Scattering investigation based on acoustical holography

Ming-Te Cheng
Iowa State University

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Scattering investigation based on acoustical holography

Cheng, Ming-Te, Ph.D.

Iowa State University, 1993
Scattering investigation based on acoustical holography

by

Ming-Te Cheng

A Dissertation Submitted to the
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Signature was redacted for privacy.

In Charge of Major Work
Signature was redacted for privacy.

For the Major Department
Signature was redacted for privacy.

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Iowa State University
Ames, Iowa
1993

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The Lord is my shepherd, I shall not be in want.
He makes me lie down in green pastures, he leads me beside quiet waters,
he restore my soul. He guides me in paths of righteousness for his name's sake.
Even though I walk through the valley of the shadow of death, I will fear no evil,
for you are with me; your rod and your staff, they comfort me.
You prepare a table before me in the presence of my enemies.
You anoint my head with oil; my cup overflows.
Surely goodness and love will follow me all the days of my life,
and I will dwell in the houses of the Lord forever. (Psalm 23)

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1. INTRODUCTION AND OUTLINE

1.1 Introduction

This study investigates the methods for determining the acoustic field scattered by an object (acoustic scattering cross-section) under different incident wave conditions. There is considerable applicability for this concept since estimation of the acoustic scattering cross-section of a scatterer is important for determining target strength [1]. The separation technique is based on measuring the composite (incident plus scattered) pressures in the near field of the scatterer using two-surface acoustical holography [2, 3, 4, 5, 6] as the candidate measurement technique. Wave number domain decomposition of measurements from these two surfaces allows separation of the scattered field, which can then be propagated to the farfield and used to predict the target strength. This research proves that it is feasible to separate the scattered element from a composite field, and to project the scattered field to the far-field with introduction of tolerable errors. Accurate far-field levels can be estimated from the processed near-field data collected over a finite, cost effective aperture. The work presented in this research provides a tool for understanding the complex scattering cross-sections of different scatterers.

The motivation for this research is that in underwater acoustics, the effectiveness of passive sonar is declining because of the improved quietness of underwater vehicles.
It is becoming essential to measure and understand the active acoustic target strength across a wide frequency range in both monostatic and bistatic conditions. This understanding is necessary to develop not only active sonar but also to ensure low target strength. Because the active sonar is directly affected by the sonar cross-section, to establish the full-scale target strength signatures is important to initiate corrective measures. However, acquiring measurement quality target strength data with conventional experimental methods may be limited by reverberation, multipath sound and other far-field considerations such as signal to noise ratio, ray bending, and interference, especially at ship beam aspect angles. These limitations are caused by the stratified layering of the ocean material properties and the air and bottom interfaces. The target strength must be in the Fraunhofer region to establish its target strength level, but this introduces complications when trying to distinguish the true echo from reverberation and multipath sound. In the past, these limitations have resulted in using approximations to the true far-field target strength. Also, detailed insight into the causes for large target strength values is difficult by conventional means due to the approximation of the scatterer as a point source [7].

Finding the accurate scattered fields of a complex shape requires solving or avoiding many of the problems with conventional approaches. The method based on acquiring the scattered fields in the near-field can eliminate several of the problems, such as reverberation and spreading loss associated with making conventional far-field measurements.

We choose acoustical holography as the main tool in the research because the data is acquired in the near-field so that many of the limitations of conventional methods are avoided. Furthermore, the advantage of acoustical holography over other
experimental methods of investigating sound scattering lies in its ability to deal with
the distributed aspects of acoustic sources [8]. In fact, detailed spatial characteristics
of fields can be recovered at any stage of holographic transformation. In addition,
the nearfield acoustical holography technique can accurately interpret acoustic fields
in the close vicinity of vibrating bodies. This feature is especially attractive because
it provides the basis for developing a method for scattering measurements that is in-
sensitive to the acoustic environment. Thus, the need for expensive anechoic facilities
that exists in other methods can be eliminated.

The payoff for this scattered field measurement approach which consists of a
near-field data collection system and signal processing to determine far field levels is
far reaching. Specifically, this method can,

1. Produce a higher signal to noise ratio.

2. Allow precise target strength measurements to be made in a controlled geometry
   environment that is less constrained by reverberation or multipath sound.

3. Allow static measurements for ensemble averaging to increase the confidence in
data.

4. Permit lower power levels for the illumination source, since the receiver is closer
to the scattering object and will see higher sound levels.

5. Allow the target strength signatures over a wide range of aspect angles to be
obtained from a single set of measurements.

6. Image the sources of scattering to investigate structural acoustic phenomena
   [7].
1.2 Goals of this thesis

Before the nearfield acoustical holography method can be used for scattering it is necessary to develop a method for decomposing the total acoustic field into the incident and scattered components. The field decomposition cannot be based on well-known time-domain gating techniques because in the nearfield both fields overlap. The goals of this research effort are to develop a generalized method for separating the scattered fields from two composite fields containing both the incident (illuminated) and scattered components. Two coordinate systems are considered. The first is a cartesian coordinate system where the measurement surfaces are cartesian planes. The reason for considering this coordinate system is because it is easier to analyze theoretically, numerically and experimentally. But the cartesian separation technique fails if the incident field comes from an oblique angle. So we consider the cylindrical coordinate system as an improvement. But this cylindrical coordinate system still can't cover the incident field from any direction. If the incident angle is oblique with respect to the normal direction of the plane to a certain limit, the separation results are not satisfactory.

1.3 Literature review

This section reviews the different decomposition techniques in both time and wavenumber domain. We also briefly review nearfield acoustical holography (NAH) because the separation technique in this research uses the same basic process as NAH and the Green's function propagators introduced in NAH are the keys to the success of the scattered field separation.
1.3.1 Decomposition technique in the time domain

Several methods have been developed for field separation [9, 10, 12, 14]. The basic idea is that if the total field is composed of both the forward (incident) and backward (scattered) propagating waves, the direction of wave propagation for the forward and backward waves are in opposite directions and the pressure is the summation of both. We can separate the field if we known the magnitude and phase difference between these two waves.

S. Takagi [9] introduced a measurement method to determine both the forward and backward intensity components of sound waves. This method uses signals through a two-microphone probe, which is necessary to decompose an intensity vector into the incident and the reflected components, to measure the reflection and absorption coefficients of a material. This method uses the auto and cross spectral densities to get the forward and backward intensity components. Takagi used this method to describe two types of estimate methods. One is applicable to a plane wave propagation along an axis in one dimensional space, and the other in two dimensional space. The method in one-dimensional space is used to measure the radiation impedance at the end of a pipe and the normal impedance of an absorptive material. The second method in two-dimensional space is applied to determine the direction of a source in the farfield. To get the forward and backward intensity components, we need to know the auto and cross spectral densities of components $G_{ff}, G_{bb}, R(G_{fb})$, and $I(G_{fb})$. We can measure the auto and cross spectral densities $G_{11}, G_{22}, R(G_{12})$, and $I(G_{12})$ through two microphones (1 and 2). The quantities $G_{11}, G_{22}, R(G_{12})$, and $I(G_{12})$ all contain the components of $G_{ff}, G_{bb}, R(G_{fb})$, and $I(G_{fb})$. By solving simultaneously four equations with four unknowns, we can obtain the desired intensity.
C. Spiekermann [10] considers the simultaneous presence of propagating and standing wave fields which add together to form the total sound field in a room (this result was reported by van Zyl et al. [11]). For a completely absorptive boundary condition, as in an anechoic room, it results in a propagating wave response. Another extreme is a completely reflected boundary condition, as in a reverberation room, which results in a standing wave response. But the mixed wave fields of absorptive and reflected waves in real systems are more complicated and make it difficult to relate to a boundary condition that is in between totally absorptive and totally reflective. Spiekermann developed an analytical method to decompose a one-dimensional acoustic pressure response associated with a specified partially absorptive boundary condition into an equivalent summation of propagating and standing waves usually associated with absorptive and reflective boundary conditions, respectively. The decomposition is accomplished by equating this mixed response to a summation of propagating and standing waves. These component responses are scaled and phase shifted by constants that depend on frequency, so that they sum to form the mixed response. The scaling factors and phase angles indicate the portions of purely propagating and standing wave components necessary to form the total mixed response. There are also four unknowns (2 magnitudes, 2 phase angles) and four equations. But these four equations are nonlinear. By solving four equations simultaneously, we can get the desired magnitudes and phases. A 1-D measurement based on a two microphone spectral analysis technique (STRIPS) is developed by Spiekermann to separate the total acoustic response [12]. This method is design especially to overcome other measurement methods which require known boundary condition. STRIPS also uses
the measured auto and cross spectral at two measurement points to decompose the propagating waves and the standing waves.

All these previous papers discuss the separation of time domain signals by transforming them into frequency domain. The following techniques obtains the signal over space and transform the signal to the wave-number domain.

1.3.2 Near-field acoustical holography

This section briefly discussed the spatial Fourier transform method and the powerful tool: nearfield acoustical holography (NAH) [2, 3, 4]. The reason we include this review is because we use the same procedure of windowing, filtering and spatial Fourier transform that has been developed for NAH for the decomposition technique in this research. Also the Green's function propagators in NAH for both the cartesian and cylindrical coordinate systems are very important in developing the separation technique. Nearfield acoustical holography involves the measurement of the sound field over an appropriate surface and the use of this measurement to uniquely determine the sound field within in a three dimensional region. It is equivalent to the case of a Dirichlet boundary condition on a surface for which the Green’s function is known [13]. The wave-number field analysis is based on the principle that spherical waves are decomposed into plane-wave components by spatial Fourier transforms. So the holographic reconstruction process is then simply the convolution (or deconvolution) of the measured boundary value with the Green’s function (propagator). The limits of any NAH are determined by the method of measuring the boundary data, in the formation of the Green’s function, and in the evaluation of the convolution integral.
For a cartesian generalized holography, the Green's function in k-space is:

\[ G(k_x, k_y, Z) = \begin{cases} 
  e^{iZ(k^2-k_x^2-k_y^2)^{1/2}} & k_x^2 + k_y^2 < k^2 \\
  e^{-Z(k_x^2+k_y^2-k^2)^{1/2}} & k_x^2 + k_y^2 > k^2 
\end{cases} \] (1.1)

which can be considered as a propagator from one plane to another plane. Note that when \( k_x^2 + k_y^2 > k^2 \), the fields are outside the radiation circle and the propagator exponentially increases with distance toward the source (decreasing \( Z \)).

For cylindrical generalized holography (which was named as GENAH) [16, 17, 18], the propagator in k-space for a cylindrical plane which covers the source is,

\[ \left[ H_m^{(1)}(k_r r_2)/H_m^{(1)}(k_r r_1) \right]. \] (1.2)

For \( k < k_z \), \( k_r \) becomes a pure imaginary number, so the propagator is

\[ \left[ K_m(k_r r_2)/K_m(k_r r_1) \right]. \] (1.3)

1.3.3 Decomposition technique in real-space

Tamura [14] used the spatial Fourier transform method (NAH) to measure the reflection coefficients of a surface with sound waves incident on the surface at oblique angles. The method involves the measurement of the complex pressure on two parallel planes lying close to the surface of a test material and decomposing each of the complex pressure distribution into plane-wave components by using the two dimensional spatial Fourier transform. The incident and reflected plane-wave components on the surface of the test material can be mathematically separated by the use of propagation theory. This separation leads to the determination of reflection coefficients at arbitrary angles of incidence.

This method is basically the same as what we use in the cartesian system separation technique. The only difference is the type of application.
1.3.3.1 Subtraction method  The total sound field is composed of the scattered field and the incident field so that no direct measurement is possible for the scattered field alone. To get the scattered field, we first measure the incident field alone, by not having the scatterer present. Then determine the total field with the scatterer present, being sure to measure over the same surface as was used for the incident field alone. The scattered field is then obtained by subtracting the pure incident field from the total field. We called this the subtraction method.

1.3.4 Summary

Review of separation techniques shows two common points. First is that the total field consists of the superposition of two different waves. These two waves must have differing phases. The second is that it is not possible to decompose the field in real-space (or the time domain) and must be transformed to wavenumber space (or the frequency domain) to do the separation.

1.4 Methodology

This section outlines the methodology used in this research to determine the scattered fields from the sound pressure measured over two surfaces. The generic methodology is the same for both cartesian and cylindrical systems. Basically this section links the nearfield acoustic holography with the wave-number decomposition technique.

Fig. 1.1 shows the sequence of the processing algorithms. The combined incident and scattered wave is measured at two measurement planes. If the incident frequencies are unknown, there is one extra step before the spatial Fourier transform.
Measurement of Window Separation Filter IFFT
Two Hologram Two Hologram Plane Plane Incidence Plane

2-U Kpatial Separation Scattered
Technique Plane

A A i A Incident Field Incident Field

^ lb'

1st Hologram Plane 1st Hologram Plane Incident Field Incident Field

2nd Hologram Plane 2nd Hologram Plane Scattered Field Scattered Field

Real Space k-Space k-Space Real Space

Figure 1.1: Procedures of getting the separated scattered field in real space
Each point must then be processed by a temporal fast Fourier transform (FFT) to form the complex spectra. This procedure will reveal what frequencies are present. Once the incident frequencies are determined, the data on two real space measurement surfaces are windowed to remove the discontinuity caused by the finite aperture size. The two-dimensional Fourier transform then transforms the data in the two measurement surfaces from two real space to wave-number space. A wave-number decomposition technique is applied to the data on the two measurement surfaces in order to separate the incident and scattered fields in k-space. The decomposition technique uses filtering to remove the undesired noise which is generated from FFT and amplified by the decomposition technique. The inverse Fourier transform (IFFT) is then used to transform the incident and scattered field back to real space resulting in the desired scattered field. Some times, windowing is applied on the real space separated data to remove errors on the edge of the surfaces on which the scattered field is determined.

Although the detailed discussion of the wavenumber decomposition technique is contained in Chapter 3 and 4, here we briefly describe these techniques. The basic idea for separating the field is similar to the methods reviewed in section 1.3.2 and 1.3.3. We know the total sound field on two measurement surfaces. Each field is composed of two different components, the incident field and scattered field. Because the Fourier transform is a linear operator, we can represent the total field in wave-number space as the summation of the incident and scattered fields. The propagator of each incident and scattered wave is known. Which means that the total fields can be represented by the incident and scattered fields of the desired separating surface. Consider the two total fields as two knowns, and the two unknowns of these two total
fields are the separating incident and scattered fields. By solving two equations with two unknowns, we can get the fields we need.

1.5 Outline of thesis

This dissertation contains seven chapters. Chapter 2 focuses on the calculation of the scattered field generated by plane or finite beam sources incident with a spherical scatterer and a prolate spheroidal scatterer illuminated by a plane wave. These fields are used to numerically verify the separation technique. The theoretical background and derivation of the scattered field is included. The geometries of the scatterers and the incident fields are used in the following chapters. The experimental verification of these fields which were measured inside an anechoic chamber are also included.

Chapter 3 concentrates on the field separation from the two cartesian nearfield measurement planes in wave-number space (k-space). It includes the theory of the separation technique and the simulated results. The simulated results are for both a spherical scatterer and a prolate spheroidal scatterer. The propagation toward the scatterer causes problems in the separation so windowing (in real-space) and filtering (in k-space) is used to produce improved results.

The following chapter talks about the separation technique for two cylindrical nearfield measurement surfaces. This case is included because the separation technique in cartesian coordinates does not work well when the incident field is from an oblique angle. The separation technique in cylindrical coordinates is being considered to improve the accuracy with waves incident at oblique angles. Besides the technical discussion and the simulated results, the propagators for both the incident and scattered fields in k-space is discussed. The implementation of the incident propagator
is not feasible in real simulation despite the theory indicating otherwise. Therefore a two plane propagation method for obtaining the propagated incident waves is developed.

Chapter 5 includes the sensitivity tests for both the cartesian and cylindrical cases. The sensitivity tests include two sections. The first section concerns the sensitivity to the separation parameters. It is useful to understand the limitation of the separation technique in order to design good experimental geometries. The parameters that are considered include the distance between measurement points in each measurement surface, the aperture size of each measurement surface, and the distance between measurement surfaces. The second section deals with the sensitivity to errors in the experimental geometry, specifically the skew, translation, offset and the finite aperture size of the measurement surfaces. The results of the sensitivity tests are helpful in deciding the accuracy of the farfield target strength results and designing experimental procedures and equipment.

In Chapter 6, the experimental results and the experimental facilities are discussed. The experimental results proved the feasibility of this technique for both coordinate systems. For the cartesian system, we use only a cast iron sphere as the scatterer illuminated by a piston speaker. For the cylindrical system, the finite source illuminating a cast iron spherical scatterer, a hard rubber spherical scatterer, a solid aluminum cylindrical scatterer and a hollow aluminum cylindrical scatterer are all considered. Because the scattered field can not be directly measured, we use a subtraction method to get the scattered field which is then compared to the separation technique.

The last chapter contains the summary, conclusions and recommendation for
future works.

We include the result of a rigorous method to calculate the field from a plane piston source [15] in Appendix A. Appendix B describes the Helmholtz Integral Equations used for the farfield projected estimate in both the cartesian and cylindrical coordinate systems.
2. BACKGROUND: CALCULATING INCIDENT AND SCATTERED FIELDS

This chapter gives details of the calculations of the incident and scattered fields used to verify the separation techniques. The verification of these calculations is also discussed.

2.1 Introduction

For investigating the scattering behaviors of an object, measurements of the scattered fields with the incident field must be done. Plane waves are the most simple incident waves that have been used for determining the scattered field, but it is difficult to experimentally generate a perfect plane wave. A circular piston source is frequently used experimentally, but the numerical calculation of the pressure distribution in the near-field of the source, which has been the subject of numerous investigations, is very difficult.

The focus of this chapter is to discuss the scattered fields of two different scatterer shapes, a sphere and a prolate spheroid, illuminated by two different sources, plane wave and the sound radiated by a plane piston source. This chapter includes two major parts. First, the incident and scattered fields of a spherical scatterer with a plane wave and a piston source illumination is discussed. Second the scattering by
a prolate spheroidal with plane wave illumination is discussed. These fields are the basis for exploring the separation technique developed in this dissertation.

Most of the theoretical work for calculating the incident and scattered field is obtained from the literature [15, 19, 20]. David Bennink [21] developed the scattering amplitude for a spherical scatterer with a piston source incident. The numerical results for the scattered field of a sphere illuminated by a piston source are developed in this research following Bennink's derivation.

2.2 Spherical scatterer

This section focuses on the incident and scattered fields for the spherical scatterer with a plane wave incident and also with the incident field produced by a plane piston source. First the sound field is derived for each case. Most of the fields are already theoretically developed [15, 19, 20, 21]. We include these derivation because they eventually lead to the development of the scattered fields for a finite piston source. The calculation of the scattered field for a finite piston source is one of the new contributions of this dissertation. The second part of this section includes the numerical analysis of the incident and scattered field for the spherical scatterer. Three major fields are examined: the incident field from a plane piston source, the scattered field of a plane incident wave and the scattered field of a plane piston source incident wave. The first two have published results to compare with. But the field from a plane piston source needs to be examined through a different approach. The nearfield results are evaluated by comparing the incident and the scattered velocity on the surface of the scatterer which should satisfy the boundary conditions. For a piston source located in the farfield, the scattered field is compared with the scattered
field from a plane wave incident.

The third part of this section compares experimental and theoretical data. A cast iron sphere is illuminated by a piston speaker as the comparison case. The coordinate system is shown in Fig. 2.1. The center of the scatterer is the origin of the coordinate system and the distance between the plane piston source and the center of the scatterer is $Z_s$. Table 2.1 lists the parameters for the spherical scatterer with plane piston source illumination.

