Government debt, Consumption, and Interest Rates: An Empirical Study of the Ricardian Equivalence Hypothesis

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Government debt, Consumption, and Interest Rates: An Empirical Study of the Ricardian Equivalence Hypothesis

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Disciplines
Behavioral Economics | Economic Theory | Finance | Political Economy | Taxation
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Abstract:

This paper investigates the effects of the government debt on private consumption and interest rates (i.e., the Ricardian equivalence hypothesis) in a nonlinear-quadratic equilibrium framework, combined with a time series analysis of the data. The model is summarized by the restrictions on the coefficients of a vector autoregressive (VAR) representation of the relevant variables. The model is not rejected for the sample period of 1947 through 1979 even though the evidence was not overwhelming. It is rejected when the sample period is extended to include the Reagan administration period up to 1987, I. A more detailed data analysis based on a VAR implies, however, that the rejection of the model does not necessarily imply rejection of the equivalence hypothesis for recent years and that the Ricardian equivalence hypothesis still provides a plausible approximation of reality.

October 1987

* I have benefited from discussions with Yasuo Amemiya, Barry Falk, Peter Orazem, John Schroeter, Chris Sims, and Neil Wallace. Any errors are my own responsibility.
In recent years, there has been increasing interest in the effects of the government debt on the economy. The basic idea on this issue is commonly attributed to the debate on the 'Ricardian equivalence hypothesis', which states that whether a given level of government purchases is tax financed or bond financed is irrelevant to the effects on the economy. The conventional view (Blinder and Solow, 1973, Feldstein, 1982) assumes that the government debt issue leads to an increase in perceived household wealth (i.e., wealth effect) and argues that an increase in public debt results in an increase in desired consumption relative to saving, hence an increase in real interest rates. But Barro (1974) and others argue that, because the future tax liabilities implicit in debt financing are foreseen by rational economic agents and private opportunities exist for intergenerational transfers and bequests, there is no wealth effect of the government debt. Therefore, changes in the relative amounts of tax and debt finance for a given amount of public expenditure would have no effect on aggregate demand (or consumption) and interest rates.

Because there have been various theoretical explanations of this issue, the resolution of the debate has become essentially an empirical matter. This has spawned a number of papers that seek to provide required empirical studies. These include Kochin (1974), Tanner (1979), Feldstein (1982), Plosser (1982), Kormendi (1983), Aschauer (1985), and Evans (1985, 1987). This paper attempts to improve upon previous studies on empirical testing of the Ricardian equivalence hypothesis in three points.

First, the Ricardian equivalence hypothesis is about the effects of a substitution of debt for taxes, for a given level of government expenditures, both on consumption expenditures and on interest rates. Plosser (1982) and Evans (1985, 1987) have studied the effects of the government deficit on interest rates, ignoring its effects on consumption,
whereas others (Kochin, 1974, Tanner, 1979, Feldstein, 1982, Kormendi, 1983, and Aschauer, 1985) have investigated the effects on private consumption, treating interest rates as constant. The tendency to focus on only one factor or the other may be due to the difficulty of specifying a consumption model with stochastic (i.e., time-varying) interest rates that provides a closed form solution. This paper examines the effects of the government debt on both private consumption and interest rates in a nonlinear-quadratic equilibrium framework with implications that include the Ricardian equivalence hypothesis to allow for time-varying interest rates. Instead of the conventional analysis, this paper employs an idea of asset pricing to determine the nature of the hypothesis in a way that leads to statistical representations with testable hypotheses.

In general, closed-form solutions for the equilibrium time paths of the variables for the nonlinear-quadratic framework can be obtained only after imposing additional assumptions on the stochastic properties of the relevant variables. To obtain an empirically tractable, closed-form characterization of the restrictions from the model, a constant relative risk aversion (CRRA) preference and the jointly covariance-stationary lognormal distribution of the relevant variables are assumed, following Hansen and Singleton (1983) and Bean (1985). The model is summarized by the restrictions on the coefficients of a vector autoregressive (VAR) representation of the relevant variables. The implied restrictions from the model are tested by using postwar data.

Second, whether there has been any empirical evidence of the regime change between a Ricardian regime and a non-Ricardian regime has become a more relevant issue these days in view of rapidly increasing government budget deficits. Various theoretical models have been suggested to
analyze a possible change in the regime.³ A sample split test is performed, adding recent data to the model. The sample period 1947 through 1979 was tested first, and the restriction of the model that reflects the Ricardian equivalence hypothesis is not rejected for this period. This also implies that the model mimics the real data fairly well, even with the restrictions mentioned above. But when the sample period is extended to include the Reagan administration period up to 1987,1 (i.e., 1947 through 1987,1), the model seems to be rejected.

Third, when the model is rejected, the cause of the rejection needs to be further investigated through more detailed data analysis. This is particularly important because rejection of the model can not automatically be interpreted as rejection of the equivalence hypothesis, due to the complex nature of the model and the restriction. To help understand possible causes of the rejection and possible differences in dynamic interactions among the variables of interest between the two sample periods, a VAR analysis of the data is performed. This is naturally derived from the covariance-stationary assumption of the variables, and provides some basis for the comparison of the outcome with previous studies. This exercise is partly intended to illustrate a close link between a non-quadratic dynamic framework and a VAR representation of the relevant variables under the assumptions mentioned as well as to test the implications of the Ricardian equivalence hypothesis with time-varying interest rates.

The data analysis based on a VAR representation implies that, faced with ever-increasing government deficits and debt, the public may have begun to adjust their consumption behavior accordingly. But, more importantly, the rejection of the model is mainly due to the change in the behavior of real interest rates - real interest rates move downward even further in response to the shocks in government debt - and government debt does not seem to have
any significant explanatory power for real interest rates and consumption, which still remain compatible with the Ricardian equivalence hypothesis. Therefore, the rejection of the model for recent years may not necessarily imply rejection of the equivalence hypothesis and the equivalence hypothesis remains a plausible approximation of the real data.

