J/Psi production in heavy ion collisions and gluon saturation

Dmitri Kharzeev
Brookhaven National Laboratory

Eugene Levin
Tel Aviv University

Marzia Nardi
Istituto Nazionale di Fisica Nucleare

Kirill Tuchin
Iowa State University, tuchin@iastate.edu

Follow this and additional works at: http://lib.dr.iastate.edu/physastro_pubs

Part of the Astrophysics and Astronomy Commons, and the Physics Commons

The complete bibliographic information for this item can be found at http://lib.dr.iastate.edu/physastro_pubs/157. For information on how to cite this item, please visit http://lib.dr.iastate.edu/howtocite.html.

This Article is brought to you for free and open access by the Physics and Astronomy at Iowa State University Digital Repository. It has been accepted for inclusion in Physics and Astronomy Publications by an authorized administrator of Iowa State University Digital Repository. For more information, please contact digirep@iastate.edu.
J/Psi production in heavy ion collisions and gluon saturation

Abstract
We calculate the inclusive J/Psi production in heavy ion collisions including the effects of gluon saturation in the wave functions of the colliding nuclei. We argue that the dominant production mechanism in proton–nucleus and nucleus–nucleus collisions for heavy nuclei is different from the one in hadron–hadron interactions. We find that the rapidity distribution of primary J/Psi production is more peaked around midrapidity than the analogous distribution in elementary pp collisions. We discuss the consequences of this fact on the experimentally observed J/Psi suppression in Au–Au collisions at RHIC energies.

Keywords
RIKEN BNL Research Center, high energy QCD, color glass condensate, gluon saturation, space-time picture and high energy, glauber approach

Disciplines
Astrophysics and Astronomy | Physics

Comments
NOTICE: This is the author’s version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
J/Ψ production in heavy ion collisions and gluon saturation

Dmitri Kharzeev \textsuperscript{a}, Eugene Levin \textsuperscript{b}, Marzia Nardi \textsuperscript{c} and Kirill Tuchin \textsuperscript{d,e}

\textit{a) Department of Physics, Brookhaven National Laboratory,}
\textit{Upton, New York 11973-5000, USA}

\textit{b) HEP Department, School of Physics,}
\textit{Raymond and Beverly Sackler Faculty of Exact Science,}
\textit{Tel Aviv University, Tel Aviv 69978, Israel}

\textit{c) Istituto Nazionale di Fisica Nucleare, Sezione di Torino,}
\textit{via P.Giuria 1, I-10125 Torino, Italy}

\textit{d) Department of Physics and Astronomy,}
\textit{Iowa State University, Ames, Iowa 50011, USA}

\textit{e) RIKEN BNL Research Center,}
\textit{Upton, New York 11973-5000, USA}

ABSTRACT: We calculate the inclusive $J/\psi$ production in heavy ion collisions including the effects of gluon saturation in the wave functions of the colliding nuclei. We argue that the dominant production mechanism in proton–nucleus and nucleus–nucleus collisions for heavy nuclei is different from the one in hadron-hadron interactions. We find that the rapidity distribution of primary $J/\psi$ production is more peaked around midrapidity than the analogous distribution in elementary $pp$ collisions. We discuss the consequences of this fact on the experimentally observed $J/\psi$ suppression in $Au − Au$ collisions at RHIC energies.

KEYWORDS: High Energy QCD, Color Glass Condensate, Gluon saturation, Space-time Picture at High energy, Glauber approach.
1. Introduction

Understanding the mechanism of $J/\psi$ production has been a challenge for over three decades. Despite a relatively large charm quark mass, the binding energy of $J/\psi$ is quite small; therefore non-perturbative corrections can be important. This is a likely cause of the difficulties encountered by perturbative QCD in describing the differential $J/\psi$ production cross section and its polarization. A significant effort has been invested into attempts to uncover the mysteries of $J/\psi$ production. Still, when confronted with the experimental data the existing approaches encounter problems that have to be cured by the introduction of additional adjustable parameters encoding the poorly understood dynamics (for a recent review see [1]).

In this paper we develop a new approach to the $J/\psi$ production in nuclear reaction suggested by two of us in [2]. It was argued in Ref. [2] that at high energies the dynamics of $J/\psi$ production is determined mostly by the strength of the coherent quasi-classical fields of the nucleus. This approach yielded a reasonable description of the experimental data, and in particular shone some light on the possible origin of $x_F$ scaling in $p(d)A$ collisions observed in the data from CERN[3], FNAL[4] and RHIC[5].

In the context of high energy nuclear physics, it is important to understand well the mechanism of $J/\psi$ production since $J/\psi$ suppression in heavy ion collisions could serve as a signal of the Quark-Gluon Plasma [6]. Motivated by the urgent necessity to understand the cold nuclear effects on the $J/\psi$ production in heavy ion collisions, we shall calculate the inclusive $J/\psi$ production in heavy ion collisions. In this paper we take into account only the cold nuclear matter effects neglecting completely the dynamical effects leading to the possible formation of the Quark Gluon Plasma.
A systematic approach to the particle production in heavy ion collisions at high energies has been developed in Refs. [7, 8, 9, 10] and will be referred to as the KLN approach. KLN assumes that the wave functions of the colliding nuclei can be described as the Color Glass Condensate (CGC) [12, 13] for which the most characteristic property is the saturation of the parton density [14, 15, 12, 16]. These ideas have passed the first check against the RHIC experimental data on multiplicities and rapidity distributions (see Refs. [7, 8, 9, 10, 11, 17]) and, in this paper, we confront them with the rapidity distribution of \( J/\psi \) mesons at RHIC. Our study in this paper is based on the following observations made in Ref. [2]: (1) The mechanism of \( J/\psi \) production in hadron-nucleus collisions is different from the one in the hadron-hadron interactions; (2) Inclusive \( J/\psi \) production and inclusive \( c\bar{c} \) production are dominated by different distance scales.

To explain our main idea, consider the \( J/\psi \) production in hadron–hadron collisions. The leading contribution is given by the two-gluon fusion

\[
G + G \rightarrow J/\psi + \text{gluon}. \tag{1.1}
\]

This process is of the order \( \mathcal{O}(\alpha_s^5) \): the partonic sub-process is of the order \( \alpha_s^3 \); two additional powers of \( \alpha_s \) arise from attaching the initial gluons to the colliding hadrons. The three-gluon fusion

\[
G + G + G \rightarrow J/\psi \tag{1.2}
\]

is parametrically suppressed as it is proportional to \( \mathcal{O}(\alpha_s^5) \). However, in hadron-nucleus collisions two of the initial state gluons can be attached to the nucleus. This brings in an additional enhancement by \( A^{1/3} \). Since in the quasi-classical approximation \( \alpha_s^2 A^{1/3} \sim 1 \) we find that the three-gluon fusion of (1.2) is actually enhanced by \( 1/\alpha_s \) as compared to (1.1). Similar conclusion holds for heavy ion collisions.

A particularly helpful insight into the nature of the contribution (1.2) is obtained if we note that three-gluon contribution (1.2) is suppressed as compared to the two-gluon one (1.1) by an additional factor \( r^2 \), where \( (2m_c)^{-1} < r < (2m_c\alpha_s)^{-1} \). This factor arises since we need to have three gluons in the area of the order of \( r^2 \). In other words, it means that this reaction originates from the higher-twist contribution. However, in the hadron - nucleus interactions the higher-twist contribution appears always in the dimensionless combination \( r^2 Q_s^2 \) with the saturation scale \( Q_s \). The saturation scale is proportional to \( A^{1/3} \) which compensates for the smallness of \( r \). The dominance of the higher twist process (1.2) is main idea of [2] and we are going to develop it in this paper in the case of heavy ion collisions. Various aspects of multi-parton interactions generating the higher twist effects in \( J/\psi \) production were considered previously in [18, 19, 20, 22, 23, 24, 21, 26, 27, 28].

The paper is structured as follows: in Section 2 we describe the production of \( c\bar{c} \) pairs in proton-nucleus and nucleus-nucleus collisions; in Sections 3 and 4 we calculate the inclusive \( J/\psi \) production cross-section in these collisions. We present our numerical results in Section 5.

2. Warm-up: inclusive production of \( c\bar{c} \) pair with fixed relative momentum

2.1 Hadron-hadron collisions

The process of \( c\bar{c} \) production in a hadron-hadron collision is shown in Fig. 1. The corresponding cross section integrated over the transverse momentum of the pair, denoted by \( \vec{p} \equiv \vec{p} (\vec{p}_2 \equiv \vec{p}) \) in Fig. 1, is equal to the square of the diagram in this figure and can be written as follows

\[
\frac{d\sigma(pp)}{dY d^2k} = \frac{1}{(2\pi)^4} \frac{1}{2(N_c^2 - 1)} \frac{4\pi^2\alpha_s}{N_c} \sum_{s,s',\lambda} \int \frac{d^2l_1}{\pi} \phi_G(x_1,l_1^2) \int \frac{d^2l_2}{2\pi l_2} \phi_G(x_2,l_2^2) \times
\]

\[
2 \int d^2 r \, dz \, \Psi_G(l_1, r, z) \left(1 - e^{il_1 \cdot z} \right) \, e^{-\frac{i}{2} l_1 \cdot z} \int d^2 r' \, \Psi_G^*(l_1, r', z) \left(1 - e^{-il_2 \cdot z'} \right) \, e^{\frac{i}{2} l_2 \cdot z'}, \tag{2.1}
\]

\[
−2−
\]

NOTICE: This is the author's version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
where $x_1 = (m_{c,t} + m_{\bar{c},t}) e^Y / \sqrt{s}$ and $x_2 = (m_{c,t} + m_{\bar{c},t}) e^{-Y} / \sqrt{s}$, with $Y$ being the rapidity of quark and antiquark pair in the center-of-mass frame, $m_{c,t}^2 = m_c^2 + k_{1t}^2$, $m_{\bar{c},t}^2 = m_{\bar{c}}^2 + k_{2t}^2$, $s, s'$ are the quark and anti-quark helicities and $\lambda$ is the gluon polarization. The function $\phi_G(x, l^2)$ is the probability to find a gluon with given $x$ and transverse momentum $l$. It is related to the gluon distribution function $xG(x, Q^2)$ as

$$xG(x, Q^2) = \int d^2 l^2 \phi(x, l^2)$$  

The factor 2 in front of (2.1) is a consequence of the $s$-channel unitarity by which the inelastic cross section equals twice the imaginary part of the elastic scattering amplitude. Eq. (1) is written in the $k_T$-factorization approach which is believed to be valid in hadron-hadron collisions at not too high energies [29, 30, 31].

