Similitude Analysis for Gas-Liquid-Fiber Flows in Cocurrent Bubble Columns

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Abstract
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Keywords
flow dynamics, fibers, bubbles

Disciplines
Complex Fluids | Fluid Dynamics | Manufacturing

Comments
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SIMILITUDE ANALYSIS FOR GAS-LIQUID-FIBER FLOWS IN COCURRENT BUBBLE COLUMNS

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ABSTRACT

Gas-liquid-fiber systems are different from conventional gas-liquid-solid systems in that the solid material (i.e., fiber) is flexible, has a large aspect ratio, and forms flocs or networks when its mass fraction is above a critical value. With its wide application to the pulp and paper industry, it is important to investigate the hydrodynamics of gas-liquid-fiber systems. In this paper, 19 parameters that influence gas holdup in gas-liquid-fiber bubble columns are critically examined and then a dimensional analysis based on the Buckingham Pi Theorem is used to derive the dimensionless parameters governing gas-liquid-fiber bubble column hydrodynamics. Seven dimensionless parameters that are related to the fiber effects on gas holdup are further analyzed, and a single dimensionless parameter combining these dimensionless parameters is derived based on a force analysis and experimental results. This dimensionless parameter is shown to be sufficient to quantify the influence of fiber on gas holdup in gas-liquid-fiber cocurrent bubble columns. It also reduces the number of parameters needed in correlating experimental gas holdup data in gas-liquid-fiber bubble columns.

INTRODUCTION

Gas-liquid-fiber systems are found in the pulp and paper industry in a variety of unit operations including flotation deinking, direct-contact steam heating, gaseous fiber bleaching and papermaking [1]. Understanding the hydrodynamics in gas-liquid-fiber systems are important to the heat and mass transport processes in the unit operations where these systems are found. In the flotation deinking process, a higher gas holdup and smaller bubble size generally imply a larger interfacial area between the gas and liquid and/or a larger gas residence time, both of which lead to higher ink removal efficiency [2, 3].

A gas-liquid-fiber system is different from conventional gas-liquid-solid systems in the fact that the solid material is fiber, which is usually flexible and has a large aspect ratio. A cellulose fiber-water suspension forms a complex slurry because the fibers have a density close to that of water and can form flocs at a fiber mass fraction as low as 0.3% and continuous fiber networks at a mass fraction larger than 1% [4]. When gas is introduced into the fiber suspension, bubble motion, coalescence, and breakup can be significantly affected by the formation of fiber networks [5] and thus, the bubble size, residence time, and gas holdup can be significantly influenced by the presence of fibers.

During the last decade, extensive experimental studies have been conducted to investigate the hydrodynamics in gas-liquid-fiber bubble columns due to newly developed column flotation deinking technology [6, 7]. Bubble size distribution in gas-liquid-fiber flows and its variation with fiber mass fraction and fiber type have been investigated in semi-batch [8-10] and cocurrent bubble columns [11]. Gas holdup in gas-liquid-fiber systems has also been studied in both semi-batch [2, 12-18] and cocurrent [2, 19-23] bubble columns. Effects of superficial gas and liquid velocity, fiber mass fraction, fiber type, and gas

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distribution method on gas holdup were studied in these investigations. A few gas holdup correlations have been presented for gas-liquid-fiber bubble columns [12, 20, 24]. All these correlations are obtained at more or less limited conditions and may not be reliable when they are used to scale up gas-liquid-fiber bubble columns because they are not based on a similitude analysis.

Gas-liquid-fiber bubble column hydrodynamics are affected by many factors. To study the influence of each factor separately requires a formidable large mount of work. A similitude analysis will reduce the number of influential factors and provide directions to simplify the problem. Experimental investigations designed around similitude analysis results will require much less cost while they provide reliable results for future applications.

Similitude analysis has been successfully applied to multiphase flow systems, such as fluidized beds [25, 26], gas-liquid bubble columns [27], and gas-liquid-solid bubble columns [28]. Glicksman et al. [29] provided an excellent review on the development and application of scaling laws for two-phase fluidized beds. Zlokarnik [30] presented a comprehensive review on the application of the Buckingham Pi method in scaling up in chemical engineering and gave a number of examples. However, no similitude analysis applicable to gas-liquid-fiber systems has been found in the open literature.

Three approaches are usually used to obtain dimensionless scaling parameters governing a process [31]. The first method is to non-dimensionalize the differential equations describing the process. However, for a complex process like gas-liquid-fiber flows in bubble columns, differential equations and boundary conditions that fully account for the relationship between the gas holdup and all influential factors are still not available. The second method is dimensional analysis, which is based on the Buckingham Pi Theorem [32]. This method is widely used because no governing equations are required before the dimensional analysis. Only dependent parameters and a complete list of independent variables that are relevant to the dependent parameters are needed. The third method is to generate dimensionless parameters by a force analysis. With this method, the forces relevant to the process are first identified and represented in terms of the parameters related to the process, and then dimensionless groups are constructed as force ratios. Although in many cases, the third method loses favor when the investigated parameters do not depend on any forces, it does provide a clear physical interpretation of the dimensionless parameters when it is applicable [31].

