1994

Dynamics and vibrations of magnetostrictive transducers

David L. Hall
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Dynamics and vibrations of magnetostrictive transducers

Hall, David L., Ph.D.
Iowa State University, 1994

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Dynamics and vibrations of magnetostrictive transducers

by

David L. Hall

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Aerospace Engineering and Engineering Mechanics
Major: Engineering Mechanics

Approved:
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In Charge of Major Work
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For the Major Department
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1994
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1. INTRODUCTION

Welcome to an investigation of Terfenol-D dynamic performance from an engineer’s perspective, i.e., "How can this material be used?" Terfenol-D is magnetostrictive, it strains in response to applied magnetic fields. Exploiting this behavior is the engineer’s job.

There is a considerable volume of literature on the physics and materials engineering of the magnetostrictive rare-earth alloys (of which Terfenol-D is but one). Reference [1] offers an authoritative overview of magnetostriction including quantum mechanics, anisotropies, rhombohedral distortions, neutron diffraction data, etc. Not much of that sort of information will appear in this dissertation. The problem at hand is to determine what the transducer designer must know in order to formulate a reasonable prediction of a Terfenol-D transducer’s dynamic behavior.

In this section of the dissertation the reader will be introduced to Terfenol-D, some (not all) of its foibles, two Terfenol-D transducer designs will be presented, general dynamic design issues discussed, and sign conventions will be introduced.

1.1 Terfenol-D a Giant Magnetostrictive Material

Magnetostriction is a variation in size of a material when it is subjected to a magnetic field. This phenomenon was discovered first in nickel. It was later found to occur in iron, cobalt, their alloys, and a number of other elements. Typical magnetostriction of these materials is limited to the order of 50 x10^-6 meters/meter. Terfenol-D is Tb_xDy_{1-x}Fe_{2+y} an alloy of terbium and dysprosium with iron. The "nol" in Terfenol stands for the Naval Ordinance Laboratory where the work began which resulted in Terfenol. Terfenol-D, as available, is a proprietary material of the ETREMA Products Division of EDGE TECHNOLOGIES, INC., Ames, Iowa. The "giant" term comes from comparing the magnetostriction of Terfenol-D with that of the materials previously mentioned. Terfenol-D exhibits strains on the order of 1000 x10^-6 m/m at room temperatures.
A "person on the street" explanation of what is occurring within the material is that the "oblong" magnetic domains (within the material) rotate from their previous orientation to align with an applied magnetic field. As a result, the material shrinks in one direction and grows in the other. Increasing the applied field increases the number of domains that are aligned in its direction, thereby increasing the strain. This continues until, for engineering purposes, all of the domains are aligned with the field and the strain approaches its maximum. In this state the material is said to be saturated. Removing the magnetic field results in the material returning (almost) to its original dimensions. Reversing the field orientation, unfortunately, does not reverse the direction of strain. The material again shrinks in one direction (the same as before) and grows in the other.

Figure 1.1 illustrates the aforementioned behavior. If one were to begin with a non-magnetized piece of Terfenol-D the curve would begin at (0,0). Increasing the magnetic field in magnitude until saturation would generate line 1. Decreasing the field would then result in line 2. Reversing the field would return the material to zero strain and then

![Diagram of applied magnetic field versus Terfenol-D strain](image)

Figure 1.1. Sketch of applied magnetic field versus Terfenol-D strain, hysteresis effects exaggerated for clarity
make line 3. Returning to zero applied field would generate line 4. Reversing again would take it back to zero strain then to saturation via line 5.

If the material were to exhibit a negative strain it would have generated the dashed lines instead of lines 3 and 4. That would then be a standard-looking hysteresis loop.

Notice the residual strain when the applied field returns to zero (line 2 or 4). This is due to magnetic hysteresis of the material, i.e., removing the applied field does not reduce the magnetic flux density within the material to zero. Different materials retain different flux densities upon removal of an applied field. Materials suitable for use as "permanent" magnets are chosen precisely because they retain relatively large flux densities. In the case of Terfenol-D, this hysteresis can be altered by changing the stoichiometry. For example, it is known that \( \text{Tb}_{0.27}\text{Dy}_{0.73}\text{Fe}_{1.97} \) exhibits less hysteresis than \( \text{Tb}_{0.3}\text{Dy}_{0.7}\text{Fe}_{1.98} \). The point is, that to a certain extent, the material can be "tuned" to an application. [2]

The "trick" in manufacturing this material is in growing the crystals so that when the material is solidified the magnetic domains are oriented correctly. That is, the domains are oriented so that the rod responds with larger net strains when it is subjected to a magnetic field. It turns out that one popular geometry used is to manufacture the Terfenol-D in rod form with the domains oriented approximately perpendicular to the rod's axis. Application of a magnetic field along the rod axis then rotates the domains, resulting in the rod getting longer (and smaller in diameter). When in rod form, a magnetic field of the correct orientation can be approximated by using wound wire solenoids.

A compressive prestress can be used to "increase" the net magnetostriction of Terfenol-D in rod form. Actually the prestress results in a prestrain which tends to rotate more domains away from an axial orientation. If one takes the compressive prestrained condition as a starting point, the displacement from there to saturation is larger than for the same rod with no prestrain. Figure
1.2 demonstrates the concept. Rods 1 and 2 represent a before and after (magnetic field application) for the no prestrain condition. Notice rod 3 is shorter than rod 1, due to the compressive prestrain. Notice also that rods 2 and 4 are almost the same length. Thus the prestrained condition allows a larger net end displacement to be realized.

Referring back to Figure 1.1, and imagining a rod/solenoid setup, it can be seen that a sine wave input current would result in, at best (recall hysteresis), a rectified sine wave displacement. Recall that reversing the current, and thus the magnetic field orientation, results in the rod lengthening again. This rectifying tendency can be avoided by providing a constant magnetic field to the material, which in effect moves "zero." See Figure 1.3.

The offset field, $H_0$, may be provided by the addition of either a DC component to the applied current in the solenoid or a permanent magnet somewhere in the magnetic circuit. Assuming provisions are made for $H_0$, the material can then be operated in what is rather
loosely termed the "linear region." (Of course, hysteresis will still exist within this region.) In this case, a sine wave input current yields approximately a sine wave output displacement.

The physical properties of Terfenol-D warrant discussion. To begin with, the material can only be described as a very "brittle" metal. Misalignment of transducer parts can easily result in a chipped or destroyed rod. A dropped rod is usually swept up. If the breaks are roughly perpendicular to the axis of the rod, one might glue the pieces back together and have the rod work just fine, assuming the following is observed: *Terfenol-D must be run in an overall state of compressive stress* (broken or not). In addition, threading, drilling, soldering, and welding should be avoided. One working with Terfenol-D should also know (in advance) that its filings are flammable. Wet grinding seems to be the machining process of preference. Terfenol-D is like aluminum in that its surface is actually an oxidized layer of the material. Normal moisture levels do not seem to result in excessive "rusting."
When dealing with giant magnetostrictive materials, one can safely say that everything is a function of everything. Of particular significance to the engineer is that the elastic modulus of Terfenol-D depends on physical and magnetic parameters. Reference [1] discusses this at length. Suffice it to say that every effort should be made to employ numerical values representative of the conditions of intended transducer operation when trying to predict transducer performance. Magnetic permeability of Terfenol-D is also a function of physical and magnetic parameters. In addition, it will be shown later that the permeability is a function of frequency of transducer operation, applied loads, and applied field strength. The importance of determining realistic "material properties" for Terfenol-D cannot be overstated.

The models developed in this study will be shown to provide reasonable approximations of experimental measurements of transducer performance. However, these will be but simulations. They will not be predictions. In order to predict transducer performance, one would need to know material properties of the Terfenol-D rod to a "high" precision, i.e., ±2 or 3 percent, before building the transducer. Attempts at quantifying material properties over the range of operation defined in this study, were unsuccessful. Variations in material properties from one test to the next (with everything that could be held constant, held constant) were usually less than about ten percent. When tests were performed at one drive level, then compared with tests at different drive levels, variations of over 30% were observed. When two rods of Terfenol-D which had been cut from the same piece of material were tested, under nearly identical conditions, their squared magnetomechanical coupling coefficient varied by about 30% (k² = 0.154 to 0.225). These types of changes did not represent a hardship in simulating transducer performance since parameters were measured for each individual simulation. However, these variations in material "constants" seriously hamper any attempts at predicting transducer performance. For the models developed here, good numbers
in mean good numbers out, equivalently: "garbage in, garbage out."
Please be advised.

1.2 Example Dynamic Transducer Designs

Figure 1.4 depicts a dynamic transducer employing Terfenol-D as the motion source. It was designed for possible use in the National Aeronautics and Space Administration's, NASA's, SELENE project (an active mirror array composed of hexagonal mirrors). There are to be three actuators for each mirror segment. Physical space constraints controlled component location. Important components of the transducer are numbered in the figure, tabulated in Table 1.1, and discussed below.

The Terfenol-D rod (8) is magnetically biased to $H_o$ via the field strength from the permanent magnet (9) directed to the rod by the high magnetic permeabilities of the steel top and bottom pieces. Thus, the DC magnetic circuit consists of components (9)-(10)-(8)-(3)-(2)-(1)-(9). An AC electric current in solenoid (7) results in an oscillating magnetic field which adds to, or subtracts from the field from the permanent magnet. In response to the changing magnetic field, the rod length changes, resulting in bidirectional motion of component (3), the motion output.

The rod (8) is maintained in a state of compressive stress by the spring washer (5). Prestress adjustment is performed by placing shims between the rod (8) and the steel bottom (10). The nonmagnetic stainless steel sleeve (6) carries tensile stress, aligns components, and is a spool for the wound wire solenoid (7). It is slit longitudinally to prevent eddy current shielding of the rod.

A diaphragm was included in this design to provide lateral support to the motion output. Only one diaphragm was incorporated because, in this application, externally applied moments to component (3) will be "small." The transducer will be coupled to the mirror via flexures. Some sort of bushing or double diaphragms can be used if the
Figure 1.4. Top and section view of Terfenol-D transducer designed for possible use in an active mirror project. Components are described in Table 1.1.
Table 1.1. Component description for Terfenol-D transducer shown in Figure 1.4

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1020 steel top</td>
</tr>
<tr>
<td>2</td>
<td>1045 steel (flame hardened) spring seat</td>
</tr>
<tr>
<td>3</td>
<td>1045 steel (flame hardened) motion output</td>
</tr>
<tr>
<td>4</td>
<td>steel diaphragm for lateral location of motion output</td>
</tr>
<tr>
<td>5</td>
<td>Belleville steel spring washer</td>
</tr>
<tr>
<td>6</td>
<td>303 stainless steel tube assembly</td>
</tr>
<tr>
<td>7</td>
<td>wound wire solenoid</td>
</tr>
<tr>
<td>8</td>
<td>Terfenol-D rod</td>
</tr>
<tr>
<td>9</td>
<td>permanent magnet</td>
</tr>
<tr>
<td>10</td>
<td>1020 steel bottom</td>
</tr>
</tbody>
</table>

the application requires them. However, if a bushing is used, the fit is critical. Very small motions at high frequencies can introduce appreciable deviations from the transducer's anticipated behavior.

Figure 1.5 shows another transducer design. The external geometry of this design was chosen to suit its intended use as a general laboratory vibration source. Note the use of a bronze bushing (b) to support any external moments applied to the motion output (a). The constant magnetic field is provided to the Terfenol-D rod by the cylindrical permanent magnet, (f). An AC electric current provided to the wound wire solenoid (g) results in an oscillating magnetic field along the longitudinal axis of the transducer. The result is bidirectional motion of (a) relative to the housing.

The path for the magnetic flux is (f)-(k)-(i)-(h)-(a)-(c)-(f). The housing components, (d) and (j), are made of aluminum since it is approximately magnetically "neutral," readily available, and low in mass. The components along the flux path are made of "high" permeability, fully annealed 1020 steel. In addition, the design is such that the cylindrical air gap in the magnetic circuit - between (c) and (a) - remains constant (radial clearance of approximately 0.010") through the entire strain cycle of the rod.
This transducer design also incorporates a very simple prestress adjusting scheme when compared to the shimming procedure of the first transducer.

Both designs introduced above feature low dynamic masses; the physical size of the motion output components have been minimized. Low internal masses translate to higher natural frequencies, thus increasing the bandwidths of the transducers. Low internal masses also help increase potential transducer output levels; less of the rod's capability is expended internally. The "small" parts are also less likely to introduce spurious resonant effects than, for example, a
transducer which employs a motion output component that extends over to the permanent magnet.

Internal damping of these transducers seems to be dominated by the damping in the Terfenol-D rod itself (2.5-4%). However, careful attention must be paid to other design aspects. Recall that Terfenol-D offers approximately \( \pm 500 \times 10^{-6} \text{ m/m strain} \). Thus, for a two-inch long rod, possible displacements are approximately \( \pm 0.001" \). It does not take much play in some component in the system to reduce transducer output to zero, for example, the play in threaded connections. These losses can be reduced by using jam nuts, greasing threads, employing flexures, etc.

The dynamics of the transducer housing must also be considered. For example, the transducer shown in Figure 1.5 exhibited several translational and rocking housing-rod modes between 2500 and 3800 Hz, including well defined structural modes with the upper housing and rod motion out of phase near both 2700 Hz and 3600 Hz. It was felt that the symmetry of a four-bolt assembly contributed to these modes of vibration. The transducer also displayed an axial resonance of the housing top near 9000 Hz. All of these resonances were in the frequency range of interest and reduced the predictability of the transducer's output. A five-bolt assembly was later built to remedy the symmetry problem and to raise the frequency of axial resonance.

1.3 Sign Conventions

The transducer will be considered a single-input, single-output system. The system input is electric current; the output is displacement, velocity, acceleration, or force. All of these are vector quantities. Most of these transducers have cylindrical geometries which suggests the use of \( r, \theta, z \) coordinates, as shown in Figure 1.6. The transducers usually consist of a rod of Terfenol-D, surrounded by a cylindrical wound wire electric solenoid, which in turn is surrounded by a cylindrical housing (made from, for example, cast Alnico V, a permanent magnet material). Typically, the origin of the coordinate system will be considered as attached to the "fixed" end of the
Terfenol-D rod. The opposite end of the rod will be the one that moves or provides a force. A positive output displacement, velocity, acceleration, and force would be directed along the axis of the transducer, in the positive z direction. A positive current would be that which causes a positive magnetic flux and results in a positive displacement of a biased rod at low frequencies (the Terfenol-D rod gets longer).

Figure 1.6. Schematic of input-output relationship and coordinate system used in analysis

To understand the dynamic behavior of magnetostrictive transducers it is necessary to investigate the general topics of transduction and electromagnetism. These concepts will be applied to the transducers under study assuming linear-systems behavior (linear in the least squares sense). Analytical electromagnetic-magnetomechanical models will be developed for predictions of transducer behavior, i.e., electrical impedance functions, displacement from current frequency response functions, FRF's, etc. Model simulations will then be compared with experimental measurements of transducer performance.
2. OBJECTIVES

1. Develop new analytical models for prediction of transducer output force, acceleration, velocity, or displacement as a function of Terfenol-D material properties, rod size, transducer geometry, input electric current, load, and frequency of operation.

2. Experimentally verify analytical models.
3. OVERVIEW OF TRANSDUCTION APPROACH AND BACKGROUND

This chapter of the dissertation will discuss mechanical systems, the transduction phenomenon, how it is measured, modelled, some of its ramifications, and how it will be pursued in this dissertation.

3.1 Mechanical Systems

Consider a simple, forced, spring-mass-damper, one-degree of freedom, underdamped system. The equation of motion for the system is given as:

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = F_0 e^{i \omega t}$$

where:
- $m$ is the effective dynamic mass of the system,
- $x$ is the degree of freedom (the output displacement),
- $b$ is the viscous damping parameter,
- $k$ is the effective linear stiffness,
- $F_0$ is the magnitude of the sinusoidal forcing function (the input),
- $j = \sqrt{-1}$,
- $\omega$ is the circular drive frequency, and
- $t$ is time.

Assuming $x(t)$ is given as, $x(t) = X e^{i \omega t}$, where $X$ is a complex valued function of frequency and drive level, the differential equation of motion is written in the standard, steady-state, impedance form:

$$\left(-m\omega^2 + j\omega b + k\right)X = z_m X = F_0$$

where the time dependence has cancelled, and the parenthetical quantity is $z_m$, the mechanical impedance of the system based on displacement. If the mechanical impedance based on velocity is sought, simply factor a $j \omega$ from the impedance shown. Specifically, impedance as force per displacement is:

$$z_m = \frac{F_0}{X} = \left(-m\omega^2 + j\omega b + k\right).$$

Impedance as force per velocity, $v$, is:
A relationship for the system output (displacement) from system input (a forcing function) is sought. Typically, the differential equation of motion evolves as follows: divide by \( m \),

\[
\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = \frac{F_0}{m} e^{j \omega t}
\]

define standard variables,

\[
\frac{d^2 x}{dt^2} + 2 \zeta \omega_n \frac{dx}{dt} + \omega_n^2 x = \frac{F_0}{m} e^{j \omega t}
\]

employ the assumed time variation of \( x \),

\[
(-\omega^2 + j 2 \zeta \omega \omega_n + \omega_n^2) x e^{j \omega t} = \frac{F_0}{m} e^{j \omega t}
\]

and divide by \( \omega_n^2 \),

\[
\left(1 - \left(\frac{\omega}{\omega_n}\right)^2 + j 2 \zeta \frac{\omega}{\omega_n}\right) x = \frac{F_0}{k}
\]

Finally, rearrange for the familiar dimensionless dynamic displacement function, aka magnification factor,

\[
\frac{X}{\frac{F_0}{k}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j 2 \zeta \frac{\omega}{\omega_n}}
\]

\( X \) is the complex valued frequency dependent dynamic displacement.

\( F_o/k \) would be the static displacement of the system if it were subjected to a constant force of magnitude \( F_0 \). Eqn. (3.1.4) is a complex valued function, thus it has a magnitude and a phase. (Recall the polar notation of a complex number: if \( z = a + jb \), \( z \) can also be written as \( z = |z| e^{j \phi} \), where the magnitude is given as \( |z| = (z \ z^*)^{1/2} = (a^2 + b^2)^{1/2} \) and the phase is \( \tan(\phi) = (b/a) \).) Noting that \( f = \omega/2\pi \), Eqn. (3.1.4) could also have been written in terms of frequency, as opposed to the circular frequency \( (f/f_n = \omega/\omega_n) \). With the previous discussion in mind, the magnitude and phase of \( X \) are given as:
\[ ||X|| = \frac{F_0/k}{\sqrt{1 - \left(\frac{f}{f_n}\right)^2 + 2\zeta \frac{f}{f_n}}} \]  
\hfill (3.1.5a)

and,
\[ \phi = \tan^{-1}\left(\frac{-2\zeta f/f_n}{1 - (f/f_n)^2}\right) \]  
\hfill (3.1.5b)

where the minus sign crept in because the complex number was in the denominator.

If the system is forced by a function in time which varies like a cosine, \( x(f, t) \) would be given as:
\[ x(f, t) = \text{Real}\left(\frac{X \frac{F_0e^{j2\pi ft}}{k}}{\frac{||X||e^{j(2\pi ft + \phi)}}{k}}\right) \]  
or, simply
\[ x(f, t) = ||X||\cos(2\pi ft + \phi). \]  
\hfill (3.1.6)

If it had been forced by a sine function, the imaginary portion would have been required; in that case, the cosine in Eqn. (3.1.6) would be replaced by a sine.

Figure 3.1 is a plot of system input and responses versus a dimensionless time scale, \( ft \), for three different frequency ratios, \( f/f_n \). The choice of this time scale allows one to easily compare response from input at three different frequencies. For the plot it was assumed that \( F_0/k = 1 \) and \( \zeta = 0.15 \). The amplitude variations in the response traces are due to the frequency dependent magnitude of Eqn. (3.1.4). The relative shifts in time between the forcing function and the responses are due to the frequency dependent phase of Eqn. (3.1.4). For the low frequency ratio, \( f/f_n = 1/3 \), the response is magnified and shifted in time only slightly when compared to the forcing function. Operation in this frequency range corresponds to the stiffness controlled range of system behavior. For the trace of \( f/f_n = 1/1 \), the output has been shifted in time (approximately one-quarter of a cycle) and magnified significantly. This trace is typical of system behavior.
when driven at frequencies near the frequency of mechanical resonance. The behavior of magnitude and phase depend primarily on the system damping in this range of operating frequencies. For $f/f_n = \frac{5}{3}$ the output magnitude is reduced and the trace is shifted in time nearly one-half cycle. Operation in this frequency range corresponds to the mass controlled range of system behavior.

Figure 3.2 is the same data as Figure 3.1, but plotted as output versus input (aka, a Lissajous plot). Note that each set of data forms an ellipse. The implication of this is that for steady-state operation, any relationship between two variables which forms an ellipse can be modelled as a complex number, i.e., a magnitude and a phase. This approximation can be used when modelling the effects of magnetic hysteresis.

![Figure 3.1](image)

Figure 3.1. System input, $F(\text{ft})/k = 1$, and system response, $x(\text{ft})$, as described by Eqn. (3.1.4), versus dimensionless time, $ft$, for frequency ratios of $1/3$, $1$, and $5/3$.
If one has an elliptical relationship between two variable, say B and H, the parameters for the functional relationships \( B(t) = B_0 \cos(2\pi ft + \phi) \), and \( H(t) = H_0 \cos(2\pi ft) \), can be estimated as follows. The magnitudes, \( B_0 \) and \( H_0 \), are simply the respective maximums. The relative phase, \( \phi \), can be estimated via

\[
\phi = \sin^{-1}(\frac{-d}{B_0}),
\]

where \( \pm d \) is the value of \( B(t) \) when \( H(t) \) is zero (the trace crosses the ordinate at \( \pm d \)). This relationship can be derived by expanding the cosine term in \( B(t) \) and it holds when the major axis of the ellipse is in quadrants 1 and 3. If the major axis is in quadrants 2 and 4, owing to the periodicity of the arcsine function, the phase should be calculated as:

\[
\phi = -\pi - \sin^{-1}(\frac{-d}{B_0}).
\]

In the form of a complex number, \( B \) from \( H \) would be: \( \frac{B}{H} = \left( \frac{B_0}{H_0} \right) e^{i\phi} \).

Figure 3.2. System response, \( x(t) = ||X|| \cos(2\pi ft + \phi) \), versus input, \( (F_0/k)\cos(2\pi ft) \). Increasing time corresponds to traveling around the ellipses in the counterclockwise direction.
3.2 Linear Transduction of Terfenol-D

Transduction is the conversion of one form of energy to another. For the transducers under study, electrical, magnetic, and mechanical energies are being converted.

The transduction phenomenon is typically modelled mathematically as a pair of linear simultaneous equations.[3, 1] For the transducing material (as opposed to a transducer containing the material, which is addressed in Section 3.3 below) the low signal linear transduction equations are given as:[1]

\[
\begin{align*}
    \varepsilon &= \frac{\sigma}{E_y^H} + qH \\
    B &= q\sigma + \mu^\sigma H
\end{align*}
\]  

(3.2.1a)

(3.2.1b)

where: \( \varepsilon \) is the mechanical strain of the material, 
\( \sigma \) is the mechanical stress in the material (positive is tensile, negative is compressive), 
\( E_y^H \) is the "Young's modulus" for the material at constant applied magnetic field strength, \( H \), 
\( q \) is the linear coupling coefficient (aka the "d constant"), 
\( H \) is the applied magnetic field strength, 
\( B \) is the magnetic flux density within the material, and 
\( \mu^\sigma \) is the magnetic permeability of the material at constant stress.

It should be noted that these are scalar equations, usually employed in the time domain. All of the quantities are those applicable in the longitudinal direction of the rod, i.e., the strain is the longitudinal strain as a function of position and time, \( \varepsilon_{33}(r, \theta, z, t, \sigma, H, B) \). The dependence on position and time is understood. The equations themselves represent the functional dependence on the other variables. Dynamic effects are completely neglected (as shown later in this chapter).

The magnetostrictive phenomenon is known to be very nonlinear[6]; however, the linear transduction equations are thought to provide
reasonable approximations over the "low signal" range of material operation. They are a linearized approximation of material behavior.

Recall that Terfenol-D needs a compressive prestress, \( \sigma_0 \), and that a magnetic bias, \( H_0 \), is typically employed in order to obtain bidirectional output motion. Equations (3.2.1) can be used to approximate either the overall states of stress, strain, etc., or just the time varying states. In the latter case, the overall state can be obtained by adding in the DC components of the variables. This can be seen by eliminating stress in the equations, yielding

\[
\varepsilon = \frac{B}{qE_y} + \left( q - \frac{\mu^\sigma}{qE_y^H} \right) H.
\]

Next, consider each state variable to be the sum of a DC and an oscillating component, e.g., \( B = B_0 + B' \) (an assumption consistent with linear systems theory), thus:

\[
\varepsilon_0 + \varepsilon' = \frac{B_0 + B'}{qE_y^H} + \left( q - \frac{\mu^\sigma}{qE_y^H} \right) (H_0 + H').
\]

Recognize that

\[
\varepsilon_0 = \frac{B_0}{qE_y^H} + \left( q - \frac{\mu^\sigma}{qE_y^H} \right) (H_0)
\]

and subtract it from both sides of the previous equation, yielding

\[
\varepsilon' = \frac{B'}{qE_y^H} + \left( q - \frac{\mu^\sigma}{qE_y^H} \right) (H').
\]

This is exactly the equation that would be obtained if Eqns. (3.2.1) had represented only the alternating components and the alternating stress had been eliminated. Thus, one can use Eqns. (3.2.1) for the overall, or simply the alternating states.

In theory, if the material properties for the rod of Terfenol-D are known, i.e., \( E_y^H \), \( q \), and \( \mu^\sigma \), along with two of the four state variables, the other two state variables can be calculated. In practice, things seem to be a little more complicated. For example, \( E_y^H \), \( q \), and \( \mu^\sigma \), all
depend on the material's state of mechanical stress along with its magnetic state and temperature.\[4, 5\] Therefore, it is imperative to know the operating state and the corresponding material properties if there is to be any hope of mathematically modelling material behavior. Published, or "nominal" values are nearly useless for simulations. Experience has shown large variations in material properties from sample to sample.

3.2.1 Discussion of variable material properties

Those steeped in transducer lore are familiar with variable material properties. The following discussion is meant for those who may need some introduction and justification for material properties that vary.

Figure 3.3 is a schematic representation of magnetostrictive "elements" for various applied field strengths. The magnetostrictive phenomenon is represented by the rotation of the elliptical bar magnets in the base of each element. Since the magnet has its own magnetic field, it will rotate in an attempt to align with any applied fields. The change in length per unit length of each element would represent the strain in the material. A stress would be a force per unit area applied to the tops and bottoms of each element (not shown). The stiffness of the non-magnetostrictive material "matrix" (in the sense of composite materials) is represented by the linear spring. The element should not be taken too literally. It was designed simply to illustrate, via simple principles from physics, some of Terfenol-D's observed behaviors.

