Appendix to 'The optimal choice of monetary policy instruments in a small open economy'

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Abstract
This is an Appendix to "The Optimal Choice of Monetary Policy Instruments in a Small Open Economy" published in The Canadian Journal of Economics / Revue Canadienne d'Economique, Vol. 41, No. 1 (February 2008): 105-137.

Keywords
monetary policy

Disciplines
Economics

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Appendix to "The Optimal Choice of Monetary Policy Instruments in a Small Open Economy"

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March 2007

Working Paper # 07005

Department of Economics
Working Papers Series

Ames, Iowa 50011

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A Appendix to “The Optimal Choice of Monetary Policy Instruments in a Small Open Economy”

A.1 The household’s problem

The household maximizes (1) subject to (4) and (6). Assuming that the equilibrium nominal interest rate is positive and therefore (6) will always hold with equality, (6) can be substituted into (4) to get

\[
P(h^t) C(h^t) = (1 - V_{t-1}) \left[ W(h^{t-1}) N(h^t) + \Pi(h^t) \right] + D(h^t)
+ D_t^* S(h^t) - \int_{h_{t+1} \in H} D(h_{t+1}, h^t) \, Q(h_{t+1} | h^t) \, dh_{t+1}
- R_t^{-1} D_{t+1}^* S_t + TR_t + v_t \left[ W_t(h^t) N_t(h^t) + \Pi(h^t) \right].
\] (A.1)

Then the household’s problem reduces to maximizing (1) subject to (A.1). The first order conditions with respect to consumption choices are given by

\[
\frac{1}{C(h^t) P(h^t)} = \lambda(h^t),
\] (A.2a)

\[
\frac{1}{C(h^{t+1}) P(h^{t+1})} f(h_{t+1} | h^t) = \lambda(h^{t+1}),
\] (A.2b)

where the law of motion of multipliers can be obtained from the choice for state-contingent assets.

\[
\frac{\lambda(h^{t+1})}{\lambda(h_t)} = Q(h_{t+1} | h^t).
\] (A.3)

Equations (A.2a) - (A.3) together yield the standard asset pricing equation

\[
\frac{C(h^t) P(h^t)}{C(h^{t+1}) P(h^{t+1})} f(h_{t+1} | h^t) = \frac{\lambda(h^{t+1})}{\lambda(h_t)} = Q(h_{t+1} | h^t).
\] (A.4)

Define \( R_t \equiv (\int Q(h_{t+1} | h^t) \, dh_{t+1})^{-1} \). Then, integrating (A.4) over \( h_{t+1} \) obtains

\[
\beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{P_t}{P_{t+1}} \right) \right\} = R_t^{-1},
\]

which is the Euler equation in the main text (after replacing arguments \( h^t \) by \( t \) subscripts).
The choice for the supply of labor is given by

\[ \chi \left[ N \left( h^t \right) \right] \phi = V_t W \left( h^t \right) \lambda \left( h^t \right) + (1 - V_t) W \left( h^t \right) \int \lambda \left( h^{t+1} \right) \, dh_{t+1}, \]  

(A.5)

which, using (A.2a) and (A.3) with (A.5), yields

\[ \chi N_t \phi C_t = \frac{W_t}{P_t} \left( V_t + (1 - V_t) R_t^{-1} \right). \]

Risk sharing The existence of complete set of domestic currency state contingent bonds obtains the following Euler condition for the rest-of-the-world households:

\[ \beta \left( \frac{C^* \left( h^{t+1} \right)}{C^* \left( h^t \right)} \right)^{-1} \left( \frac{P^* \left( h^t \right)}{P^* \left( h^{t+1} \right)} \right) \left( \frac{S \left( h^t \right)}{S \left( h^{t+1} \right)} \right) f \left( h_{t+1} | h^t \right) = Q \left( h_{t+1} | h^t \right), \]

which after combining with (A.4) obtains the following risk sharing condition:

\[ C_t = \Upsilon C_t^* Q_t, \]

(A.6)

where \( Q \equiv \frac{s}{P^*} \) is the real exchange rate; \( \Upsilon \) is a constant that depends on the initial wealth distribution of the world economy.

