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Optimal Hedging under Forward-Looking Behavior

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OPTIMAL HEDGING UNDER FORWARD-LOOKING BEHAVIOR

Introduction

With rare exceptions, previous work on hedging behavior has assumed a single production cycle. This implicitly assumes that the firm is *myopic* because such a firm is not concerned about events that occur after the end of the current production cycle. This assumption has been carried over from the risk and uncertainty literature and can be justified on the basis of simplicity. Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) applied Sandmo's (1971) model of the myopic firm under uncertainty to analyze the behavior of the firm in the presence of a forward market for output. They showed that the competitive risk-averse firm separates production from hedging decisions. They also proved that it is optimal to place a full hedge (i.e., to short hedge the entire production) if the forward price is unbiased. Otherwise, it is optimal to short hedge more (less) than total output when the forward price is greater than (less than) the expected cash price.

A straightforward consequence of full-hedge optimality under unbiased forward prices is that most farmers should place full hedges most of the time based on empirical evidence that futures prices are not significantly biased [Telser (1958), Gray (1961), Just and Rausser (1981), Martin and Garcia (1981)]. But not all farmers hedge all of their output. Extensions to the myopic hedging model have been proposed that would explain this behavior. These studies include the introduction of production risk [Chavas and Pope (1982), Losq (1982), Honda (1983), Grant (1985)], basis risk [Batlin (1983), Paroush and Wolf (1989)], hedging costs [Chavas and Pope (1982)], hedging restrictions [Antonovitz and Roe (1986), Antonovitz and Nelson (1988)], and imperfect markets [Katz (1984)].

There are instances, however, for which the myopic assumption may lead to faulty conclusions about optimal hedging behavior. An example will help make this observation clear. Consider a firm only involved in speculative storage. Because no physical transformation of the commodity occurs, the price of ending inventories at one period will equal the input price for the following period. Low output prices will then imply low input prices for subsequent storage. The positive (negative) effect of low (high) input prices for next period storage offsets the negative (positive) effect of low (high) output prices for this period storage, thereby reducing the need to hedge this period storage.

Forward-looking hedging behavior was analyzed theoretically by Anderson and Danthine (1983) and Hey (1987). Anderson and Danthine allowed the firm to revise its hedging decisions during the production cycle but assumed a single production cycle. They showed that forward-looking producers should separate production and hedging decisions but that producers should not hedge all of their output if the futures price is unbiased. Hey allowed for more than one production cycle and found that separation and suboptimality of full hedging hold. Hey's model was different from the one developed here because he assumed that (a) intertemporal utility is additive, (b) output cash prices are independently distributed and follow a constant distribution, and (c) sales decisions are taken after production and hedging decisions rather than simultaneously. Hey's results depend crucially on the sequential timing he imposed on sales, production, and hedging decisions.

The purpose of this study is to formally demonstrate that the forward-looking optimal hedge is different from the myopic optimal hedge. We postulate a risk-averse firm that maximizes expected utility of terminal wealth and derive some propositions regarding optimal hedging behavior under both myopic and forward-looking hypotheses. Because the correlation between output and input prices is most obvious for speculative storage, we first present results for the speculative firm that only stores and then for the firm that is involved in production and does not store. The last section reports the main conclusions from the analysis.

A Theoretical Model of a Speculative Storing Competitive Firm

Consider a competitive firm with a twice continuously differentiable von Neumann-Morgenstern utility function and assume that utility is strictly concave in its argument terminal wealth $[U(W_T), U'(W_T) > 0, U''(W_T) < 0]$. Terminal wealth is

$$(1.1)\quad W_T = r_{-1}\, r_0\, r_1 \dots r_{T-1}\, W_{-1} + r_0\, r_1 \dots r_{T-1}\, \pi_0 + r_1 \dots r_{T-1}\, \pi_1 + \dots + r_{T-1}\, \pi_{T-1} + \pi_T,$$

where W_t denotes monetary wealth at the end of trading date t, π_t is cash flow at time t, and r_t equals one plus the one-period interest rate prevailing at t. The interest rate need not be constant over time, but it is not random. At each trading date $0 \le t < T$, the firm can borrow and lend unlimited amounts of money for one period at the prevailing interest rate.

Input price randomness plays a key role in the model. But because of the mathematical complexity, little can be accomplished when we allow all prices to vary. We address this issue in two ways. First, we examine optimal behavior for a firm whose only productive activity is *speculative* storage. Because this firm sells the same product that it buys, the model is greatly simplified, allowing us to develop the intuition required for the second approach in which we introduce production. We solve the second approach by imposing some realistic restrictions on the technology set.

Consider the case of the firm whose activities are storing a certain commodity to profit from its resale and trading in a forward market for this product.[1] At any date t there are only two forward positions that can be negotiated: one for delivery at t+1 and the other for immediate delivery (i.e., delivery at t).[2] We use F_t to denote the net short forward position for delivery at time t+1 open at date t. There are no restrictions on the amount or sign of the forward position held by the firm, except that the firm cannot have an open position for delivery at date T+1 at the end of the terminal trading date ($F_T = 0$), and that the firm cannot hold an open position for delivery at time t at the end of trading date t ($F_{t,t} = - F_{t-1,t}$, where the first subscript denotes the opening date and the second denotes the delivery date).[3] The cash flow from opening a forward

[1]Employing a forward instead of a futures market allows us to isolate the effect of forward-looking behavior from that of basis risk.

[2]We do not require actual *delivery*, but we still use this term for clarity of exposition. Forward commitments may be canceled either by delivering the good or by undertaking an opposite transaction in the forward market.

[3]This means that at any date $0 \le t < T$ firms have only one free choice regarding the two tradable positions in the forward market. This choice is how much to commit for delivery (or receipt) at t+1.

contract lags by one period because forward trades do not create cash flows until open positions are liquidated. The forward price prevailing at t for immediate delivery is identical to the current cash price (p_t). The forward price at t for delivery at the following date t+1 (denoted by f_t), however, will be generally different from the current cash price p_t.

Under the above specifications, the firm's cash flow at any date $t \leq T$ is represented by

$$(1.2) \quad \pi_t = p_t P_t - i(I_t - P_t) + (f_{t-1} - p_t) F_{t-1} \qquad \text{s.t.} \quad I_{t+1} = I_t - P_t \geq 0,$$

where P_t represents product sales at date t, $i(\cdot)$ is a strictly convex storage cost function such that $i'(\cdot) > 0$, and I_t is beginning inventory at date t. Positive sales means that the firm sells from beginning stocks, whereas negative sales means that the firm buys to store and sell at a later date. Sales cannot exceed beginning inventory (i.e., $P_t \leq I_t$). The amount $(I_t - P_t)$ is the unsold beginning inventory at date t, which is carried over at nonrandom storage cost $i(I_t - P_t)$ to become beginning inventory at time t+1 (I_{t+1}).