It is convenient to introduce the cross section for dipole–hadron interaction in the form [32]

$$\sigma(x, r^2) = \frac{8\pi^2 \alpha_s}{N_c} \int \frac{d^2 l}{2\pi l^2} (1 - e^{i r \cdot l}) \phi(x, l^2) \ .$$  

In the DGLAP approximation the dominant contribution to the integral over $l$ comes from the region $lr < 2$ where it picks up the leading logarithmic contribution. Integrating first over all directions of the vector $l$ and then expanding the resulting Bessel function yields:

$$\sigma(x, r^2) \approx \frac{8\pi^2 \alpha_s}{N_c} \int_0^{2/r} d l \frac{1}{4} l^2 \phi(x, l^2) = \frac{\alpha_s \pi^2}{N_c} r^2 xG(x, 4/r^2) \ ,$$  

where we used (2.2).

Using (2.2) and (2.3) it is easy to rewrite (2.1) in the following form

$$\frac{d\sigma(pp)}{dY d^2 k} = \frac{1}{(2\pi)^3 2(N_c^2 - 1)} \sum_{s, s', \lambda} x_1 G(x_1, m_c^2) \times$$

$$\int d^2 r \Psi_G(m_\text{c}, r, z = 1/2) e^{-i \frac{1}{2} z \cdot k} \int d^2 r' \Psi_G^*(m_\text{c}, r', z = 1/2) e^{i \frac{1}{2} z' \cdot k} \sigma_{\text{in}}(x_2, r, r') \ .$$  

---

NOTICE: This is the author’s version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
where
\[
\hat{\sigma}_{in}(x_2, r, r') \equiv \sigma(x_2, r^2) + \sigma(x_2, r'^2) - \sigma(x_2, (r - r')^2).
\tag{2.6}
\]
(The \(\hat{\sigma}\) notation is used to distinguish the dipole cross section defined in (2.6) from the inclusive heavy quark-antiquark inelastic cross section we discuss later, see (2.12)). In derivation of (2.5) we took into account only the DGLAP contribution to \(x_1G(x_1, m_c^2)\) and we treated the c-quark as a non-relativistic particle with \(z = 1/2\). All these simplifications are not important for our main results but allow for a more compact notations.

The gluon light-cone wave function is well-known (Refs. [33, 34, 35]). It has the simplest form for a cylindrical nucleus. In Sec. 5 we perform numerical analyses with realistic nuclear density distributions.

As one can see in Fig. 2 the quark-antiquark pair production in hadron-nucleus interaction includes an additional elastic scattering of dipoles with sizes \(r\) and \(r'\) as well as inelastic interaction at points \(z_i\), which are the longitudinal coordinates of nucleons in the nucleus \(^1\). To include both processes we need to modify (2.1) in the following way
\[
\frac{d\sigma(pp)}{dY d^2k} = x_1G(x_1, m_c^2) \int d^2r \int d^2r' \Phi_G(m_c, r, r', z = 1/2) \epsilon^{1/2}(\not{z} - \not{r}) \hat{\sigma}_{in}(x_2, r, r')
\tag{2.9}
\]
In Appendix we give a detailed derivation of these formulas.

### 2.2 Hadron–heavy nucleus collisions

Production of quark-antiquark pairs in high energy proton-nucleus collisions and in DIS both in the quasi-classical approximation of McLerran-Venugopalan model [12] (summing powers of \(\alpha_s^2 A^{1/3}\)) and including quantum small-\(x\) evolution (summing powers of \(\alpha_s \ln \frac{1}{x}\)) has been calculated in Ref. [36, 37]. This process has been also considered by other authors [38, 39, 40] who obtained similar, though less general, results. Phenomenological applications have been addressed in details in [41, 42]. Using the results of [36, 37, 42] it is not difficult to generalize the formulae of the previous subsection for the case of \(pA\) collisions. The details are given in Appendix. Here we present a derivation that emphasizes the key physical issues.

As one can see in Fig. 2 the quark-antiquark pair production in hadron-nucleus interaction includes an additional elastic scattering of dipoles with sizes \(r\) and \(r'\) as well as inelastic interaction at points \(z_i\), which are the longitudinal coordinates of nucleons in the nucleus \(^1\). To include both processes we need to modify (2.1) in the following way
\[
\frac{d\sigma_{in}(pA)}{dY d^2k d^2b} = x_1G(x_1, m_c^2) \int d^2r \int d^2r' \Phi_G(m_c, r, r', z = 1/2) \epsilon^{1/2}(\not{z} - \not{r})
\tag{2.10}
\]

\(^1\)Note that \(z (z')\) appearing in (2.7), (2.8) etc. denote the fraction of the gluon’s light-cone momentum carried by the c-quark in the (complex conjugated) amplitude. \(z_i\)’s with \(i = 0, 1, 2, \ldots \) in Fig. 2 etc. denote the longitudinal coordinates of nucleons in the nucleus. These are two completely unrelated variables.
The factor $\exp\left\{-|\sigma(x_2, r^2) + \sigma(x_2, r'^2)|\rho 2 R_A\right\}$ in (2.10) describes the fact that neither dipole with the size $r$ (in the amplitude) nor dipole with the size $r'$ (in the complex conjugate amplitude) interacts inelastically with the nucleons of the nucleus between the points 0 and $z_1$ as well as between any other pair of points $z_i$ and $z_{i+1}$, where $i = 0, 1, \ldots, n - 1$. In deriving Eq. (2.10) we assumed that the collision energy is high enough so that the quark-antiquark pair is produced long before it starts to interact with the nucleus. This corresponds to coherent interaction of the pair with all nucleons of the target nucleus. It has been demonstrated in [41, 42] that this is indeed the case for charm quark production in central and forward $^2$ rapidities at RHIC and LHC.

We assume in (2.10) that the initial $c\bar{c}$ pair (with transverse momentum $l_1$ in Fig. 1) is colorless. Indeed, for large values of $k$ we can view the result of our calculation as a product of two factors: the probability to find a gluon ($l_1$) in the projectile hadron and its structure function in the target nucleus. The gluon structure function can be modeled by the interaction of a colorless probe such as dilaton or graviton ([33, 43]) with the nucleus through the splitting into the colorless $c\bar{c}$ pair. In Appendix we present a formal derivation of all the main results of this section by direct summation of the corresponding Feynman diagrams in the light-cone perturbation theory along the lines of the dipole model [33, 44].

Doing integrals over the longitudinal positions $z_i$ of nucleons and summing over $n$ in (2.10) we obtain

$$
\int_{0}^{2R_A} \rho \hat{\sigma}_{in}(x_2, r, r') dz_0 e^{-[|\sigma(x_2, r^2) + \sigma(x_2, r'^2)|]\rho 2 R_A} \sum_{n=0}^{\infty} \int_{z_0}^{2R_A} dz_1 \cdots \int_{z_{n-2}}^{2R_A} dz_{n-1} \int_{z_{n-1}}^{2R_A} dz_n \rho^n \hat{\sigma}^n_{in}(x_2, r, r')
$$

$$
= \exp \left\{-|\sigma(x_2, r^2) + \sigma(x_2, r'^2)|\rho 2 R_A\right\} \left( e^{\hat{\sigma}_{in}(x_2, r, r') \rho 2 R_A} - 1 \right)
$$

$$
= \exp \left\{-|\sigma(x_2, (r - r')^2)|\rho 2 R_A\right\} - \exp \left\{-|\sigma(x_2, r^2) + \sigma(x_2, r'^2)|\rho 2 R_A\right\}.
$$

Using (2.11) we can reduce (2.10) to the following expression

$$
\frac{d\sigma_{in}(pA)}{dY d^2 k d^2 b} = x_1 g(x_1, m^2) \int d^2 r dz e^{-i\frac{L}{2} L} \int d^2 r' dz' e^{i\frac{L}{2} L'} \Phi_G(L_1, r, r', z)
$$

$$
\times \left( \exp \left\{-|\sigma(x_2, (r - r')^2)|\rho 2 R_A\right\} - \exp \left\{-|\sigma(x_2, r^2) + \sigma(x_2, r'^2)|\rho 2 R_A\right\} \right).
$$

$^2$By forward rapidities we mean the direction of the projectile fragmentation.
This equation accounts only for the inelastic interaction and the physical meaning of (2.12) is the cross section for all possible inelastic interaction in which the $c\bar{c}$ pair is produced. We need to add the cross section for the elastic production of the quark-antiquark pair, which reads

$$\frac{d\sigma_{el}(pA)}{dY d^2k d^2b} = x_1G(x_1, m^2_c) \int d^2r e^{-i\hat{k} \cdot \hat{r}} \int d^2r' \, e^{i\hat{k}' \cdot \hat{r}'} \Phi_{G}(l_1, r, r', z = 1/2) \left\{ 1 - \exp[-\sigma(x_2, r^2) \rho \, 2R_A] \right\} \left\{ 1 - \exp[-\sigma(x_2, r'^2) \rho \, 2R_A] \right\} .$$