This paper is organized as follows. In section I, a nonlinear-quadratic equilibrium model with implications consistent with the Ricardian equivalence hypothesis is described. In section II, empirically testable restrictions are derived from the model and are tested for each sample period. An additional time series evidence based on a VAR analysis is presented and discussed in section III. Summary and concluding remarks are contained in section IV.

I. Formulation of the Hypothesis

As a theoretical foundation of the equivalence hypothesis, this paper relies on the following discrete time intertemporal optimization model. The model allows for government expenditure to enter consumer's utility function and to be a substitute for private consumption and for time varying interest rates. Consider a representative consumer who faces the following optimization problem, whose utility function is of the CRRA type:

\[
\max E_0 \sum_{t=0}^\infty \beta^t \left( C_t^{1-\gamma} G_t^\gamma \right)^{1/\gamma}, \quad 0<\gamma<1, \ 0<\delta<1, \ \gamma<1,
\]

subject to

\[
B_t/I_t + P_t C_t < B_{t-1} + P_t y_t - P_t T_t.
\]

Government budget constraint is given as

\[
P_t G_t - P_t T_t = B_t/I_t - B_{t-1}, \quad \text{and}
\]
where \( \beta = a \) discount factor

\[ C_t = \text{aggregate real per capita consumption in period } t \]
\[ G_t = \text{government expenditure in period } t \]
\[ T_t = \text{lump sum tax in period } t \]
\[ B_t = \text{one-period government-issued (nominal) debt at } t \]
\[ I_t = \text{nominal gross interest rate at } t \]
\[ P_t = \text{price level at } t \]
\[ y_t = \text{income (or output) in period } t \]
\[ E_t = \text{the mathematical expectation conditional on the information available to agents at time } t, \Omega_t. \] Hence, \( E_t(x) = E(x|\Omega_t) \).

Equation (2) states that each consumer has the opportunity in \( t \) of purchasing government bonds at a discounted price of \( 1/\Pi_t \) and each bond is redeemed in \( t+1 \) for one unit of money. Equation (3) is the government's budget constraint, which states that the government finances its expenditures by a stream of lump sum taxes \( T_t \) and by issuing one-period debt \( B_t \). The first order conditions for this problem would be

\[ \lambda_t = \beta E_t \lambda_{t+1} I_t \]

\[ (C_t^{1-\gamma} G_t^\gamma) \gamma^{-1} (1-\delta) C_t^{-\delta} G_t^{\gamma} = \lambda_t P_t, \]

where \( \lambda_t \) is a Lagrangian multiplier. Combining these two equations,

\[ \frac{1}{\beta} = E_t \left[ \frac{(C_{t+1}/C_t)^{\gamma-1} (G_{t+1}/G_t)^\gamma}{\Pi_{t+1}} \right] \]

\[ = E_t \left[ Z_{t+1} \right], \]

where \( \Pi_{t+1} = P_{t+1}/P_t \) and \( Z_{t+1} \) is all the variables in the bracket \( [..] \).

An important perspective on the dynamic nature of the model in relation to the Ricardian equivalence hypothesis is provided by the following observations. A repeated forward substitution for \( B_t \) in the consumer's
budget constraint (2) yields

\[ \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) P_t C_t = B_{-1} + \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) \left( P_t y_t - P_t G_t \right), \]

where \( I_{j,t} = \Pi I_j \) and assuming \( I_{-1} = 1 \), and \( (B_t/I_{j,t}) \to 0 \) as \( t \to \infty \).

This implies the present discounted value of private consumption expenditure is the sum of current government bond holdings and the present discounted value of disposable income. Similarly, (3) yields

\[ \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) P_t T_t = B_{-1} + \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) P_t G_t, \]

which states that the current government debt \( B_{-1} \) that matures at time 0 is supported by the present discounted value of government's budget surpluses.

By substituting (9) into (8),

\[ \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) P_t C_t = \sum_{t=0}^{\infty} \left( \frac{1}{I_{j,t-1}} \right) \left( P_t y_t - P_t G_t \right), \]

which implies that the present discounted value of consumption plus the present discounted value of government expenditure will be equal to the present discounted value of income because income (or output) is spent either by the consumer or by the government.

Maximization of the consumer's objective function (1) subject to the present value form of integrated budget constraint (10) yields the same first order condition (7). This implies that the optimizing consumer recognizes that current government debt will be financed by the future government budget surpluses. In this economy, at an equilibrium,

\[ C_t = y_t - G_t. \]

This implies that given \( \{y_t\} \) and \( \{G_t\} \) paths, \( \{C_t\} \) is not affected by government's financing decision between tax and debt. In addition, given \( \{C_t\} \) and \( \{G_t\} \) paths, equation (7) implies that real interest rates, \( (r_t/I_{t+1}) \),
are not affected by the government's financing decision either. Therefore, in this economy, the following Ricardian equivalence hypothesis holds: the equilibrium consumption and real interest rates depend only on the \((y_t)\) and \((G_t)\) paths. The government's financing decision on the time path of taxes and government bond issues has no effect on equilibrium consumption and real interest rates. This is because, as shown in equations (9) and (10), current government bonds are covered by future government budget surpluses, and the economic agents recognize this and they do not treat the current government bonds as net wealth. Therefore, their consumption behavior is not affected by the issue of government debt for given \((Y_t, G_t)\) paths (see Sargent, 1987, p. 116).

II. A Test of the Hypothesis

A. Testable Restrictions

The empirical approach of this paper is to characterize the hypothesis using the restrictions (7) which the theoretical intertemporal model places on the data. This section analyzes the first-order conditions (or Euler conditions) of an optimizing economic agent whose behavior is consistent with the Ricardian equivalence hypothesis. The nonlinear restriction (7) is linearized using additional distributional assumptions because the linear model is more easily tractable and is subject to time-series econometric analyses.