A.2 Goods and money market equilibrium

The market clearing condition for domestic output can be written as

\[ Y_t(i) = C_{H,t}(i) + C_{H,t}^*(i) + G_{H,t}(i), \]

where \( C_{H,t}^*(i) \) is the foreign demand for the home good \( i \). Using (2), (3) and (19) we can rewrite the above expression as

\[ Y_t(i) = \left( \frac{P_{H,t} \left( i \right)}{P_{H,t}} \right)^{-\epsilon} \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-1} \left( 1 - \alpha \right) C_t + \left( \frac{P_{H,t}}{S_t P^*} \right)^{-1} \alpha^* C_t^* \right] \]

\[ + \left( \frac{P_{H,t} \left( i \right)}{P_{H,t}} \right)^{-\epsilon} G_{H,t}. \]
Note that \( C_t^* = B_t^* Y_t^* \). Then, using risk sharing condition, \( C_t = \Upsilon C_t^* Q_t \) and setting \( \frac{\alpha^*}{\alpha} = \Upsilon \), we can rewrite the above expression as

\[
Y_t(i) = (P_{H,t}(i))^{-\epsilon} \Upsilon B^* Y_t^* \left[ \left( \frac{P_{H,t}}{P_t} \right)^{1-\alpha} (1-\alpha)Q_t + \left( \frac{P_{H,t}}{S_tP_t^*} \right)^{-1} \right] \\
+ (P_{H,t}(i))^{-\epsilon} \ G_{H,t}.
\]

(A.7)

Substituting (A.7) into the definition of aggregate output \( Y_t = \left[ \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}}di \right]^{\frac{1}{1-\epsilon}} \), and using (20) along with the basic identities for \( P_{H,t}, Q_t, \) and \( Z_t \), we get

\[
B_t Y_t = \Upsilon Z_t B^* Y_t^* ,
\]

(A.8)

which leads to equation (21) in the log-linear form.

### A.3 Social planner’s problem

Here we characterize the optimal allocation from the point of view of a social planner facing the resource constraints that the small open economy is subject to in equilibrium, and given our assumption of complete markets. The social planners problem is to maximize

\[
\log(C_{t+1}) - \chi \frac{N_{t+1}^{1+\phi}}{1+\phi}
\]

subject to

\[
Y_t = A_t N_t,
\]

and

\[
C_t = (B_t Y_t)^{1-\alpha} (\Upsilon B_t^* Y_t^*)^\alpha ,
\]

where the latter is obtained by combining (A.6) and (A.8). It is easy to show that the optimal allocation must satisfy \( N = (1-\alpha)^{\frac{1}{1+\phi}} \). Since \( Y = A N \), fluctuations in the efficient level of output are given by \( y_t^E = a_t \).
A.4 Loss Function

Following Woodford (2003), Gali and Monacelli (2002), and Walsh (2003), in this section we develop the loss function for the policymaker. Taking the second order Taylor approximation for the consumption path of the utility we get: and ignoring terms of order \( J^i \) for \( i \geq 2 \) we get

\[
U(C_t) = U(\overline{C}) + U_{c}(\overline{C})[c_t]. \tag{A.9}
\]

We now obtain an expression for the disutility of work. Following Walsh (2003), the second order Taylor expansion for \( V(N) \) is

\[
V(N_t) \approx V(\overline{N}) + V_{N}(\overline{N})[y_t - a_t + \frac{1}{2} \left( \frac{1}{\theta} \right) \text{var}_t y_t] + \frac{1}{2} (1 + \varphi)(y_t - a_t)^2. \tag{A.10}
\]

Combining Equations A.9 and A.10 we get

\[
U(C_t) - V(N_t) = U(\overline{C}) - V(\overline{N}) + U_{c}(\overline{C})[c_t] - V_{N}(\overline{N})[y_t - a_t + \frac{1}{2} \left( \frac{1}{\theta} \right) \text{var}_t y_t] + \frac{1}{2} (1 + \varphi)(y_t - a_t)^2. \tag{A.11}
\]

Noting that \( \frac{L}{L_H} = Z^{\alpha} \) and using (A.6) and (A.8) we can write the steady state labor market clearing condition as

\[
\frac{\overline{V}_{N} \overline{B} \overline{Y}}{U_c \overline{C}} = \frac{W \bar{X}}{P_H}, \tag{A.12}
\]

where \( \bar{X} = (\bar{V} + (1 - \bar{V}) \bar{R}^{-1}) \). We now define \( \Omega \) such that

\[
1 - \Omega = \frac{\bar{X}}{\mu B (1 - \alpha)}, \tag{A.12}
\]

where \( \Omega \) is a measure of the distortions in the economy. Equation (A.12) shows that these distortions include those arising due to market power, fiscal shocks, transaction frictions and the incentive to manipulate the terms of trade.\(^{12}\)

\[
\overline{V}_{N} \overline{Y} = U_{c} \overline{C}(1 - \Omega) (1 - \alpha). \tag{A.13}
\]
We will assume $\Omega$ to be very small such that terms like $(1 - \Omega)y_t^2 \approx y_t^2$. Substituting for $c_t$ from (22) the utility approximation can be written as