(2.13)

The sum of (2.12) and (2.13) gives

$$\frac{d\sigma_{tot}(pA)}{dY d^2k d^2b} = x_1G(x_1, m^2_c) \int d^2r e^{-i\hat{k} \cdot \hat{r}} \int d^2r' \, e^{i\hat{k}' \cdot \hat{r}'} \Phi_{G}(l_1, r, r', z = 1/2) \left\{ 1 - \exp[-\sigma(x_2, r^2) \rho \, 2R_A] \right\} \left\{ 1 - \exp[-\sigma(x_2, r'^2) \rho \, 2R_A] \right\} + \exp[-\sigma(x_2, (\vec{r} - \vec{r}')^2) \rho \, 2R_A] \right\}.$$

(2.14)

Introducing the quark saturation scale $Q_s^2$ (see (A.13) and (A.14)) we can write

$$\sigma(x, r^2) \rho \, 2R_A = \frac{1}{4} r^2 Q_s^2 \rho \, 2R_A(x).$$

(2.15)

The form of $Q_s^2$ is determined by the phenomenology of low $x$ DIS [45, 46, 47, 48, 50] and forward hadron production in $p(d)A$ collisions [49, 51, 52, 53]. Introducing a new dimensionless variable $\zeta = m_c / x$ we can rewrite (2.14) as

$$\frac{d\sigma_{tot}(pA)}{dY d^2k d^2b} = \frac{1}{\alpha_s} \frac{\alpha_s}{\pi} x_1G(x_1, m^2_c) \int d^2\zeta d^2\zeta' \, e^{i\hat{k} \cdot \zeta / (2m_c)} \left\{ \frac{1}{2 \, \zeta' \, \zeta} K_1(\zeta) K_1(\zeta') + K_0(\zeta) K_0(\zeta') \right\} \left\{ 1 - \exp[-\zeta^2 Q_s^2 / (4m^2_c)] - \exp[-\zeta'^2 Q_s^2 / (4m^2_c)] + \exp[-(\zeta - \zeta')^2 Q_s^2 / 4m^2_c] \right\}.$$

(2.16)

It was pointed out in Ref. [2] that the dominant contribution to the integrals on the r.h.s. of (2.16) is originating from the integration region $\zeta' \ll r \ll 1/m_c$, i.e. $\zeta' \ll \zeta \ll 1$ (or, equivalently, $\zeta \ll \zeta' \ll 1$). In this kinematic region (2.16) reduces to the following expression

$$\frac{d\sigma_{tot}(pA)}{dY d^2k d^2b} = \frac{1}{\alpha_s} \frac{\alpha_s}{\pi} x_1G(x_1, m^2_c) \int_0^\infty d\zeta^2 K_0(\zeta_0) J_0(\kappa \zeta / 2m_c) \int_0^\infty d\zeta'^2 K_0(\zeta') \left\{ 1 - \exp[-\zeta'^2 Q_s^2 / (4m^2_c)] \right\}.$$

(2.17)

In the saturation region $Q_s \gg m_c$ the dipole scattering amplitude reaches its unitarity limit $1 - e^{-\zeta^2 Q_s^2 / 4m^2_c} \approx 1$. Therefore, the rapidity distribution becomes

$$\frac{d\sigma_{tot}(pA)}{dY d^2k d^2b} \propto x_1G(x_1, m^2_c) \propto \exp(-\lambda Y),$$

(2.18)

while for the same process in hadron-hadron collisions we have (see (2.1))

$$\frac{d\sigma_{tot}(pp)}{dY d^2k d^2b} \propto x_1G(x_1, m^2_c) x_2G(x_2, m^2_c) \propto \text{constant}(Y),$$

(2.19)

where we assumed that $xG(x, m^2_c) \propto 1/x^\lambda$ at low $x$ (which is true if $Y$ is not too close to the proton fragmentation region). It is clear that there is a substantial difference between the rapidity distribution in these two cases.

---

NOTICE: This is the author’s version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
2.3 Nucleus-nucleus collisions in the KLN approach

Nucleus-nucleus interaction can be characterized by the saturation scale which depends on the properties of both nuclei. In the KLN approach it is assumed that multi-particle production is entirely determined by the saturation scales of the colliding nuclei \( Q^2_{s,A_1}(x_1) \) and \( Q^2_{s,A_2}(x_2) \). In the spirit of this approach we will generalize (2.14) to the case of nucleus-nucleus scattering using the Kovchegov’s conjecture [54]. In [55] Kovchegov and Mueller noted that in order that their calculation of gluon production in \( pA \) collisions be self-consistent, an entire class of the final state interactions must cancel out in the light-cone gauge. Although they did not find a physical reason for that, Kovchegov suggested that the same conclusion may hold in \( AA \) collisions as well. Using this assumption he derived an expression for gluon production in heavy ion collisions in the light-cone gauge. In Appendix we derive (A.33) along the same lines of reasoning. Here we would like to review the main steps.

The main contribution to the inclusive cross section of \( c\bar{c} \) production stems from the diagrams shown in Fig. 3. The sum of the diagrams of Fig. 3-a are proportional to

\[
x_1 G(x_1, m^2_c) \left\{ e^{-\frac{Q^2_{s,A_1}(x_1)}{8}(x - x')^2} - 1 \right\},
\]

while the diagrams of Fig. 3-b are proportional to

\[
x_1 G(x_1, m^2_c) \left\{ \left(1 - e^{-\frac{Q^2_{s,A_1}(x_1)}{8} r^2} \right) + \left(1 - e^{-\frac{Q^2_{s,A_2}(x_2)}{8} r'^2} \right) \right\}
\]

The sum of (2.20) and (2.21) gives (2.14).

For nucleus-nucleus collisions the main contribution stems from the set of the diagrams given in Fig. 4, which
can be written as

$$\frac{d\sigma_{tot}(AA)}{dY d^{2}k d^{2}b d^{2}b'} = \frac{\alpha_s N_c}{(2\pi)^{3} m_c^2 \pi} \frac{4 N_c}{\alpha_s \pi^2} \int d^{2}\zeta d^{2}\zeta' e^{i\vec{k} \cdot \left(\vec{\zeta} - \vec{\zeta}'\right)}/2m_c \left(\frac{1}{2} \frac{\zeta \cdot \zeta'}{\zeta' \zeta} K_1(\zeta)K_1(\zeta') + K_0(\zeta)K_0(\zeta')\right)$$

$$\times \left(\frac{1}{\zeta^2} \left\{1 - \exp[-\zeta^2 Q_{s,A_1}^2/8m_c^2]\right\} \left\{1 - \exp[-\zeta'^2 Q_{s,A_2}^2/8m_c^2]\right\} + \frac{1}{\zeta'^2} \left\{1 - \exp[-\zeta'^2 Q_{s,A_2}^2/8m_c^2]\right\} \left\{1 - \exp[-\zeta^2 Q_{s,A_1}^2/8m_c^2]\right\}ight) - \frac{1}{(\zeta - \zeta')^2} \left\{1 - \exp[-(\zeta - \zeta')^2 Q_{s,A_1}^2/8m_c^2]\right\} \left\{1 - \exp[-(\zeta - \zeta')^2 Q_{s,A_2}^2/8m_c^2]\right\} \right)$$

(2.22)

One can see that the first two terms in (2.22) are the same as (2.21) where factor $x_1 G(x_1, m_c^2)$ is replaced by (see (A.32))

$$\frac{\alpha_s \pi^2}{4 N_c} x_1 G(x_1, m_c^2) \rightarrow \frac{d^2 b}{r'^2} \left(1 - e^{-r'^2 Q_{s,A_1}^2/8m_c^2}\right)$$

(2.23)

or

$$\frac{\alpha_s \pi^2}{4 N_c} x_1 G(x_1, m_c^2) \rightarrow \frac{d^2 b}{r'^2} \left(1 - e^{-r'^2 Q_{s,A_2}^2/8m_c^2}\right)$$

while the last term in (2.22) is equal to (2.20) with the replacement

$$\frac{\alpha_s \pi^2}{4 N_c} x_1 G(x_1, m_c^2) \rightarrow \frac{d^2 b}{(r - r')^2} \left(1 - e^{-r'^2 Q_{s,A_1}^2/8m_c^2}\right)$$

(2.24)

Notice: This is the author's version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
The discussion and derivations of the previous section now allow us to turn to the main subject of our paper. Fig. 6

**Figure 5:** The ratio $R(Y)$ defined in (2.27), for the process of inclusive $c\bar{c}$ production with fixed relative momentum $\vec{k} = 0$ in nucleus-nucleus collisions. The calculation is performed for the Gold nuclei collision at RHIC energy $\sqrt{s} = 200$ GeV using the KLN value for the saturation scale [7].