Now let

\[ \nabla c_t = \ln C_t - \ln C_{t-1} = \epsilon_t - \epsilon_{t-1} \]
\[ \nabla g_t = \ln G_t - \ln G_{t-1} = \kappa_t - \kappa_{t-1} \]
\[ \nabla b_t = \ln B_t - \ln B_{t-1} = b_t - b_{t-1} \]
where $v = 1 - L$ is the lag difference operator, with $L$ denoting the lag operator (i.e., $Lx_t = x_{t-1}$). We assume that the stochastic process $Y_t = [v c_t, v s_t, v b_t, r_t]'$, 4x1, is a covariance stationary, indeterministic process where ' denotes transpose. Then $Y_t$ has a moving average representation by the Wold theorem

$$Y_t = D(L) u_{1t} + d_0,$$

where $D(L)$ is a 4x4 infinite order matrix polynomial in the lag operator $L$ (i.e., $D(L) = \sum_{s=0}^{\infty} [D_{ij}(s) L^s]$ for $i, j = 1, 2, 3, 4$), with $D(0) = I$, and $d_0$ is a 4 dimensional vector of constants,

$$u_{1t} = Y_t - \text{Proj.}[Y_t|Y_{t-1}, Y_{t-2}, \ldots] \text{ with Proj. denoting the linear least squares projection of } Y_t \text{ on a closed linear space spanned by } [Y_{t-1}, Y_{t-2}, \ldots].$$

By construction, $u_{1t}$ is serially uncorrelated. Further, we assume that $u_{1t}$ for $t=1, 2, \ldots , T$, is distributed as jointly normal. From this assumption it follows that $u_{1t}$ is independently normally distributed and that $Y_t$ is a stationary, Gaussian process. This implies that the distribution of $z_{t+1}$ conditional on the information set $(\Omega_{1t}) = (Y_{t-s} \text{ for } s = 0, 1, 2, \ldots)$ is normal, with a mean $E_{1t} z_{t+1}$ ($= E(z_{t+1}|\Omega_{1t})$) that is a linear function of past observations on $Y_t$ and a constant conditional covariance. From this assumption,

$$E_{1t} Z_{t+1} = E(Z_{t+1}|\Omega_{1t}) = \exp(E_{1t} z_{t+1} + 2/2), \text{ or}$$

$$\ln Z_{t+1} = z_{t+1} \sim N(E_{1t} z_{t+1}, \sigma^2).$$

From (7) and (11),
(14) \[ z_{t+1} = \ln Z_{t+1} = (\gamma - 1 - \gamma \delta) c_{t+1} + \gamma \delta g_{t+1} + r_{t+1}. \]

By taking expectations of both sides of (7) conditional on \( \Omega_{1t} \) and noting that \( \Omega_{1t} \subset \Omega_t \), (7) becomes

(15) \[ 1/\beta = E_{1t} z_{t+1}. \]

Let \( E_{1t-1} z_t = \mu_{t-1} \). By substituting (13) into (15) and taking the natural logarithms of both sides of (15), (15) becomes

\[ -\ln \beta = \ln E_{1t-1} z_t = \mu_{t-1} + \sigma^2/2, \quad \text{or} \]

\[ \mu_{t-1} = -\ln \beta - \sigma^2/2. \]

Define,

\[ u_t^* = z_t - E_{1t-1} z_t = z_t - \mu_{t-1}. \]

By substituting (14) for \( z_t \),

\[ u_t^* = (\gamma - 1 - \gamma \delta) c_t + \gamma \delta g_t + r_t - \mu_{t-1}, \]

(16) where \( a_1 = \gamma - 1 - \gamma \delta \), \( a_2 = \gamma \delta \), \( \mu_{t-1} = -\ln \beta - \sigma^2/2 \), and \( E_{1t-1} u_t^* = 0 \).

Equation (16) summarizes the restrictions among the consumption, government expenditures, and interest rates implied by the first order condition (7) of the model. Because we are interested in analyzing the effects of the government debt on private consumption and interest rates, it is assumed that the government debt \( v_b_t \) enters the system through the auxiliary equations for the prediction of government expenditure, consumption, and interest rate (i.e., \( E_{1t-1} z_t \)).

The stationary, Gaussian process \( \{c_t, g_t, v_t, r_t\} \equiv Y_t \) is assumed to have the following unrestricted vector autoregressive representation:

(17) \[ Y_t = C(L) Y_{t-1} + C_0 + u_{1t}, \]

where \( C(L) \) is a \( 4 \times 4 \) dimensional matrix with elements that are finite mth-order polynomials in the lag operator \( L \).
(i.e., $C(L) = \left[ C_{ij}(L) \right] = \left[ \sum_{s=1}^{m} C_{ij}(s) L^{s-1} \right]$ for $i,j = 1,2,3,4$.)

and $C_0 = \left[ C_{10}, C_{20}, C_{30}, C_{40} \right]'$ is a vector of constants, and

$u_{1t} = \left[ u_{11t}, u_{12t}, u_{13t}, u_{14t} \right]' = Y_t - E(Y_t | Y_{t-s}, \text{for } s=1,2,\ldots,m)$

is a sequence of mean zero, independently, jointly normally distributed random vectors. The zeros of $\det[I - C(z)z]$ are assumed to be outside the unit circle because of stationarity of the $Y_t$ process.

Equation (16) implies the following set of restrictions across the VAR coefficients in (17):

$$
\begin{align*}
& a_1 C_{11}(s) + a_2 C_{21}(s) + C_{41}(s) = 0 \\
& a_1 C_{12}(s) + a_2 C_{22}(s) + C_{42}(s) = 0 \\
& a_1 C_{13}(s) + a_2 C_{23}(s) + C_{43}(s) = 0 \\
& a_1 C_{14}(s) + a_2 C_{24}(s) + C_{44}(s) = 0 \\
& a_1 C_{10} + a_2 C_{20} + C_{40} + \ln \beta + \sigma^2/2 = 0
\end{align*}
$$

for $s=1,2,\ldots,m$, and $a_1, a_2$ are defined as before. These restrictions reflect implications of the model including Ricardian equivalence hypothesis and describe how the growth in past government debt affects real interest rates and the growth in consumption. Therefore, a finding that these restrictions are not rejected by the data implies that the model of which implications include the equivalence hypothesis is a plausible description of the real data.