The present paper identifies all parameters that influence gas holdup in a cocurrent gas-liquid-fiber bubble column, and then a dimensional analysis based on the Buckingham Pi Theorem is used to derive the dimensionless parameters relevant to this system. The force ratio approach is then used to crosscheck the dimensional analysis results and explain the significance of the dimensionless parameters. The dimensionless parameters related to the fiber influences on gas holdup are then discussed in detail. Finally, a single dimensionless parameter is derived to quantify the effects of fiber mass fraction and physical properties on gas holdup and then compared to experimental data.

**DIMENSIONAL ANALYSIS**

**Relevant Parameters**

In this analysis, gas holdup (\( \varepsilon \)), defined as the volumetric gas fraction in the fiber suspension, is the target parameter. Gas holdup in a gas-liquid-fiber bubble column is affected by many physical quantities, including: (i) the geometry of the bubble column and the gas distributor, (ii) the physical properties of the gas, liquid, and solid phases, and (iii) the process-related parameters. We limit the current analysis to isothermal flows in a bubble column and assume the gas, liquid, and fibers are all incompressible. Hence, the influence of temperature is omitted and the pressure influence on physical properties is not considered. The three groupings of the relevant parameters identified above will now be discussed.

**Geometric parameters**

It is extensively reported that the bubble column diameter (\( D \)) and height (\( H \)) have an influence on gas holdup in a bubble column [33-39]. There are also many investigations showing that the design of the gas distributor, through which the gas enters the bubble column, can dramatically affect the flow regime transition and gas holdup in a bubble column [33, 38, 40-45]. A gas distributor is usually designed as a plate or sparger with many small gas-passing orifices of the same size (\( d_0 \)). In this analysis, we assume that the orifices are uniformly distributed on the gas distributor. Thus, the distributor can be characterized by two parameters: (i) the open area ratio, \( R_A \) (defined as the ratio of the total area of all the orifices to the cross-sectional area of the bubble column), and (ii) the orifice diameter, \( d_0 \).

**Gas phase properties**

Gas density (\( \rho_g \)) and dynamic viscosity (\( \mu_g \)) can significantly affect gas holdup in a bubble column. It was reported that gas holdup increased with increasing gas density [46-49]. It has also been reported that increasing gas density delayed regime transition [49-51]. Hikita et al. [52] showed that the effect of gas density and viscosity could be significant. According to the data presented in Behkish [53], the influence of gas solubility on gas holdup is not significant.

**Liquid phase properties**

In a gas-liquid-solid bubble column, the solid particles and gas bubbles are suspended in a continuous liquid phase. The effect of liquid viscosity on gas holdup has been reported [54, 55]. The liquid surface tension has significant effects on gas holdup in a bubble column because bubble formation, coalescence, and breakup depend on the liquid surface tension [41, 56-58]. In a gas-liquid-fiber system, liquid viscosity and...
density also influence gas holdup. In the current analysis, we assume the liquid is a Newtonian fluid. Thus, the liquid properties included in the analysis are density ($\rho_l$), dynamic viscosity ($\mu_l$), and surface tension ($\sigma_l$).

**Fiber physical properties**

In the current analysis, the fiber is modeled as a cylinder, with a length-weighted average length ($L_r$) and a diameter ($d_r$). To account for the effect of fiber length distribution, a fiber length standard deviation ($S_r$) is also considered ($S_r = 0$ if a fiber has a uniform length distribution). To simplify the analysis, variations of other fiber physical properties are assumed to have insignificant effects on the hydrodynamics in gas-liquid-fiber bubble columns.

In a gas-liquid-fiber bubble column, fibers comprise the solid phase. It is well known that solid phase density can affect gas holdup in a gas-liquid-solid bubble column [59]. To make the results general, fiber density ($\rho_f$) is included as a relevant parameter, although in most gas-liquid-fiber applications using natural fiber (e.g., papermaking), fibers can be approximated as neutrally buoyant.

Fibers generally have a large aspect ratio ($r = L_r/d_r$) and can move in translation and rotation. As it rotates, a fiber can sweep out a much larger volume, exceeding its own volume by a factor of $r^2$. This results in many more collisions between fibers when they are present in the same flow field. When fibers are crowded, entanglement (or flocculation) occurs and fiber flocs form. When the fiber mass fraction in a fiber suspension is high enough, continuous fiber networks form. Fiber flocculation is a complex function of fiber volume fraction ($\phi$), length ($L_r$), aspect ratio ($r$), stiffness ($EI$, the product of the elastic modulus $E$ and the moment of inertia $I$), surface friction coefficient ($F_r$), and flow conditions [4, 60-62]. Wikstrom and Rasmuson [63] also reported that the fiber length distribution had a significant effect on fiber network strength. The presence of fibers can make the effective rheological properties of a fiber suspension significantly different from those of the suspending fluid [64-66]. Fiber flocs or networks can significantly affect bubble motion, coalescence, and breakup, and thus, gas holdup in the bubble column [2, 5, 11, 12, 20, 67-69]. Pelton and Piette [69] reported that the main reason bubbles are held up in a fiber suspension is mechanical confinement, not bubble adhesion to fibers. Thus, fiber-liquid contact angle will not be included in the dimensional analysis.

In summary, the fiber physical properties to be considered include average fiber length ($L_r$), fiber length standard deviation ($S_r$), density ($\rho_f$), diameter ($d_r$), stiffness ($EI$), and surface friction coefficient ($F_r$).