To understand this replacement we notice that the last two lines in (2.10) can be written as

$$
\int_0^{2R_A} \rho x_2 G(x_2, m_c^2) d z_0 e^{-\left[\sigma(x_2, r^2) + \sigma(x_2, r'^2)\right]} \rho (2R_A - z_0) \sum_{n=0}^{\infty} \int_0^{2R_A} d z_1 \ldots \int_0^{2R_A} d z_{n-1} \int_0^{2R_A} d z_n \rho^n \hat{\sigma}^n_{in}(x_2, r, r') \\
= \int_0^{2R_A} \rho x_2 G(x_2, m_c^2) d z_0 \exp \left\{ - \left[\sigma(x_2, r^2) + \sigma(x_2, r'^2) - \hat{\sigma}_{in}(x_2, r, r')\right] \rho (2R_A - z_0) \right\} \rightarrow (2.25) \\
\frac{x_2 G(x_2, A/|\vec{r} - \vec{r}'|)}{\sigma(x_2, r^2) + \sigma(x_2, r'^2) - \hat{\sigma}_{in}(x_2, r, r')} \left( 1 - \exp \left\{ - \left[\sigma(x_2, r^2) + \sigma(x_2, r'^2) - \hat{\sigma}_{in}(x_2, r, r')\right] \rho 2R_A \right\} \right) \\
= \sum_{s,A_1} \frac{1}{\alpha_s^2 (\vec{r} - \vec{r}')^2} \left( 1 - \exp \left\{ - \frac{(\vec{r} - \vec{r}')^2 Q^2_{s,A_1}(x_2)}{8} \right\} \right) (2.26)
$$

This corresponds to the diagram of Fig. 4-a. The sum in (2.25) reflects the fact that the dipole can scatter elastically only after the first inelastic interaction. Contribution of the diagram Fig. 4-b is treated in the same way. The low density limits, i.e. hadron-hadron or hadron–nucleus collisions are reproduced when $Q^2_{s,A_1} \ll m_c^2$ and/or $Q^2_{s,A_2} \ll m_c^2$.

In Fig. 5 we plot the ratio

$$
R(Y) = \left. \frac{d\sigma_{tot}(AA)}{dY d^2k d^2b} \right|_{k=0, \ Y=0} \\
\left. \frac{d\sigma_{tot}(AA)}{dY d^2k d^2b} \left|_{k=0, \ Y=0} \right. \right.
$$

as function of rapidity $Y$. This ratio has a much sharper maximum at $Y = 0$ than the corresponding ratio in $pp$ collisions.

### 3. $J/\psi$ production in hadron-nucleus collisions

#### 3.1 New production mechanism off nuclear targets

The discussion and derivations of the previous section now allow us to turn to the main subject of our paper. Fig. 6
Figure 6: The process of inclusive $J/\psi$ production in hadron-hadron (Fig. 6-A) and in hadron-nucleus collisions (Fig. 6-B).

displays the $J/\psi$ meson production in $pp$ and $pA$ collisions at the leading order in $\alpha_s^2 A^{1/3}$. The cross section for the latter is a direct generalization of (2.1) and reads 3

$$
\frac{d\sigma}{dY \, d^2b} \propto \int_0^{2R_A} \rho \, dz_0 \int_0^{z_0} \rho \, dz_1 \int \frac{d^2l_1}{2\pi} \phi_G(x_1, l_1) 2 \int d^2r \, dz \, \Psi_G(l_1, r, z) \otimes \Psi_V(r, z) \left( 1 - e^{i\vec{r} \cdot \vec{r}} \right) \left( 1 - e^{-i\vec{r} \cdot \vec{r}} \right) \\
\times 2 \int d^2r' \, dz' \, \Psi_G^*(l_1, r', z) \otimes \Psi_V^*(r', z') \left( 1 - e^{-i\vec{r} \cdot \vec{r}} \right) \left( 1 - e^{i\vec{r} \cdot \vec{r}} \right) \\
\times \int \frac{d^2l_2}{2\pi} \frac{l_2}{l_3} \phi_G(x_2, l_2) \int \frac{d^2l_3}{2\pi} \phi_G(x_2, l_3)
$$

(3.1)

where $\Psi_G \otimes \Psi_V$ is projection of the $J/\psi$ light-cone “wave-function” onto the virtual gluon one. Trace over all relevant quantum numbers is implied. Assuming that $\Psi_V(r, x) \propto \delta(z - 1/2) \delta(r)$ and $l_{ij}^2 z(1 - z) \ll m_i^2$, $i = 1, 2, 3$, this projection takes the following form [2, 32]

$$
\Psi_G(m_c, r, z) \otimes \Psi_V(r, z) = \sqrt{\frac{3}{48\pi\alpha_{em}}} \frac{\Gamma_{J/\psi-e^+e^-}^c}{M_{J/\psi}} \frac{m_c^3 r^2}{4} K_2(m_c r).
$$

(3.2)

At short distances $r, r' < 1/m_c$, the cross section in (3.1) is proportional to $r^2 r'^2$. This fact reflects the higher twist nature of the suggested mechanism. Since the position of the pair of nucleons is not fixed the full contribution should be proportional to $A^{1/3}$ while the mechanism of Fig. 6-A leads to an enhancement by $A^{1/3}$. The enhancement factor stems from integrations over $z_0$ and $z_1$ in (3.1)

$$
\frac{d\sigma}{dY \, d^2k} \propto \int_0^{2R_A} \rho \, dz_0 \int_0^{z_0} \rho \, dz_1 = \frac{1}{2} (\rho 2R_A)^2.
$$

(3.3)

Parametrically, the mechanism in Fig. 6-B is different from that in Fig. 6-A by the factor $\alpha_s A^{1/3}$. Therefore, in the spirit of the quasi-classical approximation in which we assume that $\alpha_s^2 A^{1/3} \sim 1$ we conclude that the mechanism in Fig. 6-B is enhanced by a big factor $1/\alpha_s$.

3.2 Propagation of the colourless $c\bar{c}$ pair through a nuclear target

In Ref. [2] detailed arguments were given which justify the application of the dipole model for calculation of $J/\psi$ production at forward rapidities at RHIC. It has been argued that the coherence length for the $c\bar{c}$ pair is sufficiently

---

NOTICE: This is the author's version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
larger than the longitudinal extent of the interaction region. This means that the development of the light-cone “wave function” happens a long time before the collision. We relied on this physical picture in our calculations in the previous section. Concerning $J/\psi$, its production is characterized by an additional formation time scale proportional to the inverse binding energy $\sim \alpha_s^2 m_c$; for a quantitative estimate, see [56]. This time is certainly larger than the charm quark pair production time (by a factor of about $1/\alpha_s^2$) implying that the formation process takes place far away from the nucleus. Therefore, in the following, we concentrate on the dynamics of $c\bar{c}$ pair interactions with the nucleus.

Figure 7: The process of inclusive $J/\psi$ production in hadron-nucleus collisions due to the interaction with an even number of nucleons.

Since the soft gluon emission processes are suppressed in the quasi-classical approximation, the $J/\psi$ meson is predominantly produced through the hadronization of the color singlet $c\bar{c}$ pair. This rules out diagrams of the type Fig. 3-b corresponding to the elastic interaction. The diagrams of the type Fig. 3-a are shown in Fig. 7. Note that since the $J/\psi$ quantum numbers are $1^-$ while those of gluons are $1^-$ an odd number of gluons must connect to the charm quark line. Consequently, each inelastic interaction of the $c\bar{c}$ pair must involve two nucleons. To take this into account we write an analogue of (2.10) in which the sum over all inelastic processes (i.e. sum over $n$) involves only even number of interactions. We have

$$
\frac{d\sigma_{in}(pA)}{dY db} = C_F x_1 G(x_1, m_c^2) \times \int_0^{2RA} \rho \hat{\sigma}_{in}(x_2, r, r') d z_0 \int dt \, d^2 \Psi_G(l_1, r, z = 1/2) \Psi_V(r) \otimes \int d^2 t' \Psi_G^*(l_1, r', z = 1/2) \Psi_V^*(r')
$$

$$
\times \left( e^{-\sigma(x_2, r^2) + \sigma(x_2, r'^2)} \rho^2 R_A \sum_{n=0}^{\infty} \int_{z_0}^{2R_A} dz_1 \int_{z_1}^{2R_A} dz_2 \ldots \int_{z_{2n}}^{2R_A} \int \right)
$$

The color factor in front comes from the calculation of Fig. 6-B, namely, it is equal to

$$
\text{Tr}(t^a t^b t^c) \text{Tr}(t^a t^b t^c') \delta_{cc'} = \frac{1}{16} \delta_{cc'} \left(f_{abc} f_{abc'} + d_{abc} d_{abc'}\right)
$$

$$
= \frac{(N_c^2 - 1)}{16 N_c} \left( N_c + \frac{N_c^2 - 4}{N_c} \right) = \frac{N_c^2 - 1}{2 N_c} \frac{N_c^2 - 2}{4} = \left( \frac{N_c^2 - 1}{2 N_c} \right)^2 \frac{N_c(N_c^2 - 2)}{2(N_c^2 - 1)} \approx C_F^2.
$$

Since $\hat{\sigma}_{in}$ is proportional to $C_F = (N_c^2 - 1)/2 N_c$, we extract this factor from the color coefficient of (3.5). The last of equations in (3.5) is written in the large $N_c$ approximation.
We argued in Sec. 2.3 (see Fig. 3) and in the Appendix that $c\bar{c}$ pair in the color octet state passes through the target with the same elastic (see Fig. 3-b and (2.21)) and inelastic (see Fig. 3-a and (2.20)) cross sections as the $c\bar{c}$ pair in the color singlet state. This is the reason we do not need to change (3.4) to include the color octet state interaction with the target.

After integration over $z_1$'s and summation over $n$ using the identity $\sum_{n=0}^{\infty} a^{2n+2}/(2n+2)! = \cosh a - 1$ we obtain the following formula

$$\frac{d\sigma_{in}(pA)}{dY d^2b} = C_F x_1 G(x_1, m_c^2) \int d^2r \Psi_G(l_1, r, z = 1/2) \Psi_V(r) \int d^2r' \Psi_G^*(l_1, r', z = 1/2) \Psi_V^*(r') \times \frac{1}{2} \left\{ \exp \left[ -2 \sigma(x_2, r') \rho 2RA \right] - 2 \exp \left[ -2 \sigma(x_2, r) \rho 2RA \right] \right\}.$$  (3.6)

The color factor in (3.4) as well as in (3.6) corresponds to the diagram of Fig. 6-B.