$$U(C_t) - V(N_t) = U(C) - V(N) - \frac{1}{2} U_c(C) (1 - \alpha) \left\{ (1 + \varphi) [y_t - a_t - \kappa^*]^2 \right\}$$  \hspace{1cm} (A.13)

$$- \frac{1}{2} U_c(C) C(1 - \alpha) \text{var}_i y(i)$$

+ terms independent of policy,

where $\kappa^* = \frac{\Omega}{1 + \varphi}$. Using the procedure detailed in Ravenna-Walsh (2003), Woodford (2003), the above expression representing the present discounted value of the utility of the representative household can be approximated by

$$\sum_{t=0}^{\infty} \beta^t U_t \approx U - F \sum_{t=0}^{\infty} \beta^t \left[ \pi_{H,t}^2 + \omega (y_t - a_t - \kappa^*)^2 \right],$$  \hspace{1cm} (A.14)

where $F = U_c(C) (1 - \alpha) \left[ \frac{\theta}{(1 - \theta)(1 - \varphi)} \right] \epsilon$, $\omega = \left[ \frac{(1 - \theta)(1 - \varphi)}{\theta} \right] \left( \frac{1 + \varphi}{\epsilon} \right) = \frac{\kappa(1 + \varphi)}{\epsilon}$, and $\kappa^*$ is the gap between flexible price steady state output level and the efficient steady state output level.

Since our focus is on stabilization policies, we will follow the literature in assuming that there are fiscal subsidies that eliminate these efficiency distortions so that $\kappa^* = 0$.

### A.5 Policy under discretion

Formally, the problem of the central bank is to choose $x_t$, $\pi_{H,t}$, and $r_t$ at $t$, such that it maximizes (33) subject to (31) and (32) while households’ expectations are taken as given. Letting $\lambda_1$ and $\lambda_2$ be the Lagrangian multiplier associated with (32) and (31) yields the following first order conditions:

$$-\omega x_t + \lambda_1 - \kappa(1 + \varphi) \lambda_2 = 0,$$  \hspace{1cm} (A.15a)

$$-\pi_{H,t} + \lambda_2 = 0,$$  \hspace{1cm} (A.15b)

$$\lambda_1 - \kappa \delta_1 \lambda_2 = 0,$$  \hspace{1cm} (A.15c)

with respect to $x_t$, $\pi_{H,t}$, and $r_t$, respectively. Eliminating $\lambda_1$ and $\lambda_2$ from the above equations gets (34).
Inflation dynamics under discretion Combining (31), (32), and (34) we get

\[ \pi_t = \omega_1 E_t \pi_{t+1} - \omega_2 a_t - \omega_3 v_t + \omega_4 b_t, \]

where

\[ \omega_1 = \frac{[\beta - \kappa \delta_1 \Theta + \kappa \delta]}{[1 + \kappa \Theta ((1 + \varphi - \delta_1))]}, \omega_2 = \frac{\kappa \delta (1 - \rho_a)}{[1 + \kappa \Theta ((1 + \varphi - \delta_1))]}, \omega_3 = \frac{\kappa \delta_0}{[1 + \kappa \Theta ((1 + \varphi - \delta_1))]}, \omega_4 = \frac{\kappa [1 - \delta_1 (1 - \rho_b)]}{[1 + \kappa \Theta ((1 + \varphi - \delta_1))]} . \]

Note \( \omega_1, \omega_2, \omega_3, \omega_4 > 0 \) and \( \Theta = \epsilon \left[ 1 - \frac{1}{1 + \varphi} \right] \). For values listed in Table 1 the absolute value of \( \omega_1 \in [0, 1) \), implying inflation has a stationary solution. Assuming assume \( a_t, v_t, b_t \) are all AR(1) processes, we can solve forward to obtain (35), where \( \phi_b = \frac{\omega_4}{1 - \omega_1 \rho_b} \), \( \phi_a = \frac{\omega_2}{1 - \omega_1 \rho_a} \), \( \phi_v = \frac{\omega_3}{1 - \omega_1 \rho_v} \).