**Process parameters**

Gas and liquid throughput rates, pressure, temperature, and fiber concentration are important process conditions and significantly affect gas holdup. In this analysis, we only consider the gas holdup in a gas-liquid-fiber bubble column under atmospheric pressure and ambient (room) temperature and neglect the effects of pressure and temperature. The superficial gas velocity ($U_g$), superficial liquid velocity ($U_l$), and fiber volumetric concentration ($\phi$, $0 \leq \phi \leq 1$) are chosen as process variables. The buoyancy term, $g(\rho_l - \rho_g)$, has been found to have a significant effect on bubble size [70] and, therefore, is included in the analysis.

**Complete relevant parameter list**

In summary, the complete list of relevant parameters for this simplified analysis includes 19 parameters, plus the target parameter, $\phi$:

$$\{\phi, \rho_f, L_r, \rho_l, \mu_l, \sigma_l, \rho_i, L_t, S_r, d_r, EI, F_r, U_g, U_l, \phi, g(\rho_l - \rho_g)\}$$

where all terms to the right of the colon influence gas holdup in gas-liquid-fiber bubble columns.

**DIMENSIONAL ANALYSIS WITH THE BUCKINGHAM PI THEOREM**

There are already four dimensionless parameters in Eq. (1):

$$\{\phi, \rho_f, F_r, \phi\}$$

We can exclude these dimensionless parameters from the dimensional analysis. It is also found that the parameters in each of the following four sets have a same dimension: (i) $D, H, R_A, d_o, \rho_g, \rho_f, \mu_l, \mu_i, \sigma_l$, $\rho_i, L_t, S_r, d_r, EI, F_r, U_g, U_l$, and (iv) $U_g, U_l$. Thus, a list of 9 dimensionless parameters can be immediately obtained representing the ratios between the parameters having the same dimensions, i.e.,

$$\left\{\frac{H}{D}, \frac{d_o}{D}, \frac{L_t}{D}, \frac{S_r}{D}, \frac{\rho_g}{\rho_f}, \frac{\rho_f}{\rho_i}, \frac{\mu_l}{\mu_i}, \frac{U_g}{U_l}\right\}$$

Only one parameter in each of the 4 sets above and three other parameters ($EI, \sigma_l, g$) are left for the dimensional analysis:

$$\{D, \rho_f, \mu_i, \sigma_l, EI, U_l, g(\rho_l - \rho_g)\}$$

There are only 3 basic dimensions (length, mass, and time) contained in all 7 parameters. Using the Buckingham Pi Theorem, the 7 parameters in Eq. (4) are reduced to the following 4 dimensionless Pi terms:

$$\begin{align*}
P_1 &= \frac{EI}{D^5 g(\rho_l - \rho_g)} \\
P_2 &= \frac{\rho_i U_l^2 D}{\sigma_l} \\
P_3 &= \frac{\rho_i U_l D}{\mu_i} \\
P_4 &= \frac{\rho_i U_l^2}{g(\rho_l - \rho_g) D}
\end{align*}$$

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The dimensionless parameters in Eq. (5), together with those included in Eqs. (2) and (3), reveal 17 dimensionless parameters:

\[
\left\{ \varepsilon, R^*, L^*, \frac{H}{D}, \frac{d_2}{D}, \frac{L_1}{D}, \frac{L_2}{L_1}, \frac{S_t}{L_t}, \frac{p_x}{p_f}, \frac{p_1}{p_f}, \frac{\mu_x}{\mu_f}, \frac{U_i}{U_s}, \frac{R_i}{R_t}, \frac{F_i}{F_t}, \frac{W_e}{W_c}, \frac{\sigma_f}{\sigma_t} \right\}
\]

(6)

The four dimensionless parameters in Eq. (5) can also be obtained by a force analysis [31]. In a gas-liquid-fiber system, there are five important forces: (i) inertial forces: \( \rho_f U_i^2 \); (ii) viscous forces: \( \mu_f U_i/D \); (iii) buoyant forces: \( (\rho_f - \rho_s)gD \); (iv) surface tension forces: \( \sigma_f/D \); and (v) fiber network strength forces: \( EI/D^4 \). Four independent dimensionless parameters can be formed by taking ratios of these 5 forces which are determined from the input variables. Equation (5) is one of the many possible resultant groups, where

\[
\Pi_3 = \frac{\rho_f U_i D}{\mu_f} = Re_p
\]

(7)

is the liquid flow Reynolds number, which represents the ratio of inertial and viscous forces;

\[
\Pi_4 = \frac{\rho_f U_i^2}{g(\rho_f - \rho_s)D} = Fr^*
\]

(8)

is the modified Froude number, which represents the ratio of inertial to buoyant forces; and

\[
\Pi_2 = \frac{\rho_f U_i D}{\sigma_f} = We
\]

(9)

is the Weber number, which is proportional to the ratio of inertial to surface tension forces and is widely used to characterize bubble dynamics in multiphase flows. These three dimensionless parameters are widely used in the literature concerning two-phase and multiphase flows.