![Image of coordinate system of a spherical scatterer](Image)

Figure 2.1: The coordinate system of a spherical scatterer

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>The radius of the piston source</td>
</tr>
<tr>
<td>$b$</td>
<td>The radius of the spherical scatterer</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Nondimensionalized wavenumber w.r.t. the source radius</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Nondimensionalized wavenumber w.r.t. the scatterer radius</td>
</tr>
<tr>
<td>$b/a$</td>
<td>Scatter/source diameter ratio</td>
</tr>
<tr>
<td>$s$</td>
<td>$s = \frac{2\pi (\frac{b}{a})}{k_a}$ Source dimensionless distance</td>
</tr>
</tbody>
</table>
2.2.1 Formulations

We begin the discussion with a general sound field. As discussed in numerous texts [20, 22, 23], radiation problems involve finding the velocity potential $\Phi(\vec{r})$ which satisfy the Helmholtz equation,

$$\nabla^2 \Phi(\vec{r}) + k^2 \Phi(\vec{r}) = 0. \quad (2.1)$$

In spherical coordinates, using an appropriate separation of variables, we can express the solution with the form

$$\Phi(\vec{r}) = \{A_{li}(kr) + B_{ni}(kr)\} Y_l^m(\theta, \phi), \quad (2.2)$$

where $l = 0, 1, 2, \cdots$ and $m = \pm 1, \pm 2, \cdots, \pm l$. The propagation of the wave can then be described as:

$$\Psi(\vec{r}, t) = \Phi(\vec{r}) e^{-i\omega t} = \{A_{li}(kr) + B_{ni}(kr)\} Y_l^m(\theta, \phi) e^{-i\omega t}. \quad (2.3)$$

and the acoustic pressure is:

$$p(\vec{r}, t) = P(\vec{r}) e^{-i\omega t} = j\omega \rho_0 \Phi(\vec{r}) e^{-i\omega t}. \quad (2.4)$$

For traveling waves, the sound pressure of Eqn.(2.4) can be expressed as

$$p(r, t) = \begin{cases} j\omega \rho_0 h_i^{(1)}(kr) Y_l^m(\theta, \phi) e^{-i\omega t} \\ j\omega \rho_0 h_i^{(2)}(kr) Y_l^m(\theta, \phi) e^{-i\omega t} \end{cases} \quad (2.5)$$

If the time dependent term is $e^{-i\omega t}$, then $h_i^{(1)}(kr)$ is the outward traveling spherical wave and $h_i^{(2)}(kr)$ is the inward traveling spherical wave. If $e^{i\omega t}$ is used for the time dependence, the role of $h_i^{(1)}(kr)$ and $h_i^{(2)}(kr)$ are reversed. The understanding of
the physical representation of these two Hankel functions helps us to determine the solution for specific waves.

We can then express the radiation of a scattered field as a superposition of spherical waves for \( r > a \) (\( a \) is the radius of the scatterer).

\[
p(r, t) = \sum_{l,m} a_{lm} h^{(1)}_l(kr) Y^m_l(\theta, \varphi)e^{-i\omega t}.
\]

Here only outgoing waves are included. Ingoing waves are omitted for source radiating sound due to the Sommerfelt radiation condition.

### 2.2.1.1 Scattering of a rigid spherical scatterer incident by a plane wave

The first case considered is the scattered field with the plane wave incident. This is the most simple case to be considered but demonstrates the basic derivation technique. Suppose a monochromatic plane wave of amplitude \( A \) moving in the \( Z \) direction impinges on a spherical obstacle of radius \( b \) with the center at \( r = 0 \) (Fig. 2.1). Considering only the spatial part of this wave, the plane wave can be expressed as

\[
P(\vec{r}) = Ae^{i\vec{k}\cdot\vec{r}} = Ae^{ikr\cos\theta}.
\]

Since the wave propagates in the \( Z \) direction, there is no dependence on \( \varphi \). Because \( m^2 = -\frac{1}{\sin^2 \varphi} \frac{\partial^2}{\partial \varphi^2} \), then \( m = 0 \). The \( Y^m_l(\theta, \varphi) \) term can then be simplified as a Legendre function, \( Y^0_l(\theta) = \sqrt{\frac{2l+1}{4\pi}} P_l(\cos \theta) \). Because a plane wave is finite at \( r = 0 \), there is no contribution from the singular term \( n_l(kr) \). The expansion of the plane wave into spherical functions can be expressed as:

\[
e^{ikr\cos\theta} = \sum_{l=0}^{\infty} (2l + 1)j_l(kr)P_l(\cos \theta).
\]
The pressure field with magnitude $A$ can be written in terms of the spherical functions as

$$p(\vec{r}, t) = A \sum_{l=0}^{\infty} (2l + 1) i^l j_l(kr) P_l(\cos \theta) e^{-i\omega t}. \quad (2.9)$$

The plane wave impinging on the spherical obstacle will produce an outgoing spherical wave (the scattered wave). With a monochromatic plane incident wave, we can expect a monochromatic scattered wave with the same frequency. Thus the total sound field will be of the linear superposition of the incident and scattered waves.

$$p = p_I + p_S = A e^{i(kr-\omega t)} + p_S, \quad (2.10)$$

where $p_S$ is a superposition of outgoing spherical waves of the type described in Eqn(2.6). Considering only the $m=0$ case, because there are only plane waves incident

$$p_S(\vec{r}, t) = A \sum_l b_l h_l^{(1)}(kr) P_l(\cos \theta) e^{-i\omega t}, \quad (2.11)$$

$$p_I(\vec{r}, t) = A \sum_l (2l + 1) i^l j_l(kr) P_l(\cos \theta) e^{-i\omega t}. \quad (2.12)$$

Because the scatterer is a hard surface, the pressure gradient is zero at the surface. Applying the boundary condition $\frac{\partial p}{\partial n} = 0$, the coefficient $b_l$ can be expressed in terms of the incident wave coefficients,

$$b_l = - (2l + 1) i^l \frac{j_l(kb)}{h_l^{(1)\prime}(kb)}. \quad (2.13)$$

Letting $T_l(kb) = - \frac{j_l(kb)}{h_l^{(1)\prime}(kb)}$, and the scattered field is

$$p_S(\vec{r}, t) = A \sum_l (2l + 1) i^l T_l(kb) h_l^{(1)}(kr) P_l(\cos \theta) e^{-i\omega t}. \quad (2.14)$$
2.2.1.2 Acoustic scattering from a rigid spherical scatterer with an incident wave from a plane piston source

This section derives the incident and scattered fields used in this research. The spatial part of the sound field radiated by a plane piston source can be expanded in terms of spherical waves,

\[ p_I = \sum_{l,m} a_{lm} j_l(kr) Y_l^m(\theta, \varphi). \]  

(2.15)

By applying boundary conditions, given in detail in appendix A, we get the coefficient \( a_{lm} [15] \):

\[ a_{lm} = 4\pi \rho_0 c_0 V_0 (-1)^l \sqrt{\frac{2l+1}{4\pi}} \delta_{m0} F_l(kz, ka). \]  

(2.16)

As in section 2.2.1.1, consider only the field symmetric about the Z axis, \( Y_l^0(\theta) \equiv \sqrt{\frac{2l+1}{4\pi}} P_l(\cos \theta) \). The incident field from the plane piston is

\[ p_I = \rho_0 c_0 V_0 \sum_l (-1)^l(2l + 1) F_l(kz, ka) j_l(kr) P_l(\cos \theta). \]  

(2.17)

The scattered field can be represented using Eqn.(2.6) as

\[ p_S = \sum_{l,m} b_{lm} h_l^{(1)}(kr) Y_l^m(\theta, \varphi). \]  

(2.18)

Apply the boundary condition for a hard surface, \( \frac{\partial p}{\partial r} \bigg|_{r=b} = 0 \) to \( p = p_I + p_S \),

\[ b_{lm} = -a_{lm} \frac{j_l(kb)}{h_l^{(1)'}(kb)} = T_l(kb) a_{lm}. \]  

(2.19)

Consider only the field symmetric about the Z axis, the scattered field from the plane piston is:

\[ p_S = (\rho_0 c_0 V_0) \sum_l (-1)^l(2l + 1) F_l(kz, kb) T_l(kb) P_l(\cos \theta) h_l^{(1)}(kr). \]  

(2.20)
Where

\[ F_i(kz, ka) = \int_{kz_0}^{kz_0 + a^2} (kr)h_i^{(1)}(kr)P_i(\xi) d(kr), \]

\[ T_i(kb) = -\frac{j_i(kb)}{h_i^{(1)}(kb)}, \]

\( h_i^{(1)}(kr) \): The spherical Hankel function of the first kind,

\( P_i(\cos \theta) \): The Legendre function,

### 2.2.2 Numerical analysis

In this section the incident and scattered fields discussed in section 2.2.1.2 are verified. We want to be sure that the fields are correct before they are used to test the separation techniques. The results of the plane piston source field and the scattered field from an incident plane wave are compared with the published results. Because the scattered fields from an incident field from a plane piston source has no published data to compare with, we test the results by comparing the velocities at the surface of the scatterer, and by comparing the scattered field with the piston source in the farfield with a plane wave incident. When the piston source is located in farfield the incident field should be close to that of a plane wave.

#### 2.2.2.1 The plane piston source

The algorithm used in calculating the wave field of a plane piston source is included in appendix A [15]. Analytical solutions only exists for the farfield and on-axis positions of the plane piston source. So we compare the computational results with the pressure amplitude on the axis of the piston and the far-field directivity pattern, using the analytic solutions given by Kinsler et al [22]. We also compare the farfield pressure with the plane wave data.
1. The pressure amplitude on the axis of the piston

We first verify the numerical results for the plane piston source by comparing the pressure amplitude on the axis of the piston with the equation from Kinsler et al. [22]. The equation from Kinsler et al. is

\[
\frac{p(r', 0)}{2\rho_0 c V_0} = |\sin\left\{\frac{1}{2} kr'\left[1 + \left(\frac{a}{r'}\right)^2 - 1\right]\right\}|
\] (2.21)

With \(r'\) is measured with respect to the center of the piston. The center of \(r\) in Eqn.(2.17) is located at the center of the scatterer(Fig. 2.1). Fig. 2.2 shows a very good agreement with \(\frac{a}{\lambda} = 4\), where \(\lambda\) is the wave length of the incident wave.

Figure 2.2: The plane piston source pressure amplitude on the axis of the piston
2. The farfield directivity pattern

The farfield pressure produced by a plane piston source is represented by Kinsler et al. [22] as

\[ P(r, \theta) = j \frac{\rho_0 c}{2} U_0 a \frac{k a e^{-ikr}}{r} \left[ \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right], \]  

(2.22)

Defining \( H(\theta) \) as the directivity pattern gives,

\[ H(\theta) = \left| \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right|. \]  

(2.23)

Fig. 2.3 shows the farfield directivity pattern calculated from Eqn.(2.17) and Eqn.(2.22). The results shows very good agreement.

Figure 2.3: The farfield directivity pattern of a plane piston source
3. The incident field from the plane wave and the piston source locate at farfield

The incident field pattern for the plane wave source and the piston wave source located in the farfield are comparable. Fig. 2.4 shows the comparison for three different frequencies: 1000Hz, 3000Hz and 5000Hz from the piston source in the near and farfield. The results are shown as the polar plots of the pressure magnitude. The sound field became very close to that of a plane wave when the piston source is in the farfield. Therefore the scattered field resulting from the piston source in the farfield can be compared to the scattering from the plane waves. This provides a check to the scattering results with a piston source.

2.2.2.2 Scattered radiation pattern for the plane incident wave

Eqn. (2.14) is used to generate the scattered field from a plane incident wave. The scattered field have also been calculated by Stenzel [24]. Stenzel expressed the scattered radiation patterns for the rigid sphere illuminated by the plane wave in terms of the variable \( kb \) which is the nondimensionalized wavenumber with respect to the radius of the spherical scatterer. Fig. 2.5 to Fig. 2.9 represent the comparison of the scattered sound field in the far field for a fixed, rigid sphere for different values of \( kb \). In the polar plots, the incident plane wave is from the left side of the scatterer. We notice that as \( kb \) increases, an omnidirectional pattern at low \( kb \) is supplemented by a forward-scattered lobe that increases with \( kb \).
Figure 2.4: The incident field pattern of plane wave and piston source: Left: nearfield; Right: farfield. —— plane piston source; --- plane wave
Figure 2.5: The polar scattered radiation pattern for $kb << 1$ with a plane incident wave. A: From Eqn.(2.14); B: From Stenzel [24]

Figure 2.6: The polar scattered radiation pattern for $kb = 2$ with a plane incident wave. A: From Eqn.(2.14); B: From Stenzel [24]
Figure 2.7: The polar scattered radiation pattern for $k_b=4$ with a plane incident wave. A: From Eqn.(2.14); B: From Stenzel [24]

Figure 2.8: The polar scattered radiation pattern for $k_b=8$ with a plane incident wave. A: From Eqn.(2.14); B: From Stenzel [24]
2.2.2.3 Scattered field illuminated by a plane piston source

No published results for the scattered field with an incident field from a finite piston source were found. This section uses several approaches to verify the computations developed in this dissertation research since there are no published results to compare with.

- The incident and scattered velocity on the surface of the scatterer

For a scatterer with an incident wave from a plane piston source, the velocity of the incident and the scattered waves at the surface of this scatterer should have the same magnitude but in opposite direction. To verify the accuracy of the incident and scattered field at nearfield, we first examine the velocity at the surface of the scatterer.
For a plane piston source, the incident and scattered velocities can be derived from acoustic pressure, Eqns. (2.17) and (2.20), with the relation \( u = \frac{1}{jk\rho_0c_0} \nabla P \):

\[
\begin{align*}
\mathbf{u}_{\text{incident}} &= \frac{1}{jk\rho_0c_0} \frac{\partial P_I}{\partial r} \\
&= \frac{v_0k}{jk} \sum_l (-1)^l (2l + 1) F_l(kz, ka) P_l(\cos \theta) j_l^0(kr), \quad (2.24)
\end{align*}
\]

\[
\begin{align*}
\mathbf{u}_{\text{scattered}} &= \frac{1}{jk\rho_0c_0} \frac{\partial P_s}{\partial r} \\
&= \frac{v_0k}{jk} \sum_l (-1)^l (2l + 1) F_l(kz, ka) P_l(\cos \theta) T_l(kb) k_l^{-1/2}(kr). \quad (2.25)
\end{align*}
\]

The results shown on Fig. 2.10 are the incident and scattered velocities on the surface of the scatterer. The left side of Fig. 2.10 shows the case for \( ka=0.5 \), \( s=100 \) and \( b/a=1 \); and the right side is the case for \( ka=1 \), \( s=10 \) and \( b/a=1 \). The polar plots represent the absolute value of the velocities at the scatterer surface. We can see that the incident and scattered velocities match very well, and the phase of the two velocities are shifted different by 180°.

- **The scattered field pattern when illuminated by a plane wave and by a piston source in the farfield**

We have shown that the field of a plane piston source is close to a plane wave when the piston source is far from the scatterer. We also discussed the scattered field of a plane incident wave. We can then compare the scattered field from a plane piston source in the farfield with the scattered field from a plane wave. The results should be very close if the computations of the scattered field from the plane piston source is accurate. Fig. 2.11 shows the scattered field for three different frequencies (1000Hz \( (ka=0.73273) \), 3000Hz \( (ka=2.19819) \) and 5000Hz
Figure 2.10: The surface velocities of the spherical scatterer with plane piston source. — incident wave velocity; - - - scattered wave velocity. A: $ka=0.5, s=100, b/a=1$; B: $ka=1, s=10, b/a=1$. 
Figure 2.11: The scattered field pattern with plane wave and piston source incident: Left: both source and the measurement plane in the nearfield; Right: both source and the measurement plane in the farfield. ——— plane piston source; ——— plane wave
(ka=3.66365)). The left side of this figure shows the scattered field in the nearfield of the scatterer with the plane piston source in the nearfield and the right shows the scattered field in the farfield with the plane piston source locates in the farfield. We can see that the results became very close when the piston source moves away from the scatter to the farfield. From the above numerical analysis, we can see that the incident and scattered fields have very good agreement with plane wave field patterns. The next section includes the comparison of the experiment data with the numerical generated data.

2.2.3 Comparisons of the numerical results with the experimental data

The purpose of this section is to compare computational and real data for the scattered and incident field of the spherical scatterer with an incident field from a plane piston source. The understandings of the difference between the numerical and experimental data can help the design of experiments. The experiments were performed in the anechoic chamber at Iowa State University. Fig. 2.12 shows the experimental setup. The scatterer is a cast iron sphere with 3.62 cm diameter and a 8 cm diameter piston speaker is used as the source. The distance between the center of the scatterer and the surface of the piston speaker is 44 cm. A 3-D cartesian scanner was used to position a quarter inch microphone at the measurement points. The sound pressure is measured.

2.2.3.1 Simulation to evaluate the pressure ratio $\frac{P_l}{P_s}$ for different parameters

Before running the experiments, the effects of each parameter on the pressure ratio $\frac{P_l}{P_s}$ is examined in order to choose parameters for the design of exper-
ments that will guarantee accurate results. A low $\frac{P_t}{P_i}$ ratio will create a situation where the scattered field can be measured most accurately.

The simulation results are shown in Table 2.2. Four parameters are used to test the sensitivity of the $\frac{P_t}{P_i}$ ratio. The first parameter is the input frequency, the second is the size of the scatterer with respect to the plane piston source, the third is the distance between the source and the scatterer and the fourth is the distance of the measurement plane from the scatterer. From these simulations, we draw the following conclusion,

1. By increasing the frequency the ratio $\frac{P_t}{P_i}$ decreases, so that by using higher frequency we can get a better scattered field.

2. By increasing the $b/a$ value the ratio $\frac{P_t}{P_i}$ decreases, so that by using scatterer larger than the piston source we can get a better scattered field.

3. By increasing the $z/a$ value the ratio $\frac{P_t}{P_i}$ decreases, but the difference is not very large. So that the increase of the distance between the piston and the scatterer will not significantly affect the results.

4. By increasing the $r/a$ value the ratio $\frac{P_t}{P_i}$ increases. So the measurement surface must be in the nearfield of the scatterer.