### 3.3 J/Ψ production in hadron-nucleus collisions in the saturation regime

In the quasi-classical approximation the gluon saturation scale is given by [33, 43], see (A.13)

$$Q_{s,A}^2(x) = 4\pi^2 \alpha_s^2 \rho T(b).$$  (3.7)

where $\rho$ is the nucleon density in a nucleus, $N_c$ is the number of colours, $b$ is the impact parameter and $T(b)$ is the optical width of the nucleus. Eq. (3.7) determines the scale of the typical transverse momenta for the inclusive gluon production [44]. Its value was extracted from the fit to the hadron multiplicities in nuclear collisions at RHIC [10, 7]. However, for the penetration of the quark-antiquark pair the typical saturation scale is about twice as small and we refer to it as the quark saturation scale $Q_{s,A}^2$, see (A.14). This scale was extracted from fits of the $F_2$ structure function in DIS [43, 45, 46, 47, 48] as we have already mentioned. Both phenomenological approaches agree with each other, so the use of either quark or a properly rescaled gluon saturation scale is merely a matter of convention.

In this paper we will use the quark saturation scale (3.7). Using this definition for the saturation momentum we have $\sigma(x_2, r) = r^2 Q_{s,A}^2(x_2)/8$ (cf. (2.15)). Substituting this expression into (3.6) we can re-write it in a more convenient form

$$\frac{d\sigma_{in}(pA)}{dY d^2b} = C_F x_1 G(x_1, m_c^2) \int d^2r \Psi_G(l_1, r, z = 1/2) \Psi_V(r) \int d^2r' \Psi_G^*(l_1, r', z = 1/2) \Psi_V^*(r') \times \frac{1}{2} \left\{ \exp \left[ -(r^2 - r')^2 Q_{s,A}^2 / 8 \right] + \exp \left[ -(r^2 + r')^2 Q_{s,A}^2 / 8 \right] - 2 \exp \left[ -(r^2 + r')^2 Q_{s,A}^2 / 8 \right] \right\}.$$  (3.8)

Integrating over the angle between $\vec{r}$ and $\vec{r}'$ we derive

$$\frac{d\sigma_{in}(pA)}{dY d^2b} = \frac{N_c (N_c^2 - 2)}{2 (N_c^2 - 1)} x_1 G(x_1, m_c^2) \int d^2r \Psi_G(l_1, r, z = 1/2) \Psi_V(r) \int d^2r' \Psi_G^*(l_1, r', z = 1/2) \Psi_V^*(r') \times \exp \left[ -(r^2 + r'^2) Q_{s,A}^2 / 8 \right] \left\{ I_0 \left( \frac{Q_{s,A}^2}{4} r r' \right) - 1 \right\}.$$  (3.9)
Deeply in the saturation region where $Q_{s,A} \gg m_c$ the typical dipole sizes are much smaller than $1/m_c$. Thus, we can expand the wave function (3.2) $\Psi_G \otimes \Psi_V \approx \text{const}$. The main contribution comes from $(r - r')^2 \leq 1/Q^2_{s,A}$ while $r \approx 1/m_c$. It gives

$$\frac{d\sigma_{in}(pA)}{dY d^2b} \propto x_1 G(x_1, m^2_c) / Q^2_{s,A}(x_2) \propto \exp (-2\lambda Y) .$$

(3.10)

In deriving (3.10) we used the same assumptions as in the case of $c\bar{c}$-pair production with fixed relative momentum (see (2.18)). One can see that (3.10) leads to a rapidity distribution that is more narrow than the distribution in hadron–hadron collisions given by (2.19).

### 4. Inclusive $J/\psi$ production in nucleus–nucleus collisions

Using the same arguments as in Sec. 2.3 which led us to (2.22) we can generalize (3.9) to obtain our main result – the formula for $J/\psi$ production in nucleus–nucleus collisions. It reads

$$\frac{1}{S_A} \frac{d\sigma(AA)}{dY d^2b} = \frac{C_F}{4\pi^2\alpha_s} \int d^2r \, \Psi_G(l_1, r, z = 1/2) \otimes \Psi_V(r) \int d^2r' \, \Psi_G^*(l_1, r', z = 1/2) \otimes \Psi_V^*(r')$$

$$\times \frac{1}{2^r \cdot l'} \left\{ \exp \left( -\frac{1}{8} (l - l')^2 (Q^2_{s,A_1} + Q^2_{s,A_2}) \right) - \exp \left( -\frac{1}{8} (l + l')^2 (Q^2_{s,A_1} + Q^2_{s,A_2}) \right) \right. - \exp \left( -\frac{1}{8} (l - l')^2 Q^2_{s,A_1} - \frac{1}{8} (r^2 + r'^2) Q^2_{s,A_2} \right) + \exp \left( -\frac{1}{8} (l + l')^2 Q^2_{s,A_1} - \frac{1}{8} (r^2 + r'^2) Q^2_{s,A_2} \right)$$

$$\left. - \exp \left( -\frac{1}{8} (l - l')^2 Q^2_{s,A_2} - \frac{1}{8} (r^2 + r'^2) Q^2_{s,A_1} \right) + \exp \left( -\frac{1}{8} (l + l')^2 Q^2_{s,A_2} - \frac{1}{8} (r^2 + r'^2) Q^2_{s,A_1} \right) \right\} ,$$

where $S_A$ is the transverse overlap area. One can check that this formula describes the hadron-nucleus $J/\psi$ assuming that $Q^2_{s,A_1}$ is small.

![Figure 8](image_url)

**Figure 8:** The process of inclusive $J/\psi$ production in nucleus-nucleus collisions due to inelastic interaction with both nuclei.

In deriving (4.1) we summed up the inelastic cross sections for both nuclei. Let us denote the nucleon coordinates in the nucleus $A_1$ by $z'_i$ and in the nucleus $A_2$ by $z_i$. The number of possible inelastic interactions (see Fig. 8) is odd for both nuclei. For a fixed number of total inelastic interactions $2n - 1$ the number of interactions in each nucleus

---

NOTICE: This is the author’s version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.
can be $1 \leq k \leq 2n - 2$. Therefore, for the term inside the curly brackets in (4.1) we have

$$
\{ \ldots \} = \int_0^{2R_{A_2}} \int_0^{2R_{A_1}} \left( \frac{1}{8} Q_{s,A_2}^2 \right) \left( \frac{1}{8} Q_{s,A_1}^2 \right) (2z_0 d_z z' 2z_0 d_z z') \exp \left\{ - \frac{1}{8} (r^2 + r'^2) (Q_{s,A_1}^2 + Q_{s,A_2}^2) \right\}
\times \sum_{k=1}^{2n-2} \sum_{k=1}^{2n-2} \int_{z_0}^{2R_{A_2}} \int_{z_1}^{2R_{A_1}} d_z d_z \ldots \int_{z_{k-2}}^{2R_{A_2}} d_z \rho^{k} \left( \frac{1}{8} Q_{s,A_2}^2 2 \mathbf{r} \cdot \mathbf{r}' \right)^k - 1
\times \sum_{n=2}^{\infty} \sum_{n=2}^{\infty} \int_{z_1}^{2R_{A_1}} d_z \ldots \int_{z_{n-k-2}}^{2R_{A_1}} d_z \rho^{n-k-1} \left( \frac{1}{8} Q_{s,A_1}^2 2 \mathbf{r} \cdot \mathbf{r}' \right)^{2n-k-2}
$$

(4.2)

Using the following mathematical identity

$$
\sum_{k=1}^{j-1} \frac{1}{k!(j-k)!} a^k b^{j-k} = \frac{1}{j!} (a+b)^{j} - \frac{1}{j!} a^j - \frac{1}{j!} b^j
$$

(4.3)

with $j = 2n - 1$ we obtain

$$
= \frac{1}{(2n-1)!} \sum_{n=2}^{\infty} \left( \frac{1}{8} Q_{s,A_2}^2 (Q_{s,A_1}^2 + Q_{s,A_2}^2) 2 \mathbf{r} \cdot \mathbf{r}' \right)^{2n-1} - \frac{1}{(2n-1)!} \left( \frac{1}{8} Q_{s,A_1}^2 Q_{s,A_2}^2 2 \mathbf{r} \cdot \mathbf{r}' \right)^{2n-1}
$$

$$
= \sinh \left( \frac{1}{8} Q_{s,A_1}^2 Q_{s,A_2}^2 2 \mathbf{r} \cdot \mathbf{r}' \right) - \sinh \left( \frac{1}{8} Q_{s,A_1}^2 Q_{s,A_2}^2 2 \mathbf{r} \cdot \mathbf{r}' \right) - \sinh \left( \frac{1}{8} Q_{s,A_1}^2 Q_{s,A_2}^2 2 \mathbf{r} \cdot \mathbf{r}' \right).
$$

(4.4)

yielding (4.1). If $Q_{s,A_2}^2 \ll Q_{s,A_1}^2$, we can expand (4.4) using $\sinh(a + b) - \sinh a - \sinh b \approx b (\cosh a - 1) + \mathcal{O}(b^2)$; then (4.1) reduces to (3.8).