In a gas-liquid two-phase bubble column, the gas holdup (\( \varepsilon \)) is a function of 9 dimensionless parameters:

\[
\varepsilon = f \left( R_A, \frac{H}{D}, \frac{d_2}{D}, \frac{p_x}{p_f}, \frac{\mu_x}{\mu_f}, \frac{U_i}{U_s}, \frac{R_i}{R_t}, \frac{F_i}{F_t}, \frac{W_e}{W_c} \right)
\]

(10)

When fibers are added to the bubble column, the presence of the fiber may have a significant effect on the bubble column hydrodynamics, and this effect is related to other dimensionless parameters listed in Eq. (6), i.e., \( \phi, F_p, L_f/D, L_d/d_2, S_t/L_t, \rho_f/\rho_s \), and \( \Pi_1 \). The dimensionless parameter \( \Pi_1 \) reflects the ratio between the fiber network strength and the buoyant force. It appears when fibers are added to gas-liquid flows and becomes important when flocculation is significant. This will be discussed in more detail in the section entitled “Fiber Effects.”

In the following two sections, an experimental program is described to study the influence of fiber mass fraction and physical properties on gas holdup in a cocurrent gas-liquid-fiber bubble columns and the dimensionless parameters derived here are used to quantify the fiber influence on gas holdup.

EXPERIMENTAL PROCEDURES

The experimental procedures used in this study have been described in detail by Tang [71] and will be briefly reviewed here. The experiments for this study are conducted in a cylindrical cocurrent bubble column, which consists of four 0.914 m tall acrylic tubes with 15.24 cm internal diameter. Collars and gaskets are used to connect the acrylic tubes for a total column height of 4 m. Figure 1 shows a schematic of the entire system. Filtered air is supplied by a compressor and enters the bubble column from the bottom via a spider sparger. The air flowrate is adjusted with a regulator and measured with one of three gas flowmeters, each covering a different flowrate range. The fiber suspension from a 379 L reservoir is pumped into the column. The pump is connected to the reservoir with a 2.44 m long 7.62 cm diameter PVC pipe. A 2.85 m long 2.54 cm diameter PVC pipe connects the pump to the column. The fiber suspension flowrate is measured with a magnetic flowmeter and varied via a pump power frequency controller.

The fiber suspension enters the column through a flow expander located immediately below the spider sparger. A gas-liquid separator is located on top of the column where air is separated from the fiber slurry while the slurry returns to the reservoir through a PVC pipe. Along the column, 5 pressure transducers (labeled as \( P_1, P_2, P_3, P_4, P_5 \) in Fig. 1) are installed, one in each of the five collars. Each acrylic tube section is numbered 1 to 4 from the bottom of the column.

The spider sparger, shown in Fig. 2, has eight arms made of 12.7 mm diameter stainless steel tubes. Thirty-three 1.6 mm diameter holes are located on one side of each arm and distributed as shown in Fig. 2. The arms are soldered to the center cylinder of the sparger such that all the holes face the same direction. Air enters the spider sparger from the central cylinder and exits from the arm holes. The sparger is installed with the holes facing upward. The dimensional analysis assumes a sparger with uniformly distributed holes. The spider sparger is used here to allow for cocurrent flow, and is designed to provide a gas distribution into the cocurrent bubble column as uniform as possible.

Three types of cellulose fibers and Rayon fiber of three lengths are used in this study. The cellulose fibers are hardwood, softwood, and bleached chemithermomechanical pulp (BCTMP). Both the hardwood and softwood fibers are kraft pulp. The key physical properties of the three cellulose fibers are listed in Table 1. The Rayon fibers used in this study have a nominal length (L) of 1, 3, or 6 mm. All Rayon fibers have a coarseness (\( \sigma \)) of 50 mg/100m, which corresponds to a fiber diameter of 20.6 \( \mu m \).
All cellulose fibers are disintegrated from dry lap fiber sheets. The fiber sheets are originally torn into small pieces and then a specified mass of oven-dry fiber is weighed. It is then soaked in tap water for 24 hours before the pieces of fiber sheet are disintegrated in a Black-Clawson laboratory hydropulper. The concentrated fiber suspension is then transferred to the reservoir and additional tap water is added to adjust the fiber mass fraction (C) to a predetermined level. Rayon fibers are prepared differently from the cellulose fibers. First, a specified mass of oven-dry fiber is weighed. Then the fiber is soaked in tap water for 24 hours before it is repeatedly washed and soaked using tap water until the surface tension of the filtered water reaches a steady value of about 70 mN/m. This process removes a majority of the proprietary additives attached to the fiber surface, which are gradually released into the fiber suspension and may affect the bubble column hydrodynamics. The washed Rayon fiber is then added into the reservoir and additional tap water is added to adjust the fiber mass fraction to a predetermined level. In this study, 0 ≤ C ≤ 100% and the volumetric fiber fraction (φ, 0 ≤ φ ≤ 1) can be obtained from φ = ρfC/(100p)0). When C is small (e.g., C < 5%), the mixture density (ρm) can be approximated as ρm = ρi = 1000 kg/m³ for fiber-water mixtures.