2.2.3.2 Directivity pattern results Two cases were examined: 1000Hz and 5000Hz. The experimental results of each case, accompany with the theoretical results, are presented by their polar plots as shown from Fig. 2.13 to 2.16. Fig. 2.13 and Fig 2.15 are the incident fields and Fig. 2.14 and Fig 2.16 are the scattered fields.
3" OD cast iron sphere

![Diagram](image.png)

Figure 2.12: The experiment setup

Table 2.2: Pressure ratio $\frac{P_t}{P_s}$ at different parameters

<table>
<thead>
<tr>
<th>$f$</th>
<th>$ka$</th>
<th>$b/a$</th>
<th>$z/a$</th>
<th>$r/a$</th>
<th>$P_t$</th>
<th>$P_s$</th>
<th>$\frac{P_t}{P_s}$ (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100Hz</td>
<td>0.073</td>
<td>0.95</td>
<td>7.5</td>
<td>1</td>
<td>$5.6 \times 10^{-3}$</td>
<td>$3.7 \times 10^{-4}$</td>
<td>23.7</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>0.95</td>
<td>7.5</td>
<td>1</td>
<td>$5.6 \times 10^{-2}$</td>
<td>$2.13 \times 10^{-2}$</td>
<td>8.4</td>
</tr>
<tr>
<td>3000Hz</td>
<td>2.198</td>
<td>0.95</td>
<td>7.5</td>
<td>1</td>
<td>0.1679</td>
<td>0.11535</td>
<td>3.3</td>
</tr>
<tr>
<td>5000Hz</td>
<td>3.664</td>
<td>0.95</td>
<td>7.5</td>
<td>1</td>
<td>0.279</td>
<td>0.0213</td>
<td>2.4</td>
</tr>
<tr>
<td>10000Hz</td>
<td>7.327</td>
<td>0.95</td>
<td>7.5</td>
<td>1</td>
<td>$5.6 \times 10^{-1}$</td>
<td>$4.65 \times 10^{-1}$</td>
<td>1.51</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>1</td>
<td>26</td>
<td>1.125</td>
<td>$1.47 \times 10^{-2}$</td>
<td>$5.55 \times 10^{-3}$</td>
<td>8.47</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>5</td>
<td>26</td>
<td>5.125</td>
<td>$1.75 \times 10^{-2}$</td>
<td>$1.392 \times 10^{-2}$</td>
<td>2.01</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>10</td>
<td>26</td>
<td>10.125</td>
<td>$2.31 \times 10^{-2}$</td>
<td>$2.07 \times 10^{-2}$</td>
<td>0.94</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>15</td>
<td>26</td>
<td>15.125</td>
<td>$3.36 \times 10^{-2}$</td>
<td>$3.08 \times 10^{-2}$</td>
<td>0.75</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>20</td>
<td>26</td>
<td>20.125</td>
<td>$6.20 \times 10^{-2}$</td>
<td>$5.63 \times 10^{-2}$</td>
<td>0.83</td>
</tr>
<tr>
<td>1000Hz</td>
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<td>5</td>
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<td>$9.29 \times 10^{-2}$</td>
<td>$3.14 \times 10^{-2}$</td>
<td>9.44</td>
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<td>1000Hz</td>
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<td>$7.08 \times 10^{-2}$</td>
<td>$3.45 \times 10^{-2}$</td>
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<td>1</td>
<td>8.75</td>
<td>1.125</td>
<td>$4.78 \times 10^{-2}$</td>
<td>$1.71 \times 10^{-2}$</td>
<td>8.95</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>1</td>
<td>10</td>
<td>1.125</td>
<td>$4.11 \times 10^{-2}$</td>
<td>$1.48 \times 10^{-2}$</td>
<td>8.86</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>1</td>
<td>12.5</td>
<td>1.125</td>
<td>$3.21 \times 10^{-2}$</td>
<td>$1.18 \times 10^{-2}$</td>
<td>8.73</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>3</td>
<td>7.5</td>
<td>3.125</td>
<td>$8.27 \times 10^{-2}$</td>
<td>$5.38 \times 10^{-2}$</td>
<td>3.74</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>3</td>
<td>7.5</td>
<td>4.125</td>
<td>$0.10625$</td>
<td>$3.22 \times 10^{-2}$</td>
<td>10.37</td>
</tr>
<tr>
<td>1000Hz</td>
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<td>3</td>
<td>7.5</td>
<td>5.125</td>
<td>$0.14795$</td>
<td>$2.30 \times 10^{-2}$</td>
<td>16.18</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>3</td>
<td>7.5</td>
<td>6.125</td>
<td>$0.2375$</td>
<td>$1.79 \times 10^{-2}$</td>
<td>22.47</td>
</tr>
<tr>
<td>1000Hz</td>
<td>0.733</td>
<td>3</td>
<td>7.5</td>
<td>7.125</td>
<td>$0.5096$</td>
<td>$1.47 \times 10^{-2}$</td>
<td>30.83</td>
</tr>
</tbody>
</table>
In the polar plots, the incident waves from the piston source are from the left side of the scatterer. There is a good comparison of the theoretical and experimental results for the 1000 Hz case. For the 5000Hz cases, the field in the direction of the incident source, the backscattered field, is distorted. The reason is yet unknown. This might be from the interference of the measurement scanner with the backscattered field or the interference of other scattered signals, especially the scattering from the measurement fixture in the anechoic chamber. These erroneous scattered fields will increase at high frequencies. The wire and connection used to suspend the spherical scatterer might also affect the results at high frequencies.

Comparing these two cases, the 1000Hz incident wave gives better results for the backscattered field. But the $\frac{P_t}{P_i}$ ratio (close to 2.8) is high. The 5000Hz wave gives a good $\frac{P_t}{P_i}$ ratio (approximately 1) but has a worse backscattered field. According to the conclusions in section 2.2.3.1, by increasing the ratio $\frac{h}{a}$, the $\frac{P_t}{P_i}$ ratio can be increased without increasing the input frequency. Since the sound source will remain constant a larger scatterer will be used. In experiments presented in chapter 6, a 21.6 cm diameter bowling ball is used.

2.3 Prolate spheroidal scatterer

This section focuses on the incident and scattered fields of a prolate spheroidal scatterer with an incident plane wave. The computer code has been generated and tested by the ORINCON Corporation [25]. Only the results for the formulations and numerical results are included.
Figure 2.13: Experimental results for 1000Hz: incident field

Figure 2.14: Experimental results for 1000Hz: scattered field
Figure 2.15: Experimental results for 5000Hz: incident field

Figure 2.16: Experimental results for 5000Hz: scattered field
2.3.1 Formulation

Because the shape of a prolate spheroid more closely models a submarine, we consider this model to test the separation technique. The solutions to the wave equation in prolate spheroidal coordinates are derived by using the separation of variables method. It is the same as the procedure in section 2.2.1. The solutions appear in terms of radial $R_{mn}^{(j)}(h, \xi)$ and angular $S_{mn}(h, \eta)$ prolate spheroidal functions. C. Flammer [26] has a very detailed description of these special functions. These functions are very complex and estimating these has been the subject of much research. The FORTRAN source code used to calculate these functions was developed from work performed by B. J. King and A. L. Van Buren [27, 28, 29, 30, 31]. The coordinate systems are shown in Fig. 2.17.

![Figure 2.17: The coordinate system for prolate spheroidal scatterer](image)
The exact solutions for calculating the complex scattered fields for both acoustically soft and hard prolate spheroidal scatterers are found in Bowman [32]. We only list the equations for a plane wave incident on the soft or hard prolate spheroidal scatterers. The acoustically soft surface means that the sound pressure is zero at the surface of the scatterer, so the boundary condition for the velocity potential is $\Phi = 0$. At the surface of a hard scatterer, the normal component of the particle velocities is zero. So the boundary condition for the total velocity potential is $\frac{\partial \Phi}{\partial n} = 0$. These are also known as the Dirichlet and Neumann boundary conditions.

- Acoustically soft prolate spheroid

If the incident field is a plane wave and the prolate spheroid is acoustically soft, then the scattered and incident fields in prolate spheroidal coordinates are given as [32]:

\[
V^s = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \left[ -2i^n \frac{\varepsilon_m}{N_{mn}} \frac{R^{(1)}_{mn}(h, \xi_1)}{R^{(3)}_{mn}(h, \xi_1)} S_{mn}(h, \eta_0) \right] \times S_{mn}(h, \eta) \cos m(\phi - \phi_0),
\]

and

\[
V^i = e^{ikr \cos \theta} = e^{ik(\xi \sin \theta + z \cos \theta)} = 2 \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \left[ i^n \frac{\varepsilon_m}{N_{mn}} S_{mn}(h, \eta_0) \right] R^{(1)}_{mn}(h, \xi) \times S_{mn}(h, \eta) \cos m(\phi - \phi_0),
\]

where

\[
i = \sqrt{-1},
\]

\[
h = \frac{kd}{2},
\]
\[ k = \frac{2\pi}{\lambda}, \]

\[ d = \text{interfocal distance}, \]

\[ \varepsilon = \begin{cases} 1 & , m = 0 \\ 2 & , m > 0 \end{cases}, \]

\[ N_{nm} = \frac{2(n+m)!}{(2n+1)!n!m!}, \]

\[ R_{mn}^{(3)}(h, \xi) = R_{mn}^{(1)}(h, \xi) + iP_{mn}^{(2)}(h, \xi), \]

\[ R_{mn}^{(3)}(h, \xi) = \begin{cases} R_{mn}^{(3)}(h, \xi_0) , \xi_0 < \xi \\ R_{mn}^{(3)}(h, \xi) , \xi_0 > \xi \end{cases}, \]

\[ R_{mn}^{(3)}(h, \xi) = \begin{cases} R_{mn}^{(3)}(h, \xi_0) , \xi_0 > \xi \\ R_{mn}^{(3)}(h, \xi) , \xi_0 < \xi \end{cases}, \]

\[ \eta_0 = \cos \theta_0. \]

- Acoustically hard prolate spheroid

If the incident field is a plane wave and the prolate spheroid is acoustically hard, then the scattered and incident fields in prolate spheroidal coordinates are given as [32]:

\[ V^* = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[ -2i^n \frac{\varphi_m}{N_{mn}} \frac{R_{mn}^{(1)}(h, \xi_1)}{R_{mn}^{(3)}(h, \xi_1)} S_{mn}(h, \eta_0) \right] R_{mn}^{(3)}(h, \xi) \]
\[ \times S_{mn}(h, \eta) \cos m(\phi - \phi_0), \quad (2.28) \]

and

\[ V^i = e^{ikr \cos \theta} = e^{ik(x \sin \theta + z \cos \theta)} = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[ i^n \frac{\varphi_m}{N_{mn}} S_{mn}(h, \eta_0) \right] R_{mn}^{(1)}(h, \xi) \]
\[ \times S_{mn}(h, \eta) \cos m(\phi - \phi_0), \quad (2.29) \]
2.3.2 Fields in the cartesian coordinate system

The FORTRAN source code used to calculate these scattered fields was developed from work performed by ORINCON Corporation [25]. Slight modifications to this code have been made for use on the DEC Vincent system. The ability to use different inter-element distances along both the X and Y-axis is also added to the code. B. Kollars [33] has a detailed descriptions of the acoustic scattering from a rigid prolate spheroid.

Because the theoretical results and the program code have been examined by other authors, here we only include four scattering patterns to show the field measured on different surfaces so that the readers can have a sense of the scattered field we will use for comparison. For all the cases, the dimension of the prolate spheroid is 100 feet along the major axis and 10 feet along the minor axis. The speed of sound in the fluid is 5000 ft/sec. The measurement plane is formed by a 36 × 36 planar array normal to the plane wave direction. The incident wave is a plane wave.

Fig. 2.18 shows the scattered fields from a hard scatterer at 100 Hz and 500 Hz with the plane wave incident at beam aspect. Fig. 2.19 shows the scattered fields from a hard scatterer at 500 Hz with the plane wave incident at bow aspect and a soft scatterer at 500 Hz with the plane wave incident at beam aspect. Referring to Fig. 2.17, beam aspect means the major axis of the scatterer lies along the y-axis and the bow aspect means the major axis lies along the z-axis. The measurement plane is 0.1 feet away from the surface of the scatterer for beam aspect and 1 foot for bow aspect. We can see that the scattered field is smoother for the soft surface at 500 Hz. The bow aspect incidence wave generated a scattered field that is not symmetric. The nonsymmetry possibly comes from the numerical integration errors.
Figure 2.18: Scattered fields of a hard scatterer: Left: 100 Hz, Right: 500 Hz (cartesian system)

Figure 2.19: Scattered fields with 500Hz incident: Left: soft scatterer at beam aspect, Right: hard scatterer at bow aspect (cartesian system)
2.3.3 Fields in the cylindrical coordinate system

The FORTRAN source code was rewritten to generate the sound field over a cylinder surrounding the prolate spheroidal scatterer. Four scattered fields are shown here for the readers to see the scattered fields that will be used in the separation process. Although the plots are calculated in a 3-D cartesian system, the field is being plotted in 2-D by spreading out the cylindrical measurement surface from the forward scattered position. So the central lobe along the Phi axis is back scattering. The two lobes on the side are the forward scattering and they should combine together to get the complete forward scattering. Because the scattering at beam aspect is more interesting for bistatic investigations, we include only beam aspect examples here.

Fig. 2.20 shows the scattered fields of a hard scatterer, the same scatterer used section 2.3.2, at 100 Hz and 500 Hz. Fig. 2.21 shows the scattered fields for a soft scatterer at 100 Hz and 500 Hz. We can see that the soft surface scatterer behaves very different at the two sides. The fields are smooth, rather than having only two lobes on the forward and backward sides.
Figure 2.20: Scattered fields of a hard scatterer: Left: 100 Hz, Right: 500 Hz (cylindrical system)

Figure 2.21: Scattered fields of a soft scatterer: Left: 100 Hz, Right: 500 Hz (cylindrical system)
3. WAVE-NUMBER DOMAIN FIELD SEPARATION FOR THE CARTESIAN COORDINATE SYSTEM

This chapter describes the decomposition technique in the wave-number domain for the cartesian coordinate system. Computational and experimental results for a spheroidal scatterer and computational results for a prolate spheroidal scatterer are shown.

3.1 Introduction

The decomposition method in the cartesian coordinate system is based on the principle that any wave form can be decomposed into plane-wave components by using a two dimensional spatial Fourier transform. By using the plane-wave propagator, we can separate the incident and scattered fields.

This technique, first introduced by Masayuks Tamuna [14] to measure the reflection coefficients at oblique incidents, is used here for the field separation. We briefly reviewed this technique in Chapter 1. Though straightforward, this separation technique has poor results when the scattered field is propagated back to the scatterer surface. When backpropagating, the propagator is an exponential function in distance for high wavenumbers. These high wave numbers will be amplified. At high wavenumbers errors and noise which was generated through the FFT computa-
tion or the experiments is often larger than the scattered field. Thus these errors are amplified by backpropagation, above the level of the low wavenumber, resulting in useless data. By applying a window in real space and a filter in wave-number space, the separation results can be improved.

Another problem is that singular point exists at the radiation circle for the separation technique; thus, the separated field in wave-number space becomes extremely large along the radiation circle. This unnatural amplification of certain wavenumber components degrades the final results. With precise filtering along the radiation circle, we can improve the separation results. But finding the best filter requires a lot of trial and error, which is unrealistic for practical use. A special treatment in the area close to the radiation circle is therefore developed.

This chapter focuses on the decomposition technique and the modifications made to the processing in order to minimize numerically generated errors. The separation technique is demonstrated for both the piston source illuminating a spherical scatterer and the plane wave illuminating a prolate spheroidal scatterer. A measurement surface that does not fully cover the scattered field (small aperture size) is used to emphasize the noise and numerical errors caused by a small aperture. The results after applying the window, filter, and radiation circle averaging are compared to results without these data processing techniques.

3.2 The separation technique

The total acoustic field existing in the source-free domain that is bounded by two parallel planes (representing either physical or imaginary surfaces) can be described
in the wave-number domain as

\[ P_T(k_x, k_y, Z) = P_I(k_x, k_y)e^{-ik_zZ} + P_S(k_x, k_y)e^{ik_zZ}, \quad (3.1) \]

where

- \( P_I(k_x, k_y) \) represents the incident field propagating in the \(-z\) direction,
- \( P_S(k_x, k_y) \) represents the scattered field propagating in the \(+z\) direction,
- \( P_T(k_x, k_y) \) is the total field in the plane \( Z \),
- \( k_z = (k^2 - k^2_x - k^2_y)^{1/2} \), and \( k \) is the wave number in the fluid.

When a plane-wave travels from one plane \( Z = Z_0 \) to another plane \( Z = Z_1 \), its complex amplitude on the plane \( Z = Z_1 \) is given in terms of the complex amplitude on the plane \( Z = Z_0 \):

\[ P(k_x, k_y, Z_1) = P(k_x, k_y, Z_0)e^{ik_z(Z_1-Z_0)}. \quad (3.2) \]

Eqn.(3.2) shows that the propagation of a plane wave is characterized by the Green's function \( e^{ik_z(Z_1-Z_0)} \) as described in the review of NAH [3, 4]. This Green's function is called the propagator of the plane wave components.

The wavenumber-space incident and scattered fields in the \( Z_1 \) plane are expressed in \( k \)-space as \( P_I(k_x, k_y, Z_1) \) and \( P_S(k_x, k_y, Z_1) \). They can be expressed in terms of fields propagating from the \( Z_0 \) plane to the \( Z_1 \) plane:

\[ P_I(k_x, k_y, Z_1) = P_I(k_x, k_y, Z_0)e^{-ik_z(Z_1-Z_0)}, \]

\[ P_S(k_x, k_y, Z_1) = P_S(k_x, k_y, Z_0)e^{ik_z(Z_1-Z_0)}. \quad (3.3) \]

Because the Fourier transform is a linear operator, the \( k \)-space total field in the \( Z_1 \) plane is expressed as a summation of the incident and scattered field:

\[ P_T(k_x, k_y, Z_1) = P_I(k_x, k_y, Z_1) + P_S(k_x, k_y, Z_1) \]
\[ P_I(k_x, k_y, Z_0) = P_T(k_x, k_y, Z_0)e^{-ik_y(Z_1-Z_0)} + P_S(k_x, k_y, Z_0)e^{ik_y(Z_1-Z_0)}. \] (3.4)

We can also represent the k-space field in the \( Z_2 \) plane by the fields in \( Z_0 \) plane as,

\[ P_T(k_x, k_y, Z_2) = P_I(k_x, k_y, Z_0)e^{-ik_y(Z_2-Z_0)} + P_S(k_x, k_y, Z_0)e^{ik_y(Z_2-Z_0)}. \] (3.5)

By measuring \( P_T(k_x, k_y, Z_1) \) and \( P_T(k_x, k_y, Z_2) \) they can be considered as known variables and the exponential functions can be numerically calculated. The two unknowns in Eqns.(3.4) and (3.5) are \( P_I(k_x, k_y, Z_0) \) and \( P_S(k_x, k_y, Z_0) \). By simultaneously solving Eqns.(3.4) and (3.5), the incident and scattered fields at the \( Z_0 \) plane are solved for:

\[
P_I(k_x, k_y, Z_0) = \frac{P_T(k_x, k_y, Z_1)e^{ik_y(Z_2-Z_0)} - P_T(k_x, k_y, Z_2)e^{ik_y(Z_1-Z_0)}}{2i \sin (k_y(Z_2-Z_1))}, \] (3.6)

\[
P_S(k_x, k_y, Z_0) = \frac{P_T(k_x, k_y, Z_2)e^{-ik_y(Z_1-Z_0)} - P_T(k_x, k_y, Z_1)e^{-ik_y(Z_2-Z_0)}}{2i \sin (k_y(Z_2-Z_1))}. \] (3.7)

Fig. 3.1 shows the schematic expression of the three planes (the total field measured on two planes and the total field on the projected plane), the source and scatterer. Table 3.1 lists all the parameters involved in the measurements and numerical simulations. Only the relative distances between measurement planes and propagating plane are needed in the decomposition process. The decomposition method involves the measurement of the complex pressure distributions at two measurement planes (\( Z_1 \) and \( Z_2 \)) and transforming each of the complex pressure distribution into k-space components by the two-dimensional spatial Fourier transform. In Eqns.(3.6) and (3.7), all the terms on the right can be measured or calculated, so the incident and scattered fields at plane \( Z_0 \) can be calculated. By using the inverse Fourier transform, these two separated fields can be transformed to real space, resulting in the scattered field separated from the incident field.
Piston source or Plane wave illuminated at frequency $f$

Figure 3.1: Geometric configuration of the source and scatterer
Table 3.1: Parameters in the system

<table>
<thead>
<tr>
<th>$Z_1$</th>
<th>Distance from the scatterer center to the piston source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_2$</td>
<td>Distance from the scatterer center to the first measurement plane</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Distance from the scatterer center to the second measurement plane</td>
</tr>
<tr>
<td>$d$</td>
<td>Spatial sampling distance in measuring plane</td>
</tr>
</tbody>
</table>

3.3 Need for signal conditioning

The right hand side of Eqns. (3.6) and (3.7), have two terms that are worth noting. The first is the propagator, $e^{ik_zZ}$. The second is the denominator, $2i \sin (k_z(Z_2 - Z_1))$. Both these terms cause problems that are exaggerated in real measurements by noise, computational errors, and a finite measurement aperture. Different signal conditioning is needed for each of these terms.

First let us consider the propagator, $e^{ik_zZ}$. In the propagator, $Z$ can be $\pm(Z_2 - Z_0)$ or $\pm(Z_1 - Z_0)$, and $k_z$ can be real or imaginary. For the case where $k_z = (k^2 - k_x^2 - k_y^2)^{1/2}$ is real, which means that $k^2$ is larger than $k_x^2 + k_y^2$, the propagator is equal to $\cos(k_zZ) + i\sin(k_zZ)$. In this case, we say that this wavenumber is inside the radiation circle. The radiation circle is defined as $\sqrt{k_x^2 + k_y^2} = k$. All the fields inside the radiation circle will radiate to the farfield \cite{3, 34}. But for the case where $k_z$ is imaginary, which occurs when $k^2$ is smaller than $k_x^2 + k_y^2$, the sound waves are evanescent. In the case of evanescent waves, the propagator is either $e^{Re(k_z)|Z|}$ or $e^{-Re(k_z)|Z|}$, depending on if $Z$ is positive or negative. If $Z = Z_1 - Z_0$ is positive, then the propagator is $e^{-Re(k_z)|Z|}$, which means that the wave propagates out to another plane and decays as it propagates. So the evanescent wave exists only in the nearfield. If the propagator is $e^{Re(k_z)|Z|}$, then $Z_1 - Z_0$ is negative. Which means that
the evanescent waves are propagated back toward the scatterer. This propagator is an exponential function that increases rapidly in the nearfield. This causes problems that require signal conditioning.