Note first that an applied field strength, $H$, tends to rotate the bar magnet from its biased state. If the mechanical stress is held constant, the strain of the element would vary, to a first approximation, like $qH$. (Recall Eqn. (3.2.1a).) Thus, $q$ is a local slope of a plot of Terfenol-D strain from applied field for a given constant stress. If the constant stress is made more negative, the resulting $q$ would be smaller. Moving the same distance against a larger force requires more work, i.e., a larger $H$ is required.\[7\]
Consider $B$ versus $H$. As shown in Eqn. (3.2.1b), it is assumed that the material has a property called "magnetic permeability," and that the permeability is a function of at least mechanical stress. In the literature one usually finds two permeabilities for the material, one for constant stress, $\mu_\sigma$, and another for constant strain, $\mu_\epsilon$. Consider what would happen to the axial flux density, $B$, if the field strength were increased with the stress held constant. The bar magnet would rotate due to the increased $H$. The axial $B$ field would increase due to the increase in $H$. In addition, the intrinsic $B$ field of the bar magnet would contribute a component to the measured axial $B$ field. Contrast that with the constant strain case where the bar magnet is not allowed to rotate and makes no significant contribution to the measured $B$ field. Thus, in the context of $B = \mu H$, the material would appear to have, and actually does have, a higher magnetic permeability.
when operated at constant stress than when operated at constant strain, i.e., $\mu^\sigma > \mu^\varepsilon$.

More can be said about the relationship between these two magnetic permeabilities. If Eqn. (3.2.1a) is solved for stress, then that relation is substituted into Eqn. (3.2.1b), in effect eliminating stress from the formulation, one obtains the following:

$$\sigma = E^H_y \varepsilon - q E^H_y H \quad \text{and} \quad B = \sigma \sigma + \mu^\sigma H \quad \Rightarrow \quad B = q E^H_y \varepsilon - q^2 E^H_y H + \mu^\sigma H$$

or

$$B = q E^H_y \varepsilon + \mu^\sigma \left(1 - \frac{q^2 E^H_y}{\mu^\sigma}\right) H = q E^H_y \varepsilon + \mu^\sigma \left(1 - k^2\right) H.$$

When the strain is constant, the variable strain component is zero, thus

$$B = \mu^\sigma \left(1 - k^2\right) H = \mu^\varepsilon H$$

or simply,

$$\mu^\varepsilon = \mu^\sigma \left(1 - k^2\right) \quad (3.2.2)$$

where the dimensionless variable $k^2$ is defined as

$$k^2 = \frac{q^2 E^H_y}{\mu^\sigma} = \frac{\sigma^2}{s^H \mu} \quad (3.2.3)$$

using variables defined above, followed by variables more familiar to physicists and transducer gurus. "k" is known as the magnetomechanical coupling factor, or one of the "figures of merit" for the material. It will be appearing fairly regularly from this point on, and will be discussed repeatedly in this dissertation. Experience has shown that $q$, $\mu^\sigma$, and $E_y^H$ vary with drive amplitude, $H$. Therefore, the reader should keep in mind that $k$, too, may vary with $H$. Calculation of $k^2$ via Eqn. (3.2.3), i.e., experimentally measuring $q$, $E_y^H$, and $\mu^\sigma$, is called the "three parameter method." The "dynamic method" for estimating $k^2$ will be discussed in the next section. (See Eqn. (3.3.15).)

Consider stress versus strain for the magnetostrictive element depicted in Figure 3.3. If the compressive stress is increased (made more negative) on a biased element and the applied field, $H$, is held
constant, the bar magnet will rotate (counterclockwise in Figure 3.3) and the spring will compress a given amount. The rotation of the bar magnet would reduce the axial B field. That trend is consistent with Eqn. (3.2.1b). Now, if H were increased to bring B back to its original value, the bar magnet would rotate clockwise, the element would become longer, and the overall strain produced by the original applied stress would be reduced. Thus, the element is stiffer for constant B than for constant H. Like magnetic permeabilities, Terfenol-D's elastic modulus is usually reported as two values, $E_y^H$ and $E_y^B$, where $E_y^B > E_y^H$.

Also like permeabilities, more can be said about this relationship. Solve Eqn. (3.2.1b) for H, substitute that into Eqn. (3.2.1a), i.e.:

$$H = \frac{B}{\mu_i^\sigma} - \frac{q\sigma}{\mu_i^\sigma} \quad \text{and} \quad \varepsilon = \frac{\sigma}{E_y^H} + qH \Rightarrow \varepsilon = \frac{\sigma}{E_y^H} + q \frac{B}{\mu_i^\sigma} - \frac{q^2 \sigma}{\mu_i^\sigma}$$

which reduces to

$$\varepsilon = \frac{1}{E_y^H} \left(1 - k^2\right) \sigma + q \frac{B}{\mu_i^\sigma}$$

then, assuming B is constant, its variable component is zero, yields

$$\varepsilon = \frac{1}{E_y^H} \left(1 - k^2\right) \sigma = \frac{1}{E_y^B} \sigma$$

or simply

$$E_y^B \left(1 - k^2\right) = E_y^H$$

(3.2.4)

One might wonder, since the magnetic permeability and elastic modulus seem to vary, what values should be used in any given simulation of an application? Good question. The answer appears to depend on many factors, including the load, transducer design, prestress, magnetic bias, drive level, and drive amplifier.

Reference [8] reports trends in permeability and compliance (the inverse of the elastic modulus) which make sense in the context of the above model. If the tricky part is minimized, that being the rotation of the bar magnet, i.e., constant strain or constant flux density (aka "blocked conditions"), the corresponding properties ($\mu_i^\sigma$ and $E_y^B$) display
the lowest variations with prestress changes and drive levels (for a good sample of Terfenol-D). The blocked properties approach those of what would be the uncoupled material.

3.2.2 Discussion of energy densities and magnetomechanical coupling

Tradition dictates a discussion of energy densities. To a first approximation, the energy density within the material can be considered as the sum of the magnetic and elastic energies. The magnetic energy density would be approximated as

$$u_{mag} = \frac{1}{2} \mu H^2,$$

while the elastic energy is approximately

$$u_{el} = \frac{1}{2} E \varepsilon^2.$$

It may not be immediately obvious, but the energy densities depend on the material's state. The maximum magnetic energy density is obtained when the material is operated such that the magnetic permeability is a maximum (constant stress), i.e., $\mu = \mu_\sigma$. Similarly, the elastic energy is a maximum when the elastic modulus is a maximum (constant induction), $E = E_y B$. In both cases, the ratio of the difference between the maximum and minimum to the maximum density is $k^2$, i.e.,

$$\frac{1}{2} E_y^B \varepsilon^2 \frac{1}{2} E_y \varepsilon^2 \quad \frac{E_y^B - E_y^H \varepsilon^2}{E_y \varepsilon^2} = \frac{E_y^B - (1 - k^2) E_y^B}{E_y \varepsilon^2} = k^2.$$

It should be noted that the above discussion ignored any losses occurring due to dynamic damping or eddy currents. Nonetheless, energy arguments are typically used for deriving $k^2$. Later (Section 3.4.1), when the material is placed in a transducer, with magnetic and strain energies stored in other transducer components, energy arguments will be used to "back-out" material parameters from experimentally measured quantities.
3.3 Linear Transduction of Terfenol-D Transducers

According to [3], the canonical form of the transduction equations for the transducer (as opposed to the material within the transducer) is:

\[ V = Z_e I + T_{em} \nu \]  
\[ F = T_{me} I + z_m \nu \]  

where: 
- \( V \) is the voltage measured over the terminals of the transducer, volts,
- \( Z_e \) is the blocked electrical impedance of the transducer, ohms,
- \( I \) is the electric current passing through the transducer, Amps,
- \( T_{em} \) is the transduction coefficient for electrical effects from the mechanical velocity, volt sec./meter (“em” => electrical from mechanical),
- \( \nu \) is the transducer output velocity, m/s,
- \( F \) is the output force of the transducer, N,
- \( T_{me} \) is the transduction coefficient for mechanical effects from the electric current, N/Amp (“me” => mechanical from electrical), and
- \( z_m \) is the mechanical impedance of the transducer, N s/m.

These equations assume a single degree of freedom, steady state operation, and that time enters the problem only in the form \( e^{\omega t} \), that is, these equations are in the frequency domain. To solve the equations, all of the coefficients of \( \nu \) and \( I \) must be known. The coefficients represent "effective" parameters.

The dependance of transducer performance on the load can be included by noting that the force applied to a load would be \( F_{load} = z_L \nu \), where \( z_L \) is the mechanical impedance of the load. The output force of the transducer would be equal and opposite, thus \( F \) of Eqn. (3.3.1b) would be given as:

\[ F = -z_L \nu. \]

Rearranging, Eqn. (3.3.1b) becomes

\[ 0 = T_{me} I + (z_m + z_L) \nu. \]  

(3.3.2)
Solving for \( v/l \),

\[
\frac{v}{l} = \frac{-T_{me}}{(z_m + z_L)}. \tag{3.3.3}
\]

The electrical impedance, as would be measured experimentally, is the complex ratio of voltage from current, \( V/l \), called \( Z_{ee} \). Dividing Eqn. (3.3.1a) by current, using Eqn. (3.3.3) for \( v/l \), and combining yields

\[
\frac{V}{l} = Z_{ee} - \frac{T_{em} T_{me}}{(z_m + z_L)} = Z_{ee}. \tag{3.3.4}
\]

As shown, the measured electrical impedance consists of two terms, the blocked electrical impedance (what one would measure if the transducer output velocity were held at zero, a very difficult task to perform in reality) and what is traditionally called the motional impedance, \( Z_{mot} \), defined as:

\[
Z_{mot} = \frac{-T_{em} T_{me}}{(z_m + z_L)}. \tag{3.3.5}
\]

The motional impedance represents a modification of the transducer's electrical impedance due to motional effects. Combining Eqns. (3.3.4) and (3.3.5):

\[
Z_{ee} = Z_{e} + Z_{mot}. \tag{3.3.6}
\]

The discussion above was general in nature. The mechanism of transduction has not yet entered into the discussion. Reference [3] continues on in a general sense, however, here attention will be turned to magnetostrictive transduction.

To determine the forms of the various coefficients, the two sets of transduction equations, Eqns. (3.2.1) & (3.3.1), will be compared. Recall that the typical transducer is basically a Terfenol-D rod, of length \( l \) and area \( A \), in a wound wire solenoid of \( N \) turns. For the comparison, the following approximations and simplifying assumptions are made. For the solenoid, it is assumed that: the rod fills the solenoid; the rod and solenoid are the same length; \( B \) and \( H \) are constant with \( r \) and \( z \); \( H = nI = Nl/l \); magnetic flux is given as \( \Phi_m = BA \); all of the turns of the solenoid have the same magnetic flux linkage, thus the total flux linkage is \( N\Phi_m \); and \( B = \mu H \). The result of these
assumptions is that the electrical inductance of the solenoid is approximately: \( L = \mu n^2 A l \). It is also assumed that stress and strain are spatially independent, thus \( F = \sigma A \) and displacement is strain times length \( (u = \varepsilon l) \). This assumption implies that the analysis is only strictly correct when operating conditions are such that the rod behaves like a linear spring. Further, it is assumed that quantities vary in time like \( e^{j\omega t} \), thus velocity and displacement are related by: \( v = j\omega u \). This form of time variation is a standard assumption for steady-state linear system analysis.

Eqns. (3.2.1) can be rearranged and simplified to obtain a form similar to Eqns. (3.3.1). This will be done because the following variables will be related, \( V \) and \( B \), \( I \) and \( H \), \( F \) and \( \sigma \), and \( v \) and \( \varepsilon \). With that aim, Eqns. (3.2.1) become:

\[
\begin{align*}
B &= \mu \sigma (1 - k^2) H + qE_y^H \varepsilon \\
\sigma &= -qE_y^H H + E_y^H \varepsilon.
\end{align*}
\]

where \( k^2 \) is defined in Eqn. (3.2.3). Making the assumptions that \( H = nl \), that \( v = j\omega u \), and that \( u = l\varepsilon \) (which implies that \( \varepsilon = v/j\omega l \)) gives

\[
\begin{align*}
B &= \left[ \mu \sigma (1 - k^2) n \right] + \left[ \frac{qE_y^H}{j\omega} \right] v \\
\sigma &= \left[ -qE_y^H n \right] + \left[ \frac{E_y^H}{j\omega} \right] v.
\end{align*}
\]

For a solenoid, the voltage drop across its leads is equal to the DC resistance times the current, \( RI \), plus the time rate of change of the total flux linkage, i.e., \( V = RI + \frac{d(NAB)}{dt} = RI + j\omega NAB \). Using \( B \) as above, \( n = N/l \), and comparing the result with Eqn. (3.3.1a) gives

\[
V = RI + j\omega NAB = \left[ R + j\omega \mu \sigma (1 - k^2) n^2 A l \right] + \left[ \frac{E_y^H}{Nq} \right] v = [Ze]_l + [Te_m]v.
\]

Therefore, the blocked electrical impedance is given as
\[ Z_e = R + j\omega \left\{ \mu \pi (1 - k^2) n A l \right\} = R + j\omega L_{\text{blocked}} \] 

(3.3.7)

where the term in the braces is recognized as an approximation of the inductance of a solenoid containing a material of permeability, \( \mu = \mu \pi (1 - k^2) = \mu \varepsilon = \text{blocked permeability of Terfenol-D.} \)

The transduction coefficient, electrical due to mechanical, is

\[ T_{em} = Nq_{k_m}^{H} = Nq_{k_m}^{H} \]  

(3.3.8)

where \( k_m^{H} \) is the mechanical stiffness of a rod of Terfenol-D, when operated at constant field strength. Ignoring losses, constant field strength translates to constant electric current. It should be mentioned that of all the simplifying assumptions outlined above, Eqn. (3.3.8) only depends upon the linear displacement profile and spatially independent B and H assumptions.

The output force of the transducer, \( F \), is equated to stress times area. Using stress as above, and comparing with Eqn. (3.3.1b), yields

\[ F = \sigma A = \left[ -q E_y^H A n \right] + \left[ \frac{E_y^H A}{j\omega} \right] v = \left[ T_{me} \right] l + [z_m] v. \]

Therefore,

\[ T_{me} = -Nq_{k_m}^{H} = -Nq_{k_m}^{H} = -T_{em} \]  

(3.3.9)

and

\[ z_m = \frac{1}{j\omega} \frac{E_y^H A}{l} = \frac{k_m^{H}}{j\omega}. \]  

(3.3.10)

Note that the transduction, mechanical due to electrical, is the negative of electrical due to mechanical. This relationship was anticipated; it is due to the inherent spatial orthogonality of electric current and magnetic field.\[3\] Note also that the mechanical impedance of the transducer consists of only a stiffness term. Dynamic effects, mass and damping, are tacitly ignored in Eqns. (3.2.1).

As a result, use of Eqns. (3.2.1) should be restricted to frequencies
well below the first axial resonance of the transducer as run with a
given load.

An equation for the electrical impedance of a magnetostrictive
transducer, \( Z_{ee} \), including motional effects, will be useful. For
example, it could be of use because the real component of \( Z_{ee} \) gives the
input electric power per squared ampere of the transducer. (Recall,
\( \text{Power} = |Z_{ee}| \cos(\phi) = |Z_{ee}| \text{Real}(Z_{ee}). \) In the present study, \( Z_{ee} \)'s
immediate utility will be the way it implies the functional form of the
magnetic permeability for the magnetostrictive rod.

To formulate \( Z_{ee} \), begin with the motional impedance, \( Z_{mot} \), as
defined in Eqn. (3.3.5). Using the relationships detailed above for \( \kappa^2 \),
\( k_m H \), \( T_{me} \), and \( T_{em} \), the motional impedance for a magnetostrictive
transducer can be written as:

\[
Z_{mot} = q^2 E^A / \mu^A \left( \sum_{m} \frac{H}{n^2 A I} \right) + j \omega \left( \mu^A \frac{k^H}{\mu^A} \frac{(z_m + z_L) \n^A A I}{j \omega} \right)
\]

It should be mentioned that, of all of the simplifying assumptions
invoked above, this expression for motional impedance is limited only
by the assumptions of constant stress and strain profiles and spatial
independence of \( B \) and \( H \). Using \( Z_e \), as defined in Eqn. (3.3.7), and \( Z_{mot} \) as
above, gives

\[
Z_{ee} = R + j \omega \left( \mu^A (1 - k^2) n^A A I \right) + j \omega \left( \mu^A \kappa k^2 m \frac{k^H}{j \omega (z_m + z_L)} \right)
\]

or

\[
Z_{ee} = R + j \omega \left( \mu^A \left( 1 + k^2 \left( \frac{k^H}{j \omega (z_m + z_L)} \right) \right) \right)
\]

(3.3.11)

where \( R \) is the DC resistance of the wound wire solenoid, the term in
the braces is the electrical inductance, and the bracketed term is \( \mu_T \),
the \textit{complex valued} magnetic permeability of the magnetostrictive rod
in the solenoid - \textit{including motional effects} as determined by the
mechanical impedance of the transducer \textit{and} the load. In particular,
the magnetic permeability of Terfenol-D in a wound wire solenoid, as a
function of material coupling, and mechanical impedances (thus frequency) is given as:

$$
\mu_T = \mu^0 \left( 1 + k^2 \left( \frac{k_m^H}{\omega (z_m + z_L)} - 1 \right) \right)
$$

(3.3.12)

It should not be a terrible surprise that the magnetic permeability is enhanced or reduced by transducer displacements. That behavior was examined earlier in connection with the magnetostrictive "elements." In the present case, displacements are enhanced or reduced due to dynamic and load effects, thus the property typically called magnetic permeability is affected.

Another form for magnetic permeability can be obtained by employing the simple, single degree of freedom mechanical impedance functions:

$$
z_m = j \omega m_m + b_m + \left( k_m^H + k_{mps}\right) / j \omega \quad \text{and} \quad z_L = j \omega m_L + b_L + k_L / j \omega
$$

where subscripts "m" refer to the transducer and "L" refer to the load.

Of the subscripted quantities: \( m \) is the dynamic mass, \( b \) is the damping coefficient, and \( k \) is the linear stiffness. The stiffness of the transducer is modelled as the parallel combination of that of the rod, \( k_m^H \), with that of the prestressing spring, \( k_{mps} \). Note that these impedances are based on velocities, as opposed to displacements.

Defining \( m, b, \omega_n^2 \), and \( 2\zeta \omega_n \) as follows: \( m = m_m + m_L, b = b_m + b_L, \omega_n^2 = (k_m^H + k_L + k_{mps})/m \), and \( 2\zeta \omega_n = b/m \); allows Eqn. (3.3.12) to be written as:

$$
\mu_T = \mu^0 \left( 1 + k^2 \left( \frac{k_m^H}{\left( k_m^H + k_L + k_{mps}\right)} \cdot \frac{1}{1 - \left( \frac{\omega}{\omega_n} \right)^2 + j 2\zeta \frac{\omega}{\omega_n}} - 1 \right) \right)
$$

(3.3.13a)

This formulation shows the mechanical impedance function in a dimensionless form, and one more familiar to some readers. It can also be written in terms of the blocked permeability and the linear coupling as:
\[
\mu_T = \mu^e + q^2 \frac{k_m^H}{(k_m^H + k_L + k_{mps})} \left[ 1 - \frac{(\omega/\omega_n)^2}{1 + j 2 \zeta \omega/\omega_n} \right] \tag{3.3.13b}
\]

Eqns. (3.3.13) are equivalent when hysteresis is ignored, i.e., \(\mu^e\) and \(q\) are real valued. The formulation of (b) allows different hysteretic behaviors to be assigned to the blocked and the motional contribution to the overall magnetic permeability. It might be useful for high drive levels.

The behavior of these functions for magnetic permeability should be checked against known trends in permeability discussed previously. For simplicity, \(k_{mps}\) is assumed to be negligible in the following discussion and attention is paid to Eqn. (3.13.3a). If \(k_L\) goes to infinity (the transducer is blocked by the stiffness of the load), the numerator of the fraction goes to zero and the permeability approaches the blocked permeability of the material \((\mu_T = \mu^o (1-k^2) = \mu^e, \text{ recall Eqn. (3.2.2)})\). This trend is in agreement with previous arguments.

Similarly, if \(k_L \ll k_m^H\), the numerator of the fraction is about one, and what happens depends upon the frequency of excitation. At frequencies, \(\omega \ll \omega_n\), corresponding to the stiffness controlled range of transducer operation, the fraction is very nearly one. Thus, the permeability is approximately \(\mu^o\), i.e., the permeability is unaffected at low frequencies. For excitation frequencies near resonance, \(\omega = \omega_n\), the permeability is increased due to the increased amplitude of the rod's displacement. In addition, a phase is present owing to the time lag between displacement and current at these frequencies. In the context of \(B = \mu_T H\), Eqns. (3.3.13) are really saying that the total time varying axial B field is the sum of that due to the time varying H, and that due to the displacement, u (the rotating "bar magnet"). This is acceptable because fields add. Expressed in the frequency domain (and assuming \(k_L = 0\):
\[ B = B_{\text{from } H} + B_{\text{from } u} = \mu^\sigma (1 - k^2) H + \mu^\sigma k^2 \left( \frac{1}{1 - \left( \frac{\omega}{\omega_n} \right)^2 + j 2 \zeta \frac{\omega}{\omega_n}} \right) H. \]

Due to dynamic effects in this frequency range, the displacement, thus the displacement related B field, is delayed in time when compared to H. In addition, the displacement amplitude varies with frequency, reaching its maximum amplification at \( \omega = \omega_n \) (assuming low damping). Thus the maximum displacement related B field will occur when \( \omega = \omega_n \).

For frequencies such that \( \omega >> \omega_n \), corresponding to the mass controlled range of transducer operation, displacements and the fraction in Eqn. (3.3.13a) go to zero. In this case, the permeability again approaches the blocked value, i.e., \( \mu_T = \mu^\sigma (1 - k^2) = \mu^\varepsilon \).

Finally, an increase in coupling, \( k^2 \), increases the effects of static and dynamic displacements. Most of the above trends agree with those expected of the magnetic permeability as explained in connection with Eqns. (3.2.1). The frequency related affects are a consequence of fields adding, and the displacement related field being delayed in time due to dynamic effects.

Using the permeability defined in Eqn. (3.3.13a), \( Z_{ee} \) can be written as below.

\[
Z_{ee} = R + j \omega \left\{ \mu^\sigma \left( \begin{array}{c}
\frac{k^H}{k_m^H + k_L + k_{mps}} \\
1 - \left( \frac{\omega}{\omega_n} \right)^2 + j 2 \zeta \frac{\omega}{\omega_n}
\end{array} \right) \right\} n^2 A l.
\]

Figure 3.4 is a plot of \( Z_{ee} \) versus frequency. The upper plot is the magnitude and the lower is the phase. The following values were used: \( R = 6 \) \( \Omega \), \( k_L = k_{mps} = 0 \), \( n = 23264 \) turns/meter, \( A = \pi (3.175 \times 10^{-3})^2 \) meters\(^2\), \( l = 55.88 \times 10^{-3} \) meters, \( \mu^\sigma = 5 \) \( \mu_0 \) Tesla meter per amp turn, \( \zeta = 0.04 \), and \( k^2 = 0.0, 0.1, \) and \( 0.2 \). Discussion of the natural frequency, \( \omega_n \), will be delayed temporarily.
Figure 3.4. Plots of $Z_e$, Eqn. (3.3.14), versus frequency. Upper plot is magnitude, lower is phase.
As shown in the figure, when \( k^2 = 0 \), i.e., no magnetomechanical transduction is present, the transducer behaves electrically like a simple R-L circuit. The magnitude increases with frequency and the phase starts at zero, then approaches \(+90^\circ\) with increasing frequency. For the traces where \( k^2 > 0 \), the plots display all of the distinctive characteristics of a coupled electromechanical transducer.[3] As is typical of magnetostrictive transducers, the magnitudes reach a local maximum followed by a local minimum. The phase approaches \(+90^\circ\), dips to a local minimum, then approaches \(+90^\circ\) again. Note how the variations in magnitude and phase increase with increasing coupling. Note also how the frequency range over which these fluctuations occur increases with increasing coupling. For the magnitude plot, the frequency at which the local minimums occur is approximately the same, whereas the frequency where the local maximums occur decreases with increasing coupling. To a first approximation, these frequencies are related by \( k^2 \).

Consider for a moment the equation for the natural frequency of any rod behaving as a linear spring:[10]

\[
\omega_n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} = \frac{1}{2\pi} \sqrt{\frac{AE}{I}}.
\]

For Terfenol-D, the Young's modulus, \( E \), varies; thus there are at least two "natural" frequencies to consider:

\[
\omega_B = \frac{1}{2\pi} \sqrt{\frac{AE_B}{Im}} \quad \text{and} \quad \omega_H = \frac{1}{2\pi} \sqrt{\frac{AE_H}{Im}},
\]

but the two moduli are related by Eqn. (3.2.4). Thus

\[
\omega_H = \frac{1}{2\pi} \sqrt{\frac{A(1-k^2)E_B}{Im}} = \sqrt{(1-k^2)} \omega_B
\]

solving for \( k^2 \),

\[
k^2 = 1 - \left( \frac{\omega_H}{\omega_B} \right)^2.
\]  

(3.3.15)

Therefore, if one can measure these frequencies, they can calculate the coupling for the rod operating at that prestress, that magnetic
bias, with that load, and at that drive level. Calculation of $k^2$ via Eqn. (3.3.15) is the dynamic method mentioned in the discussion of Eqn. (3.2.3). In the literature, one finds $f^H$ (aka $f_r$ and $f_{oz}$) called the "resonance" frequency, and $f^B$ (aka $f_a$ and $f_{oy}$) called the "antiresonance" frequency.[5, 6]

It should be noted parenthetically that Eqn. (3.3.15), though derived above assuming that the stress and strain in the rod were independent of axial position, also holds for a "free-free" rod.[1] However, in the free-free case, stress and strain are not independent of axial position. For occasions like these, one finds in the literature expressions for a $k_{33}$, which is termed a "geometry independent" coupling. The ratio of the elastic energy calculated assuming a half-sine displacement profile (mode one axial vibration of a free-free rod) to the elastic energy calculated assuming a linear displacement profile, is $\pi^2/8$. The relation $k_{33} \approx (\pi/\sqrt{8})k = 1.11k$ represents an attempt to compensate for the axial variations of stress and strain when the rod is run with free-free end conditions.

For Figure 3.4 it was assumed that $f^B = 5000$ Hz, where $f^B$ is the natural frequency based on Terfenol-D stiffness $E_y^B$ (the modulus reported to be closest to a constant value [8]). As a consequence, the natural frequency based on Terfenol-D stiffness $E_y^H$ varied with $k^2$ like $f^H = f^B \sqrt{1-k^2}$. In the derivation of Eqns. (3.3.13) it was assumed that $\omega_n$ was based on $k_{33}^H$, the rod stiffness calculated using $E_y^H$, thus, for the simulations in the figure, it was assumed that the resonant frequency of the mechanical system was $\omega_n = 2\pi f^H$.

