Cov}(XP', P')$ is included, the results can be different. As stated before, a negative $\text{Cov}(XP', P')$ means that a borrower expects the real value of his/her house to rise with inflation. If this term is negative and sufficiently large in absolute value, the optimal value of $h$ will stay at zero, suggesting that only index-linked mortgages will be used. Thus, if the real value of the house is expected to rise with the inflation, the borrowers tend to prefer index-linked mortgage even if their nominal labor income is fixed. In Statman’s view, this effect is of special importance, because mortgage bankers usually ignore it and judge a borrower’s ability to pay the mortgage by his labor income.

In conclusion, Statman emphasizes that borrowers’ choice for fixed-rate and index-linked mortgages depend on the signs of the covariance between the rate of changes in labor income and the rate of inflation ($\text{Cov}(YP', P')$) and the signs of the covariance between changes in the net value of house and the rate of inflation ($\text{Cov}(XP', P')$).
Donald Smith's 1987 study extended borrowers' mortgage choice theory to include adjustable-rate mortgages. Building on Baseal & Biger and Statman's studies, Smith further examines the impact of covariance between the real interest rate and, in turn, real income and real asset values on consumers' mortgage decisions. This analysis focuses on the directional impact the various factors have on the borrowers' choice moving toward an all fixed-rate or an all adjustable-rate contract. The study is based on the following assumptions:

1. A mean-variance expected utility maximizing borrower in a single-period framework.

2. The borrower's objective is to maximize the expected utility of end-of-period real wealth.

The equations for calculating real wealth \( W \) are

1. For a fixed-rate mortgage:
\[
W^F = YP^t - FMP^t + XP^t
\]

2. For an adjustable-rate mortgage:
\[
W^A = YP^t - AMP^t + XP^t
\]

where \( Y \) = the borrower's nominal income at the end of the period;
\( F = 1 + \) fixed interest rate;
\( A = 1 + \) interest rate to be adjusted at the end of the period;
\( M = \) loan amount;
\( P = \) price level index at the end of the period;
\( X = \) the proceeds of the sale of the asset financed by the loan.

The borrower's utility function is assumed to be a simple function of the expected mean and variance of real wealth.

\[
EU(W) = E(W) - LVar(W)
\]

where \( L \) is the measure of constant absolute risk aversion. The borrower's optimal decision depends on which loan contract provides the higher expected utility of real wealth. Thus, if \( EU(W^A) - EU(W^F) > 0 \), an adjustable-rate mortgage will be chosen; if \( EU(W^A) - EU(W^F) < 0 \), a fixed-rate mortgage will be preferred. If the result is zero, the borrower is indifferent to these two mortgages. Adjustable-rate mortgages are tied to some reference market interest rate or index of rates. The nominal
adjustable rate can be decomposed into the inflation rate and the real rate. The focus of Smith’s study is to examine the extent to which variability in the real rate can affect the borrower’s choice between the two alternative mortgages. If inflation does not change the reference rate in an one-to-one fashion, the real interest rate will vary. Substituting 3.19 and 3.20 into 3.21 and using the expression for the variance of sums of random variables:

\[
EU(W^A) - EU(W^F) = E(FP^i) - E(AP^i) + LMF^2 \text{Var}(P^i)
\]

\[-LM \text{Var}(AP^i) + 2LM \text{Cov}(YP^i, AP^i) - 2LM \text{Cov}(YP^i, P^i)
\]

\[+ 2L \text{Cov}(XP^i, AP^i) - 2LM \text{Cov}(XP^i, P^i) \quad 3.22\]

The first two terms in 3.22 may be written as follows:

\[
E(FP^i) - E(AP^i) = [F - E(A)]E(P^i) - \text{Cov}(A, P^i) \quad 3.23
\]

The rate spread between fixed-rate and adjustable-rate \([F - E(A)]\) is positive, and the \(\text{Cov}(A, P^i)\) term in 3.23 will be negative as long as market rates and inflation move in the same direction. The result is \(E(FP^i) - E(AP^i) > 0\).

The next two terms in 3.22 may be combined as

\[
LM[\text{Var}(FP^i) - \text{Var}(AP^i)] \quad 3.24
\]

Since the market reference rate is positively correlated with inflation, changes in the real rate will be smaller than changes in the inflation rate. Thus the variance of the real cost of a fixed rate loan will exceed the variance of the real adjustable rate, indicating a positive result for equation 3.24. The remaining terms in 3.22 are

\[
2L[\text{Cov}(YP^i, AP^i) - FCov(YP^i, P^i)] + 2L[\text{Cov}(XP^i, AP^i) - FCov(XP^i, P^i)] \quad 3.25
\]

The implications of \(\text{Cov}(YP^i, P^i)\) and \(\text{Cov}(XP^i, P^i)\) were stated in Baessel and Biger’s and Statman’s theories respectively. The impact of variability in the real interest rate, which can be tested from term \(\text{Cov}(YP^i, AP^i)\) and \(\text{Cov}(XP^i, AP^i)\) is examined here. A positive covariance between the real
income and the real interest rate \([\text{Cov}(\text{YP}^t, \text{AP}^t)>0]\) shifts borrower’s preference toward the adjustable-rate mortgage. Similarly, a positive covariance between the real asset value and the real interest rate \([\text{Cov}(\text{XP}^t, \text{AP}^t)>0]\) increases the probabilities of ARM loans.

Smith’s conclusion is that borrowers’ choice for ARM loans depends on a number of factors including: a wide and positive rate spread, increases in real income and real asset value due to inflation and real interest rates, and a relatively low degree of risk aversion.

Jan K. Brueckner in 1986 further developed the theory of borrowers’ choice on adjustable-rate mortgages. In his view, a borrower is faced with a trade-off between mortgage cost and interest rate risk in choosing between an adjustable-rate mortgage (ARM) and a fixed-rate mortgage (FRM). Since the initial interest rate of an ARM is generally low, it may be assumed lower as compared to a FRM. But, the ARM borrower must bear the risk of future rate variability. The borrower thus evaluates the market trade-off between cost and risk and chooses the mortgage with the most favorable combination of these features.

The coexistence of different types of mortgages in today’s market indicates that borrowers react differently to the cost-risk trade-off. This suggests that individual characteristics play an important role in mortgage decision. Brueckner’s theoretical study aims to find out how the optimal mortgage choice depends on borrowers’ characteristics and the market condition.

Brueckner’s study includes three parts. First, a price equation relating the ARM margin \((M)\) and the rate cap \((K)\) is derived. The equation shows that these two parameters are inversely related: a lower margin relates to a higher rate cap. This is an indication that lenders are willing to lower the ARM margin to secure less interest rate risk.

The borrower’s indifference curve between the margin and the rate cap parameters, showing the demand-side trade-off between interest rate risk \((K)\) and mortgage cost \((M)\) is then developed. The borrower’s mortgage
choice will be the optimal margin-cap combination, which is a point of tangency between an indifference curve and the price curve.

Given the choice framework constructed above, a comparative static analysis is made about how a change in borrower characteristics influences the change in optimal mortgage. The analysis shows that:

1. Borrowers who place a high value on future consumption prefer a tight interest rate cap and are likely to favor FRMs.
2. Borrowers with rapidly rising income streams prefer a loose rate cap and are likely to choose ARMs.
3. Wealthy borrowers with a higher income level path usually have large housing consumption and thus favor tightening rate cap. They are likely to be FRM borrowers.
4. The change in the market environment also has some effect on borrower's mortgage decision. If both lenders and borrowers expect that the future interest will fall as the case of a recession, the optimal rate cap falls, and the borrowers will choose FRMs.

Brueckner and Follain's empirical study of 1988 estimates an econometric model of choice between adjustable-rate and fixed-rate mortgages. The purpose of the study is to "identify household characteristics and variables that are important determinants of the probability that a borrower will choose an ARM over fixed-rate mortgage."  

The data for the study includes a working sample of 475 mortgages in 1985, 316 of them FRMs and 159 ARMs. This sample is drawn from the Residential Mortgage Finance Database compiled by the National Association of Realtors. The mean contract interest rates for FRMs is 12.21 percent and for ARMs 10.44. The majority of the ARMs are one-year adjustable, the rest are six-month, three-year, and five-year adjustable. The average

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adjustment period in the sample is 1.29 years. Almost all ARMs have both adjustment-period and life-of-loan rate caps, with the average values being 2.34 percent and 4.98 percent, respectively.

A borrower’s choice between a FRM and an ARM depends on a number of factors. The first three factors are a borrower risk aversion, personal discount rate for future consumption, and the strength of his/her demand for housing. The theoretical background of these three assumptions is based on the following beliefs: An individual with low risk aversion and a high discount rate can live with the uncertainty of what ARM future payments will be and is likely to be an ARM borrower, while an individual with a high demand for housing will have a large mortgage loan and is highly sensitive to the risk of fluctuating interest rates and thus likely to be a FRM borrower. Moreover, when borrowers have dependent children, their risk aversion is expected to rise, while their discount rate on future consumption is expected to fall in response to concerns with future spending, such as college educational expenses. The dummy variable FAMILY is then used to represent the three borrower’s characteristics: high risk aversion, low discount rate and high demand for housing. All three are expected to have a negative impact on the choice of an ARM.

Another key factor is the borrowers’ income path characteristics consist of the current level as well as the rate of increase in their income stream. This is based on Brueckner’s conclusions of the previous theoretical study that the rate of increase of borrower’s income stream has a positive impact on ARM choice, whereas the level of increase of income has a negative effect. In this model borrower’s income (INC) is used to measure the level of the income, and the age of the borrower (AGE) is used as a proxy for the rate of increase of the income assuming that young borrowers have a higher rate of increase of income as compared to older borrowers.

Mobility also plays a critical role on ARM choice. Since borrowers with higher mobility usually prepay their mortgage, their interest-rate
risks will be much lower than other ARM borrowers, hence mobile individuals prefer ARM loans. The variable used in the model, identified as NEWMETRO, indicates that the borrower is new to the metropolitan area, and "the expectation is that individuals who have moved at least once between metropolitan areas are more likely to move again than those who have not."^8

In addition, the market interest-rate differential between FRMs and ARMs has an important effect on Borrower's mortgage choice. A higher differential raises the probability of ARM choice. The variable RATEDIFF is used to measure the rate differential.

Furthermore, the general level of rates tends to have a great impact on ARM choice. This hypothesis is based on the fact that in an inflationary environment, high nominal rates make the resulting initial payments more expensive under FRM. Since ARMs, which require lower initial payments, offer a way to ease the liquidity problem, the demand for ARMs can be expected to increase with the rise of inflation. The variable AFIXRT representing the FHLBB average rate is used as a measure of the general affordability of mortgages.

Another affordability effect on mortgage choice is measured by the variable REPTBUYR, which indicates whether the borrower is a repeat homebuyer. Since repeat buyers are able to make a large down payment out of housing equity, they are unlikely to be constrained by higher payment-to-income qualification standards under FRMs. On the other hand, many first-time buyers may not be able to meet the qualification and thus be pushed toward ARMs. REPTBUYR is expected to have an inverse relationship with ARM choice.

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The interest-rate expectations of borrowers also affect mortgage choice greatly. Different individual expectation on interest-rate leads to different mortgage choice. The problem is that the differences in individual expectation are unobservable. Thus expectation effects would be related to the error term in the mortgage choice equation.

Finally, the regional dummy variables NE, NC and SO\(^9\) are included in the model to explain the possibility that consumers in certain parts of the country especially prefer ARMs. The econometric model used is of the form

\[ V_i = X_i'B - E_i \]

where,

- \( X_i \): A vector consisting of the explanatory variables discussed above for observation \( i \).
- \( B \): A coefficient vector.
- \( E_i \): A standard normal error term.

An ARM is chosen if \( V_i > 0 \), whereas a FRM is chosen if \( V_i < 0 \).

One problem in estimating equation 3.26 is that the sample does not reflect the rate on the mortgage not chosen. Therefore, the FRM-ARM rate differential actually faced by the borrower is unknown. In order to solve this problem, RATEDIFF (rate differential) is estimated using available data to correct selectivity bias. The empirical results of their estimation of RATEDIFF, however, showed little variation in the FRM-ARM rate differential. This is an indication of consistent pricing policies within markets. Since the FRM-ARM rate differential is essentially non-random, the problem of selectivity bias can be ignored.

The estimated coefficients for both an equation using the selectivity-corrected RATEDIFF and an equation without the selectivity-corrected RATEDIFF suggest the following facts:

\(^9\) The transactions are roughly divided among the northeast, north central, southern, and western parts of the country. The first three regions are represented by the dummy variables NE, NC, and SO.
1. The coefficient of RATEDIFF is significantly different from zero and positive in sign, indicating that the ARM choice probability rises with the increase of FRM-ARM rate differential as expected.

2. The coefficient of AFIXRT also shows a positive relationship between ARM choice and the general level of rates. This means that for a given rate differential, the higher the general level of rates, the higher the probability of choosing an ARM. In other words, as the general level of rates rises, mortgages in general become less affordable for homebuyers. Therefore, the relatively cheaper ARMs become more attractive.

3. Among borrower characteristic variables, only INC and NEWMETRO are statistically significant. The positive coefficient of NEWMETRO supports the expectation that intermetropolitan movers with high mobility have a higher ARM choice probability.

4. In contrast to the expectation, the result shows a positive effect of the level of income stream (INC) on ARM choice, although the strength of the income effect is modest.

5. All the rest of the variables regarding borrower’s characteristics are statistically insignificant, suggesting that those variables (AGE, FAMILY, REPTBYR) have little impact on homebuyer’s mortgage selection.

6. All three of the regional dummy variables have negative coefficients, indicating lower probabilities of ARM choices for borrowers in nonwestern areas.

The above empirical results suggest that the most important determinants on mortgage choice are the FRM-ARM differential and the level of the FRM rate. An increase in either of these variables leads to an increase in ARM choice probability. In addition, mobile borrowers with high-income prefer adjustable-rate mortgages, and western borrowers have higher ARM choice probabilities than others.
Finally, the 1987 empirical study by Upinder S. Dhillon, James D. Shilling and C.F. Sirmans examines the impact of pricing and borrower characteristics on the choice between fixed-rate and adjustable-rate mortgages. The data for Dhillon's study were collected from the Baton Rouge, Louisiana office of a national mortgage banker on loans closed in January 1984. During this period, the lender maintained a consistent policy of originating and selling single-family loans in the secondary mortgage market at par. The total sample contains 78 observations with 46 fixed-rate and 32 uncapped adjustable-rate loans. All fixed-rate loans are nonassumable with 30-year maturities. The interest rates on the adjustable rate mortgages are tied to intermediate Treasury securities. The frequency of interest rate adjustment is between six months and one year.

The study assumes the interest-rate risk premium can be measured by the spread between long-term and short-term securities, and a standard probit probability specification. The model to be estimated is

\[ P_i = \Pr(I_i \geq I^*) \]  

where

\[ P_i = \text{dependent variable for borrower i that takes the value one if the borrower chooses an adjustable rate mortgage and zero otherwise; } I_i = \text{"mortgage choice" index, e.g., the index } I_i \text{ measures the propensity of borrower i to choose an adjustable-rate mortgage.} \]

Variations in \( I_i \) are explained by the following linear equation:

\[ I_i = f(\text{FI, M, YLD, PTS, MAT, AGE, SCH, FTB, FO, MC, SE, MOB, PR, NW, LA, STL}) \]

where

\[ \text{FI = fixed interest rate; } \]
\[ M = \text{margin on the adjustable rate mortgage; } \]
\[ YLD = \text{the difference between the 10-year Treasury rate less the 1-year Treasury rate; } \]
**PTS** = the ratio of points paid on adjustable to fixed rate mortgages;

**MAT** = the ratio of points paid on adjustable to fixed rate mortgages;

**AGE** = age of the borrower;

**SCH** = number of years of school;

**FTB** = a dummy variable of one if the borrower is a first-time homebuyer, zero otherwise;

**CO** = a dummy variable of one if there is a co-borrower, zero otherwise;

**MC** = a dummy variable of one if the borrower is married, zero otherwise;

**SE** = a dummy variable of one if the borrower is self-employed, zero otherwise;

**MOB** = number of years at present address;

**PR** = the ratio of mortgage payments to income;

**NW** = net worth of the borrower;

**LA** = liquid assets;

**STL** = short-term liabilities.

The coefficients reported in the Dhillon's study are estimated probability index changes with respect to changes in the independent variables. Since the probability of choosing an adjustable rate mortgage is a definite increasing function of the estimated probability index, the coefficients do indicate the directions in which the independent variables affect the actual probability of taking out an adjustable rate mortgage.

The results show that:

1. All the price variables are significant, indicating that those variables play a dominant role in the choice decision. For instance, the independent variables margin (**M**) and points (**PTS**) have a negative effect on the choice of adjustable-rate mortgages.
2. Some classes of borrowers, like households with co-borrowers, married couples, and those with short expected housing tenures, have a tendency to prefer adjustable-rate mortgages. However, other borrower characteristics, such as age, education, first-time homebuyer and self-employment are insignificant. Taken together, individual borrower characteristics have a weak influence on mortgage choice.

3. Holding everything else constant, the effect of variation in mortgage payment ratios on ARM choice is insignificant. This implies that borrowers "look beyond" initial payments.

4. The positive sign of variable NW (net worth of the borrower) indicates the borrowers with greater wealth tend to prefer ARM loans.

The conclusion is that pricing variables have a significant effect on mortgage choice decision. Households with co-borrowers, married couples, and short expected housing tenures have the greatest probability of taking out adjustable-rate mortgage loans.

Principles of Mortgage Pricing

There are two basic methodologies for mortgage pricing. The option pricing theory realizes that all mortgage contracts include some option elements. FRMs as well as ARMs grant the borrower the option of prepaying the loan at any time prior to maturity. Another obvious feature of a FRM, and one of the reasons for the popularity of ARMs among lenders, is that the borrower can retain the loan if interest rates rise but may refinance at a lower rate, a call option, if rates decline. ARMs offer options of their own: for example, if the market interest rate increases beyond the rate caps on the ARM, the borrower only faces a maximum rate hike as dictated by the interest rate caps. The lender, in essence, has granted the borrower the "option of not paying the difference between the market rate and the cap on the adjustment date for the length of the adjustment
period" if the former lies above the cap ceiling. Option pricing theory is then useful in determining mortgage prices based on the claims given up by the lender or the borrower.

Simulation, or Monte Carlo techniques, employs the known or assumed characteristics of changes in interest rates to perform "what if" experiments on ARMs with different terms. Different interest rate generating processes may be assumed. For example, the mean reverting assumption implies that long terms interest rates change at random but tend to revert to the mean average. The Wiener process, on the other hand, is one in which random interest rate changes, defined in terms of a trend and volatility, are generated and simply added to the previous rate over a several-year period. Regardless of the rate-generating process, different ARMs may be compared by observing their payment and loan balance behavior. If, for example, the scenario calls for a substantial upward trend with large volatility in interest rates, the ARM with small rate caps may not perform well for the lenders while those with more liberal rate change caps perform better. Of course, a more liberal rate cap may result in a smaller margin or lower initial rate, and the resulting ARM, in turn, will not perform well if rates do not rise or rise slowly with little volatility.

Michael Asay,10 emphasized the possibility of paying less or more than the market rate of interest because of the cap or floor. With some simplifying assumptions, he was able to find the price of ARM caps in terms of a discount from a simple ARM—an adjustable rate mortgage without caps that adjusts completely and quickly to the current index rate. This ARM grants no options and has a value equal to the discounted future value of its payments. The discount from the ARM with no caps is the "price" of the


cap on a regular ARM. Asay found that a 1 percent annual cap required a 2.86 percent discount from par. A 2 percent cap reduced the discount by .94 percent to 1.92. For an ARM with a 1 percent annual and a 5 percent lifetime cap, changes in the assumed volatility of rates resulted in further discounts. An increase of 1 percent in volatility, from 2 to 3 percent, resulted in a jump in the discount to 6.15 percent and a 1 percent decline in volatility lowered the discount to .37 percent. The volatility of interest rates, then, is a primary factor in pricing ARMs.

As an alternative to a discount from a simple ARM, the caps of a regular ARM may be priced in terms of the margin over index, more restricted caps demanding a larger margin. With an average life of thirteen years and an initial 12 percent interest rate, a 1 percent point discount in the ARM price translated to an additional 20-basis-point margin.

Kau, Keenan, Muller, and Epperson focused on the margin required by lenders to make the value of an ARM to them equal to its value to borrowers. The latter is equal to the loan amount plus the value of the prepayment option. They assumed the interest rate was a mean reverting process of the following form:

\[ dr = s.(t - r)dt + (r.dvar) \]

where \( t \) is the long-term interest rate, \( r \) is the short-term rate, and \( s \) is the speed of adjustment.

After establishing the boundary conditions for the value of the ARM components, they followed a simulation experiment to find the optimal trade-off of the margin and the annual and lifetime rate caps. For ARMs originating at the assumed long-term rate of 12 percent, imposing a 2

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percent annual cap required a margin increase of 42 basis point from 116 to 158. Moreover, the placement of a 5 percent life-of-loan cap on a previously uncapped loan demanded a 15-basis-point increase from 143 to 158 basis points. This indicates the lower value of the 5 percent lifetime cap compared with an annual rate cap of 1-2 percent.