The second problem is caused by the denominator, which is zero when \( k_z(Z_2 - Z_1) \) is equal to \( n\pi, n = 0, \pm 1, \pm 2 \ldots \). At the radiation circle \( k_z = 0 \), so the denominator of each equation is zero at the radiation circle. Thus at the radiation circle Eqns. (3.6) and (3.7) have a singularity. This problem will also require signal conditioning.

If the field before transformation is perfectly covered (infinite aperture size), each plane wave component is a Dirac delta function in k-space. But the real measurement surface can not cover an infinite plane, so all the measurement surfaces have a finite aperture size. For a finite aperture size, the Fourier transform of this real space plane wave component becomes a sinc function:

\[
\int_{-\infty}^{\infty} dz e^{-ik_z z} = \int_{-\frac{L}{2}}^{\frac{L}{2}} dz e^{-ik_z z} = \frac{2 \sin (k_z L/2)}{k_z}. \tag{3.8}
\]

Each sinc function should be applied with the same propagator as the delta function for the original plane wave component. But in digital signal processing, we can only apply the field at the position \((k_{x0}, k_{y0})\) with the propagator at \( k_{z0} = \sqrt{k_{x0}^2 + k_{y0}^2} \). So the evanescent waves are mistakenly propagated. Besides improperly propagating the evanescent wave, the exponential functions also enlarges the error and noise generated from computations or experimental procedures and make the separation unacceptable. The solution for this is to apply the filtering in k-space field in order to remove the noise and error.

Section 3.4 discusses the problems that are caused by the propagator and explains the effect of windowing (applied in real space) and filtering (applied in k-space) on the decomposition technique. A piston source illuminating a spherical scatterer is
used to explain why windowing and filtering is necessary. Section 3.5 discusses the limitations of using the filter at the radiation circle for the singularities and the solution for this problem.

### 3.4 Windowing and filtering

Windowing refers to multiplying data in real space with a window function and filtering refers to multiplying data in the wave-number domain by a filter function. The Fourier transform algorithm considers the input field as a periodically extended function. The finite aperture size generates the discontinuity in the function which windowing removes. Filtering is necessary because of the enlargement of the noise and errors, and unproper propagation as discussed in the previous section. The filtering is applied after the wave-number domain fields are separated into the scattered and incident fields.

The window and filter used in this research is adjustable and has the general form:

\[
W(X, Y) = \begin{cases} 
    1 - \frac{e^{-\alpha}}{2} & \text{for } r < c, \\
    e^{\alpha} & \text{for } r > c
\end{cases} 
\]  

(3.9)

where,

- in real-space, \(X=x, Y=y\) are the coordinate for each point,
- in wave-number space, \(X=k_x, Y=k_y\),
- \(\alpha\): determines the aperture size of the window or filter function (input parameter),
• slp: slope of the window or filter function (input parameter),

\[ \alpha = \frac{1 - \frac{r}{c}}{slp}, \]

• \( r = \sqrt{X^2 + Y^2} \),

• \( c = \text{apt} \times \delta d \times n/2 \) is the half aperture size of the window or filter. The variable \( \delta d \) is the increment between points and \( n \) is the number of points to each side of the measurement surface.

Four examples of the window or filter, with various input parameter values, are presented in Fig. 3.2 and Fig. 3.3. The larger that the value "apt" is, the wider the window or filter. And a smaller value for "slp" corresponds to a sharper window or filter. The value of "slp" is usually kept under 0.2 to avoid the deformation at the central part of the window or filter.

3.4.1 Case study of the effect of filtering

The following case is used to demonstrate the effect of filtering on the separation technique. A spherical scatterer with an incident field from a plane piston source is used as the example case. The parameters used are (see Table 2.1):

• The radius of the plane piston source is \( a=4 \text{ cm} \)

• Source dimensionless radius is \( ka=5 \) \( \Rightarrow \ k=125 \) \( \Rightarrow \ f=6823 \text{ Hz} \)

• Source dimensionless distance is \( s=10 \) \( \Rightarrow \ Z_s=31.83(\text{cm}) \)

• Scatter/source ratio is \( b/a=1 \)
Figure 3.2: The window(filter): Left: apt=0.9, slp=0.1; Right: apt=0.6, slp=0.1

Figure 3.3: The window(filter): Left: apt=0.6, slp=0.01; Right: apt=0.6, slp=0.2
The total acoustic pressure is calculated over an array of $32 \times 32$ points with an increment of 1.0 cm between points, on two different planes, $Z_1$ and $Z_2$, where $Z_1=6$ cm and $Z_2=8$ cm (see Fig. 3.1). The incident and scattered fields are separated at the plane $Z_0=5$ cm.

Fig. 3.4 shows the total fields calculated at $Z_1$ and $Z_2$. We can see that in this case the field did not go to zero at the edge of the aperture. Windowing in real-space is then used to remove the discontinuity at the edge caused by the non zero value. The fields are also zero padded before applying the FFT to avoid overlap.

Fig. 3.5 shows the calculated incident and scattered field at the separating position $Z_0$. Fig. 3.6 and Fig. 3.7 are the separated scattered and incident fields at $Z_0$ with and without filtering. The filter parameters are “apt”=0.4 and “slp”=0.15.

Figure 3.4: $ka=5$, the total field: Left: at $Z_1$; Right: at $Z_2$
The improvement is obvious after applying the filter in the wave-number space. Fig. 3.8 is the percentage error for the separated fields. To better understand the effect of the exponential functions on the separation, Fig. 3.9 and Fig. 3.10 compare the Green’s functions with and without applying the filter. Fig. 3.9 is the propagator $e^{ik_z Z}$ with $Z=4$ cm. This propagator inside the radiation circle has a magnitude of 1.0. Outside the radiation circle, the propagator is $e^{-|\text{Im}(k_z Z)|}$, an exponentially decaying function, since the evanescent wave will not propagate out to the farfield. Filtering has no effect on this propagator (Fig. 3.9 right). Fig. 3.10 is the propagator $e^{-ik_z Z}$ ($Z=4$ cm is positive). Inside the radiation circle, the magnitude is still 1.0. But outside the radiation circle, the propagator becomes exponentially increases, with the form $e^{\text{Im}(k_z Z)}$. This exponential increase is significant as shown on the left of Fig. 3.10. The plot on the right side of Fig. 3.10 shows the propagator after applying the filter ("apt"=0.4, "slp"=0.15). We can see the change of the propagator outside the radiation circle.

Fig. 3.11 and Fig. 3.12 compare the separated scattered and incident fields in k-space at $Z_0$ with the theoretically calculated fields. Comparing the left plots of Fig. 3.11 and Fig. 3.12, we can see that the separated incident field shows a very good agreement with the theoretical data. Only small peaks exist around the radiation circle. But the separated scattered field has significant peaks around the radiation circle. This is an effect due to the singularity in Eqns. (3.6) and (3.7). Otherwise the fields are very comparable with the theoretical data.

From these plots, we can see that the scattered fields are distorted without using the filter (Fig. 3.6). This is because the Green’s function becomes extremely large outside the radiation circle for the backward propagation (Fig. 3.10). When applying
Figure 3.5: $ka=5$, the fields at $Z_0$: Left: incident field; Right: scattered field

Figure 3.6: $ka=5$, the separated scattered field at $Z_0$: Left: no filtering; Right: with filtering
Figure 3.7: \( ka = 5 \), the separated incident field at \( Z_0 \): Left: no filtering; Right: with filtering

Figure 3.8: \( ka = 5 \), the error function for the separated field: Left: incident; Right: scattered
Figure 3.9: \( \kappa a = 5 \), the k-space Green's function, \( e^{ik_z Z} \), \( Z = 4 \) cm: Left: without filtering; Right: with filtering

Figure 3.10: \( \kappa a = 5 \), the k-space Green's function, \( e^{-ik_z Z} \), \( Z = 4 \) cm: Left: without filtering; Right: with filtering
Figure 3.11: \( \text{ka}=5 \), the k-space scattered field: Left: separated; Right: theory

Figure 3.12: \( \text{ka}=5 \), the k-space incident field: Left: separated; Right: theory
the separation technique, both the evanescent waves and the noise are multiplied by the Green's functions. The noise which is higher than the real signal at high wavenumbers, is then amplified so that it dominates the separation results, leading to the distorted results. This explains why it is necessary to use heavy filtering (apt=0.4, slp=0.15) in the processing of results (Fig. 3.6 Right).

3.5 Averaging around the radiation circle

This section considers the singularity in the denominator in the separation equations, Eqns.(3.6) and (3.7). The denominator is zero when \( k_z(z_2 - z_1) \) equals 0 or an integer multiplication of \( \pi \). Because the distance between the two separation planes is small (0.5 cm in this case), \( k_z(z_2 - z_1) \) would not be larger than \( \pi \) except for a very high \( k \) values. So the only practical possibility for a singularity is when \( k_z = 0 \), which corresponds to the radiation circle. We use the following case to demonstrate the effect of the singularity.

Consider the case where \( ka=1.0, d=5 \text{ cm}, s=50 \) and a 32 \( \times \) 32 point separation plane is used. For \( a=4 \text{ cm}, \) the frequency is 1365Hz in air. The distance between the two measurement planes is 0.5 cm. With \( s=50 \) the piston source is 31.8 cm away from the scatterer. We choose \( ka=1.0 \) for the consideration of a low \( ka \) value. By using the old separation program, the distortion which includes a large peak around the radiation circle was present especially for the scattered field in k-space, Fig. 3.13 and Fig. 3.14. The figures in this case are represented in two dimension; the origin of the x axis was set to be in the center of the measurement plane. The peaks in k-space generate a big increase on the edge of the field in real-space, (right plots of Fig. 3.13 and Fig. 3.14). When using a filter with apt=0.38 and slp=0.15, we can
Figure 3.13: $ka=1$, the separated incident field, no filter: Left: k-space; Right: real-space. Simulation; - - - separation

Figure 3.14: $ka=1$, the separated scattered field, no filter: Left: k-space; Right: real-space. Simulation; - - - separation
Figure 3.15: $ka=1$, the separated incident field, with filter(apt=0.38, slp=0.15):
Left: k-space; Right: real-space. — simulation; - - - separation

Figure 3.16: $ka=1$, the separated scattered field, with filter(apt=0.38, slp=0.15):
Left: k-space; Right: real-space. — simulation; - - - separation
get reasonable results, (Fig. 3.15 and Fig. 3.16). The filter actually removes all the information on and outside the radiation circle. But to get the right filter, a lot of trial and error is needed to determine the filter that reduces the peaks along the radiation circle without removing needed information. This is not a practical process.

D. Groutage [7] uses the wave-vector removing methods in k-space to filter out the plane wave component. Inspired by this, we implement several tests of spectrum averaging around the radiation circle over the range of $\Delta k$ or $2\Delta k$ to eliminate the large peaks. Fig. 3.17 graphically describes the five averaging method that are discussed below:

Figure 3.17: The method of averaging the spectral around the radiation circle
1. Averaging the Green's functions and the denominator of the separating equation over the four points nearest the desired point (Fig. 3.17 I). This is the simplest test we considered. All the points within the $\Delta k$ distance to $k_z = 0$ are replaced by the averaging value of the four closest points.

2. Averaging the Green's functions and the denominator of the separating equation over the eight points nearest the desired point (Fig. 3.17 II). This test uses all the adjacent points for averaging.

3. Similar to II, but each point is averaged like I (Fig. 3.17 III). This is the most complicated test. The four corner points (e,f,g,h) of "CC" are averaged first by their four closest points. Then the "CC" is replaced by the eight point averaged value.

4. Repeat 1, 2, 3 for the incident and scattered fields. This test attempts to average the separated incident and scattered fields before transforming them back to real space.

5. Repeat 1 to 4 but use $2\Delta k$ instead of $\Delta k$. All the points within a $2\Delta k$ range of the radiation circle need to be replaced by the averaging values.

After the exploring all these methods, we found that the method III for the Green's functions and the denominator of the separating equation averaging within a $2\Delta k$ range provided the best results. Fig. 3.18 and Fig. 3.19 show the separated fields after using this averaging method. The results are a significant improvement.

By using the same averaging method for the study case in section 3.3, we see that the peaks are removed from the separated incident and scattered fields in k-space for $ka=5$. The real space results also show good improvement (Fig. 3.20 and Fig. 3.21).
Figure 3.18: $ka=1$, the separated incident field, averaging method: Left: k-space; Right: real-space. — simulation; - - - separation

Figure 3.19: $ka=1$, the separated scattered field, averaging method; Left: k-space; Right: real-space. — simulation; - - - separation
Figure 3.20: $k_a=5$, the separated incident field, averaging method; Left: k-space; Right: real-space. --- simulation; - - separation

Figure 3.21: $k_a=5$, the separated scattered field, averaging method; Left: k-space; Right: real-space. --- simulation; - - separation
3.6 Numerical simulations

3.6.1 Piston source illuminating a rigid sphere

We include two sets of separation results in this section. The first is the same case as in section 3.3. All the parameters are kept the same except the aperture size is doubled by change the sampling distance \(d\) to 0.02m. The detailed sensitivity test results will be included in Chapter 5. From Fig. 3.22 and Fig. 3.23, we can see the effect of the aperture size on the separation method. The scattered field after separation is very close to the theoretical result even without filtering if the aperture size is large enough to cover the whole field. The second case is when the incident wave comes in from an oblique angle. Because the separation technique uses only the normal components of the fields, the separation results are not as good when the oblique angle increases. Fig. 3.24 shows the separated scattered field. The window parameters are “apt=0.8” and “slp=0.05” and the filter parameters are “apt=0.3” and “slp=0.01”. The separation is not good even if all the optimization techniques are used. Thus, the cartesian separation technique does not work with waves incident at an oblique angle.

3.6.2 Plane wave illuminating a prolate spheroidal scatterer

This section focuses on the separation results for a prolate spheroid scatterer with an incident plane wave. The geometry configuration of the scatterer is shown in Fig. 3.1. Because the numerical data from ORINCON Corp. cannot generate a good plane wave field, we use the analytical result to get the plane wave field. From Eqn.(2.27), the plane wave incidence at an angle \(\theta\) with respect to the positive z-axis...
Figure 3.22: The separated incident field at $Z_0$ (no filtering): Left: separated; Right: theory

Figure 3.23: The separated scattered field at $Z_0$ (no filtering): Left: separated; Right: theory
Figure 3.24: The separated scattered field of an oblique incident wave: Left: separated; Right: theory

is given as $V_i = e^{ikr \cos \theta} = e^{ik(x \sin \theta + z \cos \theta)}$. This plane wave is generated in real space and combined with the scattered field from the ORINCON software to get the total field.

Four cases are presented in this section. The prolate spheroidal scatterer used for all the cases is 100.0 feet along the major axis and 10.0 feet along the minor axis. The sound speed is 5000.0 feet per second. The measurement plane is formed by a $36 \times 36$ point planar array normal to the plane wave direction. These four cases include different frequencies, soft and hard scatterers, and plane waves impinging at beam and bow aspect. Referring to Fig. 3.1, the beam aspect corresponds to the major axis of the scatterer being along the $y$-axis and the bow aspect corresponds to the major axis being along the $z$-axis.

Case I shows a plane wave impinging on a hard prolate spheroid at beam aspect.
The center of the array is at the coordinates (5.1,0.0,0.0) and has an angle $\theta = 0$ with respect to the z-axis. The frequency is 100 Hz and its incident direction is $\zeta=270$ degrees. The separation distances are 5 feet along the major axis and 2 feet along the minor axis. The distance from the first measurement plane to the center of the scatterer is $Z_1 = 5.1 ft$, and from the second measurement plane is $Z_2 = 7.1 ft$. The first measurement plane and the separation plane are the same. After separation, the scattered field is very close to the simulated field as shown in Fig. 3.25. The window parameters are “apt=0.8” and “slp=0.1” and the filter parameters are “apt=0.7” and “slp=0.01”.

**Case II** shows a 500 Hz plane wave impinging on a hard prolate spheroid at beam aspect. The separation distances are 5 feet along the major axis and 1.5 feet along the minor axis. The distance from the first measurement plane to the center of the scatterer is $Z_1 = 5.1 ft$, and from the second measurement plane is $Z_2 = 6.1 ft$. The frequency is 500 Hz. The separated and simulated scattered fields are shown in Fig. 3.26. The window parameters are “apt=0.8” and “slp=0.15” and the filter parameters are “apt=0.5” and “slp=0.01”.

**Case III** shows a 500 Hz plane wave impinging on a soft prolate spheroid at beam aspect. All the separated and simulated scattered fields are shown in Fig. 3.27.

**Case IV** shows a 500 Hz plane wave impinging on a hard prolate spheroid at bow aspect. The separation distance is 2.0 feet on both sides. The distance from the second measurement plane to the scatterer center is $Z_2 = 51.5 ft$. The first measurement plane and the separation plane are the same, at $Z_1 = 50.5 ft$. The separated and simulated scattered fields are shown in Fig. 3.28. The window parameters are “apt=0.8” and “slp=0.15” and the filter parameters are “apt=0.7”.
and "slp=0.01".

All the separated fields in k-space have been optimized by averaging the values around the radiation circle. The separated results are even better for the prolate spheroidal scatterer illuminated by the plane wave. The reason is that the plane wave in these cases are at normal incidence and all the wave-number components are used for the separation.
Figure 3.25: The scattered field for prolate spheroidal scatterer, Case I: 100 Hz, beam aspect, hard scatterer; Left: separated; Right: theory

Figure 3.26: The scattered field for prolate spheroidal scatterer, Case II: 500 Hz, beam aspect, hard scatterer; Left: separated; Right: theory
Figure 3.27: The scattered field for prolate spheroidal scatterer, Case III: 500 Hz, beam aspect, soft scatterer; Left: separated; Right: theory

Figure 3.28: The scattered field for prolate spheroidal scatterer, Case IV: 500 Hz, bow aspect, hard scatterer; Left: separated; Right: theory
4. WAVE-NUMBER DOMAIN FIELD SEPARATION IN CYLINDRICAL COORDINATE SYSTEM

The previous chapter proves the feasibility of separating the incident and scattered fields for cartesian coordinates. The key to this success is that the propagator is exactly known and incident and scattered fields propagate in opposite directions. However the separation technique in cartesian coordinate doesn't work well when the incident field come from an oblique angle. Implementing the separation technique in cylindrical coordinates is considered as an improvement of the accuracy with waves incident at oblique angles. The problem that accompanies the cylindrical system is that the incident field contains both incoming and outgoing wave, while the scattered field is only outgoing waves. Thus the outgoing waves include both the incident and scattered waves. So the approach to the separation technique in cylindrical coordinates is much different than in cartesian coordinates.

In the first and second sections of this chapter, we discuss the propagators for the scattered and incident fields and the problems that were encountered when applying these propagators with the digital fast Fourier transform (FFT). The third section describes the separation technique in cylindrical coordinates. The fourth section shows the results for applying the separation technique to a spherical scatterer illuminated by a plane piston source and a prolate spheroidal scatterer illuminated
by a plane wave.

The cylindrical coordinate system is shown in Fig. 4.1. The two measurement planes are located at \( r_1 \) and \( r_2 \). And the separated scattered field is to be determined at \( r_0 \). The axis of the cylindrical measurement plane is aligned with the major axis of the prolate spheroid. The measurement plane begins in the forward propagation direction (-x axis) and moves counterclockwise with angle \( \phi \).