On the other hand, equation (16) yields the following restricted VAR system:

(18) $A_0 Y_t = B_1(L) Y_{t-1} + B_0 + u_{2t}$, or

$Y_t = A_0^{-1} B_1(L) Y_{t-1} + A_0^{-1} B_0 + A_0^{-1} u_{2t}$, where...
The matrix lag polynomial $B_1(L)$ is given in partitioned form by

$$B_1(L) = \begin{bmatrix} B_{kl}(L) \\ 0 \end{bmatrix}$$

with $\left[ B_{kl}(L) \right] = \left[ \sum_{l=1}^{m} B_{kl}(S) L^{s-1} \right]$ for $i=1,2,3$, $j=1,2,3,4$, and $0$ is a $1 \times 4$ zero (row) vector, and

$u_{2t} = [u_{21t}, u_{22t}, u_{23t}, W_t^*]'$. That is, the fourth row of the equation (18) is given by

$$a_1 v_{ct} + a_2 v_{gt} + r_t = -(\ln \beta + \sigma^2/2) + W_t^*$$

with $E_{1t} = W_t^* = 0$.

**B. A Test of the Hypothesis**

Because $u_{1t}$ for $t=1,2,...,T$, in (17) is independently normally distributed, the likelihood function of a sample of $u_{1t}$ for $t = 1,2,...,T$ is given by

$$L(\theta_1 | \{u_{1t} \}) = (2\pi)^{-4T/2} \left| V_1 \right|^{-T/2} \exp \left[ (-1/2) \sum_{t=1}^{T} u_{1t}' V^{-1} u_{1t} \right]$$

where $\theta_1 = [C_{ij}(L), C_0, V_1 - L=1,2,...,m, i,j = 1,2,3,4]$ and $V_1 = E u_{1t}' u_{1t}'$. Minimizing (19) subject to (17) yields unconstrained estimates of the coefficients $\theta_1$. Under the restriction (18), the likelihood function (19) becomes a function of $\theta_2 = [a, B_{ij}(L), B_0, V_2, L=1,2,...,m, i,j = 1,2,3,4]$, where $V_2 = E u_{2t}' u_{2t}'$. As Wilson (1973) and Bard (1974) have noted, maximum likelihood estimates with an unknown general (non-diagonal) covariance matrix $V$ for $i=1,2$ are obtained by minimizing with respect to the coefficients the criterion

$$(T/2) \ln |V_i| = (T/2) \ln \det(T^{-1} \sum_{t=1}^{T} u_{1t}' u_{1t}')$$

for $i = 1, 2$. 

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where the $\mathbf{u}_{it}'s$ are the estimated vectors of residuals. Let $L_u$ be the value of likelihood function at its unconstrained maximum while $L_r$ is the value of likelihood function under the restriction (18). Then, under the null hypothesis that the restriction (16) (or (18)) is correct,

$$-2 \ln \left( \frac{L_r}{L_u} \right) = -2 \left( -\frac{T}{2} \right) \ln \left( \frac{|\mathbf{V}_r|}{|\mathbf{V}_u|} \right) = T \left( \ln |\mathbf{V}_r| - \ln |\mathbf{V}_u| \right)$$

is asymptotically distributed as $X^2(q)$, where $q$ is the number of restrictions imposed, and where $\mathbf{V}_r$ and $\mathbf{V}_u$ are the constrained and unconstrained estimates of $\mathbf{V}$, respectively (see Wilson, 1973). Because the unconstrained version of the system in (17) has $4(1+4m)$ regressors, and the number of free parameters in the system is equal to $3+3(1+4m)$, the likelihood ratio statistic $-2 \ln \left( \frac{L_r}{L_u} \right)$ is distributed in large samples as $X^2(q)$ random variable with $q = 4(1+4m)-3-3(1+4m) = 4m-2$. Thus, $q = 2$ when $m = 1$, $q = 6$ when $m = 2$.

The empirical procedure is conducted by full information maximum likelihood (FIML) estimation to obtain $|\mathbf{V}_r|$ and $|\mathbf{V}_u|$. The actual estimation was carried out by using the GAUSS computer package (specifically, MAXIMUM program), which provides various algorithms for computing numerical gradients and hessian matrices to minimize the value of $|\mathbf{V}|$'s. Estimations and the tests were obtained by using quarterly data for two time periods: 1947,I through 1979,IV and 1947,I through 1987,I. The results are reported in Tables 1 and 2. During 1980 - 1987, government deficits and debts have increased significantly as a result of a fiscal revolution created by the Reagan administration that includes a program of tax cuts.

Consumption expenditure ($\mathbf{C}_t$) is measured, following the usual practice of excluding durables from measured consumption, by per capita personal
Table 1.1  FIML ESTIMATION OF EQUATIONS (17) and (18) with m=1.
1947:III to 1979:IV

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<td>(0.0697)</td>
<td>(0.0942)</td>
</tr>
<tr>
<td>B3j</td>
<td>-0.0071</td>
<td>-0.2378</td>
<td>-1.246</td>
<td>0.1723</td>
<td>1.922</td>
</tr>
<tr>
<td></td>
<td>(0.0044)</td>
<td>(0.3435)</td>
<td>(0.0868)</td>
<td>(0.0905)</td>
<td>(0.1224)</td>
</tr>
</tbody>
</table>

\[
\mu = -0.0802 \\
\mu_1 = -21.6101 \\
\mu_2 = 0.4905
\]

\[
|V_u| = 10^{-15} \times 0.361
\]

\[
|V_\mu| = 10^{-15} \times 0.374
\]

Likelihood ratio test statistic = 129 x ln (0.374/0.361) = 4.5537

^a Marginal significance level = 0.1021

Source: Citibank economic data base

Notes: Estimated standard errors in parentheses.