Clauretie and Sklar\textsuperscript{13} reported the result of a simulation on 10 ARMs available in the marketplace at the time the study was done. The ARMs varied according to the frequency of rate change, types of caps, first-year rates, discounts, and margins. They were tested under four different interest rate scenarios based on annual trends and volatility. The authors assumed that for each month during a 12-month period, ARM payments were invested in a fund earning the conventional rate of interest. This fund was added to the loan balance at the end of the period to form the loan's terminal value which is turn was used to rank the ARMs. In addition, a FRM with three discount points was included and compared with the ARMs.

The results showed that for moderate upward trend and variability, the fixed rate mortgage was quite competitive, it was ranked first under two scenarios and fourth in another. Plus, the FRM could also perform well for declining rates; the authors, however, only considered low, moderate and large upward trend, and omitted the simulation of a negative trend. ARMs with no rate change caps, or if the caps were present, with large discounts to compensate for the caps, performed consistently well. The results confirmed that the lender may profit from trading off the caps for less frequent change intervals. Loans with a combination of annual and lifetime interest rate caps and attractive first-year rates performed poorly regardless of the interest rate scenario. The poor performing ARMs also tended to have low margins and low initial rates in combination.

Clauretie\textsuperscript{14} noted that most ARM terms that can be altered to increase investment returns, rate caps and frequency of rate change for example, also increased the variability of investment performance. Discount points and margins, however, may be increased to produce larger returns without increasing the variance. He showed that ARMs with above average discounts and margins are mean-variance efficient when simulated over various interest rate scenarios. Lenders who design the ARM should then consider pricing caps in terms of larger discounts and margins.

Crane and Lea\textsuperscript{15} reported on the results of two simulation procedures. In the first case, the interest rate generating process for any period began with the forward rate implied by the yield curve and then randomly generated changes were used to adjust that rate. The second procedure generated interest rates based on the prior year's rate. The authors were able to indicate the discount necessary to equate capped with uncapped ARMs from the difference in the present value of the income stream generated by each type. The resulting discounts implied by the two methods of generating interest rates were significantly different for a 1 percent payment capped ARM. The discounts for both the 1 and 2 percent caps under the second method were virtually identical to those suggested by Asay using the option pricing methodology.

Buser, Henderson, and Sanders\textsuperscript{16} employed the standard option pricing method to establish an interest rate generating procedure for their simulation in an attempt to estimate the discount required for different sizes of the lifetime cap. Under a normal yield curve, only 24-28 basis

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\textsuperscript{14} T. Clauretie, "ARM's Investment: Variance and Returns," \textit{Mortgage Banking,} Summer 1986: 45-56.


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points were required as the discount for a high variability scenario. When the yield curve was assumed steeply rising, high volatility required a discount of 555-70 points and moderate volatility demanded 30-40 basis point discount.
CHAPTER 4: FUTURES MARKETS

In the early 1800s, it was common for farmers to bring their crop to centralized markets for sale, where often times they faced temporary lack of demand. Within a few months, however, the demand for such products would rise considerably above the available supply, thus creating a large increase in prices. Such adverse price movements were disruptive to the market, and realizing the seasonal nature of the disturbance, traders began making contracts for forward or future deliveries. The Chicago Board of Trade was founded in 1848, and by the late 1800s, the rules of futures trading were formalized by creating standardized futures contracts, and establishing rules of conduct, clearing, and settlement.

The Chicago Mercantile Exchange was started in 1874, followed by the Kansas City Board of Trade (1856), the Mid-America Commodities Exchange (1868), and the New York Commodities Exchange, Inc. in 1933.

Futures contracts are legal obligations to make or accept delivery of a specific commodity at a predetermined time and place. The terms and conditions of the futures contracts are nonnegotiable by the trading parties, except for the contract price which is determined through open bidding on the floor of exchanges. This allows transactions to be conducted quickly, and increases liquidity of futures contracts trading.

The two most general measures of liquidity and depth of futures markets are trading volume and open interest. Trading volume is the number of contract-unit transactions during a given period of time. Open interest is the number of open contracts at any given point in time.

Futures trading volume is reported as one side only--buys and sells are not added together--although the number of contracts bought always equals the number of contracts sold. A large volume of trading is a good indication of a liquid market.
Open interest is the number of contracts which have not been offset by an opposite trade, nor settled by delivery at the end of the business day. Open interest is also recorded on a one-side basis.

**Interest Rate Futures**

The need for the organized trading of futures contracts on fixed income securities was evident after the sharp rise and fall of interest rates in 1969-1970, and 1973-1974. Frequent fluctuations in rates since 1979, have made the advantage of trading interest rate futures quite clear and caused a tremendous growth in the volume of trade for such instruments.

In October 1975, the Chicago Board of Trade began marketing the Government National Mortgage Association—Collateralized Depository Receipt (GNMA-CDR) futures contracts, followed by the Chicago Mercantile Exchange introduction of 90-day U.S. Treasury Bills futures market. Currently, the CBT lists various interest-rate futures contracts including those on U.S. Treasury bonds, 5 and 10 year Treasury notes, as well as Municipal Bond futures, stock index futures, and most recently (June 11, 1990) 2-year U.S. Treasury note futures.

Treasury bonds are obligations of the United States government to make semi-annual coupon payments, along with the lump sum payment of the principal at a predetermined maturity date to the holder of such securities. All the specifications of Treasury securities, except the price, are known at the time of the sale. The price is determined at competitive bidding to purchase the securities.

The U.S. government auctions Treasury bills, Treasury notes, and Treasury bonds with a specific maturity through the 12 Federal Reserve Banks and the U.S. Treasury Department Bureau of Public Debt. Small investors, wishing to invest less than $1 million, submit noncompetitive

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17 The institutional aspects of the interest rate futures contracts may be found in "The Treasury Futures for Institutional Investors" and "CBOT Financial Instrument Guide" published by the Chicago Board of Trade.
bids, agreeing to pay the price established by competitive bidders who must specify quantity, and minimum yield to maturity at which they are willing to buy.

The Treasury fills all noncompetitive bids first, and awards the remaining issues to competitive bidders starting with the lowest yield (highest price). The average of the yields awarded is calculated, and the coupon of the newly issued security is set to the nearest 1/8 of a percent below the average yield. For Treasury bills, however, the full face value must be paid at the auction, with the difference between the face value and the average price set at the auction refunded to the purchaser in about two days. The interest income for T-bill will be realized when the full face value is received at maturity.

Treasury bills auctions are held weekly, while Treasury bonds and notes are sold about once a month. The maturities range from 91 days to a year for bills, and two to thirty years for notes and bonds. The size of an issue is as much as $10 billion dollars, accounting for the exceptional liquidity of these markets.

Approximately $100 billion worth of government securities are traded daily. In addition to the large size of the offerings, the frequency of auctions, relative absence of credit risk, and the intense competition among government securities dealers, leading to narrow bid-ask spreads, are the causes of such high liquidity.

The secondary market for Treasury securities is a dealer market. Security dealers purchase a large sum and a wide array of securities for their own account, and with their own capital. They report to the Federal Reserve System, and trade either directly with the public, or indirectly through a network of brokers.

The standardized features of the CBT Treasury futures require the delivery of the underlying Treasury instrument with a face value of $100,000 during the months of March, June, September, and December. These months can extend to more than two years in the future. At any given time,
several Treasury securities may qualify as deliverable for a CBT Treasury futures contract.

The Treasury bond futures contract stipulates delivery of $100,000 face value, 8 percent coupon, U.S. Treasury bonds with at least 15 years to maturity, or if callable, with at least 15 years to call date. However, it should be realized that yields in financial markets are "generally" quoted on a hypothetical, 8 percent, 20 year Treasury bond. This information is used only for reference purposes, and although 8 percent is used as the basis of the CBOT T-Bond futures contract, other bonds with different coupon rates may be delivered in to the correlating futures as long as they meet contract specifications with regard to minimum time to maturity. It is the choice of the short, the seller, to determine which issue to deliver, and logic dictates that the short would choose to deliver the cash instrument which is most economical.18

The seller of the futures contract must identify the least costly cash instrument to deliver in cases of contract settlements. For example, a $100,000 T-bond, with a 12 percent coupon maturing May 15, 2005, has greater market value than a $100,000 T-bond with a 10 3/4 percent, maturing August 15, 2005. The 12 percent T-bond offers $12,000 in annual interest compared to $10,750 received by holding the 10 3/4 percent T-bond. The most "advantageously" priced instrument for delivery is known as the cheapest-to-deliver (CTD). With a variety of bonds trading at any time, with different coupon rates and maturities, the seller has a difficult task in identifying the cheapest-to-deliver instrument. The ability to accurately compare the cash and futures prices is imperative in making the selection.

For the purposes of comparing the cash and the futures prices, and in order to allow the delivery of any outstanding U.S. T-bonds for settlement

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of futures contracts, a "conversion factor" system has been developed by
the CBOT. A conversion factor represents the price, as percentage of par,
at which a particular bond or note will yield 8 percent. If the above
mentioned 12 percent T-bond of May 15, 2005, were to be delivered in March
1988, a conversion factor of 1.3682 would signify that this bond is
approximately 137 percent more valuable than the 8 percent futures contract
standard. Similarly, the 10 3/4 percent bond of August, 2005, deliverable
in March 1988, is 125 percent more valuable than the contract standard.

If a futures contract position remained open following the last day
of trading for this contract, the dollar value of any specific T-bonds to
be delivered against the open position would be obtained according to the
following formula, and is termed the "invoice price."

\[
\text{Invoice Price} = \text{Contract Size} \times \text{Futures Contract Settlement Price} \times \text{Conversion Factor} + \text{Accrued Interest.}
\]

Assuming a settlement price of $80,000 or 80 percent of par for March
1988 T-bond futures contract, the invoice price of the 12 percent, May
2005, T-bond would equal $109,465, ($100,000 \times 0.8 \times 1.3682) + accrued
interest. Cash equivalent price of a futures contract, or adjusted futures
price, is determined by multiplying the futures price by the appropriate
conversion factor, i.e., invoice price without regard to accrued interest.
Conversely, when the cash price of any deliverable bond or note is divided
by its appropriate conversion factor, the resulting price is called a
futures equivalent price or adjusted cash price.

The conversion factor for an 8 percent bond is approximately one.
The eligible instruments for delivery, with a coupon lower than 8 percent,
have a conversion factor of less than one, signifying the discount.
Similarly, T-bonds with a coupon of more than 8 percent, show a premium
over par by a conversion factor which is greater than one. Since invoice
prices incorporate conversion factors, they will also reflect any premiums
or discounts.
Chicago Boards of Trade Treasury futures contract prices are quoted in points, as a percentage of par value. The minimum price increment is called a "tick", and is equal to one thirty-second of one percent or $31.25 (1/32 X .01 X $100,000). A T-bond future contract priced at 91-10 indicates a dollar value of 91 10/32 percent of par or $91,312.50 (.91315 X $100,000). A 5-year T-note futures contract quoted at 85-06 has a dollar price of $85,187.50.

Through the conversion factor system, a general relationship between the cash price and its equivalent futures price is established. This is critical in determining the cheapest-to-deliver instrument in cases of settlement. Specifically, the price of the futures contract approximates the price of the cheapest-to-deliver cash instruments if its coupon were 8 percent while holding its maturity and yield constant. In other words, throughout its life, a futures contract tends to track the adjusted cash price of the cheapest-to-deliver instrument. In reality however, there is a price differential between the two. The difference between the cash price and the futures price of the same instrument is known as the basis. It is an important concept because basis changes tend to be more stable than changes in either cash or futures prices.

The basis can be either positive, signifying an excess of cash over futures price, or negative in cases where cash price is lower than the futures price. Theoretically, the basis should equal "the cost of carry." Assuming that financial market participants finance a cash position by borrowing at the short term rates, and lock in a selling price by writing futures contracts, the difference between interest received from coupon payments and the short term financing rate is the net financing charges to hold the cash position.

If the investor earns more interest income on the cash bond than he paid to finance it, he experiences a net interest income or "positive carry." The difference between adjusted cash price and the futures price—the basis—is thus also positive. Positive carry is characteristic of an
upward slopping yield curve, where long term rates are higher than short term rates. In a positive interest rate environment, thus, cash prices exceed futures price. When the observed yield curve is inverted, there is negative carry due to excess of long-term over short-term rates. The investor would be spending more on a net basis to finance the purchase of the cash instrument, hence, the futures price would be higher than cash price.

The Theory of Hedging with Financial Futures

With the understanding of the basics of futures markets and specifications of Treasury securities futures contracts, the next step is to analyze hedging opportunities in the mortgage markets offered by interest rate futures.

Futures markets provide several advantageous possibilities for hedgers. Among them are continuous trading on an exchange floor, firm prices displayed instantaneously, immediate offsetting positions available without any need to renegotiate terms of the commitment, a narrow price spread between bid and ask prices for most futures contracts (typically 1/32 of 1 percent), and the existence of a clearing house to insure satisfactory settlement of contracts.

Working\textsuperscript{19} suggests that futures markets have traditionally been viewed as speculative markets. The hedging function of the futures markets was regarded as a useful by-product. To many people, however, the hedging function of the futures market is what is desirable and appealing. Any individual facing a price risk—the risk that the value of a cash position will change from its current level—may consider the hedging benefits of the futures markets.

Hedging classification schemes are either based on the hedger's purpose or based on the hedger's choice of hedging instrument. A direct

hedge is a hedge in futures of the same commodity as the hedger has a current or anticipated cash position. A cross hedge is a hedge in the futures of a different, but related commodity. The criteria for judging the "sameness" of the asset may be the grade, the quantity, or the time of delivery.

There are several types of hedges in the futures markets, of which three are more important in application to financial markets. A selective, or discretionary hedge, is part hedge and part speculation. Its primary purpose is to prevent large losses in the value of commodity stocks, or cash financial instruments, by attempting to forecast the direction of price. An expected decline in price is motivation for placing a full hedge, while expectations of higher prices would encourage investors to lift all or portions of their short hedge to benefit from a rise in the value of their asset. In other words, the motivation of hedging is not risk avoidance in the strict sense, but avoidance of loss.

The second type of hedge used in financial markets is the risk-avoidance hedging, often called an "insurance hedge." This type of hedge is undertaken to avoid any changes in the market value of the cash position. To protect themselves from adverse price or interest rate fluctuations, commercial interests typically sell a quantity of futures sufficient to hedge their full inventory, thus this hedge is often "complete."

An anticipatory, or forward pricing, hedge is undertaken to "lock in" and protect a cash position that is expected to be taken in the future. For example a bank planning to borrow funds in six months might sell a financial futures contracts to lock in its future borrowing rate. To the extent that interest rates and futures prices move inversely, an increase in the cost of borrowing caused by higher rates would be offset by the gain from purchasing the futures contracts at a lower price than it was originally sold.
Successful hedging strategies to reduce exposure to interest rate risk using financial futures requires the hedgers to first determine the exact dollar amount of the position to be hedged. Then an appropriate futures contract has to be selected to minimize basis risk; the number of contracts needed to provide a full hedge must also be determined. Then the buy or sell in the futures markets would be executed, with a reversal of the position at the end of the hedging horizon.

The most important step in successful futures hedging is to devise a way to manage the differences in cash and futures price movements, commonly called basis risk. This occurs due to different price sensitivities exhibited by the futures and cheapest-to-deliver instruments with regard to yield changes. Minimizing the basis risk is achieved through the use of the appropriate hedge ratio, also called the cash-futures equivalency ratio. The hedge ratio represents the number of futures contracts providing a desired change in the value of the futures position when the value of the cash position changes. Figures 4-1 and 4-2 show graphically the cases of a perfect and a partial hedge using futures contracts.

In case of a perfect hedge, the seller of the futures contracts fully offsets any changes in the cash position when price movements affect his securities portfolio. Similarly an investor who has a short position in the security market may fully protect his position by buying the appropriate number of futures contracts. A partial hedge on the other hand would protect the long or short cash position only partially. Thus, choosing the best futures contract and the correct number of such contracts becomes essential in futures hedging.

In choosing the appropriate futures contracts as the hedging instrument, the hedger must determine the price correlation between the futures and cash instruments, as well as the hedging time horizon, the contract option months, and the liquidity of the futures contract.

If the hedger expects that a change in the market value of the cash instrument to be offset by an opposite change in the price of the futures
Figure 4-1. A complete hedge using futures contracts
Figure 4-2. A partial hedge using futures contracts
market instrument, the magnitude of the change in values relative to one-
another must be examined. Regression analysis may be used to determine the
correlation of the futures price movements to the yield, or price changes
in the cash market. The "best" or the most closely related futures
contracts will then be identified. The regression coefficient may also be
used as the appropriate hedge ratio. This subject is further examined in
Chapter 7 and is used to determine the hedge ratio when attempting to use
T-bond futures contracts to hedge changes in mortgage values.

In addition to regression coefficients mentioned above, other formal
methods exist to determine the appropriate hedge ratios. They include
conversion factor weighing, basis point value weighing, and duration
comparisons.

Conversion factors, as measures of relative price sensitivity of the
cash instrument to the futures contract, may be used as equivalency ratios.
For example, the conversion factor used in an earlier example (1.3682 for a
12 percent bond maturing in May 2005) signifies that price sensitivity of
cash instrument is approximately 136 percent of the futures price
sensitivity. So the conversion factor shows how many futures contracts are
needed to hedge $100,000 face value of the cash position. Inherent in this
analysis is that the cash instrument being hedged is the cheapest-to-
deliver into the futures contract. If not, the futures contract is tracing
one bond, while the conversion factor is adjusting for the volatility of
another. The result is basis point risk.

A basis point value, BPV, is the change in price of a debt instrument
resulting from one basis point change in the yield of that instrument,
holding maturity constant. Thus, the hedge ratio is calculated as the
ratio of basis point value of cash security to basis point value of the
futures contract. This ratio, thus, provides us with the change in cash
instrument price relative to the change in futures contract price.

A point of concern in calculating basis point value of the futures
contract is the familiar assumption that the futures contract price tracks
the price of the cheapest-to-deliver cash security. As a result, the BPV of the futures contract is simply the BPV of the cheapest-to-deliver instrument divided by its conversion factor.

Duration is the weighted average of the present value of all the cash flows, with years of cash flow used as the appropriate weights. Alternatively, duration may be defined as weighted average time until cash flow payment, using the present value of cash flows as the relevant weights. Regardless of which definition is used, duration is a means of comparing interest rate risk between securities with different coupons and different maturities. As the maturity of a bond increases, duration also increases. Similarly a decrease in the yield of the security, lower coupon payments, and less frequent cash flows increase duration.

Duration represents the price sensitivity of a security to changes in yield, with higher duration representing a riskier bond. If there is an expected change in the discount factor (yield), the understanding of the concept of duration, and the effective use of it, may help the investor in choosing the correct security to purchase. For example, the duration of a 14 percent bond with maturity of seven years is 4.99 with the assumption of discount factor of 8 percent. Similarly, a 7 year, 7 percent coupon bond, yielding 8 percent has a duration of 5.61 years. If there is an expected rate increase, the 7 percent bond should be avoided due to its relatively higher interest rate sensitivity.

Since duration measures the percentage change in price of a security in response to a given percentage change in yield, knowing the duration and the change in yield, the change in price of that security may be calculated.

After identifying a series of futures contracts, the hedging time horizon needs to be examined. A portfolio of securities with 1 year to maturity may best be hedged using T-bill futures, whereas portfolios of longer duration should be hedged with T-note or T-bond futures.
Liquidity and contract month are closely related, and must be considered next. Generally, the more distant months in the futures contract are much less liquid than the nearby months. The use of nearby months, thus, provides more liquidity, and also allows for rolling the position forward thus provide for hedging of long term risk.

The Hedger as A Utility Maximizer

The analysis of how large a portion of the cash market a hedger would choose to hedge may be studied in the utility maximization framework. Suppose, as a consequence of decisions made at time $t$, an investor anticipates acquiring $q_{t,T}$ quantity of cash instrument in time $T$. The cost of the instrument at time $t$ is denoted $c(q_{t,T})$, and $x_{t,T}$ represents the quantity of output hedged in time $t$ by selling T-bond futures. The investors overall profit at time $T$, denoted by $\Pi_t$ is given by

$$\Pi_t = P_t \cdot (q_{t,T} - x_{t,T}) + (\Phi_t + F_{t,T}) \cdot x_{t,T} - c(q_{t,T})$$  

where $P_t$ is the investor's revenue from an unhedged unit of the instrument, and $\Phi_t + F_{t,T}$ represent investor's revenue from a hedge unit of the assets.

$\Phi_t$ denotes maturity basis, which by definition is $P_t - F_{t,T}$. Thus, the first term on the right hand side is the revenue from the unhedged portion of output, the second term is the revenue from the hedged portion of the output, and the third term is cost.

Then,

$$E_t(\Pi_t) = E_t(P_t) \cdot (q_{t,T} - x_{t,T}) + (E_t(\Phi_t) + F_{t,T}) \cdot x_{t,T} - c(q_{t,T})$$  

$$\text{Var}_t(\Pi_t) = \text{Var}_t(P_t) \cdot (q_{t,T} - x_{t,T})^2 + \text{Var}_t(\Phi_t) \cdot x_{t,T}^2 + 2 \cdot (q_{t,T} - x_{t,T}) \cdot x_{t,T} \cdot \text{cov}(P_t, \Phi_t)$$

---

where $E_t(\Pi_T)$ is the expected terminal profit and $\text{var}_t(\Pi_T)$ is the variance of profit.