We hypothesize that at any date $0 \leq t \leq T$ the firm selects the levels of sales (P_t) and hedging (F_t) that maximize expected utility of terminal wealth, given the available information. Optimal decisions at the current date t = 0 solve the following set of recursive equations

$$(1.3) \quad M_T[r_{T-1} W_{T-1} + (f_{T-1} - p_T) F_{T-1}, I_T; \mathbf{p}_T]$$

$$= \max_{\mathbf{d}_T} U[r_{T-1} W_{T-1} + p_T P_T - i(I_T - P_T) + (f_{T-1} - p_T) F_{T-1}],$$

$$(1.4) \quad M_t\{r_t \dots r_{T-1} [r_{t-1} W_{t-1} + (f_{t-1} - p_t) F_{t-1}], I_t; \mathbf{p}_t\}$$

$$= \max_{\mathbf{d}_t} E_t\{M_{t+1}[r_{t+1} \dots r_{T-1} (r_t W_t + (f_t - p_{t+1}) F_t), I_{t+1}; \mathbf{p}_{t+1}]\}, \; t = 0, 1, \dots, T-1,$$

where: $\mathbf{d}_t = (P_t, F_t)$ if $0 \leq t < T$, $\mathbf{d}_T = (P_T, 0)$,

$\mathbf{p}_t = (p_t, f_t),$

$\underline{\mathbf{p}}_t = (\mathbf{p}_0, \ldots, \mathbf{p}_t).$

$E_t(\cdot)$ denotes the expectation operator based on information available at t, the vector $\mathbf{d}_t$ contains the firm's decision variables corresponding to date t, and the matrix $\underline{\mathbf{p}}_t$ comprises the cash and futures prices up to (and including) time t. Terminal wealth is as defined in (1.1), and cash flows are given by (1.2). The solution to the problem summarized by expressions (1.3) and (1.4) can be obtained by recursively solving the Lagrangian functions

(1.5) $£_T = U[r_{T-1} W_{T-1} + p_T P_T - i(I_T - P_T) + (f_{T-1} - p_T) F_{T-1}] + \eta_T (I_T - P_T),$

(1.6) $£_t = E_t\{M_{t+1}[r_{t+1} \ldots r_{T-1} (r_t W_t + (f_t - p_{t+1}) F_t), I_{t+1}; \underline{\mathbf{p}}_{t+1}]\} + \eta_t (I_t - P_t), \ t = 0, 1,\ldots, T\text{-}1,$

where η_t is the Lagrangian multiplier.

The first-order conditions (FOCs) corresponding to the terminal date (t = T) are[4]

(1.7) $$\frac{\partial £_T}{\partial P_T} = (p_T + i') M_T' - \eta_T = 0,$$

(1.8) $$\frac{\partial £_T}{\partial \eta_T} = I_T - P_T \geq 0, \eta_T \geq 0, \eta_T \frac{\partial £_T}{\partial \eta_T} = 0,$$

where M_T' denotes U' evaluated at the optimum. Condition (1.7) requires that the Lagrangian multiplier (η_T) be strictly positive because $(p_T + i') M_T' > 0$. Hence, $(I_T - P_T)$ must equal zero to satisfy the Kuhn-Tucker condition (1.8), i.e., the optimal sales policy at the terminal date is to sell all beginning inventories ($P_T = I_T$). Therefore, $\mathbf{d}_T = (I_T, 0)$, the optimal cash flow for the terminal date reduces to $\pi_T = p_T I_T + (f_{T-1} - p_T) F_{T-1}$, and the maximum value function is

[4]Recall that $F_T = 0$ by assumption.

(1.9) $M_T[r_{T-1} W_{T-1} + (f_{T-1} - p_T) F_{T-1}, I_T; \mathbf{p}_T] = U[r_{T-1} W_{T-1} + p_T I_T + (f_{T-1} - p_T) F_{T-1}]$.

The FOCs for dates previous to the terminal time ($0 \leq t < T$) are (see Appendix A)

(1.10) $$\frac{\partial \pounds_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} [r_t (p_t + i') M_t' - E_t(p_{t+1} M_{t+1}')] - \eta_t = 0,$$

(1.11) $$\frac{\partial \pounds_t}{\partial F_t} = r_{t+1} \cdots r_{T-1} [f_t M_t' - E_t(p_{t+1} M_{t+1}')] = 0,$$

(1.12) $$\frac{\partial \pounds_t}{\partial \eta_t} = I_t - P_t \geq 0, \eta_t \geq 0, \eta_t \frac{\partial \pounds_t}{\partial \eta_t} = 0,$$

where $M_t' = E_t(M_{t+1}')$ evaluated at the optimum corresponding to date t (note that $M_t' > 0$). The solution to expressions (1.10) through (1.12) is a unique absolute constrained maximum because the objective function is strictly concave, and the constraint set is convex.[5] Expressions (1.9) through (1.12) provide the framework needed to analyze the behavior of the forward-looking risk-averse firm.

Before proceeding with the analysis, it is helpful to better define myopic and forward-looking behavior. A myopic decision maker cares only about two dates: the present and one date in the future. Such an agent neglects the possibility of updating and/or taking decisions at all other times. Hence, in our notation the myopic firm is one making decisions at date T-1. Myopic behavior is inconsistent if the decision maker stays in the market for more than two trading dates. At the first trading date (T_1-1), the myopic agent behaves as if the next trading date (T_1) is the last one. But when date T_1 arrives and the agent decides to stay in the market for one more trading time (denoted as T_2), T_1 becomes the trading date preceding the (new) terminal time T_2, i.e., T_1 is

[5]We will assume for the remainder of the analysis that the solution to (1.3) and (1.4) exists. The conditions for this are given in Bertsekas (1976, p. 375).

now T_2-1. In other words, T_1 is the terminal date from the perspective of T_1-1, but it is not terminal when the firm is at T_1.

The inconsistency of myopic behavior arises as follows. By FOCs (1.7) and (1.8), we know that $E_{T_1-1}(P_{T_1}) = I_{T_1}$, and by FOCs (1.10) through (1.12) we know that there is a positive probability of $P_{T_2-1} < I_{T_2-1}$ and that it is impossible to have $P_{T_2-1} > I_{T_2-1}$. But decision dates T_1 and T_2-1 correspond to the same *calendar* date, hence the expectation of next-period sales is always upwardly biased. The bias occurs because the firm expects next-date sales to match beginning inventories, but when the next date actually arrives the firm sometimes finds it optimal not to sell the entire beginning stock. In contrast, a forward-looking (or nonmyopic) firm is any firm making decisions at $t < T-1$ and that cares about at least two dates in the future at which time it will revise decisions.

To summarize, a myopic firm can be defined as one whose *planning horizon* is the same as its *decision horizon* [Merton (1982, p. 656)].[6] A forward-looking firm, in contrast, is one whose planning horizon comprises at least two decision horizons. There are striking differences in the qualitative behavior of forward-looking firms compared to myopic firms, and this is the issue addressed in most of what follows.

It is necessary to know the determinants of the optimal physical decisions (i.e., the variables that affect optimal storage I_1 or, equivalently, optimal sales P_0). The main results regarding this issue are summarized in Proposition 1.

PROPOSITION I: STORAGE AND SALES BEHAVIOR. *In the presence of a forward market, optimal storage (or sales) for a risk-averse firm is determined independently from the subjective joint distribution of random variables, from the decision maker's degree of risk aversion, and from the optimal hedging decision. If positive, optimal storage is such that discounted current forward*

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[6]According to Merton, the planning horizon "is the maximum length of time for which the investor gives any weight in his utility function," and decision horizon is "the length of time between which the investor makes successive decisions, and it is the minimum time between which he would take any action."

price equals current cash price plus marginal storage cost. These results hold for both myopic and forward-looking firms.

Proof. According to FOC (1.11), at the optimum the equality

(1.13) $E_0(p_1 M_1') = f_0 M_0'$

holds. Substituting (1.13) into FOC (1.10) and rearranging yields

(1.14) $f_0 - r_0 [p_0 + i'(I_1)] = - \eta_0/(r_1 \ldots r_{T-1} M_0')$.

Hence:

a. If $f_0 < r_0 [p_0 + i'(0)]$, then $\eta_0 > 0$, and therefore $I_1 = 0$.

b. If $f_0 = r_0 [p_0 + i'(0)]$, then $\eta_0 = I_1 = 0$.

c. If $f_0 > r_0 [p_0 + i'(0)]$, then $\eta_0 = 0$, and therefore $I_1 > 0$ satisfying $f_0 = r_0 [p_0 + i'(I_1)]$. Q.E.D.