During data acquisition, surface tension and pH of the water filtrate from the fiber suspensions are measured with a Sigma 703 digital tensiometer and a Milwaukee SM 802 pH/EC/TDS meter, respectively. The pH and surface tension in different fiber suspensions at various conditions are close to that of water except in BCTMP fiber suspensions, where surface tension decreases significantly with increasing fiber mass fraction in the range 0.05% ≤ C ≤ 0.8% and remains

---

**Table 1. Properties of the cellulose fibers used in this study.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Hardwood</th>
<th>Softwood BCTMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Species</td>
<td>Eucalyptus</td>
<td>65-75% Northern Black Spruce, 20-25% Jackpine, 5-10% Balsam Fir</td>
</tr>
<tr>
<td>LA (mm)</td>
<td>0.69</td>
<td>1.2</td>
</tr>
<tr>
<td>Lr (mm)</td>
<td>0.78</td>
<td>2.31</td>
</tr>
<tr>
<td>ω (g/m)</td>
<td>6.9 × 10⁵</td>
<td>13.1 × 10⁵</td>
</tr>
<tr>
<td>nr (1/g)</td>
<td>21.4 × 10⁶</td>
<td>6.37 × 10⁶</td>
</tr>
</tbody>
</table>
relative constant at about 50 mN/m when \(1.0% \leq C \leq 1.5\%\). More details are presented in [22].

All experiments in this study are carried out under atmospheric pressure and ambient temperature. The superficial gas velocity range is \(0 \leq U_g \leq 20\) cm/s, and the superficial liquid velocity range is \(0 \leq U_l \leq 10\) cm/s. Fiber mass fraction \(C\) is defined as the ratio of the oven-dry fiber mass to the suspension mass. In this study, the fiber mass fraction range is \(0 \leq C \leq 1.5\%\) for all fiber types except 6 mm Rayon fibers, which was \(C \leq 0.4\%\) because of clogging in the 2.54 cm PVC pipe at fiber mass fractions higher than 0.4\%.

To acquire gas holdup data at a given \(U_g\) and \(U_l\), 4800 readings are collected by a computer data acquisition system from each instrument every 10 ms and averaged after quasi-steady conditions are reached. With five pressure signals, the time-averaged gas holdup in each section is calculated from

\[
e_i = 1 - \frac{\Delta P_i}{\Delta P_{0,i}}
\]

where \(\Delta P_i = P_{li} - P_{hi}\) is the pressure difference between the lower \((P_{li})\) and higher \((P_{hi})\) ends of column section \(i\) \((i = 1, 2, 3, 4)\); \(\Delta P_{0,i}\) is the corresponding pressure difference when the column is filled only with the specified water-fiber suspension flowing at the same \(U_l\). Equation (11) accounts for the effects of wall shear stress but neglects the effect of liquid acceleration due to void changes that may influence gas holdup in cocurrent bubble columns [72, 73]; however, these effects are estimated to be negligible for the conditions of this study [71, 74]. The overall column gas holdup is defined as \(\varepsilon = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)/3\), the average gas holdup in the three lower sections. The gas holdup in the top section is not included in the overall gas holdup because of measurement error due to the void caused by large bubbles escaping the column top, which is significant during some experimental conditions [21].

Measurement uncertainties are estimated following the method provided by Figliola and Beasley [75] and details are presented by Tang [71]. The typical uncertainties associated with superficial velocities are ±2-4\% for \(U_g\) and ±1.5-5\% for \(U_l\), respectively. The corresponding absolute gas holdup uncertainty is estimated to be ±0.005-0.01.

**FIBER EFFECTS**

Tang and Heindel [23] investigated the influences of fiber mass fraction and type on gas holdup in the same cocurrent air-liquid-fiber bubble column used in the present study. They reported that for different fiber types, neither the crowding factor \(N_c\) nor the fiber number density \(N_l\) was sufficient in quantifying fiber effects on gas holdup. However, a combined parameter of \(N_c^{-a}N_f^p\), was sufficient to quantify the fiber influence on gas holdup in a cocurrent gas-liquid-fiber bubble column, i.e., the gas holdup data points collapse on a single curve on the \(\varepsilon\) vs. \(N_c^{-a}N_f^p\) plot for a given \(U_l\) and \(U_g\). The index parameter \(a\) was determined (by a trial and error method) to be ~0.2 for all investigated conditions. The slight difference between the physical implications of \(N_c\) and \(N_f\) (i.e., \(N_f\) only accounts for the average fiber length while \(N_c\) together with \(N_f\) provides both average and standard deviation of fiber length distribution) was used to justify the application of the combination of these two parameters in characterizing the fiber effects. Although fiber length distribution is an important reason that \(N_f\) should be used together with \(N_c\) to quantify the fiber effects on gas holdup, it is not a sufficient one, i.e., there may be other reasons because \(N_f\) alone does not characterize gas holdup effects when the fibers have a uniform length (1, 3, and 6 mm Rayon), while \(N_c^{-a}N_f^p\) with \(a = 0.2\) does [23].

The physical significance of \(N_c^{-a}N_f^p\) is expanded upon below.

When a gas bubble moves in a fiber suspension, if the bubble is larger than the void between fibers (or fiber spacing), it will collide with the fibers. The bubble-fiber contact affects bubble movement. This interaction is significant when the fiber mass fraction is high and fiber networks form. Bubbles will be entrained in the fiber network when the following two criteria for gas bubble holdup in a fiber suspension are satisfied: (i) the fiber spacing is smaller than the bubble diameter, and (ii) the fiber network is sufficiently strong to sustain the pressure exerted on the fiber network by the bubble due to its buoyancy force [69]. Hence, fiber spacing in a fiber suspension and the ratio between the network strength and the pressure exerted on the fiber network by the bubble due to its buoyancy force are two important factors to bubble movement through a fiber suspension. For multiple bubble motions in a fiber suspension, these two factors also play important roles [5, 22]. It is expected that these two factors should also significantly affect gas holdup in a gas-liquid-fiber bubble column.