Assuming that the basis has a mean of zero for all terminal values of spot price, $E_t(\phi_t) = 0$ for all $P_T$, implies that $\text{cov}(P_T, \phi_T) = 0$, and $E_t[F_{t,T}] = E_t[P_T]$. Then,

$$E_t(\Pi_T) = E_t(F_{t,T}) \cdot (q_{t,T} - x_{t,T}) + F_{t,T} \cdot x_{t,T} - c(q_{t,T})$$ \hspace{1cm} (4.4)

$$\text{var}_t(\Pi_T) = \text{var}_t(P_T) \cdot (q_{t,T} - x_{t,T})^2 + \text{var}_t(\phi_t) \cdot X_{t,T}^2$$ \hspace{1cm} (4.5)

rearranging terms in equation 4.4 yields

$$E_t(\Pi_T) = E_t(F_{t,T}) \cdot q_{t,T} - (E_t(F_{t,T}) - F_{t,T}) \cdot x_{t,T} - c(q_{t,T})$$ \hspace{1cm} (4.6)

Assuming that there is a cost to hedging, $E_t(F_{t,T}) - F_{t,T} > 0$, and the basis risk is less than the price risk, we can graph the relationship between $E_t(\Pi_T)$ and $x_{t,T}$, and the one between $\text{var}(\Pi_T)$ and $x_{t,T}$, Figure 4-3 and the one between $\text{var}(\Pi_T)$ and $X_{t,T}$, Figure 4-4. In graphing these relationships, we assume that $\text{var}_t(P_T)$, $\text{var}_t(\phi_t)$, $E_t(F_{t,T})$, $q_{t,T}$, $F_{t,T}$, and $c(q_{t,T})$ are held constant.

The above two graphs can be combined to show the relationship between the producers expected profit, $E_t(\Pi_T)$, and the variance of profit as shown in Figure 4-5.

Figure 4-6 shows that the optimum profit-risk trade-off may be found by overlaying the risk-return indifference map and Figure 4-5. We may then refer to Figure 4-3 or 4-4 to find the size of optimal hedge, denoted by $X_{t,T}$ amount of time $T$-delivery futures at time $t$.

Next, if $F_{t,T}$ were to decline so that the cost of hedging $(E_t(F_{t,T})-F_{t,T})$ were to rise, expected profit would fall while the variance of profit would not change. As a result, the profit/risk curve of Figure 4-6 pivots downward, thus increasing the amount of risk. Comparison of points $M$ and $M'$, in Figure 4-7 show that the amount of the instrument that the producer
Figure 4-3. Expected profit as a function of quantity hedged
Figure 4-4. Variance of profit as a function of quantity hedged
Figure 4-5. Profit/risk trade-off as a function of quantity hedged
Figure 4-6 The optimal profit/risk trade-off for a risk-averse utility maximizer
Figure 4-7. The profit/risk line when cost of hedging increases
has chosen to hedge declines which makes sense in lieu of higher hedging cost.

If the cost of hedging is zero, the profit/risk curve would be a horizontal line, Figure 4-8. In such a case, all risk-averse producers would hedge their entire output.

Johnson\(^{21}\) showed that basis risk and price risk are related according to the following equation

\[
\text{Var}(\delta_T) = (1 - P_{PF}) \cdot \text{Var}_s(P_T)
\]

where \(P_{PF}\) is the correlation coefficient between \(P_T\), and \(F_{TT}\), and \(P_{PF}\) is the coefficient of determination, and shows the effectiveness of hedge. For example, a \(P_{PF} = 1\) implies a perfect correlation between spot and futures prices, and basis risk is zero, hence the hedge is perfectly effective.

If the effectiveness of the hedge increases, the variance of the profit would decline in response to the decline in the variance of the basis (equation 4.6). The profit/risk thus pivots upward, as shown in Figure 4-9, with the resulting optimal hedge at point M\(^{*}\). Since the new optimal hedge has a higher expected profit, the amount of hedge has declined. In other words, as the hedge becomes more effective, the investor requires less of it for the desired level of risk reduction.

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Figure 4-8. The optimal quantity hedged when hedging is costless
Figure 4-9. The optimal quantity hedged when hedging effectiveness increases
CHAPTER 5: THE OPTIONS MARKETS

Chapter 5 is composed of two sections. First, various options market instruments, including Treasury bond and Treasury note options, options on financial futures, and options on interest rate indexes, will be introduced. Then, the Black's options pricing theory on financial futures will be discussed.

Options Markets Instruments

A T-bond option contract covers $100,000 principal amount of a specific issue of the two or three most recently issued bonds with approximately 30 years to maturity. These options are generally introduced for trading two business days following the auction of the particular T-bond, and have four consecutive monthly expiration dates following the auction month for the underlying securities. Trading of options on a particular issue will be replaced by options on more recent and hence more liquid Treasury bonds of the same type.

T-bond options strike prices bracket the current market price of the underlying security at intervals of 1/2 point, ensuring the existence of a near-the-money option at all times. Moreover, strike prices generally will cover an interval at least two points above, and two points below an issue's current market price. However, with low volatility of interest rates, a one-point strike price interval could be set, and with increased volatility, the interval could be expanded to four points above and below the security's market price.

Premiums for options on T-bonds are quoted in terms of points and 32nds of a point, with minimum fluctuations of 1/32 of a point. Each point is equal to one percent of the underlying amount (.01 x $100,000 = $1,000). Thus, the minimum fluctuation equals $31.25 (1/32 x $1,000). For example,

22 For a more detailed discussion of the institutional aspects of T-bond and T-note option contracts see "Understanding Treasury Bond and Note Options" published by the Chicago Board Option Exchange.
a quote of 3.04 equals $3,125.00 (3 4/32 x $1,000). If exercised, settlement of these options occurs on the second business day following exercise. This allows market participants to cover positions in the cash market as the option on T-bonds is based on a specific issue concept. The only deliverable security is the one carrying a particular rate of interest, maturing at a specific time.

**T-note options contracts** are very similar to options on the bonds in characteristics and uses. The underlying securities are specific issues of the most recent 5 year Treasury notes of $100,000. Strike prices are set at one-point intervals as percentage of par and bracket the current market of the individual issue. Expiration is generally in the two nearby months on the March quarterly cycle (March, June, September, December), and final settlement is the Saturday following the third Friday of the expiration month. Premiums are stated in points and 32nds of par, with minimum of 1/32.

**Options on financial futures contracts** are written on a debt security futures contract as the underlying asset, rather than the debt security itself. Trading of options on Treasury bond futures began in October 1982, on the floor of the CBOT, and was followed by the introduction of options on T-note futures in May 1985, and options on Municipal Bond Index futures in June 1987.

The trading unit for the CBOT options on U.S. Treasury bond/note futures is a $100,000 face value CBOT U.S. Treasury bond/note futures contract of a specified delivery month. The long investor in T-security futures option may exercise the option on any business day prior to expiration by notifying the clearing corporation who in turn assigns the notice to an option writer through its clearing members. A futures position is thus established for the options buyer (long futures position

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23 See "CBOT Financial Instruments Guide" for more of a detailed discussion.
for call buyers and short futures for put holders), and an opposite futures position for the seller at the futures option strike price. After exercise, the investors may liquidate the futures position immediately or at a later time, and they also may hold the contract until expiration and make or accept delivery.

T-bond futures options strike prices bracket the current T-bond futures contract price at integral multiples of 2 points, while strike prices are set at intervals of 1 point per T-note futures contracts. The trading months for futures options are the same as those for CBOT T-bond/T-note futures contracts, March, June, September, and December. Premiums are stated in points and sixty-fourths of a point, with minimum price fluctuations of one sixty-fourth of a point or $15.625 per contract ($100,000 x 1/64%). A price quote of 3.04 for an option on T-bond futures contract, thus, equals $3,062.50.

Critics of futures options point out that investors in such instruments are twice removed from the cash market, making hedging strategies more complex compared to options on cash instruments. However, when it is advantageous to exercise options on Treasury futures, investors move from one leveraged position to another. While exercising the option on bonds requires payment of the full market price of the security in cash, the exercise of a security futures option requires only margin money to establish the futures position. Furthermore, protection continues through the futures position even after the option on futures is exercised.

Other advantages of options on futures over options on cash bonds include the availability of an array of deliverable bonds against a futures contract versus the specific bond issues deliverable against a T-bond option. Moreover, the underlying asset for futures options is extremely liquid, and premiums for such options reflect the competitive, continuous pricing of T-bond futures. Finally, futures contracts, and options on futures are traded on the same floor whereas cash securities and options written on them are not.
Options on interest rates, introduced by the Chicago Board Options Exchange on June 23, 1989, are "cash settled, yield driven" instruments which provide another alternative for traders in financial instruments in terms of hedging and speculative activities. These options are based on an underlying composite value which is calculated using interest rate yields rather than security or security futures prices. Moreover, delivery of bonds, or bond futures is not necessary in cases of exercise, as options on interest rates are settled in cash. The contract size is the same $100 multiplier as options on equities and stock indexes, and are only to be exercised at maturity, unlike other financial options.

The short-term rate option is based on the yield of the most recently auctioned 13-week Treasury bill, while the options on long-term interest rates are based on the average yield to maturity of the two most recently issued 7, 10, and 30-year Treasury securities. The value of the composites are 10 times the underlying Treasury rates, T-bill rates for the short-term interest rate options, and the average of the six longer-term notes and bonds for options on long-term interest rates. An average yield-to-maturity of 10.55 percent would place the long term composite at 105.5, and an annualized yield of 7.5 percent on newly issued 13-week Treasury bills translates to a composite value of 75.00. When the Treasury rate changes one point the composite changes by a multiple of 10, or 10 points.

The strike prices of interest rate options are set at 2 1/2 point intervals to bracket the current value of the composite. Each interval represents 1/4 of one interest rate percentage point, or 25 basis point. Strike prices represent the underlying interest rates, a strike price of 97 1/2 represents 9.75 percent interest rate.

Exercise of an interest rate option gives the holder the cash difference between $100 times the strike price and that day's closing composite value multiplied by $100. For example, if interest rates average to 9.75 percent at expiration, the holder of a call option on interest rates with strike price of 92 1/2 would exercise the option and receive
$500, the difference between the current composite and the strike price (97 1/2 - 92 1/2 = 5) times $100. If the composite closes below the strike price of a call, or above the strike price of the put, the option would expire unexercised and the holder would lose any premium paid.

Premium for interest rate options are quoted in points and fractions with a minimum fraction of 1/16 if the premium is below 3 points, and a minimum of 1/8 for all other premiums. Each point equals $100; a premium of 6 3/8 is thus equal to $637.50. Because interest rate options are yield-driven, a change in the yield of the underlying Treasury security changes their value. Specifically, a rise in the 13-week Treasury bill rates or in the average of the 6 long term rates would increase the value of the short term and the long term interest rate options respectively, while lowering the premiums of the corresponding put options.

Options Pricing Theory

The theory of pricing options on futures contracts is derived by Black (1976), based on the Black-Scholes stock option study of 1973. Under certain assumptions, the value of futures options will depend on the price of the futures contract and variables that can be taken to be known constants such as options exercise price, time to maturity, risk free rate of interest, and variance of the fractional change in the futures prices.

The major assumptions of the Black model are:

1. The markets for futures and options are frictionless, i.e., there are no restrictions on short sales, no transactions costs, and no taxes;

2. the risk-free rate of interest is known and constant over the life of the option;

---

24 The extent of literature on option pricing is vast. See the "Pricing of Options and Corporate Liabilities" by Fisher Black and Myron Scholes. Also, "Option Pricing: A Review," by Clifford Smith, Jr. are good references.
3. The fractional change in the futures price over any interval is distributed log-normally with a known variance which remains constant over the life of the option;

4. The option can only be exercised at maturity (European type).

It is then possible to create a hedged position whose value will depend only on time and the values of known constants by taking a long position in the option, and a short position in the futures contracts. Denoting $C(F,T)$ as the value of the European call option expressed as a function of futures price, $F$, and time, $t$, in order for the hedge to be riskless, the hedger must adjust the position continuously by holding a number of contracts equal to $1 / (\partial C/\partial F)^{25}$ for every futures contract sold short. Moreover, since futures contracts are settled daily with gains added to the trader's account and losses deducted from his/her account, the value of the futures contract is reset to zero each day. The value of the equity of the hedged position is then just the value of the option.

The change in the value of the hedged position over the time interval $\Delta t$ is:

$$\Delta C = \frac{\partial C}{\partial F} \cdot \Delta F$$ (5.1)

where

$$\Delta C = C(F + \Delta F, t + \Delta t) - C(F, T)$$ (5.2)

Assuming that the short position is continuously adjusted, $\Delta C$ may be expanded by the use of stochastic calculus:

$$\Delta C = \frac{\partial C}{\partial F} \cdot \Delta F + \frac{1}{2} \frac{\partial^2 C}{\partial F^2} \cdot \sigma^2 \cdot F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t$$ (5.3)

---

25 The Black model assumes continuous adjustment of the hedge at zero cost. In order for the hedge to be riskless, the change in the value of the position must be zero; thus the hedge ratio is $H = \partial C/\partial F$, since an instantaneous unit change in the futures price causes change in the price of the call option of this magnitude. There is no theoretical justification for assuming that $H$ is constant at alternative values of $F$, so, as the price changes, the hedge ratio must be adjusted to keep $H$ equal to $\partial C/\partial F$. 

substituting 5.3 into 5.2, the change in the value of the hedged position is:

\[
\frac{\partial C}{\partial F} \cdot \Delta F = \frac{\partial C}{\partial F} \cdot \Delta F + \frac{1}{2} \frac{\partial^2 C}{\partial F^2} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t - \\
\frac{\partial C}{\partial t} \cdot \Delta F = \frac{1}{2} \frac{\partial^2 C}{\partial F^2} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t \quad 5.4
\]

Since for a riskless hedge, the value of the equity must equal the value of equity time \( r \Delta t \),

\[
\frac{1}{2} \frac{\partial^2 C}{\partial F^2} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t = Cr \Delta t \quad 5.5
\]

Solving 5.5 for \( \frac{\partial C}{\partial t} \) yields the following differential equation

\[
\frac{\partial C}{\partial t} = Cr - \frac{1}{2} \frac{\partial^2 C}{\partial F^2} \cdot \sigma^2 F^2 \quad 5.6
\]

with

\[
C(F,T) = F - E \quad \text{if } F > E \quad 5.7
\]

\[
= 0 \quad \text{if } F \leq E
\]

as the boundary conditions, i.e., if the futures price, \( F \), is greater than the exercise price, \( E \), the option holder may attain monetary gain by exercising the option and buying the futures contract below its current price, the holder would leave the option unexercised.

Equations 5.6 and 5.7 may be solved for \( C \), which is the Black formula for the value of a European call option on a futures contract:

\[
C(F,T) = e^{-rT} \left[ F \cdot N(d_1) - E \cdot N(d_2) \right] \quad 5.8
\]

where

\[
d = \frac{\ln(\frac{F}{E}) + \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}} \quad 5.9
\]
\[ d = \frac{\ln \left( \frac{F}{E} \right) - \frac{1}{2} \cdot \sigma^2 \cdot T}{\sigma \sqrt{T}} = d - \sigma T \quad 5.10 \]

where \( T = \) Time to expiration
\( E = \) Exercise price
\( F = \) Futures price
\( r = \) risk-free rate of return
\( \sigma^2 = \) variance of futures price
\( N(d) = \) cumulative normal density function for \( d; \)

The implications of the model may be examined by evaluating the effects of changes in the value of the model's parameters on the options price.\(^{26}\)

\[ \frac{\partial C}{\partial F} = e^{-rT} \cdot N(d_1) > 0 \quad 5.11 \]

\[ \frac{\partial C}{\partial E} = -e^{-rT} \cdot N(d_2) < 0 \quad 5.12 \]

\[ \frac{\partial C}{\partial r} = -T \cdot e^{-rT} \cdot [F \cdot N(d_1) - E \cdot N(d_2)] < 0 \quad 5.13 \]

\[ \frac{\partial C}{\partial \sigma} = e^{-rT} \cdot T \cdot F \cdot N'(d_1) > 0 \quad 5.14 \]

\[ \frac{\partial C}{\partial \text{var}} = \frac{1}{2} T e^{-rT} \sigma \cdot N'(d_1) + e^{-rT} \sigma \cdot T \cdot N(d_1) > \frac{1}{2} T e^{-rT} \sigma \cdot N(d_1) = 0 \quad 5.15 \]

\(^{26}\) The derivations of the effects of changes in each of the model's parameters on the value of the call option are based on the work in Option Pricing by Robert A. Jarrow and Andrew Rudd, Homewood, Illinois: Richard D. Irwin, Inc., 1983: 119-120.
The valuation of a European put option follows directly from the Black formula for a call option on futures, and the application of the put-call parity. Assuming the construction of a portfolio made of long position in put options, short position in call options, and a long position in the futures contract, there will be a terminal cash flow that does not depend on the relationship between the futures price and the exercise price of the option as shown in Table 5-1.

Table 5-1. Cash flow from the arbitrage portfolio

<table>
<thead>
<tr>
<th>Position</th>
<th>Opening</th>
<th>$F_t &gt; E$</th>
<th>$F_t &lt; E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write a call</td>
<td>$C_t$</td>
<td>$E-F_t$</td>
<td>0</td>
</tr>
<tr>
<td>Buy a put</td>
<td>$-P_t$</td>
<td>0</td>
<td>$E-F_t$</td>
</tr>
<tr>
<td>Buy a futures contract</td>
<td>0</td>
<td>$F_t-F_t$</td>
<td>$F_t-F_t$</td>
</tr>
<tr>
<td>Total</td>
<td>$C_t-P_t$</td>
<td>$E-F_t$</td>
<td>$E-F_t$</td>
</tr>
</tbody>
</table>

The initial value of the arbitrage portfolio must be zero, if there are no riskless profits to be made, so, the present discounted value of the cash flows must be zero

$$0 = C_t - P_t + e^{-rT}(E-F_t)$$  \[5.16\]

which may be rearranged as the put call parity

$$P = C - e^{-rT} \cdot (E-F)$$  \[5.17\]

The put-call parity equation may be used to solve for the price of the put option on the same futures contract, and with the same exercise price and expiration date as the call option by using the Black call option model.

\[N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}\]
The effect of a change in any of the parameters of the model on the put option price can be derived from the impact on the call option price and the put-call parity equation as follows:

\[
\begin{align*}
\frac{\partial P}{\partial F} &< 0 \quad 5.18 \\
\frac{\partial P}{\partial E} &> 0 \quad 5.19 \\
\frac{\partial P}{\partial r} &< 0 \quad 5.20 \\
\frac{\partial P}{\partial \sigma} &> 0 \quad 5.21 \\
\frac{\partial P}{\partial T} &= 0 \\
\frac{\partial P}{\partial T} &< 
\end{align*}
\]
CHAPTER 6: THE APPLICATION OF FINANCIAL FUTURES AND OPTIONS AS HEDGING INSTRUMENTS IN MORTGAGE MARKETS

Chapter 6 begins with position diagrams as introductory tools in understanding the use of financial futures and options in mortgage markets. The return distribution to the unhedged mortgage and those mortgages "covered" with various futures and options is then derived mathematically. Finally, the results of an empirical study on the use of such instruments in mortgage markets are reported.

Position Diagrams

Position diagrams are used to represent basic strategies on buying or writing of put and call options.

1. Buy call, Figures 6-1a and 6-1b. Having acquired the right but not the obligation to buy at strike price, $E$, if the market price of the underlying security appreciates over the strike price, the holder would gain by exercising the option. If the market price of the underlying security remains at or below the strike price, the option will be allowed to expire, and the buyer will have lost the premium.

2. Buy put, Figure 6-2. Since the buyer of the put has the right, but not the obligation to sell at specific strike price, $E$, if interest rates rise and cause market prices to fall below $E$, he/she would exercise the option and gain in the process. A fall in interest rates causing market prices to rise above $E$, gives no incentives for the exercise of the option and the paid premium is lost.

3. Write call, Figure 6-3. Any increases in interest rates cause the call option to remain unexercised, hence the writer will earn the

---

27 There is an inverse relationship between interest rates and security prices. For example, assuming that a bond with 30 years remaining to maturity was issued at par (100) with a coupon of 8% to reflect market rates at the time of issue, if market rates rise to 10%, it would be traded below par of 87.07 to have a yield comparable to market rates.
Figure 6-1. Net capital gain from the the buy-call strategy: a. As a function of yield to maturity of the underlying security; b. As a function of the underlying security's value.
Figure 6-2. Net capital gain from the buy-put strategy
Figure 6-3 Net capital gain from the sell-call strategy
premium received. A decline in rates and the resulting higher security prices, however, gives the holder of the option an incentive to exercise his/her right to buy the security at E. The writer of the option, in this case, incurs a loss equal to the difference between the market price and exercise price. The loss, however, is modified by the call option premium which is already received.