At what frequency, $f_o$ (called the "resonant" frequency), might one expect to measure the largest transducer output displacements or velocities? That depends upon with what the transducer is being electrically driven. Assume for a moment that a given transducer/rod combination has, for example, $k^2 = 0.2$. Terfenol-D responds to the applied magnetic field strength, $H = nI$; therefore, the response will depend on the current passing through the transducer windings. If one drives the above transducer with a constant current source, the largest response will occur near $f^H$, i.e., $f_o \approx f^H$. On the other hand, if
one drives the transducer with a constant voltage source, (assuming $k^2$ is constant) the largest current will pass through the windings when the electrical impedance is a local minimum (refer to Figure 3.4). As a result, the response will reach its maximum at a frequency higher than $f^H$, i.e., $f_o > f^H$. Reference [6] reports that the designer may anticipate $f^H < f_o < f^B$, which at least narrows the range of possible resonant frequencies a bit.

Recall that the superscript on a variable indicates the quantity that is held constant. If one extrapolates that to the quantity which varies the least, $f^H$ and $f^B$ can be estimated from a plot of electrical impedance. Bearing in mind that $V = Z_{ee}I$, that $B$ is related to $V$, and $H$ is related to $I$, it can be argued that when the magnitude of $Z_{ee}$ is a maximum, $B$ varies much more than does $H$. Thus, compared to $B$, $H$ is nearly constant, i.e., $f^H$ is approximately the frequency at which the magnitude of $Z_{ee}$ reaches a local maximum. Similarly, when the magnitude of $Z_{ee}$ is a minimum, $B$ varies less than does $H$. Therefore, $f^B$ is approximately the frequency at which $Z_{ee}$ assumes a local minimum.

How good is this approximation? It depends on the relative values of the mechanical damping and magnetomechanical coupling. If the coupling is high and damping is low, the approximation is better than when the coupling is low and the damping is high. For the data displayed in Figure 3.4, the estimate provided by Eqn. (3.3.15), using the frequencies of maximum and minimum $Z_{ee}$ magnitudes, exceeded the values used in the simulations by 65 and 19 percent, for $k^2 = 0.1$ and 0.2, respectively. Recall that all of the discussion about coupling ignored losses. Mechanical damping and eddy currents are of particular concern in these transducers. So far, eddy currents have been ignored. (See Chapters 4 and 5.) However, mechanical damping tends to reduce the magnitude of, and frequency at which mechanical resonance occurs.[10] It also spreads out the resonant peak. This wider peak and lower resonant frequency translate into an early arrival of $f^H$ and a delayed arrival of $f^B$, as would be measured experimentally using the above approximation technique. It also reduces the fluctuations in magnitude and phase. As the coupling is increased with the damping
held constant, the coupling estimate from the above procedure improves. For example, for $k^2 = 0.5$, the estimated coupling was 0.508, a difference of less than two percent.

At this point, no experimental evidence has been offered to substantiate the correctness of Eqn. (3.3.14), the expression derived for the transducer's observed electrical impedance. The expression was developed by comparing two different sets of linear transduction equations and invoking a substantial number of simplifying assumptions (the worst of which is likely the approximation of electrical inductance of a solenoid). It should be recognized that if the same approximation for the inductance of a coil is used, $Z_{ee}$ given by Eqn. (3.3.14) is equivalent to that given by Eqn. (3.3.4). Both equations will display behavior similar to that of the experimentally measured electrical impedance functions. As shown in Figure 3.4, Eqn. (3.3.14) did display the distinctive characteristics of a coupled electromechanical transducer. It also provided trends in coupling estimates which are consistent with the basic assumptions employed by the underlying theory and the mechanics of the situation. However, perhaps its greatest value is the way in which it betrayed the secrets of the magnetomechanical coupling. From the global view of the transducer as an R-L circuit, where L involves a magnetic permeability of the material in the core of the solenoid, the coupling must appear as a variation of that permeability. The expressions for $\mu_T$ given by Eqns. (3.3.13) are, therefore, the most useful equations of the lot. They will be employed later when the effects of eddy currents are included and better approximations of electrical inductance are used. Eqns. (3.3.13) constitute magnetomechanical models of Terfenol-D.

3.4 Vector-Impedance and Admittance Analysis

Linear transduction equations have been introduced and are being approached assuming a single degree of freedom and using lumped parameters. If the linear transduction equations are to be of use, one must know the blocked electrical impedance, transduction coefficients, and the mechanical impedances of the transducer and the
load. In this section of the dissertation, the theory of vector-impedance and admittance analysis will be introduced and methods for estimating the "effective" parameters from experimental electrical impedance measurements will be reported.

The material in this section draws heavily upon the work reported by [3], Frederick V. Hunt (1905-1972), in his authoritative monograph, *ELECTROACOUSTICS: The Analysis of Transduction, and Its Historical Background*, published by the American Institute of Physics for the Acoustical Society of America. It was reasonably priced, interesting, and has proven to be invaluable.

It was shown in Eqn. (3.3.9) that for these magnetostrictive transducers \( T_{me} = - T_{em} \). Defining \( T \) as \( T = T_{em} = ||T|| \exp(-j\beta) \), allows rewriting the motional impedance as

\[
Z_{mot} = \frac{T^2}{(z_m + z_L)} = \frac{||T||^2 e^{-j2\beta}}{(z_m + z_L)}
\]  

(3.4.1)

where \( z_m \) and \( z_L \) are the mechanical impedances (based on velocity) of the transducer and the load, respectively. The mechanical impedances are given as

\[
z_m = j\omega m_m + b_m + k_m / j\omega
\]

and

\[
z_L = j\omega m_L + b_L + k_L / j\omega.
\]

Their sum is \( Z_m \), defined as:

\[
Z_m = z_m + z_L = j\omega m + b + k / j\omega
\]  

(3.4.2a)

or, equivalently:

\[
Z_m = \frac{k}{j\omega} \left(1 - \left(\frac{\omega}{\omega_0}\right)^2 + j2\zeta \frac{\omega}{\omega_0}\right)
\]  

(3.4.2b)

where:

\[
m = m_m + m_L \quad b = b_m + b_L \quad k = k_m + k_L \quad \zeta = \frac{b}{2\sqrt{km}} \quad \text{and} \quad \omega_0 = \sqrt{\frac{k}{m}}.
\]

\( \zeta \) is the dimensionless damping coefficient and \( \omega_0 \) is the circular frequency of mechanical resonance. If \( b \) in Eqn. (3.4.2a) is a constant, a plot of \( Z_m \) in the complex plane is simply a vertical line positioned at
Real\{Z_m\} = b \ (\text{refer to Figure 3.5}). A plot of 1/Z_m is a circle of radius 1/b, centered at 1/(2b) + 0j.

The inverse of Z_m is called the mechanical admittance, Y_m, defined as:

\[
Y_m = \frac{1}{Z_m} = \frac{Z_m^*}{Z_m Z_m^*} = \frac{b - j\omega \left( m - k / \omega^2 \right)}{b^2 + \omega^2 \left( m - k / \omega^2 \right)^2}
\]

(3.4.3)

where the asterisk denotes a complex conjugate. Consider the value of this function with varying frequencies. For \( \omega = 0 \), the real component of Y_m is nearly zero while the imaginary component is positive and increasing. As the frequency is increased, the top half of the admittance circle is traced out. When the frequency reaches the point that \( \omega^2 = k/m = \omega_0^2 \), i.e., the frequency of mechanical resonance, Y_m is real valued, specifically, \( Y_m = 1/b \). When the frequency is increased further, the imaginary component becomes negative and the bottom half of the admittance circle is traced out. As the frequency is increased to positive infinity, Y_m approaches zero from quadrant four.

For a lightly damped system, special significance is placed on the "half-power" points, occurring at frequencies \( \omega_1 \) and \( \omega_2 \). These frequencies are those at which the real and imaginary components of

\[Z_m\quad 1/Z_m\quad ||T||e^{-j2\beta}/Z_m\]

Figure 3.5. Schematic representation of complex plane plots of the displayed functions
the mechanical impedance, or its inverse, are equal in magnitude. When this is the case, Eqn. (3.4.3) becomes

\[
\frac{1}{Z_m} = \frac{b \pm jb}{b^2 + b^2} = \frac{1}{2b} \pm \frac{j}{2b}
\]

which has a magnitude of \(\sqrt{2/(2b)} = 0.707\) times the magnitude at resonance, \(1/b\). Note that these are the coordinates of the top and bottom points of the admittance circle. The frequencies \(\omega_1\) and \(\omega_2\) can be obtained from Eqn. (3.4.3) by setting the real component equal to plus or minus the imaginary component, assuming light damping, and recognizing the equivalence of negative and positive frequencies. With those stipulations, \(\omega_1\) and \(\omega_2\) are approximated as:

\[
\omega_2 = (1 + \zeta)\omega_0
\]

and

\[
\omega_1 = (1 - \zeta)\omega_0.
\]

Subject to the same stipulations, the mechanical quality, \(Q\), is defined as:

\[
Q = \frac{1}{2\zeta} = \frac{\omega_0}{\omega_2 - \omega_1}.
\]

For the motional impedance, Eqn. (3.4.1), the effects of the squared transduction coefficient are a scaling and a rotation (in the complex plane) of the circle resulting from \(Y_m\). Thus, the diameter of the impedance loop would be, \(D = ||T||^2/b\), and the dip angle of the line from zero to \(\omega_0\) would be \(2\beta\). This evolution is depicted in Figure 3.5. It should be mentioned that the frequency scale around the circle is not uniform. Typically, most of the data points are located near the origin. Also, it is typically assumed that \(T\) and \(\beta\) are independent of frequency, though that may not be true.

For the transduction coefficients given in Eqn. (3.3.9), repeated here for reference,

\[
T_{me} = -Nq\frac{E^H_A}{1} = -Nqk_m^H = -T_{em}
\]

there is no reason to anticipate any phase since \(N\), \(q\), and \(k_m^H\) are all real numbers, at least to a first approximation. The most likely source
of phase from these parameters will be $q$, the local slope of a plot of strain versus applied field strength, at constant stress. To a first approximation, $q$ is simply a real valued constant applicable for transducer operation at "low" drive levels. As drive levels are increased, it will be shown later that $q$ also increases, owing to nonlinearities in the magnetostrictive material. Another consequence of higher drive levels is that magnetic hysteresis becomes appreciable. Thus, a better approximation of $q$ would be a complex valued "constant," with its magnitude and phase dependant on drive level amplitudes. (In essence, this approach is approximating a hysteresis loop as an ellipse, a fairly common approximation, outlined above in Section 3.1.)

Another contribution to the dip angle, $2\beta$, will come from the effects of eddy currents occurring in various components of the transducer. The eddy currents will affect the measured electrical impedance, causing the measured resistance (the real portion of $Z_{ee}$) to increase with frequency, and the measured inductance to decrease. The effects of eddy currents (on $Z_{ee}$) are shown schematically in Figure 3.6b, and discussed in more detail in Chapters 4 and 5.

Experimental measurements of voltage and current will not yield simply the motional impedance, but rather $Z_{ee}$, the sum of $Z_{mot}$ and $Z_e$. Using Eqn. (3.4.1) for the motional impedance, the expression for the electrical impedance of the transducers under study becomes:

$$Z_{ee} = Z_e + Z_{mot} = (R_e + j\omega L_e) + \frac{||T||^2 e^{-j2\beta}}{z_m + z_L}$$

(3.4.4)

where: $R_e$ is the DC electrical resistance of the coil windings and $L_e$ is the blocked electrical inductance. Figure 3.6 shows Eqn. (3.4.4) schematically. In the figure, sketch (a) assumes no eddy current effects are present; sketch (b) reflects typical changes when eddy currents are appreciable.

At this point, the investigator who wants to estimate effective transducer parameters via a plot of the motional impedance, has basically three options for generating the plot.
Option 1) They can attempt to block the transducer's output motion and experimentally measure the blocked electrical impedance, then measure the impedance when operating normally, producing a motional impedance circle via $Z_{\text{mot}} = Z_{\text{ee}} - Z_{e}$. This option has the very real disadvantages of: a) running the risk that the motion is not truly blocked, especially during contraction of the rod; and b) changing the material properties significantly as a result of changing the prestress in the material while attempting to block the output motion.

Option 2) Estimate a blocked impedance from normal impedance measurements at frequencies where the motional effects are thought to be small, i.e., at frequencies well below, and well above resonance.

Figure 3.6. Schematic representations of the combination of blocked and motional impedances forming the measured impedance of a magnetostrictive transducer. (a) assumes the absence of eddy currents, (b) assumes the presence of appreciable eddy currents.
In reality, one is typically limited in the choice of these frequencies by the occurrence of additional spurious resonances, e.g., housing resonances which show up in the measured electrical impedance function. Regardless, it is recommended that the estimates of real and imaginary components of the blocked impedance, as functions of frequency, be obtained separately. Changes in blocked impedance values due to eddy current effects are difficult to take into account with this method since the variation of resistance and inductance are not linear with frequency.

Option 3) They can do what is normally done, neglect the changes in $Z_e$ over the relatively short frequency range required to produce the motional impedance circle. Employing this approach, the blocked electrical impedance is usually assumed to be the value where the impedance loop begins and ends, that is, where it crosses itself, $Z_e(\omega_0)$. $Z_{\text{mot}}$ is then calculated as $Z_{\text{mot}} = Z_{ee} - Z_e(\omega_0)$. This approach seems most justifiable when dealing with a lightly damped resonance, i.e., the shorter the frequency range of appreciable motional effects, the better the approximation.

Instead of generating $Z_e$ to estimate transducer parameters via one of the three options above, one can use $Z_{ee}$ and obtain essentially the same information. This approach closely parallels Option 3 above. One does a best fit of a circle to the loop (Figure 3.6b), noting the circle diameter that intersects $Z_{ee}$ where $Z_{ee}$ crosses itself. The frequency at the other end of that diameter is $\omega_0$. The frequencies at $\pm 90^\circ$ from this diameter are $\omega_1$ and $\omega_2$. The angle formed by the diameter and the horizontal is the effective $2\beta$ (including effects of eddy currents). The diameter itself is $||T||^2/b$.

The easiest technique for putting numbers to the effective parameters of a transducer, using electrical impedance analysis, is to take impedance measurements with two different known mass loads. For this scenario, in one case the mass would be $m = m_m + m_1$, the measured resonant frequency would be $\omega_01$, the half-power frequencies would be $\omega_1$ and $\omega_2$, the diameter of the motional impedance loop would be $D$, and the dip angle would be $2\beta$. In the second case the mass
would be \( m = m_m + m_2 \) and the measured resonant frequency would be \( \omega_{02} \). In theory, for the second measurement, only the added mass and the resulting mechanical resonant frequency need be known.

The internal effective dynamic mass of the transducer, \( m_m \), can be estimated from the expressions for resonant frequencies by assuming that the stiffness is the same in both cases. Thus:

\[
m_m = \frac{m_2 \omega_{02}^2 - m_1 \omega_{01}^2}{\omega_{02}^2 - \omega_{01}^2}.
\] (3.4.5)

The effective linear stiffness of the transducer is then given as either:

\[
k = k_m = \omega_{01}^2 (m_m + m_1) \quad \text{or} \quad k_m = \omega_{02}^2 (m_m + m_2).\] (3.4.6)

From the definitions of \( \zeta \), \( \omega_0 \), and \( Q \), the effective damping parameter, \( b \), can be estimated via:

\[
b = b_m = (m_m + m_1)(\omega_2 - \omega_1).\] (3.4.7)

From the diameter of the motional impedance circle comes an expression for the transduction coefficient (recall, \( D = \|T\|^2/b \)),

\[
\|T\| = \sqrt{bD},\] (3.4.8)

and the effective rotation angle is given as:

\[
\beta = \frac{\text{dip angle}}{2}.
\] (3.4.9)

That was the theory. In practice, one would likely want to conduct several tests with several different mass loads in an attempt to reduce the uncertainties in the estimated transducer mechanical impedance parameters. In addition, attention should be paid to the drive and operation parameters. As justification for this statement, recall that: due to nonlinearities in the material, the drive levels are important; due to the interaction of the transducer and the power supply, the type of power supply (voltage vs. current source) is important; due to the increasing effects of eddy currents with frequency, the test conditions and loads should resemble the intended operating conditions as closely as possible.

With the above parameters and estimates of the blocked electrical impedance in hand, transducer behavior with different loads, \( z_L \), can be
estimated. Add $z_L$ to the transducer impedance, $z_m$, to obtain a new $Z_m$. Then calculate velocity per ampere via Eqn. (3.3.3), modified to read:

$$\frac{v}{I} = \frac{-T_{me}}{Z_m} = \frac{||T|| e^{-j\beta}}{Z_m}. \quad (3.4.10)$$

Note that $\beta$ (which is due to hysteretic and eddy current effects) introduces an additional phase lag over that due to the mechanical impedance. As a result, one cannot simply look at the phase of $v/I$, and assume that mechanical resonance occurs at 0° (or -90° for $u/I$, displacement/current).

The electrical impedance could be calculated as:

$$\frac{V}{I} = Z_e - \frac{T_{em} T_{me}}{(z_m + z_L)} = Z_e + \frac{||T||^2 e^{-j2\beta}}{Z_m}. \quad (3.4.11)$$

Output force per ampere would be $(-vz_L)/I = -z_L(v/I)$, etc. It is of particular significance that the above estimates would include magnitudes and phases between the variables. This sort of information is necessary for applications like active vibration control, where phase makes all the difference. On the other hand, phase is less critical in other applications, for example, sonar transducers.[11]

All in all, the above procedure yields only "effective" parameters. A mechanical model was assumed, the theory was developed, and parameters were estimated to yield the best fit of experimental transducer behavior to the assumed model.

To this point, all analysis was based on looking at the electrical impedance function, $Z_{ee}$, as available at the transducer's electric terminals. Another form of analysis is also available. It entails examining the electrical admittance function, $Y_{ee} = I/V = 1/Z_{ee}$. Like the impedance method, one obtains a motional loop. However, different information is available from the study of the electrical admittance, and a comparison of the information from the two methods allows the definition of an effective electromechanical coupling coefficient, $k^2$.[3] Ignoring losses, this is the same $k^2$ developed previously, Eqns. (3.2.3) and (3.3.15), where $f^H$ is called $f_{oZ} = \omega_o/2\pi$, from impedance analysis (thus the "Z"), and $f^B$ is called $f_{oY} = \omega_o/2\pi$, from admittance
analysis. Actually, from the admittance analysis, one obtains the
diametral frequency, \( \omega_0 \), as the frequency corresponding to the far side
of the horizontal diameter of the admittance circle. Theoretically, the
admittance circle is not rotated as it was in impedance analysis. If it
is, there are significant leakage reactance terms in the transducer
under study.[3]

Traditionally, the two forms of analysis, impedance and admittance,
are presented in terms of equivalent circuits, circuit transformations,
and imaginary transformers linking the electrical and mechanical
circuits. That approach will not be pursued here. However, for
completeness, the impedance approach stems from a force-voltage
analogy, while the admittance approach is from a force-current
analogy, aka, a mobility representation.

Electrical admittance analysis is facilitated by defining the total
admittance, \( Y_{ee} \), blocked admittance, \( Y_e \), and motional admittance, \( Y_{mot} \),
as shown.

\[
Y_{ee} = \frac{1}{Z_{ee}} = \frac{1}{V} \quad Y_e = \frac{1}{Z_e} \quad Y_{mot} = Y_{ee} - Y_e
\]  

(3.4.12)

Note that the motional admittance is not the reciprocal of the
motional impedance, \( Z_{mot} \).

Reference [5] has developed an ingenious technique for obtaining
realistic coupling estimates of the magnetostrictive material
Terfenol-D. Instead of using the voltage and current measurements of
the drive coil, they measured the electric current in the drive coil, and
the voltage of a separate internal pick-up coil (wound directly on the
Terfenol-D rod). This approach drastically reduced the leakage flux
problems of a large drive coil and the distortions introduced into the
analysis by said leakage. A variation on their approach was employed
in this investigation.

Leakage flux would be the magnetic flux affecting coil voltages
which does not actually pass through the material. A drive coil's
voltage is affected by all of the flux linkage seen by all of its layers.
Thus, the flux passing through the coil windings and the cylindrical air
gap normally provided between the coil and the Terfenol-D rod, would
be considered leakage flux. (A cylindrical air gap is a necessary condition for long term operation of these transducers since the material physically moves and would otherwise eventually ruin the coil.) Flux is roughly \( BA = \mu H A \), thus the ratio of permeability times the cross-sectional area (A) of the coil windings plus the air gap, to the permeability times area of the magnetostrictive rod, would be a measure of the relative amount of leakage flux. Terfenol-D has a relative permeability of roughly between 2 and 10, while the relative permeability of copper and air is approximately one. Thus for a thick-walled coil (or large air gap), the leakage flux can become appreciable. On the other hand, if the core material were a steel with a relative permeability of, say 1000, the same leakage flux would be quite small compared to the flux carried by the rod.

3.4.1 Magnetomechanical coupling versus effective coupling

There is a difference between the coupling of the material, \( k^2 \) of Eqn. (3.2.3), and the effective coupling of the material in a given transducer, \( k^2_{\text{eff}} \). As demonstrated above, the realities of trying to use the material will introduce deleterious effects, i.e., \( k^2_{\text{eff}} < k^2 \).

Following the lead of [11], energy arguments can be used to derive effective coupling coefficients for rods in transducers, including the effects of leakage flux and the stiffness of the prestressing mechanism. In the spirit of everything being some kind of a squared \( k \), define:

\[
  k^2_{\text{eff}} = k^2_M k^2_E k^2 \Rightarrow k^2 = \frac{k^2_{\text{eff}}}{k^2_M k^2_E} \quad (3.4.13)
\]

where: \( k^2_{\text{eff}} \) is the measured coupling of the transducer containing the magnetostrictive rod,

\( k^2_M \leq 1 \) and is a term attributable to energy stored in the leakage flux of the experimental apparatus (the transducer, in the present case), and

\( k^2_E \leq 1 \) and stems from elastic energy being stored in the transducer's prestressing mechanism.
It remains now to derive expressions for the magnetic and elastic storage terms.

Consider the ratios of the difference of the maximum and minimum stored energies to the maximum (recall, that was how \( k^2 \) was defined in Section 3.2.2). For a transducer with a leakage inductance, \( L_{\text{leak}} \), the magnetic formulation gives

\[
\frac{1}{2} \frac{L_{\text{max}}^2 - \frac{1}{2} L_{\text{min}}^2}{L_{\text{max}}^2} = \frac{L_{\text{max}} - L_{\text{min}}}{L_{\text{max}}} = \left( \frac{1}{L_{\text{max}}} \right) \left( L_{\text{max}} + L_{\text{leak}} \right) = \left( L_{\text{max}} + L_{\text{leak}} \right) = \frac{L_{\sigma} - L_{\varepsilon}}{L_{\sigma} + L_{\text{leak}}},
\]

where \( L_{\sigma} \) and \( L_{\varepsilon} \) are the inductances based on the maximum and minimum magnetic permeabilities, \( \mu_{\sigma} \) and \( \mu_{\varepsilon} \), respectively. Multiplying by \( L_{\sigma} \) implies

\[
k_{\text{eff}}^2 = \frac{L_{\sigma}}{L_{\sigma} + L_{\text{leak}}} \left( \frac{L_{\sigma} - L_{\varepsilon}}{L_{\sigma}} \right) = k_{M}^2 k^2.
\]

Thus,

\[
k_{M}^2 = \frac{L_{\sigma}}{L_{\sigma} + L_{\text{leak}}}. \tag{3.4.14}
\]

Note that \( k_{M}^2 \) approaches unity as \( L_{\text{leak}} \) approaches zero. As a result, reducing the leakage inductance of the transducer will improve the performance of the transducer, that is, it will behave as though it had higher coupling. Higher coupling means more of the input energy is transduced.

On the mechanical/elastic side, where the stiffness of the transducer is \( k_{m} \), which is the sum of the stiffnesses of the rod and prestressing mechanism, \( k_{mps} \):

\[
k_{\text{eff}}^2 = \frac{1}{2} \frac{k_{m, \text{max}}^2 u^2 - \frac{1}{2} k_{m, \text{min}}^2 u^2}{k_{m, \text{max}}^2} = \frac{\left( k_{m}^B + k_{mps} \right) - \left( k_{m}^H + k_{mps} \right)}{k_{m}^B + k_{mps}} = \frac{k_{m}^B}{k_{m}^B + k_{mps}} \frac{k_{m}^H}{k_{m}^H + k_{mps}},
\]

where \( u \) is the displacement and \( k_{m}^B \) and \( k_{m}^H \) are the linear stiffnesses of the magnetostrictive rod based on the maximum and minimum moduli, \( E_{y}^B \) and \( E_{y}^H \), respectively. Thus
\[ k_{\text{eff}}^2 = k_E^2 k^2 \Rightarrow k_E^2 = \frac{k_B}{k_m + k_{\text{mps}}} \]  \hspace{1cm} (3.4.15)

As shown, reducing the stiffness of the prestress mechanism will increase the transducer's effective coupling. Eqn. (3.4.15) can also be written in terms of \( k^2 \) and \( k_m^B \), by noting that \( k_m^B = k_m^H / (1 - k^2) \).

For a transducer possessing both a leakage inductance and a prestressing mechanism, in other words, for any real transducer, the effective coupling is as defined above in Eqn. (3.4.13). For those who seek the material coupling from measurements of effective coupling and knowledge of the leakage inductance and the prestressing stiffness of the transducer, omitting the algebra, \( k^2 \) can be estimated as:

\[ k^2 = \frac{k_{\text{eff}}^2}{k_m^2} \frac{(k_m^H + k_{\text{mps}})}{(k_m^H + k_{\text{eff}}^2 k_{\text{mps}} / k_M^2)} \]  \hspace{1cm} (3.4.16)

where the above relation for \( k_m^B \) has been used. If the effective coupling measurement were made with the transducer driving a load with a stiffness \( k_L \), Eqn. (3.4.16) can still be used if one substitutes \( (k_{\text{mps}} + k_L) \) everywhere \( k_{\text{mps}} \) appears.

It was mentioned previously that reducing the leakage flux will increase the effective coupling of the transducer. Leakage flux is a rather complicated function of the coil and the transducer. Techniques for reducing leakage flux will not be addressed in this dissertation. On the mechanical side, where it was shown that reducing the stiffness of the prestressing mechanism tends to increase the effective coupling, the designer must make some trade-offs. It was found in this investigation that spring washers were the best of the available options. Compared to coil springs, deforming housings, piano wires, etc., the spring washers were compact, of low mass, simple, inexpensive, and readily available in a variety of sizes and stiffnesses.
3.5 The Combined Approach to the Analysis of Linear and Near-Linear Transduction of Terfenol-D Transducers

Linear transduction of Terfenol-D has been discussed, as has the "effective" or holistic view of transduction of Terfenol-D transducers. The linear equations for the material were shown to be limited to low frequencies; the stiffness control zone of dynamic material operation. They ignored damping and inertial effects of the rod itself. This is true, even if one were to improve the output force approximation, used in the derivation of Eqns. (3.3.9) and (3.3.10), to include the dynamic effects of internal transducer parts (e.g., components 3, 4, and 5 of Figure 1.4). The effective approach allowed for a more realistic dynamic mechanical impedance model to be employed. In addition, the effective approach was couched in variables that are easily measured (force, velocity, voltage, and current) and somewhat more familiar (to most readers) than magnetic parameters (flux or flux density and field strength).