A.6 Policy under full commitment

The problem of the policymaker can be expressed as:

\[ -\frac{1}{2} \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \pi_{H,t}^2 + \omega x_t^2 \right\} + 2 \Theta_t [x_t - E_t x_{t+1} + (r_t - (E_t \pi_{H,t+1}) - u_t] + 2 \Psi_t [\pi_{H,t} - \beta E_t \pi_{H,t+1} - \kappa (1 + \varphi) x_t - \kappa \delta_1 r_t + \kappa \delta_0 v_t - \kappa b_t] \right\} , \]

where \( 2 \Theta_t \) and \( 2 \Psi_t \) are the state contingent multipliers associated with the two constraints respectively. The first order conditions with respect to \( x_t, \pi_{H,t}, \) and \( r_t \) are:

\[ \omega x_t + \Theta_t - \frac{\Theta_{t-1}}{\beta} - \Psi_t (1 + \varphi) \kappa = 0, \quad (A.16a) \]

\[ \pi_{H,t} - \frac{\Theta_{t-1}}{\beta} + (\Psi_t - \Psi_{t-1}) = 0, \quad (A.16b) \]

\[ \Theta_t - \kappa \delta_1 \Psi_t = 0, \quad (A.16c) \]

A full commitment plan is defined as a bounded solution \( \{ \pi_{H,t}, x_t, r_t, \Theta_t, \Psi_t \}_{t=0}^{\infty} \) to the system of equations (32), (31), (A.16a) - (A.16c) along with the initial conditions \( \Theta_{-1}, \Psi_{-1} = 0 \). Combining (A.16a) and (A.16c) rewrite

\[ \Psi_t = \frac{1}{\kappa [(1 + \varphi) - \delta_1]} \left( \omega x_t - \frac{\Theta_{t-1}}{\beta} \right). \]
Use the above with (A.16b) to obtain
\[ x_t - x_{t-1} = -\frac{\kappa}{\omega} [(1 + \varphi) - \delta_1] \pi_{H,t} + \kappa [(1 + \varphi) - \delta_1] \frac{\Theta_{t-1}}{\omega} + \frac{\Theta_{t-1} - \Theta_{t-2}}{\omega}, \]
which using \( \omega = \frac{\kappa(1+\varphi)}{\epsilon} \) is expressed as (36) in the main text.

### A.7 Fixed exchange rates

Using (37) to substitute for \( z_t \) in (21), and using the resulting expression in (25) yields
\[ m_{ct} = -(1 + \varphi) p_{H,t} - (1 + \varphi) a_t - \delta_0 v_t - \varphi b_t, \]
which combined with (18) obtains
\[ \Lambda_e p_{H,t} = p_{H,t-1} + \beta E_t \{ p_{H,t+1} \} - \kappa(1 + \varphi) a_t - \kappa \delta_0 v_t - \kappa \varphi b_t, \]
where \( \Lambda_e = 1 + \beta + \kappa(1 + \varphi) \). Taking shocks one at a time, it can be shown that
\[ p_{H,t} = \Psi_e p_{H,t-1} - \Omega_{ex} x_t, x = a, v, \text{ and } b. \]
where \( \Psi_e = \frac{1}{\beta^2} \left( \Lambda_e - \sqrt{\Lambda_e^2 - 4\beta} \right) \) and \( \Omega_{ea} = \frac{\kappa(1+\varphi)}{\Lambda_e - \beta \psi_e - \beta p_a}, \Omega_{ev} = \frac{\kappa \delta_1}{\Lambda_e - \beta \psi_e - \beta p_v}, \text{ and } \Omega_{eb} = \frac{\kappa \psi_e}{\Lambda_e - \beta \psi_e - \beta p_b}. \) Under our parameter assumptions (see Table 1) \( \Omega_{ea}, \Omega_{ev}, \Omega_{eb} > 0. \)

### A.8 Monetary targeting

#### A.8.1 Domestic prices

Analogous to the case of fixed exchange rates, we can combine (18), (21), (25), (40) to obtain
\[ \Lambda_M p_{H,t} = p_{H,t-1} + \beta E_t p_{H,t+1} + \xi_v v_t - \xi_a a_t + \xi_b b_t, \]
where
\[ \Lambda_M = (1 + \beta + \kappa(1 + \varphi)) ; \quad \xi_v = \left( \frac{\kappa(1 + \varphi - \delta_1) + \kappa \delta_1 \rho_v}{(1 - V)} - \kappa \delta_0 \right); \]
\[ \xi_a = \kappa(1 + \varphi); \quad \xi_b = \kappa (1 - \delta_1 + \delta_1 \rho_b). \]
For the parameter values listed in Table 1, \( \Lambda_M, \xi_v, \xi_a, \xi_b > 0 \). Taking shocks one at a time, it can be shown that

\[
p_{H,t} = \Psi_M p_{H,t-1} + \Omega_M \chi_t, \quad \chi = v, \text{ and } b,
\]

where

\[
\Psi_M = \frac{1}{2\beta} \left( \Lambda_M - \sqrt{\Lambda_M^2 - 4\beta} \right) \in (0, 1), \quad \Omega_Mv = \frac{\xi_v}{\Lambda_M - \beta \Psi_M - \beta \rho_v} > 0, \quad \Omega_Mb = \frac{\xi_b}{\Lambda_M - \beta \Psi_M - \beta \rho_q} > 0 \quad \text{under our parameter assumptions (see Table 1).}
\]