4. Write put, Figure 6-4. Higher rates will result in the exercise of the put option and the writer must accept delivery of the underlying securities at higher than market value and loses in the process. The put premium, as in the case of writing calls, modifies the losses. A decline in market interest rates and the increase in securities' prices, leaves the put option unexercised and the writer will gain the premium.

With this background, some of the choices available to financial institutions may be examined. The institution may hedge against the loss of the portfolio value stemming from increases in interest rates by offering ARMs, Figure 6-5. As is shown in Figure 6-5, for a simple ARM with no interest rate caps or floors, this strategy fully insures the financial institution against lower portfolio value but also totally eliminates any gains in value that may result from lower interest rates.

The public demand for FRMs is still present, and for competitive purposes if nothing else, the institution may desire to make such mortgage loans, Figure 6-6. If interest rates rise, mortgage value drops, and a decline in rates translate in to higher portfolio value. Thus, offering FRMs clearly involves a risk which must be analyzed more carefully.

The risk structure of a FRM portfolio may be altered with the use of financial futures and/or options on financial futures. This chapter will examine some of the choices available to financial institutions in hedging interest rate risk exposure of their mortgage portfolio. Position diagrams are employed to show the effect of each hedging strategy on the mortgage
Figure 6-4. Net capital gain from the sell-put strategy
Figure 6.5. Net capital gain from an uncapped ARM position
Figure 6-6 Net capital gain from the FRM position
position as rates change, and the return on the unhedged position as well as the return on alternative hedged positions are derived mathematically.

Return Distribution to Alternative Positions

The return distribution to the unhedged mortgage position as well as the mortgage portfolio hedged with various hedging strategies is derived in this section. These include hedging the mortgage portfolio with a short position in financial futures, a short position in call options on financial futures, a long position in financial futures put options, and a combination of the latter two strategies.

The unhedged mortgage position

If the cash instrument's prices at time \( t_1 \) and \( t_2 \), are \( M_1 \) and \( M_2 \) respectively, where \( t_2 > t_1 \), the value of gains or losses from the unhedged mortgage position is

\[
U = X_M (M_2 - M_1) \quad 6.1
\]

where \( X_M \) = size of the cash position in multiples of $100,000

The expected value and variance of the unhedged position may then be defined as

\[
E(U) = X_M E(M_2 - M_1), \quad 6.2
\]

\[
\text{Var}(U) = X_M^2 \text{Var}(M) \quad 6.3
\]

where \( \text{Var} (M) \) is the subjective variance of the cash instrument and \( E \) is the expectations operator.

\[28\] This strategy may be viewed as a speculative position since the financial institution would use it under specific expectations of the potential interest rates movements. It may be more accurate to refer to this strategy as an "insurance policy" rather than a hedging strategy. For continuity and simplicity, however, this position will be referred to as a hedging strategy in the remainder of this study.
Hedging with financial futures

Based on the traditional hedging theory, emphasizing the pure risk-avoidance characteristics of futures markets, the hedger would take a position in the futures market equal to, but opposite of, his/her position in the cash market. This argument is based on the assumption that cash and futures instruments' prices generally move together; thus, the gain or loss on the hedged position would be less than for an unhedged position, Figure 6-7.\textsuperscript{29}

Denoting the futures instrument's price at times $t_1$ and $t_2$, $F_1$ and $F_2$ respectively, the profit/loss position of the cash market instrument hedged with a short position in the futures contract on a one-to-one basis is

$$R = XM [(M_2 - M_1) + (F_1 - F_2)]$$  

If the prices of the cash and futures instruments move together, the variance of the hedged position would be smaller than that of the unhedged position. This risk reduction potential is often discussed in terms of basis, $F_t - M_t$, as discussed in Chapter 4. Allowing $B_t$ to represent the basis at time $t$, equation 6.4 may be written as:

$$R = -XM [(F_2 - M_2) - (F_1 - M_1)] = -XM AB$$

\textsuperscript{29} It will be shown later in this chapter that even if the assumption of cash and futures instruments' prices moving together is not satisfied, Figure 6-7 would hold true by using an optimal hedge rather than a one-to-one hedge.
Figure 6-7. Net capital gain from the FRM covered with the short-futures strategy.
Traditional hedging was based on the argument that changes in the basis were quite small relative to the price of the instruments because of the possibility of making or taking delivery of the commodity.

The pure risk-minimizing assumption of the traditional hedging theory was criticized beginning in 1953 by H. Working who argued that hedging was undertaken primarily to maximize profits. The holders of a long cash position would only sell futures contracts if they expected a narrower basis.

Changes in basis may be especially important when a cross-hedge—where the futures contract instrument is different than the cash instrument—is being undertaken. In such cases as hedging mortgage loans with T-bond futures contracts, the possibility of a change in the basis over the hedging period is of special importance.

Johnson\(^{30}\) and Stein\(^{31}\) developed a unified theory of hedging by applying basic portfolio theory to incorporate the traditional risk-minimizing criteria and the maximization aspects of expected profit theory. A number of alternative measures of risk focusing on the disutility of uncertainty may be used, including the probability of loss, the expected value of loss, and the variance of the expected return. Ederington (1979), assumed hedging of only one cash instrument, and subsequently derived the optimal proportion of the cash instrument to be hedged in the futures markets when changes in the basis are not necessarily equal to zero.

If \( R \) represents the change in the market value of the portfolio which contains \( X_m \) and \( X_f \) holdings of the cash and futures market instruments


respectively, the expected return and variance of the hedged positions are:

\[
E(R) = X_M E(M_2 - M_1) + X_F E(F_2 - F_1) - C(X_F)
\]

\[
\text{Var}(R) = X_M^2 \text{var}(M) + X_F^2 \text{var}(F) + 2X_M X_F \text{cov}(M,F)
\]

where \(C(X_F)\) represents brokerage and other costs of undertaking futures contracts, and \(\text{var}(M)\), \(\text{var}(F)\), and \(\text{cov}(M,F)\) represent the subjective variance and covariance of the possible price changes between periods one and two. Although margin costs are not known with certainty, they have traditionally been stable over time. It is thus assumed that the variance of \(C(X_F)\) is zero.

Let \(n\) represent the number of futures contracts traded for each unit of the cash instrument held, i.e., \(n = -X_F/X_M\). Substituting \(n\) into equations 6-6 and 6-7 gives

\[
E(R) = X_M E(M_2 - M_1) - n X_M E(F_2 - F_1) - C(X_M,n)
\]

\[
\text{Var}(R) = X_M^2 \text{var}(M) + n^2 X_M^2 \text{var}(F) - 2n X_M \text{cov}(M,F)
\]

Letting the expected change in basis be \(E(\Delta B) = E((F_2-M_2)-(F_1-M_1))\), the expected return on the hedged position is

\[
E(R) = X_M[(1-n)E(\Delta M) - nE(\Delta B)] - C(X_M,n)
\]

where \(E(M_2-M_1)\) is the expected change in the price of one unit of the cash instrument.

---

\[32\] The terms expected return and expected capital gains are used interchangeably in the rest of the study.
Equation 6.10 shows that if the expected change in the basis is zero, as in the traditional theory, the expected gain/loss of the hedged position, \( E(R) \), is reduced as \( n \) approaches one. It may also be seen that the changes in the basis can add to, or reduce, the return that would have been expected on the unhedged position where \( E(U) = X_M(M_2 - M_1) \).

Since the size of the holding of the cash instrument is assumed to be constant, the effect of a change in \( n \) on the variance of the return is

\[
\frac{\partial \text{var}(R)}{\partial n} = X_M^2[2n\text{var}(F) - 2 \text{cov}(M, F)]
\]

and the risk minimizing hedge ratio is

\[
n^* = \frac{\text{cov}(M, F)}{\text{var}(F)}
\]

The numerical value of \( \text{cov}(M, F)/\text{var}(F) \) may be estimated from historical data with an ordinary least square regression of \( (M_2 - M_1) \) observations on \( (F_2 - F_1) \) observations. The regression coefficient for \( (F_2 - F_1) \) is the estimate of \( \text{cov}(M, F)/\text{var}(F) \). If the risk minimizing hedge ratio, \( n^* = \frac{\Delta M}{\Delta F} \), is substituted in equation 6.8, the expected return on the optimally hedged portfolio may be found as shown in equation 6.13:

\[
E(R) = E[X_M(\Delta M - n^*\Delta F)] = E[X_M(\Delta M - \Delta M/\Delta F \cdot \Delta F)] = 0
\]

For simplicity, in the rest of this study, the term \( C(X_M, n) \) is omitted from the return distribution of mortgages hedged with both futures and futures options contracts. Since the cost of entering a futures position and that of entering option markets are quite comparable, the comparative effect on the return distribution of mortgages hedged with futures or options is negligible.
Figure 6-7, then, also shows the return on the hedged portfolio using an optimum number of futures contracts, where the curve for the return on the cash holding is weighted by $1/n^*$ in order to make the analysis consistent. The return is thus for $1/n^*$ units of the cash instrument and one unit of the futures instrument.

In Figure 6-7, the slopes of the return lines are of equal absolute size, since for each one unit change in the price of the futures contract, the value of $1/n^*$ units of the cash instrument will be $n^* \times 1/n^* = 1$. The return on the sale of a futures contract is positive when $F^2 < F^1$, because the seller of the contract can buy it at a lower price than its initial selling price.

The total return for the hedged position is then zero at each alternative price of the futures contract. It has been implicitly assumed that $n^* > 0$ in this discussion, so that the cash position is hedged with a short position in the futures instrument. Moreover, it has been assumed that the basis is stable over time, i.e., although the prices of the cash and futures instruments may not move together, the relationship between the two, once determined, remains constant.

**Hedging with options on financial futures**

Prepayment is a substantial problem for financial institutions attempting to hedge in the futures market: as interest rates fall, the value of the mortgage portfolio will rise, but the consumers will begin to prepay their mortgages and take out new ones at the lower rates. Therefore, the value of the mortgage portfolio may not appreciate for the institution, while the futures position is declining in value. The net result may be a loss in the futures market, without a corresponding gain in the portfolio of mortgages. So, institutions are able to protect
themselves from rising interest rates but not from losses during periods of portfolios of mortgages.\textsuperscript{34}

Mortgage positions may also be hedged with various option market strategies. One advantage of using futures options as hedging instrument is that the option offers protection only on one side of the market compared to a futures market position that is affected by both sides of the market.\textsuperscript{35} If the financial institution chooses the "appropriate" option hedging strategy, it can protect itself only against the one-sided movement of rising interest rates.

Moreover, if the institution chooses to engage in purchasing an option as the hedging method, it may benefit from the fact that margin requirement on options on futures are not subject to daily settlements. The institution may, then, not be adversely affected in cases of large movements of the futures contract price. In other words, the institution need not mark the account to market at the end of every day, thus eliminating the need to have large sums of cash at hand to avoid the risk of frequent margin calls. If the institution, on the other hand, decides to write an option as the hedging strategy, it remains subjected to daily settlements.

Hedging mortgages with a short position in call options on financial futures, Figure 6-8,\textsuperscript{36} is a strategy for bearish bankers and is effective in a relatively flat market. It limits the upside potentials of the price movements in order to lower the loss in the event of falling values.

\textsuperscript{34} Marshal and Colwell, "Hedging Mortgage Portfolios with Options on Futures", \textit{Real Estate Development}, Fall 1986: 7.

\textsuperscript{35} Recall that an option is defined as the right, but not the obligation to buy or sell an asset at a predetermined price.

\textsuperscript{36} The figure is drawn for either a one-to-one hedge, if the basis is zero, or for an optimal hedge of $1/n$ units of the cash instrument with one call option, if the basis is not assumed to be zero. In the latter case, the slopes of the curve for the return on the cash instrument may be viewed as having been weighted by $1/n$. 

Figure 6-8. Net capital gain from the FRM covered with the sell-call strategy
Although there are other possible strategies to limit the loss, writing calls is considered in this study because the premium received is an effective "insurance policy" when the price of the underlying security declines by a relatively small amount. As is shown in Figure 6-8, the banker who is involved in this strategy will lose only if the value of the underlying security falls below $k'$. Any decline in value smaller than this amount, are fully offset by the call premium received.

One disadvantage of this strategy, as for any other option writing strategy, is the daily settlement requirement.\(^{37}\) If the call is "uncovered", the writer must deposit the premium plus 20 percent of the underlying security's value and may thus be subjected to margin calls. If the call is out-of-the-money, the margin is reduced by the amount by which the option is out-of-the-money.

For "covered" calls, if the option's exercise price is at least equal to the underlying security price, the banker need not deposit any additional margin beyond that required on the underlying security. If, however, the exercise price is less than the security price, the maximum amount the investor may borrow on the futures contract is based on the call's exercise price rather than the futures contract price.\(^{38}\)

It is assumed here that the banker has a portfolio of financial assets that include T-bond and T-bond futures contracts, also the primary options considered are the at-the-money calls. So, it is assumed that the daily settlement of call writing strategy is not of great consequence to the financial institution.

Assuming that the exercise price of the call option is equal to the initial selling price of the futures contract, $F_c$, and the call option

\(^{37}\) Additional information on margin requirements on option writing strategies may be obtained from Robert W. Kolb, *Options: An Introduction*, Miami, Florida: Kolb Publishing Company, 1990.

premium at the time of initiating the hedge, \( t \), is denoted \( C_t \), the profit/loss position of selling the option is determined as follows.

The option would expire unexercised if futures price were to decline over the hedge period, or if the futures price at the end of the hedging period, \( F_T \), were below the exercise price of the option, \( E \). The investor would then earn the call option premium. If futures prices were to increase so much as to surpass the options strike price, the exercise of the option would create a loss equal to the difference between \( F_T \) and \( E \).

The expected return to the mortgage hedged with selling call options is

\[
R_c = (M_f - M_i) + C_t \quad \text{for} \quad F_T < E
\]
\[
= (M_f - M_i) - (F_T - E) + C_t \quad \text{for} \quad F_T > E
\]

(6.14)

As was discussed earlier in this chapter, the prices of the cash and futures contracts do not necessarily move together. To correct for the existence of a nonzero basis, the previously derived hedge ratios may be also used in options hedging. So, instead of hedging one mortgage contract by writing one call option, it is assumed that \( \frac{1}{n'} \) units of the cash instrument is hedged with selling call options. Intuitively, it is easier to do this rather than assuming that partial option contracts are written to cover one unit of the cash instrument. Equation 6.14 would then change as follows:

\[
R'_c = \frac{1}{n'} \left( M_f - M_i \right) + C_t \quad \text{for} \quad F_T < E
\]
\[
= \frac{1}{n'} \left( M_f - M_i \right) + (F_T - E) + C_t = C_t^{39} \quad \text{for} \quad F_T > E
\]

(6.15)

\[^{39}\text{It is assumed that at the time the hedge is initiated, } E \text{ is equal to } F_t. \text{ So, } (F_T - E) \text{ is the same as } (F_T - F_t) \text{ which will be canceled out by } \frac{1}{n'} \left( M_f - M_i \right), \text{ assuming that } n' \text{ is the optimal hedge ratio previously derived. The expected return on the mortgage portfolio hedged with writing call options, when } F_T > E, \text{ is then } C_t.\]
Purchase of put options on T-bond futures as hedging tool. Figure 6-9, sets a limit on the loss in mortgage value resulting from higher interest rates without limiting the potential for gains if interest rate were to fall. Specifically, the put option purchased with an exercise price equal to the futures contract price at time of initiating the hedge, \( t \), at a premium of \( P \), would only be exercised if the futures price were lower at the time the hedge was lifted. This would create a gain of \( F - E \). The option would expire unexercised otherwise. The expected return to the hedged position is then

\[
R_p = (M_2 - M_1) + (E - F_T) - P, \quad F_T < E
\]
\[
= (M_2 - M_1) - P, \quad F_T > E
\]

Hedging the mortgage with put options according to the optimal hedge ratio, i.e., hedging \( 1/n^* \) units of the mortgage with one put option contract, would yield

\[
R_p' = 1/n^* (M_2 - M_1) + (E - F_T) - P, = P^* \quad F_T < E
\]
\[
= 1/n^* (M_2 - M_1) - P, \quad F_T > E
\]

Finally, combining a short position in financial futures call options with a long position in the put options on the same contract provides a total return similar to the hedge with selling futures contracts,

\[ (E - F_T) = (F_I - F_T), \] assuming the purchase of an at-the-money put at the start of the hedge period. If \( n^* \) is the optimal hedge ratio, then \( (F_I - F_T) \) and \( 1/n^* (M_2 - M_1) \) cancel each other. The expected return on the mortgage position hedged with the buy put strategy when \( F_T < E \) is simply \( P, \).
Figure 6-9. Net capital gain from the FRM covered with the buy-put strategy
Figure 6-10. The long put hedges against a rise in rates and the short call trades away some of the seller's upside potential in a rally in return for premium income that will offset some or all of the cost of the put. The expected return for the hedged position is then

\[
R_{CP} = \begin{cases} 
(M_2 - M_1) + (E - F_T) - P_i + C_i & F_T < E \\
(M_2 - M_1) - (F_T - E) - P_i + C_i & F_T > E
\end{cases}
\]

Hedging \(1/n\) units of the mortgage using this strategy changes the expected return as follows,

\[
R_{CP} = \frac{1}{n} (M_2 - M_1) + (E - F_T) - P_i + C_i \\
\frac{1}{n} (M_2 - M_1) - (F_T - E) - P_i + C_i \\
= -P_i + C_i
\]

Assuming the equality of put and call premia for all at-the-money options,

\[
R_{CP} = 0
\]

The Results of the Empirical Study

The result of empirical studies of the four hedging strategies using historical data is presented in this section.

Data specification

The length of each hedge period is six months and a new hedge was initiated on the 15th day of every month beginning January 1983 and ending
Figure 6-10. Net capital gain from the FRM covered with the synthetic futures position
in December 1991. Therefore, a total of 102 hedged positions are considered. The year 1983 was chosen as the starting date since options on futures contracts were first introduced three months prior to that time and by the beginning of that year there was considerable volume of trade in the market.

Mortgage rates used in the study were those on fixed rate conventional, fully amortized first mortgages on single family homes closed in the third week of every month for all lender types. This mortgage rate series was obtained from the Federal Home Loan Bank Board of Des Moines for the period 1983-1988 and then from the Office of Thrift Supervision for the remainder of the period under study.

The mortgage rate series was converted into a price series with the use of "The prepayment mortgage value table for the 30-year mortgage prepaid in twelve years" extracted from Thorndike Encyclopedia of Banking and Financial Tables, Revised Edition. Prepayment in twelve years was assumed because it roughly corresponds to the expected life of recently originated conventional mortgages.

The Treasury bond futures contract chosen was the one with maturity lasting at least to the last day of the hedge period. The futures price used was the settlement price for the 15th day of the month and in those cases where the 15th day was not a trading day, the price for the closest alternative day was used. The premium for put and call options on T-bond futures contracts with strike prices closest to the futures price and maturities of equal or greater than those of the underlying futures contract were also obtained for a trading day as close to the middle of the month as possible. Similar data were gathered for the next in-the-money and out-of-the-money puts and calls.

The futures price series and put and call option premia were collected from the Statistical Annual published by the Chicago Board of Trade for the period October 1982 to December 1986, and from the Wall Street Journal for the years 1987 through 1991.
It was then assumed that the financial institution was hedging a portfolio of thirty-year mortgages with an interest rate of eight percent and expected prepayment in twelve years. This mortgage type was used since it is essentially on the same footing as the T-bond futures contracts which are based on an eight-percent-coupon Treasury bond.

**Estimation of optimal hedge ratios**

It was shown in Chapter 4 that the optimal number of futures contracts used to hedge mortgage portfolio values may be estimated as the regression coefficient of the change in mortgage value on the change in price of futures contracts. The result of risk minimizing hedge ratio estimation are presented in this section.

It was assumed that each position was maintained for six months. The data used in the estimations, then, were monthly observations for the six-month changes in the mortgage and Treasury bond futures contract prices. It was further assumed that the first hedge was established on January 15, 1983, and the last hedge lifted on December 15, 1991.

The slope coefficient for the regression of the changes in mortgage prices on changes in the futures contract prices using the six-month periods was .7821. This means that in order to minimize the risk of the position, each $100,000 of mortgage holdings would be hedged by selling .7821 Treasury bond futures contracts. The coefficient of determination for this regression was .2922 signifying that a large portion of the movement in mortgage values is not accounted for by the changes in futures prices. The value of Durbin-Watson statistic, .4943, suggested a large degree of autocorrelation among the residuals. When the estimates were corrected for autocorrelation, the optimal hedge ratio dropped to .6132 with $r^2$ of .4076.

---

41 The coefficient was significant at the .0001 level.
In both of the above cases, a noticeable portion of mortgage value changes was found not to associate with changes in futures contract price. It was shown by Hillard and Haney\(^2\) that during the latter parts of the 1970s, changes in mortgage interest rates lagged behind changes in the yield on long-term government bonds by approximately one month. Since the yield on government bonds move parallel to the movement in futures prices, the relationship between changes in mortgage prices and changes in future prices were re-estimated incorporating a one month lag. In other words, it was assumed that mortgages were hedged a period earlier by selling futures contracts—the mortgage portfolio of 2/15/83 was hedged in 1/14/83\(^3\) as an example.