Proposition 1 shows that separation between physical and hedging decisions is a robust result because it holds for both myopic and forward-looking decision makers. Our findings extend those by Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) to the forward-looking scenario, relaxing the simplifying assumptions used by Hey (1987). Optimal storage (and sales) behavior is completely characterized in the proof to Proposition 1, and comparative statics follow easily from total differentiation of $f_0 - r_0 [p_0 + i'(I_1)] = 0$.[7]

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[7]Note that in the forward-looking scenario we cannot use jointly normally distributed prices to justify mean-variance analysis. The quantities stored after the current date (i.e., $I_2, \ldots, I_T$) are random but cannot follow a normal distribution because firms do not store if $f_t < r_t [p_t + i'(0)]$.

We turn now to the focus of this paper, i.e., the characterization of the optimal hedge. To this end, it is useful to rewrite one of the components of FOC (1.11) in an alternative way, namely:[8]

$$(1.15)\quad E_0(p_1\, M_1') = E_0[p_1\, E_0(M_1'|p_1)]$$

$$= E_0(p_1)\, E_0(M_1') + Cov[p_1, E_0(M_1'|p_1)],$$

where: $E_0(M_1'|p_1) = \int_{f_1} M_1' f_1(f_1|\mathbf{p}_0, p_1)\, df_1 > 0,$

$f_1(f_1|\mathbf{p}_0, p_1)$ = conditional density function of f_1, given $(\mathbf{p}_0, p_1)$.

Employing (1.15) and $E_t(M_{t+1}') = M_t'$, we can state FOC (1.11) as follows:

$$(1.16)\quad [f_0 - E_0(p_1)]\, M_0' = Cov[p_1, E_0(M_1'|p_1)].$$

Inspection of the sign of the covariance term in expression (1.16) will allow us to prove the results summarized in Propositions 2 and 3. To show Proposition 6, we will use the following model of cash price behavior:

$$(1.17)\quad p_t = \alpha + \beta\, p_{t-1} + e_t,\ 0 \le \beta \le 1,\ e_t \text{ i.i.d. zero-mean random variable.}$$

Expression (1.17) nests the cases of serially independent prices ($\beta = 0$), random walk ($\beta = 1$), and autoregressive process of order 1 ($0 < \beta < 1$). In addition, by unbiased forward prices we will mean that forward prices are always unbiased, i.e., $E_t(p_{t+1}) = f_t$, t = 0,..., T-1.

[8]Recall that for any pair of random variables x and y, $E(x\, y) = E(x)\, E(y) + Cov(x, y)$.

PROPOSITION 2: MYOPIC STORAGE HEDGE. *The optimal hedge for a myopic risk-averse firm that perceives the forward price to be unbiased is to (short) sell forward the total amount stored. This hedge is independent from the myopic firm's degree of risk aversion.*

Proof. According to (1.16), at the optimum, $[f_0 - E_0(p_1)]$ and $\text{Cov}[p_1, E_0(M_1'|p_1)]$ must bear equal signs because $M_t' > 0$. In particular, $\text{Cov}[p_1, E_0(M_1'|p_1)] = 0$ if $f_0 = E_0(p_1)$. For the myopic firm date 0 is T-1, and from (1.9) we have $E_{T-1}(M_T'|p_T) = M_T'$. Then,

$$(1.18)\quad \frac{\partial E_{T-1}(M_T'|p_T)}{\partial p_T} = (I_T - F_{T-1})\, M_T'' \gtrless 0 \text{ as } F_{T-1} \gtrless I_T$$

because $M_T'' < 0$. But p_T is monotonically increasing in p_T and $E_{T-1}(M_T'|p_T)$ is monotonically increasing (decreasing) in p_T if $F_{T-1} > (<)\, I_T$. Hence, applying Theorem 43 in Hardy, Littlewood, and Pólya (1967) we obtain

$$(1.19)\quad \text{Cov}[p_T, E_{T-1}(M_T'|p_T)] \gtrless 0 \text{ as } F_{T-1} \gtrless I_T.$$

In particular, if $f_{T-1} = E_{T-1}(p_T)$, it must be true that $F_{T-1} = I_T$. Q.E.D.

PROPOSITION 3: FORWARD-LOOKING STORAGE HEDGE. *(1) The optimal hedge for a forward-looking risk-averse firm that perceives the forward price to be unbiased is not necessarily to sell forward the entire quantity stored. Furthermore, the optimal forward-looking hedge depends upon the firm's degree of risk aversion.*

(2) If the firm is constant absolute risk averse (CARA) and cash prices behave as in expression (1.17), then the optimal forward-looking hedge under unbiased forward prices is strictly smaller than the entire amount stored.

Proof. We show only part (2) of Proposition 3 because it implies part (1). For a forward-looking CARA firm with cash prices behaving as in (1.17) and unbiased forward prices, we get

$$(1.20)\quad \frac{\partial E_0(M_1'|p_1)}{\partial p_1} = r_1 \dots r_{T-1}\,(I_1 - F_0)\,E_0(M_1''|p_1) - r_2 \dots r_{T-1}\,(r_1 - \beta)\,E_0(I_2\,M_1''|p_1)$$

$$- r_3 \dots r_{T-1}\,(r_2 - \beta)\,\beta\,E_0(I_3\,M_2''|p_1) - \dots - r_{t+1} \dots r_{T-1}\,(r_t - \beta)\,\beta^{t-1}\,E_0(I_{t+1}\,M_t''|p_1)$$

$$- \dots - (r_{T-1} - \beta)\,\beta^{T-2}\,E_0(I_T\,M_{T-1}''|p_1),$$

where M_T'' equals $U'' < 0$ evaluated at the optimum and M_t'' denotes $E_t(M_{t+1}'')$ evaluated at the optimum. Expression (1.20') follows from (1.20) because the other terms in the right-hand side of (1.20) vanish (see Appendix B).

Assume that $I_1 \leq F_0$. Then $\partial(M_1'|p_1)/\partial p_1 > 0$ because $E_0(M_1''|p_1) < 0$, $(r_t - \beta) > 0$, and $E_0(I_{t+1}\,M_t''|p_1) < 0$.[9] But if $\partial(M_1'|p_1)/\partial p_1 > 0$ then $\mathrm{Cov}[p_1, E_0(M_1'|p_1)] > 0$, which violates FOC (1.16) under unbiased forward prices. Therefore, it must be true that $I_1 > F_0$. Q.E.D.

The results reported in Proposition 2 are analogous to those obtained by Holthausen (1979) and Feder, Just, and Schmitz (1980) and demonstrate that our model is consistent with the standard literature. Our findings about the optimal forward-looking hedge (Proposition 3) reveal that full-hedge optimality under unbiased forward price is not a robust result. From Propositions 2

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[9]The proof of $E_0(I_2\,M_1''|\,p_1) < 0$ follows from the fact that $I_2 = 0$ if $f_1 \leq r_1\,[p_1 + i'(0)]$, and $I_2 > 0$ otherwise (see proof of Proposition 1). Hence,

$$E_0(I_2\,M_1''|\,p_1) = E_0[I_2\,M_1''|\,p_1 \geq f_1/r_1 - i'(0),\,]\,\mathrm{Prob}_0[p_1 \geq f_1/r_1 - i'(0)]$$

$$+ E_0[I_2\,M_1''|\,p_1 < f_1/r_1 - i'(0)]\,\mathrm{Prob}_0[p_1 < f_1/r_1 - i'(0)]$$

$$= E_0[I_2\,M_1''|\,p_1 < f_1/r_1 - i'(0)]\,\mathrm{Prob}_0[p_1 < f_1/r_1 - i'(0)],$$

where $\mathrm{Prob}_t(\cdot)$ is the probability of $(\cdot)$, given the information at date t. Therefore, $E_0(I_2\,M_1''|\,p_1) > 0$ {unless $\mathrm{Prob}_0[p_1 < f_1/r_1 - i'(0)] = 0$}. The proof of $E_0(I_{t+1}\,M_t''|\,p_1) < 0$ for $1 < t \leq T-1$ is analogous.

and 3, it is clear that the simplicity of the optimal myopic hedge under unbiased forward price is attributable to the myopic firm assuming with certainty that whatever it stores now will be completely sold in the next trading time and that it will store nothing at the next date (i.e., $I_{2=T+1} = 0$). Also, the myopic firm does not plan to hedge at the next trading time (i.e., $F_{1=T} = 0$). In contrast, the forward-looking firm assigns a positive probability to storing and/or trading forward at the next trading date. But next-date optimal storage and hedge are correlated with next-date cash price, and therefore they serve as partial substitutes for current hedging. It is this substitution effect that leads to full-hedge suboptimality in the forward-looking scenario.