![Geometric configuration of the source and scatterer in cylindrical coordinates](image)

**Figure 4.1** Geometric configuration of the source and scatterer in cylindrical coordinates

### 4.1 Wave propagation in general cylindrical coordinate

Consider the propagation of a general field \( p(r, \phi, z) \) in cylindrical coordinates. For a monochromatic field, it satisfies the Helmholtz equation outside the source volume:

\[
\nabla^2 p(r, \phi, z) + k^2 p(r, \phi, z) = 0,
\]

where \( k = \omega/c \).
By solving this equation, using separation of variables, we can get the general field in real space as:

\[
p(r, \phi, z) = \sum_m \{ \cos(m\phi + \alpha_m)H_m^{(1)}(k_r r) \left[ A_m e^{-ik_{zz}z} + B_m e^{ik_{zz}z} \right] + \cos(m\phi + \beta_m)H_m^{(2)}(k_r r) \left[ C_m e^{-ik_{zz}z} + D_m e^{ik_{zz}z} \right] \},
\]

where \( k_{zz} \) is the axial wave number (with the units of radian per meter) and \( m \) is the circumferential wave number (the number of wavelengths which fit around the circumference of the cylindrical plane) and \( k_r = \sqrt{k^2 - k_{zz}^2} \). The coefficients \( A_m, B_m, C'_m \) and \( D'_m \) are real. By taking the Fourier transform of both sides of Eqn.(4.1), we can get \( P_m(r,n,k_z) \) in k-space as

\[
P_m(r,n,k_z) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-i\phi} \int_{-\infty}^{\infty} dz e^{-ik_z z} p(r, \theta, \phi)
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-i\phi} \int_{-\infty}^{\infty} dz e^{-ik_z z} \sum_m \{ \cos(m\phi + \alpha_m)H_m^{(1)}(k_r r) \left[ A_m e^{-ik_{zz}Z} + B_m e^{ik_{zz}Z} \right] + \cos(m\phi + \beta_m)H_m^{(2)}(k_r r) \left[ C'_m e^{-ik_{zz}Z} + D'_m e^{ik_{zz}Z} \right] \}.
\]

Consider the integral containing the \( \phi \) terms:

\[
\int_0^{2\pi} d\phi e^{-i\phi} \cos(m\phi + \alpha_m)
\]

\[
= \left[ e^{i\frac{m}{m} \alpha_m} e^{-i\phi} \left[ -i \cos m\phi + m \sin m\phi \right] \right]_{-\frac{m}{m}}^{2\pi + \frac{m}{m}}
\]

\[
= e^{-2\pi i} \frac{-1}{m^2 - n^2} (-i \cos \alpha_m + m \sin \alpha_m).
\]

This term is non zero only when \( m = n \), so

\[
\int_0^{2\pi} d\phi e^{-i\phi} \cos(m\phi + \alpha_m) = \delta_{mn} \pi e^{i\alpha_n},
\]

(4.2)
where $\delta_{mn}$ is the Kronecker delta.

The $k$-space generalized field can be expressed as the summation of the components at $k_z = \pm k_{zz}$ and $m=n$:

$$P_m(r, n, k_z) = \frac{1}{2\pi} \delta_{mn} \left[ \pi \delta(k_z + k_{zz}) A_m e^{i\alpha_m} H_m^{(1)}(k_z r) \right. $$

$$+ \left. \pi \delta(k_z - k_{zz}) B_m e^{i\alpha_m} H_m^{(1)}(k_z r) \right. $$

$$+ \pi \delta(k_z + k_{zz}) C_m e^{i\alpha_m} H_m^{(2)}(k_z r) $$

$$+ \pi \delta(k_z - k_{zz}) D_m e^{i\alpha_m} H_m^{(2)}(k_z r) \right]. \quad (4.3)$$

In Eqn.(4.2), we replace the $\frac{1}{2} A_m e^{i\alpha_m}$ term with a complex coefficient $A_m$ and do the same for $B_m, C_m$ and $D_m$.

$$P_m(r, n, k_z) = \delta_{mn} \left[ \delta(k_z + k_{zz}) [A_m H_m^{(1)}(k_z r) + C_m H_m^{(2)}(k_z r)] \right. $$

$$+ \delta(k_z - k_{zz}) [B_m H_m^{(1)}(k_z r) + D_m H_m^{(2)}(k_z r)] \right]. \quad (4.4)$$

where $\delta(k_z - k_{zz})$ is the Dirac delta function.

The Dirac delta functions exist only when the fields used in the Fourier transformation are infinite in the $z$-direction. If the measurement aperture size is finite from $z = L/2$ to $z = -L/2$, the Fourier transform of this real space incident plane wave will instead be a sinc function:

$$\int_{-\infty}^{\infty} dz e^{-i(k_z-k_{zz})z} = \int_{-\pi}^{\pi} dz e^{-i(k_z-k_{zz})z}$$

$$= 2 \sin \left( \frac{(k_z - k_{zz}) L}{2} \right) \frac{L}{(k_z - k_{zz})}. \quad (4.5)$$

Thus in real measurements the Dirac delta function in Eqn.(4.3) is replaced by the sinc function.
4.2 The propagator for the scattered field

We first consider the propagation of a scattered field [16, 17]. For the scattered field, because there is only outgoing waves, the \( H_m^{(2)}(k_r r) \) term is ignored. The scattered field in k-space consists of only the \( H_m^{(1)} \) term and can be expressed as

\[
P_{sm}(r, n, k_z) = S_m H_m^{(1)}(k_r r), \tag{4.6}
\]

where \( S_m \) is a complex coefficient for the scattered field.

The connection between two k-space fields, \( P_{sm}(r_1, n, k_z) \) and \( P_{sm}(r_2, n, k_z) \), is the propagator, so the pressure \( P_{sm}(r_2, n, k_z) \) can be expressed as

\[
P_{sm}(r_2, n, k_z) = \left[ H_m^{(1)}(k_r r_2)/H_m^{(1)}(k_r r_1) \right] P_{sm}(r_1, n, k_z). \tag{4.7}
\]

The ratio of the Hankel functions is called the propagator.

When \( k < k_z \), \( k_r \) becomes a pure imaginary number and Eqn.(4.6) becomes the ratio of two modified Bessel functions \( K_m \),

\[
P_{sm}(r_2, n, k_z) = \left[ K_m(k_r r_2)/K_m(k_r r_1) \right] P_{sm}(r_1, n, k_z). \tag{4.8}
\]

Just like the exponentially decaying functions in the previous chapter, the propagator when \( k < k_z \) decays exponentially when the pressure component travels away from the surface,

\[
K_\lambda(x) = \frac{\pi}{2} i e^{\frac{1}{2} \lambda n} H_\lambda^{(1)}(ix) \xrightarrow{x \to \infty} \sqrt{\frac{\pi}{2x}} e^{-x}. \tag{4.9}
\]

The propagator is applied to the field scattered from a rigid sphere with an incident field from a plane piston source. Fig. 4.2 and Fig. 4.3 show the propagated real and k-space scattered field of a spherical scatterer illuminated by a plane piston source. As was described in section 2.5, the central lobe of the figures represents
the backpropagating waves. The plots on the left are the theoretical data. The nondimensionalized wave number $ka$ is 5, $r_1$ is 1.1a, $r_2$ is 1.3a and $\Delta d$ is 0.25a. The variable "a" is the radius of the source. For a field with large aperture size that covers the whole field, the propagator is very good. The results prove to be correct and workable in the simulated and propagated fields.

4.3 Using a Bessel function to propagate the incident field

For a general incident field, the situation is more complicated than the scattered field because it contains both incoming and outgoing waves. Consider the term $A_m H_m^{(1)}(kr) + C_m H_m^{(2)}(kr)$ in Eqn.(4.4). We can represent the Hankel functions with the Bessel function $J_m(kr)$ and the Neumann function $N_m(kr)$:

$$A_m [J_m(kr) + iN_m(kr)] + C_m [J_m(kr) - iN_m(kr)]$$

$$= (A_m + C_m)J_m(kr) + i(A_m - C_m)N_m(kr).$$

Because the field must be finite at $r=0$. $N_m(kr)$ is infinite at $r=0$, then $N_m(kr)$ term must be zero. The coefficient $A_m$ must be equal to $C_m$. The general field can then be simplified as,

$$P_m(r,n,k_z) = G_m J_m(K_zr).$$

(4.10)

Here $G_m$, which is equal to $\delta_{mn}(\delta(k_z + k_{zz})A_m + \delta(k_z - k_{zz})C_m)$, is the complex coefficient of the generalized incident field.

The propagator of a generalized incident wave then can be expressed as:

$$P_{im}(r_2,n,k_z) = [J_m(k_zr_2)/J_m(k_zr_1)] P_{im}(r_1,n,k_z) \quad k > k_z,$$

(4.11)
Figure 4.2: The real space scattered field: Left: theory; Right: propagated

Figure 4.3: The k-space scattered field: Left: theory; Right: propagated
\[ P_{im}(r_2, n, k_z) = \frac{[I_m(k, r_2)/I_m(k, r_1)] P_{im}(r_1, n, k_z)}{k < k_z} \]  

(4.12)

where \( I_m \) is the Modified Bessel function. The large argument expansion for \( I_\lambda(x) \) is,

\[ I_\lambda(x) = e^{-\frac{1}{2}\lambda \pi i} J_\lambda(iz) \frac{1}{\sqrt{2\pi x}} e^x. \]  

(4.13)

Therefore this modified Bessel function exponentially increases for large \( k, r \), similar to the case of backpropagation in section 3.3. Although the Bessel function is the propagator for any incident wave, the numerical implementation of this propagator is not possible for a complicated field. The following two subsections will discuss the situation when the propagator does not work and present an alternative solution for a general incident field.

### 4.3.1 Propagating a plane wave at normal incidence with the Bessel function

A plane incident wave is the simplest case beside the cylindrical wave propagation. The performance of the propagator with a plane wave propagator is helpful to understand how the propagator will work with a more complicated field. Fig. 4.4 shows the geometric configuration of the general oblique plane incident wave. The variable \( \alpha \) is the angle between the vector \( \vec{k}_{rr} \) and the X-axis and \( \beta \) is the angle between the vector \( \vec{k} \) and the Z-axis.

If the plane wave is at normal incidence \( (\alpha = 0^\circ, \beta = 270^\circ) \), the cylindrical wave expansion of this plane wave can be represented by the summation of the Bessel functions [20]:

\[ p_i(r, \phi, z) = \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\phi}. \]  

(4.14)
After Fourier transformation, the k-space field becomes:

\[
P_{im}(k, n, r) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-im\phi} \int_{-\infty}^{\infty} dz e^{-ik_z z} \sum_{m=-\infty}^{\infty} i^m J_m(kr)e^{im\phi}
\]

\[
= \delta(k_z)2\pi i^n J_n(kr) \quad n = -\infty \cdots \infty.
\]  

(4.15)

The delta function exists if the aperture size is finite or infinite. The reason is that this numerical incident wave only has components at \( k_z = 0 \), since \( k_r = k \), so that the \( e^{-ik_z Z} \) term in the integral becomes 1.0.

Fig. 4.5 shows the k-space fields from direct simulation and through propagation. The Bessel function is used as the propagator (Eqns.(4.11) and (4.12)). The simulated and propagated data match well. The field in k-space exists only along \( k_z=0 \). The parameters used here are the same as in section 4.2 except this is for a plane incident wave. From the results of this case, the correct Bessel function propagator is applied to each component field and gives correct propagation results.

![Diagram](image)

Figure 4.4: Generalized oblique incident plane wave with wave number \( k \)
4.3.2 Propagating a plane wave at oblique incidence with the Bessel function

For the plane wave at oblique angles of incidence, the wave vector can be decomposed into three components $k_x$, $k_y$, and $k_z$. The plane wave can then be expressed as,

$$e^{i\mathbf{k} \cdot \mathbf{r}} = e^{i(k_{rr} \cos \alpha \hat{x} + k_{rr} \sin \alpha \hat{y} + k_{zz} \hat{z})}(r \cos \phi \hat{x} + r \sin \phi \hat{y} + Z \hat{z})$$

$$= e^{ik_{zz}Z} e^{ik_{rr} \cos(\phi - \alpha)}$$

$$= e^{ik_{zz}Z} \left[ \sum_{m=-\infty}^{\infty} i^m J_m(k_{rr}r)e^{im(\phi - \alpha)} \right], \quad (4.16)$$
where \( k_{rr} \) is the component of wave vector \( \vec{k} \) in the \( \vec{r} \) direction and \( k_{zz} \) is the component in the \( z \)-direction. The \( k \)-space field is,

\[
P_{im}(k, n, r) = \delta(k_z - k_{zz})2\pi i^n J_n(k_{rr} r)e^{-im\alpha} \quad n = -\infty \cdots \infty. \tag{4.17}
\]

For a finite aperture size, the \( k \)-space field becomes

\[
P_{im}(k, n, r) = \frac{2\sin [(k_z - k_{zz})L/2]}{(k_z - k_{zz})}2\pi i^n J_n(k_{rr} r)e^{-im\alpha} \quad n = -\infty \cdots \infty. \tag{4.18}
\]

This case is not as simple as the normal incident wave. The \( k \)-space field should exist only along \( k_z = k_{zz} \) (\( k_{zz} \) is the wave number of the incident field in the \( z \)-direction) for an infinite aperture size. But if the aperture size is finite, the Fourier transform of this real space plane incident wave becomes a sinc function instead of a Dirac delta function. The sinc function generates a continuum of wave numbers in \( k \)-space, replacing what should be a single wave number if an infinite aperture size were used. To get the right results for propagation, these higher wave number components must use the same propagator as those for \( k_z = k_{zz} \). Fig. 4.6 shows the \( k \)-space fields of an oblique incident wave \((\alpha = 20^\circ, \beta = 70^\circ)\) from direct simulation and propagation. The result is a sinc function. The parameters used here is the same as the previous normal incident plane wave case. The propagation results are correct for this plane wave incident case because we apply each wavenumber component with the propagator for the wavenumber \( k_{zz} \). This means that although the finite aperture size effect spreads this wave component into a sinc function, all the wave component still need the same propagator.

The steps we have used here to get accurate results are incorrect. If we know a priori that there is only one plane wave incident from a known direction then we can use this technique to get accurate results. But this is not acceptable because in
general we will know very little about the incident wave. In other words the wrong propagators need to be applied to each wave number component to get the right propagation.

4.3.3 Propagating a general incident field with the Bessel function

For the plane piston source or any other generalized field, the propagator should still be \[ J_m(k \cdot r_0)/J_m(k \cdot r_1) \] for \( k > k_r \) and \[ I_m(k \cdot r_0)/I_m(k \cdot r_1) \] for \( k < k_r \). But the direct use of this propagator does not work with a finite aperture.

With an infinite aperture size, each cylindrical wave is a single traveling wave component in the k-space. But with the finite aperture size, it becomes the continuum of wave numbers, as discussed in section 4.1 (a sinc function). This is very important because with a general incident wave the k-space field at each wavenumber, \( k_x \) and \( k_y \),
includes many different waves because of the overlapping of the sinc functions. The method, used in section 4.3.2, for the propagation of a plane wave with an oblique angle of incidence cannot be applied here.

Fig. 4.7 shows a simplified k-space field near the radiation circle $k_z = k$. Each component in this field needs to have its own propagator applied, $[J_m(k_r r_0)/J_m(k_r r_1)]$ if $k > k_r$, or $[I_m(k_r r_0)/I_m(k_r r_1)]$ if $k < k_r$. So point a should be propagated with $[J_m(k_r r_0)/J_m(k_r r_1)]$ even though it is outside the radiation circle and point b should be propagated with $[I_m(k_r r_0)/I_m(k_r r_1)]$ even though it is inside the radiation circle. The same situation happened in the scattered field propagator. The difference is the modified Bessel function $I_m$ (incident field propagator) and $K_m$ (scattered field propagator). The modified Bessel function $K_m$ for the scattered field propagation is a decaying function (Eqn.(4.9)) so the field (from the sinc function) and the noise (generated from the computation process) outside the radiation circle after propagation will not greatly affect the results. But the modified Bessel function $I_m$ used for the incident field propagation is an increasing exponential function (Eqn.(4.13)). Some of the fields outside the radiation circle is greatly increased by the increasing exponential function. The noise/error outside the radiation circle is thus enlarged by the $I_m$ function. This seems similar to the situation we discussed in section 3.3. We used windowing and filtering to improve the results. But applying the filtering in the case of a general incident field does not give good results. The wave is still distorted. Thus an alternative method is needed to accurately propagate a general incident field.
4.4 Two plane method to propagate the incident wave

An alternative method for propagating the incident wave is to use the original expression for the incident field: \( P_{im}(r, n, k_z) = A_m H^{(1)}_m(k_z r) + C_m H^{(2)}_m(k_z r) \), where \( A_m \) is equal to \( C_m \) for an infinite aperture size. To get the propagated incident field, we use a method similar to the separation technique that was used. This technique needs the incident fields to be known on two surfaces:

\[
P_{im}(r_1) = (A_m)H^{(1)}_m(k_z r_1) + C_m H^{(2)}_m(k_z r_1), \quad (4.19)
\]
\[
P_{im}(r_2) = (A_m)H^{(1)}_m(k_z r_2) + C_m H^{(2)}_m(k_z r_2). \quad (4.20)
\]

The incident fields \( P_{im}(r_1) \) and \( P_{im}(r_2) \) can be measured or calculated, and the Hankel functions \( H^{(1)}_m \) and \( H^{(2)}_m \) can be calculated. The two unknowns in Eqn.(4.19) and Eqn.(4.20) are \( A_m \) and \( C_m \). By solving Eqn.(4.19) and Eqn.(4.20), we can get \( A_m \) and \( C_m \). Then any propagated field at \( r_0 \) is,

\[
P_{ipm}(r_0) = (A_m)H^{(1)}_m(k_z r_0) + C_m H^{(2)}_m(k_z r_0). \quad (4.21)
\]
Fig. 4.8 and Fig. 4.9 represent the real and k-space propagated fields using one-plane propagation (Eqn.(4.11) and Eqn.(4.12)) and two-plane propagation (Eqn.(4.21)) for the case of a plane piston source with a spherical scatterer \((ka=3)\). The one-plane results are very bad but the two-plane results are good. These results clearly show that the two-plane propagation method is necessary.

**4.5 The separation technique**

The propagators for both the scattered and incident fields have been discussed. The difficulty in using these propagators in a real measurement where a finite aperture is used is the mixing of propagating and evanescent waves around the radiation circle. In this section we develop the separation technique and show that the problem around the radiation circle effects the separation results.

To develop the field separation in cylindrical coordinates, we begin by representing the total field at different positions \(r_1\) and \(r_2\). The k-space expression of the total field is the summation of the incident and scattered fields. Thus the total field at two radial distances is

\[
P_T(r_1) = A_m H_m^{(1)}(k_r r_1) + C_m H_m^{(2)}(k_r r_1) + S_m H_m^{(1)}(k_r r_1)
\]

\[
= (A_m + S_m) H_m^{(1)}(k_r r_1) + C_m H_m^{(2)}(k_r r_1)
\]

\[
= (A_T^m) H_m^{(1)}(k_r r_1) + C_m H_m^{(2)}(k_r r_1), \tag{4.22}
\]

\[
P_T(r_2) = A_m H_m^{(1)}(k_r r_2) + C_m H_m^{(2)}(k_r r_2) + S_m H_m^{(1)}(k_r r_2)
\]

\[
= (A_m + S_m) H_m^{(1)}(k_r r_2) + C_m H_m^{(2)}(k_r r_2)
\]

\[
= (A_T^m) H_m^{(1)}(k_r r_2) + C_m H_m^{(2)}(k_r r_2), \tag{4.23}
\]

where \(A_T^m = A_m + S_m\).
Figure 4.8: The propagated plane piston source from theory: Left: k-space; Right: real-space

Figure 4.9: The propagated plane piston source: Left: Eqn.(4.21); Right: Eqn.(4.11) and Eqn.(4.12)
The unknowns in Eqn.(4.22) and Eqn.(4.23) are \( A_m^T = (A_m + S_m) \) and \( C_m \). Solving Eqns.(4.22) and (4.23) simultaneously with \( P_T(r_1) \) and \( P_T(r_2) \) known, \( A_m^T \) and \( C_m \) are written as:

\[
A_m^T = \frac{P_T(r_2)H_m^{(2)}(k,r_1) - P_T(r_1)H_m^{(2)}(k,r_2)}{H_m^{(1)}(k,r_2)H_m^{(2)}(k,r_1) - H_m^{(2)}(k,r_2)H_m^{(1)}(k,r_1)},
\]  

(4.24)

\[
C_m = \frac{-P_T(r_2)H_m^{(1)}(k,r_1) + P_T(r_1)H_m^{(1)}(k,r_2)}{H_m^{(1)}(k,r_2)H_m^{(2)}(k,r_1) - H_m^{(2)}(k,r_2)H_m^{(1)}(k,r_1)}.
\]  

(4.25)

To get the scattered field, we need to know \( S_m \). Since \( A_m = C_m \) and \( A_m^T = A_m + S_m \), the scattered coefficient can be solved for as \( S_m = A_m^T - C_m \). The separated scattered field can then be calculated as,

\[
P_s(r_0) = S_mH_m^{(1)}(k,r_0) = (A_m^T - C_m)H_m^{(1)}(k,r_0).
\]  

(4.26)

Now the scattered field can be evaluated at any position \( r_0 \).