The results of the re-estimation showed the optimal hedge ratio as .8991 with \(r^2\) equal to .3542 for the uncorrected, and .7993 with coefficient of determination at .4256 for the corrected series. The result of the regression analysis is summarized in Table 6-1.

Table 6-1. Regression estimation for changes in mortgage and futures prices

<table>
<thead>
<tr>
<th></th>
<th>CONTEMPORANEOUS HEDGE</th>
<th></th>
<th>LAGGED HEDGE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\Delta M = a + b \Delta F)</td>
<td></td>
<td>(\Delta M = a + b \Delta F_{t-1})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>UNCORRECTED</td>
<td>CORRECTED</td>
<td>UNCORRECTED</td>
<td>CORRECTED</td>
</tr>
<tr>
<td></td>
<td>FOR AUTOCORRELATION</td>
<td>AUTOCORRELATION</td>
<td>FOR AUTOCORRELATION</td>
<td>AUTOCORRELATION</td>
</tr>
<tr>
<td>a</td>
<td>0.0831</td>
<td>0.3571</td>
<td>-0.5567</td>
<td>0.1265</td>
</tr>
<tr>
<td>b</td>
<td>0.7821</td>
<td>0.6132</td>
<td>0.8991</td>
<td>0.7993</td>
</tr>
<tr>
<td>n</td>
<td>102</td>
<td>102</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>d.f</td>
<td>100</td>
<td>100</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>t</td>
<td>7.5412</td>
<td>14.5851</td>
<td>3.2563</td>
<td>10.295</td>
</tr>
<tr>
<td>R-SQUARED</td>
<td>.2922</td>
<td>.4076</td>
<td>.3542</td>
<td>.4256</td>
</tr>
<tr>
<td>D-W</td>
<td>.4943</td>
<td>1.6210</td>
<td>.9821</td>
<td>1.9230</td>
</tr>
</tbody>
</table>


\(^3\)January 15, 1983 was not a trading day.
Return distribution to alternative strategies

The period under the current study is one in which interest rate levels generally declined, and the mortgage and futures values consequently increased. There was considerable variation within the changes in mortgage values. Of the 102 hedge periods, 62 were periods in which mortgage values increased and the remaining 40 periods showed a decline in values. The maximum increase in value, 9.05, occurred between November 1985 and May 1986 and the minimum gain was .060 points, for the period August 1987 to February 1988. The maximum loss in value of 6.300 points happened in 1987, April to October, and the minimum loss was .050 points, between March and September 1983.

One of the main advantages of using options on futures is that the holder has the right, but not the obligation to take a position in the futures market. So, the return to mortgages hedged with a long position in futures put options, as an example, rather than futures contracts should be relatively more favorable when mortgage values and futures prices generally rise. Of course, the mortgage value must increase by at least an amount large enough to offset the premium paid for the put option before the benefits of this hedging strategy materialize. Referring back to Figure 6-9, mortgage value must increase above k'' before any gains are realized.

The return on mortgages hedged through writing call options, for relatively moderate price changes, could be more favorable than hedging with futures, if mortgage values decline. For example, if mortgage values fall down to k' in Figure 6-8, the loss in mortgage value is more than offset by the call premium received, thus generating more favorable results than hedging with futures as depicted in Figure 6-7. If the mortgage value falls below k', on the other hand, the call premium does not offset the

All mean returns and standard deviation of returns are expressed as thousands of dollars. 9.050 points, then, equals $9,050.
full amount of the loss in mortgage portfolio. Writing calls in this case will not be preferable to hedging with futures.

In order to evaluate the risk-return of alternative strategies under rising interest rates, a simulation method was used to reverse the mortgage and futures price series. Consequently, the mortgage and futures prices at the time the hedge was initiated became the end of period prices, and prices at the time of hedge terminations were used as the initial price series. The strike prices of the options were then adjusted to represent the new initial futures prices and the premia for the options were obtained from the sources mentioned previously. In cases when options with the new strike prices were not traded, the option premia was estimated according to the Black futures options pricing formula using the one month historical variance and the six-month Treasury-bill rate.

When the two series are combined, the resulting sample would represent a series in which there is no trend in the price of either mortgages or the futures contracts, but there remains substantial variation in these prices within the period.

Return distribution for the combined series The result of the hedging exercise for the combined series is reported in Table 6-2 for the contemporaneous and in Table 6-3 for the lagged series. The first entry in each column shows the mean capital gains, or mean return, from an unhedged position in a portfolio of $100,000 mortgages, as well as the mean return from the same portfolio if hedged with various hedging strategies. Brokers fees are not considered in this study, so the returns may be slightly overestimated. The second number in each column refers to the standard deviation of returns for the hedged and unhedged mortgage positions.

The return for the unhedged mortgage and mortgages hedged with futures are zero for both the contemporaneous and the lagged relationships. This is expected as reversing the mortgage and the futures price series
Table 6-2. Return distribution for the hedged and unhedged mortgage positions, combined series, contemporaneous relationship

<table>
<thead>
<tr>
<th></th>
<th>One-to-One Hedge Ratio</th>
<th></th>
<th>Optimal Hedge Ratio</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Capital Gain</td>
<td>Standard Deviation of Return</td>
<td>Mean Capital Gain</td>
<td>Standard Deviation of Return</td>
</tr>
<tr>
<td>UNHEDGED MORTGAGES</td>
<td>0 4.823</td>
<td>0 4.823</td>
<td>0 4.823</td>
<td></td>
</tr>
<tr>
<td>MORTGAGES +</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FUTURES</td>
<td>0 4.212</td>
<td>0 3.504</td>
<td>0 3.071</td>
<td></td>
</tr>
<tr>
<td>MORTGAGES +</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CALL OPTIONS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.162 3.445</td>
<td>2.257 3.196</td>
<td>2.345 3.808</td>
<td></td>
</tr>
<tr>
<td>E=F</td>
<td>0.322 3.229</td>
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<td>LONG PUT +</td>
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Table 6-3. Return distribution for the hedged and unhedged mortgage positions, combined series, lagged relationship

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<th>OPTIMAL HEDGE RATIO</th>
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<td>MORTGAGES + FUTURES</td>
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<td>MORTGAGES + CALL OPTIONS</td>
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<td>E=F</td>
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<td>E=F</td>
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<tr>
<td></td>
<td>E&gt;F</td>
<td>-0.477</td>
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would simply reverse the sign of returns without changing the magnitude. The standard deviations of return for the hedged position are lower for both series and for all hedge ratios although the lagged series show a larger reduction on the variability of returns. The one-to-one hedge of the lagged mortgage series with financial futures reduced the standard deviation of return by 24 percent, from 4.823 to 3.651 points. Using the uncorrected optimal ratio of .8991 lowered the standard deviation to 3.307 points and the use of optimal hedge ratio corrected for autocorrelation, .7993, accounted for standard deviation of 3.005 points. These numbers translate into a reduction of 31 and 38 percent respectively.

For the contemporaneous series, the variability of returns were reduced by 13, 27, and 36 percent respectively for the one-to-one, uncorrected, and corrected hedge ratios.

Hedging mortgage portfolios with selling at-the-money call options on financial futures generated interesting results. The return on all cases was positive with a minimum of .322 points for the one-to-one hedge of the contemporaneous mortgage series and a maximum of 1.506 points for the same series hedged with the uncorrected hedge ratio. Standard deviation of returns were lowered considerably in all cases. The smallest reduction in variability was a standard deviation of 4.068 points, 15 percent lower than that of the unhedged mortgage. The best result was a reduction of 41 percent in standard deviation of return to 2.861 points, for the uncorrected optimal hedge ratio of the lagged series. In general the lagged series showed a better reduction of the standard deviation for all cases of hedging with the short position in the call option than the

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45 A standard deviation of $3,651 on capital gains from a $100,000 position of 8 percent mortgages.
contemporaneous series. This was true for the options with exercise price different than the initial futures price as well.

The employment of in-the-money call options, $E < F$, increased the return at the expense of higher standard deviation in all but two cases. The variability of return for the lagged series hedged with the corrected hedge ratio, and that of the contemporaneous series hedged with the uncorrected ratio were lower when the in-the-money option was used. The use of the out-of-the-money option lowered the return and increased the standard deviation in all cases of optimal hedging, but increased the return slightly and lowered variability by a small amount for the one-to-one hedge.\(^{46}\)

In conclusion, hedging with a short position in call options on T-bond futures contracts lowered the standard deviation of return compared to the unhedged mortgage and increased the return under all circumstances considered in this study. These results are consistent with the fact that this position may be more correctly viewed as an "insurance policy" rather than a hedge since the banker would undertake it only if he/she has formed some expectations on the future directions of interest rates.

The buy-put strategy was also quite successful in lowering the variability of return when compared with the unhedged mortgage position. For at-the-money puts, the standard deviations of return were lowered in all cases with the smallest reduction of 20 percent for the contemporaneous series hedged with the corrected ratio, and the best result of 41 percent lower standard deviation in the case of uncorrected optimal hedge of the lagged series. However in all cases, the return to the hedged positions were negative, with the one-to-one hedge showing a smaller decline in return for both series as compared to the optimal hedge.

\(^{46}\) The possibility of margin calls are not considered in this study. It is implicitly assumed that the financial institution owns a portfolio of financial assets, including Treasury bonds and Treasury bond futures contracts. The margin requirement is thus similar to those on futures contracts.
The use of the in-the-money put option, \( E > F_t \), as the hedging instrument produced curious results. The standard deviation of returns were generally higher compared to the at-the-money put hedge, but returns were surprisingly lower also. On the other hand, the out-of-the-money put created a better return, i.e., a smaller loss, at the expense of higher variability in five out of six cases for both series and hedge ratios. These results may be attributed to a possible overpricing of put options and will be discussed later in this chapter when comparing the result of the study for the original and the reverse series.

The synthetic futures hedge, selling calls and buying puts with the same strike price and assuming equal put and call premia, is expected to create a return similar to that of the mortgage position hedged with futures contracts. Indeed for all cases of optimal hedges for the lagged and contemporaneous series with the at-the-money options this seems true. The returns to this position are .080 and .081 points for the uncorrected and corrected optimal hedges of the lagged series respectively. For the contemporaneous series these values are .103 and .007 points for the corrected and uncorrected hedges. On average, then, there is approximately $7 of positive return to this position.

The standard deviation of returns are lower than that of the unhedged mortgage in all but one case. When the contemporaneous series is hedged with the corrected optimal ratio, the return is above zero by a relatively large amount, .103 points, and the variability of return has increased by approximately 5 percent compared to the unhedged position. The other cases show a standard deviation which is 17 to 25 percent lower than the unhedged position.

When compared to the short futures hedging, however, the synthetic futures does not perform as well in reducing variability if optimal hedge ratios are used. In the most disappointing case, the futures hedge creates a standard deviation which is approximately 64 percent lower than the one offered by the sell call-buy put strategy and in the most promising
condition the futures hedge is more successful in lowering variability by only 7 percent. The one-to-one hedge with the put-call strategy fared better than futures hedging in lowering variability of return although it created a considerable loss.

The move to a higher exercise price created a substantial loss for all optimal hedges using this strategy and increased the standard deviation by a large amount as well. The use of options with smaller exercise prices created a considerable profit although it came at the expense of quite larger variabilities. This may be due to relative overpricing of put and/or relative underpricing of calls and will be mentioned later in comparisons of the original and reverse price series.

**Return distribution for the original and the reverse series**

This section will focus on a more detailed analysis of each hedging strategy. The returns to the original and the reverse series are looked upon separately in an attempt to distinguish and explain the return distributions as they respond to the direction of the price movements.

Tables 6-4 and 6-5 report the results of the experiment for the original series under both relationships while Tables 6-6 and 6-7 present the results for the reverse series.

The specific hedging scenarios are then as follows: On the 15th day of the month the financial institution aims in managing mortgage-portfolio price risk resulting from potential interest rate changes. On the same day, the institution may decide to enter an opposite position in the futures markets. Thus, a number of futures contracts will be sold on that day for each $100,000 of mortgage portfolio. The exact number of the these contracts may be one if the hedger follows a one-to-one hedging strategy, or other than one if the hedger employs an optimal ratio derived from the regression method explained earlier. This position will be maintained for six months at which time the institution will take an offsetting position in the futures market, thus realizing a gain or a loss depending on the direction and magnitude of changes in futures prices. This change,
Table 6-4. Return distribution for the hedged and unhedged mortgage positions, original series, contemporaneous relationship

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<td>1.182 4.823</td>
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Table 6-7. Return distribution for the hedged and unhedged mortgage positions, reverse series, lagged relationship

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<td>MORTGAGES + CALL OPTIONS</td>
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ideally, should wholly or partly offset the gain or loss in mortgage values.

For the original series, falling rates and rising values, futures hedging lowers potential gains for the benefit of lower variation of returns. Net capital gains are much closer to zero when mortgages are hedged with futures contracts. They range from a low of −.296 points (a loss of $294 on a $100,000 position) for the one-to-one hedge of the lagged series to a high of .275 points for the corrected optimal hedge of the contemporaneous series. Standard deviations of return is lower by 13 to 38 percent as well.

Optimal hedging with futures performs better than the one-to-one hedge, returns are closer to zero and standard deviations are lower. This may be expected, as the discussion of the basis in Chapter 4 suggested that the use of an optimal number of futures contracts when the basis is not zero would increase the effectiveness of the hedge. Furthermore optimal hedging of the lagged relationship is more successful in lowering the variation of return compared to the contemporaneous series. Finally, when optimal hedge ratios are corrected for autocorrelation, both the return and the standard deviation improves considerably for the lagged as well as the contemporaneous series.

The magnitude of the change in mean expected capital gains as compared to the magnitude of the corresponding change in the standard deviation of returns is next discussed to better understand the effectiveness of this hedge.

The one-to-one contemporaneous hedge of the mortgage portfolio lowered the mean expected return by 124 percent for only a 13 percent reduction in standard deviation. The optimal hedges, however, performed better: 98 percent lower return and 27 percent lower standard deviation for the uncorrected hedge ratio of the contemporaneous series, and a 77 percent reduction in expected capital gains coupled with a 37 percent
smaller standard deviation for the corrected-for-autocorrelation hedge ratio of the same series.

The optimal hedge of the lagged relationship, although lowering the variability of returns by a larger amount than the contemporaneous hedge, paid for it by a larger reduction in mean returns: A reduction of 31 percent in standard deviation at the expense of 109 percent lower return for the uncorrected, and 38 percent lower variability costing 97 percent lower mean capital gains for the corrected optimal hedge ratio. The one-to-one hedge of this series, lowered the variability by 24 percent and the return by 121 percent.

The reverse series provides collaborating results. The magnitude of returns for futures market hedging are always the same as the original series and have the opposite sign. Standard deviations are exactly the same. In general, then, standard deviation of returns are lowered if the futures hedge of the reverse series is undertaken, and the mean expected returns are more favorable and closer to zero as compared to the large loss of the mortgage value.

So it is concluded that given the circumstances under this study, futures hedging is successful in lowering the variability of returns. Optimal hedge ratios are superior to the one-to-one ratio, correction for autocorrelation improves the results, and the lagged relationship performs better than the contemporaneous when mortgage values are falling.

As an alternative strategy, an at-the-money call option may be sold with the strike price nearest the futures price on that day. If interest rates increase in the next six months resulting in lower mortgage and futures prices, the option remains unexercised and the institution may apply the call premium already received to soften the blow of lower mortgage portfolio value. In periods of falling prices, then, the hedger with the sell-call option strategy must do better on average than the unhedged banker. If interest rates decline and values rise, the loss incurred from the exercise of the call by the holder would only lower the
gain from the increase in portfolio value. The banker in this case is guaranteed a positive return, although limited in magnitude depending on the call option premium.

The empirical results of the study support this theory. The return distribution for both the contemporaneous and lagged studies for the original series—falling rates and rising prices—show a positive return for short call hedging under all conditions involved. The one-to-one hedge of the contemporaneous series provides the lowest positive return, 1.096 points, for the at-the-money option compared to an average of 1.182 points for the unhedged mortgage. When the optimal mortgage amount is hedged with the call option, the gains increase substantially 2.539 and 2.593 points for the optimal hedge ratio uncorrected and corrected for autocorrelation respectively. Similar increases in return are evident when the lagged relationship is considered, although the contemporaneous series provide slightly higher returns.

Since the original series is characterized by rising mortgage values, and yet the position of mortgages plus sales of call options has a higher mean return than the unhedged position during this period, the results need to be further analyzed. An examination of option prices will aid in continuing the study.

As Figure 6-8 showed, during periods of rising mortgage and futures prices, the holder of the call option incurred a loss in the option position modified by the premium received. This loss, coupled with the gain in mortgage value, resulted in a guaranteed position of earning a positive return equal to the call option premium. Had interest rates declined by a substantial amount resulting in mortgage values increasing by an amount larger than the call premium, the holder of the call would end up with a mean return that was inferior to the unhedged position. Relatively small increases in mortgage value, on the other hand, would enhance the returns if the position is covered by selling call options.
The average at-the-money call option premia for this period was 2.798 points or $2,798. Moreover, in agreement with the Black option pricing theory, the premia for in-the-money and out-of-the-money options were 3.894 and 1.922 respectively.\textsuperscript{47} The average gain in the mortgage position, however, is only 1.182 points or $1,182. So, in accordance with the above discussion, and as is evident from Tables 6-4 and 6-5, the return for the covered-with-call position is close to the average call premium. Thus the sell-call strategy provides for an average capital gains which is better than the unheded mortgage position.

Variability of returns are lowered considerably when compared to the unhedged mortgage as well as mortgages hedged with futures contracts. When standard deviation is considered, the lagged series performs much better than the contemporaneous. For example, standard deviation is lowered by 63 percent for the uncorrected ratio of the lagged relationship compared to the unhedged mortgage, and by 46 percent compared to the future hedge. For the contemporaneous series these values are at 53 and 36 percent respectively.

So, for the case of rising prices, the short call strategy performs better than the unhedged and the hedged with futures series both in terms of the magnitude and the variability of returns.\textsuperscript{48}

The reverse series represent a simulated case for the general rise in interest rates over the entire period under study. Again, in all cases the hedger is doing better with call options than without. A loss of -1.182 points for the contemporaneous relationship is changed to much lower losses or even gains, -0.071 for the corrected optimal ratio hedge with the short position in at-the-money call and 0.418 for the uncorrected optimal hedge.

\textsuperscript{47} Recall that $\frac{\partial C}{\partial E} < 0$ as was previously shown.

\textsuperscript{48} Comparison of Figures 6-6 and 6-7 similarly suggests that during periods of rising values the sell-call strategy should provide better capital gains than the sell-futures strategy.
There is even a potential gain of 1.452 points for the one-to-one hedge. For the lagged series, optimal hedges always provide positive net returns except when the corrected ratio is used for the out-of-the-money call.

Referring to Figure 6-8, the above results are as predicted. During periods of falling values, if the mortgage is covered by selling call options, the loss in mortgage value is to be lower by an amount equal to the call premium. This is because when futures prices are falling, the holder of the call does not exercise the option. The writer, then, simply gains the premium which in turn will soften the loss of mortgage value. For example, the premia for the at-the-money-option, 2.193 points or $2,193 is roughly equal to the difference between the returns on the hedged and the unhedged mortgage positions. In other words, the loss in the hedged mortgage position is less than the loss in the unhedged portfolio by an amount which is close to the call option premia.

Standard deviation results are not as promising under a falling price scenario when short call hedging is used. Although in all cases this strategy lowers variability compared to the unhedged mortgage, the futures hedging may be a better choice. Generally the short call position for the reverse series generates returns that are much closer to the futures return while the standard deviation is higher than that of futures for all cases of optimal hedging. The one-to-one hedge has a better than futures standard deviation.

Although these results apply to this particular series, the graphical representation is supported nevertheless. Hedging mortgages with selling

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49 For optimal hedges, the corrected optimal hedge of the contemporaneous reverse series as an example, the loss of unhedged mortgage, -$1,182, is reduced by selling call options to a loss of- $71; a reduction in loss equal to $1,111 which is smaller than the option premia. However, since the optimal hedge ratio of .6132 was used in this case to hedge $163,079 of mortgages with selling one call option, the loss in mortgage value is, on average, higher at -1.182/.6132 or -1.927. The reduction in loss is then much closer to the call premium.
call options reduces the potential loss from falling prices while ensuring a positive return, although of limited magnitude, when mortgage values rise sharply. Moreover, the variability of returns are lowered with this type of hedge.

Hedging mortgage risk by buying put options should limit the losses to the put premium if prices and values decline. Rising portfolio value due to lower interest rates will, however, be also modified. The banker who hedges with the buy put strategy thus substitutes lower gains in return for limiting losses.

Over the period under the current study, the investor who chose this method of hedging did not do well in term of returns. For the original series, lagged or contemporaneous hedges with either one-to-one or optimal hedge ratios, the gain in mortgage portfolio value was substantially lowered. The one-to-one hedge with at-the-money put created a loss of -1.050 points compared to a potential gain of 1.182 points for the contemporaneous unhedged mortgage, a decline in mean capital gains of $2,232 or 188 percent. The optimal hedge ratios resulted in losses of -.919 and -.449 points for the uncorrected and corrected for autocorrelation respectively. In percentage terms the above losses were smaller, 177 and 138 percent respectively, as compared to the one-to-one hedge.