An alternative interpretation of the full-hedge suboptimality result is that we have formalized a common behavioral pattern known as *anticipatory* hedging. The firm may operate in the forward market to speculate and/or to place two types of hedges, namely *risk-avoidance* and anticipatory hedges.[10] If the forward price is unbiased, the firm does not speculate and trades forward only to hedge. The risk-avoidance hedge consists of selling current storage forward to reduce its price risk, whereas the anticipatory hedge is placed to avoid the price risk of next-date storage. Therefore, the risk-avoidance hedge is identical to current storage. In contrast, the size of the anticipatory hedge depends, among other factors, on the distribution of the random quantities stored and hedged at next-date, the agent's degree of risk aversion, and the joint distribution of random prices. Hence, the sum of risk-avoidance and anticipatory hedges generally differs from current storage and depends on the degree of risk aversion. This is true unless the firm currently knows exactly how much it will store and hedge at next-date, so next-date storage and hedge are nonrandom. The myopic case is an example of the latter situation in which next-date storage and hedge are known to be exactly zero ($I_{2=T+1} = F_{1=T} = 0$).

In Figure 1, we show the magnitudes of the optimal storage and hedging positions for a forward-looking firm at decision date T-2 (which corresponds to calendar time t_0). In this example, it is assumed that prices at calendar times t_1 and t_2 (i.e., decision dates T-1 and T,

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[10]Risk-avoidance and anticipatory hedges are defined in Marshall (1989, p. 198).

respectively) have the discrete distributions reported in Table 1. Optimal decisions are then found by numerical maximization of the expected value of utility, where the utility function is $[-\exp(-\lambda W_T)]$. To simplify the analysis, it is assumed that the storage cost function is quadratic $[i(I_{t+1}) = 0.001\, I_{t+1}^2]$ and that the interest rate is zero (i.e., $r_{t_0} = r_{t_1} = 1$). Figure 1 illustrates the case of $p_{t_0} = 9$ and unbiased forward price (i.e., $f_{t_0} = 10$). For this reasonable scenario, the magnitude of the optimal forward-looking hedge lies between 78 percent and 93 percent of the optimal storage for slightly and severely risk-averse decision makers ($\lambda = 0.0001$ and $\lambda = 0.01$, respectively).[11] This result contrasts with that of the optimal myopic hedge, which equals 100 percent of the optimal storage, irrespective of the degree of absolute risk aversion.

The general suboptimality of the full hedge under unbiased forward prices is an important result. It is widely accepted that full hedging is optimal when the forward price is unbiased. The full hedge is appealing because of its simplicity. Also, its normative content is easy and broadly applicable because it makes complete abstraction of the agent's degree of risk aversion. Our model shows that, despite these appealing characteristics, full-hedge optimality depends crucially upon assuming myopic behavior and/or independence of output and material input prices.

Given the previous discussion, it is easier to understand why the full hedge overestimates the optimal forward-looking hedge under unbiased forward prices when the firm is CARA and cash prices behave as in (1.17). The value of ending inventories is negatively associated with input costs for next period storage; therefore, next-date storage reduces the cash price risk associated with current storage: revenue from current storage will be low if p_1 is low, but then the firm will be able to buy material input to store at a low price, thereby partly offsetting the lower revenue. This means that next-date storage is an imperfect substitute for current hedging (when viewed in the context of multiperiod profitability), so that the hedge required to achieve a certain level of reduction in next-date cash price risk is smaller than it would be if next-date storage did not contribute to risk reduction.

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[11] As $\lambda \to \infty$, the optimal hedge converges asymptotically to 489.58 from below.

In general, for CARA firms we can express $\partial E_0(M_1'|p_1)/\partial p_1$ as

$$(1.21)\quad \frac{\partial E_0(M_1'|p_1)}{\partial p_1} = r_1 \ldots r_{T-1}\,(I_1 - F_0)\,E_0(M_1''|p_1) - r_1 \ldots r_{T-1}\,E_0(I_2\,M_1''|p_1)$$

$$- \int_{f_1} M_1' \,\frac{\partial f_1(f_1|\mathbf{p}_0, p_1)}{\partial p_1}\, df_1 - \int_{f_1} [\int_{\mathbf{p}_2} M_2' \,\frac{\partial h_2(\mathbf{p}_2|\underline{\mathbf{p}}_1)}{\partial p_1}\, d\mathbf{p}_2]\, f_1(f_1|\mathbf{p}_0, p_1)\, df_1$$

$$- \ldots - \int_{f_1} \{ \int_{\mathbf{p}_2} [\int_{\mathbf{p}_3} \ldots \int_{\mathbf{p}_{T-1}} (\int_{\mathbf{p}_T} M_T' \,\frac{\partial h_T(\mathbf{p}_T|\underline{\mathbf{p}}_{T-1})}{\partial p_1}\, d\mathbf{p}_T)$$

$$h_{T-1}(\mathbf{p}_{T-1}|\underline{\mathbf{p}}_{T-2})\, d\mathbf{p}_{T-1} \ldots h_3(\mathbf{p}_3|\underline{\mathbf{p}}_2)\, d\mathbf{p}_3]\, h_2(\mathbf{p}_2|\underline{\mathbf{p}}_1)\, d\mathbf{p}_2\}\, f_1(f_1|\mathbf{p}_0, p_1)\, df_1,$$

where $h_{t+1}(\mathbf{p}_{t+1}|\underline{\mathbf{p}}_t)$ is the conditional density function of p_{t+1} and f_{t+1}, given $\underline{\mathbf{p}}_t$. Expression (1.21) is helpful in that it allows us to separate clearly the two main components of the optimal forward-looking hedge. The first term in the right-hand side of (I.21) is the risk-avoidance component, whereas the other terms are the anticipatory component. The risk-avoidance term vanishes if $F_0 = I_1$. The anticipatory component can be further divided into *direct* and *indirect* anticipation terms (i.e., the second and the remaining terms in the right-hand side of (1.21), respectively). The direct anticipation component is due to the effect of next-date storage (I_2). The direct anticipation term is strictly positive, irrespective of risk attitudes or price distributions, and requires a long hedge ($F_0 < 0$) to equal zero. Finally, the indirect anticipation component involves the impact on current hedging attributable to the interaction between the risk attitude and the price distribution, and it has an ambiguous sign.

The indirect anticipation component vanishes when forward prices are unbiased and cash prices behave as in expression (1.17). If forward prices at future decision dates are allowed to be biased, the optimal forward-looking CARA hedge may be larger than the amount stored, even if the current forward price is unbiased. This may happen because next-date cash price indirectly affects the current hedge through its relationship with next-date forward price. The sign and magnitude of this indirect effect depends on the size of the next-date hedge, which in turn may be positive or negative and large enough to cause the current hedge to exceed the amount stored.

It is interesting to note that when cash prices behave as in (1.17), the optimal forward-looking hedge under unbiased forward prices is strictly negative if nothing is stored (e.g., $F_0 < I_1 = 0$), and the forward-looking firm establishes a long forward position. In contrast, the optimal myopic hedge in the same situation is $F_0 = 0$. This is an interesting result because it explains the existence of anticipatory hedging under unbiased forward prices without resorting to any ad-hoc assumptions. In the standard myopic framework, anticipatory hedging is modeled by assuming that the firm currently knows exactly how much it will store and hedge at the next date. This assumption is clearly inconsistent. If the firm is myopic, we have shown that it is suboptimal to expect next-date sales to be anything less than beginning stocks. If the firm is forward-looking but knows next-date storage and hedge with certainty, then either prices are nonstochastic or the firm does not behave optimally.