4.6 Numerical simulation

The cases considered here are the sound field from a plane piston source incident on a rigid spherical scatterer and a plane wave incident on a prolate spheroidal scatterer. The results at two frequencies for each case are shown to demonstrate the validity of the cylindrical separation technique.

Windowing and filtering are used throughout the process. Windowing is only used along the Z-axis in real space because in the \( \phi \) direction the field is always continuous so that no edge effect exists along the \( \phi \) direction. In k-space, an oval filter is used in the analysis.
4.6.1 Oval filtering

The oval filter used in the cylindrical separation technique is a variation of the filter described in section 3.4. With the oval filter the aperture size is adjustable along both the m-axis and the $k_z$-axis.

The general form of the oval filter can be expressed as (same as Eqn.(3.9)),

$$W_{\text{oval}}(k_z, m) = \begin{cases} 
1 - \frac{e^{-\alpha_{\text{oval}}}}{2} & \text{for } r < c, \\
\frac{e^{rac{\alpha_{\text{oval}}}{2}}}{2} & \text{for } r > c 
\end{cases}$$

(4.27)

Where

- $k_z, m$ are the coordinates of the field in k-space.
- $apt_z$ is the ratio for the aperture size of the filter in $k_z$ direction (input parameter).
- $apt_m$ is the ratio for the aperture size of the filter in m direction (input parameter).
- slp is the slope of the filter (input parameter).
- $\alpha_{\text{oval}} = \frac{(1 - r)}{\text{slp}}$.
- $r = \sqrt{k_z^2 + m^2}$ is the length from the field point to the center.
- $c = \sqrt{c_m^2 + c_z^2}$ is the half aperture size of the filter. The variables $c_m$ and $c_z$ are the half aperture size at each side (m and $k_z$).

For the oval filter $c_m$ and $c_z$ satisfy the oval function,

$$\frac{c_z^2}{r_z^2} + \frac{c_m^2}{r_m^2} = 1,$$
where

\[ c_z = \sqrt[2]{\frac{r^2 r_m}{r^2 \tan \theta^2 + r_m^2}}, \]

\[ c_m = \sqrt[2]{\frac{r^2 r_m \tan \theta^2}{r^2 \tan \theta^2 + r_m^2}}, \]

- \( r_m = apt_m \times nrow/2 \).

- \( r_z = apt_z \times \delta d \times mcol/2 \): \( \delta d \) is the increment between points in z direction and \( mcol \) is the number of points at z-direction.

- \( \tan \theta = \frac{m}{k_z} \).

Four examples of the oval filter, with various input parameters, are presented in Fig. 4.10 and Fig. 4.11. The larger the apt value is, the wider the filter is. And the smaller the input variable slp is the sharper filter is. The value of the input variable slp is usually kept under 0.2 to avoid deformation at the central part of the filter. The filter is always centered at \( k_z = 0 \) and \( m=0 \).

4.6.2 The spherical scatterer illuminated by a plane piston source

The parameters for the plane piston source illuminating a spherical scatterer are \( k_a=3 \) and \( s=30 \). The inter-element distance along the Z-axis is 0.15 and the first and second measurement surfaces are located at \( r_1=1.05 \) and \( r_2=1.15 \). All these parameters are nondimensionalized by the radius of the plane piston source. The array size is \( 32 \times 32 \) points. Fig. 4.12 shows the comparison for the separated scattered field. The windowing parameters are “apt”=0.9 and “slp” =0.1 before k-space processing and “apt”=0.8, “slp” =0.1 after k-space processing. The filtering parameters are “apt_z”=0.5, “apt_m”=0.8 and “slp” =0.01.
Figure 4.10: The oval filter: Left: \( a_{ptz} = 0.9, a_{ptm} = 0.5, \text{slp} = 0.1 \); Right: \( a_{ptz} = 0.5, a_{ptm} = 0.9, \text{slp} = 0.1 \)

Figure 4.11: The oval filter: Left: \( a_{ptz} = 0.5, a_{ptm} = 0.9, \text{slp} = 0.2 \); Right: \( a_{ptz} = 0.5, a_{ptm} = 0.5, \text{slp} = 0.01 \)
The results are comparable to the quality of results in cartesian coordinates, except that additional windowing is necessary. The windowing is applied to the separated scattered field in real space. The reason is that there are some errors along the edge of the surface after the separation processing. This windowing is necessary for all the plane piston source cases. High peaks are also noticed around $k_z = k$ in k-space which is similar to the cartesian coordinate system. The edge effects may come from these peaks.

4.6.3 The prolate spheroidal scatterer illuminated by a plane wave

The prolate spheroidal scatterer parameters are 360.0 feet along the major axis and 33.0 feet along the minor axis. The incident plane wave has a frequency of 150 Hz and the sound speed is 5000 ft/sec. The inter-element distance along the Z-axis is 12.5 feet and two measurement surfaces have radii 18.0 feet and 21.5 feet. Fig. 4.13 is the separated scattered field compared with the theoretical prediction. The windowing is applied only before the k-space processing with the parameters, “apt”=0.8 and “slp” =0.1. The filter is used in k-space with “aptz”=0.7, “aptm”=0.8 and “slpm” =0.01. We can see that the result after separation is better than for the plane piston case. Again this is because the input plane wave is a more simple field compared to the incident field from a plane piston. Filtering is also needed less for this case.
Figure 4.12: Plane piston source with the rigid spherical scatterer: Left: theory; Right: separated

Figure 4.13: Plane incident wave with the prolate spheroidal scatterer: Left: theory; Right: separated
5. SENSITIVITY TESTS

This chapter focuses on the analyses that was done to understand the influence of the parameters in the separation process. Two class of issues are considered. The first is to determine which parameters will affect the performance of the separation technique. The second issue is what influence does errors in the measurement setup have on the separation results. For the first section, the incident frequency, the aperture size, the distance between the two measurement surfaces and the inter-element spacing are considered. The spherical scatterer with the plane piston source is the study case. For the second section, the measurement surface skew, offset, horizontal translation and the measurement surface curvature are considered. The example case used is the prolate spheroidal scatterer with a plane wave incident. The results from the finite aperture size study are also included. Both the cartesian and cylindrical coordinate systems are considered and the comparisons are made with the scattered field from theoretical data.

5.1 Sensitivities of the separation parameters

A series of sensitivity analyses are conducted to determine the effects that the measurement parameters have on the accuracy of the separation technique. These analyses use a numerical approach in which the parameters of interest are varied over
a range of values while keeping the remaining parameters constant. The parameters which are identified as potentially significant are the aperture size with respect to the size of the scatterer, the separation between measurement points in a measurement surface, the nondimensionalized wavenumber, and the separation distance between two measurement surfaces. The spherical scatterer illuminated by a plane piston source is used as the study case. The far-field projection (see appendix B) is used as a reference to compare the sensitivity. The scale for the projection is a fixed projection distance from a to 1000a in log scale (a is the sphere radius).

The following sections present detailed descriptions of the analyses. The results for both the cartesian and cylindrical coordinate systems are considered.

5.1.1 Parameter description in cartesian coordinates

Measurement aperture size. Aperture size with respect to the scatterer has practical interest from both the view points of the separation technique and the experimental implementation. In performing the sensitivity tests, the variable is the measurement aperture size while holding the inter-element distance equal to \( \lambda/8 \) and \( ka \) equal to \( 2\pi \). No windowing or filtering is used, and the distances of the first and second measurement surfaces are \( \lambda/16 \) and \( \lambda/8 \) from the scatterer surface.

The ratio of the length of the side of the measurement aperture to the diameter of the sphere (L/d) is chosen as a metric to indicate the relationship between the measurement plane and scatterer size. The range of L/d is from 0.5 to 4.

Inter-element distance of a measurement surface. This concerns the spatial sampling of the measurement surface. Although the projection errors due to
aliasing in the under-sampled case are well known, it is desired to have a clear understanding of the effects of over-sampling the measurement surface on the separation results. In all cases, the array size is kept constant with a side length equal to $10\lambda$ and the sphere diameter is set to be $5\lambda$. (The aperture size twice the size of the scatterer.) No windowing or filtering is used. In all cases, $k\alpha$ is held constant at $2\pi$. The four cases of this study have defined element separations of $2\lambda$, $\lambda$, $\lambda/2$ and $\lambda/4$.

The nondimensionalized wavenumber $k\alpha$. This indicates the frequency range that will gave the best separation results. We set the array size constant (2d), and inter-element distance $\lambda/2$. The first and second measurement plane are located at $\lambda/16$ and $\lambda/8$ away from the sphere surface. In the cases studied, the $k\alpha$ value increases from $\pi/2$ to $4\pi$.

The distance between two measurement planes. With the first measurement plane fixed at $\lambda/16$ from the scatterer surface, the second measurement plane is moved $\lambda/8$, $\lambda/4$, $\lambda/2$, $\lambda$, and $2\lambda$ away from the first measurement plane. In all cases, the array size is kept constant with a side length is equal to $10\lambda$ and the sphere diameter is equal to $5\lambda$. No windowing or filtering is used. The frequency, $k\alpha$ is held constant at $2\pi$.

5.1.2 Results for the cartesian coordinate system

Fig. 5.1 to Fig. 5.4 are the results of the sensitivity tests for each parameter for the cartesian coordinate system. Each figure shows the difference in the farfield projected target strength between the theoretical and the separated results. The unit
is dB.

In Fig. 5.1, the parameters at the far-field are presented as a function of the aperture size/scatterer diameter (L/d). As expected, the error tends to decrease as the measurement size increase. The reason for this is that the edge effect (difference between the edge pressure and zero) decreases with an aperture size that is large compared to the scatterer; in which case, measurement surface more completely covers the field. The errors due to the aperture size can be improved by using the windowing technique as discussed in Chapter 4.

Fig. 5.2 depicts the effect of inter-element spacing. The pressure is expressed as a function of the ratio of the inter-element distance and the wavelength. The error of under-sampling is obvious and there is little difference between λ/2 and λ/4 at the far-field. So that there is little to gain by spatially over-sampling the measurement surface, unless you want more spatial information.

The nondimensionalized ka is an important factor. The farfield pressure (at a distance 1000a from the scatterer) is expressed as a function of ka (Fig. 5.3). The aperture size has a direct relation with ka. With a 2d aperture size, the range of ka above π results in good separation. This is very useful to decide the frequency range with a limited scanner size.

Fig. 5.4 is the separation result with different separation distances. For the separation distance less than 3λ/16, the results are good (within 0.6 dB error). When the separation distance increases, the separation accuracy decreases. The reason is that when the measurement surfaces moves far away from the scattering center, the evanescent waves are not included. This makes the separation (reconstruction)
Figure 5.1: Effects of the aperture size on separation technique (cartesian system)

Figure 5.2: Effects of the inter-element distance on separation technique (cartesian system)
Figure 5.3: Effects of the nondimensionalized wavenumber $ka$ on separation technique (cartesian system)

Figure 5.4: Effects of the separation distance between two measurement planes on separation technique (cartesian system)
procedure impossible.

5.1.3 Results for the cylindrical coordinate system

The same parameters discussed for the cartesian coordinate system are investigated for the cylindrical system. The conclusions from the cartesian coordinate sensitivity tests are directly applicable to axial components (z-axis), which means that a large ratio between the aperture size and the scatterer diameter is needed, the inter-element distance must be within $\lambda/2$ to prevent under sampling, a high nondimensionalized input wavenumber is needed and the two measurement planes must be close. In a cylindrical coordinate system, the sensitivity results cannot be compared by the farfield projected target strength of a theoretical data. The reason is that the cylindrical separation must use filtering in k-space for most cases, but there is no reference to set the filtering parameters. Because the farfield projected target strength depends on the nearfield separated field, the filtering parameters can greatly influence the projected results. The reason for considering the cylindrical coordinate system is because the separation method of this coordinate system can separate the wave that incident from an oblique angle. But the trade off is that the measurement procedure must be very precise for certain parameters.

As discussed in Chapter 4, the complexity of the separation technique in the cylindrical coordinate system comes from the propagators. Because the Hankel function propagator is a function of $k_r r = (\sqrt{k^2 - k_r^2})r$, there are three parameters that are important to the separation results. The wavenumber $k$, the measurement surface radius $r$ and the distance between measurement points along the Z-axis. The considerations of these three parameters is different from the cartesian coordinate system.
The reason is that these three parameters mixed together and effect the value of the Hankel function. So the sensitivities of the circumferential components cannot be analyzed separately. Also the Hankel functions are not continuously increasing or decaying functions for small arguments which makes them very sensitive to the input $k^r$ value. These combined effects make it impossible to provide general sensitivity guidelines for the cylindrical coordinate system. Thus a case by case sensitivity analyses is needed when experiments are being designed.

5.2 Sensitivities of errors in parameters

This section focuses on the effect of errors in measurement parameters on the separation techniques, especially how they affect the target strength in the farfield. Errors in the measurement parameters occur due to the measurement surfaces not being exactly what the experimenter think they are. For example, the currents in water can distort a measurement array or move it from its assumed position. The following analyses can help understand the sensitivities of the measurement setup and improve the experimental design. With these results, an experimental apparatus can be designed so that the parameters that cause most errors in the separation technique can be designed with the most accuracy possible at the expense of parameters that the separation results are less sensitive to.

The prolate spheroidal scatterer illuminated with a harmonic plane wave impinging from the broad side is used as the sample case. By using the Helmholtz Integral Equation (H.I.E.), the scattered field is projected to the far field. Comparisons are made between the theoretical (true value) and separated projected scattered field values along an axis centered at the center of the scatterer and extending along a
180° arc in the far field (appendix B).

Section 6.2.1 focuses on the cartesian coordinate system. Section 6.2.1.1 discusses the measurement error parameters which include: skew, offset, horizontal translation, measurement surface curvature, and finite aperture. The results are presented in section 6.2.1.2. Section 6.2.2 focuses on the cylindrical coordinate system. The discussion in this section is very similar to the cartesian system, but only the on-axis projected pressure is compared. The error parameters that are discussed are skew, shift, offset, horizontal translation and aperture size. The results are discussed in section 6.2.2.2.

5.2.1 Cartesian coordinate system

For the cartesian coordinate system, the sensitivities due to errors in parameters which include the measurement skew, offset, horizontal translation, finite aperture size and surface curvature are considered. For the offset and horizontal translation, the wavelength is used as a reference parameter. For the skew and measurement surface curvature, the angle (degree) is used as the reference parameter. For the finite aperture size, the size of the scatterer is used as the reference parameter.

We choose the study case of a 33 feet (minor axis) by 360 feet (major axis) rigid prolate spheroid with an incident plane wave at 150 Hz. The measurement surfaces are 15 point × 45 point plane arrays with the number of rows (along minor axis) equal to 15 and the number of columns (along major axis) equal to 45. The center of the array is at the coordinate (xc,yc,zc) and makes an angle θ with respect to the z-axis. The angle θ is equal to zero for this study case. The center of the first measurement plane is located at (29,0,0) and the second is located at (32.5,0,0). The inter-element
spacing of the array is 6.25ft along the minor axis ($\Delta_r$) and 12.5ft along the major axis ($\Delta_c$). For the incident plane wave, the incident angle is 270° with respect to the positive z-axis (Fig. 4.1). The speed of sound in water is 5000 ft/sec. At 150 Hz, the wave number $k$ is 0.1885 and the wave length $\lambda$ is 33.33 feet. The physical parameters for the scatterer are an interfocal distance of the spheroid, $d$ of 358.5 feet, a nondimensionalized variable $h=kd/2$ of 33.8 and a mean radius $r = (\text{major axis} + \text{minor axis})/4$ of 98.25 feet. An important parameter for the comparison of the true model to the far-field projection is the distance $R$, which is measured from the z-axis to the center of the separated plane. For the far-field projection, the log ration of $R/r$ from $10^0$ to $10^3$ is used as the scale.

5.2.1.1 Parameter description A series of sensitivity analyses are conducted to determine the effects of the separation results when varying the parameters involved in measurements. This analyses is performed using a numerical approach in which the measurement planes are varied while parameters input to the separation technique remain constant. The following paragraphs present detailed descriptions of the analyses cases.

Offset The "offset" of the measurement surface is defined as a parallel shift of the measurement planes by a certain distance from the expected location (Fig. 5.5a). Four cases were studied. Both the first and second measurement planes were shifted by $\pm \lambda/4$ and $\pm \lambda/8$. An offset is likely to occur when the measurement of the distance between the center of the measurement plane is not accurately controlled.
a) Offset

b) Horizontal Translation

c) Skew

d) Measurement Surface Curvature

Figure 5.5: Offset, horizontal translation, skew and measurement surface curvature (cartesian system)
Horizontal translation This defines the shift of the center of the measurement planes by a certain distance up or down from the expected position (Fig. 5.5b). The array size is kept constant. Because of the symmetry of the model, we only considered translation in one direction. The first and second measurement planes were moved together by $\lambda/4$ and $\lambda/8$. This case happens when the center of the scanner does not align with the center of the scatterer.

Skew This case involves the "skew" of a measurement plane. The measurement surfaces are tilted with respect to the plane perpendicular to the incident field direction (Fig. 5.5c). The plane tilted at 1 degree and 2 degree is considered. In these two cases, the array size is kept constant. Tilt occurs when the scanner and the scatterers axis is not correctly aligned.

Measurement surface curvature For the measurement surface curvature, we examine the phase difference in the acoustic pressure between the center and the end of the array (Fig. 5.5d). The point source at a desirable distance was used as a source to generate the surface curvature. To correctly match the scattered fields with the plane wave cases, the $20\log_{10} R$ spread factor is added to yield the scattered field level. Four cases were studied here with the measurement surface curvature phase difference of $10^\circ$, $30^\circ$, $50^\circ$ and $70^\circ$. To obtain the right location of the point source for a certain measurement surface curvature, Eqns. 5.1 and 5.2 are applied:

$$\frac{y}{\lambda} = \frac{\phi}{360^\circ},$$  \hspace{1cm} (5.1)

$$R^2 + \left(\frac{L}{2}\right)^2 = (R + y)^2 \implies R = \frac{1}{2y} \left(\frac{L^2}{4} - y^2\right).$$ \hspace{1cm} (5.2)
Where:

- $\phi$ is the desired phase difference between the center and the end of the array,
- $R$ is the calculated distance from the point source to the center of the measurement plane,
- $\lambda$ is the wavelength,
- $L$ is the measurement plane size along major axis.

This case happens when for example the movement of the water that the measurement is being made in warps the array.

**Finite aperture** The aperture size with respect to the scatterer dimension is studied to understand the sensitivity of the forward projection to the measurement plane size. The aperture size is varied from 1 to 1.3 times the dimension of the scatterer with 0.1 times the scatterer dimension as the resolution.

5.2.1.2 Results Fig. 5.6 through 5.15 show the results for each parameter discussed in the previous section. The projected estimate from the separated scattered field of a $15 \times 45$ points array is compared with the data directly from theory and the projected value using the $45 \times 45$ points measurement surface using the H.I.E.