### A.8.2 Exchange rates and Nominal interest rates under monetary targeting

**Velocity shocks** To obtain the path of nominal exchange rates under monetary targeting when there are shocks to velocity we combine (13), (21), and (40) to get nominal exchange rate as

\[
s_t = p_{H,t} + y_t = \frac{V}{1-V} v_t.
\]

The path of nominal interest rates

\[
r_t = \frac{V}{1-V} (E_t v_{t+1} - v_t)
\]

can be obtained by combining (31) and (40).

**Fiscal shocks** To understand the response of the nominal exchange, we first combine (13), (21) with (40) to obtain

\[
s_t = p_{H,t} + y_t + b_t = b_t, \quad \text{(A.17)}
\]

where the second equality follows from (40) after substituting \( v_t = 0 \). The path of the nominal interest rates

\[
r_t = (E_t b_{t+1} - b_t)
\]

can be obtained by combining (31), and (40).

### A.9 Domestic inflation targeting

Setting \( r_t = \tau \pi_{H,t} \) and plugging into (31) and (32) the equilibrium conditions can be summarized by means of the difference equation

\[
\begin{bmatrix}
x_t \\
\pi_{H,t}
\end{bmatrix} = A_T \begin{bmatrix}
E_t \{x_{t+1}\} \\
E_t \{\pi_{H,t+1}\}
\end{bmatrix},
\]

8
where

\[ A_T = \begin{bmatrix}
\frac{-\tau \kappa_x + 1}{\kappa_x + \tau \kappa_x + 1} & -\beta & \frac{-\tau \kappa_r + 1}{\kappa_r + \tau \kappa_r + 1} \\
\frac{-\tau \kappa_r + 1}{\kappa_r + \tau \kappa_r + 1} & -\frac{\beta}{\kappa_r + \tau \kappa_r + 1} & \frac{-\beta}{\kappa_r + \tau \kappa_r + 1}
\end{bmatrix}, \]

where \( \kappa_x = \kappa (1 + \varphi) \) and \( \kappa_r = \kappa a_1 \). For the values listed in Table 1, it is easily verified that both the eigenvalues of \( A_T \) lie inside the unit circle thereby establishing determinacy.

Combining (31), (32) and (27) obtains

\[ p_t = \Psi_{T1} p_{t-1} + \Psi_{T2} E_t p_{t+1} - \Psi_{T3} E_t p_{t+2} + \Omega_T \chi_t, \quad (A.18) \]

where \( \Psi_{T1} = \frac{1}{\tau} \left[ \frac{(1 - \kappa \delta_1 \tau)}{\kappa (1 + \varphi)} + \tau \right], \Psi_{T2} = \frac{1}{\tau} \left[ \frac{(1 - \kappa \delta_1 \tau)}{\kappa (1 + \varphi)} + 1 + \frac{2\beta}{\kappa (1 + \rho)} \right], \Psi_{T3} = \frac{\beta}{\kappa (1 + \varphi)} \),

\[ \Upsilon = \left[ \frac{2(1 - \kappa \delta_1 \tau)}{\kappa (1 + \varphi)} + \tau + 1 + \frac{\beta}{\kappa (1 + \varphi)} \right], \Omega_{Ta} = -\frac{(1 - \rho_a)}{\Upsilon}, \Omega_{Tv} = -\frac{\kappa \delta_0}{\Upsilon \kappa (1 + \varphi)} [(1 - \rho_v)], \Omega_{Tb} = -\frac{1}{\Upsilon} \left[ \frac{\varphi (1 - \rho_b)}{(1 + \varphi)} \right] \] and \( a_t, v_t, \) and \( b_t \) are all AR(1) processes with autocorrelation coefficients \( \rho_a, \rho_v, \) and \( \rho_b, \) respectively. Equation (A.18) can be rewritten as

\[ p_t = \frac{\Psi_{T2}}{\Psi_{T3}} p_{t-1} - \frac{1}{\Psi_{T3}} p_{t-2} + \frac{\Psi_{T1}}{\Psi_{T3}} p_{t-3}. \quad (A.19) \]

The characteristic polynomial associated with (A.19) is

\[ 1 - ax + bx^2 - cx^3 = 0. \]

For values listed in Table 1, the three roots of the above equation are found to lie inside the unit circle implying domestic price level exhibits a non-stationarity process.