The two sets of linear equations were compared, allowing expressions to be developed which linked the two approaches. The equation developed for $Z_{ee}$, Eqn. (3.3.14), is subject to several simplifying assumptions. Other than ignoring the effects of eddy currents, the most limiting assumption was that it neglected leakage flux contributions to the solenoid's measured voltage. To make it more realistic, a leakage inductance term should be added. However, all of the leakage would be outside of the rod, thus the expression shown is accurate for the portions of the solenoid containing Terfenol-D. The relations derived for the magnetic permeability of Terfenol-D, including dynamic effects, Eqns. (3.3.13), are limited only by the assumptions of spatially constant stress-strain and B-H profiles. Eqns. (3.3.13) are, in essence, magnetomechanical models for the magnetostrictive material within the core of the wound wire solenoid. Electromagnetic models for the transducer will be developed in Chapter 4. Combining the electromagnetic models with the magnetomechanical models will allow one to calculate estimates of the transducer's electrical impedance functions.
For these transducers, quantities of engineering interest include, but are not limited to, V/l, v/l, F/l, power consumption, and power delivered to the load. For the combined approach, assuming certain parameters are known, the quantities of interest can be estimated. The parameters which must be known are: the type of power supply to be used, the material coupling (k^2), or its constituents, magnetic permeabilities (μ^o or μ^e for the Terfenol-D rod and those of the other transducer components), electrical conductivities (for use in the electromagnetic models developed in Chapter 4), mechanical models of the load and the transducer, and the geometry and specifications of transducer components (e.g., diameters, lengths, coil turns, coil resistance, etc.).

Assuming Terfenol-D permeability is that of Eqns. (3.3.13), that a reasonable approximation for the inductance of a solenoid is employed, and that the model of the electromagnetic circuit reflects the realities of the transducer under study (see Chapter 4), one can calculate a reasonable approximation of Z_{ee}. One can then calculate an estimate of the blocked electrical impedance, Z_0, by numerically disabling the magnetomechanical coupling, i.e., use μ_T = μ^ε. Following this procedure allows one to estimate Tem = T via:

$$T_{em} = \sqrt{(Z_{ee} - Z_0)Z_m}$$

(3.5.1)

where Z_m is the combined impedance of the transducer and the anticipated load. At this point, other quantities of interest can be calculated, for example, velocity per ampere is available as:

$$v = \frac{T_{em}}{Z_m}$$

As mentioned previously, the magnetostriction phenomenon is nonlinear. Typical transducer displacement from electric current trends (assuming low frequencies of excitation) are depicted in Figure 3.7 for the three general ranges of transducer operation. For low amplitude signals, displacement from current is modelled well by a linear relationship. For the medium signal range, the effects of magnetic hysteresis become appreciable. In this, the "near-linear"
range, an elliptical relationship will be assumed to exist between displacement and current - even at low frequencies. Of course an ellipse cannot be relied upon to produce or mimic the harmonic frequencies present in the displacement plot as evidenced by the sharp tips of the hysteresis loop. Thus, this approach is limited to applications where the harmonic frequencies can be ignored, or treated as noise. At the highest drive levels, even an elliptical assumption is poor. Therefore, predictions via this modelling approach are extremely suspect.

It should be noted that the trends in Figure 3.7 are not independent of excitation frequency. As one might expect, dynamic effects tend to round the sharp corners of the hysteresis loops as excitation frequencies increase. In effect, improving the accuracy of the elliptical approximation and extending the drive amplitude range over which this modelling procedure might be applicable.

Figure 3.7. Schematic representation of displacement from electric current for low frequency operation of a Terfenol-D transducer
4. ELECTROMAGNETIC MODELS

Recall that the theory of transducer operation is roughly as follows:
1) An electric current in the solenoid begets a magnetic field along its axis. 2) A magnetic field permeating Terfenol-D tends to cause the magnetic domains within the material to rotate (in an effort to align with the applied field). 3) Rotation of the magnetic domains causes the Terfenol-D rod to change length (strain). (It works the other way, too. If you change its length, you rotate magnetic domains.) Thus, if you vary the electric current in the solenoid (the input variable), to a first approximation, you vary the length of the Terfenol-D rod.

From the above description one might anticipate an electromagnetic model coupled to a magnetomechanical model resulting in an electromagnetic-magnetomechanical model of the transducer. The expressions developed for the magnetic permeability of Terfenol-D, including motional effects, Eqns. (3.3.13), constitute the magnetomechanical models. Electromagnetic models will be developed in this chapter.

A review of the technical issues will be followed by the development of an electromagnetic model for applied field strength as a function of radial position for a cylindrical conductor. The model will then be applied to two different physical models of the transducer. The first physical model yields the classic solution for a cylindrical conducting rod in a solenoid. The second is a little more complicated. Both physical models allow predictions of, among other things, the complex valued electrical impedance of the transducer - including motional and eddy current effects.

4.1 Review of Technical Issues

A review of the technical issues will begin with a general discussion of magnetism in matter and the approximations associated with the material property commonly called magnetic permeability. Then, an introduction to magnetism in magnetostrictives will be presented. Finally, the causes of eddy currents and their effects on
the electrical impedance of a wound wire solenoid containing a conductor, specifically a solenoid with Terfenol-D, will be discussed.

4.1.1 Introduction to magnetism in matter

For quasi-static conditions,

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu M$$

(4.1.1)

where $\mathbf{B}$ is the magnetic induction vector, or flux density vector at a point within the material, $\mathbf{H}$ is the applied magnetic intensity vector, and $M$ is the magnetization vector of the material (more on $M$ follows). This equation can be written because fields (and their densities) add. One often finds in the literature (see, for example, [9]) in one dimension:

$$\mathbf{B} = \mu_\ast \mathbf{H}$$

(4.1.2)

Now the question is, what is reasonable to assume about $\mu_\ast$?

Traditionally, a material's permeability is given as:

$$\mu_\ast = \mu_0 (1 + \chi_m)$$

(4.1.3)

where $\chi_m$ is called the magnetic susceptibility of the material, often defined as

$$\chi_m = ||M||/||H||$$

(4.1.4)

Note that it has been tacitly assumed that $M$ and $H$ are linearly related. If $-1 < \chi_m < 0$ for a given substance, it is called a diamagnetic material. If $0 < \chi_m < 1$, the material is termed paramagnetic. Typically, for paramagnetic and diamagnetic materials, $\mu_\ast = \mu_0$ is "good enough" ($\chi_m$'s magnitude is on the order of $10^{-5}$ for typical example materials). If $\chi_m > 1$ for a material, it is called a ferromagnetic substance (more on ferromagnetism follows). Thus, for paramagnetic and diamagnetic materials, and as a first approximation for ferromagnetic materials, $\mu_\ast$ is a real valued constant. As a consequence, one is making a linear assumption, i.e., one is assuming that the normal hysteresis loop for the material in question is approximated well by a straight line over the range of interest.

What is $M$? It is the vector sum, per unit volume, of the material's magnetic moments. It is thus a macroscopic measure of the alignment of magnetic moments within the material. Due care is being exercised
to avoid Quantum Mechanics (and other such necromancy) in this development. It will be correspondingly non-rigorous. Vaguely speaking, on the atomic level there are electrons zipping about (charge is flowing) resulting in magnetic fields being produced (recall Faraday). In addition, each electron has associated with it an intrinsic magnetic moment owing to its "spin" (not to be taken too literally). The net magnetic moment of an atom is a vector combination of these moments. With the exception of diamagnetic effects (discussed below and usually negligible), the magnitude of $M$ is varied only by aligning existing magnetic moments in the material. The magnitudes of individual magnetic moments within the material are not changed by application of an H.

On the interatomic level, if the interaction of atomic moments is weak (due to temperature effects, low magnitudes, separation, etc.), the material is paramagnetic. Here, the application of an H has little effect on the overall alignment of the moments which thus exhibit a correspondingly small effect on the overall flux density, i.e., $M \ll H$ so $B \approx \mu_0 H$.

Diamagnetic materials are repelled by either pole of a permanent magnet; bismuth is the classic example. In this case, the application of an H induces a flux density contribution in the opposite direction ($\chi_m$ is negative). It is thought that all materials exhibit diamagnetism but that the effects are usually smaller than, thus masked by, paramagnetic or ferromagnetic effects. Diamagnetism is an interesting consequence of "Lenz's law." For a discussion, see reference [13].

Ferromagnetism arises from strong interatomic interaction of the atomic moments. These materials are usually thought of as containing magnetic domains. Domains can be thought of as zones in the material where a high degree of alignment exists between the magnetic moments of the atoms within the zone. Domains and their behaviors are examined more closely in the next section of the dissertation. Until then, with the application of a field, domains might rotate or domain walls can move (one domain grows at the expense of another).
Usually the application of a small $H$ can induce a high degree of alignment of magnetic moments throughout the specimen, resulting in a huge $M$, i.e., $B \approx \mu_0 M$. Note, however, that the quantities of interest are magnetic moments, angular momentums, cross products and such things for many millions of atoms; $M$ can be a very nonlinear function of $H$. Thus, Eqn. (4.1.4) may be a poor assumption. (A glance at a normal hysteresis loop should confirm this.) Nonetheless, the typical example given for ferromagnetism is iron where $M$ can be around two-thousand times the $H$ value. For most ferromagnetic materials, the application and subsequent removal of an $H$ can result in a high degree of alignment of the domains - it has just been "magnetized." Such materials find applications as permanent magnets if this residual flux density is large enough. Magnetic analysis of ferromagnetic materials becomes fairly complicated because the state of the material is now a function of where it has been; $B$ depends on $H$ and the prior state of $M$.

For those situations and materials where Eqn. (4.1.3) might be a reasonable approximation, but one wants to improve the approximation by modelling the hysteresis loop as an ellipse,

$$\mu_I = \mu_0 (1 + \chi_M) e^{i\phi}$$

is traditionally one's first attempt.[12] The phase, $\phi$, is thought to be independent of frequency, at least below the frequencies where eddy currents become "important."[3] After that? Unknown to the author.

According to those in the field of Ferromagnetodynamics: the dynamics of magnetic bubbles, domains, and domain walls, the above analysis is simplistic. They speak of limiting velocities of domain walls, damping terms, wall accelerations, ferromagnetic resonances, localized eddy currents from domain wall motion (another damping and inertial mechanism), etc.[14] The point to be made is that changing the magnetization of most any material by varying the field strength has associated with it damping and inertial effects. Along these lines, perhaps something like

$$\frac{d^2 M}{dt^2} + 2\zeta \omega_n \frac{dM}{dt} + \omega_n^2 M = kH$$

(4.1.6)
would be a better model than Eqn. (4.1.4). When applied to Terfenol-D, this sort of local and general magnetic behavior may be completely dwarfed by the effects of the magnetomechanical coupling in the material. This statement will be justified in the next section.

4.1.2 Introduction to magnetism in magnetostrictives

Terfenol-D does not fall squarely into any of the aforementioned material types - when one views only the magnetic permeability. Relative permeabilities of Terfenol-D range between approximately 2 and 10, which means that the magnetic susceptibilities of the material vary between 1 and 9. Despite that, Terfenol-D is "highly" magnetic. *Its permeability appears "low" due to the transduction of magnetic energy to elastic energy.*

As mentioned previously, Terfenol-D has magnetic domains and the "trick" in manufacturing the material is in growing the crystals so that the domains are aligned when cooled. Figure 4.1 is a schematic representation of an idealized four domain Terfenol-D rod. Each view represents the rod when subjected, in turn, to various conditions. The corresponding values of axial (33) stress, $\sigma$, strain, $\varepsilon$, applied field, $H$, and material magnetization, $M$, are displayed below each view.

View (a) shows the rod as manufactured. Note that the bottom two domains are perpendicular to the axis of the rod, while the top two are nearly perpendicular. In this view, there is no stress on the rod and the applied field and magnetization of the material are zero. View (b) depicts the rod with a compressive prestress large enough to result in all four domains being perpendicular to the axis of the rod. Note that the rod is slightly shorter than the rod in view (a), that the strain is now defined as zero, that there is still no applied field strength, and that the net magnetization of the sample is still zero. In the progression from view (a) to (b), the rod strained but the magnetization did not change. This will not be the case in the progression from (b) to (c) to (d). View (c) shows the rod in its magnetically biased state (assuming a positive $H$ points up). The rod is now longer due to the magnetomechanical coupling and the rotation of
the magnetic domains. View (d) depicts the rod when the field strength is increased so that the magnetization is approximately at its saturation value, which corresponds with the maximum strain of the rod.

The magnetization process consisting of domain rotation, as displayed in Figure 4.1, views (b) through (d), is very nearly a reversible process. If such a rod existed, it would display almost no magnetic hysteresis when operated in this range. Magnetic hysteresis is the result of domain wall motion. This phenomenon is discussed next.

Figure 4.2 depicts a less than perfect Terfenol-D rod, view (a). The rod is then subjected to a prestress (b). However, the prestressing was not sufficient to cause the top two domains to rotate back until they were perpendicular to the rod's axis.

View (c) depicts what occurs with an increase in H. (H = + implies an increase, ++ is a further increase.) Note that the bottom two domains have rotated slightly, resulting in a slight strain. Contrast
that with the top two domains where the net strain is still zero. These domains have not rotated, instead, the domain wall has moved. (Domain walls "move" when individual magnetic moments near the "wall" reverse, or flip.) The second domain is oriented favorably with respect to the applied field, so it grew at the expense of the top domain. If the applied H had been directed down, the top domain would have grown at the expense of the second. In either case, no net strain is realized from the top two domains because no rotation has occurred. However, they do contribute to the net axial magnetic moment of the sample. It is important to note that at this point, view (c), if the applied field were reduced significantly, the top domain would grow at the expense of the second. However, if the field were completely removed, the top domain would not resume its original size; it would be smaller and M would not return to zero. This is the primary source

\[
\begin{align*}
\sigma &= 0 \\
\varepsilon_{\text{top}} &= + \\
\varepsilon_{\text{bot}} &= 0 \\
H &= 0 \\
M &= 0
\end{align*}
\]

\[
\begin{align*}
\sigma_0 &= 0 \\
\varepsilon_{\text{max}} &= + \\
H_{\text{max}} &= + \\
M_{\text{sat}} &= +
\end{align*}
\]

\[
\begin{align*}
\sigma_0 &= + \\
\varepsilon_{\text{max}} &= + \\
H_{\text{max}} &= + \\
M_{\text{sat}} &= +
\end{align*}
\]

Figure 4.2. Schematic representations of a less than perfect four domain Terfenol-D rod depicting domain changes when the free rod (a) is subjected to a prestress that is too low (b), then an increasing H from (b) to (e). Also shown are the strains, referenced to the prestressed lengths, of the top and bottom pairs of domains.
of magnetic hysteresis. Energy is lost when domain walls move (individual moments flip).

For view (d), H has increased to the point that the domain wall has moved completely through the top domain leaving, in essence, only three domains in the sample. As shown, the three domains have also rotated, resulting in strain from the top and bottom domains. The progression from (d) to (e) is a rotation, thus it is a reversible process and all domains contribute to the strain of the rod.

The discussion above was incredibly simplistic. Commercially available Terfenol-D is composed of "billions and billions" of domains with a statistical distribution of domain orientations. As a result, all combinations of the behaviors discussed above (and tens of others omitted from the discussion) are likely to occur within any given rod. (Like most magnetic subjects, if you remove the "net," you are left with pure "magic.")

An experimental correlation between strain and magnetic moment has been reported by [8]. They report that the relationship is nonlinear, but very nearly single valued (no hysteresis was observed in their near DC tests). They were working with a very good sample and sufficient prestress levels. Not every rod/application will behave that nicely - especially if one buys a rod with a large number of domains oriented at 77° (one of the "behaviors" omitted from the previous discussion) instead of 180°. In this case, magnetization and strain are not single valued; hysteresis is almost always present.

The topic of ferromagnetodynamics was introduced in the last section. Reference [21] presents experimental evidence which suggests that ferromagnetodynamic effects can be neglected in Terfenol-D applications. In their investigation they concluded that the magnetization processes are independent of frequency, at least at frequencies below 1000 Hz. They also concluded that, "... anomalous eddy current behavior, which usually occurs around domain walls, is not significant."[21 p. 6176] Interestingly, their tests were nearly a worst case scenario; they were performed on rods with no prestress. As depicted above, a prestress on the material tends to orient the
magnetic domains within the material so that the magnetization process is a rotation of the intrinsic magnetic moments, as opposed to a hysteretic domain wall motion. Thus, in samples where the likelihood of domain wall motion was high, they still found the effects to be negligible.

4.1.3 Electromagnetism, the wound wire solenoid, and eddy currents

Recall "Faraday's law," a time varying magnetic flux within a circuit induces a voltage in that circuit. Further, according to Lenz, this induced voltage will be such that the resulting current flow will contribute a flux that opposes the change in the imposed flux. Restated: flow of an electric charge results in a magnetic flux density perpendicular to the charge flow and the electric field induced by varying a magnetic flux density will be perpendicular to the flux. In the case of a wound wire solenoid, a current in the solenoid, flowing in the positive \( \theta \) direction, results in a magnetic flux density vector in the positive \( z \) direction. Varying this density with time gives an electric field directed in the negative \( \theta \) direction. In the present case, the flux within the Terfenol-D rod is oscillating due to the current in the solenoid varying with time. Terfenol-D is a conductor, thus any annular ring within the material forms a circuit and will carry an induced current due to the induced electric field. These induced currents are known as "eddy currents." Their presence tends to decrease the applied field's penetration of the material, i.e., inner portions of the rod are magnetically "shielded." Also, since the eddy currents flow in a material with some resistance, there is a heating/ohmic loss associated with their presence. That is why induction furnaces work.

Consider, for a moment, an air solenoid. The complex valued electrical impedance of a long, tightly wound air solenoid is typically approximated as \( Z = V/I = R + j\omega L \), where \( Z \) is the complex valued impedance, \( V \) is the voltage across while \( I \) is the current through the solenoid, \( R \) is the solenoid's DC resistance, \( j = \sqrt{-1} \), \( \omega \) is the circular frequency, and \( L \) is the self inductance of the solenoid. Inductance, \( L \),
is defined as the total magnetic flux linkage divided by the impressed current: \( L = N\Phi_m/l \) where \( N \) is the number of turns of the solenoid. In the case of an air solenoid, \( L \) can also be approximated as the product of the magnetic permeability of free space, the square of the turns per unit length of the solenoid, and the solenoid's volume, i.e., \( L = \mu_0 n^2 \text{Vol} \). Equivalently, \( L = \mu_0 N^2 A/l \) where \( A \) is the area (of the bore) and \( l \) is the solenoid's length. Note that both of these representations for the inductance of an air solenoid are real valued constants which represent physical parameters of the solenoid. If one experimentally measured \( Z \) (over the audible frequency range, say, 0 to 40000\( \pi \) rad./sec) they should find that its real component was basically constant with frequency and that its imaginary component increased linearly with frequency (with a slope of \( L \)). An air solenoid should be a good approximation of a first order system, i.e., its magnitude increases at 20 dB per decade and its phase begins at zero and goes to +90°. The real part of \( Z \) represents the energy dissipation mechanism, i.e., the ohmic losses, while the imaginary part represents the energy storage mechanism (energy is stored in the magnetic field down the bore of the solenoid and the field present in the windings).

Matters become more complicated with a conductor present. In particular, if one is driving a piece of Terfenol-D with an electric solenoid and then measures the input voltage and current of the solenoid, the ohmic losses due to eddy currents will appear as a resistance that increases with drive frequency, i.e., Real(\( Z \)) increases with frequency. The reduction in penetration (reduction in flux, thus flux linkage) will appear as an inductance that decreases with frequency: Imag(\( Z \))/\( \omega \) generally decreases with frequency. Following the lead above, the electrical impedance of a wound wire solenoid with Terfenol-D (or any conductor) as the core material is modelled as: \( Z = R_{\text{DC}} + j\omega L_T \) where \( R_{\text{DC}} \) is the DC resistance of the coil while the complex valued inductance is defined as \( L_T = a + jb = N\Phi_m / l \). Thus, \( Z = \{(R_{\text{DC}} - \omega b) + j\omega a\} \) where it is anticipated that both \( a \) and \( b \) are real numbers that should vary with frequency, \( a > 0 \), \( b \leq 0 \). The quantity \( \{-\omega b\} \) represents the ohmic losses due to eddy currents within the
conducting core; thus, as discussed above, it should increase with frequency. Likewise, \( \{a\} \) should decrease with frequency. No longer can the inductance be calculated as a constant, dependant primarily on the geometry of the coil.

The temptation to attribute eddy current effects to some strange permeability change of Terfenol-D, though traditional, will be resisted. What? One may be tempted to say \( L = \mu N^2/A/l \) where \( \mu \) is now an area averaged, complex valued function of frequency, calculated from simple theory or experimental impedance measurements of a solenoid with Terfenol-D in its bore. *This will not be done here.*

For this investigation, the permeability of Terfenol-D will be that given by Eqn. (3.3.13a). That expression will be used for low signal operation. For higher drive amplitudes, Eqn. (3.3.13b) could be used, where a frequency independent phase lag would be introduced in an attempt to model the magnetic hysteresis and the hysteresis existing between displacement and drive current. In either case, the permeability of Terfenol-D will be affected by the dynamics of the transducer (assumed fixed) and by the dynamics of the load (which presumably vary from one application to the next). Since the load affects the permeability, and the permeability effects the eddy currents present within the transducer components (shown in the next section), eddy current losses, though they occur within the transducer, are not an unchanging characteristic of the transducer. Eddy current losses also depend upon the load. In this investigation, once armed with one of the above forms of Terfenol-D permeability, the effects of eddy currents on the applied field strength, \( H \), will be mathematically modelled. Then, electrical impedance estimates will be calculated.

The magnetic permeabilities of the various other components of the transducer will be assumed to take the form of Eqn. (4.1.3), a linear relationship for low signal operation. Eqn. (4.1.5), a frequency independent elliptical relationship, could be used for higher drive amplitudes.
4.2 Electromagnetic Model Development

Begin with Maxwell's equations and neglect the displacement current terms (assumed negligible in a conductor).

\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \]  \hspace{1cm} (4.2.1)

\[ \nabla \times \mathbf{H} = \mathbf{J} \]  \hspace{1cm} (4.2.2)

\[ \nabla \cdot \mathbf{B} = \mathbf{V} \cdot \mathbf{B} = 0 \]  \hspace{1cm} (4.2.3)

where: \( \mathbf{E} \) is the electric field vector, volts/meter, 
\( \mathbf{B} \) is the magnetic flux density vector, Tesla, 
\( \mathbf{H} \) is the magnetic field strength vector, amp-turns/meter, and 
\( \mathbf{J} \) is the current density, amps/meter².

Eqn. (4.2.1) is "Faraday's law," Eqn. (4.2.2) is "Ampere's law," and Eqn. (4.2.3) is "Gauss' law of magnetism." Eqn. (4.2.4) is "Ohm's law" and Eqn. (4.2.5) is the standard linear B-H magnetic model.

\[ \mathbf{J} = \sigma_e \mathbf{E} \]  \hspace{1cm} (4.2.4)

\[ \mathbf{B} = \mu \mathbf{H} \]  \hspace{1cm} (4.2.5)

where: \( \sigma_e \) is the conductivity of the material, mho/meter, and 
\( \mu \) is the magnetic permeability of the substance, 
Tesla meter/amp turn = henries/meter.

Cylindrical coordinates, \( r, \theta, z \), and the following assumptions will be employed in the model development:

i) no axial (z) variations (neglects end effects in transducer)

ii) displacement current is negligible

iii) a linear magnetic model, Eqn. (4.2.5)

iv) no \( \theta \) dependence

v) spatially constant physical properties, i.e., \( \mu, \sigma_e \), etc.

vi) \( \mathbf{H}(r,\omega,t) = \mathbf{H}(r,\omega) e^{i\omega t} \mathbf{e}_z \), i.e., using separation of variables (\( \mathbf{e}_z \) is the unit vector in the z direction)

vii) neglect ferromagnetodynamic effects

Rationale of assumptions:

i) For the transducers in question, it was felt that the solenoid was a reasonable approximation of the ideal. For example, for \( H = nI = 23500 \) amp-turns per meter, the experimentally measured field strength at the center of the solenoid, approximately on the center line
of the solenoid, was $\frac{294 \times 1000}{(4\pi)} = 23400$ amp-turns/meter, a
difference of less than one percent. Probably a better indication of the
applicability of this assumption is a comparison of inductance values
implied by different formulae, for example, $L = \mu \pi r^2 n^2 \left\{ \frac{(l^2 + r^2)^{1/2}}{2} \right\}$. The reduction in inductance due to end effects for these solenoids is
approximately six-percent. Material properties are generally not
known to that accuracy. A third justification for this assumption is
that finite element programs have shown that the magnetic field
within a solenoid equipped with steel end caps is nearly constant along
the length of the solenoid, at least at low frequencies. The last
argument for this assumption is detailed in Chapter 5, but says in
essence, the rod is centered lengthwise within the wound wire
solenoid and that that portion of the solenoid is where the axial
variations in applied field strength are the least pronounced.

ii) Within the conductors present in the transducer, neglecting
displacement currents is a standard assumption.\[12\] It is also
applicable for those regions of the transducer where air is the medium.

iii) A linear magnetic model is a place to begin. Results of this
model will be compared with experimental evidence gathered when the
transducer was operating in the linear region (that where linear
systems analysis provides very good predictions of transducer
behavior). Drive current amplitudes will be "small" implying that it
will be operating magnetically within a stable minor hysteresis loop.
A complex permeability, as in Eqn. (4.1.5), could be used for modelling
transducer behavior at higher current amplitudes. Dynamic effects
will be included via a complex, frequency dependant permeability, that
is, $\mu$ in Eqn. (4.2.5) will be $\mu T$, as given in Eqns. (3.3.13).

iv) No angular dependence seems reasonable due to the physical
symmetry of the transducer. This assumption is consistent with the
ideas of a homogeneous continuum, i.e., local variations in material
behavior are considered "small," whether they are or not.

v) Physical properties are assumed to be spatially constant.
Terfenol-D properties are hoped to average to some meaningful value
(not necessarily the published values). If it turns out that the
properties do not average to meaningful values, there does not seem to
be any advantage in trying to formulate an analytical solution for the
electromagnetic side of the transducer. The equations become very
complicated when one uses tensor conductivities and permeabilities.
Hopes of gaining insight from the mathematical model wane in that
case.

vi) Separation of variables is the standard technique for solving
partial differential equations when seeking a steady-state solution by
assuming a sinusoidal time variation of the quantities. In the present
case, the input current will be varying sinusoidally and it is assumed
that other quantities will do likewise. This development is for the
time varying or alternating field strength: \( H(r,\omega,t, ...) = H_{\text{total}} - H_0 \).

vii) Ferromagnetodynamic effects will be neglected owing to having
no idea how they may enter the problem at high frequencies. (Recall
that reference [21] worked at frequencies up to only 1000 Hz.)

Assumptions i and iv reduce the problem to one-dimension, which is
why assumption vi shows only one spatial variable, \( r \). In the following
discussion the time dependance cancels and the radial and frequency
dependance is assumed. The object of this endeavor is to obtain a
solution for \( H \) as a function of radial position, frequency, and time.