A Productive Competitive Firm

The main results discussed in the preceding section were obtained by assuming the cash flow presented in expression (1.2) and are attributable to the contemporaneous relationship between revenue and input cost at each date. In this section, we will show that similar conclusions apply to firms characterized by less restrictive cash flows. The complications that arise from allowing for random input prices in a forward-looking context are attributable to the possibilities of stochastic production and/or input substitution. Hence, we can apply our basic model to other types of cash flows by constraining the production function to be nonstochastic and such that inputs with random prices cannot be substituted.

It is straightforward to extend the analysis performed in the previous section to competitive firms with the Leontief-type short-run production function

(2.1) $Q_t = \min[Q_t^s/\Phi, q(\mathbf{V}_t)]$,

where Q_t denotes production of final good at date t, $Q_t \geq 0$, Q_t^s represents material input use, Φ is a fixed input-output coefficient ($\Phi > 0$), $\mathbf{V}_t$ is a vector of nonmaterial inputs, and $q(\cdot)$ is a strictly concave production function. Output Q_t cannot be sold before date t+1; in other words, the firm starts production at time t and finishes output right before date t+1.

According to (2.1), adding Φ units of material input increases production by one unit over the range in which the vector of nonmaterial inputs does not constrain production. If enough units of material input are added, the set of nonmaterial inputs eventually becomes binding and production cannot increase. The fact that there is no substitutability between material input and $q(\cdot)$ does not mean that substitution among the nonmaterial inputs in vector $\mathbf{V}_t$ is prevented. For example, it may be feasible to substitute capital for labor in wheat milling, even though substitutability of wheat for either of these other two inputs combined or alone is negligible for all practical purposes. Note also that material input becomes nonbinding as Φ tends to zero, resulting in a standard production function $q(\cdot)$. In other words, the standard production function is nested in (2.1). Storage, transportation, refining and/or purifying of raw materials (e.g., oil, sugar, and metals), grain milling (e.g., wheat and rice), oilseed crushing, alloy preparation, energy generation, meat packing, and livestock production are examples of processes that comply with this Leontief function. In the farm sector, feedlot, hog, and poultry production are but some of the production processes that can be modeled by this function with reasonable accuracy.

Diewert (1971) has shown that the cost function dual to (2.1) is

(2.2) $C = \Phi\, s_t\, Q_t - c(Q_t; \mathbf{v}_t)$,

where C is variable cost, s_t is material input price, $c(\cdot)$ is a strictly convex nonmaterial cost function such that $c'(\cdot) > 0$, and $\mathbf{v}_t$ is a vector of nonmaterial input prices. We will assume that nonmaterial input prices are constant, and we will simply write $c(Q_t)$ instead of $c(Q_t; \mathbf{v}_t)$ because we are not concerned with nonmaterial input prices. Assuming that material input price is stochastic whereas that nonmaterial input prices are constant is not unrealistic because in many situations material input constitutes the largest share of variable cost. Also, nonmaterial input prices are generally less volatile, and substitutability among nonmaterial inputs should cause variable cost changes far less pronounced than those caused by material input price changes.

Because output and material input prices are different from each other in this scenario, to make the analysis more interesting we will hypothesize that forward markets exist for both output and material input. We will denote the forward price and forward position corresponding to material input by f_t^s and F_t^s, respectively. Then, the cash flow for a firm with the Leontief production function (2.1) can be represented by

$$(2.3)\quad \pi_t = p_t\, Q_{t-1} - \Phi\, s_t\, Q_t - c(Q_t) + (f_{t-1} - p_t)\, F_{t-1} + (f_{t-1}^s - s_t)\, F_{t-1}^s \qquad \text{s.t.}\quad Q_t \geq 0,$$

and the optimal decisions at the current date satisfy

$$(2.4)\quad M_T[r_{T-1}\, W_{T-1} + p_T\, Q_{T-1} + (f_{T-1} - p_T)\, F_{T-1} + (f_{T-1}^s - s_T)\, F_{T-1}^s;\, \mathbf{p}_T]$$

$$= \max_{\mathbf{d}_T} U[p_T\, Q_{T-1} - \Phi\, s_T\, Q_T - c(Q_T) + (f_{T-1} - p_T)\, F_{T-1} + (f_{T-1}^s - s_T)\, F_{T-1}^s],$$

$$(2.5)\quad M_t\{r_t \ldots r_{T-1}\, [r_{t-1}\, W_{t-1} + p_t\, Q_{t-1} + (f_{t-1} - p_t)\, F_{t-1} + (f_{t-1}^s - s_t)\, F_{t-1}^s];\, \mathbf{p}_t\}$$

$$= \max_{\mathbf{d}_t} E_t\{M_{t+1}[r_{t+1} \cdots r_{T-1}\, (r_t\, W_t + p_{t+1}\, Q_t + (f_t - p_{t+1})\, F_t + (f_t^s - s_{t+1})\, F_t^s);\, \mathbf{p}_{t+1}]\},$$

$$t = 0, 1, \ldots, T-1,$$

where: $\mathbf{d}_t = (Q_t, F_t, F_t^s)$ if $0 \leq t < T$, $\mathbf{d}_T = (Q_T, 0, 0)$, $Q_t \geq 0$,

$\mathbf{p}_t = (p_t, s_t, f_t, f_t^s)$,

$\underline{p}_t = (p_0, \ldots, p_t)$.

Terminal wealth and cash flows are given by expressions (1.1) and (2.3), respectively.

To distinguish the firm represented by cash flow (1.2) from the firm represented by cash flow (2.3), we will refer to the latter as a *manufacturing* firm. The analysis of optimal production and hedging for the manufacturing firm can be performed by using similar procedures as those used for the speculative storing firm. To avoid repetition, we outline the main results here and focus on the major behavioral differences between speculative storing and manufacturing firms.

It can be shown that optimal production at the terminal date is zero, yielding the maximum value function

$$(2.6)\quad M_T[r_{T-1}\, W_{T-1} + p_T\, Q_{T-1} + (f_{T-1} - p_T)\, F_{T-1} + (f^s_{T-1} - s_T)\, F^s_{T-1}; \underline{p}_T]$$

$$= U[r_{T-1}\, W_{T-1} + p_T\, Q_{T-1} + (f_{T-1} - p_T)\, F_{T-1} + (f^s_{T-1} - s_T)\, F^s_{T-1}].$$

The FOCs for any date preceding the terminal time are

$$(2.7)\quad \frac{\partial £_t}{\partial Q_t} = r_{t+1} \cdots r_{T-1}\, [E_t(p_{t+1}\, M_{t+1}') - r_t\, (\Phi\, s_t + c')\, M_t'] \leq 0,\ Q_t \geq 0,\ Q_t\, \frac{\partial £_t}{\partial Q_t} = 0,$$

$$(2.8)\quad \frac{\partial £_t}{\partial F_t} = r_{t+1} \cdots r_{T-1}\, [f_t\, M_t' - E_t(p_{t+1}\, M_{t+1}')] = 0,$$

$$(2.9)\quad \frac{\partial £_t}{\partial F^s_t} = r_{t+1} \cdots r_{T-1}\, [f^s_t\, M_t' - E_t(s_{t+1}\, M_{t+1}')] = 0,$$

where: $£_t = E_t\{M_{t+1}[r_{t+1} \cdots r_{T-1}\, (r_1\, W_1 + p_{t+1}\, Q_t + (f_t - p_{t+1})\, F_t + (f^s_t - s_{t+1})\, F^s_t); \underline{p}_{t+1}]\}$.