^a Marginal significance level is defined as Prob [X ≥ x] under the null hypothesis, where X is a chi-square distributed random variable and x is the test statistic.
### Table 1.2  FIML ESTIMATION OF EQUATIONS (17) and (18) with m=1.
1947:III to 1987:I

<table>
<thead>
<tr>
<th>j=0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_{1j})</td>
<td>.0052</td>
<td>.1087</td>
<td>-.0323</td>
<td>.0284</td>
</tr>
<tr>
<td></td>
<td>(.0010)</td>
<td>(.0803)</td>
<td>(.0216)</td>
<td>(.0213)</td>
</tr>
<tr>
<td>(C_{2j})</td>
<td>.0042</td>
<td>.0350</td>
<td>.6007</td>
<td>-.0880</td>
</tr>
<tr>
<td></td>
<td>(.0029)</td>
<td>(.2366)</td>
<td>(.0636)</td>
<td>(.0628)</td>
</tr>
<tr>
<td>(C_{3j})</td>
<td>-.0084</td>
<td>-.1567</td>
<td>-.0913</td>
<td>.2319</td>
</tr>
<tr>
<td></td>
<td>(.0037)</td>
<td>(.2998)</td>
<td>(.0806)</td>
<td>(.0795)</td>
</tr>
<tr>
<td>(C_{4j})</td>
<td>.0032</td>
<td>.1342</td>
<td>-.0096</td>
<td>-.0139</td>
</tr>
<tr>
<td></td>
<td>(.0019)</td>
<td>(.1513)</td>
<td>(.0407)</td>
<td>(.0401)</td>
</tr>
</tbody>
</table>

#### Unconstrained model (Equation (17))

#### Constrained model (Equation (18))

\(\mu = -.0574\)  \(\sigma_1 = -22.5513\)  \(\sigma_2 = -.5220\)

\(|V_u| = 10^{-15} \times .758\)

\(|V_u| = 10^{-15} \times .835\)

Likelihood ratio test statistic = 158 \(\times \ln (.835/.758) = 15.2862\)

Marginal significance level = .0005

**Notes:** See Table 1.1.
Table 2.1  FIML ESTIMATION OF EQUATIONS (17) AND (18) WITH \( m=2 \).
1947:III to 1979:IV

<table>
<thead>
<tr>
<th>( j=0 )</th>
<th>( m=1 )</th>
<th>( m=2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

**Unconstrained model (Equation (17))**

\[
C(m)_{1j} = \begin{pmatrix}
0.0054 & 0.1189 & -0.0741 & 0.0222 & 0.1173 \\
0.0122 & 0.0877 & 0.0291 & 0.0227 & 0.0648 \\
\end{pmatrix} 
\]

\[
C(m)_{2j} = \begin{pmatrix}
0.0080 & -0.1381 & 0.5895 & -0.0603 & -0.1559 \\
0.0037 & 0.2707 & 0.0898 & 0.0701 & 0.2000 \\
\end{pmatrix} 
\]

\[
C(m)_{3j} = \begin{pmatrix}
-0.0092 & -0.2712 & -0.2822 & 0.2376 & 0.0546 \\
0.0047 & 0.3422 & 0.1136 & 0.0886 & 0.2529 \\
\end{pmatrix} 
\]

\[
C(m)_{4j} = \begin{pmatrix}
0.0016 & -0.3606 & -0.1814 & -0.0368 & 0.7204 \\
0.0016 & 0.1202 & 0.0399 & 0.0311 & 0.0888 \\
\end{pmatrix} 
\]

**Constrained model (Equation (18))**

\[
B(m)_{1j} = \begin{pmatrix}
0.0039 & 0.0048 & -0.0198 & 0.0007 & 0.0301 \\
0.0122 & 0.0924 & 0.0307 & 0.0239 & 0.0682 \\
\end{pmatrix} 
\]

\[
B(m)_{2j} = \begin{pmatrix}
0.0072 & -0.1721 & 0.6270 & -0.0622 & -0.1789 \\
0.0037 & 0.2710 & 0.0899 & 0.0702 & 0.2002 \\
\end{pmatrix} 
\]

\[
B(m)_{3j} = \begin{pmatrix}
-0.0101 & -0.3090 & -0.2590 & 0.2366 & 0.0513 \\
0.0047 & 0.3424 & 0.1136 & 0.0887 & 0.2531 \\
\end{pmatrix} 
\]

\[
\mu = -0.1210 \\
a_1 = -29.9910 \\
a_2 = -0.9309 \\
\]

\[
\left| V_u \right| = 10^{-15} \times 0.242 \\
\left| V_v \right| = 10^{-15} \times 0.269 \\
\text{Likelihood ratio test statistic} = 128 \times \ln \left( \frac{0.269}{0.242} \right) = 13.5390 \\
\text{Marginal significance level} = 0.0352 \\
\]

Notes: See Table 1.1.
Table 2.2 FIML ESTIMATION OF EQUATIONS (17) and (18) WITH m=2.
1947:III to 1987:I

<table>
<thead>
<tr>
<th>j=0</th>
<th>m=1</th>
<th>m=2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconstrained model (Equation (17))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C(m)_{1j}$</td>
<td>.0060</td>
<td>.0823</td>
</tr>
<tr>
<td></td>
<td>(.0011)</td>
<td>(.0011)</td>
</tr>
<tr>
<td>$C(m)_{2j}$</td>
<td>.0029</td>
<td>.7320</td>
</tr>
<tr>
<td></td>
<td>(.0033)</td>
<td>(.2430)</td>
</tr>
<tr>
<td>$C(m)_{3j}$</td>
<td>-.131</td>
<td>-.1995</td>
</tr>
<tr>
<td></td>
<td>(.0041)</td>
<td>(.2999)</td>
</tr>
<tr>
<td>$C(m)_{4j}$</td>
<td>.0016</td>
<td>.7716</td>
</tr>
<tr>
<td></td>
<td>(.0021)</td>
<td>(.1526)</td>
</tr>
<tr>
<td>Constrained model (Equation (18))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(m)_{1j}$</td>
<td>.0036</td>
<td>.0047</td>
</tr>
<tr>
<td></td>
<td>(.0012)</td>
<td>(.0851)</td>
</tr>
<tr>
<td>$B(m)_{2j}$</td>
<td>.0025</td>
<td>-.0326</td>
</tr>
<tr>
<td></td>
<td>(.0033)</td>
<td>(.2431)</td>
</tr>
<tr>
<td>$B(m)_{3j}$</td>
<td>-.143</td>
<td>-.2458</td>
</tr>
<tr>
<td></td>
<td>(.0041)</td>
<td>(.3003)</td>
</tr>
</tbody>
</table>

| $\mu = -1.066$ | $a_1 = -30.0369$ | $a_2 = -1.0238$ |
|               | (.0198)     | (2.5115)     | (.6588)     |