Eqn. (4.2.2) becomes:

\[
-\frac{\partial H}{\partial r} e_\theta = J = \sigma_e E = \sigma_e E e_\theta, \quad \text{or}
\]

\[
E = \frac{1}{\sigma_e} \frac{\partial H}{\partial r},
\]

(4.2.6)

where: \( e_\theta \) is the unit vector in the \( \theta \) direction,
an equation number below an equals sign indicates the
equation employed in writing the equality, and
\( E \) is the magnitude of the vector \( E \), which in the one-
dimensional case represents no loss of information, that
is, it is now known that the only component of \( E \) exists in
the \( \theta \) direction.

Eqn. (4.2.1) evolves as:
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E}{\partial t} \right)_z = -\frac{\partial B}{\partial t} = -\frac{\partial B}{\partial t} e_z = -\mu \frac{\partial H}{\partial t} e_z = -j \omega \mu H e_z \quad (4.2.7)
\]

since the partial with respect to time of \(\mu H\) is simply \(j \omega \mu H\) via assumption vi. Dropping the unit vector (note, first, that the only component of \(H\) is in the \(z\) direction, along the axis of the transducer) and using Eqn. (4.2.6) for \(E\) in Eqn. (4.2.7) yields a second order partial differential equation for \(H\).

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \left\{ -\frac{1}{\sigma_0} \frac{\partial H}{\partial r} \right\} \right) = -j \omega \mu H \rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H}{\partial r} \right) = j \omega \mu \sigma_0 H
\]

thus,

\[
\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \frac{\partial H}{\partial r} - j \omega \mu \sigma_0 H = 0. \quad (4.2.8)
\]

One now typically defines \(k\) (not to be confused with any of the \(k\)'s in Chapter 3) as:

\[
k = \sqrt{j \omega \mu \sigma_0} \quad (4.2.9)
\]

so Eqn. (4.2.8) becomes

\[
\frac{\partial^2 H}{\partial r^2} + \frac{1}{r} \frac{\partial H}{\partial r} - k^2 H = 0 \quad (4.2.10)
\]

Friedrich Wilhelm Bessel (1784-1846) used series methods to find the solution for partial differential equations like this one. As a consequence, Eqn. (4.2.10) is known as a modified Bessel equation of order zero with a general solution of the form:

\[
H(r, \omega) = c_1 I_0(\text{kr}) + c_2 K_0(\text{kr}) \quad (4.2.11)
\]

where: \(I_0(\text{kr})\) is known as the \textit{modified Bessel function of the first kind of order zero}, i.e., \(I_0(\text{kr}) = (-j)^0 J_0(j\text{kr}) = J_0(j\text{kr})\), and \(K_0(\text{kr})\) is known as the \textit{modified Bessel function of the second kind of order zero}. (Definitions of this function vary between authors.[15-18])

Regardless, as \(\text{kr} \to 0\), \(I_0 \to 1\) and \(K_0 \to +\infty\). Note, \(k\) has dimensions of \(1/\text{length}\) while \(r\)'s dimensions are \text{length}; therefore, \(\text{kr}, I_0(\text{kr}),\) and \(K_0(\text{kr})\) are all dimensionless.

Bessel functions, aka cylindrical harmonic functions, are generally found in "special functions" sections in physics texts or one can find
entire books written about them. The problem here was simply to be certain that the calculations of the functions were performed correctly on a digital computer. The software employed was a student version of MATLAB. It included routines for calculating integer order Bessel and Hankel functions, both of the first type. Recall, Hankel functions are the complex combination of Bessel functions of the first and second type (the second type also known as a Neumann function). Things get interesting rather quickly. "Important" results and characteristics of Bessel functions will be quoted from [16] as needed below.

Eqn. (4.2.11) is the solution sought for the applied field in a cylindrical conductor. How (much more?) complicated does the model need to be? Are most of the eddy effects due to currents within the Terfenol-D rod? How about eddy currents in the steel end caps of these transducers (about which this model implies nothing)? Must the effects of eddy currents occurring in the cylindrical permanent magnet (or any conducting housing) be included? If so, which permanent magnet eddy currents are important? Those due to varying the flux within the cylindrical electric circuit formed by the magnet, or those due to the return flux (again, ignored by the model)?

To answer questions like these, physical models of the transducer were assumed, analytical expressions were derived for magnetic flux as a function of radial position, then electrical impedance functions were calculated employing standard definitions of inductance and magnetic flux linkage. These functions were then compared with experimentally measured electrical impedances ($Z_{ee}$). A good match of experiment with model implied a good model.

Figure 4.3 is a section view of the Terfenol-D transducer in this study. Shown is the view one would see if one were looking down the z axis of the transducer with its end caps removed. Also shown in the figure is the nomenclature used for various radii. Recall the discussion of leakage flux in Chapter 3. As shown in the figure, there is an air gap between the rod and solenoid, and a substantial thickness to the wound wire solenoid. As a result, there is a considerable
amount of magnetic flux affecting the voltage over the solenoid which does not pass through the Terfenol-D rod.

4.3 Transducer Model 1

Considering only the rod, that is, ignoring the external permanent magnet and the air-gap between the Terfenol-D rod and the wound wire solenoid, yields the following boundary conditions applicable to Eqn. (4.2.11):

i) \( H(0) = \text{finite} \Rightarrow c_2 = 0 \), and

ii) \( H(R_{To}) = nI \Rightarrow c_1 = nI/I_0(kR_{To}) \).

Boundary condition ii) implies: \( H \) must be a continuous function of \( r \), that \( H_{\text{solenoid}} = nI \), and that the field resulting from the eddy currents within the Terfenol-D rod is zero outside the rod. Thus, for this transducer model, Eqn. (4.2.11) becomes:
This is the classic solution for a conducting rod in a wound wire solenoid,[19] though it is traditionally written in terms of Kelvin functions, ber and bei, and their derivatives, ber' and bei', when applied to a Terfenol rod.[20, 11] The approach detailed here will be left in terms of modified Bessel functions since they have been studied extensively and computer programs for calculating these functions are usually available.

Assuming each layer of the solenoid has the same flux linkage, and that the rod fills the solenoid, the expression for the electrical inductance is:

\[ L(\omega) = \mu n^2 (\pi R_{to}^2) \left\{ \frac{2}{kR_{to}} \frac{l_1(kR_{to})}{l_0(kR_{to})} \right\} \]  

(4.3.2)

where the term in the parentheses is recognized as the volume of the solenoid and that in the braces represents the effects of eddy currents on the standard inductance of a wound wire solenoid. Because the arguments of the modified Bessel functions, \( kR_{to} \), are complex, the inductance as predicted by the model is also a complex valued function (recall the previous discussion of \( L_T = a + jb \)).

4.4 Transducer Model 2

The cylindrical conducting permanent magnet was ignored in the development of the previous model. A simplistic investigation of the possible effects of eddy currents induced in the magnet (see Figure 4.3) was performed. A resistance for the permanent magnet was estimated, say \( R_{pm} \). The time rate of change of magnetic flux within the circuit formed by the cylindrical permanent magnet, say \( d\Phi_m/dt \), was estimated by integrating \( \mu H(r) \) over the area of the rod (\( H(r) \) from the first model), plus the area of the air gap (\( H = \text{constant} = nI \)), plus the area of the windings of the solenoid (\( H(r) = nI(R_{so} - r) / (R_{so} - R_{si}) \)), i.e., linear with \( r \). The eddy current within the permanent magnet was estimated via Faraday's induction formula: \( V = (-d\Phi_m/dt) \), so \( l_{\text{eddy pm}} = \)
\[ V/R_{pm} = (-d\Phi_m/dt) / R_{pm} \]. \( H \) from the eddy current in the permanent magnet was compared with \( H \) from the solenoid \( (H_{sol} = nI = Nl/length, H_{eddy\_pm} = 1.0 I_{eddy\_pm} / length \), since there is only one "turn" of the permanent magnet). Some representative values are shown in Table 4.1 for the transducer under study. As demonstrated by these values, the effects of eddy currents within the permanent magnet become increasingly significant with increasing frequency of excitation. A more rigorous development follows.

For the second transducer model, the effects of the eddy currents in the external cylindrical permanent magnet will be included. In addition, it will be assumed that \( H(R_T) \neq nI \). Maxwell's equations will be applied to each region of material within the transducer (recall Figure 4.3). The magnetic field strength, \( H \), will be assumed to be continuous with \( r, 0 \leq r \leq R_{pm} \), though none of its derivatives need be continuous at the material interfaces. Note, there will be parameters for each region of material, i.e., there will be a \( k_T \) and \( k_{pm} \) applicable to the Terfenol-D and permanent magnet, respectively.

Boundary conditions (BC) for this problem are:

i) \( H(0, \omega) = \text{finite} \),

ii) \( H(R_{pm}, \omega) = 0 \), and

iii) \( H(r, \omega) \) is continuous at material interfaces.

Table 4.1. Estimates of relative magnetic field strength, magnitude and phase, due to induced eddy currents in conducting external cylindrical permanent magnet

| Frequency, \( f \), Hz | || \( H_{eddy\_pm} / H_{sol} || \) | phase \( H_{eddy\_pm} / H_{sol} \) |
|------------------------|-------------------------------|------------------|
| 1                      | 0.0                           | -90°             |
| 100                    | 0.003                         | -90°             |
| 1000                   | 0.03                          | -94°             |
| 5000                   | 0.14                          | -106°            |
| 7000                   | 0.19                          | -111°            |
| 10000                  | 0.24                          | -117°            |
The second boundary condition is an approximation which assumes that most of the action is occurring within the transducer.

**REGION ONE: 0 ≤ r ≤ R_{T0} (material in region is Terfenol-D)**

The governing differential equation is, as in the first transducer model, Eqn. (4.2.10), with solution Eqn. (4.2.11). Boundary condition i) implies that \( c_2 \) is again zero. Evaluating at \( r = R_{T0} \) implies:

\[
H(R_{T0}) = c_1 I_0(k_T R_{T0}) \quad \Rightarrow \quad c_1 = \frac{H(R_{T0})}{I_0(k_T R_{T0})}
\]

thus

\[
H(r) = H(R_{T0}) \frac{I_0(k_T r)}{I_0(k_T R_{T0})} \quad 0 \leq r \leq R_{T0}. \tag{4.4.1}
\]

Keep in mind, however, that as yet the value of \( H(R_{T0}) \) is not known.

**REGION TWO: R_{T0} < r ≤ R_{si} (material in region is air)**

In air, the current density, \( J \), is zero. Thus, Eqn. (4.2.2) becomes:

\[
cur(H) = 0 \quad \text{or} \quad -\frac{\partial H}{\partial r} = 0.
\]

As a result,

\[
H(r) = \text{constant} = H(R_{T0}) \quad R_{T0} < r \leq R_{si}. \tag{4.4.2}
\]

**REGION THREE: R_{si} < r < R_{so} (material in region is copper wire)**

The behavior of the applied field within the solenoid will be approximated by assuming the current density is known, i.e., \( J = I/A_{cu} \varepsilon_\theta \), where \( I \) is the input current and \( A_{cu} \) is the cross sectional area of the copper wire. With this approximation, Eqn. (4.2.2) becomes:

\[
cur(H) = \frac{I}{A_{cu}}\varepsilon_\theta \quad \text{or} \quad -\frac{\partial H}{\partial r} = \frac{I}{A_{cu}}.
\]

Integrating to obtain \( H \) yields:

\[
H(r) = -\frac{I}{A_{cu}} r - c_3.
\]

Evaluating at \( r = R_{si} \), where \( H \) is known via Eqn. (4.4.2), gives an expression for \( c_3 \) as:

\[
c_3 = -H(R_{T0}) - \frac{I}{A_{cu}} R_{si}.
\]
\[ H(r) = -\frac{1}{A_{cu}} \left( -H(R_{T0}) \right) - \frac{1}{A_{cu}} R_{si} = H(R_{T0}) - \frac{1}{A_{cu}} (r - R_{si}). \] (4.4.3)

It is not likely to be immediately apparent, but \( A_{cu} = (R_{so} - R_{si})/n \) to the order of the above approximation. As a convenient result, Eqn. (4.4.3) becomes:

\[ H(r) = H(R_{T0}) - n \frac{1}{(R_{so} - R_{si})} (r - R_{si}) \quad R_{si} < r < R_{so}. \] (4.4.4)

Note that this analysis resulted in a linear relationship for the field strength as a function of radius through the wound copper wires of the solenoid.

REGION FOUR: \( R_{so} < r < R_{pmi} \) (material in region is air)

Like in region two, \( H(r) \) is constant. The constant is obtained by invoking BC iii) at \( r = R_{so} \) in Eqn. (4.4.4). Thus:

\[ H(r) = H(R_{T0}) - n \frac{1}{(R_{so} - R_{si})} \quad R_{so} < r < R_{pmi}. \] (4.4.5)

REGION FIVE: \( R_{pmi} < r < R_{pmo} \) (material in region is cast Alnico V, a permanent magnet)

The governing differential equation is again Eqn. (4.2.10) with a general solution of the form:

\[ H(r) = c_4 I_0(k_{pm} r) + c_5 K_0(k_{pm} r), \] (4.4.6)

where \( k_{pm} \) is simply Eqn. (4.2.9) using the properties of the permanent magnetic material. BC ii) gives \( c_5 \) in terms of \( c_4 \) as:

\[ c_5 = -c_4 \frac{I_0(k_{pm} R_{pmo})}{K_0(k_{pm} R_{pmo})}. \] (4.4.7)

Continuity of \( H(r) \) at \( r = R_{pmi} \) yields \( c_4 \) in terms of \( n_l \) and \( H(R_{T0}) \) (and a few assorted modified Bessel functions). In particular:

\[ H(R_{pmi}) = \frac{H(R_{T0})}{(4.4.5)} - n \frac{I_0(k_{pm} R_{pmi})}{K_0(k_{pm} R_{pmo})} K_0(k_{pm} R_{pmi}), \]

solving for \( c_4 \) gives:

\[ c_4 = \frac{|H(R_{T0}) - n l|}{\left\{ I_0(k_{pm} R_{pmi}) - \frac{I_0(k_{pm} R_{pmo})}{K_0(k_{pm} R_{pmo})} K_0(k_{pm} R_{pmi}) \right\}}. \] (4.4.8)
Combining Eqns. (4.4.7), (4.4.8) and/into (4.4.6) results in the expression shown in Eqn. (4.4.9) for the alternating component of the field strength within the permanent magnet.

\[
H(r) = \left\{ \begin{align*}
\frac{I_0(k_{pm}R_{pm})}{K_o(k_{pm}R_{pm})} & \quad 0 < r \leq R_{pm} \\
\frac{I_0(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} & \quad R_{pm} < r \leq R_{pmo}
\end{align*} \right. 
\]

Expressions have been derived for \(H(r)\) throughout the transducer in terms of \(n_l\), the drive field strength, and \(H(R_{To})\), the yet unknown field strength at the surface of the Terfenol-D rod. To determine \(H(R_{To})\) another equation is needed. Following the lead of Stoll,[12] apply Faraday's induction relation at the inner radius of the cylindrical permanent magnet, i.e.,

\[
[V]_{R_{pmi}} = \left[ -\frac{d\phi_m}{dt} \right]_{R_{pmi}} 
\]  
\[
(4.4.10)
\]

where the LHS is calculated as:

\[
[V]_{R_{pmi}} = 2 \pi R_{pmi} [E]_{R_{pmi}} = 2 \pi R_{pmi} \left[ -\frac{1}{\sigma_{epm}} \frac{\partial H}{\partial r} \right]_{R_{pmi}} 
\]

\[
(4.4.6)
\]


\[
[V]_{R_{pmi}} = \frac{-2 \pi R_{pmi}}{\sigma_{epm}} \left[ (H(R_{To}) - n_l) \frac{\partial}{\partial r} \left\{ \frac{I_0(k_{pm}R_{pm})}{K_o(k_{pm}R_{pm})} \right\} \right. \\
\left. \{-I_0(k_{pm}R_{pmo})/K_o(k_{pm}R_{pmo})\} \right]_{R_{pmi}} 
\]

(substituting, \(x = k_{pm}r\) and using \(\partial / \partial r = (dx/dr) \partial / \partial x = k_{pm} \partial / \partial x\))

\[
[V]_{R_{pmi}} = \frac{-2 \pi R_{pmi}}{\sigma_{epm}} \left\{ \left\{ \frac{I_0(k_{pm}R_{pmo})}{K_o(k_{pm}R_{pmo})} \right\} \right. \\
\left. \{-I_0(k_{pm}R_{pmo})/K_o(k_{pm}R_{pmo})\} \right\} \] 

(assuming standard modified Bessel function relations: \(\partial I_0(x)/\partial x = I_1(x)\), and \(\partial K_0(x)/\partial x = -K_1(x)\))
\[ |V|_{R_{\text{pmi}}} = -\frac{2\pi R_{\text{pmi}} k_{\text{pmi}}}{\sigma_{\text{pmi}}} \left[ \frac{I_0(k_{\text{pmi}} R_{\text{pmi}})}{K_0(k_{\text{pmi}} R_{\text{pmi}})} + \frac{I_0(k_{\text{pmi}} R_{\text{pmi}})}{K_0(k_{\text{pmi}} R_{\text{pmi}})} \right] (H(R_{\text{To}}) - n1) \]

\[ \sigma_{\text{pmi}} = \frac{\sigma_{\text{pmi}}}{k_{\text{pmi}}} \]

(4.4.11)

\[ |V|_{R_{\text{pmi}}} = -c_x (H(R_{\text{To}}) - n1). \]

The RHS of Eqn. (4.4.10) is calculated by taking the time derivative of the total magnetic flux occurring within the four regions described by \(0 \leq r < R_{\text{pmi}}\). The time derivative is simply \(j_0\) times the flux, and the total flux is

\[ \phi_m = \int_0^{R_{\text{pmi}}} \mu_0 H(\rho)2\pi \rho d\rho = \Phi_{m1} + \Phi_{m2} + \Phi_{m3} + \Phi_{m4} \] (4.4.13)

where each of the fluxes come from integration of the appropriate kernel with the corresponding limits.

REGION ONE: \(0 \leq \rho \leq R_{\text{To}}\) (material in region is Terfenol-D) \(H(\rho)\) via Eqn. (4.4.1)

\[ \Phi_{m1} = \int_0^{R_{\text{To}}} \mu \pi R_{\text{To}}(\rho) 2\pi \rho d\rho = \int_0^{R_{\text{To}}} \mu_0 H(R_{\text{To}}) \frac{I_0(k_T \rho)}{I_0(k_T R_{\text{To}})} 2\pi \rho d\rho \]

\[ \Phi_{m1} = \mu_0 \pi R_{\text{To}}^2 \left[ \frac{2}{k_T R_{\text{To}} I_0(k_T R_{\text{To}})} \right] H(R_{\text{To}}) = c_{10} H(R_{\text{To}}) \] (4.4.14)

REGION TWO: \(R_{\text{To}} < \rho \leq R_{\text{si}}\) (material in region is air) \(H(\rho)\) via Eqn. (4.4.2)

\[ \Phi_{m2} = \int_{R_{\text{To}}}^{R_{\text{si}}} \mu_0 H(R_{\text{To}}) 2\pi \rho d\rho = \mu_0 \pi \left( \frac{R_{\text{si}}^2 - R_{\text{To}}^2}{R_{\text{si}}^2 - R_{\text{To}}^2} \right) H(R_{\text{To}}) = c_{20} H(R_{\text{To}}) \] (4.4.15)

REGION THREE: \(R_{\text{si}} < \rho < R_{\text{so}}\) (material in region is copper) \(H(\rho)\) via Eqn. (4.4.4)

\[ \Phi_{m3} = \int_{R_{\text{si}}}^{R_{\text{so}}} \mu_0 \left[ H(R_{\text{To}}) - n1 \right] \frac{(\rho - R_{\text{si}})}{(R_{\text{so}} - R_{\text{si}})} 2\pi \rho d\rho \]

\[ = \mu_0 \pi \left( \frac{R_{\text{so}}^2 - R_{\text{si}}^2}{R_{\text{so}}^2 - R_{\text{si}}^2} \right) H(R_{\text{To}}) \cdot n1 \]

\[ = c_{30} H(R_{\text{To}}) - c_{31} n1 \] (4.4.16)

REGION FOUR: \(R_{\text{so}} < \rho < R_{\text{pmi}}\) (material in region is air) \(H(\rho)\) via Eqn. (4.4.5)
Eqn. (4.4.13) can now be written in terms of the constants defined in Eqns. (4.4.14) through (4.4.17):

\[
\Phi_m = (c_{10} + c_{20} + c_{30} + c_{40})H(R_{To}) - (c_{31} + c_{40})nI \tag{4.4.18}
\]

with a time derivative given as:

\[
\frac{\partial \Phi_m}{\partial t} = j\omega \Phi_m = j\omega (c_{10} + c_{20} + c_{30} + c_{40})H(R_{To}) - j\omega (c_{31} + c_{40})nI. \tag{4.4.19}
\]

Eqn. (4.4.10) can now be rewritten using Eqns. (4.4.12) and (4.4.19):

\[
c_x(H(R_{To}) - nI) = j\omega (c_{10} + c_{20} + c_{30} + c_{40})H(R_{To}) - j\omega (c_{31} + c_{40})nI.
\]

solving for the only unknown, H(R_{To}), yields:

\[
H(R_{To}) = \frac{c_x - j\omega (c_{31} + c_{40})}{c_x - j\omega (c_{10} + c_{20} + c_{30} + c_{40})} nI. \tag{4.4.20}
\]

One can now calculate H(R_{To}) as a function of the parameters of the problem. Use of this quantity will enable estimates of H(r,\omega) via Eqns. (4.4.1), (4.4.2), (4.4.4), (4.4.5), and (4.4.9). In addition, the electrical impedance of the transducer can now also be estimated - including the effects of eddy currents. If one ignores the magnetic flux occurring within the copper windings of the solenoid (equivalent to assuming that the solenoid is "thin"), an estimate of the inductance for a solenoid of N turns would be:

\[
L = \frac{Nd_m}{I} = \frac{N(\Phi_{m1} + \Phi_{m2})}{I}, \tag{4.4.21}
\]

where \(\Phi_{m1}\) and \(\Phi_{m2}\) are defined above in Eqns. (4.4.14) and (4.4.15), respectively.

A more complicated, but better approximation of the inductance of the transducer is obtained by considering the flux linkage of each layer of the solenoid. All of the layers of the solenoid link with \(\Phi_{m1}\) and \(\Phi_{m2}\). However, the outer layer experiences more flux linkage than the inner layer, owing to the flux occurring within the coil windings. An approximation of the flux as a function of radial position within the windings, \(\Phi_{mi}\), can be obtained by integrating the expression for \(\Phi_{m3}\).
above from $R_{si}$ to $r_i$, where $r_i$ is the inner radius of the $i$th layer of the coil. The resulting expression is:

$$\Phi_{mi} = \mu_0 \pi \left( r_i^2 - R_{si}^2 \right) \left[ H( R_{To} ) + \frac{R_{si}}{R_{so} - R_{si}} n_1 \right] - \frac{2 \mu_0 \pi}{3} \frac{r_i^3 - R_{si}^3}{R_{so} - R_{si}} n_1. \quad (4.4.22)$$

Assuming that the solenoid has $N$ turns and $p$ layers ($N/p$ would be an approximation of the turns per layer), an estimate of the inductance would be calculated via:

$$L = \frac{N \Phi_m}{I} = \sum_{i=1}^{p} \frac{N}{p} \left( \Phi_{m1} + \Phi_{m2} + \Phi_{mi} \right). \quad (4.4.23)$$

Eqn. (4.4.23) was the relation used to calculate the inductances reported in this dissertation.

For either of the eddy current models to be useful in a general application, one would need to know the material parameters beforehand. It is hoped that a general form of, for example, $\mu_T( ||H||, f, \text{prestress}, \text{displacement}, \text{magnetic circuit}, \text{etc.})$ can be identified for a given Terfenol-D stoichiometry. That was not done in this study; however, if that relation can be identified, then perhaps a couple of measurements, the results of which would be supplied with a piece of Terfenol-D from the manufacturer, could allow the ultimate user to employ this model to predict electrical impedance, and other quantities of engineering interest, before the transducer is built. This subject is addressed again in Section 5.8 and again in Appendix E.

For the present investigation, material properties will be taken from the literature where possible. The balance of the parameters will come from experimental measurements.
5. APPLICATION OF MODELS AND COMPARISONS WITH EXPERIMENTS

Magnetomechanical models were derived in Chapter 3. Two electromagnetic models were derived in Chapter 4. The goal of this chapter of the dissertation is to experimentally verify the combination of the magnetomechanical and electromagnetic models. Towards that end, this chapter will begin with a discussion of the assumptions inherent to the models, introduce the experimental set-up, discuss the primary transducer used in this investigation, and explain what was done to approximate the assumptions experimentally. Attention will then turn to estimating model parameters from easily obtained transducer measurements. Time domain measurements of transducer performance are presented for a variety of drive currents to demonstrate the range over which a linear model might be expected to provide reasonable estimates of transducer performance. These measurements reveal two types of nonlinearities in the material. Transducer performance is then investigated in the frequency domain. Three types of simulations are presented for electrical impedance, \( Z_{ee} \), and displacement from current. The first ignores all eddy current effects, the second type is one where eddy currents occur in only the rod, the third type includes the effects of eddy currents in the external cylindrical housing. By comparing these simulations, the effects of eddy currents on transducer performance is examined. Finally, model simulations of the transducer's electrical impedance and displacement from current are compared with experimental measurements for the case where the transducer has a conducting cylindrical external housing (transducer Model 2 of Chapter 4). The other two cases are addressed in Appendix E. Transducer Model 1 is approximated experimentally with a longitudinally slit ("C" shaped) external magnet and a one-piece magnetostrictive rod. The no eddy current case is approximated experimentally by using a laminated rod in the slit housing.
5.1 Discussion of Model Assumptions as Applied to Experiment

The magnetomechanical models developed in Chapter 3 came from a comparison of the transduction equations for the magnetostrictive material with the equations for the transducer containing the material. The models are mathematical statements for the magnetic permeability of the magnetostrictive material within the transducer. They are given by Eqns. (3.3.13).

The major assumptions in the derivation of the models were, low signal, linear operation, and that stress, strain, magnetic flux density, and applied field strength were all independent of axial position. Therefore, if there is hope of the model approximating experiment, one must run the transducer in the low signal range and the mechanical load must be such that the rod behaves like a linear spring (if it does not already when the transducer is unloaded).

For the simulations presented below, typical low frequency strains (100 to 300 Hz) were limited to approximately $\pm 4$ to $30 \times 10^{-6} \text{ m/m}$, which are small when compared to the "giant" strains of $\pm 500 \times 10^{-6} \text{ m/m}$ mentioned in Chapter 1. Thus, the transducer was run in the "low" signal range. Strains at resonant frequencies were usually about six or seven times the low frequency strains, yielding a more respectable $\pm 200 \times 10^{-6} \text{ m/m}$.

The transducer built for this investigation was designed with dynamic operation in mind; thus the internal dynamic mass was minimized. As a consequence, an external mass was required to cause the Terfenol-D rod to behave as a linear spring.