For a qualitative discussion it is instructive to rewrite (4.1) in the region $r' \ll r \approx 1/Q_{s,A} \ll 1/m_c$. Expanding expression in the curly brackets we derive

$$
\{ \ldots \} = \frac{1}{64} Q_{s,A_1}^2 Q_{s,A_2}^2 (Q_{s,A_1}^2 + Q_{s,A_2}^2) (\mathbf{r} \cdot \mathbf{r}')^3 (1 + \mathcal{O}(Q_{s,A}^2 r'^2, Q_{s,A}^2 r^2, Q_{s,A}^2)) \exp \left\{ - \frac{1}{8} (Q_{s,A_1}^2 + Q_{s,A_2}^2) r^2 \right\}
$$

(4.5)

In this approximation Eq. (4.1) becomes (after integration over the angle between $\mathbf{r}$ and $\mathbf{r}'$)

$$
\frac{1}{S_A} \frac{d\sigma(\AA)}{dY} \propto \int d^2 r \Psi_G(l_1, l, z = 1/2) \otimes \Psi_V(r) \int d^2 r' \Psi_G^*(l_1, l', z' = 1/2) \otimes \Psi_V^*(r')
	imes Q_{s,A_1}^2 Q_{s,A_2}^2 (Q_{s,A_1}^2 + Q_{s,A_2}^2) r^2 r'^2 \exp \left\{ - r^2 (Q_{s,A_1}^2 + Q_{s,A_2}^2) / 8 \right\}
$$

(4.6)

$$
\times \frac{Q_{s,A_1}^2 Q_{s,A_2}^2}{(Q_{s,A_1}^2 + Q_{s,A_2}^2)^3}
$$

(4.7)

In (4.7) we replaced the wave functions of (3.2) by a constant as already discussed in Sec. 3.3 and took the integral over the angle between $\mathbf{r}$ and $\mathbf{r}'$.

From (4.7) one can see that the spectrum of $J/\psi$'s in ion-ion collisions is more narrow than the one in hadron-hadron. Explicitly

$$
\frac{d\sigma(\AA)}{dY} \propto \frac{d\sigma(pp)}{dY} \left( \frac{1}{Q_{s,A_1}^2 + Q_{s,A_2}^2} \right)^3 \propto \frac{d\sigma(pp)}{dY} e^{-2\Lambda|Y|},
$$

(4.8)
Figure 9: Rapidity dependence of the ratio \( R(Y) = \frac{d\sigma}{dY}(Y) / \frac{d\sigma}{dY}(Y=0) \) for the gold-gold collision at RHIC. For the saturation momenta the KLN expression was used.

where we use that \( Q_s^2(x) \propto (1/x)^3 \) and \( Y \) is the rapidity of \( J/\psi \) in the center-of-mass frame. Therefore, our prediction is that the rapidity distribution of \( J/\psi \) is much more narrow in nucleus-nucleus collisions than in the proton–proton ones.

To evaluate how close we are to the saturation region at RHIC energies in this process we first rewrite the general formula for the kinematic region \( r \gg r' \). It takes the following form:

\[
\frac{1}{S_A} \frac{d\sigma(AA)}{dY d^2b} \propto Q_{s,A_1}^2(x_1) Q_{s,A_2}^2(x_2) \left[ Q_{s,A_1}^2(x_1) + Q_{s,A_2}^2(x_2) \right] \int_0^\infty d\zeta \, \zeta^9 K_2(\zeta) e^{-\frac{c^2}{\zeta \pi^2 \xi_s^2}} \left[ Q_{s,A_1}^2(x_1) + Q_{s,A_2}^2(x_2) \right] (4.9)
\]

In Fig. 9 we plot the result of our calculation for the ratio \( R(Y) = \frac{d\sigma}{dY}(Y) / \frac{d\sigma}{dY}(Y=0) \) using (4.9). For \( Au-Au \) collision at RHIC with \( \sqrt{s} = 200 \text{ GeV} \) taking the KLN value for the saturation momentum \( Q_s^2(y=0) = 2.2 \text{ GeV}^2 \) for central collisions we find that the rapidity distribution turns out to be very narrow although not quite to an extent suggested by the approximate expression (4.8) (see Fig. 9). The rapidity distribution in Fig. 9 is driven by the ratio \( Q_{s,A_1}^2/Q_{s,A_2}^2/(Q_{s,A_1}^2 + Q_{s,A_2}^2)^3 \). The cross section decreases with the increase of the value of the saturation momentum \( Q_{s,A_1}^2 + Q_{s,A_2}^2 \). However, this decrease is much milder than in (4.8).

5. Numerical calculations

In this section we perform numerical calculations of inclusive \( J/\psi \) production using (4.1). First of all, we reinstall the impact parameter dependence of the saturation scales and consider a realistic distribution density for nuclei. Recall that \( Q_s^2 \propto \rho_T(b) \). Denote the impact parameter between centers of two nuclei as \( b \). The position of a nucleon inside nucleus \( A_1 \) with respect to its center denote by \( x_1 \). Then the position of a nucleon in the nucleus \( A_2 \) is given by \( b - x_2 \). We have

\[
Q_{s,A_1}^2 \rightarrow Q_{s,A_1}^2(\frac{x_1}{z}) \quad Q_{s,A_2}^2 \rightarrow Q_{s,A_2}^2(\frac{b - x_2}{z}). \quad (5.1)
\]

In our Glauber-type approximation (see e.g. [57]) we neglect the impact parameter dependence in nucleon-nucleon interactions considering their range much smaller than the size of nuclei. The observable that we are going to calculate is the number of \( J/\psi \)'s inclusively produced in nucleus–nucleus collisions at a given rapidity \( Y \) and a
Figure 10: $J/\psi$ rapidity distribution in Au-Au collisions for different centrality cuts. Experimental data from [59].

centrality characterized by the impact parameter $b$. The corresponding expression reads

$$
\frac{dN^{AA}(Y,b)}{dY} \propto \int d^2s \ Q^2_{s,A_1}(x_1,\bar{s}) Q^2_{s,A_2}(x_2,\bar{b} - \bar{s}) [Q^2_{s,A_1}(x_1,\bar{s}) + Q^2_{s,A_2}(x_2,\bar{b} - \bar{s})] \\
\times \int_0^\infty d\zeta \zeta^6 K_2(\zeta) \exp \left\{-\frac{\zeta^2}{8m_c^2} [Q^2_{s,A_1}(x_1,\bar{s}) + Q^2_{s,A_2}(x_2,\bar{b} - \bar{s})]\right\} \, .
$$

(5.2)

where

$$
\frac{dN^{AA}(Y,b)}{dY} = \frac{dN^{pp}(Y)}{dY} \frac{\sigma_{AA}^{s}}{\sigma_{AA}^{pp}}
$$

and

$$
x_1 = \frac{m_{J/\psi}}{\sqrt{s}} e^{-Y}, \quad x_2 = \frac{m_{J/\psi}}{\sqrt{s}} e^{Y}
$$

(5.3)

We can also write down (5.2) in the following way:

$$
\frac{dN^{AA}(Y,b)}{dY} = C \frac{dN^{pp}(Y)}{dY} \int d^2s \ T_{A_1}(\bar{s}) T_{A_2}(\bar{b} - \bar{s}) [Q^2_{s,A_1}(x_1,\bar{s}) + Q^2_{s,A_2}(x_2,\bar{b} - \bar{s})] \frac{1}{m_c^2} \\
\times \int_0^\infty d\zeta \zeta^6 K_2(\zeta) \exp \left\{-\frac{\zeta^2}{8m_c^2} [Q^2_{s,A_1}(x_1,\bar{s}) + Q^2_{s,A_2}(x_2,\bar{b} - \bar{s})]\right\} \, .
$$

(5.4)

The overall normalization constant $C$ includes the color and the geometric factors $C_F/(4\pi^2\alpha_s S_p)$ where $S_p$ is interaction area in proton–proton collisions; it also includes a rather poorly known amplitude of charm quark–antiquark transition into $J/\psi$ and a gluon in the case of $pp$ collisions (see Fig. 6-A).

To calculate the multiplicity of $J/\psi$'s in $AA$ collisions using (5.4) we need to know (i) rapidity distribution of $J/\psi$ multiplicity $dN^{pp}/dY$ in $pp$ collisions and (ii) the overall normalization constant $C$. We fitted the rapidity distribution of $J/\psi$'s in $pp$ collisions to the experimental data of Ref. [58] with a single gaussian. The global normalization factor $C$ is found from the overall fit.

Now we can compute the nuclear modification of the $J/\psi$ rapidity distribution in $AA$ collisions at all centralities using (5.4). In figure 10 we compare our results with the experimental data of PHENIX Collaboration [59] for Au-Au collisions at $\sqrt{s} = 200$ GeV. The agreement between our calculation and the experimental data is reasonable.

In figure 11 we show the same results in the “measured/expected” form, namely the experimental data divided by our calculations. It is tempting to use the difference between this ratio and the unity as a measure of the magnitude
of the final state effect for $J/\psi$ production. However, at the moment we prefer to refrain from a premature conclusion about the size of the final state effect as such a conclusion would crucially depend on the value of the $C$ factor. In addition we are aware of the limitations due to the accuracy of the experimental data. To give a firm conclusion we need to perform a comparison with a high precision $dAu$ data, where the $C$ factor can eventually be fixed with a higher accuracy.

To emphasize the nuclear dependence of the inclusive cross sections it is convenient to introduce the nuclear modification factor

$$R_{AA}(y, N_{\text{part}}) = \frac{dN_{AA}^{y}}{dy}{N_{\text{coll}}} \frac{dN_{pp}^{y}}{dy}. \quad (5.5)$$

It is normalized in such a way that no nuclear effect would correspond to $R_{AA} = 1$. In Fig. 12 we plot the result of our calculation. The nuclear modification factor exhibits the following two important features: (i) unlike the open charm production, $J/\psi$ is suppressed at $y = 0$. This is not very surprising since the probability of the $J/\psi$ formation is reduced due to multiple interactions with the gluons; (ii) cold nuclear effects account for a significant part of the $J/\psi$ suppression observed in heavy ion collisions. On the other hand, the cold nuclear effects discussed in this paper may not be sufficient to account for a very low $R_{AA}$ in the most central events. Higher precision $dA$ and $AA$ data will allow to tell whether there is a suppression of $J/\psi$ in quark–gluon plasma, or the directly produced $J/\psi$’s survive [60].