The most important results regarding this type of firm are obtained by means of expressions (2.6) through (2.9). These results are summarized as Propositions 4, 5, and 6, which are the respective counterparts of Propositions 1, 2, and 3. To prove Proposition 6, we will use the following expression regarding the relationship between output and material input cash prices:

(2.10) $s_t = \gamma + \delta\, p_t + u_t$, u_t i.i.d. random variable.

PROPOSITION 4: PRODUCTION BEHAVIOR. *In the presence of an output forward market, optimal production for a risk-averse manufacturing firm is independent from the subjective joint distribution of random variables, from the decision maker's degree of risk aversion, and from the optimal hedging decision. If positive, optimal production is such that discounted current output forward price equals (weighted) current material input cash price plus marginal production cost. These results hold for both myopic and forward-looking firms.*

Proof. By substituting FOC (2.8) into FOC (2.7) and rearranging, we obtain

(2.11) $f_0 - r_0\, [\Phi\, s_0 + c'(Q_0)] \leq 0,\ Q_0 \geq 0,\ \ Q_0\, \{f_0 - r_0\, [\Phi\, s_0 + c'(Q_0)]\} = 0.$

Therefore,

a. If $f_0 \leq r_0\, [\Phi\, s_0 + c'(0)]$, then $Q_0 = 0$.

b. If $f_0 > r_0\, [\Phi\, s_0 + c'(0)]$, then $Q_0 > 0$ and $f_0 = r_0\, [\Phi\, s_0 + c'(Q_0)]$. Q.E.D.

PROPOSITION 5: MYOPIC PRODUCTION HEDGE. *The optimal hedge for a myopic manufacturing risk-averse firm that perceives output and material input forward prices to be simultaneously unbiased is to sell the entire production in the output forward market and to sell nothing in the material input forward market. This hedge is independent from the myopic firm's degree of risk aversion.*

Proof. Rewrite FOCs (2.8) and (2.9) as

(2.12) $[f_0 - E_0(p_1)]\ M_0' = \mathrm{Cov}[p_1, E_0(M_1'|p_1)]$,

(2.13) $[f_0^s - E_0(s_1)]\ M_0' = \mathrm{Cov}[s_1, E_0(M_1'|s_1)]$,

where: $E_0(M_1'|p_1) = \int_{s_1} \int_{f_1} \int_{f_1^s} M_1'\ k_1(s_1, f_1, f_1^s|\mathbf{p}_0, p_1)\ df_1^s\ df_1\ ds_1 > 0$,

$E_0(M_1'|s_1) = \int_{p_1} \int_{f_1} \int_{f_1^s} M_1'\ l_1(p_1, f_1, f_1^s|\mathbf{p}_0, s_1)\ df_1^s\ df_1\ dp_1 > 0$.

$k_t(s_t, f_t, f_t^s|\mathbf{p}_{t-1}, p_t)$ is the conditional density function of s_t, f_t and f_t^s, given $\mathbf{p}_{t-1}$ and p_t, and $l_t(p_t, f_t, f_t^s|\mathbf{p}_{t-1}, s_t)$ is the conditional density function of p_t, f_t and f_t^s, given $\mathbf{p}_{t-1}$ and s_t.

If the firm is myopic and both forward prices are simultaneously unbiased, we need $\mathrm{Cov}[p_T, E_{T-1}(M_T'|p_T)] = \mathrm{Cov}[s_T, E_{T-1}(M_T'|s_T)] = 0$. This is satisfied if $F_{T-1} = Q_{T-1}$ and $F_{T-1}^s = 0$ because such a hedge yields M_T' independent from both p_T and s_T. Q.E.D.

PROPOSITION 6: FORWARD-LOOKING PRODUCTION HEDGE. *(1) The optimal hedge for a manufacturing forward-looking risk-averse firm is generally different from the optimal myopic hedge. Furthermore, the optimal forward-looking hedge depends on the firm's degree of risk aversion.*

(2) If the manufacturing forward-looking firm is CARA and output and material input cash prices behave as in (1.17) and (2.10), then the optimal hedge under unbiased forward prices consists of buying forward contracts of material input and, if cash prices are unrelated ($\delta = 0$) and serially uncorrelated ($\beta = 0$), selling the entire production in the forward market.

Proof. See Appendix B.

Proposition 4 confirms the robustness of the separation result, showing that it applies to a cost function that characterizes many production processes, even when the firm is forward-looking. It is also important to note that separation holds irrespective of the existence of a forward market for material input. Propositions 5 and 6 highlight the differences between myopic and forward-looking hedging behavior and confirm the weakness of the full-hedge optimality result.

Even for the myopic case, it will generally be true that $F_0 \neq Q_0$ and $F_0^s \neq 0$ simultaneously if output or material input forward prices (or both) are biased. This can be seen from

$$(2.14)\quad \frac{\partial E_{T-1}(M_T'|p_T)}{\partial p_T} = (Q_{T-1} - F_{T-1})\, E_{T-1}(M_T''|p_T) + \int_{s_T}\int_{f_T}\int_{f_T^s} M_T' \frac{\partial k_T(s_T, f_T, f_T^s|p_{T-1}, p_T)}{\partial p_T}\, df_T^s\, df_T\, ds_T,$$

$$(2.15)\quad \frac{\partial E_{T-1}(M_T'|s_T)}{\partial s_T} = - F_{T-1}^s\, E_{T-1}(M_T''|s_T) + \int_{p_T}\int_{f_T}\int_{f_T^s} M_T' \frac{\partial l_T(p_T, f_T, f_T^s|p_{T-1}, s_T)}{\partial s_T}\, df_T^s\, df_T\, dp_T.$$

For example, if the output forward price is unbiased but the material input forward price is biased, FOCs require that $\partial E_{T-1}(M_T'|s_T)/\partial s_T \neq 0$ and $\partial E_{T-1}(M_T'|p_T)/\partial p_T = 0$. This will generally mean a nonzero forward position in the input market ($F_{T-1}^s \neq 0$) and an output hedge different from total production ($F_{T-1} \neq Q_{T-1}$). In fact, a full output hedge ($F_{T-1} = Q_{T-1}$) does not yield $\partial E_{T-1}(M_T'|p_T)/\partial p_T = 0$ if $F_{T-1}^s \neq 0$ unless p_T and s_T are independently distributed.

Proposition 6 clarifies our previous discussion about the storage case. With unbiased forward prices, and independent and serially uncorrelated cash prices ($\delta = 0$ and $\beta = 0$), the optimum hedge consists of the risk-avoidance hedge ($F_0 = Q_0$) and the anticipatory hedge ($F_0^s < 0$). In terms of payoff with respect to alternative forward prices, the net effect of both forward positions (F_0, F_0^s) is similar to a less than fully hedged output position. In the real world, however, having forward markets for both output and material input is the exception rather than the rule. Proposition 7 summarizes some important results concerning the situation in which either the output forward market or the material input forward market is missing.

PROPOSITION 7: FORWARD-LOOKING PRODUCTION HEDGE IN THE ABSENCE OF FORWARD MARKETS. *Assume that the manufacturing firm is CARA, that material input and output cash prices behave as in (1.17) and (2.10) with* $0 < \delta \geq \beta/\Phi$, *and that forward prices are unbiased. Then:*

(1) The optimal forward-looking hedge in the absence of a forward market for material input is strictly less than the entire current production.

(2) The optimal forward-looking hedge in the absence of a forward market for output is strictly less than the amount of material input imbedded in the entire current production.

Proof. See Appendix C.