As to the last assumption, the independence of flux density and field strength with axial position was not experimentally verified. However, a numerical approach yielded encouraging results. Reference [9, p. 682], derives a formula for the flux density along the axis of a single layer, tightly wound air filled solenoid. This formula was employed in a Fortran program, "itsolenoid.f" in Appendix C, to estimate the axial flux density as a function of axial position along the center-line of the solenoid. The results of the calculation are shown in Figure 5.1. Over the length of the solenoid which would
contain the Terfenol-D rod, the average flux density was calculated as 0.95 times the maximum value (the maximum occurs along the centerline, in the lengthwise center of the coil). As mentioned in Section 4.2, finite element programs have shown that steel end caps tend to decrease the axial variations of a solenoid. Figure 5.1 is applicable to an air solenoid - without steel end caps. Therefore, the figure is something like a worst case, at least for low frequencies. At higher frequencies, one might anticipate that eddy currents in the steel ends would tend to reduce their apparent permeability; thus axial fluctuations would likely return. All in all, though the axial independence of flux density (and field strength) is probably not strictly correct, it was thought to be a reasonable engineering assumption.

Figure 5.1. Plot of calculated normalized axial field, $H_z(z)/H_{z\,\text{max}}$, versus axial position referenced to the lengthwise center of the solenoid and normalized so that the rod occupies (-1, 1). Calculations were performed using "itsolenoid.f," in Appendix C, assuming a simple air solenoid.
5.2 Experimental Set-Up

Electric current is the variable most closely related to the magnetic side of these transducers (recall, \( H = nl \)). Therefore, in this study, quantities of engineering interest are related to drive current, as opposed to drive voltage. With that aim, the transducer under study was powered by a high impedance amplifier, something akin to a "current source." Specifically, a Techron 7520 power supply amplifier was used. It was fitted with their optional current control module, 75A08. When properly adjusted, the amplifier/module system was supposed to adjust the output voltage as required to make the output current follow the input signal's amplitude and its wave form. Output current amplitudes typically varied from +0 to -30% over the frequency range of interest in this study (0 to 10 kHz) while driving the transducer. Fortunately, it was more faithful to the wave form.

Data is presented below (Section 5.4) to substantiate this statement.

The experimental set-up is shown in Figure 5.2. As shown in the figure, a Fourier analyzer (Tektronix 2630) was used to digitally sample/monitor the transducer drive voltage and current, the pick-up coil voltage, and the output voltage of the displacement sensor (MTI 1000 Fotonic Sensor with KDP-062 H plug-in module). Between the analyzer and the personal computer, PC, various numerical operations were performed on the digital samples. For the tests reported here, the analyzer was exclusively AC coupled, which means that an internal blocking capacitor was placed in the measurement circuit of each channel. Blocking capacitance values were estimated experimentally, from which the -3 dB point while AC coupled was calculated as approximately three hertz. Channel-to-channel matching information is published for DC coupled measurements, where both channels are at the same full-scale voltage setting. Real test conditions, AC coupling at various full scale settings, are not published. Therefore, tests were performed to estimate the errors one might expect when performing these types of measurements. Typical errors in magnitude and phase of below one percent were found for frequencies between 10 Hz and 20 kHz.
There were several technical difficulties to overcome with the experimental set-up shown in Figure 5.2. To begin with, the negative output of the amplifier was not the electrical ground - it floated. The inputs on the analyzer were single-sided, i.e., they were not differential inputs; thus, "where" ground was mattered. The inputs were also limited to ±10 V, while the amplifier was capable of producing approximately ±35 V. To overcome these difficulties, ±12 V operational amplifier circuits were built (and calibrated) to allow monitoring of the transducer's drive current and voltage, and the voltage of the integral pick-up coil. The amplifiers for the drive current and voltage were both differential inputs with input impedances of 200 kΩ (the same as the analyzer's). In addition, they both inverted the signal. The current monitor amplifier circuit had an overall sensitivity of (-5.010 ±0.2%) volts per ampere. The drive
current amplifier circuit sensitivity was \((-0.2726 \pm 0.1\%)\) volts per drive-volt. Compared to the uncertainties of other quantities, for example, any Terfenol-D material property, the uncertainties introduced by these amplifiers were very small. The amplifier for the pick-up coil was primarily a buffer (input impedance \(= 10^{12}\Omega\)) with either a gain of 0.994 or 10.95 \(\pm 0.2\%\) volts per pick-up volt (it was adjustable).

Displacement measurements were performed using an MTI 1000 Fotonic Sensor. The instrument employs fiber optics to send and receive white light. Basically, a probe is positioned over the target, a piece of reflective tape on the item of interest, and the amount of collected light is related to the separation of the probe and the target. The instrument can be fitted with different probes of different sensitivities and frequency ranges. Regardless which probe is used, the measurements are prone to errors if the room lights are on (60 Hz galore) or if the probe is allowed to move in relation to the target (either axially or if the probe is tipped). It was necessary to construct a massive, rubber isolated, precision positioning apparatus to hold the probe over the transducer. It was also necessary to calibrate the instrument with each use (despite the fact that each probe is furnished with a published sensitivity). Experimental uncertainties associated with displacement measurements were thought to be less than about five percent.

The instrument had three distinct advantages. First, it was a "non-contacting" transducer; the effects on target displacements due to the impact of light were assumed negligible. Second, light is unaffected by stray magnetic fields produced by the magnetostrictive transducer (the same cannot be said for some accelerometers). And third, it was available.

5.2.1 The transducer and parameter corrections

The transducer in this study resembled that shown in Figure 1.5 with two notable exceptions: it had five housing bolts, instead of four, and a pair of diaphragms instead of the bronze bushing. The five bolt
set-up reduced the symmetry (and significant rocking modes of the housing top) and increased the first axial resonant frequency of the housing top beyond the frequencies of interest (to over 10 kHz). The use of a pair of diaphragms eliminated the binding, or stiction, associated with a sliding rod in a bushing. A subtle, but important consequence of this is that the top of the transducer is subjected to lower excitation force amplitudes, resulting in smaller motion amplitudes. In addition, the frequency content of the excitation force is reduced by eliminating the stiction (the force from stiction would resemble a step function). As a result, low frequency operation of the transducer is not feeding high frequency forces to the housing top and the rod. The diaphragms also effectively eliminated side-to-side motion and rocking of the motion output component of the transducer. Their disadvantages include the increased complexity of the transducer (more precision parts) and they introduced a comparatively minor axial resonance near 3300 Hz (the first mode of vibration of the diaphragms).

As mentioned previously, the transducer also featured a special double coil (see Appendix A for details). The inner most layer of the solenoid was used as a pick-up coil. The other ten layers were the drive coil. The pick-up coil allowed improved experimental estimates of transducer/rod parameters because, as discussed earlier, when compared to the drive coil, the pick-up coil offered a significant reduction in leakage flux. The leakage flux correction term of Section 3.4.1 was estimated as $k^2_M = 0.97^2$ for the pick-up, and $k^2_M = 0.90^2$ for the drive coil. The estimates were obtained from [11, p. 120].

The transducer in this study used Terfenol-D rods 6.38 mm in diameter and 50.8 mm long (0.251 x 2.000 inch) with a mass of approximately 14.7 grams. The mechanical compressive prestress of the material was approximately 14 MPa (2 ksi). The total mass of the transducer was adjustable, it ranged from 0.7 to 2.21 kg, depending upon how much mass was attached to the aluminum base.

The transducer included a permanent magnet for biasing the magnetostrictive rod (an art). It remains a problem area of these
transducers that the magnetic bias point is, in the final analysis, empirical. Instead of fighting that battle, a summing amplifier circuit was built to allow the addition of a DC voltage to the signal provided by the generator (Figure 5.2). The Techron amplifier converted the DC voltage to a DC current, which was used to adjust the magnetic bias of the transducer. Unless noted otherwise, the bias current was 0.605 ±0.005 amperes for measurements reported in this study. This current amplitude represents a compromise between the problems due to ohmic heating, and the "optimal" magnetic bias point for the magnetostrictive rod.

Classical methods of estimating effective transducer parameters were presented in Chapter 3. However, to use the models in this study, it is necessary to go further. The first magnetomechanical model, Eqn. (3.3.13a), is repeated here.

\[
\mu_T = \mu^0 \left[ 1 + k^2 \left( \frac{k_m^H}{(k_m^H + k_L + k_{mps})} \right) \left( 1 - \left( \frac{\omega}{\omega_0} \right)^2 + j2\zeta \frac{\omega}{\omega_0} \right) \right]^{-1}
\]

(3.3.13a)

where: \( \mu^0 \) is the magnetic permeability of the magnetostrictive material when operated at constant stress,
\( k^2 \) is the square of the magnetomechanical coupling factor,
\( k_m^H \) is the linear stiffness of the magnetostrictive rod when operated at constant field strength, i.e., \( k_m^H = AE_y^H/l \),
\( k_L \) is the linear stiffness of the load,
\( k_{mps} \) is the linear stiffness of the transducer's prestressing mechanism (for example, component (e) of Figure 1.5),
\( \omega \) is the circular frequency of excitation,
\( \omega_0 \) is the circular frequency of mechanical resonance - including the effects of the load (not necessarily the frequency at which the transducer produces the largest output), and
\( \zeta \) is the dimensionless damping coefficient - also including the effects of the load.

Note that the expression for permeability contains a "mixed" batch of parameters. The first three, \( \mu, k^2 \), and \( k_m H \) (thus \( E_Y^H \)) are derived from linear theory applied to the magnetostrictive material within the transducer. Other parameters stem from linear theory applied to the transducer itself. The net result is that measurements of parameters must encompass both realms.

In a nutshell, \( \mu \) was calculated, \( k^2 \) was calculated from the effective coupling (which is calculated from experimental frequencies), \( k_m H \) was calculated from the resonant frequency and the mechanical model of the transducer under study, \( \omega_0 = \omega_0 Z \) from impedance analysis, \( k_L = 0 \), \( k_m^p_s \) was obtained from the literature, \( \omega \) was known, and \( \zeta \) was estimated from impedance/admittance analysis. The methods used in this study for obtaining these model parameters from experimental measurements are detailed below.

A mechanical model of the transducer was formulated, as opposed to estimating "effective" internal mass and stiffness. Masses, areas, and the as-run resonant frequency, \( \omega_0 Z \), were measured from which the rod's stiffness and modulus of elasticity could be estimated. However, to this point it has been tacitly assumed that the "base" of the transducer was fixed. For the simulations below, that was not the case. It was found to be very difficult to secure the base without introducing spurious resonances in the frequency range of interest. Instead of trying to explain those away, it was decided to approximate the fixed end condition with a relatively large base mass, i.e., the base was actually a seismic mass. Then the entire transducer was placed on a "soft" foam pad (made from household weatherstrip), and operated in the upright position (simulating the operating conditions of standard laboratory shakers). Figure 5.3 shows lumped parameter models of the transducer as tested, sketch (a), and as assumed in the development of the magnetomechanical models, sketch (b).

Neglecting the stiffness and damping of the foam pad, the equations of motion for the system in Figure 5.3a are given as:
where: $k = k_{\text{mps}} + k_{\text{m}} = \text{the stiffness of the prestress mechanism plus}
\text{the stiffness of the Terfenol-D rod when operated at}
\text{constant field strength},$

$F_T$ is the current dependant force from the magnetostrictive
rod (the stiffness related term has been included in $k$), and

$F$ is the output force of the transducer.

The system's resonant frequencies and modes of vibration are
approximated by examining the free vibrations of the undamped system
described by:

$$
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_m & -b_m \\
-b_m & b_m
\end{bmatrix} \frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} k & -k \\
-k & k
\end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -F_T \\
F + F_T
\end{bmatrix} \quad (5.2.1)
$$

Figure 5.3. Schematics of lumped parameter mechanical models of the
transducer as tested (a) and as assumed in model
development (b)
A shift to the frequency domain is performed by substituting $x = X e^{j\omega t}$ in Eqn. (5.2.2). Cancelling the complex exponent yields

$$\begin{bmatrix} -\alpha^2 m_1 & 0 \\ 0 & -\alpha^2 m_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(5.2.3)

where $X_1$ and $X_2$ are frequency dependent complex numbers. Non-trivial solutions for $X_1$ and $X_2$ are possible only when the determinant of the coefficient matrix is zero. Thus

$$\det \begin{bmatrix} k - \omega^2 m_1 & -k \\ -k & k - \omega^2 m_2 \end{bmatrix} = 0$$

which implies, omitting the algebra:

$$\omega^2 \left( \omega^2 m_1 m_2 - k(m_1 + m_2) \right) = 0.$$

The frequencies which satisfy this equation are approximately the natural frequencies of oscillation of the lightly damped mechanical system. (Their squares are the eigenvalues for the system of equations.) The two possible solutions are $\omega = 0$, corresponding to the first mode of vibration of the system, i.e., rigid body motion where $X_1/X_2 = +1$, and

$$\omega = \omega_{0Z} = \sqrt{\frac{k}{\frac{m_1 m_2}{m_1 + m_2}}} = \sqrt{\frac{k_{mps} + k_m^H}{\frac{m_1 m_2}{m_1 + m_2}}}$$

(5.2.4)

which corresponds to the second mode of vibration for the 2-degree of freedom system. This is the frequency of mechanical resonance in this study.

The dimensionless mode shape (the eigenfunction) for the second mode can be determined by solving either of the relations in Eqn. (5.2.3) while using $\omega$ as given in Eqn. (5.2.4). Solving the first equation:

$$X_1 \left( k - \omega^2 m_1 \right) = X_2 k \quad \Rightarrow \quad \frac{X_1}{X_2} = \frac{k}{k - \omega^2 m_1} = \frac{1}{1 - \frac{m_1 + m_2}{m_2}} = -\frac{m_2}{m_1}$$

thus, for any output motion, $X_2$, the base motion, $X_1$, will be given as
This relation implies that if one wants to reduce the base motion, they should choose \( m_1 \gg m_2 \). Note also that the motions are out of phase (the two masses move in opposite directions) and that \( X_1/X_2 \) is independent of frequency.

Equation (5.2.5) was investigated experimentally and found to be a good approximation of the experimental set-up. For \( m_2 = 0.150 \text{ kg} \) and \( m_1 = 1.685 \text{ kg} \), the ratio of accelerations (velocities, or displacements) should have been \( 0.09 e^{j\pi} \). Experimental acceleration measurements averaged around \( 0.08 \), and were out of phase, for frequencies below 6 kHz. For \( m_2 = 0.108 \text{ kg} \) and \( m_1 = 2.214 \text{ kg} \), theory suggests \( 0.049 e^{j\pi} \). Experiment yielded \( 0.05 e^{j\pi} \). These results were thought to be in good agreement with theory.

This change in mechanical models has some influence on how transducer parameters (permeability, coupling, etc.) are estimated. Recall Eqn. (3.2.3):

\[
k^2 = \frac{q^2 \varepsilon_y^H}{\mu_\sigma} \tag{3.2.3}
\]

where: \( k^2 \) is the square of the magnetomechanical coupling factor, \( q \) is the linear coupling, which is the local slope of a plot of strain from applied field strength at a particular constant stress, \( \varepsilon_y^H \) is the modulus of elasticity at a particular constant field strength, and \( \mu_\sigma \) is the magnetic permeability of the magnetostrictive rod, as measured at a particular constant stress. Knowledge of any three implies the fourth. For the simulations that follow, \( \mu_\sigma \) was calculated from the other three parameters.

\( k^2 \) is available from Eqn. (3.4.16) (repeated below) where the effective coupling is estimated from electrical impedance and admittance analyses using the dynamic method of Eqn. (3.3.15). To begin with, \( \omega_0 z \) is estimated from the experimental impedance loop.
Then one obtains $\omega_0Y$ from the horizontal diameter of the admittance loop. It is next assumed that the frequency of mechanical resonance given by Eqn. (5.2.4) is: $\omega_0Z = 2\pi f^H$, where $f^H$ is the frequency of mechanical resonance of the magnetostrictive rod when excited with a constant field strength. This was felt to be a reasonable assumption since the transducer was driven with a "current source." It is further assumed that $\omega_0Y$ corresponds to $2\pi f^B$. The effective coupling is then calculated via Eqn. (3.3.15):

$$k_{eff}^2 = 1 - \left(\frac{f^H}{f^B}\right)^2 = 1 - \left(\frac{\omega_0Z}{\omega_0Y}\right)^2.$$  

Finally, $k^2$ is calculated as:

$$k^2 = \frac{k_{eff}^2}{k_M^2} \left(\frac{k_m^H + k_{mps}}{k_m^H + k_{eff}^2 k_{mps} / k_M^2}\right)$$

where $k_M^2$ is due to leakage inductance of the experimental set-up, Eqn. (3.4.14), and the procedure for estimating $k_m^H$ is discussed next.

The mechanical stiffness of the magnetostrictive rod is available by solving Eqn. (5.2.4) for $k_m^H$:

$$k_m^H = \omega_0^2 \left(\frac{m_1 m_2}{m_1 + m_2}\right) - k_{mps}.$$  

One can now estimate the magnetomechanical coupling factor.

The modulus of elasticity is obtained from the stiffness assuming the rod behaves as a linear spring. Thus, from Eqn. (5.2.6)

$$E^H = \frac{1}{A} \left(\frac{1}{\omega_0^2 \left(\frac{m_1 m_2}{m_1 + m_2}\right) - k_{mps}}\right)$$

where $l$ is the length and $A$ is the area of the rod.

It is now desired to estimate the linear coupling of the material, $q$, using transducer displacement from current measurements, $u/l$, at low frequencies of excitation. Here, low frequencies are those at which the effects of eddy current shielding of the material and dynamic effects on displacements are thought to be small. In theory, $q$ should be determined from magnetostrictive rod behavior at a given constant stress. The measurements of $u/l$ were not at constant stress because
there was a prestressing spring attached to the end of the rod. The force seen by the material from the spring varied with the strain of the rod; specifically, \( F_s = -k_{mps} \varepsilon \). Thus, the varying stress on the rod was \((-F_s/A)\). Using Eqn. (3.2.1a), the equation for the material relating strain, stress, and applied field strength, implies:

\[
\varepsilon = \frac{\sigma}{E_y^H} + qH = \frac{-k_{mps} \varepsilon}{AE_y^H} + qH,
\]

solving for \( q \):

\[
q = \frac{\varepsilon}{H} \left( 1 + \frac{k_{mps}}{k_m^H} \right)
\]  \( (5.2.8) \)

where the expression for the linear stiffness of the rod has been used.

Another variation between theory and experiment needs to be considered before \( q \) can be estimated. Measurements of displacement, \( u \), were actually measurements of \( x_2 \) (from Figure 5.3). Eqn. (5.2.5) implies that dynamic rod strain is related to the measured displacement, by

\[
\varepsilon = x_2 - x_1 = \frac{x_2}{l} \left( 1 - \frac{x_1}{x_2} \right) = \frac{u}{l} \left( 1 + \frac{m_2}{m_1} \right)
\]  \( (5.2.9) \)

Using this, and the fact that at low frequencies (no eddy effects) \( H = nI \), allows formulation of the material property \( q \) in terms of the measured parameters.

\[
q = \frac{u}{l} \left( 1 + \frac{k_{mps}}{k_m^H} \right) \left( 1 + \frac{m_2}{m_1} \right) \frac{1}{(n)}
\]  \( (5.2.10) \)

With \( q \), \( E_y^H \), and \( k^2 \) in hand, \( \mu^\sigma \) can be calculated by rearranging Eqn. (3.2.3). Specifically:

\[
\mu^\sigma = \frac{q^2 E_y^H}{k^2}
\]  \( (5.2.11) \)

The final parameter required for the magnetomechanical model is \( \zeta \), the damping coefficient. Effective damping was estimated from the mechanical quality (recall, \( \zeta = 1/(2Q) = (\omega_2 - \omega_1)/(2\omega_0) \)). However, there are, yes, two dimensionless damping coefficients to consider.
The first is available from electrical impedance analysis, $\zeta_Z$, and the second from admittance analysis, $\zeta_Y$. In the spirit of compromise, and because it gave reasonable results, the average of the two was used; $\zeta = (\zeta_Z + \zeta_Y)/2$. It will be shown later that one cannot obtain a reasonable estimate of the mechanical damping coefficient by looking at a plot of displacement from current.

5.3 Time Domain Experimental Measurements

Experimental measurements were performed in the time domain to facilitate characterization of transducer behavior. Figure 5.4 displays six different measurements of transducer displacement as a function of time. All measurements were fifty time-averages of transducer response to a 200 Hz sinusoidal current excitation. The amplitudes of the current excitations are shown in milliamperes on the plots. The upper traces (a) represent transducer operation in the low to medium signal linear range (recall Figure 3.7). For these amplitudes, 50, 100, and 200 mA, the harmonic displacement amplitudes were, for practical purposes, buried in the noise base.

The lower plot displays displacement in response to 400, 800, and 1270 mA (medium to high) current excitation levels. Note the change in ordinate scales from (a) to (b). As shown, much larger displacements were realized at the higher current levels. All three traces in (b) have approximately zero for a mean value; displacements are shifted up because of the AC coupling of the analyzer (the blocking capacitor performs a time average). For this to occur, the wave forms had to distort significantly, as is easily seen in the 1270 mA trace. (Distortion is also addressed in Figure 5.7 below.) In fact, the highest drive current was sufficient to overcome the magnetic bias of the rod. This can be seen more clearly in Figure 5.5 where the first cycle of the 400 and 1270 mA displacement data of Figure 5.4 are plotted versus input electric current. Arrows have been placed in the figure to indicate the direction of increasing time. Note that for the most
Figure 5.4. Experimental measurements of displacement, $u$, versus time for six current excitation levels (values in milliamperes). 200 Hz sinusoidal current excitation was used in all cases. Mass load = $m_2 = 0.1087$ kg, base mass = $m_1 = 1.66$ kg
negative displacements of the large amplitude trace, the rod collapsed to its shortest length and actually started to increase in length again (the displacement became less negative). The loop then crosses itself, returns to a minimum displacement and climbs the right-hand side of the loop. Recalling the previous discussion of near-linear behavior and approximating relationships as ellipses, the data in Figure 5.5 suggests that there is little hope of an elliptical approximation of the 1270 mA displacement from current relationship. However, the 400 mA is more like an ellipse.

The first cycle of both the 400 and 200 mA data of Figure 5.4 is plotted as displacement versus current in Figure 5.6. The broad lines represent the experimental data while the narrow lines are best-fit ellipses (see Appendix A for calculation details). A comparison was made between the experimental data and the elliptical model.
Maximum errors were related to the maximum experimental displacement via:

\[
\text{%error} = 100 \max \left( \frac{u_{\text{model}} - u_{\text{exp.}}}{u_{\text{exp.}}} \right)
\]

For the data in Figure 5.6, the maximum errors were, respectively, 6 and 10 percent for the 200 and 400 mA excitations. For 500 mA, the error was about 15%. At higher currents, the curves exhibited progressively "sharper ends," resulting in 40-50% errors. Thus, the accuracy of the elliptical approximation increases with decreasing current amplitude. This trend was exhibited as long as the displacements were large enough to be out of the noise base.

Figure 5.7 shows a single cycle of the 400 mA data of Figure 5.4 and the corresponding cycle of the elliptical model. In the figure the broad

![Figure 5.6. Displacement from current at 200 Hz for the first cycle of the 200 and 400 mA data of Figure 5.4. Broad lines are experimental, light lines are "best-fit" ellipses](image)

- 3
  - 2
  - 1
  - 0
  - 1
  - 2
  - 3
-500
  0
  500

Current, i(t), mA

Displacement, \(u \times 10^{-6} \text{ m}\)
line is experiment, the thin line is the model. Shown in this fashion, the distortion of the wave form can be seen. This distortion requires the presence of multiple frequencies within the signal. Both traces have the same peak-to-peak values and zero time average values. Note, however, that the experimental measurement appears to go more positive than negative. Again, this is due to the AC coupling of the measurement system. The transducer traversed the negative displacements more slowly than the positive; thus the curve is shifted up.

If the transducer were linear in a least squares sense, i.e., a sinusoid input results in a sinusoid output, elliptical relationships would be exact. That is clearly not the case - especially at the higher drive amplitudes. For the transducer of this study, 500 mA seems to be approximately the end of the "near-linear" range of transducer

![Figure 5.7. Displacement versus time from experiment and elliptical time domain model. The experimental data is from Figure 5.4](image)
operation. After that, the elliptical approximation is considered poor. Regardless of current amplitude, in this study, attention is paid only to the drive frequency. Any other frequencies are treated as noise. Keep in mind, however, that they exist and, as evidenced above, that they very likely increase with increasing drive amplitude. This will be documented in the next section via autospectral density functions. To be honest, if a motion source is sought for applications requiring medium to large displacements, and where harmonic distortion is an important issue, Terfenol-D should probably not be one's first choice.

It should be noted that another form of nonlinearity in Terfenol-D transducers is apparent by examining Figures 5.5 and 5.6. Imagining ellipses, the effective slopes of the relationships increase with increasing current amplitude. Thus, doubling the current amplitude, more than doubles the displacement in this range of transducer operation.

5.4 Frequency Domain Experimental Measurements

Autospectral density functions and frequency response functions (FRF's) were calculated for, and between, the various quantities of interest in this study. Autospectral density functions were calculated primarily to display the presence of harmonic frequencies within the output acceleration, drive voltage, and drive current signals. For these measurements the transducer was driven by a sinusoidal current of primarily one frequency. It was the job of the current control module to ensure that the current was but one frequency. A worst case example will be presented to demonstrate that this was approximately the case. Output acceleration autospectral density functions will be used to display the increase in harmonic content with increasing drive current amplitudes.

FRF's were calculated so model parameters could be estimated, and so there was something with which to compare model simulations. FRF's were calculated via a swept-sine technique. For these tests, the analyzer's integral signal generator would produce a reference signal at a single frequency. The amplifier/module would convert the signal
to a drive current. The analyzer would wait a couple of seconds for transients to decay, then calculate values of drive coil voltage from current, displacement from current, and pick-up coil voltage from drive coil current, at the frequency of excitation. The analyzer would then increase the frequency of excitation, wait a couple of seconds, and do it again. Thus, it swept through a frequency range, one frequency at a time.

5.4.1 Experimental autospectral density measurements

It was mentioned previously that the amplifier/current control module assembly was more faithful to the input signal's wave form than its amplitude (recall that the amplitude varied by about 30%). Figure 5.8 displays how well the amplifier/control module did in maintaining wave form. The figure is data from 100 averages of autospectral density functions calculated from the drive current and drive voltage while running the transducer at 1.115 amperes, in response to a 200 Hz input signal. (This was a very high current for the transducer.) Each set of data was normalized by the corresponding 200 Hz amplitude; thus both display 0 dB at 200 Hz. The amplitudes of the harmonics represent distortions of the wave forms. As shown in the figure, the largest harmonic amplitude of the current is -40 dB, which corresponds to approximately one-percent of the current amplitude at 200 Hz (all amplitudes are zero to peak). The voltage, on the other hand, displays much larger harmonic amplitudes. For example, the -17 dB at 600 Hz is approximately fourteen-percent of the 200 Hz voltage amplitude.