6. Conclusions

In this paper we have developed a model for $J/\psi$ production in heavy ion collisions. Assuming that contributions of strong color fields of the two colliding nuclei do not interfere we summed all the diagrams proportional to the positive powers of the large parameter $\alpha_s^2A^{1/3} \sim 1$ in both nuclei. Our main result is given by Eq. (4.1). In the
RHIC kinematic region this formula can be simplified and is given by (4.9). We used this equation in our numerical calculation with the realistic nuclear profiles described in Sec 5 in details. The results are presented in Figs. 10–12.

We can see that the rapidity and centrality dependence are reproduced quite well. This observation implies that an appreciable amount of the $J/\psi$ suppression in high energy heavy ion collisions comes from the cold nuclear effects. Fig. 12 demonstrates that the nuclear modification factor for $J/\psi$ production is strongly suppressed even at zero temperature. While $J/\psi$ suppression in the forward direction is not a surprise (the nuclear modification factor for light and, perhaps, heavy hadrons is known to be suppressed), similar behavior in the central rapidity region is a peculiar feature of the $J/\psi$ production. The reason is that multiple scattering of $c\bar{c}$ pair in the cold nuclear medium increases the relative momentum between the quark and antiquark, which makes the bound state formation less probable. Formally, the sum rule proven in [11], which guarantees emergence of the Cronin enhancement for single partons, is broken for the bound states.

Our result strongly suggests that the cold nuclear matter effects play a very important role in $J/\psi$ production in heavy ion collisions. The final nuclear modification factor, which is measured in experiment, is undoubtedly a result of a delicate interplay between the cold and hot nuclear matter effects.

We consider the present work as the first step towards understanding the role of the cold nuclear effects in $J/\psi$ production in high energy heavy ion collisions. We calculated the parametrically enhanced contribution coming from even number of scatterings of $c\bar{c}$ pair in the nuclei. However, we neglected other contributions that may be phenomenologically important though parametrically small. These include soft gluon radiation in the final state and color octet mechanism of $J/\psi$ production. Moreover, for peripheral collisions these contributions become of the same order as the one discussed in this paper. Therefore, we plan to perform a detailed investigation of these contributions in the future.

**Acknowledgments.** The work of D.K. was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886. K.T. was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371; he would like to thank RIKEN, BNL, and the U.S. Department of Energy (Contract No. DE-AC02-98CH10886) for providing facilities essential for the completion of this work. This research of E.L. was supported in part by a grant from Ministry of Science, Culture & Sport, Israel & the Russian Foundation for Basic research of the

---

**Figure 12:** Nuclear modification factor for $J/\psi$ production in heavy ion collisions for different rapidities.
A. Heavy quark production

Figure 13: Diagrams contributing to the $q\bar{q}$ production in pA collisions at high energies. Transverse momenta of the produced quark and antiquark $\vec{k}_1$, $\vec{k}_2$, their transverse coordinates in the amplitude $\vec{x}_1$, $\vec{x}_2$ and in the complex conjugated one $\vec{y}_1$, $\vec{y}_2$ and the gluon transverse coordinate in the amplitude $\vec{u}$ and in the complex conjugated one $\vec{v}$ are shown in boldface. Vertical dashed lines indicate all possible interaction times of incoming proton with the nucleus (shown by crosses). More detailed discussion can be found in [36, 37]. Notations follow [37].

In this appendix we derive the cross section for production of a $c\bar{c}$ pair with fixed relative momentum in pA collisions. A general problem of $q\bar{q}$ pair production in pA collisions including all possible nonlinear evolution effects was solved in Refs. [36, 37]. Let us introduce the following notations: $\vec{k}_1$ and $\vec{k}_2$ are the produced quark and anti-quark transverse momenta, $\vec{q}$ is the gluon transverse momentum, $z = k_+/q_+$ is a fraction of the light-cone momentum of gluon carried by the produced quark; $\vec{x}_1$ and $\vec{y}_1$ are the transverse coordinates of the produced quark in the amplitude and in the complex conjugated amplitude respectively; $\vec{x}_2$ and $\vec{y}_2$ are the corresponding coordinates of the antiquark. Transverse coordinates of gluon in the amplitude $\vec{u}$ and in the complex conjugated amplitude $\vec{v}$ are given by $\vec{u} = z\vec{x}_1 + (1 - z)\vec{x}_2$ with $u = |\vec{u}| = r$ and $\vec{x}_{12} = \vec{x}_1 - \vec{x}_2 \equiv \mathcal{R}$, $(x_{12} = |\vec{x}_{12}|)$ and analogously for $\vec{v}$. With these notations the double inclusive quark–anti-quark production cross section is given by [36, 37]

$$\frac{d\sigma}{d^2k_1 dk_2 dy_1 dy_2} = \frac{1}{16(2\pi)^6} \int d^2x_1 d^2x_2 d^2y_1 d^2y_2 \int_0^1 dz \, e^{-i(\vec{k}_1 \cdot \vec{y}_1 - \vec{k}_1 \cdot \vec{y}_2)}$$

$$\times \sum_{i,j=1}^{3} \Phi_{ij}(\vec{x}_1, \vec{x}_2; \vec{y}_1, \vec{x}_2; z) \Xi_{ij}(\vec{x}_1, \vec{x}_2; \vec{y}_1, \vec{x}_2; z), \quad (A.1)$$

The products of the light-cone “wave functions” are detailed as follows [37]

$$\Phi_{11}(\vec{x}_1, \vec{x}_2; \vec{y}_1, \vec{y}_2; z) = 4 \, C_F \left( \frac{\alpha_s}{\pi} \right)^2 \left\{ F_2(\vec{x}_1, \vec{x}_2; z) F_2(\vec{y}_1, \vec{y}_2; z) \frac{1}{x_{12} y_{12} uv} [(1 - 2z)^2$$

$$\times (\vec{x}_{12} \cdot \vec{u}) (\vec{y}_{12} \cdot \vec{v}) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + F_1(\vec{x}_1, \vec{x}_2; z) F_1(\vec{y}_1, \vec{y}_2; z) m^2 \frac{u \cdot v}{uv}$$

$$+ 4z^2 (1 - z)^2 F_0(\vec{x}_1, \vec{x}_2; z) F_0(\vec{y}_1, \vec{y}_2; z) - 2z (1 - z) (1 - 2z) \left( \frac{\vec{x}_{12} \cdot \vec{u}}{x_{12} u} F_2(\vec{x}_1, \vec{x}_2; z) \right.$$  

$$\times F_0(\vec{y}_1, \vec{y}_2; z) + \frac{\vec{y}_{12} \cdot \vec{v}}{y_{12} v} F_2(\vec{y}_1, \vec{y}_2; z) F_0(\vec{x}_1, \vec{x}_2; z) \bigg) \bigg\}, \quad (A.3)$$
\[ \Phi_{22}(x_1, x_2; y_1, y_2; z) = 4CF \left( \frac{\alpha_s}{\pi} \right)^2 m^2 \left\{ K_1(m x_{12}) K_1(m y_{12}) \frac{1}{x_{12} y_{12} u^2 v^2} [(1 - 2 z)^2 \right. \\
\times (x_{12} \cdot u) (y_{12} \cdot u) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + K_0(m x_{12}) K_0(m y_{12}) \frac{u \cdot v}{u^2 v^2} \right\}, \tag{A.4} \]

\[ \Phi_{12}(x_1, x_2; y_1, y_2; z) = -4CF \left( \frac{\alpha_s}{\pi} \right)^2 m \left\{ F_2(x_1, x_2; z) K_1(m y_{12}) \frac{1}{x_{12} y_{12} u^2 v^2} [(1 - 2 z)^2 \right. \\
\times (x_{12} \cdot u) (y_{12} \cdot u) + (\epsilon_{ij} u_i x_{12j}) (\epsilon_{kl} v_k y_{12l})] + m F_1(x_1, x_2; z) K_0(m y_{12}) \frac{u \cdot v}{u^2 v^2} \right\} \\
- 2 z (1 - z) (1 - 2 z) \frac{y_{12} \cdot u}{y_{12}^2 v^2} F_0(x_1, x_2; z) K_1(m y_{12}) \right\}, \tag{A.5} \]

\[ \Phi_{33}(x_1, x_2; y_1, y_2; z) = \Phi_{11}(x_1, x_2; y_1, y_2; z) + \Phi_{22}(x_1, x_2; y_1, y_2; z) + \Phi_{12}(x_1, x_2; y_1, y_2; z) \\
+ \Phi_{21}(x_1, x_2; y_1, y_2; z) = \Phi_{13}(x_1, x_2; y_1, y_2; z) - \Phi_{12}(x_1, x_2; y_1, y_2; z) \tag{A.6} \]

\[ \Phi_{23}(x_1, x_2; y_1, y_2; z) = -\Phi_{21}(x_1, x_2; y_1, y_2; z) - \Phi_{22}(x_1, x_2; y_1, y_2; z) \tag{A.7} \]

\[ \Phi_{13}(x_1, x_2; y_1, y_2; z) = \Phi_{31}(y_1, y_2; x_1, x_2; z) \tag{A.9} \]

The auxiliary functions \( F_1, F_2 \) and \( F_0 \) are defined as

\[ F_2(x_1, x_2; z) = \int_0^\infty dq J_1(q u) K_1 \left( x_{12} \sqrt{m^2 + q^2} z(1 - z) \right) \sqrt{m^2 + q^2} z(1 - z), \tag{A.10} \]

\[ F_1(x_1, x_2; z) = \int_0^\infty dq J_1(q u) K_0 \left( x_{12} \sqrt{m^2 + q^2} z(1 - z) \right), \tag{A.11} \]

\[ F_0(x_1, x_2; z) = \int_0^\infty dq J_0(q u) K_0 \left( x_{12} \sqrt{m^2 + q^2} z(1 - z) \right), \tag{A.12} \]

where \( u = |u|, x_{12} = x_1 - x_2, x_{12} = |x_{12}|, \) and \( q = k_1 + k_2. \)