$|V_u| = 10^{-15} \times .578$

$|V_u| = 10^{-15} \times .654$

Likelihood ratio test statistic = $157 \times \ln (0.654/0.578) = 19.3947$
Marginal significance level = .0035

Notes: See Table 1.1.
consumption expenditure on nondurables and services in constant 1982 dollars. Government expenditure \( (G_t) \) is measured by per capita government expenditure (federal, state, and local) on goods and services in constant 1982 dollars. Government debt \( (b_t) \) is measured by per capita total marketable interest-bearing public debt in constant 1982 dollars. Real interest rates \( (r_t) \) were computed following the formula in (11), where \( I_t \) is measured by the three-month return on Treasury bill yields and \( \Pi_t \) is measured using GNP deflator. All the data were obtained from the Citibase data file.

First, the sample period 1947,1 - 1979,IV was used to estimate the system (17) and (18), and the likelihood ratio statistics were computed. The period 1947,1 - 1979,IV yields a total of 132 quarterly observations. One observation, however, is used to compute inflation \( (\Pi_t) \), and another to compute differences \( (\nu_{ct}, \nu_{gt}, \nu_{bt}) \). Also, \( m \) number of observations are lost for the lags in estimation. Therefore, \( T = 132 - 2 - m = 130 - m \).

Similarly, for the period 1947,1 - 1987,1, \( T = 161 - 2 - m = 159 - m \).

As shown in Table 1.1, when the the lag length was taken \( m = 1 \), the value of the log-likelihood ratio statistic is \( -2 \ln(L^2/L^1) = T (\ln V_r - \ln V_u) = 129.1 \) (Table 1.1). The test statistic is less than the 10 percent critical value of the \( X^2(2) \) distribution 4.61, hence the hypothesis is not rejected at this conventional level of significance, even though the evidence is not overwhelming. With the lag length \( m = 2 \), the value of the likelihood ratio statistic is 13.5390 and the marginal significance level is .0352 for the period 1947 - 1979 (Table 2.1).

When the sample period is extended to include the Reagan administration period up to 1987 (1947,1 - 1987,1), the lag length \( m = 1 \) yields the value of the likelihood ratio statistic 15.2862, with marginal significance level
The test statistic is far above the 1 percent critical value of the $X^2(2)$ distribution 9.21, hence the hypothesis is firmly rejected even at the 0.1 percentage level. With $m=2$ for the extended period 1947, I - 1987, I, the marginal significance level is .0035 (Table 2.2), hence the hypothesis is again rejected at any conventional level of significance.

From the estimated coefficients for the period 1947 - 1979 with $m=1$, $\mu = -.0802$, $a_1 = -21.8101$, $a_2 = -.9867$, it follows that the point estimate for the substitutability of government spending for private consumption, $\delta$, is 0.0453 ($\delta = .0311$ when $m=2$). This implies the government expenditure substitutes very little for private consumption. This is not surprising because the government expenditure $G_t$ includes federal expenditures that are generally thought to be a poor substitute for private consumption. This finding is in accord with Kormendi (1983) and Aschauer (1985), who also found that government spending substitutes poorly for private consumption in utility, but disagrees with Feldstein (1982), who found a large crowding out of private consumption expenditure. The point estimate for the risk-aversion parameter $\gamma$ is -21.7968 ($\gamma = -29.9219$ when $m=2$).

If the model was firmly rejected for both sample periods, it would be very hard to identify what features or assumptions of the model were contradicted by the data. This is because the restrictions of the model reflect such assumptions as the specification of the preference (time-additive CRRA type), the specification of the distribution (the covariance stationary log normal distribution), and the auxiliary equational assumptions as well as implications of the Ricardian equivalence hypothesis. Any of these factors could be the source of the rejection of the model.
Because the model was not rejected for the period up to 1979, however, it is implied that the Ricardian equivalence hypothesis reflected in the model appears to mimic the real data fairly well, at least until 1979.

III. Data Analysis based on a VAR representation.

What are the implications of these empirical results and what caused rejection of the model in the extended period up to 1987? An immediate implication would be that some changes may have occurred in the coefficients of the VAR system (17) when the sample period was extended from 1979 to 1987. The VAR coefficients, in general, are difficult to give meaningful economic interpretations. Instead, equivalent information is preserved in the moving average coefficients, which are easier to interpret. This leads to the so-called VAR analysis of the data pioneered by Sims (1980) [For a brief discussion of the innovation accounting that provides the basis for the VAR analysis in this section, see Appendix B] Recently, some economists (e.g., Barro, 1980, Plosser, 1982, and Hirschhorn, 1984) have tended to analyze the equivalence hypothesis in terms of the effects of the shocks (or unanticipated changes) in the government debt. It would be interesting to apply the innovation accounting method to the data and to compare the outcome with that of previous works.

Before getting into the data analysis based on a VAR, a Chow test of each equation in (17) is implemented as a preliminary step to identify which equations of the four-variable (τc, τg, τb, r) system are mainly responsible for the change in the test result of section II. In Table 3, splitting the sample at 1980 shows that significant differences between the two parts of the sample are associated with the equations for interest rates and
Table 3. Sample Split Test

<table>
<thead>
<tr>
<th>Equation</th>
<th>F Statistics</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) With 1 lag</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>$F(5,148) = 1.4765$</td>
<td>.2009</td>
</tr>
<tr>
<td>$\nu_g$</td>
<td>$= 2.5998$</td>
<td>.0276</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>$= 1.2720$</td>
<td>.2790</td>
</tr>
<tr>
<td>$r$</td>
<td>$= 4.0928$</td>
<td>.0016</td>
</tr>
<tr>
<td>(b) With 2 lags</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>$F(9,139) = 1.4974$</td>
<td>.1545</td>
</tr>
<tr>
<td>$\nu_g$</td>
<td>$= 2.2861$</td>
<td>.0201</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>$= 1.0525$</td>
<td>.4019</td>
</tr>
<tr>
<td>$r$</td>
<td>$= 2.2157$</td>
<td>.0244</td>
</tr>
</tbody>
</table>

Note: see (11) for notations.
Also see footnote 8.