Figs. 5.6 and 5.7 present the results with no distortion to the measurement plane using a $15 \times 45$ point array. Fig. 5.6 presents the result for the on-axis comparison between the projection estimate and the true model. The difference is 3.12 dB at the far field (1000a). By comparing the result with the $45 \times 45$ points array (0.371 dB), it can be concluded that the coverage of the field by a $15 \times 45$ points array is not
enough for an accurate separation. But with limitations on the real implementation, the 15 × 45 array is chosen to do the sensitivity tests. The possible extension of this analysis would be to minimize the far-field error by optimizing the inter-element spacing, the distance between the measurement planes, and the signal processing involved. Because all the on-axis comparisons are very similar, the results are not included in this dissertation. Table 5.1 shows the on-axis comparison in the farfield (1000a). The difference between the true pressure and the projected estimate is always within 2 dB. This results shows that the monostatic target strength are not sensitive to the measurement plane errors in the cartesian coordinate system.

Fig. 5.7 is the comparison of the far-field data as a function of angle at 1000a (appendix B). The two extreme ends of this figure shows deviation between the true model and the projected estimate. This results is expected due to the finite aperture size used in the H.I.E. estimate.

The comparison of the far-field data as a function of angle for different sensitivity parameters are included from Fig. 5.8 to Fig. 5.15. Fig. 5.8 and Fig. 5.9 show the cases for “offset”. Fig. 5.11 shows results for “horizontal translation”. Fig. 5.10 shows the results for the “skew” of the measurement planes. The “measurement surface curvature” results are presented in Figs. 5.12 and 5.13. And the “finite aperture” results are shown in Figs. 5.14 and 5.15.

In Fig. 5.8, Fig. 5.9 and Fig. 5.11, the offset and horizontal translation do not have a large effect on the far-field estimates of the scattered field.

From Fig. 5.10, the directivity plots show that the far-field pressure pattern is rotated with respect to the true model. Which means that the skew of the measurement plane can rotate the far-field pressure pattern by a small angle.
Table 5.1: On-axis comparison between true pressure and projected estimate at the farfield (1000a)

|                         | True (dB) | Separation (dB) (H.I.E.) | $|\Delta|$ (dB) |
|-------------------------|-----------|--------------------------|--------------|
| **Skew**                |           |                          |              |
| 1                       | -61.127   | -61.866                  | 0.739        |
| 2                       | -61.127   | -62.412                  | 1.285        |
| **Offset**              |           |                          |              |
| $\lambda/4$            | -61.127   | -60.074                  | 1.053        |
| $\lambda/8$            | -61.127   | -60.699                  | 0.428        |
| $-\lambda/8$           | -61.127   | -62.755                  | 1.628        |
| $-\lambda/4$           | -61.127   | -63.499                  | 2.372        |
| **Translation**        |           |                          |              |
| $\lambda/8$            | -61.127   | -61.659                  | 0.532        |
| $\lambda/4$            | -61.127   | -61.653                  | 0.526        |
| **Measurement**        |           |                          |              |
| 10                      | -61.127   | -60.915                  | 0.212        |
| 30                      | -61.127   | -61.233                  | 0.106        |
| 50                      | -61.127   | -61.963                  | 0.836        |
| 70                      | -61.127   | -62.376                  | 1.249        |
| 90                      | -61.127   | -62.486                  | 1.359        |
| **Finite Aperture**    |           |                          |              |
| 1.0                     | -61.127   | -63.109                  | 1.982        |
| 1.1                     | -61.127   | -61.217                  | 0.09         |
| 1.2                     | -61.127   | -60.71                   | 0.417        |
| 1.3                     | -61.127   | -60.092                  | 1.035        |
| **Exact (15 × 45)**    |           |                          |              |
|                         | -61.127   | -61.666                  | 0.539        |
| **Exact (45 × 45)**    |           |                          |              |
|                         | -61.127   | -61.268                  | 0.141        |

$20\log_{10}(P_{\text{scattered}}/P_{\text{incident}})$
Figs. 5.12 and 5.13 conclude that only within certain angular ranges it is possible to get acceptable accuracy. This shows rather high sensitivity of the separation technique to the measurement surface curvature.

For the finite aperture case in Figs. 5.14 and 5.15, the larger the aperture size, the better fit between the true and the projected estimate.

5.2.2 Cylindrical coordinate system

For the cylindrical coordinate system, the sensitivities of the measurement surface offset, shift, horizontal translation and skew are considered. The study case is the same as was used for the cartesian system. The cylindrical measurement surface is a 44 point x 44 point array. The central line of this cylindrical array is coincident with the major axis of the prolate spheroid. The inter-element spacing along the z-axis is 12.5 ft. The plane wave is incident at 270° with respect to the positive z-axis.

5.2.2.1 Parameter description

Offset The “offset” of a cylindrical measurement surface is different than the cartesian coordinate system. The cylindrical center doesn’t move, but the radius of this surface is changed by \( \pm \lambda/4 \) and \( \pm \lambda/8 \) (Fig. 5.16a). This occurs when the microphone used for measurements is not positioned correctly or the measurement of the radius of motion is incorrect.

Shift The “shift” considers the case where the center of the measurement surface is shifted by a certain distance with the radius of the surface remaining unchanged. Because the incident wave comes in from the x-axis, the effects are different for
Figure 5.6: Exact: on-axis comparison (cartesian system)

Pressure Level at Different Angle
98250 ft From Center of Separated Plane

Figure 5.7: Exact: farfield comparison at different angles (cartesian system)
Pressure Level at Different Angle 98250 ft From Center of Separated Plane

Figure 5.8: Offset ($\lambda/4$): farfield comparison at different angles (cartesian system)

Pressure Level at Different Angle 98250 ft From Center of Separated Plane

Figure 5.9: Offset ($-\lambda/4$): farfield comparison at different angles (cartesian system)
Figure 5.10: Horizontal translation ($\lambda/4$): farfield comparison at different angles (cartesian system)

Figure 5.11: Skew ($2^\circ$): farfield comparison at different angles (cartesian system)
Figure 5.12: Measurement surface curvature (10°): farfield comparison at different angles (cartesian system)

Figure 5.13: Measurement surface curvature (70°): farfield comparison at different angles (cartesian system)
Figure 5.14: Finite Aperture (1.0): farfield comparison at different angles (cartesian system)

Figure 5.15: Finite Aperture (1.3): farfield comparison at different angles (cartesian system)
shifts along the x-axis or the y-axis. If shifting occurs along the x-axis, the field along the circumferential axis is still symmetric. But if shifting occurs along the y-axis, the field along the circumferential axis becomes nonsymmetric. Both cases are considered by moving the center line of the measurement surface by \( \lambda/4 \) and \( \lambda/8 \) (Fig. 5.16b).

**Horizontal translation** The horizontal translation of a measurement surface is the same as in the cartesian coordinate system. The measurement surfaces are moved up by \( \lambda/4 \) and \( \lambda/8 \) (Fig. 5.16c). This happens when the center of the cylindrical scanner is not aligned with the center of the scatterer.

**Skew** Fig. 5.17 shows four different skew cases. For case a), the measurement surface was tilted with respect to the plane perpendicular to the incident field. This happens when the vertical scanner is not vertical to the turn table. Case b) occurs when the whole plane is tilted with an angle. This happens when the turn table is not aligned correctly. Case c) occurs when the plane wave is incident from an angle and case d) occurs when the scatterer not aligning along the z-axis.

**5.2.2.2 Results** Fig. 5.18 shows the on-axis comparison of the projected estimate with the true pressure. The parameters that are used include a skew of 1 degree, an offset of \( \lambda/8 \), a horizontal translation of \( \lambda/8 \), and a shift along the x-axis of \( \lambda/8 \). The separation results are not sensitivity to the shift and horizontal translation. But the effect of the offset and skew are significant. The offset results cause significant errors in the near-field and the far-field. The skew causes errors
Figure 5.16: Offset, shift and horizontal translation (cylindrical system)
Figure 5.17: Skew (cylindrical system)
in the far-field projection. The skew case here is the case a) in Fig. 5.17. The sensitivities of the offset, shift and the skew of the measurement surface are now discussed in detail.

Fig. 5.19 shows the effect of the offset on the separation technique with different offset values. The radius of the measurement surface varies from λ/16 to λ/4. Even for the λ/16 case, the errors are significant. This is reasonable because the propagators for the cylindrical coordinate system are all functions of the surface radius. These functions are very sensitive to the input values, resulting in significant errors in the separation results. This result emphasize the importance of a good measurement of the radii of the measurement surfaces. Fig. 5.20 gives a comparison of the separated scattered field with λ/8 offset with the theoretical scattered field with the surface radius at 18 feet.

Fig. 5.21 shows the effect of the shift on the separation technique with different shift values. The center of the measurement surface is shifted λ/8 and λ/4 along the x-axis and y-axis. The separation results in the farfield are not significantly sensitive to the shift. But for the y-axis shifting of λ/4, a big drop occurs at the near-field, so the nonsymmetry of the field does affect the separation results. Fig. 5.22 gives a comparison of the separated scattered field with a λ/8 shift along the y-axis with the theoretical scattered field with the surface radius at 25.5 feet.

Fig. 5.23 and Fig. 5.24 are two different investigations of the skew sensitivity. Fig. 5.23 investigates the sensitivities of different skew angles for the skew case a). The larger the skew angle, the more significant the error in the far-field. Fig. 5.24 is the comparison of three different skew cases b), c) and d). The skew cases b) and c) have the worst projected estimations. Case d) has a smaller error. When
looking back to Fig. 5.17, if the scatterer is the fixed object of the measurement, case b) changes the relative position of the measurement surface and case c) changes the relative position of the incident wave. But case d) changes both the relative positions of the measurement surface and the incident wave. This emphasize the importance of the correct alignment of the prolate spheroidal scatterer.

All the analyses above did not apply any windowing after the separation. The edge effects cause significant problems at the ends of the z-axis as seen in Fig. 5.25. Applying the windowing on the separated scattered field in real space, the sensitivity comparison with the exact data improves dramatically as shown in Fig. 5.26. The parameters for the windowing is “apt=0.9” and “slp=0.1”. Because the skew effect is common if the vertical scanner is not aligned perfectly, the measured data usually contains the effects due to skew. Consequently, windowing becomes necessary for the separated fields to improve the error caused by skew.
Figure 5.18: Comparison between the exact, skew, horizontal translation, offset and shift along x-axis (cylindrical system)
Figure 5.19: Effect of the offset on the separation technique (cylindrical system)

Figure 5.20: Offset: Left: theory; Right: separated (cylindrical system)
Figure 5.21: Effect of the shift on the separation technique (cylindrical system)

Figure 5.22: Shift at y-axis: Left: theory; Right: separated (cylindrical system)
Figure 5.23: Skew at different angles (cylindrical system)

Figure 5.24: Skew at different cases (cylindrical system)
Figure 5.25: Skew: Left: no windowing; Right: windowing (cylindrical system)

Figure 5.26: Comparison of skew with and without windowing (cylindrical system)
6. EXPERIMENT PROCEDURES AND RESULTS

6.1 Introduction

This chapter focuses on experimentally verifying the feasibility of the separation technique. The experiments were performed in both the cartesian and cylindrical coordinates. All the measurements are accomplished by using a cartesian and a cylindrical 3-D scanner inside an anechoic chamber. For the cartesian experiments, a cast iron spherical scatterer is illuminated by a plane piston speaker. For the cylindrical cases, the piston speaker illuminates four different scatterers: a cast iron sphere, a hard plastic sphere, a solid aluminum cylinder and a hollow aluminum cylinder. The experimental setup and results for both the cartesian and the cylindrical systems are described separately in two sections.

6.2 Cartesian system

6.2.1 Experiment facilities

For a complete measurement, the total pressure field at two surfaces is needed for the separation technique. The measurement of the scattered field at the separation surface is also needed for comparison with the separation results. The subtraction method that is used to determine the scattered field was outlined in section 1.3.3.1.
This method requires the measurement of the total pressure field and the incident field at the separation surface. The scattered field is calculated by subtracting the incident field from the total field.

Fig. 6.1 shows the experiment setup. Experiments are conducted inside an anechoic chamber that contains an automatic scanner and a baffled piston. The dimension of the anechoic chamber is 4.12 m x 4.75 m x 2.11 m. The interior surfaces of the chamber is covered with sound absorptive wedges made of fiber glass. The wedge size, 61.0 cm x 20.0 cm x 34.9 cm, provides the chamber with a cutoff frequency of 175 Hz. The absorptive lining minimizes the acoustic reflection and reverberation such that nearly free-field conditions can be achieved. The piston is a flat-topped honeycomb diaphragm base speaker with a 4 cm radius. This piston speaker simulates the plane piston source discussed in appendix A.

A computerized data acquisition and processing system is used for measuring the two-dimensional complex sound field. The system consists of a Masscomp computer workstation, an amplifier, a filter, a charge amplifier, a precision microphone and a 183 cm x 168 cm x 114 cm X-Y-Z scanner with a resolution of 5 x 10^-6 m that is driven by three stepping motors. In addition to the stepping motor controller, a stepping motor driver is needed due to the heavy load on the slides of the scanner. The protection of the scanner during motion is provided by limit switches and fuses. The scanner was mounted on four aluminum pipes and reinforced by eight steel cables.

All the functions of the integrated system were monitored and controlled through an instrumentation-oriented interface (IEEE-488 standard) by the Masscomp workstation. The Masscomp workstation executes the data acquisition procedure, and controls the movements of the scanner. The entire control cycle starts with activa-
Figure 6.1: Geometric configuration of the 3-D cartesian scanner
ting the scanner by sending positioning commands, i.e. motor speed, direction, and number of step from the computer to the stepping motor controller. As soon as the sensor reaches the desired position, the computer acquires the complex sound pressure data with the use of the one-microphone technique. Depending on the number of motor steps and data averaging, an appropriate time delay is included by the computer in order to insure that the data acquiring process is completed before the scanner is moved to the next position; which, is typical of open-loop control.

The amplifier and filter handle the modification of the input signals. The microphone probe and charge amplifier are the data collecting devices.

A 3" OD cast iron sphere is illuminated by the baffled piston speaker. A sinusoidal signal for the speaker is generated by the signal generator which is also used as the reference signal. The measured signals from the probe go into the charge amplifier and was amplified and filtered before being processed by the Masscomp workstation. It should be noted that the Masscomp workstation cannot measure the complex acoustic pressure directly. Consequently, the autospectrum of the reference signal and the frequency response function between the reference signal and the desired acoustic pressure were calculated by the computer. The complex acoustic pressure is then recovered from these two functions. All measured signals are measured with respect to this reference signal in order to obtain both the magnitude and phase of the acoustic pressure.

For the backscattering measurement, we are limited by the design of the scanner. One measurement can not get the whole total field because the vertical scanner will hit the scatterer when crossover. The alternative is to divide the whole plane into two half planes for measurements. These two half planes are then combined to get a
full total field.

6.2.2 Results

This section includes the results of the measurements. Only the cast iron sphere was used as the scatterer for the cartesian system experiment. We compare the separated scattered field at the same position as the first measurement plane. Fig. 6.2 shows the magnitude and phase of the total field on first measurement plane and Fig. 6.3 is for the second measurement plane. The input frequency is 4000 Hz and the distance between measurement points is 3 cm, and the measurement plane contains $32 \times 32$ points.

The first measurement plane is 1 cm away from the surface of the scatterer and the second measurement plane is 2 cm away from the scatterer surface (see Fig.4.1). Comparing the total fields at the two surfaces, there is a significant increase in the center of the field on the first measurement plane, which is from the scattered field. The second measurement surface does not have this significant increase. It is very important for the measurement surfaces to be in the nearfield in order to obtain such a difference.

Fig. 6.4 is the separated and measured incident fields and Fig. 6.5 shows the separated and subtracted scattered fields. The window used in real space is $apt=0.9$ and $slp=0.1$. The filter used in k-space is $apt=0.85$ and $slp=0.1$. In Fig. 6.4 there is a side lobe in the measured incident field which should not exist for a perfect surface piston speaker. This side lobe effect is removed because of the windowing and filtering. The results for the separation are not affected by the distortion of this incident wave because the separation technique is not effect by the kind of incident
Figure 6.2: The first measurement total field: Left: magnitude; Right: phase

Figure 6.3: The second measurement total field: Left: magnitude; Right: phase
wave that is present as long as they are consistent for all the measurements.

Fig. 6.5 shows the separated and subtracted scattered fields. These two fields are very comparable and proved the feasibility of the separation technique in the cartesian coordinate system. The peak magnitudes are not shown on the plots, because the filtering used in the separation technique distorts the results.

6.3 Cylindrical system

6.3.1 Experimental facilities

As shown in Fig. 6.6, the cylindrical scanner consists of the vertical scanner mounted on top of a turn table. This vertical scanner is 1.83 meters high and the turn table has a 48.3 cm radius. Both the turn table and the vertical scanner are driven by the stepping motors. The rest of the setup is the same as the cartesian system. The sound speed in air is 343 m/s and the distance between the speaker and the scatterer center is 96.2 cm. Four different scatterers are used. The dimensions of each scatterer are:

1. Cast iron spherical scatterer: 7.62 cm in diameter.

2. Hard plastic spherical scatterer (bowling ball): 21.6 cm in diameter.

3. Solid aluminum cylindrical scatterer: 8.26 cm diameter and 21.0 cm long.

4. Hollow aluminum cylindrical scatterer: 7.62 cm outer diameter and 33.2 cm long with a thickness of 15.9 mm.

For the scatterers with small cross sections (1,3,4), we used 4000Hz as the incident frequency. But for the bowling ball, we used 1200Hz as the incident frequency
Figure 6.4: The incident field: Left: separated; Right: measured

Figure 6.5: The scattered field: Left: separated; Right: subtracted
Figure 6.6: Geometric configuration of the cylindrical scanner
for this larger size in order to keep a lower $\frac{P_1}{P_2}$ ratio, as discussed in section 2.2.3.1.

Unlike the cartesian system, we can measure the sound pressure of the spherical scatterers (1,2) over the whole 2-D finite cylindrical measurement surfaces. But for the cylindrical scatterer (3,4), their lengths are too long and the scanner will block the source when moving in between the source and the scatterer. Similarly to the cartesian system, the measurement surfaces were divided into the upper and lower half surfaces. Then we combine these two half cylinders to get the total fields.

6.3.2 Results

The results are shown in Fig. 6.7 to Fig. 6.13. All the separated scattered fields are located at the first measurement plane. Each data set is compared with the results from subtraction method. These plots represent the cylindrical surface spread out into a cartesian plane. The central lobe along the phi axis is the backpropagation and the two side lobes should combine together to form the forward scattered field.

Case I: Cast iron spherical scatterer

The two measurement surfaces are 0.9 cm and 2.0 cm away from the surface of the scatterer. The distance between points along z-axis is 1.5 cm. The array size is 36 by 36 points. Windowing is used before and after the separation process in real space. The windowing parameters before the separation process are “apt”=0.6 and “slp”=0.23 and the parameters after the separation process are “apt”=0.8 and “slp”=0.1. The oval filter parameters used here are “apt_x”=0.58, “apt_m”=0.7 and “slp”=0.01.

Fig. 6.7 and Fig. 6.8 are the results for the cast iron spherical scatterer illuminated at 4000Hz. Fig. 6.7 shows the separated and subtracted scattered fields. These
two data set are very comparable. The backscattered lobe has more noise for the separated results. One reason is that the scanner interferes with the backscattered field when it moves inbetween the piston speaker and the scatterer. The subtraction technique seems to gave better results. The reason is because the subtraction technique does not need any special signal processing techniques such as windowing or filtering, and is not sensitive to the measurement noise.

Fig. 6.8 shows the theoretical scattered field with the same parameters and another subtracted scattered field with a 44 by 44 measurement array. Comparing the two subtracted fields, we can see that the measurement is repeatable. The 44 by 44 array has better resolution along the phi axis and gives a more flat backscattering. It is very comparable with the theoretical data.

**Case II: Hard plastic spherical scatterer**

The two measurement surfaces are 1.0 cm and 2.0 cm away from the surface of the scatterer. The distance between points along the z-axis is 2.5 cm. The array size is 36 by 36 points. Windowing is used before and after the separation process in real space. The windowing parameters before the process are “apt”=0.6 and “slp”=0.3 and the parameters after the process are “apt”=0.7 and “slp”=0.15. The oval filter parameters used here are “apf.”=0.25, “apfm”=0.6 and “slp”=0.1.