Proposition 7 reminds us that the standard full hedge optimality result depends crucially on (i) the firm being myopic, or on (ii) output cash prices being serially uncorrelated and output and material input cash prices being independent from each other. Full-hedge suboptimality under forward-looking behavior and unbiased forward prices is important and especially relevant for empirical work. Many studies have been conducted to obtain empirical estimates of the "optimal hedge" when there is a futures rather than a forward market [e.g., Ederington (1979), Anderson and Danthine (1980), Cecchetti, Cumby, and Figlewski (1988), Myers and Thompson (1989)]. The normative content of these studies is usually emphasized on the basis that the optimal hedge under unbiased futures prices is independent of the decision maker's degree of risk aversion [Batlin (1983), Benninga, Eldor, and Zilcha (1984)]. Our results suggest that this would be the case only if (i) the agent is myopic or if (ii) output cash prices are serially uncorrelated and output and material input cash prices are unrelated to each other.

Conclusions

In this study, we have shown that separation between production (or speculative storage) and hedging is a robust result because it holds even if firms are forward-looking. In the presence of forward markets, optimal production (storage) for a forward-looking firm is identical to an otherwise equivalent myopic firm. Optimal production (storage) is determined solely by nonrandom factors and is independent from the agent's price expectations and degree of risk aversion.

In contrast, full-hedge optimality under unbiased forward prices holds only if (i) the decision maker is myopic or if (ii) output cash prices are serially uncorrelated and output and material input cash prices are independent from each other. Full hedging is suboptimal when the firm is forward-looking and (i) output cash prices are serially correlated or (ii) output and material input prices are contemporaneously related. In this instance, suboptimality arises because the firm foresees that at next decision date it will stay in the market and it will take decisions based on the observed values of the relevant random variables. Hence, next-date decisions are random and affect the current risks faced by the firm and therefore will have an impact on the optimal current hedge.

Most real-world situations are characterized by the absence of either a forward market for (a) output or (b) material input. If the forward-looking firm exhibits constant absolute risk aversion, forward prices are unbiased, and some realistic conditions hold regarding the behavior of cash prices, the firm will hedge less than its entire current production under case (b), and will hedge less than the material input embedded in its entire current production under case (a).

Our results may help explain why firms do not fully hedge, even when there is empirical evidence that futures prices are generally unbiased. Also, full-hedge suboptimality under forward-looking behavior and unbiased forward prices raises questions about the normative properties of studies concerned with the empirical estimation of optimal myopic hedges in the presence of futures rather than forward markets.

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Appendix A. Derivation of FOC (1.10)

The FOCs corresponding to the Lagrangian for $0 \leq t < T$ are (A1) plus (1.11) and (1.12).

$$\text{(A1)} \quad \frac{\partial \pounds_t}{\partial P_t} = E_t\left[\frac{\partial M_{t+1}\{r_{t+1} \cdots r_{T-1}\,[r_t\, W_t + (f_t - p_{t+1})\, F_t];\, I_{t+1};\, \underline{\mathbf{p}}_{t+1}\}}{\partial I_{t+1}} \; \frac{\partial I_{t+1}}{\partial P_t}\right]$$

$$+\, r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, (p_t + i')\, M_t'\{r_t \cdots r_{T-1}\,[r_{t-1}\, W_{t-1} + (f_{t-1} - p_t)\, F_{t-1}],\, I_t;\, \underline{\mathbf{p}}_t\} - \eta_t = 0.$$

But note that

$$\text{(A2)} \quad \frac{\partial I_{t+1}}{\partial P_t} = -1,$$

$$\text{(A3)} \quad \frac{\partial I_{t+1}}{\partial I_t} = 1,$$

$$\text{(A4)} \quad \frac{\partial M_t}{\partial I_t} = E_t\left[\frac{\partial M_{t+1}}{\partial I_{t+1}} \; \frac{\partial I_{t+1}}{\partial I_t}\right] - r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, i'(\cdot)\, M_t' + \eta_t$$

$$\text{(A4')} \qquad = r_t\, r_{t+1} \cdots r_{T-2}\, r_{T-1}\, p_t\, M_t',$$

where (A4') is obtained by using expressions (A1) through (A3). It follows from (A4') that

$$\text{(A5)} \quad \frac{\partial M_{t+1}\{r_{t+1} \cdots r_{T-1}\,[r_t\, W_t + (f_t - p_{t+1})\, F_t];\, I_{t+1};\, \underline{\mathbf{p}}_{t+1}\}}{\partial I_{t+1}}$$

$$= r_{t+1}\, r_{t+2} \cdots r_{T-2}\, r_{T-1}\, p_{t+1}\, M_{t+1}'\{r_{t+1} \cdots r_{T-1}\,[r_t\, W_t + (f_t - p_{t+1})\, F_t],\, I_{t+1};\, \underline{\mathbf{p}}_{t+1}\}.$$

Substitution of (A2) and (A5) into FOC (A1) and rearrangement yields expression (1.10).

Appendix B. Proof of Proposition 6

If the firm is CARA, forward prices are unbiased, and output and material input prices behave as in (1.17) and (2.10), we have

$$\text{(B1)}\quad \mathrm{Cov}[s_1, E_0(M_1'|s_1)] = \mathrm{Cov}[\gamma + \delta\, p_1 + u_1, E_0(M_1'|\gamma + \delta\, p_1 + u_1)]$$

$$= \delta\, \mathrm{Cov}[p_1, E_0(M_1'|p_1)] + \mathrm{Cov}[u_1, E_0(M_1'|u_1)],$$

$$\text{(B2)}\quad \frac{\partial(M_1'|p_1)}{\partial p_1} = r_1 \ldots r_{T-1}\,(Q_0 - F_0 - \delta\, F_0^s)\, E_0(M_1''|p_1)$$

$$- r_2 \ldots r_{T-1}\,(r_1\,\Phi\,\delta - \beta)\, E_0(Q_1\, M_1''|p_1) - r_3 \ldots r_{T-1}\,(r_2\,\Phi\,\delta - \beta)\,\beta\, E_0(Q_2\, M_2''|p_1)$$

$$- \ldots - r_{t+1} \ldots r_{T-1}\,(r_t\,\Phi\,\delta - \beta)\,\beta^{t-1}\, E_0(Q_t\, M_t''|p_1) - \ldots - (r_{T-1}\,\Phi\,\delta - \beta)\,\beta^{T-2}\, E_0(Q_{T-1}\, M_{T-1}''|p_1),$$

$$\text{(B3)}\quad \frac{\partial(M_1'|u_1)}{\partial u_1} = - r_1 \ldots r_{T-1}\, F_0^s\, E_0(M_1''|u_1) - r_1 \ldots r_{T-1}\,\Phi\, E_0(Q_1\, M_1''|u_1).$$

Under unbiased forward prices, FOCs (2.12) and (2.13) require $\mathrm{Cov}[p_1, E_0(M_1'|p_1)] = \mathrm{Cov}[s_1, E_0(M_1'|s_1)] = 0$. Therefore, from (B1) it must also be true that $\mathrm{Cov}[u_1, E_0(M_1'|u_1)] = 0$.

Assume that $0 \le F_0^s$. Then $\partial(M_1'|u_1)/\partial u_1 > 0$ because $E_0(M_1''|u_1) < 0$ and $E_0(Q_1\, M_1''|u_1) < 0$, thus implying that $\mathrm{Cov}[u_1, E_0(M_1'|u_1)] > 0$. But this contradicts the requirement that $\mathrm{Cov}[u_1, E_0(M_1'|u_1)] = 0$. Hence, it must be true that $0 > F_0^s$.

Assume that $Q_0 < (>)\, F_0$. If $\beta = \delta = 0$, $\partial(M_1'|p_1)/\partial p_1 > (<)\, 0$ and therefore $\mathrm{Cov}[p_1, E_0(M_1'|p_1)] > (<)\, 0$, which contradicts the necessary condition that $\mathrm{Cov}[p_1, E_0(M_1'|p_1)] = 0$. Hence, it must be true that $Q_0 = F_0$.