The purchase of in-the-money puts magnified the negative effects which may be explained by guessing that the higher premium paid for this option more that offset the benefit of exercising it. In other words, since in this period interest rates declined and prices increased, the exercise of the option would be rarely warranted. Thus, a person who paid an average premium of 2.627 points ($2,627) for the at-the-money put or 3.636 points ($3,636) for the in-the-money option eliminated potential gains in mortgage value at a high cost. Conversely, the out-of-the-money put with average premium of $1,829 provided for smaller declines in portfolio value as compared to the unhedged position.
The strategy, however, was quite successful in lowering the variability of returns. The one-to-one hedges lowered the standard deviation of return from an unhedged value of 4.823 points to 2.734 and 3.113 points for the lagged and contemporaneous relationships respectively. These numbers mean a reduction of 43 and 35 percent for each case. The comparable futures hedging accounts for a smaller reduction in variability. In other words, the one-to-one hedge with futures has a standard deviation of return which is 26 percent higher than the contemporaneous buy-put hedge and 25 percent higher than the lagged relationship's variability. This is true for the put with exercise price other than \( E = F_t \) as well.

Optimal hedges with the put option were also very helpful in lowering the standard deviation of returns. The lagged relationship performed better than the contemporaneous. In the best case, uncorrected hedge of the lagged relationship, variability of the unhedged return was lowered to 2.889 points, a reduction of 40 percent. The corrected optimal hedge of the contemporaneous series was the least successful and lowered the standard deviation by 9 percent to 4.408 points. On average, variability was lower by 25 percent when mortgage portfolio was hedged with the buy-put strategy.

When compared to the futures hedging, the results are a bit puzzling. It is expected that under the original series of rising prices the strategy should perform better than the futures position. For the one-to-one and the uncorrected optimal hedges this seems to be true. Although in both cases, and for the lagged as well as the contemporaneous relationships, returns are lower than the futures case, standard deviation of returns are smaller as well. For both relationships, however, the corrected optimal hedge accounts for a higher variability that the future hedge.

Falling prices represented by the reverse series resulted in a loss of -1.182 points in the value of the unhedged contemporaneous position. The price did not, however, fall by enough to compensate for the premium paid for the put option. Had prices fallen by a large amount, according to
Figure 6-9, the loss would not have exceeded the paid premium. In this case, the average loss for the optimal hedge ratios of both relationships is indeed on par with the average premium paid for the put option. The average premium paid to buy the put option is 3.440 points ($3,440) and the average loss is 3.040 points ($3,040). So, in fact optimal hedges did limit the loss at a level very close to average put premium, while the one-to-one hedges showed a smaller loss. This is expected as a one-to-one hedge covers a $100,000 mortgage with one option contract while the optimal hedge represents the hedge of a larger value with the same contract, \( n^* < 1 \). The optimal hedge should then account for a larger loss.\(^5^0\)

The move to in-the-money puts with an average premium of 4.522 points magnified the losses as well as increasing the variability of returns. Lower priced out-of-the-money puts, with average premium of 2.478 points, created smaller losses but higher standard deviations. This, in general, is not to be expected since in a period of falling prices the put is exercised. It must then be theorized that either due to a lower than expected rise in interest rates or as a result of overpricing of puts for the period under study, this hedging method was not successful for the reverse series. In other words, mortgage prices did not fall by enough to create large enough gains from the exercise of the costly put to compensate the buyer of the option for the paid premium.

The sell call-buy put strategy using at-the-money options for the original mortgage series generally provide returns and standard deviations similar to the unhedged position. The optimal hedge of the lagged and contemporaneous series increased returns by a low of .220 (18 percent) and

\(^5^0\) For the corrected hedge ratio of the contemporaneous series, for example, the hedge ratio of .6132 means that \((1/.6132)* 100,000\) of mortgage portfolio is being hedged with the purchase of one put option contract. Consequently, the average loss of $1,182 per $100,000 of mortgages would on average equal to 1/.6136 times as high for this optimal hedge. The loss, in other words is 1.631 * $1,182, or $1,928 for the optimal hedge. So, the average loss for the one-to-one hedge is smaller than the average loss for the optimal hedges.
a high of .750 points (63 percent), accompanied with changes in
variability of 12 percent to 27 percent lower. In this period of rising
prices, the exercise of the call created losses that were more than offset
by the gains in mortgage value modified by the call premia received and the
put premia paid. Thus a positive return was generated and variability was
reduced in all cases.

The reverse series generated large losses for optimal hedges compared
to the unhedged position, although the variability of returns were lowered
as well in all cases of hedging with the at-the-money options. The losses
were increased between 11 and 45 percent and standard deviations were
lowered by 16 to 26 percent. The one-to-one hedge lowered the mean capital
loss as well as lowering the variability.

When compared to the futures hedge, the synthetic futures position
created more erratic results. Return were much higher that the futures
position for the original series and lower considerably for the reverse
series. Standard deviations were similar in all cases.

So, although for the combined series the results were quite similar,
the original and reverse series compare differently to the futures hedge.

The use of options with an exercise price different than the initial
futures price created higher fluctuations in returns and increased the
standard deviation greatly. When E is smaller than F, the profits for the
original series soar and the losses for the reverse series diminish. This
may be due to the fact that the premium received for the new in-the-money
call far exceeds the premium paid for the out of the money put, on average
3.893 versus 1.829 points. Rising prices triggered the exercise of the
call but did not create big enough losses to offset the large net premium
received as well as the gain in mortgage value.
CHAPTER 7: THE COMPARISON OF FIXED AND ADJUSTABLE RATE MORTGAGES BASED ON OPTIONS HEDGING STRATEGIES

The choices that a financial institution has available in hedging the interest rate risk associated with its portfolio of fixed rate mortgages include offering adjustable rate mortgages and "covering" the fixed rate mortgage with various options on T-bond futures. If the institution chooses to offer ARMs solely, changes in the short term interest rates translate into similar changes in mortgage rates, thus leaving the portfolio value relatively intact. If, on the other hand, the choice was to cover the risk of the FRM with options on financial futures, the additional cost of hedging needs to be addressed more carefully.

The empirical study of the previous Chapter 6 clearly showed the benefits that exist in hedging mortgage portfolios with options on futures. This chapter continues the study by simulating possible changes in mortgage rates and analyzing the effects on the unhedged FRM portfolio as well as portfolios hedged with various options strategies. Had mortgage rates changed, as allowed by the limitations of an ARM, the value of the ARM portfolio would remain relatively constant while the FRM value would fluctuate. If the mortgage bankers were able to "insure" against adverse and undesirable changes in the value of their FRM, they should be willing to hold them as well. This "insurance policy" may be in the form of hedging strategies with options on financial futures contracts.

Marshal and Colwell compared the fixed and variable mortgage rates based on the cost of hedging the interest rate risk of FRMs with options on futures:^51

What if the SSL decides to issue both fixed rate mortgages and variable rate mortgages? What should be the interest rate spread between the two mortgages? The fixed rate mortgage could be "converted" to a variable rate mortgage simply by purchasing a put option on a futures contract and rolling it over until the fixed rate

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^51 David W. Marshal and Peter P. Colwell, "Hedging Mortgage Portfolios with Options on Futures," Real Estate Development, Fall 1986: 8-9.
mortgage is paid off. If the only difference between a fixed rate mortgage and a variable rate mortgage is the fact that a fixed rate mortgage has an additional expense of purchasing options, then the spread in the interest rate between the two types of mortgages should be no more than the cost of the option. If every nine months the S&L has to spend an additional $600, for example, in order to convert an $80,000 fixed rate mortgage, an extra cost of $600 for the required option could be covered by charging a rate which is 1 percent higher than the variable rate, \( \frac{($600\times1.333)}{$80,000} = 0.01 \). This 1 percent, then, should be the spread between the fixed and the variable rates.

Since the default risk is generally greater for ARMs due to the fact that lower income buyers may qualify easier as a result of lower initial rates, mortgage bankers may prefer FRMs under comparable cost and risk structure. They should be indifferent to hedging the risk with ARMs or by hedging with futures options as long as the cost of options hedging is not prohibitive and is taken into account when fixed mortgage rates are determined. If the same purpose is served by either of the two hedging methods, ARMs or options, then the cost of the option as the "insurance instrument" should be the difference between the ARM rate and the FRM rate. This amount represents the "conversion cost" of "neutralizing" the FRM risk structure similar to that of an ARM.

The current study continues by performing a simulation study similar to the one performed by Marshal and Colwell to determine the benefits of hedging with futures options under various interest rates scenarios, followed by a historical comparison of the fixed-adjustable rate differential to the cost of hedging with interest rate put options. "If the spread in the marketplace is larger than the spread as determined by hedging with options, then the S&L can benefit from the situation by offering "overpriced" fixed rates and converting them to variables at a cheaper cost than is implied by the interest rate spread. The spread ought to narrow as S&Ls compete against each other in this way."

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52 The $600 is spent every nine months which translates into a yearly cost of $800. In order to raise $800 on an $80,000 loan, the rate must be increased by 1 percent.

Each simulation exercise involves generating a random series of interest rates by allowing the rates to change randomly within predetermined annual and lifetime caps. The new interest rate series is then used to analyze the effects of random changes in interest rates on the unhedged mortgage value as well as the value of the FRMs hedged with the three options strategies discussed in Chapter 6: writing calls, buying puts, and the combination of the two. This exercise will be different from the one performed in the preceding chapter as the changes in interest rate are completely randomized, with no trends in the direction of the changes.\textsuperscript{54}

It is assumed that the value of the mortgage and the T-bond futures are correlated as determined in Chapter 6 for the lagged series corrected for auto correlation. In other words, an optimal hedge ratio of .7993 is used in hedging the mortgage portfolio with futures options. This choice was made because R-squared was the largest for that particular series. Marshal and Colwell had assumed a one-to-one hedge ratio.

As an example, on 1/15/83, the price of a $100,000 mortgage with an 8 percent coupon, 30 years to maturity and prepayment in 12 years is known to have been 70.884 percent of par from the empirical study of Chapter 6. If the interest rates were to increase by 1 percent during the six-month hedge period, 1/83-7/83, the value of the mortgage would drop to 66.595 percent of par, a decline of $4,289.00. However, since one option contract is being used to hedge an optimal mortgage amount of $(1/.7993)\times$100,000 = $125,109, the actual loss in mortgage value is magnified to $5,365.\textsuperscript{55}

\textsuperscript{54} The "combined" mortgage and futures price series in Chapter 6 provided similar circumstances of random interest rate changes, although it was still only one of the many possible interest rate scenarios. The current exercise, however, is performed for 42 different simulated interest rate and price series.

\textsuperscript{55} Recall from Chapter 6 that to account for the basis in hedging with futures options, 1/n units of the cash instrument were assumed to be hedged with one option contract on T-bond futures.
On the same day, three options market strategies might have been considered: a call option with strike price nearest the current price of the T-bond futures contract could have been sold for 2.547 points or $2,547; an at-the-money put option could have been purchased at a cost of $2,391; and finally a combination of the above actions at a net cost of - $156 could have been taken. As a result of the 1 percent change in interest rates, the futures price would decline, thus prompting the exercise of the put and would cause the call to remain unexercised. The new price for the futures contract may be calculated as follows.

Since the price of the futures contract on 1/15/83 was 75.656 percent of par, it provided for a yield of 11.457 percent (the coupon rate of 8 percent and maturity of 15 years are known). The price at the new rate of 11.475+1 or 12.475 percent may then be calculated at 70.279 percent of par. The exercise of the put option would generate $3,330.00 of profits for the mortgage banker, 76.00 -70.279 -2.391 = 3.330 points, which offsets some of the loss in mortgage value. The financial institution loses only $2,065 compared to potential loss of $5,365.00 for the unhedged mortgage. The call strategy would lower the loss in mortgage value by the amount of premium received, thus the banker would incur a net loss of $2,818.00 ($5,365 - $2,547). Finally, the strategy of buy put-sell call, in turn, would provide for a profit of $482.00 instead of the large loss of the unhedged mortgage portfolio.

For the case of a decline in interest rates, 1 percent lower for example, the value of one mortgage contract climbs by 4.701 points to 75.585 percent of par, creating a gain of (1/.7993)*$4,701 = $5,881 in the mortgage position. The value of the futures contract at the same time increases to 81.677 percent of par. In this case, the put will remain unexercised, lowering the capital gains of the unhedged mortgage by the paid premium of 2.391 points to $3,490. The loss incurred from the exercise of the call by its holder, on the other hand, results in a net
gain of $2,407. The combined put-call strategy lowers the return to a modest gain of $16.

For both of the above cases of change in interest rates, the standard deviation of returns may also be calculated and compared to the return distribution of the unhedged mortgage portfolio.

The current study considers 42 scenarios of random changes in interest rates over the period January 1983 to December 1991. For each of the 101 six-month hedge periods, an initial series of mortgage prices, futures prices, and put and call option premia is assumed as will be discussed in the next section. Interest rates are then allowed to vary randomly during each of the hedge periods, resulting in net capital gains or losses in the unhedged mortgage value or mortgages hedged with options strategies. The mean capital gains and standard deviation of returns on an annual basis and for the entire period under study are then calculated and analyzed.

Next, the put option hedging strategy will be discussed further as the potential hedging choice that may best suit the financial institution's objectives. The cost of buying put options on financial futures will, in turn, be used to compare the fixed and adjustable mortgage rates and to test a hypothesis regarding the spread between the two rates.

The Results of the Simulated Study

The simulation study involves generating 42 random series of interest rates and comparing the value of the original mortgage portfolios to the average of values from each of the new interest rate series. The change in the value of the unhedged mortgage when interest rates change at random may then be evaluated. Similarly, the change in value of the mortgage portfolio hedged with one of the option strategies under the same scenarios of random changes in interest rates may be examined and compared to the change in the value of the unhedged mortgage. The results of the simulation study are reported in the next section.
Data specification

It is assumed that the financial institution continuously owns a portfolio of 8 percent, 30 year mortgages with 12 years to prepayment. At the start of each hedge period of six months duration, the mortgage value and the price of the T-bond futures contracts are the same as the ones used in the empirical study of Chapter 6. Moreover, at the beginning of each period, the S&L may choose to engage in one of the three options strategies, also with the specifications identical to those explained in Chapter 6. It is further assumed that after the hedge initiation, long term interest rates may fluctuate by a random amount within given bounds. The current study assumes an interest rate cap of 1 percent per six-month period. The FRM rate is thus allowed to increase by a maximum of 1 percent during the hedge period or it may decline by as much.56

Strictly for comparison purposes, and only pertinent to the particular period under this study, a simple regression analysis was performed to compare changes in short term rates and the long term mortgage rates.57 For the period January 1983 to December 1991, monthly data on the three and six-month Treasury bill rates were collected from the Federal Reserve Bulletin. These rates were looked upon as the index rate to which a margin is added to determine the ARM rate. Two situations are considered: for the contemporaneous series, the relationship between the changes in mortgage rates and the rate change on Treasury bills in the same month are compared. The lagged relationship considers the relation of the

56 The interest rate cap on mortgage rates was chosen at 1 percent per six-month hedge period after an examination of the movements in rates over the period under consideration. The comparison of the changes in the short term and the long term interest rates that follow are for "rough" comparisons and have no bearing on the study.

57 The literature on the term structure of interest rate is vast. The current study does not attempt to capture the relationship between the long term and the short term rates with the simple regression performed. The slope coefficient of the regression line is only used as a "rough" relation factor between the two rates.
change in mortgage rate to the change in T-bill rate of one month earlier. The result of regressing FRM rates with specifications identical to those of Chapter 6 on the three or six-month T-bill rates for both relationships are reported in Table 7-1. The mortgage rate is denoted as 'm', and 'tb' represents the Treasury bill rate. The regression coefficient for the T-bill rate, b, is the measure used in translating the short term rate changes to the corresponding change in the long term rates. Thus, the 2 percent annual cap assumed for the FRM rate is a response to a cap range of 2.63 to 2.73 percent annually for the ARM, depending on which slope coefficient is considered.

Table 7-1. Regression estimates for changes in mortgage rate and the Treasury bill rate

<table>
<thead>
<tr>
<th>CONTEMPORANEOUS T-BILL RATES</th>
<th>LAGGED T-BILL RATES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta m_t = a + b \Delta tb_t )</td>
<td>( \Delta m_t = a + b \Delta tb_{t-1} )</td>
</tr>
<tr>
<td>THREE-MONTH BILL</td>
<td>SIX-MONTH BILL</td>
</tr>
<tr>
<td>a</td>
<td>5.35</td>
</tr>
<tr>
<td>b</td>
<td>.72</td>
</tr>
<tr>
<td>n</td>
<td>102</td>
</tr>
<tr>
<td>d.f.</td>
<td>100</td>
</tr>
<tr>
<td>R-SQUARED</td>
<td>.50</td>
</tr>
</tbody>
</table>

The structure of the simulation study performed is as follows: There are 101 hedge periods considered in this part of the study for the case of the lagged mortgage rate and futures price series. During the first hedge period, interest rates are allowed to move within the given bounds. The value of the unhedged mortgage and the mortgage portfolios covered with the option strategies will thus change. During the subsequent period, the exercise is repeated starting with the actual mortgage rate at the beginning of the period as reported in Chapter 6. Similarly, for each of the remaining hedge periods, the study begins with the mortgage rate
reported in Chapter 6 and allows for a random change in rates within the
given bounds and measures the change in the unhedged and "covered" mortgage
portfolios. The sum of the changes in value for all of the hedge periods
is then reported as the change in value for the entire period under study.
The exercise is then repeated 42 times.

Return distribution for the "worst case" scenario

Under the worst case scenario, interest rates increase by the maximum
allowed. This means a maximum rate hike of 1 percent per hedge period.
The value of the mortgage portfolio, hence, drops during each of the hedge
periods as discussed in the introduction to Chapter 7, and so does the
price of the futures contract. The buy-put hedging strategy limits the
losses to the extent of the paid premium and the sell-call strategy softens
the loss by the amount of premium received. The results for the entire
period under study are summarized in Table 7-2. Minimum and maximum
returns to each strategy for the entire period under study are presented in
the first and second columns respectively, and the next two columns in
Table 7-2 show the mean return and the standard deviation of returns for
the unhedged as well as those for mortgages hedged with the three option
strategies.

As is shown in Table 7-2, the mean return for the worst case scenario
is -5.396 points, or a loss of $5,396. Since mortgage values fell in
every hedge period as a result of higher rates, there was a continuous loss
throughout the entire period under study as is shown by the annual return
distribution, Table 7-3. The maximum loss (minimum return) for the entire
period under study is -6.362 points, occurring in 1991, and the minimum
loss (maximum return) for the entire period happened in 1984 at -4.227
points. Although rates increased by a constant 1 percent per hedge period,
the standard deviation of changes in values (returns) is .679 points.

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58 Mean expected return and mean capital gains are used
 interchangeably in this chapter.
Table 7-2. Comparison of the unhedged mortgage position under the worst case scenario of changes in interest rates to various hedged positions...lagged relationship

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>MINIMUM CAPITAL GAIN</th>
<th>MAXIMUM CAPITAL GAIN</th>
<th>MEAN CAPITAL GAIN</th>
<th>STANDARD DEVIATION OF RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNHEDGED MORTGAGE</td>
<td>-6.362</td>
<td>-4.227</td>
<td>-5.396</td>
<td>0.679</td>
</tr>
<tr>
<td>MORTGAGE E&lt;F + SELL CALL OPTION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-5.363</td>
<td>0.654</td>
<td>-1.490</td>
<td>0.954</td>
</tr>
<tr>
<td>E=F</td>
<td>-5.760</td>
<td>-0.612</td>
<td>-2.594</td>
<td>0.871</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>-6.501</td>
<td>-1.702</td>
<td>-3.460</td>
<td>0.824</td>
</tr>
<tr>
<td>MORTGAGE + BUY PUT OPTION</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-5.376</td>
<td>-0.695</td>
<td>-2.795</td>
<td>0.852</td>
</tr>
<tr>
<td>E=F</td>
<td>-4.064</td>
<td>0.508</td>
<td>-1.593</td>
<td>0.849</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>-3.855</td>
<td>1.555</td>
<td>-0.601</td>
<td>0.951</td>
</tr>
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<td>MORTGAGE + SELL CALL + BUY PUT</td>
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It may seem that in the case of a continuous increase in interest rates, there should not be a standard deviation to returns. However, since the starting mortgage values at the beginning of each period were quite different in response to different initial mortgage rates, and because a given change in interest rate does not translate into an equal change in mortgage value as determined from the Thorndike Encyclopedia of Banking and
Table 7-3. Annual return distribution and the minimum-maximum pairs for the hedged and unhedged mortgage positions under the worst case scenario of changes in interest rates...at the-money options reported only

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>MINIMUM CAPITAL GAIN</th>
<th>MAXIMUM CAPITAL GAIN</th>
<th>MEAN CAPITAL GAIN</th>
<th>STANDARD DEVIATION OF RETURN</th>
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<tr>
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<td>2.004</td>
<td>1.397</td>
<td>0.361</td>
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</table>
Financial Tables, Revised Edition, there remained a standard deviation to returns of the worst case scenario. 59

The sell-call strategy lowered the loss from the unhedged mortgage portfolio by 52 percent to -2.594 points. As expected, the decrease in loss in the amount of $2,802 is very close to the average premium paid for the call options, $2,788. The standard deviation of returns was increased by 25 percent to .871 points. The maximum loss under this hedging strategy is -5.760 points occurred in 1987, and the minimum loss is -.612 points, occurring in 1986.