By definition, the gluon saturation scale \( Q_s^2 \)

\[ Q_s^2 = 4 \pi \alpha_s^2 \rho T(b) \tag{A.13} \]

with \( \rho \) the nucleon number density in the nucleus and \( T(b) \) the nuclear profile function. We also use the quark saturation scale \( Q_s^2 \) give by

\[ Q_s^2 = \frac{C_F}{N_c} Q_s^2 \sum_{i=5}^{N} \frac{1}{2} Q_i^2. \tag{A.14} \]

Throughout the theoretical discussion we assumed for simplicity that the nuclear profile is cylindrical with \( T(b) \approx 2R_A \) for \( b^2 \leq R_A^2. \) However, numerical calculations in Sec. 5 are performed with an accurate parameterization as discussed there in detail. In the large \( N_c \) approximation we write

\[ \Xi_{11}(x_1, x_2; y_1, y_2; z) = e^{-\frac{1}{2} (x_1 - y_1)^2 Q_s^2 \ln(1/|x_1 - y_1|)} - \frac{1}{2} (x_1 - y_2)^2 Q_s^2 \ln(1/|x_1 - y_2|), \tag{A.15} \]

\[ \Xi_{22}(x_1, x_2; y_1, y_2; z) = e^{-\frac{1}{2} (u - z)^2 Q_s^2 \ln(1/|u - z|)} \tag{A.16} \]

\[ \Xi_{33}(x_1, x_2; y_1, y_2; z) = 1, \tag{A.17} \]

\[ \Xi_{12}(x_1, x_2; y_1, y_2; z) = e^{-\frac{1}{2} (x_1 - y_2)^2 Q_s^2 \ln(1/|x_2 - y_2|)} - \frac{1}{2} (x_1 - y_2)^2 Q_s^2 \ln(1/|x_2 - y_2|), \tag{A.18} \]

\[ \Xi_{23}(x_1, x_2; y_1, y_2; z) = e^{-\frac{1}{2} u^2 Q_s^2 \ln(1/|u|)} \tag{A.19} \]

\[ \Xi_{13}(x_1, x_2; y_1, y_2; z) = e^{-\frac{1}{2} z^2 Q_s^2 \ln(1/|z|)} - \frac{1}{2} z^2 Q_s^2 \ln(1/|z|) \tag{A.20} \]
All other $\Xi_{ij}$’s can be found from the components listed in (A.15)-(A.20) using

$$\Xi_{ij}(x_1, x_2; y_1, y_2; z) = \Xi_{ji}(y_1, y_2; x_1, x_2; z)$$  \hspace{1cm} (A.21)

similar to (A.9).

If the typical gluon momentum $q$ is much smaller than the produced quark mass, the above expressions can be significantly simplified. Indeed, since $z(1-z) \leq 1/4$ we get

$$q^2z(1-z) \ll m^2,$$  \hspace{1cm} (A.22)

and the auxiliary functions read

$$F_2(x_1, x_2; z) = K_1(x_{12}m) m^{-1},$$  \hspace{1cm} (A.23)
$$F_1(x_1, x_2; z) = K_0(x_{12}m) u^{-1},$$  \hspace{1cm} (A.24)
$$F_0(x_1, x_2; z) = 0.$$  \hspace{1cm} (A.25)

In this approximation the only non-vanishing products of “wave functions” are given by

$$\Phi_{11}(x_1, x_2; y_1, y_2; z) = \Phi_{22}(x_1, x_2; y_1, y_2; z) = -\Phi_{12}(x_1, x_2; y_1, y_2; z)$$
$$= 4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 \frac{m^2}{u} \left\{ K_1(x_{12}m) K_1(y_{12}m) \frac{1}{x_{12} y_{12} u^2 v^2} \left[ (1 - 2z)^2 \left( x_{12} \cdot y_1 \right) (y_{12} \cdot x_1) \right] \right.$$
$$+ (\epsilon_{ij} u_i x_{12j} (\epsilon_{kl} v_k y_{12l})) + K_0(x_{12}m) K_0(y_{12}m) \frac{u - u}{u^2 v^2} \left\}. \hspace{1cm} (A.26) \right.$$

Averaging over all directions of gluon emission from the valence quark using $\langle \epsilon_{ij} u_i x_{12j} (\epsilon_{kl} v_k y_{12l}) \rangle = u \cdot v \frac{x_{12} \cdot y_{12}}{u^2 v^2}$ we arrive at the well-known result [36, 40]

$$\Phi_{11}(x_1, x_2; y_1, y_2; z) = 4 C_F \left( \frac{\alpha_s}{\pi} \right)^2 \frac{m^2}{u} \left\{ \frac{x_{12} \cdot y_{12}}{x_{12} y_{12}} \left[ (1 - z)^2 + z^2 \right] K_1(x_{12}m) K_1(y_{12}m) + K_0(x_{12}m) K_0(y_{12}m) \right\}$$

Let’s now introduce the following notations, see Fig. 1:

$$p = k_1 + k_2, \quad k = k_1 - k_2 + (1 - 2z) \left( k_1 + k_2 \right),$$  \hspace{1cm} (A.27)

or, equivalently,

$$k_1 = z p + \frac{1}{2} k, \quad k_2 = (1 - z) p - \frac{1}{2} k.$$  \hspace{1cm} (A.28)

At $z = 1/2$, momentum $k$ becomes the relative momentum of the $cc$ pair. In this case integration over the total momentum $p$ results in the delta function $\delta^{(3)}(u - v)$. This delta-function simplifies expressions in the exponents of (A.15)–(A.20). For the sum over all rescattering factors including the signs of $\Phi_{1j}$’s we get (omitting the logarithms $\ln(1/x\mu)$ for brevity)

$$\Xi(x_{12}, y_{12}; z = 1/2) = e^{-\frac{1}{2} (x_{12} - y_{12})^2 Q_1^2} + 1 - e^{-\frac{1}{2} x_{12} Q_1^2} - e^{-\frac{1}{2} y_{12} Q_1^2}$$  \hspace{1cm} (A.29)

Using the delta function in (A.2) to integrate over $\underline{u}$ and integrating over $\underline{v}$ in the leading logarithmic approximation we derive

$$\frac{d\sigma}{d^2k dy dB} = \frac{C_F \alpha_s^2 m^2}{4\pi^5} \int_0^1 dz \int d^2x_{12} \int q^2 y_{12} e^{-i \frac{1}{2} (x_{12} - y_{12}) \ln(1/\mu) |x_{12} - y_{12}|}$$

$$- 21 -$$
\[ d\sigma_{pA}/d^2k dy dB = \frac{\alpha_s m^2}{8\pi^4} \int d^2r' \int d^2r e^{-\frac{i}{2} \vec{k} \cdot \vec{r}'} xG(x_1, m_c^2) \left\{ \frac{1}{2} \frac{r \cdot r'}{rr'} K_1(rm/\ell) K_1(r'm) + K_0(rm)K_0(r'm) \right\} \times \left\{ e^{\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} + 1 - e^{-\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} - e^{-\frac{i}{2} Q_s^2 r'^2} \right\} \]

(A.31)

Finally, using the \( x_{12} = \frac{1}{\ell}, y_{12} = \frac{1}{\ell}' \) notation obtain the final expression for the \( cc \) pair production with fixed relative momentum (at \( z = 1/2 \)):

\[ \frac{d\sigma_{pA}}{d^2k dy dB} = \frac{\alpha_s m^2}{8\pi^4} \int d^2r' \int d^2r e^{-\frac{i}{2} \vec{k} \cdot \vec{r}'} xG(x_1, m_c^2) \left\{ \frac{1}{2} \frac{r \cdot r'}{rr'} K_1(rm/\ell) K_1(r'm) + K_0(rm)K_0(r'm) \right\} \times \left\{ e^{\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} + 1 - e^{-\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} - e^{-\frac{i}{2} Q_s^2 r'^2} \right\} \]

(A.32)

Therefore, taking the large \( N_c \) approximation \( Q_s^2 \approx \frac{1}{g^2} Q^2 \) and \( C_F \approx N_c/2 \), we derive

\[ \frac{dN_{AA}}{d^2k dy dB} = \frac{\alpha_s m^2}{8\pi^4} \int d^2r' \int d^2r e^{-\frac{i}{2} \vec{k} \cdot \vec{r}'} xG(x, 4/r) \left\{ \frac{1}{2} \frac{r \cdot r'}{rr'} K_1(rm/\ell) K_1(r'm) + K_0(rm)K_0(r'm) \right\} \times \left\{ e^{\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} + 1 - e^{-\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} - e^{-\frac{i}{2} Q_s^2 r'^2} \right\} \cdot \frac{8C_F}{\pi^2\alpha_s} \left( 1 - e^{-\frac{i}{2} (\vec{r} - \vec{r}')^2 Q_s^2} \right) \left( 1 - e^{-\frac{i}{2} Q_s^2 r'^2} \right) \cdot \left( 1 - e^{-\frac{i}{2} Q_s^2 r'^2} \right) \right\} \]

(A.33)

References


[19] B. Z. Kopeliovich and F. Niedermayer, JINR-E2-84-834


NOTICE: This is the author's version of a work that was accepted for publication in Nuclear Physics A. Changes resulting from the publishing process, such as peer review, editing, corrections, structural formatting, and other quality control mechanisms may not be reflected in this document. Changes may have been made to this work since it was submitted for publication. A definitive version was subsequently published in Nuclear Physics A, v.826 (2009): doi: 10.1016/j.nuclphysa.2009.06.016.