Table 4. Four Variable Innovation Accounting:
Percentage of 24 quarter Forecast Error Variance Explained in each Period
(1947 to 1979) / (1947 to 1987)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Explained</th>
<th>Std.Error</th>
<th>By Innovations in</th>
<th>$\nu_g$</th>
<th>$r$</th>
<th>$\nu_c$</th>
<th>$\nu_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_g$</td>
<td>.02/.02</td>
<td>72/85</td>
<td>15/6</td>
<td>9/6</td>
<td>4/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>.02/.02</td>
<td>1/2</td>
<td>73/84</td>
<td>22/11</td>
<td>4/3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>.01/.01</td>
<td>5/6</td>
<td>9/9</td>
<td>82/81</td>
<td>3/4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>.02/.02</td>
<td>5/6</td>
<td>4/14</td>
<td>8/7</td>
<td>82/73</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Std.Error reports forecast standard error for the variable explained when sampling error in the estimated coefficients is ignored.
FIGURE 1. RESPONSES TO SHOCK IN DEBT

1947-1979 period

1947-1987 period
FIGURE 2. RESPONSES TO SHOCK IN G

- 1947-1979 period
- 1947-1987 period

15.3
government expenditures. In the context of the equivalence hypothesis, the change in interest rate-behavior is more important.

If the market is efficient and economic agents are rational, so that they anticipate the future tax liabilities associated with government debt, it is expected that past information on debt should have little, if any, predictive power for current interest rates and the growth in consumption. The restriction (16) of the model also implies that, once the growth in government expenditure and the interest rates are accounted for, the information on government debt should be of no help in predicting the growth in consumption (or, once it is adjusted for the growth in government spending and consumption, government debt should not help predict real interest rates). Plosser (1982), in particular, argued that surprise increases in government debt should not drive interest rates up based on the assumption of rational expectations and market efficiency.

As reported in error decomposition based on the four variable innovation accounting with four lags [Table 4], innovations in debt explain only 3 and 4 percent of forecast error variance in the growth in consumption, and 4 and 3 percent of forecast error variance in real interest rates in 24 quarters in the first (1947-1979) and second (1947-1987) sample periods, respectively. This implies that the change in debt does not have any significant predictive power for interest rates and the growth in consumption in either period. This result seems to be fairly robust to changes in ordering of the variables in the system.

Further investigation of a possible difference in impulse responses in two periods is illustrated in Figure 1 by the impulse responses to shocks in debt for two sample periods: 1947-1979 and 1947-1987. It indicates that, for a given change in government debt, the public tends to consume more, and
a positive response of private consumption tends to be more persistent than before. Government expenditures tend to recover faster than before. In contrast, real interest rates tend to move downward even further in response to shocks in government debt.\(^9\)

It remains an open question why real interest rates respond negatively to the shocks in debt; this does not seem to nicely fit the conventional view or the Ricardian equivalence view, although only a small fraction of real interest rates is explained by debt. This finding, however, agrees with Evans (1985, 1987), who found that there is no statistically significant positive association between interest rates and budget deficits and instead found that several associations are significantly negative. This also is consistent with Plosser’s finding (1982) that surprise increases (or innovations) in government debt do not lead to upward movements in real interest rates.

In recent years real interest rates have been relatively high, and it was suspected that this was partly due to a higher level of government debt issue. The preceding discussion, however, shows that innovations in debt may not be an important source of recent high interest rates. The responses to shock in government expenditure are plotted in figure 2. In Figure 2, it is noted, however, that interest rates responded positively to the shocks in government spending during the first six quarters. For the extended period up to 1987, interest rates responded more strongly to the government spending shocks. In other words, interest rates seem to respond positively to shocks in government spending rather than to shocks in debt [see Plosser (1982) for similar results]. In short, government debt does not seem to have any significant predictive power for interest rates and the growth in consumption in either period and, in response to a given change in debt, the negative effect on real interest rates seems to be even stronger in recent
years. These are, as a first approximation, not at variance with the implications of the equivalence hypothesis. It is still not clear from this analysis what really causes the strong rejection of the model for recent years.

IV. Summary and Concluding Remarks

This paper has investigated whether the Ricardian equivalence hypothesis holds for the postwar U.S. data in a general equilibrium framework. A nonlinear-quadratic, intertemporal framework was employed to incorporate time-varying interest rates into the model. The implications were summarized by the restrictions on a linear system of the variables, VAR coefficients, under certain assumptions. The restriction was not rejected for the sample period 1947,1 through 1979,IV; it was, however, rejected when the sample period was extended up to 1987,1.

An attempt was made to explore implications of the rejection through data analysis based on a VAR approach, even though the cause of rejection could not be fully ascertained because of the complexity of the hypothesis. The evidence seems to indicate a possible change in the behavior of interest rates. Whether this signals the shift in the regime is not clear due to limitations of the analysis. The data analysis based on a VAR does not indicate any strong violation of the equivalence hypothesis. From the methodological point of view, this work illustrates a close link between a nonlinear-quadratic structural model and a linear VAR representation of the data under certain circumstances.
Appendix A (data)

\( g_t \): per capita government expenditure (federal, state and local) on goods and services in constant (1982) dollars

\( i_t \): three month Treasury bill rates

\( b_t \): per capita total marketable interest-bearing public debt in constant (1982) dollars.

\( c_t \): per capita personal consumption expenditure on nondurables and services in constant (1982) dollars

\( p_t \): GNP deflator.

source: Citibase data file.

Appendix B. Innovation accounting.