The buy-put strategy performed very well under the worst case scenario. The average loss of the unhedged position was lowered to -1.593 point, a decrease of approximately 70 percent, while the standard deviation increased by 28 percent. The minimum return of the position was -$4,064 and the maximum was actually a moderate profit of $508.

The sell call-buy put hedging strategy created much improved returns at a cost of 28 percent higher variability. The mean return from the mortgage position hedged with buying put and selling call options was a profit of $1,200, an increase of 122 percent. Annual return distribution results show that the continuous loss from the unhedged mortgages were transformed into a profit for every year in the period under study for this hedging choice.

The use of options with an exercise price other than the current futures price created return distributions that are consistent with the above results. The in-the-money call, E<F, and put, E>F, improved the returns without much adverse effects on the standard deviations.

59 For example, in January 1983, mortgage value as found from Thorndike was 70.884 percent of par to yield 13.354 (the current rate at that time). A 1 percent increase in rates lowered the mortgage value to 66.594 percent of par, a loss in value equal to $4,289. Starting with a mortgage value of 90.041 percent of par, in January 1987, to yield the then current 10.354 percent rate, an increase of 1 percent in rates would cause a fall in mortgage value in the amount of $6,008 to 84.033 of par.
The mean return of the call strategy with the smaller exercise price is -1.490 points, a loss smaller than the at-the-money call position by 1.104 points at the expense of .083 points higher variability. The overall reduction of loss compared to the unhedged mortgage is 3.906 points or 72 percent, but the standard deviation, at .954 points, is higher by approximately 40 percent. This means an increase in variability by .275 points which may not be alarming due to the fact that the standard deviation of returns for the unhedged mortgage were quite low to begin with. For the in-the-money put, the mean return is a loss of -.601 points, smaller than the loss of the unhedged position by 4.795 points and better than the at-the-money put by .992 points. The variability is higher than the call with E=F by .102 points and larger than the unhedged position by .272 points.

Return distribution for the "best case" scenario

Under the best case scenario, interest rates decline by 1 percent per hedge period, thus raising the mortgage value as shown for the entire period under study in Table 7-4 as well as raising the futures contract price. The call strategy, in this case, would set a limit on the gains and the put strategy should lower potential gains to the extent of the premium paid.

The mean unhedged return for this period is a gain of 5.938 points with a corresponding standard deviation of .755 points. On an annual basis, the unhedged mortgage portfolio exhibits a positive mean return for every year in the period 1983-91 as is shown in Table 7-5. The minimum return, occurring in 1984, is 4.620 and the highest gain is 7.012 in 1991.

Hedging with the call option in this case lowers the return considerably. The mean return, $1,506, is lower by $4,430 and the standard deviation of returns is higher at $941. Minimum and maximum returns are both much smaller. The maximum return from this position is only $3,319
Table 7-4. Comparison of the unhedged mortgage position under the best case scenario of changes in interest rates to various hedged positions...lagged relationship

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
<th>MEAN</th>
<th>STANDARD</th>
</tr>
</thead>
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<td>CAPITAL</td>
<td>CAPITAL</td>
<td>DEVIATION OF RETURN</td>
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<td>5.938</td>
<td>0.755</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-2.341</td>
<td>2.381</td>
<td>0.610</td>
<td>0.956</td>
</tr>
<tr>
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<td>0.941</td>
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<tr>
<td></td>
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</tr>
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<td>0.959</td>
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<td>-5.435</td>
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</table>

with the minimum return actually well on the negative side at -$1,135. During three of the hedge periods starting in 1985, and two periods beginning in 1987, the institution incurred a loss by engaging in this strategy. The largest loss of -$1,135 occurred for the hedge period of October 1985 to April 1986. The much larger premia of in-the-money calls made the results less desirable, mean return of only $610 with standard deviation of $956.
Table 7-5. Annual return distribution and the minimum-maximum pairs for the hedged and unhedged mortgage positions under the best case scenario of changes in interest rates...at-the-money options reported only

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>MINIMUM CAPITAL GAIN</th>
<th>MAXIMUM CAPITAL GAIN</th>
<th>MEAN CAPITAL GAIN</th>
<th>STANDARD DEVIATION OF RETURN</th>
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<td>-0.294</td>
<td>-1.397</td>
<td>0.556</td>
</tr>
<tr>
<td>91</td>
<td>-1.684</td>
<td>-0.775</td>
<td>-1.230</td>
<td>0.339</td>
</tr>
</tbody>
</table>
The buy-put strategy lowered the mean return by $2,669 which as previously shown in Figure 6-9 (page 94) is close to the average put premium of $2,627 points. The rise in standard deviation was not as dramatic as was for the sell-call strategy, a 16 percent increase to $876 compared to the 25 percent increase in variability for the call position. During none of the hedge periods did the S&L lose any money, although the minimum return was at a low of $1,314. Since in this period of rising prices the put was generally not exercised, the higher average premia of the in-the-money option lowered the mean return to $2,287 with a corresponding 37 percent increase in standard deviation.

The combined put-call strategy is not desirable under the best case scenario. The mean return from this strategy is a loss of -$1,127 and the banker loses money in 88 of the 101 hedge periods. In the years 1986 and 1989-1991, in fact, there was never any profits despite declining rates when this strategy was utilized. The mean return was positive only in 1984, but the standard deviation was also 2.5 times as large in the same year as compared to the unhedged position. In general, the standard deviation for the entire period was raised by 49 percent to $1,012.

The above two cases represent extreme interest rate changes and may not be likely to happen often. During the period 1983-1991, mortgage rates increased in 40 of the 101 six-month hedge periods, and declined in 61 periods. The increase in rates was larger than 1 percent per six-month on 9 occasions: 4 hedge periods starting in the latter part of 1985, 2 in the mid-1986 and two in the spring of 1989. The largest six-month increase in mortgage rates, 1.625 percent, was for the hedge period November 1985 to May 1986, followed by 1.470 percent for the hedge period starting in October 1985 and 1.450 percent for the period starting in December of the same year. The mortgage rate declined by 1 percent only between April and October of 1987.

During the same period, 1983-1991, the three-month Treasury bill rate increased by more than 2.5 percent in only 2 of the 101 hedge periods.
considered, and declined by more than that amount on 5 occasions. The six-
month T-bill rate climbed over the 2.5 percent mark 3 times and deceased by a
larger magnitude in six periods. Even allowing a tighter cap of 2 percent annually on short term interest rates, the increase in rates surpassed the ceiling only 4 times for the three-month bill and also 4 times in the case of the longer term security. The floor cap was reached on 9 occasions for the three-month and 11 times for the six-month bill.

The study of these adverse cases is necessary, however, both because of the possibility of occurrence and more importantly due to the boundaries they set for the return distribution of various strategies. Table 7-6 shows the lower and upper bounds for the mean return with and without options hedging. The minimum and maximum returns are also presented for each case. It is noticeable that the range of returns has decreased dramatically when the FRM portfolio is hedged with options on futures.

**Return distribution for the general simulated case**

The general case of changes in interest rates is considered next by simulating a rate change within the rate cap of +1 percent and the floor of -1 percent per hedge period. So, starting with the actual mortgage rate at the beginning of each period, rates are adjusted upward or downward by a simulated amount for the 102 cases of the contemporaneous and the 101 hedge periods of the lagged relationships. The mean and standard deviation of returns are reported in Table 7-7, and the minimum and maximum net capital gains are shown in Table 7-8. Finally, the annual return distribution for the unhedged mortgage and the hedged portfolios are depicted in Table 7-9.

---

60 As an additional exercise, and for comparison purposes, the return distribution for the contemporaneous hedge of the mortgage portfolio, corrected for autocorrelation is also calculated in this section.
Table 7-6. The limits of return for the unhedged and hedged mortgage positions...at-the-money options

<table>
<thead>
<tr>
<th>LOWER BOUND</th>
<th>UPPER BOUND</th>
<th>RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNHEDGED MORTGAGE MEAN</td>
<td>-5.396</td>
<td>5.938</td>
</tr>
<tr>
<td>MIN\MAX</td>
<td>-6.362</td>
<td>7.012</td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL OPTION MEAN</td>
<td>-2.594</td>
<td>1.506</td>
</tr>
<tr>
<td>MIN\MAX</td>
<td>-5.760</td>
<td>3.319</td>
</tr>
<tr>
<td>MORTGAGE + BUY PUT OPTION MEAN</td>
<td>-1.593</td>
<td>3.296</td>
</tr>
<tr>
<td>MIN\MAX</td>
<td>-4.064</td>
<td>5.456</td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL + BUY PUT MEAN</td>
<td>-1.127</td>
<td>1.200</td>
</tr>
<tr>
<td>MIN\MAX</td>
<td>-4.251</td>
<td>3.340</td>
</tr>
</tbody>
</table>

The lagged and the contemporaneous series resulted in very similar outcomes, thus for continuity purposes, only the lagged relationship is discussed in the rest of this section.

The unhedged mortgage portfolio under simulated changes in rates has a mean return of $170 for the entire period under study, and the standard deviation of returns is $3,286. The minimum and maximum returns are -$6,358 and $6,975 respectively.

Hedging with options on financial futures lowers the standard deviation considerably for all three strategies and for all exercise prices. The best results in lowering the variability occurred when a
Table 7-7. Return distribution for the hedged and unhedged mortgage positions resulting from simulated changes in interest rates...entire hedge period under study

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>CONTEMPORANEOUS RELATIONSHIP</th>
<th>LAGGED RELATIONSHIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MEAN RETURN</td>
<td>STANDARD DEVIATION OF RETURN</td>
</tr>
<tr>
<td>UNHEDGED MORTGAGE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BEST CASE</td>
<td>0.170</td>
<td>3.286</td>
</tr>
<tr>
<td>WORST CASE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SIMULATED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL OPTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>0.902</td>
<td>1.154</td>
</tr>
<tr>
<td>E=F</td>
<td>0.934</td>
<td>1.583</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>0.912</td>
<td>2.080</td>
</tr>
<tr>
<td>MORTGAGE + BUY PUT OPTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-0.781</td>
<td>2.217</td>
</tr>
<tr>
<td>E=F</td>
<td>-0.715</td>
<td>1.692</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>-0.568</td>
<td>1.279</td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL + BUY PUT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E&lt;F</td>
<td>0.025</td>
<td>0.945</td>
</tr>
<tr>
<td>E=F</td>
<td>0.122</td>
<td>0.937</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>0.247</td>
<td>1.004</td>
</tr>
</tbody>
</table>

The combined strategy of selling at-the-money calls and buying at-the-money puts was undertaken.

The standard deviation of returns was lowered by 70 percent to $985 with only a 25 percent drop in the mean return. Combining the in-the-money put with out-of-the-money call actually increased the mean return despite 68 percent lower variability.
Table 7-8. Minimum and maximum values for the hedged and unhedged mortgage positions resulting from simulated changes in interest rates...entire hedge period under study

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>CONTEMPORANEOUS RELATIONSHIP</th>
<th>LAGGED RELATIONSHIP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MEAN RETURN</td>
<td>STANDARD DEVIATION OF RETURN</td>
</tr>
<tr>
<td>UNHEDGED MORTGAGE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BEST CASE SIMULATED</td>
<td>-6.358</td>
<td>6.975</td>
</tr>
<tr>
<td>WORST CASE SIMULATED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL OPTION</td>
<td>-2.987</td>
<td>3.626</td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-3.599</td>
<td>3.968</td>
</tr>
<tr>
<td>E=F</td>
<td>-4.138</td>
<td>4.422</td>
</tr>
<tr>
<td>MORTGAGE + BUY PUT OPTION</td>
<td>-4.759</td>
<td>4.820</td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-4.037</td>
<td>4.159</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>-4.747</td>
<td>3.362</td>
</tr>
<tr>
<td>MORTGAGE + SELL CALL + BUY PUT</td>
<td>-3.203</td>
<td>2.345</td>
</tr>
<tr>
<td>E&lt;F</td>
<td>-3.070</td>
<td>2.408</td>
</tr>
<tr>
<td>E&gt;F</td>
<td>-3.762</td>
<td>2.600</td>
</tr>
</tbody>
</table>

When used individually, the sell-call or buy-put strategies performed similarly in lowering the standard deviation of returns, 1.437 points lower for the put and a 1.561 points drop in the case of the call. The mean return was considerably lower than that of the unhedged mortgage when the put strategy was employed, and was increased by a large amount for the sell-call position. The new returns are -$911 and $1,098 respectively for the put and call strategies.
Table 7-9. Annual return distribution for the hedged and unhedged mortgage positions resulting from simulated changes in interest rates...at-the-money options reported only

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>YEAR</th>
<th>MEAN RETURN</th>
<th>STANDARD DEVIATION OF RETURN</th>
<th>MEAN RETURN</th>
<th>STANDARD DEVIATION OF RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNHEDGED MORTGAGE</td>
<td>1983</td>
<td>0.123</td>
<td>2.381</td>
<td>0.123</td>
<td>2.381</td>
</tr>
<tr>
<td></td>
<td>84</td>
<td>0.017</td>
<td>2.407</td>
<td>0.017</td>
<td>2.407</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>0.211</td>
<td>2.607</td>
<td>0.211</td>
<td>2.607</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>0.147</td>
<td>3.000</td>
<td>0.147</td>
<td>3.000</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.137</td>
<td>3.235</td>
<td>0.137</td>
<td>3.235</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>0.144</td>
<td>3.044</td>
<td>0.144</td>
<td>3.044</td>
</tr>
<tr>
<td></td>
<td>89</td>
<td>0.525</td>
<td>3.354</td>
<td>0.525</td>
<td>3.354</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.166</td>
<td>3.291</td>
<td>0.166</td>
<td>3.291</td>
</tr>
<tr>
<td></td>
<td>91</td>
<td>0.177</td>
<td>3.342</td>
<td>0.177</td>
<td>3.342</td>
</tr>
<tr>
<td>MORTGAGE +</td>
<td>1983</td>
<td>1.129</td>
<td>1.154</td>
<td>1.115</td>
<td>1.132</td>
</tr>
<tr>
<td>SELL CALL OPTION</td>
<td>84</td>
<td>0.964</td>
<td>1.349</td>
<td>0.946</td>
<td>1.295</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>0.813</td>
<td>1.451</td>
<td>0.866</td>
<td>1.363</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>1.697</td>
<td>1.486</td>
<td>1.688</td>
<td>1.402</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>0.994</td>
<td>1.829</td>
<td>1.043</td>
<td>1.724</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>1.260</td>
<td>1.580</td>
<td>1.166</td>
<td>1.536</td>
</tr>
<tr>
<td></td>
<td>89</td>
<td>0.543</td>
<td>1.338</td>
<td>1.292</td>
<td>1.297</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.349</td>
<td>1.682</td>
<td>0.676</td>
<td>1.630</td>
</tr>
<tr>
<td></td>
<td>91</td>
<td>0.448</td>
<td>1.514</td>
<td>0.473</td>
<td>1.487</td>
</tr>
<tr>
<td>MORTGAGE +</td>
<td>1983</td>
<td>-0.759</td>
<td>1.373</td>
<td>-0.827</td>
<td>1.286</td>
</tr>
<tr>
<td>BUY PUT OPTION</td>
<td>84</td>
<td>-0.872</td>
<td>1.370</td>
<td>-0.902</td>
<td>1.313</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>-0.931</td>
<td>1.380</td>
<td>-0.983</td>
<td>1.305</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>-1.436</td>
<td>1.608</td>
<td>-1.471</td>
<td>1.527</td>
</tr>
<tr>
<td></td>
<td>87</td>
<td>-0.955</td>
<td>1.878</td>
<td>-1.052</td>
<td>1.771</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>-0.822</td>
<td>1.612</td>
<td>-0.909</td>
<td>1.564</td>
</tr>
<tr>
<td></td>
<td>89</td>
<td>-0.151</td>
<td>1.809</td>
<td>-0.295</td>
<td>1.760</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>-0.284</td>
<td>1.578</td>
<td>-0.435</td>
<td>1.542</td>
</tr>
<tr>
<td></td>
<td>91</td>
<td>-0.260</td>
<td>1.744</td>
<td>-0.057</td>
<td>1.559</td>
</tr>
<tr>
<td>MORTGAGE +</td>
<td>1983</td>
<td>0.255</td>
<td>0.641</td>
<td>0.215</td>
<td>0.634</td>
</tr>
<tr>
<td>SELL CALL +</td>
<td>84</td>
<td>0.144</td>
<td>0.311</td>
<td>0.105</td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>-0.135</td>
<td>1.028</td>
<td>-0.111</td>
<td>0.914</td>
</tr>
<tr>
<td></td>
<td>86</td>
<td>0.110</td>
<td>1.311</td>
<td>0.113</td>
<td>1.231</td>
</tr>
<tr>
<td>BUY PUT +</td>
<td>87</td>
<td>0.377</td>
<td>1.129</td>
<td>0.358</td>
<td>1.027</td>
</tr>
<tr>
<td></td>
<td>88</td>
<td>0.338</td>
<td>0.699</td>
<td>0.312</td>
<td>0.652</td>
</tr>
<tr>
<td></td>
<td>89</td>
<td>-0.197</td>
<td>0.900</td>
<td>-0.179</td>
<td>0.868</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>0.112</td>
<td>0.785</td>
<td>0.114</td>
<td>0.739</td>
</tr>
<tr>
<td></td>
<td>91</td>
<td>0.134</td>
<td>0.729</td>
<td>0.074</td>
<td>0.718</td>
</tr>
</tbody>
</table>
The $741 drop in the mean return when the put is purchased, may be looked upon as the cost of lowering the standard deviation of return by hedging with the at-the-money put option. While it may seem like hedging with call options actually enhances profits, closer analysis of Table 7-7, reveals that the lower limits of return allow for a smaller loss in the case of the put hedging while providing for greater upper bounds on profits when compared to hedging with the call option.

Moreover, the minimum loss is smaller in the case of covering the FRM with puts and the maximum gain is higher for this strategy. These results, combined with the fact that financial institutions' goal is primarily to protect their mortgage value from falling, may point to the put strategy as the more advisable choice. Buying put options sets a limit on losses while the sell-call choice only lowers the potential loss if interest rates increase.

The re-examination of the worst case scenario outcome show that buying the put resulted in much smaller losses than selling the call, -$2,594 versus -$4,593. In addition, with more relaxed caps or no cap at all on interest rate changes, the loss from the unhedged FRM or mortgage plus call option would be unlimited. So, if the purpose of the banker is to curb the negative effects of rising interest rates, buying put options on financial futures to cover the FRM portfolio is recommended.

**THE FRM-ARM Rate Differential**

The previous simulation study shows that hedging the interest rate risk of a FRM portfolio with options on T-bond futures is quite beneficial in lowering the variability of returns. Moreover, with the primary purpose of protecting mortgage value from declining, hedging the FRM with purchasing put options on financial futures provides similar benefits to offering ARMs. A hypothesis is thus formed to suggest that the difference between the FRM and the ARM should not be higher than the cost of hedging.
with put options if the two strategies performs similar functions. This hypothesis is further analyzed in this section.

Comparison of FRMs and ARMs

ARMs as a group may be compared to FRMs in terms of their characteristics and intended functions. FRMs offer the borrower the right to retain the mortgage if rates increase and the option to refinance if interest rates decrease. This is similar to a call option. ARMs are designed to protect mortgage loan investors from rising interest rates subsequent to origination and the resulting decline in mortgage value. A question may arise as to the possibilities of declining rates and the fact that ARMs prohibit the potential advantage of this situation. It may, at first, be argued that ARM investors are no worse than FRM lenders since borrowers have the option to refinance at lower rates, making FRMs equivalent in this case to ARMs except for the initial contract rate.

However, since most FRMs have at least some origination fee and discount points, refinancing an existing high-rate FRM with a new lower-rate loan is not costless. By industry standards, rates may need to drop by 2 or 3 percentage point before refinancing becomes a financially reasonable option for the borrower. In other words, fixed-rate loans have "sticky" rates in the case of moderately falling yields, but ARMs have their rate adjusted. In fact, if the implicit "call option" characteristic of FRMs could have been revoked by lenders in terms of requiring large prepayment penalties, the superiority of FRMs over ARMs under moderately falling rates would be evident. However, "since prepayment penalties are unpopular with borrowers and illegal in many states, some lenders have substituted large up-front discount points and/or origination fees to lower the possibilities of refinancing FRMs prior to their original maturity."61

Substantial similarities, then, exist between fixed rate mortgages

and adjustable rate mortgages with interest rate caps and floors. Mortgage lenders/investors will earn below market returns on both instruments should rates rise more sharply than expected, while interest rate floors on ARMs and costly "calls" of FRMs will cause lenders to earn above-market rates of return if interest rates decline more than expected. Because of these similarities, the fundamental determinants of the spreads between the market rates of interest and the fixed and adjustable mortgage rates should be the same. Moreover the relationship between the two mortgage rates should be easily deducible based on the same determinants.