Fig. 6.9 and Fig. 6.10 are the results for the hard plastic spherical scatterer illuminated at 1200Hz. Fig. 6.9 shows the magnitude and 2-D contour of the subtracted scattered field. Fig. 6.10 shows the magnitude and 2-D contour of the separated scattered field. From the 2-D contour plots, we can notice the asymmetry along the Z-axis, for the subtracted scattered field. This imbalance is not significant. But for the separated scattered field, the distortion is significant. The reason for this
Figure 6.7: Scattered fields (cast iron sphere, 36x36): Left: separated; Right: subtraction

Figure 6.8: Scattered fields (cast iron sphere): Left: theory; Right: subtraction; 44x44
distortion might be that at this lower frequency, the field are not as directive as at high frequencies. In addition, the measured scattered field is more noisy than with other objects. It is believed that the noise is caused by the sound reflected from the turn table which interferes with the measurement. For the subtracted scattered field, the effect is not significant because the reflected field from the turn table is small compared with the real scattered field in the nearfield of the scatterer. But the separation results is very sensitive to the accuracy of the total field measurements. The noise is then enlarged by the modified Bessel function and directly effects the results of the separation technique. This effect from the turn table actually exists for all the cases, but it is not significant for the high frequency cases.

**Case III: Solid cylinder scatterer**

The two measurement surfaces are 1.0 cm and 2.0 cm away from the surface of the scatterer. The distance between points along z-axis is 2.0 cm. The array size is 56 points along z-axis by 36 points along phi-axis. Windowing in real space is used before and after the separation process. The windowing parameters before the separation process are “apt”=0.9 and “slp”=0.01 and the parameters after the separation process are “apt”=0.7 and “slp”=0.1. The oval filter parameters are “aptz”=0.4, “aptm”=0.8 and “slp”=0.01.

Fig. 6.11 and Fig. 6.12 are the results for the solid cylindrical scatterer illuminated at 4000Hz. Fig. 6.11 shows the comparison of the subtracted scattered field with the theoretical scattered field calculated from simulation programs. The simulated scattered field is generated from the prolate spheroid program but the edges of the cylinder shape are not as square as the solid cylinder. Consequently the simulated scattered field is smoother on the edge of the cylinder. Fig. 6.10 shows the
comparison of the separated scattered field from the experimental data with the separated scattered field from simulated data. The results are very comparable. In the measured scattered field, the edge effect after the separation are significant but they are removed by windowing. The reason for this edge effect was discussed in the sensitivity analyses.

**Case IV: Hollow cylindrical scatterer**

The two measurement surfaces are 1.0 cm and 2.0 cm away from the surface of the scatterer. The distance between points along z-axis is 2.0 cm. The array size is 56 points along z-axis by 36 points along phi-axis. Windowing is used in real space before and after the separation process. The windowing parameters before the process are “apt”=0.9 and “slp”=0.01 and the parameters after the process are “apt”=0.9 and “slp”=0.01. The oval filter parameters used here are “apt_z”=0.35, “apt_m”=0.65 and “slp”=0.01.

Fig. 6.13 shows the results for the hollow cylinder scatterer illuminated at 4000 Hz. The separated scattered field is compared with the subtracted scattered field. We can see that the separation result are not very good. The reasons is that the cylinder is too long for the scanner to cover the appropriate aperture size. As discussed in section 5.2, this is a very important factor. For the subtracted data, the scattered field has a more symmetric and smooth field without being affected by the aperture size. The reason is that the subtraction method does not require extensive numerical processing.
Figure 6.9: Subtracted scattered fields (bowling ball): Left: magnitude; Right: 2-D contour

Figure 6.10: Separated scattered fields (bowling ball): Left: magnitude; Right: 2-D contour
Figure 6.11: Scattered fields (solid cylinder): Left: subtracted; Right: theory

Figure 6.12: Separated scattered fields (solid cylinder): Left: experiment; Right: simulation
Figure 6.13: Scattered fields (hollow cylinder): Left: separated; Right: subtraction
7. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary and conclusions

A method to determine the acoustic fields scattered by an object under different incident wave conditions is developed. With the estimate of the scattered field, the acoustic scattering cross-section of the scatterer and the target strength of the scatterer can be calculated. The method is proven to be feasible for both the cartesian and the cylindrical coordinate system. The separation techniques have advantages over the conventional experimental methods for acquiring the target strength. The conventional methods are limited by reverberation, multipath sound and other farfield considerations. In this research, the data is acquired in the near-field so that many of the limitations of conventional methods are avoided. Furthermore, detailed spatial characteristics of fields can be recovered at any stage of the 2-D spatial Fourier transformation. Through the wave number domain decomposition of these two measurement planes the separated scattered field is propagated to the farfield to predict the target strength. This research proves that it is feasible to calculate the scattered field in the far-field within a tolerable error using data collected in the nearfield over a finite, cost effective aperture. The work presented in this research provides a tool for understanding the complex scattering cross-sections of different scatterers.
The separation technique

The decomposition method in the cartesian coordinate system is based on the principle that any wave form can be decomposed into plane-wave components by using a two dimensional spatial Fourier transform. By using the plane-wave propagator theory, the incident and scattered fields can be separated. With the use of windowing, filtering, and averaging around the radiation circle, the separation results are good. The key to the success of the cartesian coordinates system is that the propagator is exactly available and incident and scattered fields are in opposite directions. However the separation technique in the cartesian coordinate does not work well when the field incident from an oblique angle. So the cylindrical coordinate system is considered as an improvement. In the cylindrical coordinate system, the propagator is not as easily implemented as in the cartesian coordinate system. In the cylindrical coordinate system, the incident field contains both the incoming and outgoing waves while the scattered field is only outgoing waves. Thus the outgoing waves include both the incident and scattered waves. So the separation technique in cylindrical coordinates is different than in cartesian coordinates.

For the scattered field in the cylindrical coordinate system, the Green’s function for the k-space propagation is the Hankel function of the first kind. This propagator is applied to the scattered field of a spherical scatterer with a plane piston source, and the results proved to be correct and workable in the simulated and propagated fields. For the incident wave propagator in cylindrical coordinate system, the Green’s function for the k-space propagation is the Bessel function. Direct use of this Bessel function cannot provide a good propagation because of the finite aperture size. Therefore, the two plane propagation method is used to get the accurate propagation
of the incident field.

For the separation technique in the cylindrical coordinate system, the two plane separation method is used. This method measures the total fields at two nearfield surfaces. Each total field can be represented as the combination of the Hankel functions of the first and second kind. The two unknowns are the coefficients of these two Hankel functions. With two equations and two unknowns, we can solve the equation and get the two coefficients of the Hankel functions. The solutions of these two coefficients give the incoming and the outgoing waves which compose the total field in the separated surface. The outgoing wave includes components from both the scattered and the incident fields. The incoming wave includes only the incident field component. Because the incoming and outgoing waves component should be equal for the incident wave, the scattered field is separated out by removing the incident field outgoing wave.

**Finite aperture size**

For the cartesian coordinate system, two problems arise from the nature of the separation equations. The first is due to the finite aperture size of the measurement surfaces. Another problem is the singularity in the denominators of the separation equation which results in the singular points around the radiation circle for the separated fields.

Because of the finite measurement aperture size, problems of improper propagation and exponential increase of the noise and numerical errors arise. The result is that the waves are actually mistakenly propagated. In addition to improperly propagating the fields, the exponential functions for backpropagation propagates the evanescent wave in the nearfield and enlarges the error and noise generated from
computations or the experimental process. These problems make the separation unacceptable. To solve this problem, windowing and filtering are used in the real and wave-number domains, respectively.

The averaging method is used to remove the singularity of the denominators. This method averages the Green's functions and the denominator of the separating equation for points around the radiation circle. Results show that the separated field by using this averaging method is significantly improved.

For the cylindrical coordinate system, the finite aperture size measurement still causes the major inaccuracies in the separation technique. For an infinite aperture size, each cylindrical wave is a single traveling wave component in the k-space. But with the finite aperture size, it becomes the continuum of wave numbers. Similar to the cartesian coordinate system, some of the fields outside the radiation circle are wrongly magnified because of the sinc function overlapping and the propagation with an increasing exponential function. The noise and errors outside the radiation circle are enlarged.

Windowing and filtering

For the cartesian coordinate system, windowing is necessary because of the discontinuity at the edges of the finite measurement surface in real space. Filtering removes the enlargement of the noise and errors and the improper propagation in wave-number space. The filtering is applied after the fields are separated into the scattered and incident fields.

For the cylindrical coordinate system, windowing and filtering are used throughout the process. Windowing is only used along the Z-axis in real space because in the circumferential direction the field is always continuous so that no edge effect exists.
In k-space, an oval filter is used in the analyses.

Windowing and filtering are necessary in the separation process for obtaining good separation results.

**Sensitivity**

To understand the influence of the parameters in the separation process, two class of issues are considered. The first is to determine which parameters will affect the performance of the separation technique. The second issue is the influence that errors in the measurement setup has on the separation results.

The parameters that are considered are the inter-element distance in a measurement surface, the nondimensionalized wavenumber, the aperture size, and the separation distance between two measurement surfaces. From the analyses, the following conclusions are drawn for the cartesian coordinate system:

1. The error tends to decrease as the aperture size increase. The reason for this is that the edge effect decrease with increasing aperture size. The effect of a small aperture size can be improved by using the windowing technique.

2. The error of under-sampling is obvious and little is gained by spatially over-sampling the measurement plane. The inter-element spacing should be \( \lambda/2 \).

3. The separation technique works better for high wave-number (\( ka \)) case. With the aperture size twice the dimension of a scatterer, \( ka \) value above \( \pi \) results in good separation.

4. When the separation distance increases, the separation accuracy decreases. The reason is because the evanescent waves are not included when the measurement
surfaces moves far away from the scattering center. A separation distance of $3\lambda/16$ should be maintained.

For the cylindrical coordinate system, the conclusions from the cartesian coordinate sensitivity tests still hold for axial components (the Z-axis). The complexity of the separation technique in the cylindrical coordinate system comes from the propagators (Hankel functions). The value of the wavenumber $k$, the measurement surface radius $r$ and the inter-element distance between points along the axial axis are mixed together and effect the value of a Hankel function. So the sensitivities of the circumferential components cannot be analyzed separately and makes it impossible to provide general sensitivity guidelines. Simulations for each case should be used to establish appropriate measurement parameters.

For the second issue, the factors considered are the skew, shift, offset, horizontal translation and the measurement surface curvature. The considerations for the cartesian and cylindrical coordinate systems are different. For the cartesian coordinate system, three interesting cases are the skew, the measurement surface curvature and the finite aperture size. With skew the far-field pressure pattern is rotated with respect to the true model. For the finite aperture case, the larger the aperture size, the better fit between the true and the projected estimates. From the results of the measurement surface curvature, only within certain angular ranges it is possible to get a good comparison. This shows rather high sensitivity of the separation technique to the measurement surface curvature.

For the cylindrical coordinate system, the shift and translation does not bias the results significantly. But the skew and the offset have significant effects on the separation results. For the offset, the radius of the measurement surface is incorrectly
measured from. Because the propagators in the separation technique vary greatly as a function of the radius of the measurement surface, an accurate measurement of the surface radius can improve the separation results. For the skew, the bias comes from the edge effect after the separation. By using windowing after the separation, the results are improved significantly.

Results

For the simulated wave field, both a spherical scatterer illuminated by a plane piston source and a prolate spheroidal scatterer illuminated by a plane wave are used for separation. The separated results are comparable with the use of a window and filter.

For the experiment results, in the cartesian coordinate system, the field after separation is comparable for a cast iron spherical scatter illuminated by a plane piston source. The backscattering lobe has more noise for the separated scattering field. The reason is because the scanner interferes with the field when moving inbetween the piston speaker and the scatterer.

For the cylindrical measurements, four different scatterers are used. The separation results are comparable. But they are also affected by the sensitivity parameters such as the aperture size, the incident wave frequency, and the skew of the scatterer. Windowing and filtering are necessary for getting good results.

The subtraction technique seems to give better results. The reason is because the subtraction technique does not need any signal processing and is not sensitive to the measurement noise. But the separation technique does not need the scatterer to be removed during the measurement process.
Conclusions

From the above summary, we conclude the feasibility of this separation technique for obtaining the scattered field experimentally. The cartesian coordinate separation technique is less sensitive to the measurement errors but gives poor results when the incident field is at an oblique incident angle. The cylindrical coordinate separation technique is very sensitive to the measurement errors but can provide the separation for the field with an oblique incident angle. Windowing and filtering is necessary for the separation technique.

7.2 Suggestion for future research

The future work for the continuation of this research can be broken into two parts. The improvement of the separation technique and the application of this technique.

Through the analyses of numerical and experimental data, we concluded that the separation techniques are feasible. But the separation process is not yet perfect. First, in the numerical analyses, for the cartesian coordinate system, the problems generated from the singularity of the separation technique and the enlarged noise and error outside the radiation circle cause distortions in the separation results. In the cylindrical coordinate system, the overlapping of the wave number components can lead to mispropagating the fields resulting in separation results that are distorted. There is still plenty of room for improving this technique. A possible approach is to apply an adaptive filter technique [37] to replace the traditional windowing and filtering. Developing new optimization methods to remove the singularity is also a possible direction for future research. In the cylindrical separation technique, we use
the theoretical incident field coefficients to decompose the scattered field. But from the two plane incident wave propagation, we know that the numerical processing makes the coefficients unequal. A further investigation about what happens to each terms can help to improve the cylindrical separation technique.

Secondly the errors from the experimental data are significant. The backscattering fields measured are interfered with by the scanner cross section when the scanner moves between the scatterer and the plane piston source. A new design of the scanner with a smaller cross section should improve the measured data.

For the applications of this technique, the scattered field in the nearfield of a structure can be investigated. We notice that the subtraction method gives better scattered fields. The combination of the subtraction method and the near-field holography method should be able to give important data for analysis of structural acoustics. But for the case when the scatterer cannot be moved. The combination of the separation technique and NAH should also provide enough information for the structural acoustic analyses of complicated scatterers.

Another possible application is in the target strength signature identification. Because this technique allows parallel processing to obtain the target strength signature over a wide range of aspect angles from a single set of measurement data, a data base can be generated for the target strength signature. With the application of the artificial neural network [37] to this data base, a system of signature identification is possible.
APPENDIX A.  THE PLANE PISTON SOURCE SURROUNDED BY
A RIGID FLANGE

This appendix summarizes the calculation of the sound radiation from a plane piston source embedded in a rigid baffle. The velocity potential \( \Phi_P \) at a field point P for a plane piston source surrounded by an infinite rigid baffle is given by [15],

\[
\Phi_P = \frac{V_0}{2\pi} \int e^{-ikR} \frac{dS}{R}.
\]  

(A.1)

From Fig. A.1, we can see the distance \( R \) between point P and point \( P_i(r_1, \theta_1, \phi_1) \) on the radiating surface is given by:

\[
R^2 = r_2^2 + \rho^2 + \rho_1^2 - 2\rho \rho_1 \cos(\phi - \phi_1) = r^2 + r_1^2 - 2rr_1 \cos \Gamma.
\]

The term inside the integral of Eqn.(A.1) can be expanded in terms of Legendre polynomials as,

\[
\frac{e^{-ik(x^2 - 2xy \cos \theta + y^2)^{\frac{1}{2}}}}{(x^2 - 2xy \cos \theta + y^2)^{\frac{1}{2}}} = -ik \sum_{n=0}^{\infty} (2n + 1) j_n(kx) h_n^{(2)}(ky) P_n(\cos \theta).
\]  

(A.2)

Using this expression the velocity potential can be represented as,

\[
\Phi = \frac{-V_0i}{k} \sum_{l} (2l + 1) j_l(kr) P_l(\cos \theta) \int_{r_0}^{r_a} r_1 h_l^{(2)}(kr_1) P_l(\cos \theta_1) dr_1.
\]  

(A.3)

Here \( \theta_1 = \frac{\nu_1}{r_1} \) and \( r_a = (r_0^2 + a^2)^{\frac{1}{2}} \). Representing \( \int_{kr_0}^{kr_a} z h_l^{(2)}(z) P_l(\frac{kr}{z}) dz \) as a new function \( F_l \), the velocity potential can be written as,

\[
\Phi = \frac{-V_0i}{k} \sum_{l} (2l + 1) j_l(kr) P_l(\cos \theta) F_l.
\]  

(A.4)
This expression states that the velocity potential is represented as the summation of spherical waves. The spatial part of the acoustic pressure can be expressed as:

\[ P = ik\rho_0c_0\Phi = \rho_0c_0V_0 \sum_l (2l + 1) j_l(kr) P_l(\cos \theta) P_l. \]  

(A.5)
APPENDIX B. HELMHOLTZ INTEGRAL EQUATION

The sensitivity results are compared in the farfield by using the projected field from the Helmholtz Integral Equation. This appendix describes the H.I.E. used in both the cartesian and cylindrical coordinate system.

B.1 Cartesian coordinate system

The approximation to the Helmholtz Integral Equation over a planar surface is given by [35] (Fig. B.1):

\[ \hat{p}(r) = \sum_{n=1}^{N} p(r') G_p(r, r'). \]  
\[ (B.1) \]

Where

\[ G_p(r, r') = j \cos \theta (1 - \frac{j}{kR} \frac{e^{-jkR}}{\lambda R} \Delta S), \]  
\[ (B.2) \]

\[ \Delta S = |R| \] is the length of \( R \) vector,
\[ R = |R| \] is the length of \( R \) vector,
\[ \theta \] is the angle from the outward surface normal \( n(\text{at } r') \) to \( R \),
\[ \Delta S \] is the surface area element at each point, which is equal the total area divided by the number of total array elements.
B.1.1 Comparison between true model and projected estimate

Two kinds of farfield projected estimate comparisons are used in the sensitivity tests. The first comparison is between the scattered on-axis true value with the projected estimate from the separated scattered field using the Helmholtz Integral Equation. The second comparison is between the far-field true value with the projected estimate at the far-field from H.I.E. for different angles: the directivity pattern.

The on-axis true value versus the projected estimate

The far-field, on-axis projected pressure level is defined at a projected distance of one thousand times the mean radius \((r)\) of the scatterer from the measurement plane. Fig. B.2 gives an example of the behavior for the forward projected levels as a function of projected distance. The true pressure is plotted on the same axis. The nominal predicted pressure converges to the true model as range increases with the
Figure B.2: Comparison: true model vs. projected estimate (H.I.E.)

The far-field true value versus the projected estimate as a function of angle (directivity pattern)

This comparison involves the far-field projected estimate with the true value at different angles with respect to an axis of the scatterer (Fig. B.3a). The angle ranges from 0 to 180 degrees with a 2 degrees resolution. The distance is 1000r from the scatterer and the angles are defined with respect to the negative z-axis. Fig. B.3b shows an example of this comparison. At the two extreme ends of this plot, the projected estimate does not match well with the true model, as expected, due to the finite aperture size used in the H.I.E. estimate.
Figure B.3: The far-field true vs. the projected estimate at different angles: a) geometry; b) comparison
B.2 Cylindrical coordinate system

The approximation to the Helmholtz Integral Equation over a cylindrical surface is given by (Fig. B.4):

\[ \hat{\mathbf{p}}(r) \approx - \sum_{n=1}^{N} \left\{ j \omega p \mathbf{u}(r'_n) - \cos \theta_n (j k + \frac{1}{R_n}) p(r'_n) \right\} \frac{e^{-jkR_n}}{4\pi R_n} \Delta S. \] (B.3)

Where

- \( \mathbf{R} = r - r'_n \),
- \( R = |\mathbf{R}| \) is the length of \( \mathbf{R} \) vector,
- \( \theta \) is the angle from the outward surface normal \( \mathbf{n}(\mathbf{r'}_n) \) to \( \mathbf{R} \),
- \( \Delta S \) is the surface area element at each point.

For the sensitivity test in the cylindrical coordinate system, only the on-axis true value is compared with the projected estimate. Fig. B.5 shows an example.
Figure B.4: Geometry used in H.I.E.: cylindrical measurement surface S

Figure B.5: True pressure compared with the projected estimate
BIBLIOGRAPHY