Fiber spacing in a fiber suspension can be represented by the crowding factor, a dimensionless parameter defined as the number of fibers in a spherical suspension volume with a diameter equal to the average fiber length [4]:

\[
N_c = \frac{2}{3} \frac{\phi}{d_f} \left( \frac{L_c}{d_f} \right)^2
\]

which is a combination of two dimensionless parameters identified in Eq. (6): \(\phi\) and \(L_c/d_f\).

In a bubble column with the gas phase distributed by a perforated plate or sparger at the column bottom, coalescence between the newly generated bubbles at the aeration zone due to the slowing down or trapping effects of fiber networks is the major mechanism that fibers affect gas holdup in a bubble column [22]. Because \(d_f\) is directly related to the size of newly generated bubbles [42], the sparger orifice diameter \(d_f\) is used as the characteristic length in quantifying the bubble buoyancy pressure.

When a bubble rises upward in a fiber suspension, there are significant interactions between the bubble and fiber network. The bubble pushes on the portion of the fiber network that contacts the top of the bubble, attempting to relocate this fiber region to a higher position. Meanwhile, the portion of the
network pushed by the bubble is linked to other regions in the fiber network via fiber-fiber friction. The frictional forces provide resistance to fiber relocation. However, if the bubble is sufficiently large, the fiber network may break up into two or more regions along a weak fiber network connection. The collective effect of the frictional forces is to provide the fiber network a certain level of tensile strength. To some degree, the action of the rising bubble on the fiber network is very similar to the action of an upward moving ball on a large flexible sheet covering the ball, which causes significant stretching of the sheet along the contacting location. Hence, the tensile strength represents the strength of the fiber network in this situation.

Assuming a uniform fiber volume fraction, Farnood et al. [76] derived the tensile strength in a fiber network to be

$$\tau_{\text{tensile}} \approx 0.0062F_t \left( \frac{E}{I} \right) L_r \phi^3/d_r^2$$

(13)

The dimensionless parameter representing the ratio between tensile strength and bubble buoyancy force with a characteristic length $d_o$ is

$$\Pi' = \frac{\tau_{\text{tensile}}}{\left( \rho_t - \rho_g \right) g d_o} = \frac{F_t \left( \frac{E}{I} \right) L_r \phi^3}{\left( \rho_t - \rho_g \right) g d_o d_r^2}$$

(14)

which is a combination of $\Pi$, $L_r/D$, $d_r/D$, $\phi$, $F_t$.

For a fiber of uniform length, the fiber number density, $N_f$, (i.e., the number of fibers per unit volume of fiber suspension) is related to $N_c$:

$$N_f = 6N_c/\pi L_r^3 \propto N_c/L_r^3$$

(15)

Note, however, that for cellulose fiber systems, the fiber number density is actually calculated from $N_f = \rho_m n_C/100$. Hence, Eq. (14) can be written as

$$\Pi' = \frac{F_t \left( \frac{E}{I} \right) d_r}{\left( \rho_t - \rho_g \right) g d_o} N_e^{4/3} N_f^{2/3}$$

(16)

The ratio of the indices of $N_e$ and $N_f$ in Eq. (16) is 4/5, which is significantly different from the value (4/3) found in the experimental study of Tang and Heindel [23], indicating that $\Pi'$ alone is not sufficient to characterize the fiber effects on gas holdup. This is because $\Pi'$ only accounts for the ratio of bubble buoyancy force to fiber network strength. The fiber effects on gas holdup also result from fiber crowding. Thus, consider combining $N_e$ and $\Pi'$ to derive a new dimensionless parameter:

$$\Pi^* = \Pi'^{(4/3)} = \left( \frac{F_t \left( \frac{E}{I} \right) d_r}{\left( \rho_t - \rho_g \right) g d_o} \right)^{(4/3)} N_e^{4/3} N_f^{2/3}$$

(17)

To determine $\alpha$, let the index ratio

$$\frac{4\beta/3 + \alpha}{5\beta/3} = 4$$

(18)

and

$$\frac{4\beta/3 + \alpha + 5\beta/3} = 1$$

(19)

Note that Eqs. (18) and (19) come from the conditions imposed by Tang and Heindel [23] such that the index ratio on $N_e$ and $N_f$ is 4 and the indices sum to 1. Therefore, $\alpha = 16/25$ and $\beta = 3/25$ and

$$\Pi^* = \left( \frac{F_t \left( \frac{E}{I} \right) d_r}{\left( \rho_t - \rho_g \right) g d_o} \right)^{325} N_e^{4/3} N_f^{5/3}$$

(20)

Equation (20) shows that the dimensionless parameter $\Pi^*$ deviates from the fiber effect characterization parameter reported in Tang and Heindel [23] by a multiplier,

$$\left( \frac{F_t \left( \frac{E}{I} \right) d_r}{\left( \rho_t - \rho_g \right) g d_o} \right)^{3/25}$$

(21)

which is a function of fiber stiffness ($EI$), surface friction coefficient ($F_r$), fiber diameter ($d_r$), gas and liquid density difference ($\rho_t - \rho_g$), and orifice diameter ($d_o$). In the present study, ($\rho_t - \rho_g$), $g$, and orifice diameter ($d_o$) do not change.