The mortgage markup from short-term rates is largely determined by the slope of the term structure of interest rates and the longer-run volatility in short-term rates. "The more upward-sloping is the term structure, the more lenders expect rates to rise; and the more averse lenders are to increases in rates, the greater will be the markup on the fixed or capped adjustable mortgage rates. Moreover, even if the term structure is relatively flat, high long-run volatility in short term rates means a reasonable likelihood of significantly higher interest rates during some future periods. So, the higher is long-run rate volatility, the greater is the markup."  

Setting looser rate caps on ARMs, introducing ARM rate floors, and making the implicit FRM call option more costly through larger refinancing fees, modifies the relationship between the markup and its determinants. The looser are the caps and the more costly is the "call" or the prepayment option, the less the markup will be for a given slope of the term structure and level of long-run volatility. With loose caps, lenders lose less relative to market rates when rates rise; with costly FRM prepayment and ARM floors, lenders gain relative to market rates when rate decline. The decrease in markup varies with the slope of the term structure and the

---

long-run volatility in rates. Pricing mortgages, then, depends crucially on the assumptions made regarding possible future interest rate paths, as well as the terms of mortgage contract.

The difference between the adjustable mortgage rates and those on fixed rate mortgages, in the current study, is discussed in terms of hedging the interest rate risk of the FRM portfolio with options on financial futures. As shown in the previous empirical studies, in addition to offering ARMs, the risk of declining mortgage value as a result of higher interest rates may be successfully hedged by purchasing put options on T-bond futures contracts. Figure 7-1 is a reproduction of Figures 6-5 and 6-9 and may be used to compare the two strategies in the simplest case of no interest rate caps and floors. As discussed earlier, since the purpose of insuring the mortgage value may be achieved through either offering ARMs or covering FRMs with put options, the rate differential between the two mortgage types should be no larger than the cost of option hedging. For example, if the cost of covering $100,000 of mortgage value with put options were $4,000 annually, the maximum markup of the FRM relative to the ARM should be 4 percent per year ($4,000/$100,000 = .04). The remainder of this section explores the above idea in some detail.

The experiment scenario and comparative results

It is assumed that the S&L may purchase one at-the-money put option contract on T-bond futures in the month preceding the start of a one-year hedge period, an anticipatory hedge. Once expired, the option may be rolled over to cover the remainder of the year. The amount of the mortgage to be hedged with one put option is determined based on the hedge ratio technique developed earlier. A one-to-one hedge translates to the coverage of $100,000 of mortgage value with one option while the employment of the optimal hedge ratio, \( n^* \), means covering \((1/n^*)(100,000)\) with the same contract. For the current study, the optimal hedge ratios of .7993 and
Figure 7-1. Net capital gain from the uncapped ARM position versus net capital gain from the FRM covered with the buy-put strategy.
.8991, the uncorrected and corrected for autocorrelation ratios of the lagged series respectively, are used. The optimal hedging choice in this case, then, is to cover either $111,222 or $125,109 of mortgage value with a single put option contract.

Two cost structures are considered. The ex-ante cost simply considers the sum of the option premia for the options used in the annual hedge as the hedging cost. The ex-post cost adjusts the above simple cost by the possible benefits of exercising the option(s). If neither of the options is exercised during the course of the year, the two cost structures are identical. If, on the other hand, one or both options are exercised, the resulting gains are subtracted from the option premia to determine the net cost of hedging the FRM portfolio. The following example shows the calculation of the two hedging costs.

On February 15, 1988, one at-the-money put option with maturity of September 1988 and exercise price of 92 was purchased at the cost of 3.625 points, or $3,625. In September of the same year, the option was exercised for a gain of $3,188 and a net profit of -$438. In the same month, another at-the-money option was purchased for $2,766 to cover the FRM portfolio for the next six months. In March of the next year, the option was out-of-the-money and expired unexercised for a net loss of -$2,766. The ex-ante hedging cost for the year was $6,391 ($3,625+$2,766), and the ex-post cost of the hedge was $3,203 ($3,625+$2,766-$3,188). If these options were used to hedge $100,000 of mortgage portfolio, the ex-post cost, for example, would translate to 3.203 percent annually. Hedging the optimal mortgage amounts would lower the cost to 2.880 percent and 2.560 percent respectively for the uncorrected (n″=.8991) and corrected (n″=.7993) hedge ratios. So, if the mortgage portfolio was hedged with put options on financial futures, the annual cost in percentage terms would fall in the 2.560 to 3.203 percent range depending on the hedge amount. For reference and comparison purposes, the fixed mortgage rate during February 1988 was
at 9.900 percent and the three-month T-bill rate was 5.690. A 1.5 percent margin between the T-bill rate and the rate set on the ARM would then lead to a 2.710 percent difference between the fixed and adjustable mortgage rates and a 2 percent margin would cause 2.210 percent annual rate differential.

The average annual hedging cost in percentage terms are reported in Tables 7-10, 7-11, and 7-12 for the three hedge ratios. The results emphasize the following facts.

First, the cost of hedging in percentage terms declines as the one-to-one hedge is replaced by optimal hedging. This is expected as the movement from the one-to-one hedge ($100,000 of mortgage value) to the optimal hedge ratios of .7993 ($111,222 of mortgage value) and .8991 ($125,109 of mortgage value) represents more cost effective hedges—the same hedging cost is incurred to cover a larger dollar value of mortgage portfolio. Second, the result of hedging with the next in-the-money option for each of the three hedge ratios shows a rise in hedging cost. Recall from Table 7-7 that hedging FRMs with in-the-money puts provides for a better coverage in terms of improved mean and standard deviation of returns as compared to the at-the-money option. The improved coverage, however, demands a higher cost. Similarly, smaller coverage may be obtained by purchasing out-of-the-money puts at a lower cost. Finally, the comparison of the ex-ante and ex-post costs show a decline in hedging cost for every year in the study. In some cases, at-the-money hedges of 1983 and 1989 for example, hedging the fixed rate mortgage portfolio with put options on financial futures would be costless if the exercise of the option is taken into account. Since the true cost/benefit of option hedging includes the possibilities of exercise, the ex-post cost will be used in the reminder of this section.
Table 7-10. Average annual cost of covering $100,000 of mortgage value with one put option contract on financial futures (one-to-one hedge ratio)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>EX-ANTE COST</th>
<th>EX-POST COST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E&lt;F E=F E&gt;F</td>
<td>E&lt;F E=F E&gt;F</td>
</tr>
<tr>
<td>1983</td>
<td>2.600 4.240 6.371</td>
<td>0.569 -0.417 -1.694</td>
</tr>
<tr>
<td>1984</td>
<td>2.796 4.411 6.523</td>
<td>2.605 3.906 5.609</td>
</tr>
<tr>
<td>1986</td>
<td>5.440 7.078 9.418</td>
<td>2.444 2.117 1.686</td>
</tr>
<tr>
<td>1987</td>
<td>4.310 5.685 7.551</td>
<td>2.670 2.681 2.092</td>
</tr>
<tr>
<td>1989</td>
<td>3.326 4.947 6.651</td>
<td>0.359 -0.296 -1.112</td>
</tr>
<tr>
<td>1990</td>
<td>3.166 4.817 6.855</td>
<td>2.716 3.304 3.659</td>
</tr>
</tbody>
</table>

Table 7-11. Average annual cost of covering $111,222 of mortgage value with one put option contract on financial futures (uncorrected hedge ratio for the lagged series)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>EX-ANTE COST</th>
<th>EX-POST COST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E&lt;F E=F E&gt;F</td>
<td>E&lt;F E=F E&gt;F</td>
</tr>
<tr>
<td>1983</td>
<td>2.338 3.812 5.728</td>
<td>0.512 -0.375 -1.523</td>
</tr>
<tr>
<td>1984</td>
<td>2.514 3.966 5.865</td>
<td>2.343 3.512 5.043</td>
</tr>
<tr>
<td>1986</td>
<td>4.891 6.364 8.468</td>
<td>2.197 1.903 1.516</td>
</tr>
<tr>
<td>1987</td>
<td>3.875 5.111 6.789</td>
<td>2.401 2.410 1.881</td>
</tr>
<tr>
<td>1989</td>
<td>2.990 4.448 5.980</td>
<td>0.323 -0.266 -1.000</td>
</tr>
<tr>
<td>1990</td>
<td>2.847 4.331 6.163</td>
<td>2.442 2.970 3.290</td>
</tr>
</tbody>
</table>

Table 7-12. Average annual cost of covering $125,109 of mortgage value with one put option contract on financial futures (corrected hedge ratio for the lagged series)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>EX-ANTE COST</th>
<th>EX-POST COST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E&lt;F E=F E&gt;F</td>
<td>E&lt;F E=F E&gt;F</td>
</tr>
<tr>
<td>1983</td>
<td>2.078 3.389 5.092</td>
<td>0.455 -0.333 -1.354</td>
</tr>
<tr>
<td>1985</td>
<td>3.188 4.555 6.173</td>
<td>2.871 3.934 5.151</td>
</tr>
<tr>
<td>1987</td>
<td>3.445 4.544 6.035</td>
<td>2.135 2.143 1.672</td>
</tr>
<tr>
<td>1989</td>
<td>2.658 3.954 5.316</td>
<td>0.287 -0.236 -0.889</td>
</tr>
<tr>
<td>1990</td>
<td>2.531 3.850 5.479</td>
<td>2.171 2.641 2.925</td>
</tr>
</tbody>
</table>
Table 7-13 summarizes the results for the purpose of comparing the FRM-ARM rate differential to the ex-post cost of hedging with at-the-money puts. The FRM series is the same as the one used in the previous empirical studies. The ARM is constructed very simply by adding a margin to the three-month Treasury bill rate and is used mainly for illustration purposes. No assumptions are made on various features of the ARM.

The results of the table are analyzed by performing the appropriate tests on the following three sets of hypothesis:

\[
\begin{align*}
H_0 : & \quad m_1 = m_2 \\
H_1 : & \quad m_1 \neq m_2
\end{align*}
\]

Table 7-13. Comparison of the FRM-ARM rate differential and the ex-post cost of hedging mortgage portfolio with at-the-money put options on T-bond futures

<table>
<thead>
<tr>
<th></th>
<th>FRM-ARM</th>
<th>EX-POST COST OF OPTION HEDGING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MARGIN</td>
<td>1.5 PERCENT</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5 PERCENT</td>
</tr>
<tr>
<td>1983</td>
<td>2.495</td>
<td>1.995</td>
</tr>
<tr>
<td>1984</td>
<td>2.344</td>
<td>1.844</td>
</tr>
<tr>
<td>1985</td>
<td>2.223</td>
<td>1.723</td>
</tr>
<tr>
<td>1986</td>
<td>2.098</td>
<td>1.593</td>
</tr>
<tr>
<td>1987</td>
<td>1.879</td>
<td>1.479</td>
</tr>
<tr>
<td>1989</td>
<td>1.768</td>
<td>1.268</td>
</tr>
<tr>
<td>1990</td>
<td>1.737</td>
<td>1.237</td>
</tr>
</tbody>
</table>

where \( m_1 \) is the mean cost of the hedging strategy and \( m_2 \) is the mean annual difference between the adjustable and fixed rates. The appropriate test statistics used was of the following form:

\[
t = \frac{(x_1-x_2) \cdot \sqrt{n_1+n_2-2}}{\sqrt{(n_1-1)s_1^2 + (n_2-1)s_2^2}/(n_1+n_2-2) \cdot [1/n_1+1/n_2]}^{1/2}
\]

where \( x_1 \) and \( x_2 \) are sample means, and \( s_1^2 \) and \( s_2^2 \) are sample variances.

The above tests were preceded by an analysis of variance test of the following form to validate the equality of population variances.
The statistic used was:

\[ F = \frac{s_1^2}{s_2^2} \]

where \( s_1^2 \) and \( s_2^2 \) are sample variances for the hedging cost and the FRM-ARM rate differentials respectively.

In cases where the null hypothesis of equal variances was rejected, the Mann-Whitney U-test of equal distribution with the following test statistic was used.\(^63\)

\[
U = n_1n_2 + \frac{[n_1(n_1+1)]}{2} - \text{sum}(R_i)
\]

\[
U' = n_1n_2 - U
\]

where \( n_1 \) and \( n_2 \) are the sample sizes, and \( \text{sum}(R_i) \) is the summation of the ranks of sample 1, the hedging cost.

The above tests were applied to the outcomes of the study with the following conclusions. The difference between the fixed mortgage rate and either of the adjustable rates were smaller than the cost of the one-to-one and uncorrected optimal hedges for 1984, 1985, and 1990 and equal for 1986 and 1987. The results for the corrected optimal hedge differed only by the fact that the difference between the FRM rate and the rate on the ARM with the 1.5 percent margin was larger than the hedging cost, although it was smaller if a 2 percent margin were used. On a monthly basis, the rate difference was smaller than the cost of hedging on 64 of the 96 periods for the 1.5 percent ARM and on 65 occasions for the 2 percent margin.

The above results suggest that the difference between the fixed and adjustable rate mortgage rates indeed are not larger than the cost of hedging FRM portfolios with put options. In other words, it may cost the mortgage banker more if option hedging were to replace hedging with ARMs. This may in fact point to the efficient pricing of the two mortgage types in relation to one-another. However, further study of the hypothesis is needed before conclusive statements may be made. There is a great deal of difference among adjustable mortgage loans. The existence of the annual and life-time interest rate caps and floors, for example, as well as the greater possibility of default for borrowers of ARMs may complicate the determinants of rate differentials between the two mortgage types. Moreover, hedging with put options may represent "over-insuring" the portfolio since the potential for gain in a declining rate environment is not eliminated with the use of this strategy as it would for the ARM hedging. The hedging cost under the current study, then, may be an over-estimation of the cost of converting the risk structure of a FRM portfolio to one similar to that of an ARM.
CHAPTER 8: SUMMARY AND CONCLUSIONS

The study of Chapter 6 suggests that hedging mortgage portfolios with financial futures and options is quite beneficial in improving the mean-standard deviation estimates.

For the combined series, a simulated sample with no trend in the price of either mortgages or futures despite substantial variation in both prices, the results show that assuming no broker fees, futures hedging lowers the variability of returns substantially although the mean return is identical to that of the unhedged mortgage, zero on average. Hedging with the sell-call strategy is also quite successful in lowering the standard deviation, and, it increases mean return by a large amount as well. The buy-put strategy for the period under study creates a loss for the benefit of lower variability in returns. Finally, the synthetic futures position, sell call-buy put, provides for returns similar to that of futures hedging although standard deviation estimates are slightly higher.

For both the put or call strategies, the employment of the in-the-money option magnifies the return outcomes, higher gains for the call strategy and larger losses for the put position. Selling the in-the-money call, however, lowers the variation of returns while buying the put with larger exercise price increases the variability as compared to the at-the-money option. The buy-put sell call strategy is not recommended for other than at-the-money options since the resulting standard deviation is higher than that of the unhedged position.

In general, hedging with T-bond futures and options on such futures contracts, lowers the variability of return in all cases of hedging, for both a contemporaneous or a lagged (anticipatory) hedge, and for at-the-money, as well the next near-the-money options with the exception of near-money synthetic futures hedges.
The original series of mortgage and futures prices represented a declining interest rate environment and rising mortgage values. Futures hedging under this scenario lowered potential gains for the benefit of lower variation of returns. Mean return was much closer to zero when the mortgage portfolio was hedged with futures and the standard deviation of returns were much lower.

The call strategy provided for both a considerable decrease in variation and an increase in returns. In fact, as the position diagram predicted, there was a positive return in the case of the sell-call hedge for both hedge relationships, and all exercise prices and hedge ratios. The standard deviation of returns were also lower under all circumstances, by as much as 63 percent in one case. Moreover, the call strategy also out-performed simple futures hedging in all cases, both in terms of mean return and standard deviation of returns.

The put strategy modified the potential gains in this case by the extent of the paid premium. Returns were generally lower than that of the unhedged and the hedged-with-futures positions. Variability of returns were always smaller as compared to the unhedged mortgage, but showed various comparative results against the futures hedge. The standard deviation of returns under the buy-put hedge of the original series were between 25 percent lower and 23 percent higher than that of the mortgages hedged with futures. On average for both relationships and all hedge ratios, however, the standard deviation of returns were quite similar for both strategies.

Finally, the synthetic futures position created comparative or better returns than the simple futures hedge, but also resulted in higher standard deviation.

So, based on the current study, when interest rates are expected to fall leading to potentially higher mortgage and futures prices, the four hedging strategies are successful in lowering the risk as measured by
standard deviation of returns. Moreover, selling calls, or selling calls combined with buying puts also improve returns while selling futures or put options on futures contracts lowers the mean return.

In the case of rising rates and falling values, as represented by the reverse series, the futures hedge performed similar to the case of the original series in the magnitude of changes, but with the opposite signs for mean returns. In other words, standard deviations were lower in all cases by the amount equal to the decrease for the original series, but the returns were increased and were near zero. The returns for the mortgages hedged with call options were better than those of the unhedged mortgage portfolios because the premium received cushioned the drop in value, and standard deviations were lower. When compared with the futures hedge, however, mean returns were generally higher, but so were the variability of those returns. The put strategy created large losses, generally in line with the average premium paid, but it also lowered the variability of returns by as much as 41 percent compared to the unhedged mortgage and 14 percent compared to the futures hedge. The synthetic futures position lowered the mean return but increased the standard deviation as compared to hedged-with-futures case.

It may be concluded from the result of the empirical study of Chapter 6 that hedging mortgages with Treasury bond futures, and option on T.bond futures contracts, is generally helpful in improving the risk-return distributions. The hedging choice depends on the risk attitude of the investor and the directions of price (interest rate) movements. Of the four strategies considered in the current study, the futures choice seems to perform equally well under both decreasing and increasing interest rate scenarios. The call strategy performs better that the futures position both in terms of mean and variability of returns when interest rates decline, but improves the returns only at the expense of higher standard deviations when rates rise. The put strategy, on the other hand, limits
maximum losses to the extent of the premium paid, but lowers the variation in returns considerably when rates increase as compared to all other strategies. The sell call-buy put strategy performs much better, in terms of mean return, than the simple futures hedge if rates decline and much worse if interest rates increase. On average, however, the mean return from the two strategies are quite comparable, but the synthetic futures results in higher variability of returns.

The empirical study of Chapter 6 showed the benefits of hedging mortgage portfolios with purchasing put options on financial futures in an increasing interest rate environment. The position diagrams further suggested that the risk structure of a FRM may be manipulated to resemble that of an ARM with the buy-put strategy. If the S&L aims in deterring the risk of lower mortgage value, this hedging strategy seems quite suitable for the purpose.

The results of the empirical study of Chapter 7 even further explored this idea. The FRM portfolio was subjected to random changes in value in response to simulated interest rate variations.

The worst case scenario, when interest rates increased by the maximum allowed, shows a substantial decline in the value of the unhedged mortgage portfolio, with little variability, however, due to the constant nature of the changes. Both the call and the put strategies reduced the losses by a large amount for a small increase in the standard deviation. In the declining price environment, the put strategy did better than the call as expected from the theory and the position diagrams. The synthetic futures created a large gain. The use of the in-the-money options improved the returns although it also increased the standard deviation of returns.

The best case scenario of maximum declines in interest rates also resulted in expected outcomes. Large gains of the mortgage portfolio were modified by the put and call options. In this case, however, in-the-money options, with substantially higher average premia, lowered the return as
compared to the at-the-money option. Variability of returns were higher in all cases.

The two extreme rate change examples set the limit on gains or losses of the hedged and unhedged mortgage portfolios, while the general case of interest rate change further examined the hedging strategies. Standard deviation of returns were lowered considerably for all three hedging and for all exercise prices, in one case by as much as 70 percent. When used individually, the sell-call or buy-put strategies performed similarly in lowering the variability of returns, while the call position created a profit and the put caused a loss. The synthetic futures position was most successful in lowering the standard deviation of returns and generated returns that were significantly near-zero as compared to the other two strategies.

Although the above exercise showed that all three strategies perform well as hedging instruments, the buy-put strategy was pursued in the remainder of Chapter 7 where the pricing of the two mortgage types and their relationship were considered. The reason for choosing this particular hedging strategy was the fact that the financial institutions' primary goal, in this case, was to protect their mortgage value from falling, and this purpose is best served with a buy-put option strategy as was predicted by position diagrams and proved by the empirical study.

Mortgage pricing techniques follow either the options pricing principles based on the claims given up by the lenders or the borrowers; or simulation methods that perform "what if" experiments on ARMs based on assumptions of rate change. The literature examined in this study pointed to the fact that due to substantial similarities that exist between capped ARMs and FRMs with large up-front fees to prohibit refinancing, the fundamental determinants of the spread between the fixed and adjustable mortgage rates and their relationship to short term market rates of interest are the same.
The last part of the empirical work in the current study examined the difference between the fixed and adjustable mortgage rates in terms of the hedging cost of "converting" the FRM to an ARM with regard to its interest rate risk characteristics. The ex-post cost of the hedge, option premia adjusted by gains from the exercise, was compared to the difference between the fixed and adjustable mortgage rates. Several hypothesis regarding the two costs were tested on an annual and monthly basis. The results suggested that the rate differential between the two mortgage rates is generally not larger than the cost of hedging.
REFERENCES


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