Why are there any harmonics in the voltage and the current? Terfenol-D is nonlinear - especially at large drive amplitudes (recall Figure 5.4) - and it (Terfenol-D) "works both ways." Due to nonlinearities in the magnetomechanical coupling, multiple frequencies appear in the displacement, velocity, and acceleration. Because of the coupling, these frequencies are transduced back to the electromagnetic side of the transducer. Thus, they appear in the drive voltage and current. In the present example, the amplifier/module
assembly was actively trying to control the current's wave form, so the relative amplitudes of the harmonics were lower in the current than in the voltage. The opposite would be the case if the amplifier/module assembly had been operating in the voltage control mode. If a standard amplifier had been used, the relative amplitudes would depend on the impedance of the amplifier. In addition, one would likely find that the harmonic frequencies were fed all of the way back to the signal generator.

Figure 5.9 displays relative output acceleration amplitudes versus frequency over a range of 10 to 400 mA drive current amplitudes. Figure 5.10 shows the results for 400 to 1115 mA drive currents. In generating the figures, autospectral density functions were calculated
Figure 5.9. Normalized acceleration autospectral density functions for 200 Hz sinusoidal current excitations. 0 dB and %HD values are in Table 5.1

at each drive current amplitude. All autospectral density functions were calculated: using a 200 Hz sinusoidal drive current, using a rectangular window, and with $m_1 = 2.214$ kg and $m_2 = 0.108$ kg. Each set of datums were then normalized by their value at the fundamental frequency (200 Hz); thus each set displays 0 dB at 200 Hz. All values reported were at least 5 dB above the noise floor of that particular measurement, then the noise floors were removed from the plots.

Considering Figure 5.9, the only acceleration value above the noise floor for the 10 mA excitation was that of the fundamental frequency, 200 Hz, called the "first harmonic." The 20 mA level elevated a 600 Hz component out of the noise floor (called the "third harmonic" since it is at 3 times the fundamental's frequency). Increasing to 50 mA resulted in components for the third, fifth, and seventh harmonics above the noise floor. 100 mA gave those, plus the ninth harmonic.
200 mA included all of the odd harmonics plus the second harmonic. 400 mA drive current resulted in all of the odd plus the first two even harmonics. These trends all indicate that increasing the drive amplitude generally increases the distortion of the output waveform. These trends are in agreement with those suggested by the time domain displacement measurements discussed in the previous section.

Figure 5.10 begins where 5.9 left off (400 mA) and continues up to an 1115 mA drive current amplitude. The data in the figure further validates the aforementioned trends. Note that the third harmonic of the 1115 mA drive current function is only down one dB. That corresponds to a 600 Hz signal at 90% of the 200 Hz acceleration's amplitude. Very large distortions are occurring to the wave forms at the high amplitude drives.

![Figure 5.10](image)

Figure 5.10. Normalized acceleration autospectral density functions for displayed amplitudes of 200 Hz sinusoidal current excitations. 0 dB and %HD values are in Table 5.1
Percent harmonic distortion, %HD, was calculated for the data sets displayed in Figures 5.9 and 5.10. Included in the calculations were all of the data points which were at least 5 dB above the noise floor of the measurement (there were points included in the calculations which were not shown in either of the plots). The distortion was calculated as:

\[
%HD = 100 \frac{\sum_{\text{all amplitudes}} (\text{all amplitudes}) - 200 \text{ Hz amplitude}}{\sum_{\text{all amplitudes}}}.
\]  

(5.4.1)

The calculated values are displayed in Table 5.1. Also shown in the table are the 0 dB values for each set of data displayed in Figures 5.9 and 5.10. Note that %HD increases with drive current, and that doubling the current amplitude more than doubled the 200 Hz acceleration amplitude (further evidence of the second type of nonlinearity seen in the time domain data).

### Table 5.1. Statistics for datums in Figures 5.9 and 5.10.

| Drive Current mA | 0 dB Value milligra
vities | %HD Eqn. (5.4.1) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3.9</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>8.2</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>28.5</td>
<td>16</td>
</tr>
<tr>
<td>100</td>
<td>63.0</td>
<td>21</td>
</tr>
<tr>
<td>200</td>
<td>146</td>
<td>28</td>
</tr>
<tr>
<td>400</td>
<td>407</td>
<td>49</td>
</tr>
<tr>
<td>500</td>
<td>613</td>
<td>56</td>
</tr>
<tr>
<td>800</td>
<td>1440</td>
<td>75</td>
</tr>
<tr>
<td>1115</td>
<td>2310</td>
<td>83</td>
</tr>
</tbody>
</table>

#### 5.4.2 Experimental electrical impedance measurements

Experimental measurements of a single transducer's electrical impedance functions were performed at various drive current amplitudes, but all with the same mechanical load. Recall that the electrical impedance is the complex ratio of drive voltage per ampere.
Thus, each function has a magnitude and a phase. Figure 5.11 displays experimental measurements of what amount to $Z_{ee}(f,l)$, at three different drive current amplitudes, 5, 100, and 300 mA. All data was gathered using the swept-sine technique discussed earlier. For the tests shown, the compressive prestress was approximately 15 MPa, $m_2 = 0.125$ kg, and $m_1 = 2.19$ kg. Graph (a) displays the magnitudes while (b) shows the relative phases. Both plots are reminiscent of Figure 3.4.

Referring to Figure 5.11, note that at the three different drive amplitudes, there were three different mechanical resonant frequencies; the large variations in amplitude and phase seem to occur over different frequency ranges for each drive amplitude, and the mechanical resonant frequency, $f_{oZ}$, is in that range (somewhere). Increasing the current amplitude reduced the stiffness of the magnetostrictive rod.

Recall in the discussion concerning Figure 3.4, it was pointed out that the modulus at constant flux density, $E_y^B$, was supposed to be nearly a constant, and that the frequency at which the magnitude displayed a local minimum, $f^B$, was also supposed to be nearly constant. The trends shown in Figure 5.11, where the frequency of the local minimum in the magnitude plot reduces with increasing amplitude, is an anomaly. However, this trend was seen in all samples of Terfenol-D tested in the course of this study. It seems to be a reality of the material - one not usually discussed in pleasant company - and it is particularly pronounced at low excitation amplitudes. It severely complicates one's attempts at attaining reproducible results, makes attempts at general classification of material properties a bit of a problem, and adds months to a degree program. This behavior is not mentioned in linear theory (perhaps because engineers like constants?). With Terfenol-D, there really are not any constants to be had. Interestingly, strange changes in "material properties" upwards of 25% are termed "small" in the literature.[8]
Figure 5.11. Magnitude (a) and phase (b) of experimental measurements of $Z_{ee}$ generated using current control and a swept-sine technique. All three were measured using the same load on a single transducer.
One must exercise caution when interpreting FRF's of Terfenol-D transducers. As shown above, and will be demonstrated again below, each FRF is drive amplitude dependent. In that light, they really are not frequency response functions in a least-squares linear sense. Nonetheless, they will be displayed.

5.4.3 Experimental displacement from current functions

Figure 5.12 shows experimental measurements of displacement per ampere, which were gathered at the same time as the data in Figure 5.11. Graph (a) is the magnitude and (b) is the phase. Looking at the magnitude traces, the change in mechanical resonant frequencies is clearly seen. Note also the change in the low frequency magnitudes from one drive level to the next. Increasing the drive amplitude increased the displacement per ampere over the low to medium signal range of transducer operation. This phenomenon was seen in the time domain data, and the acceleration, so it was expected here. All three traces resemble the classic, underdamped, one degree of freedom, spring mass damper system's dynamic displacement plot, aka magnification factor.[10] They traverse the low frequency range staying nearly constant in magnitude. As dynamic effects become important, the displacement increases, reaching a maximum near mechanical resonance, thereafter decreasing in magnitude.

The phase also resembles what one might anticipate, with a couple of notable exceptions. First, at low frequencies, it approaches, but does not seem to reach zero. There is a vertical shift from one plot to the next. This is the frequency independent phase shift discussed earlier in connection with magnetic hysteresis. It is due to magnetic hysteresis in the material. As shown, with increasing drive current amplitude, thus increasing field strength, the vertical shift, or the phase lag due to hysteresis, increases. The phase calculated for the 5 mA drive current is showing the effects of the datums being perilously close to the noise floor of the measurement.

The second exception is that all of the traces seem to be tipped down. This effect stems from the phase lag between electric current
Figure 5.12. Magnitude (a) and phase (b) of experimental measurements of displacement per ampere, $u/l$, generated using current control and a swept-sine technique. All three were measured using the same load on a single transducer.
in the solenoid and applied field strength on the rod; \( H \neq nI \) due to eddy currents within the transducer. Terfenol-D strains in response to \( H \), not in response to current. Thus, the eddy induced phase between \( H \) and \( I \) appears in \( \varphi/\varphi \). The results are that, instead of passing through \(-90^\circ\) at resonance and approaching \(-180^\circ\) at high frequencies, the phase is closer to \(-120^\circ\) at resonance and seems to be heading for the basement at high frequencies.

The analogy between the traces in Figure 5.12 and the classic magnification factor ends here. If one tries to estimate the mechanical system damping from these plots, by calculating the magnification factor at resonance, the values estimated will be high. The values obtained from these plots would include other effects - it would be a damping for the entire electromechanical system, not just the mechanical system. The damping term required for simulations using the models detailed in this dissertation is that of only the mechanical system.

It was mentioned in Section 3.3 that the mechanical resonant frequency was somewhere between the local maximum and minimum of the impedance magnitude plot, and that where it occurred was a function of with what the transducer was being electrically driven. A comparison of Figures 5.11 and 5.12 reveals a method for estimating the frequency at which the largest mechanical motions occur, using the electrical impedance function's phase. From the data it seems that, with this experimental set-up, the frequency at which the electrical impedance phase reaches a local minimum is the best match with the frequency of maximum mechanical motion.

5.5 Comparisons of Electrical Impedance Functions From Experiment and Models

This section of the dissertation will begin with an example of model parameter estimation using electrical impedance and admittance analysis, and the relations developed above. Then simulations of transducer electrical impedance will be compared with the experimental measurements. There are three simulation types of
interest. One will totally ignore the presence of eddy currents. Another simulation will include eddy effects occurring within the magnetostrictive rod; this uses the electromagnetic model developed in Section 4.3 (the classic solution). The third type of simulation is that which includes the effects of eddy currents in the external cylindrical permanent magnet (developed in Section 4.4).

Model parameters will be estimated from frequency response functions calculated between the pick-up coil voltage and the current in the drive coil. These functions will be called the "pick-up impedance," $Z_{pu}$. The pick-up electrical admittance is then defined as $Y_{pu} = 1/Z_{pu}$. The 100 mA test from the previous section will be detailed.

Figure 5.13 is $Z_{pu}$ plotted as imaginary versus real. Shown on the figure are the lines corresponding to the by-eye best fit circle (recall Figure 3.6) and the frequencies of interest, $f_{oZ} = 2640$ Hz, $f_{1Z} = 2540$ Hz, and $f_{2Z} = 2740$ Hz. Each datum is marked with a "+". For these tests, measurements over the first 2000 Hz were taken every 100 Hz. These are the points starting near zero and continuing up into the circle. Measurements at frequencies between 2000 and 4000 Hz were taken every 20 Hz. That is why the data points seem very dense on the left-hand side of the circle. Note that measurements every 100 Hz would have resulted in only three data points for the entire right-hand side of the circle. The frequency range from 4000 to 6000 Hz was covered at 100 Hz increments. These frequencies correspond to the sparsely populated region heading out of the top of the plot.

A damping estimate was calculated from the half-power and resonant frequencies as:

$$ \zeta_Z = \frac{f_2 - f_1}{2f_o} = \frac{2740 - 2540}{2(2640)} = 0.0379 \text{ or } 3.79\%.$$ 

$2\pi f_{oZ}$ is used in Eqn. (5.2.6) to estimate the linear stiffness of the rod:

$$ k_m^H = (2\pi 2640)^2 \left( \frac{2.19 \times 0.1247}{2.19 + 0.1247} \right) - 1.5 \times 10^6 = 30.96 \times 10^6 \text{ N/ m.} $$
Figure 5.13. Imaginary versus real components of $Z_{pu}$ showing the motional impedance circle, half-power point frequencies and mechanical resonance frequency.

Equation (5.2.7) can now be used to estimate the modulus of elasticity at constant field:

$$E^H = \frac{1}{A} \frac{m k^H}{\pi (0.003175)^2} = 49.67 \text{ GPa.}$$

The pick-up coil's admittance, $Y_{pu}$, was calculated. It is shown in Figure 5.14. Also shown on the plot are the horizontal diameter and the diameter perpendicular to it. From these lines are found $f_{0Y} = 2780$ Hz, $f_{1Y} = 2700$ Hz, and $f_{2Y} = 2855$ Hz. From these frequencies comes another damping estimate, $\zeta_y = 0.0279$ (the average of the two damping estimates is $\zeta = 0.0329$), and an estimate of the effective coupling:

$$k_{\text{eff}}^2 = 1 - \left( \frac{f_0}{f_{0Y}} \right)^2 = 1 - \left( \frac{2640}{2780} \right)^2 = 0.09818.$$
Figure 5.14. Imaginary versus real components of $Y_{pu}$ showing motional admittance circle, half-power point frequencies and the diametral frequency

The parameter round-up is completed with an estimate of $u/l$ for use in the calculation of $q$, the linear coupling, given in Eqn. (5.2.10). Displacement per ampere was estimated as the average of the first ten datums (100 to 1000 Hz.) of the experimental measurement (shown in Figure 5.12a). That procedure resulted in $u/l = 3.96 \times 10^{-6}$ m/A and $q = 3.712 \times 10^{-9}$ m/(A turn).

Model simulations were all performed using MATLAB software (see Appendix C) run on a Macintosh. A program was written to calculate the analytical expression developed for the applied field strength at the outer radius of the Terfenol-D rod, $H(R_{To})$, Eqn. (4.4.20). The models developed in this dissertation can be run on any calculator that can handle modified Bessel functions. If they were but as common as sines and cosines, everyone would have one. Anyway, the program actually calculated the dimensionless ratio $H(R_{To})/nl$ at each frequency.
of interest. It then added the DC resistance of the coil to the complex valued transducer inductance, $L$, which was calculated via the analytical expressions developed for magnetic flux as a function of radial position. The inductance is given by Eqn. (4.4.23) and was calculated based on a coil the same length as the Terfenol-D rod - the other 9% of the solenoid's length was ignored. In the final analysis, what was really happening in the end-caps of the transducer was unknown; thus they were completely ignored. This unknown would also effect any measurements of magnetic permeability that might be attempted, so none are reported. Magnetic permeabilities for Terfenol-D were all calculated via Eqn. (5.2.1). That value was then modified to include motional effects via Eqns. (3.3.13).

The above program can be used to calculate all three types of simulations. Eddy currents can be completely ignored by changing the conductivities of Terfenol-D and the permanent magnet, from values around $10^6$ to values around 1. This has the effect of making the material very resistive to the flow of electric charge. Restoring the conductivity of Terfenol-D to its published value allows simulation of transducer performance with eddy currents occurring only in the rod. Restoring the conductivity of the cylindrical permanent magnet to its published value has the effect of including eddy currents occurring in the external housing. These calculations were performed, and the resulting electrical impedance functions, $Z_{ee\text{ model}}$, are displayed in Figure 5.15.

Consider the magnitude traces for "No Eddy" and "Rod Only." Note the reduction in magnitude due to the eddy effects at the highest frequencies, i.e., $f > 4000$ Hz. This trend makes sense since, at the high frequencies, the eddy currents in the rod are shielding the inner portions of the rod from the imposed field. There is less field towards the center of the rod than at the outside edge. The inductance is reduced because eddy currents reduce the total flux linkage. Eddy losses are related to the $k$ parameter defined in Eqn. (4.2.9), that is, the quantity $(\omega \mu \sigma_e)^{1/2}$. Increasing any of the components has the effect of increasing the eddy currents. At high frequencies, losses
Figure 5.15. Model simulations of transducer electrical impedance functions, \( Z_{ee\ model} \), for no eddy current effects, eddy effects only in Terfenol-D rod, and eddy currents in rod and housing. (a) is magnitude, (b) is phase.
increase because $\omega$ increases. However, eddy current effects are also exhibited near $f^H$, the frequency of the local maximum impedance magnitude. This is because of the increased permeability due to mechanical effects.

A plot of the relative magnetic permeability of the Terfenol-D rod used in all three simulations, $\mu_T/\mu_0$, magnitude and phase, is shown in Figure 5.16. At frequencies around 2500 Hz, the permeability is increasing in magnitude; thus the eddy losses and eddy shielding increase in significance and the blocking observed at high frequencies is also displayed around 2500 Hz. In addition, around $f^B$, the frequency of the local minimum of the impedance magnitude trace, the magnitude of the permeability is reduced below even the blocked value (calculated as $4.46\mu_0$ for these simulations). That is why "No Eddy" and

Figure 5.16. Magnitude and phase of the relative permeability of Terfenol-D used in the simulations shown in Figure 5.15. This is not the permeability that would be measured in the presence of eddy currents. See text
"Rod Only" in Figure 5.15 nearly overlap between approximately 2600 and 4000 Hz. In that zone, the reduction in permeability has made eddy current losses appear insignificant. A bit of a surprise.

This result suggests that the designer should consider operating their transducer at frequencies above resonance - even if they seek to build a classic resonant transducer. As shown in Figure 5.12, operating slightly above mechanical resonance still yields appreciable displacement amplitudes. Of course, if the transducer were built to resonate at, for example, 25 kHz, eddy losses would likely not be reduced to insignificance simply by the variation in Terfenol-D permeability. However, eddy losses at 25 kHz would be higher than at, say, 25.5 kHz. There is still a savings to be had.

Consider now the phase traces for the simulated electrical impedance functions, Figure 5.15b. The phase of a complex number is calculated from the ratio of the imaginary component to the real component. It is expected that eddy currents will increase the real component (due to the increase in ohmic losses) and decrease the imaginary (because of shielding). As a result, the phase in the presence of eddy currents should be reduced. As shown in the figure, the models display that trend. For the "No Eddy" trace, the phase approaches +90° with increasing frequency. That is expected for an R-L circuit. The addition of eddy currents in the rod, reduces the phase. The addition of eddy currents in the rod and housing, further reduces the phase and the magnitude of the electrical impedance.

At first glance, one might like the idea of reducing the impedance magnitude. For a given voltage, one could force more current through the transducer. Unfortunately, the extra current would be transduced to heat, not motion. The impedance is reduced primarily due to destructive interference of magnetic fields - once again reducing the flux linkage seen by the drive coil. The fields are due to the current in the drive coil, and those currents induced in the rod and the cylindrical housing. The amplifier running the transducer is supplying the energy for all of these currents. When possible, eddy currents are to be avoided.
It should be noted that the permeability displayed in Figure 5.16 is not the permeability one would measure in the presence of eddy currents. It is impossible to separate the effects of eddy currents on experimentally measured magnetic permeabilities from the "true" permeability of the substance. What is shown in Figure 5.16 is the assumed form for the material's "true" permeability. The best method for implying that this assumed form may be correct is to show that it is capable of reproducing experimental results. Along those lines, the "Rod & Hsg." simulation will be compared with the 100 mA experimental measurement of Figure 5.11 next.

Figure 5.17 shows both the experimentally measured electrical impedance for the transducer, when operated at 100 mA current amplitude, and the model simulation that includes the effects of the rod and housing. Except near resonance, typical errors in the simulated magnitudes average less than five-percent of the experimental measurement. The maximum error in the simulated phase, other than near resonance, is five-percent at about 400 Hz. The simulation was performed using the as-run measured values detailed at the beginning of this section, plus published values for electrical conductivities, permeability of the permanent magnet, stiffness of the preload spring, etc.

As shown in the figure, the simulation bears a striking resemblance to the experimental measurement. This simulation's accuracy was about average. Some were slightly better, others were a bit worse. A particularly good example of a bad simulation would be one of the 300 mA trace in Figure 5.11. In that case, there was trouble identifying model parameters from experimental measurements. For that test, a plot of the impedance loop (as in Figure 5.13) was quite distorted, making the estimated frequencies suspect. 300 mA was the largest current amplitude the experimental set-up could produce over the entire frequency range of interest. (The amplifier ran out of voltage.) The limits of the amplifier are quite likely one source for the distortion of the experimental impedance loop. At these "high"
Figure 5.17. Comparison of $Z_{ee}$ magnitude and phase between experiment and simulation. The experimental data is the 100 mA trace of Figure 5.11. The simulation includes the effects of eddy currents in the cylindrical housing.
currents, the variation in current amplitude with frequency was a maximum (about 30%). Recalling Terfenol-D's propensity for changing properties with drive amplitude, it seems likely that the variations would yield a distorted curve.

A comparison of the three different simulations displayed in Figure 5.15 with the results shown in Figure 5.17 confirms that the transducer model 2, developed in Section 4.4, the one labeled "Rod & Hsg." in the figures, provides the best estimate of the experimental measurement. That is as it should be since the transducer tested had a conducting cylindrical external housing, and the other models ignored its presence.

5.6 Comparisons of Experiment and Model Displacement from Electric Current Frequency Response Functions

In this section of the dissertation, the procedure for estimating displacement per ampere will be presented. Three types of simulations will again be shown, this time to demonstrate the effects of eddy currents on the classic transduction coefficient, Tem, and displacement per ampere. Then simulation results will be compared with experimental measurements of u/I.

In this study, the transduction coefficient, Tem, was estimated via Eqn. (3.5.1), repeated here:

\[
T_{em} = \sqrt{Z_{ee} - Z_e}(z_m + z_L)
\]  

(3.5.1)

where: \( Z_{ee} \) = the electrical impedance of the transducer, as run, \( Z_e \) = the blocked electrical impedance of the transducer, and \( z_m + z_L \) = the mechanical impedance of the transducer and the load.

As discussed in Section 3.5, knowledge of \( T_{em} \) allows calculation of displacement per ampere via:

\[
\frac{u}{I} = \frac{1}{j\omega} \frac{v}{I} = \frac{T_{em}}{j\omega (z_m + z_L)}.
\]  

(5.6.1)

The displacement per ampere simulations in this study were calculated as follows. \( Z_{ee} \) was estimated as discussed above. The simulations included motional effects, and the effects of eddy
currents. $Z_e$ was also calculated by the same program; however, this time through the permeability of the Terfenol-D rod was assumed to be the blocked value, i.e., $\mu_T = \mu^e = \mu^T(1-k^2)$. This was done because motional effects should not be included in the simulation of the blocked electrical impedance. However, eddy current effects should be included. The mechanical impedance functions were known for the calculation of $Z_e$, so they were available for this calculation. Once $T_{em}$ is known, $u/l$ is calculated via Eqn. (5.6.1).

Figure 5.18 displays the magnitude and phase for the transduction coefficient, as a function of frequency, for the three simulations displayed in Figure 5.15, "No Eddy," "Rod Only," and "Rod & Hsg."

For the simulation which ignored eddy effects entirely, $T_{em}$ reduced to that given by Eqn. (3.3.8), i.e. $T_{em} = Nqk_mH = 1180(3.71 \times 10^{-9})30.96 \times 10^6 = 136 \text{ N/A}$, with no phase. Ignoring eddy currents results in a constant, real valued transduction coefficient. Including the effects of eddy currents occurring within the Terfenol-D rod has the tendencies of reducing the magnitude and introducing a frequency dependant phase lag. As one might expect, eddy currents reduce the abilities of the transducer to transduce. Including the effects of eddy currents in the rod and housing, reduces the magnitude and phase even further.

The displacement enhanced permeability, thus enhanced eddy currents, contributed to the disturbances shown in the magnitude and phase of the two simulations which included eddy effects. As frequencies increase towards resonance, eddy currents increase; thus the transduction coefficient decreases. Just after resonance, eddy currents decrease and transduction increases. This is more evidence that one should consider operating their "resonator" at a frequency slightly higher than that of mechanical resonance. As frequencies are increased even further, eddy current losses return due to the dual effects of increasing frequencies and increasing permeability (recall Figure 5.16).
Figure 5.18. Model simulations of transducer transduction coefficients, $T_{em}$, for no eddy current effects, eddy effects only in Terfenol-D rod, and eddy currents in rod and housing. (a) is magnitude, (b) is phase.
Classical transduction theory assumes that $T = \| T \| e^{i\beta} = T_{em}$ and that it is a constant for the transducer. That is a convenient result since a small number of tests on the transducer will allow the designer to estimate transducer performance when it is subjected to a different load. The results presented in Figure 5.18 agree with theory, so long as eddy currents are absent. Recalling the effects of a load on the transducer's output motion, the effects of transducer output motion on the magnetic permeability of the magnetostrictive rod, and the magnetic permeability's effects on eddy currents, one can see that classical theory may provide a poor approximation, i.e., $T \neq$ constant.

Consider what would happen in the present case for the "Rod & Hsg." transducer if $T$ were estimated as a constant from these "tests." An average value around resonance would be about 98 N/A. If the transducer were then loaded such that it resonated at 1000 Hz, the value estimated at 2800 Hz would likely be about 20% low (picture the disturbance near resonance in Figure 5.18a "sliding up" to 1000 Hz). Similarly, if the new load caused resonance to occur at 5000 Hz, the estimate obtained here would be about 20% high. One point to be made is that $T$ is not really a constant if eddy currents are present, and if $\pm20\%$ is not "good enough," the designer is destined to perform simulations like these.

Recall also that it is assumed in classical theory that $\beta$, the tip angle of the motional loop, is a constant. Figure 5.18b shows that this, too, was not really the case. What might be the consequences of neither $T$ nor $\beta$ being constant over the range of frequencies required to trace out the motional impedance circle? A rather complex variation of the effective diameter and tip angle resulting in strangely warped circles? Possibly. That sort of behavior was observed. Some of it, however, was attributed to variations in actual drive levels during the tests. Warped circles were most prevalent during high drive level tests. Not coincidentally, permeabilities varied the most, thus eddy effects varied the most, during the high drive level tests. Perhaps the most valuable consequence of these trends is the reminder it offers to the reader that this approach is an approximate technique.
Parameters obtained via this technique are also approximate. It seems to be the best approximation available; however, it is still an approximation.

Recall that the phase of displacement from current (Figure 5.12b) went beyond the negative one-hundred and eighty degrees one might have anticipated for a second order system. It was mentioned that the phase plots seemed "tipped." Considering Eqn. (5.6.1), one can see that the phase displayed in Figure 5.18b, will be added to that from the mechanical admittance. Thus, the model provides a mechanism to account for that extra phase measured experimentally. Plots of displacement per ampere, as predicted by the three different simulations, are shown in Figure 5.19.

The trends displayed in Figure 5.19 are consistent with those expected. As shown in the magnitude traces, eddy currents reduce the displacement magnitudes predicted near mechanical resonance. The frequencies at which the peak displacements occur are the same at which the transduction coefficients reached their local minimums (Figure 5.18). The decrease in transduction, due to eddy currents, manifests itself as reduced displacements.

The phase traces of Figure 5.19 are also of interest. When eddy currents are not present, the phase predicted is indeed that of a simple second order system. The addition of eddy currents in transducer components results in an extra phase lag, i.e., the traces with eddy current effects are tipped.