No direct measurements of $E$, $F_t$, $d_r$ have been performed in this study, but data from the literature are cited to evaluate the significance of this multiplier. Direct measurements performed by Andersson and Rasmuson [77] showed that the inter-fiber friction coefficient did not vary significantly between most fibers used in the current study (i.e., $F_t \approx 0.6$ for wet Rayon and kraft fibers). Amelina et al. [78] also reported a friction coefficient of $F_t \approx 0.5$ for wet cellulose fibers. Hence, we assume $F_t \approx 0.6$ in this study. The elastic moduli ($E$) and fiber wall thickness to fiber diameter ratio ($r_e$) are included in Table 2. The fiber diameter ($d_r$), moment of inertia ($I$), stiffness ($EI$), and ($F_t d_r E I^{3/25}$) are calculated and are also included in Table 2.

Cellulose fibers have lumens. The lumens affect fiber stiffness. In this paper, the influence of the fiber lumen is considered by assuming cellulose fibers are hollow cylinders with a constant fiber wall thickness to fiber diameter ratio ($r_e$) for each fiber type, which are estimated from literature and shown in Table 2. Thus, the fiber diameter ($d_r$) is calculated by

$$d_r = \sqrt[3/2]{\frac{4 \omega}{\pi \rho_f \left[ 1 - (1 - 2r_e)^2 \right]}}$$

(22)

with $\rho_f = 1500$ kg/m$^3$ and the moment of inertia ($I$) is estimated by

$$I = \frac{1}{64} d_r^4 \left[ 1 - (1 - 2r_e)^4 \right]$$

(23)

According to Table 2, the values of the term ($F_r d_r E I^{3/25}$) are very similar for hardwood, softwood, BCTMP, and Rayon fibers. Since ($\rho_t - \rho_g$) $g d_o$ is the same for all fiber types at all operating conditions, Eq. (21) has a similar value for all investigated fibers. Thus, the dimensionless parameter $\Pi^*$ is equivalent to $N_e^{4/3} N_f^{5/3}$, and when the same gas holdup data
Table 2. Wet fiber physical properties.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Fiber Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hardwood</td>
</tr>
<tr>
<td>E (Pa)</td>
<td>1.0e7&lt;sup&gt;16&lt;/sup&gt;</td>
</tr>
<tr>
<td>r&lt;sub&gt;0&lt;/sub&gt;</td>
<td>0.12&lt;sup&gt;16&lt;/sup&gt;</td>
</tr>
<tr>
<td>ω (kg/m)</td>
<td>6.9e-8</td>
</tr>
<tr>
<td>d&lt;sub&gt;r&lt;/sub&gt; (m)</td>
<td>1.18e-5</td>
</tr>
<tr>
<td>I (m&lt;sup&gt;4&lt;/sup&gt;)</td>
<td>2.00e-22</td>
</tr>
<tr>
<td>EI (Nm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>2.00e-15</td>
</tr>
<tr>
<td>[F&lt;sub&gt;rd&lt;/sub&gt;d(EI)]&lt;sup&gt;1/25&lt;/sup&gt;</td>
<td>4.15e-3</td>
</tr>
</tbody>
</table>

Table notes:
<1> No data has been found for wet kraft hardwood fiber. This is an estimated value. It is expected to be larger than that of kraft softwood fiber because fiber of smaller diameter tends to have a larger modulus [79].
<2> For kraft softwood fiber at a yield of ~70% [80].
<3> Ground wood fiber has an elastic modulus of ~9.0 MPa [80]. Here we use 4.5 MPa for BCTMP fiber because it is further chemically pulped and bleached.
<4> From Mauersberger [81].
<5> Estimated from Scallan and Green [82].
<6> An estimation based on the consideration that the BCTMP fiber has lignin attached to the fiber wall.

from Tang and Heindel [23] is plotted as ε vs. Π'<sup>1</sup><sub>f</sub> for constant U<sub>g</sub> and U<sub>l</sub>, it is expected that the data from different fiber types will gather close to a single curve (i.e., different fiber types possess a very similar ε vs. Π'<sup>1</sup><sub>f</sub> trend). This is shown in Fig. 3 with ρ<sub>f</sub> = 1000 kg/m<sup>3</sup>, ρ<sub>g</sub> = 1.29 kg/m<sup>3</sup>, and d<sub>f</sub> = 1.6 mm. The trends in Fig. 3 show that the dimensionless number Π'<sup>1</sup><sub>f</sub> formulated here can sufficiently characterize the fiber effect on gas holdup in a gas-liquid-fiber bubble column. Furthermore, Eq. (21) does not vary with fiber length, so it is constant for Rayon fibers of three different lengths.

Using the parameters in Eq. (1), the dimensionless parameter Π'<sup>1</sup><sub>f</sub> is formulated as

\[ Π'_{f} = φF_{f}^{1/25} \left[ \frac{EI}{(ρ_f-ρ_g)gd_0} \right]^{2/25} \left( \frac{L_{r}}{d_r} \right)^{4/25} \left( \frac{D}{L_{r}} \right)^{12/25} \left( \frac{d_0}{D} \right)^{-3/25} \]  \( (24) \)

This parameter includes d<sub>f</sub>/D and all dimensionless fiber properties except S<sub>r</sub>/L<sub>r</sub> and ρ<sub>f</sub>/ρ<sub>l</sub>. The influence of the fiber properties is mainly on fiber suspension properties, i.e., fiber crowding and network strength, which can be sufficiently quantified by Π'<sup>1</sup><sub>f</sub>. Therefore, the roles of the dimensionless parameters d<sub>f</sub>/D, φ, F<sub>f</sub>, L<sub>r</sub>/d<sub>r</sub>, and Π<sub>l</sub> in Eq. (6) can be replaced by a single dimensionless parameter Π'<sup>1</sup><sub>f</sub>.