Appendix C. Proof of Proposition 7

In the absence of a forward market for material input, the analysis can be performed by setting $F_t^s = 0$, in which case only FOCs (2.7) and (2.12) apply. Then, we have

$$\text{(C1)} \quad \frac{\partial(M_1'|p_1)}{\partial p_1} = r_1 \dots r_{T-1} (Q_0 - F_0) E_0(M_1''|p_1) - r_2 \dots r_{T-1} (r_1 \Phi \delta - \beta) E_0(Q_1 M_1''|p_1)$$

$$- r_3 \dots r_{T-1} (r_2 \Phi \delta - \beta) \beta E_0(Q_2 M_2''|p_1) - \dots - r_{t+1} \dots r_{T-1} (r_t \Phi \delta - \beta) \beta^{t-1} E_0(Q_t M_t''|p_1)$$

$$- \dots - (r_{T-1} \Phi \delta - \beta) \beta^{T-2} E_0(Q_{T-1} M_{T-1}''|p_1).$$

Following the arguments in the proof of Proposition 6, we must have $Q_0 > F_0$ if $0 < \delta \geq \beta/\Phi$.

Similarly, if there is no forward market for output, F_t must be set equal to zero and only FOCs (2.7) and (2.13) apply. In this instance,

$$\text{(C2)} \quad \mathrm{Cov}[s_1, E_0(M_1'|s_1)] = \delta \,\mathrm{Cov}[p_1, E_0(M_1'|p_1)] + \mathrm{Cov}[u_1, E_0(M_1'|u_1)] = 0,$$

$$\text{(C3)} \quad \frac{\partial(M_1'|p_1)}{\partial p_1} = r_1 \dots r_{T-1} (Q_0 - \delta F_0^s) E_0(M_1''|p_1) - r_2 \dots r_{T-1} (r_1 \Phi \delta - \beta) E_0(Q_1 M_1''|p_1)$$

$$- r_3 \dots r_{T-1} (r_2 \Phi \delta - \beta) \beta E_0(Q_2 M_2''|p_1) - \dots - r_{t+1} \dots r_{T-1} (r_t \Phi \delta - \beta) \beta^{t-1} E_0(Q_t M_t''|p_1)$$

$$- \dots - (r_{T-1} \Phi \delta - \beta) \beta^{T-2} E_0(Q_{T-1} M_{T-1}''|p_1),$$

$$\text{(C4)} \quad \frac{\partial(M_1'|u_1)}{\partial u_1} = - r_1 \dots r_{T-1} F_0^s E_0(M_1''|u_1) - r_1 \dots r_{T-1} \Phi E_0(Q_1 M_1''|u_1).$$

Assume $Q_0 \leq \delta F_0^s$. Then $\partial(M_1'|p_1)/\partial p_1 > 0$ and $\partial(M_1'|u_1)/\partial u_1 > 0$, which implies that $\mathrm{Cov}[s_1, E_0(M_1'|s_1)] > 0$. But if $\mathrm{Cov}[s_1, E_0(M_1'|s_1)] > 0$, then FOC (2.12) cannot hold. Hence, $Q_0 > \delta F_0^s \geq \beta/\Phi\, F_0^s \geq F_0^s/\Phi$, implying that $\Phi Q_0 = Q_0^s > F_0^s$.

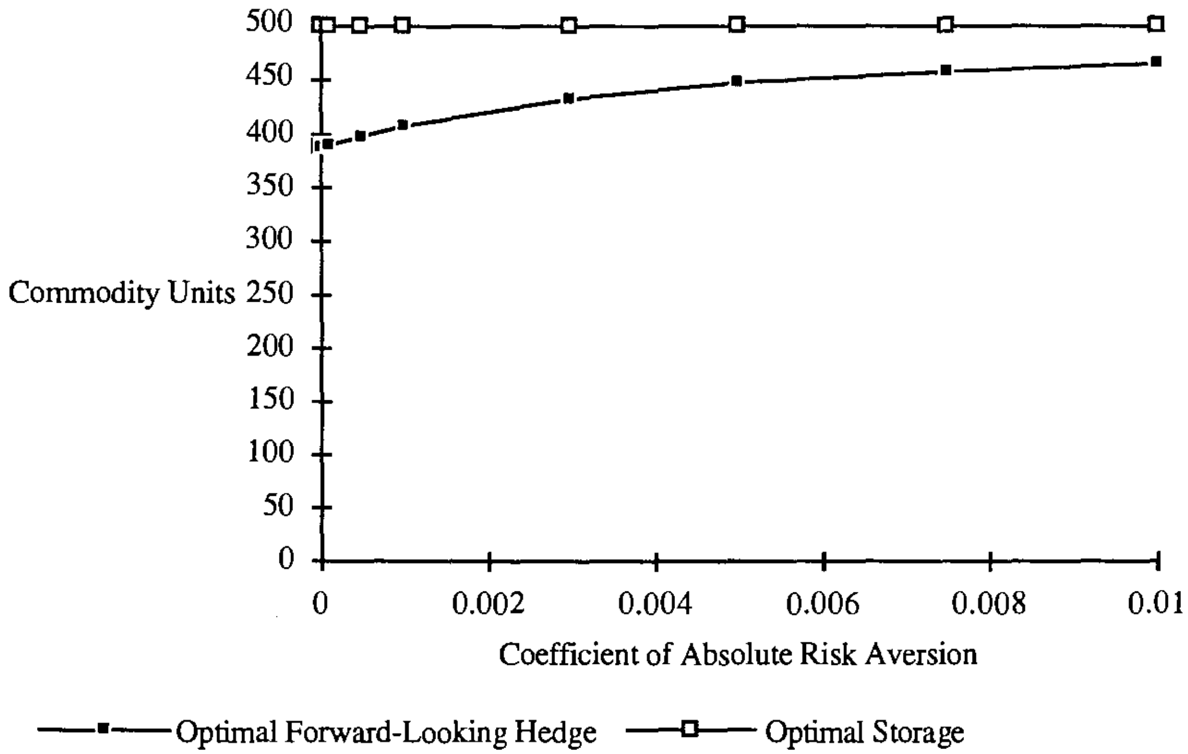

Figure 1. Optimal storage and forward-looking hedge under unbiased forward prices

Table 1. Probability distribution of prices at dates t_1 and t_2

Date t_1				Date t_2		
Event	p_{t_1}	f_{t_1}	Probability	Event	p_{t_2}	Probability conditional on event at t_1
1	13	12.5	0.0625	1	14.5	0.25
1	13	12.5	0.0625	2	11.5	0.50
1	13	12.5	0.0625	3	9.5	0.25
2	13	11.5	0.1250	4	14.5	0.25
2	13	11.5	0.1250	5	11.5	0.50
2	13	11.5	0.1250	6	9.5	0.25
3	13	10.5	0.0625	7	14.5	0.25
3	13	10.5	0.0625	8	11.5	0.50
3	13	10.5	0.0625	9	9.5	0.25
4	10	11	0.1250	10	13	0.25
4	10	11	0.1250	11	10	0.50
4	10	11	0.1250	12	7	0.25
5	10	10	0.2500	13	13	0.25
5	10	10	0.2500	14	10	0.50
5	10	10	0.2500	15	7	0.25
6	10	9	0.1250	16	13	0.25
6	10	9	0.1250	17	10	0.50
6	10	9	0.1250	18	7	0.25
7	7	9.5	0.0625	19	11.5	0.25
7	7	9.5	0.0625	20	8.5	0.50
7	7	9.5	0.0625	21	5.5	0.25
8	7	8.5	0.1250	22	11.5	0.25
8	7	8.5	0.1250	23	8.5	0.50
8	7	8.5	0.1250	24	5.5	0.25
9	7	7.5	0.0625	25	11.5	0.25
9	7	7.5	0.0625	26	8.5	0.50
9	7	7.5	0.0625	27	5.5	0.25