From the moving average representation of \( Y_t \) in (12), \( Y_t = D(L) u_{1t} + d_0 \), the \( D_{ij}(s) \), the \( i,j \) th component of \( D(s) \), represents the dynamic response of each endogenous variable \( Y_i \) after \( s \) periods to an initial shock in \( Y_j \).

Although \( u_{1t} \) is serially uncorrelated by the construction, the components of \( u_{1t} \) may be contemporaneously correlated so that the interpretation of \( D(s) \) may be misleading. Therefore, an orthogonalizing transformation to \( u_{1t} \) is done as follows.

\[
Y_t = \sum_{s=0}^{\infty} D(s) G^{-1} G u_{1t-s} + d_0 = \sum_{s=0}^{\infty} D(s) G^{-1} w_{t-s} + d_0
\]

\[
= \sum_{s=0}^{\infty} H(s) w_{t-s} + d_0,
\]

where \( \text{var}(u_{1t}) = V, \ V^{-1} = G'G, \) and \( \text{var}(w_t) = \text{var}(G u_{1t}) = I. \)

When \( G \) is taken to be a lower triangular matrix, the coefficients of \( H(s) \)
represent responses to shocks (or innovations) in particular variables. The variance of each element in $Y_t$ can be unambiguously allocated to sources in elements of $w$ because $w$ is now serially and contemporaneously uncorrelated. The orthogonalization provides $\sum_{s=0}^{T} H(s)_{i,j}^2$, which is the component of the error variance in the $T+1$ step ahead forecast of $Y_i$ that is accounted for by the innovations in $Y_j$.

Because a vector autoregressive representation of $Y_t$ is already derived in (17), the moving average representation of $Y_t$ is thought to be obtained from the VAR representation in (17). One possible problem would be the interpretation of the innovation in $Y_t$, because $Y_t$ includes the differenced variables such as $v_{ct}, v_{gt}, v_{bt}$. It is noted, however, that the innovations in these variables are, for example,

$$(E_{lt} - E_{lt-1})v_{bt} = E_{lt} (b_t - b_{t-1}) - E_{lt-1} (b_t - b_{t-1}) = (E_{lt} - E_{lt-1}) b_t.$$  

That is, the innovation in $v_{bt}$ amounts to the innovation in $b_t$, hence it can have the usual interpretations.
Footnotes


2. For example, to derive a testable hypothesis of the implications of the Ricardian equivalence hypothesis, Aschauer (1985) assumes a quadratic preference and a constant interest rate.


4. The assumption of a covariance stationary process of \( Y_t = [\tau_t, \gamma_t, \tau_{b_t}, \tau_{r_t}] \) seems to me a reasonable one. It is a common practice in time series statistics to difference a seemingly nonstationary process (i.e., with no fixed mean) to produce (hopefully) a stationary process (for example, see Box and Jenkins, 1976). For an interesting recent discussion of a differencing versus a detrending of the economic variables, see Nelson and Plosser (1982). A logarithmic transformation of the data helps to stabilize the variations (i.e., to reduce heteroscedasticity) that tend to increase with the level of the data. Also, a Gaussian process that is covariance stationary, which is assumed for \( Y_t \) in this paper, will be (strictly) stationary because its distribution is determined by its first two moments. Even though differencing of series \( c_t \) and \( g_t \) was dictated by the model to derive a closed form restriction, it may be unnecessary to difference all the series simultaneously when considering several nonstationary series jointly because it may result in more complications in model fitting (see Tiao and Box, 1981, p. 804).

5. For example, using the notation that will be introduced later in equation (17),

\[
E_{t-1} \tau_{c_t} = C_{11}(L) \tau_{c_{t-1}} + C_{12}(L) \tau_{g_{t-1}} + C_{13}(L) \tau_{b_{t-1}} + C_{14}(L) \tau_{r_{t-1}}
\]

\[
E_{t-1} \tau_{g_t} = C_{21}(L) \tau_{c_{t-1}} + C_{22}(L) \tau_{g_{t-1}} + C_{23}(L) \tau_{b_{t-1}} + C_{24}(L) \tau_{r_{t-1}}
\]

where \( C_{ij}(L) \), \( i=1,2, \ j=1,2,3,4 \), is a matrix polynomial in the lag operator.

6. More precisely, stationary process \( Y_t \), which has a moving average representation in (12), \( Y_t = B(L) u_t + \mu_t \), also has a vector autoregressive (VAR) representation (17) when it is invertible (i.e., all the zeros of the determinantal polynomials of moving average representation coefficients \( B(L) \) are outside the unit circle).

7. To implement the FIML estimation procedure more efficiently, first the estimates from the OLS were used as the starting values for the unrestricted model estimation. Second, for the restricted model estimation, the estimates from the unrestricted model were used again as the starting values, and various other starting values also were tried for the starting values, including \( \mu, a_1, \) and \( a_2 \), to ensure the global optimization points.
8. It can be shown that \( \frac{(RES-URSS)/k}{URSS/(n_1+n_2-2k)} \) is distributed as \( F(k, n_1+n_2-2k) \), where URSS (unrestricted residual sum of squares) is obtained by adding the residual sum of squares for each equation corresponding to each sample period which has d.f. \( n_1+n_2-2k \). RES (restricted residual sum of squares) is obtained by estimating a single equation over the whole sample period which has d.f. \( n_1+n_2-k \), where \( n_1 \) and \( n_2 \) are the numbers of observations in the first and the second sample period, respectively, and \( k \) is the number of coefficients.

With one lag, \( k=5 \), \( n_1=129 \) (1947,4-1979,4), \( n_2=29 \) (1980,1-1987,1), hence \( n_1+n_2-2k=148 \). With two lags, \( k=9 \), \( n_1=128 \) (1948,1-1979,4), \( n_2=29 \), hence \( n_1+n_2-2k=139 \) in table 3.

9. There have been several attempts to reconcile this finding with the Ricardian regime. For example, Hirschhorn (1984) provides a rational expectations model with limited information that shows that a positive correlation between unanticipated government debt and output does not imply that government bonds are net wealth. Barro (1980) also found that the unanticipated part of government debt affected output.
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