Figure 3 Variation of gas holdup with Π'<sup>1</sup><sub>f</sub> (Eq. (20)): (a) U<sub>l</sub> = 2 cm/s and (b) U<sub>l</sub> = 10 cm/s
This results in a simplified dimensionless parameter list of 11 parameters that affect gas holdup in cocurrent gas-liquid-fiber systems:

\[
\left\{ \varepsilon, R_A, \frac{H}{D}, \frac{S_t}{L_t}, \frac{\rho_g}{\rho_l}, \frac{\rho_c}{\rho_l}, \frac{H_k}{U_L}, \Pi_t^*, \Pi_r, \Pi_t, \Pi_l^*, Fr^*, We \right\} \tag{25}
\]

This list can be shortened if \(S_t/L_t\), which represents the fiber length distribution and also has a significant effect on fiber suspension properties [63], is included in a modified expression for \(\Pi_t^*\).

Since the gas holdup versus \(\Pi_t^*\) is the same for different fiber types in Fig. 3 and gas holdup monotonically decreases with increasing \(\Pi_t^*\) (Fig. 3), gas holdup in different fiber suspensions can be quantitatively analyzed by calculating and comparing just \(\Pi_t^*\) for different fiber suspensions. Table 3 summarizes the dimensional and dimensionless values used in Fig. 3.

From Eq. (24), it is easy to see the relative importance of the different factors influencing gas holdup in gas-liquid-fiber bubble columns. As expected, the fiber aspect ratio and fiber volumetric fraction are the two most significant factors.

**CONCLUSIONS**

An overview of all the important parameters influencing gas holdup in cocurrent gas-liquid-fiber bubble columns was conducted and 19 parameters have been identified. A dimensional analysis based on the Buckingham Pi Theorem was completed to derive the dimensionless parameters governing cocurrent gas-liquid-fiber bubble column hydrodynamics. Force analysis was also used to crosscheck the results and explain the physical implications of important dimensionless parameters. Seven dimensionless parameters that were related to the fiber effects on gas holdup were further analyzed and a single dimensionless parameter combining these dimensionless parameters was derived to characterize the overall effect of fiber mass fraction. This dimensionless parameter was demonstrated to be sufficient to quantify the fiber influence on gas holdup in a gas-liquid-fiber cocurrent bubble column. Hence, a method to quantitatively compare gas holdup data from different fiber suspensions was provided, and this method significantly reduced the number of parameters needed to correlate experimental gas holdup data in gas-liquid-fiber bubble columns.

**NOMENCLATURE**

- **a**: exponential parameter in \(N_{c}^{a}N_{l}^{a}\)
- **C**: fiber mass fraction, \(\% (0 \leq C \leq 100)\)
- **D**: bubble column diameter, m
- **d_f**: fiber diameter, m
- **d_o**: gas distributor orifice diameter, m
- **E**: fiber elastic modulus, Pa
- **Fr**: Froude number, \(\rho_{U/g}D/g(\rho_l - \rho_g)\)
- **g**: acceleration due to gravity, \(m/s^2\)
- **H**: bubble column height, m
- **I**: moment of inertia of a fiber, \(m^4\)
- **L**: nominal fiber length, m
- **L_A**: arithmetic average fiber length, m
- **L_f**: length-weighted average fiber length, m
- **N_c**: fiber number density in fiber suspensions, \(l/m^3\)
- **N_f**: number of fibers per unit mass, \(l/g\)
- **p**: pressure, Pa
- **R_A**: gas distributor open area ratio
- **Re_l**: liquid phase Reynolds number, \(\rho_{l}U_{l}/\mu_{l}\)
- **r**: fiber aspect ratio
- **r_6**: fiber wall thickness to fiber diameter ratio
- **S_f**: fiber length standard deviation, m
- **U_g**: superficial gas velocity, \(m/s\)
- **U_l**: superficial liquid velocity, \(m/s\)
- **We**: Weber number, \(\rho_{l}U_{l}^2D/\sigma_l\)

**Greek symbols**

- **\(\Delta p\)**: pressure difference between two column axial locations, Pa
- **\(\epsilon\)**: gas holdup
- **\(\mu_g\)**: gas dynamic viscosity, \(Pa\cdot s\)
- **\(\mu_l\)**: liquid dynamic viscosity, \(Pa\cdot s\)
- **\(\omega\)**: fiber coarseness, \(kg/m\)
- **\(\phi\)**: fiber volume fraction, \(0 \leq \phi \leq 1\)
- **\(\Pi_t\)**: dimensionless parameters, Eq. (5)
- **\(\rho_f\)**: fiber wall material density, \(kg/m^3\)
\( \rho_g \) gas density, kg/m³
\( \rho_l \) liquid density, kg/m³
\( \rho_m \) mixture density, kg/m³
\( \sigma \) surface tension, N/m
\( \tau_{\text{tensile}} \) tensile strength, Pa

Subscripts
0 without aeration
g gas
H at the higher end
i identification number of a column section
L at the lower end
l liquid

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