The "Rod & Hsg." simulation is compared in Figure 5.20 with the 100 mA experimental measurement of displacement per ampere. These simulation results were not adjusted to match experiment. As to the magnitudes, recall that what was measured experimentally was \( u = x_2 \). However, the model assumes the rod is fixed at one end. Thus, the displacement predicted by the model is high by the factor \( (1 + m_2/m_1) \). This comes from Eqn. (5.2.9), where

\[
\varepsilon = \frac{u_{\text{model}}}{u_{\text{exp}}} = \frac{u_{\text{exp}}}{l} \left(1 + \frac{m_2}{m_1}\right) \Rightarrow u_{\text{model}} = u_{\text{exp}}(1.057).
\]
Figure 5.19. Model simulations of transducer displacement per ampere for no eddy current effects, eddy effects only in the Terfenol-D rod, and eddy currents in rod and housing. (a) is magnitude, (b) is phase.
Figure 5.20. Comparison of $u/l$ magnitude and phase between experiment and simulation. The experimental data is the 100 mA trace of Figure 5.12. The simulation includes the effects of eddy currents in the cylindrical housing.
With this in mind, the model should over predict displacements by about six percent. Even considering that, the model error is in the ±10% range. Like the electrical impedance simulation of the 100 mA experiment, the errors in this displacement from current simulation were typical of the average simulation using the models presented in this study. Some simulations were a little closer, others were a little further away.

As to the phase shown in Figure 5.20b, the model was capable of producing the same trends as were measured. However, as run, the model had no provision for the frequency independent phase lag discussed earlier (the one due to magnetic hysteresis). Thus, the trace of the model phase appears to be shifted up in relation to the experimental measurement. That can be "fixed" by multiplying Eqn. (5.6.1) by el DC angle, where the DC angle is found by fitting a line to the first few phase measurements, and extrapolating back to 0 Hz. In this case, one is, in essence, installing a βHysteresis.

A technique for simulating transducer output displacement from electric current has been presented in this section of the dissertation. A variation on this technique is presented in Appendix D.

5.7 Discussion of Eddy Current Trends Using a Constant Permeability

It should be mentioned that eddy current theory, as it is traditionally applied to Terfenol-D, makes the assumption that eddy current losses are usually the most significant at "high" frequencies, when the transducer is operating in the mass controlled region. As a result, it is assumed that the eddy current losses should be estimated using the blocked permeability of Terfenol-D. The idea of basing the eddy current losses on the blocked permeability was investigated using the model which included the effects of the rod and housing.

To this point in this study, χT has been given by Eqn. (3.3.13a). This permeability was used everywhere that the permeability of Terfenol-D was required. Recall, for the second transducer model, the eddy current parameter given by Eqn. (4.2.9) was κT = √(j ω μT σeT), and this
$k_T$ was used as part of the arguments of a number of different modified Bessel functions. Modified Bessel functions were required to account for the effects of eddy currents. Simulations were attempted using the blocked permeability in $k_T, \mu_T$ from Eqn. (3.3.13a) was used in Eqn. (4.4.14), the only place the permeability of Terfenol-D appears outside of a Bessel function and it does differ from the blocked permeability for mechanical (i.e., non-eddy current) reasons.

The results of these simulations were unsatisfactory (a sample calculation is presented in Appendix D). To begin with, these simulations over predicted electrical impedance magnitudes around the frequency of mechanical resonance (approximately the frequency of the local maximum impedance magnitude). This trend suggests that using the blocked permeability results in an underestimation of eddy current losses near resonance. Lower losses translate to higher flux linkages, requiring larger voltages for a given current. Thus, the predicted impedance magnitude is larger.

The second unsatisfactory observed trend was a further increase in the magnitude of the displacement from current simulations at frequencies around resonance (when compared with the previous technique, which already overestimates the peak by about 10%). This trend also suggests that using the blocked permeability results in an underestimation of eddy current losses near resonance. Lower losses mean higher transduction resulting in larger displacement predictions.

The last unsatisfactory trend was that the phase of displacement from current, as predicted via this assumption, did not follow the experimental phase as well as the simulations resulting from the use of the dynamic permeability.

All of these trends suggest that the formulation for the magnetic permeability given by Eqns. (3.3.13) should be included in estimating eddy current effects. The trends also add credence to the approach taken in this study, i.e., motional effects do change the magnetic permeability of Terfenol-D. Including these motional effects results in improved estimates of transducer performance when compared to classical theory.
Classical eddy current theory is usually couched in terms of a penetration depth, $\delta$, where $\delta$ is used to imply the thickness of component laminations required to reduce eddy effects to near zero. That was not done here. (For those interested, $\delta = (1+j)/k$ where $k$ is given by Eqn. (4.2.9) for the various components of the transducer.[12]) However, the results above suggest that if the transducer designer opts for the classical approach in estimating eddy current losses, they should at least include an enhanced permeability in their calculations. For example, if the transducer is to be operated near mechanical resonance, instead of estimating the eddy current losses using, say, $\mu_T \approx 5\mu_0 = \text{a constant}$, calculate the estimate using $\mu_T = 2\mu^\sigma \approx 10\mu_0$. If the transducer is to be operated at frequencies well below mechanical resonance, use $\mu_T = \mu^\sigma = 5\mu_0$; above resonance use $\mu^e$, etc.

5.8 Repeatability of Transducer Performance Using Terfenol-D as the Motion Source

There is a very significant difference between predicting Terfenol-D transducer performance and simulating it. The comparisons made so far have been between model simulations and transducer performance. For each simulation, model parameters (Terfenol-D material properties) were derived/calculated from the corresponding experimental measurements of transducer performance. The substantial agreement between experimental measurements and model simulations is due primarily to the consistent formulation of the model. This agreement adds credence to the modelling procedure. It remains a very different task, however, to predict a transducer's performance. In order to use the models developed in this study for that purpose, one must be able to predict the material properties of the magnetostrictive rod within the transducer as functions of "everything," i.e., drive amplitude, prestress, magnetic bias point, temperature (discussed below), and quite possibly other undiscovered/ignored, but significant factors. The transducer in this study used only Terfenol-D as the motion source. It was not the purpose of this study to generate general functional relationships for
material properties of Terfenol-D. However, a discussion of the variabilities observed in material parameters will be the subject of this section of the dissertation. The discussion will begin with a comparison between a model prediction, simulation, and an experimental measurement of \( Z_{\text{ee}} \).

Figure 5.21 shows magnitude and phase of the electrical impedance of the transducer, run at 100 mA with a mass load of 0.108 kg, as predicted, as measured experimentally, and as simulated. For the figure, the prediction was calculated assuming that the material parameters were those as measured for the 100 mA experiment shown in Figure 5.17 (where the mass load was 0.125 kg). Specifically: \( q = 3.712 \times 10^{-9} \text{ m/A}, E_y^H = 49.67 \text{ GPa}, k^2 = 0.1087, \) and \( 2\zeta = 0.0658 \). For Figure 5.21, however, the mass load of the transducer was changed. Compare the agreement between simulation and experiment, and the agreement between prediction and experiment as displayed in the figure. (The simulation very nearly overlaps the experiment.) The relatively poor agreement between prediction and actual performance was thought to be due primarily to the changes in Terfenol-D material parameters from run-to-run and day-to-day. It seems to be a problem of the magnetostrictive material Terfenol-D, not a problem of the modelling procedure. (It represents a problem for this, and any modelling procedure.)

As one might expect, the largest errors in both the prediction and the simulation occurred at frequencies near resonance. However, the largest error in the simulation was about 8%, whereas the largest error in the prediction was over 40%. The root-mean-square error in the simulation was about 2%; for the prediction it was about 10%.

The parameters measured from the experimental data shown in Figure 5.21, and used in the calculation of the displayed simulation, were: \( q = 3.150 \times 10^{-9} \text{ m/A}, E_y^H = 48.51 \text{ GPa}, k^2 = 0.0861, \) and \( 2\zeta = 0.0781 \). A comparison with those used in the prediction reveals that the linear coupling, \( q \), changed by 18%, the elastic modulus at constant field strength changed by 2%, the square of the magnetomechanical
Figure 5.21. Comparison of model prediction and simulation with experimental measurement of $Z_{ee}$, (a) magnitude and (b) phase. See text.
coupling coefficient, $k^2$, changed by 26%, and the damping changed by 16%. These were changes experienced using the same rod, at the same prestress, with the same magnetic bias, in the same transducer, with different mass loads, and measured on different days. According to theory, at least as presented in this dissertation, changing the mass load (or the day) should have had no effect on the material parameters. Changing the mass load should have changed the performance of the transducer, but not the Terfenol-D magnetostrictive rod's material parameters. Nonetheless, they changed. As shown in Figure 5.21, these changes had a detrimental effect on the accuracy of the prediction of the transducer's performance. As a result, the designer who uses the models developed in this study, should anticipate larger errors in their predictions than those displayed in the simulations presented in this dissertation.

To this point in this study, the effects of temperature changes on Terfenol-D performance have been neglected. According to [2], at the low drive levels used in the present study, one may anticipate the magnetostriction to remain very nearly constant over the temperature range of 0 to at least 40 °C. Since the transducer in this study always remained "cool" to the touch, it was thought to have remained below body temperature, i.e., less than 40 °C. Nonetheless, a series of tests were performed to determine temperature effects on the transducer's performance. Eight swept-sine experimental measurements were performed using an excitation level of 100 mA and seven tests were performed using 20 mA drive current amplitudes. All measurements were made with the same rod, same prestress, same load, and in a fashion so that temperature effects (due to ohmic heating of the coil, rod and housing) should have been discernable. Model parameters $E_y^H$, $q$, $k^2$, and $\zeta$ were calculated from each experimental measurement. The only identifiable trend due to temperature was a reduction in damping with increasing temperature - but that was displayed only by the 100 mA tests. None of the other parameters at either drive level displayed reproducible trends with the changes in temperature experienced for these tests ($20 \leq \text{temperature} \leq 40 \, ^\circ\text{C}$).
The statistics for these measured transducer parameters are displayed in Table 5.2. For the table, ave. = the arithmetic mean of the data, std. dev. = the standard deviation of the data, based on "N", as opposed to "N-1", and %unc. = 100(std. dev./ave.), which is a measure of the relative uncertainty in the measured quantities.

The uncertainties displayed in Table 5.2 for the moduli of elasticity, the 20 mA damping, and the 100 mA k² were likely due to the uncertainties in the measured frequencies used in the calculations. The frequency resolution of the graphical technique employed was approximately ±5 Hz, and a change on that order would result in larger uncertainties than are actually shown. However, the 100 mA damping and the 20 mA k² uncertainties are larger than what might be reasonably attributed to experimental frequency measurement error.

As mentioned previously, the 100 mA damping varied with temperature. The 20 mA k² simply varied, and it varied more than the

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$E_y^H$ (GPa)</th>
<th>$2\zeta$ ($\times 10^{-9}$ m/A)</th>
<th>$q$ (m/A)</th>
<th>$k^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 mA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ave.</td>
<td>48.08</td>
<td>7.49</td>
<td>3.23</td>
<td>0.0900</td>
</tr>
<tr>
<td>std. dev.</td>
<td>0.23</td>
<td>0.35</td>
<td>0.08</td>
<td>0.0028</td>
</tr>
<tr>
<td>%unc.</td>
<td>0.5</td>
<td>4.7</td>
<td>2.5</td>
<td>3.1</td>
</tr>
<tr>
<td>20 mA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ave.</td>
<td>56.23</td>
<td>8.47</td>
<td>2.53</td>
<td>0.0735</td>
</tr>
<tr>
<td>std. dev.</td>
<td>0.81</td>
<td>0.08</td>
<td>0.10</td>
<td>0.0031</td>
</tr>
<tr>
<td>%unc.</td>
<td>1.4</td>
<td>0.9</td>
<td>4.0</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Table 5.2. Statistics for Terfenol-D material parameters as measured experimentally for $N = 8$ transducer tests at 100 mA, and $N = 7$ tests at 20 mA drive current amplitudes. Test conditions were identical with the exceptions of drive current amplitudes and a possible variation in temperature from 20 to 40 °C.
100 mA value. The same conclusion can be drawn from the experimental measurements of q. While the uncertainty in the numerical value of q from the experimental set-up was on the order of five-percent, it was thought that the relative changes were subject only to random instrumentation errors, i.e., much less than one-percent uncertainty. Thus, it is shown that the linear coupling varies from test-to-test.

With the exception of the damping parameters, the 20 mA tests displayed larger variations than the 100 mA tests. This trend is likely due to anomalies within the Terfenol-D rod. It seems likely that at low drive amplitudes, fewer domains are participating in the the transduction process than at the higher drive amplitudes. Thus, the anomalous behavior of a small percentage of domains is more significant at lower than higher drive amplitudes.

Material parameters also seem to vary from one Terfenol-D rod to the next. This behavior was demonstrated by testing two, brand-new, two-inch long rods which were cut from the same piece of stock. It was thought that two rods from the same stock would provide the best possible match of material parameters. The two rods were tested under as close to identical conditions as was physically possible (same bias, same prestress, same drive amplitude, 100 mA, 0.108 kg mass load, etc.). Each rod was tested twice, one test right after the other. Variations between samples of 9% were found in $E_y H$, 17% in $\zeta$, 22% in q, and 32% in $k^2$. These types of variations from one sample to the next imply the futility in employing published values of Terfenol-D parameters when modelling Terfenol-D actuators.

Table 5.3 summarizes the types of variations discussed above. All of the datums are percentages and were calculated by comparing the maximum to the minimum parameter measured under the corresponding conditions, i.e., $100(\text{maximum} - \text{minimum})/\text{maximum} = \text{reported values}$. Variations from day-to-day (measured with different mass loads), test-to-test (one right after the other), and rod-to-rod are shown. As shown by the data in the table, it is not unreasonable to anticipate
significant variations in Terfenol-D behavior. These variations in Terfenol-D parameters represent an area where further research is needed. In the mean time, user be advised.

Table 5.3. Summary of percentage variations in measured Terfenol-D material parameters from day-to-day (at 100 mA), test-to-test, and rod-to-rod (also at 100 mA)

<table>
<thead>
<tr>
<th>Parameter:</th>
<th>Parameter:</th>
<th>Parameter:</th>
<th>Parameter:</th>
<th>Parameter:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_y^H$</td>
<td>$2\zeta$</td>
<td>$q$</td>
<td>$k^2$</td>
</tr>
<tr>
<td>Condition:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>day-to-day</td>
<td>2</td>
<td>16</td>
<td>18</td>
<td>26</td>
</tr>
<tr>
<td>test-to-test</td>
<td>1</td>
<td>13</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>(100 mA)</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>(20 mA)</td>
<td>9</td>
<td>17</td>
<td>22</td>
<td>32</td>
</tr>
<tr>
<td>rod-to-rod</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. CONCLUSIONS

New analytical models for simulation of transducer electrical impedance and output displacement per ampere, as functions of magnetostrictive rod dimensions, transducer geometry, load, and frequency of excitation, have been developed and experimentally verified. The models provide information about both magnitude and phase for quantities of engineering interest.

The magnetomechanical model was derived by comparing the low signal transduction equations for the magnetostrictive material, with those for the transducer containing the material. The result was an expression for the magnetic "dynamic" permeability of the magnetostrictive material within the transducer. The permeability was found to be a function of the traditional low signal material parameters: $k^2$, $q$ (aka the "d" constant), $E_yH$, and $\mu^\sigma$, and a function of the stiffness of the prestress mechanism, applied load, transducer mechanical impedance, frequency of excitation, and amplitude of excitation. The dynamic permeability, i.e., the magnetomechanical model, is given by Eqns. (3.3.13). The effects of eddy currents within various components of the transducer were included by solving Maxwell's equations in cylindrical coordinates. The electromagnetic model derived for Transducer Model 2, Eqn. (4.4.20), was the most general since it included the effects of a conducting external cylindrical housing.

The transducer's electrical impedance function, $Z_{ee}$, is estimated by using the magnetomechanical model in the electromagnetic model, wherever the permeability of the magnetostrictive material appears. The transducer's blocked impedance, $Z_e$, is estimated by using the magnetostrictive material's *blocked* permeability in the electromagnetic model, wherever the permeability of the magnetostrictive material appears. The transducer's motional impedance, $Z_{mot}$, is estimated by subtracting $Z_e$ from $Z_{ee}$. The transducer's transduction coefficient, $T_{em}$, is calculated from the definition of $Z_{mot}$. Output displacement per ampere is then calculated
using the mechanical impedances of the transducer and the load, along with the transduction coefficient.

The models are thought to be reasonable for use in the low to medium signal amplitude range of transducer operation. In the present study, the experimental set-up limited the range of operation (it was incapable of maintaining 300 mA current amplitudes over the frequency range of interest). Time domain measurements suggested, however, that current amplitudes of approaching 500 mA could have been modelled. This represents magnetostrictive rod operation up to about one-third of its saturation strain.

The simulations reported here used published values for all material properties except for the transducer's effective damping factor, and the following Terfenol-D parameters: magnetomechanical coupling, magnetic permeabilities, linear coupling, and moduli of elasticity. These quantities were estimated by using the approximation techniques of electrical impedance and admittance analysis. It was thought to be encouraging that none of these measured quantities were required to be ridiculous values in order for the models to provide reasonable results. For example, if it had been required to use magnetomechanical coupling factors greater than one, or even approaching one, that would have cast reasonable doubt upon the model formulations.

The transducer in this study used Terfenol-D as the motion source. Time domain and frequency domain measurements show two types of nonlinearities of which a user should be aware. The first type of nonlinearity pertains to harmonic frequencies, aka overtones. When the material is excited with a field strength resulting from a sinusoidal current in the wound wire solenoid, the output displacement (velocity and acceleration) contains harmonic frequencies, and the relative amplitudes of the harmonics increase with increasing drive amplitude. The second type of nonlinearity deals with amplitude dependance of transducer performance. Doubling the drive amplitude more than doubles the displacement amplitude when operating in the low to medium signal amplitude range.
The models are limited in several, possibly important, ways:

1) They deal with the low to medium signal linear range of transducer operation.

2) They ignore the presence of harmonic frequencies. If harmonic signal content is sought, the methods presented here will not supply that information.

3) They assume that the rod behaves as a linear spring. This is considered to be a reasonable assumption for most applications since, in the process of prestressing the rod and supplying some mechanism for attaching a load to the transducer, the magnetostrictive rod is generally mass loaded. However, in some transducers, the mass load may not be large enough for this to be a reasonable assumption.

4) End effects within the solenoid were ignored, recall, B and H were assumed to be independent of axial position. In the results presented in this dissertation, the ends of the solenoid were ignored (except in Appendix E). Specifically, the inductance was calculated based on a solenoid exactly as long as the magnetostrictive rod. This was done despite the fact that the solenoid was actually nine-percent longer than the rod. Calculations were attempted based on the actual length of the solenoid. The assumption for this attempt was that the portions of the steel ends which extended into the solenoid (recall components "i" and "a" of Figure 1.5), behaved exactly as did the Terfenol-D rod. When that was done, there was a consistent ten-percent over-prediction of electrical impedance amplitudes. Neither assumption of solenoid behavior, of course, was strictly correct. In that sense, the models were "calibrated" to the transducer under study. It remains a limiting assumption of the models presented in this dissertation, that B and H are independent of axial position. Two-dimensional electromagnetic models H(r, z), were not developed.

5) The ends of the transducer, which constituted portions of the overall magnetic circuit, were ignored. It might be concluded that the effects of eddy currents occurring within these components were insignificant compared to the large-scale eddy currents within the rod and external housing. If they had been significant, they might have
resulted in deviations, between experiment and model simulations, much like those caused by ignoring the external housing. Then again, perhaps no strange unexplained trends were apparent because the steel ends were, for all practical purposes, doing all of the damage they were going to do by 100 Hz (the first measurement frequency of these simulations). If that were the case, they would have been buried in the aforementioned "calibration" of the model. The effects of the steel ends remain an unknown.

6) The models were formulated for a common transducer geometry, i.e., a solid magnetostrictive rod in a wound wire solenoid which was within a cylindrical external housing. If one were trying to estimate behavior for a different geometry, for example, a hollow rod with a bolt down the center, the equations in Chapter 4 would need to be solved employing the correct boundary conditions.

The models as presented are capable of generating reasonable reproductions of experimental measurements. It is important to note that these were simulations, not predictions (recall Section 5.8). However, the success of the simulations suggest that if one had reasonable estimates of material and transducer parameters \( \mu^\sigma, k^2, E_y^H, k_{mps}, q \), etc., as functions of amplitude of excitation, and everything else), reasonable predictions of transducer behavior could be calculated before the transducer was built. General formulations for material parameters were not found as part of this investigation. That work constitutes further research which should be conducted.

Assuming parameters are known, the models could be useful as design tools. Along those lines, if a transducer designer believes that eddy currents will not be a problem with their design, i.e., the rod has been laminated (an expensive proposition), the housing has been slit longitudinally to interrupt the cylindrical conducting path, etc., the model formulation ignoring all eddy currents can be used. (See Appendix E.) If the rod is not laminated, but the housing has been slit (or does not exist), the model including eddy effects of only the rod can be used. (See Appendix E.) If nothing is done to reduce eddy
effects within transducer components, the model including eddy
effects of the rod and housing provide the most reasonable results.

The models presented in this study represent an improvement in the
field of impedance modelling of magnetostrictive actuators. Previous
investigations have used an "eddy current factor" (another $\chi$) defined
using Kelvin functions, which are equivalent to the Bessel functions
used here, but where the permeability is always assumed to be a
constant, usually the blocked permeability of the Terfenol rod. The
result of that assumption is an under prediction of eddy current losses
as frequencies increase to mechanical resonance. At these
frequencies, enhanced motion results in increased permeability and
larger than expected eddy current losses. Eddy current theory as
traditionally applied to Terfenol-D transducers, thus assumes that
eddy current losses are an unchanging feature of the transducer. The
success of the simulations presented here, which assume otherwise,
indicates that constant permeability does not always provide the best
estimate of eddy current losses. (See Appendix D.)

The general areas of linear transduction, magnetism,
magnetostriction, and electromagnetism, as applied to cylindrical
magnetostrictive transducers, have been investigated. They have been
explained and developed assuming the reader has just a general
dynamics and mechanics background. Thus, the work presented in this
dissertation should help bridge the gap between the
physicists/materials scientists, who developed the giant
magnetostrictive material Terfenol-D, and the engineer who may wish
to utilize the material as a motion source.
REFERENCES


2 Clark, A. E., "Giant Magnetostriction From Cryogenic Temperatures to 200° C," U. S. Naval Surface Weapons Center, Silver Springs, MD.


ACKNOWLEDGMENTS

Financial support was received, and is gratefully acknowledged. Thanks to the National Aeronautics and Space Administration for their generous support through the Graduate Student Research Program. In particular, the support of the Sensors Branch at the George C. Marshall Space Flight Center, in Huntsville Alabama is appreciated. Thanks also to the others who have supplied financial support during the course of this work: The Roy J. Carver Foundation, National Science Foundation, the Department of Aerospace Engineering and Engineering Mechanics at Iowa State University, the Guaranteed Student Loan Program, Carol W. Hall, Betsy Olberding and Stan Hall, Millard E. Hall, and Joreen J. Hall.

The help, support, advice, and insight provided by the brave souls on The Committee was appreciated.

Alison B. Flatau, aka "Dr. Al," served as Major Professor during this effort. (I still think she deserves an annual parade.) Thank you for your guidance, time, patience, and friendship. Thank you for your trust, and for believing in me. I will always be grateful.

Kenneth G. McConneil, aka "Mr. Vibs," taught me about experimental errors. You have me so nervous about force transducers that I will probably never use one again. As to your other advice, I believe it will serve me well throughout my career, and life. Thank you.

Bruce R. Munson is simply the finest instructor I have ever had the pleasure of meeting. You, sir, are amazing! You have ability, humility, and extend a genuine respect to your students. You patiently wait for your students to understand. Your standards are high, and your students strive to meet them. I count myself among the lucky few who have taken classes from you. Thank you.

Peter J. Sherman, my controls guru, also extended his respect, insight, and friendship. You, too, are good. When I was taking controls, I actually thought I understood the material! Thanks. By the way, in the course of working with Terfenol-D, I decided that all of the self-sensing control techniques we discussed would be of little value. At present, there seem to be too many deleterious realities of the
material to make any of our ideas of value. It seems that one's best
bet at controlling this stuff lies with strain gages, external sensors,
big amplifiers, etc. Self-sensing will likely have to wait until such
time as predictable material exists.

Lalita Udpa served as my magnetics specialist. She kept me honest
in my magnetism and electromagnetism work. Thank you for not
writing me off as an idiot. When I could figure out what questions to
ask, you were kind enough to answer them. Thank you.

The Department of Aerospace Engineering and Engineering Mechanics
at Iowa State University has been very, very good to me. Dr. David K.
Holger (the Chair), Gayle Faye (the one in charge), Sally Van De Pol
(second in command), and Tammy Boyd (third in command? What did
you do?) have all been supportive and understanding during my stay in
Iowa. However, it went further than that. You folks were looking out
for me, went to bat for me, and were on my side. I never felt like just
another graduate student. I actually felt welcome. Thank you all.

It is safe to say that this work would not have been possible
without the kind support and understanding of my family. My "PhD
Widow," Joreen, has not seen much of me lately. Thank you for your
understanding whilst I have been "gone." You picked up the slack,
carried the load, listened to my gibberish, and your importance cannot
be overstated.

Every member of my family has been supportive during my decade of
college. Thank you all so very much. I will try to be better company in
the future.
APPENDIX A: TIME DOMAIN MEASUREMENTS OF FLUX DENSITY, MAGNETIC PERMEABILITY, MAGNETIC FIELD STRENGTH, AND OUTPUT DISPLACEMENT

Though none were reported, B-H measurements were performed for the following reasons:

1) Magnetic permeability can be used as an input of the electromagnetic-magnetomechanical model discussed in the body of this communication. For this type of measurement the transducer was operated at frequencies where the eddy current and dynamic effects were thought to be negligible.

2) It was desired to show the functional relationship between applied field strength, frequency of excitation, and magnetic permeability. This information could be used by the electromagnetic-magnetomechanical model for excitation amplitudes where magnetic hysteresis cannot be ignored; thus extending the range of operating conditions under which reasonable estimates of transducer behavior might be calculated by the model.

B-H measurements were performed in a nearly standard fashion. Voltage measurements from a pick-up coil were digitally sampled, then numerically integrated and scaled to obtain a discrete version of B(t). Drive current measurements were taken simultaneously with the pick-up voltages, allowing calculation of H(t). The average magnetic permeability for the volume enclosed by the pick-up coil was estimated by fitting an ellipse to the B(t) vs. H(t) data. More on the numerical techniques follows later.

For dynamic B-H measurements, over long periods of time, using various rods, drive levels, frequencies, etc., it was thought that winding a pick-up coil directly on the magnetostrictive rod was inadvisable. Recall that the rod strains; if the coil had been wound directly onto the rod, it too would have been required to move. This would have added mass to the rod, would have been asking for electrical problems when the insulation wore off of the wire, and would have made assembly of the transducer quite complicated. Thus, a coil assembly was wound with the inside layer being the pick-up coil.

This construction, though physically sound, had at least one implication that had to be taken into consideration. The induced voltage of the pick-up coil was due to the time rate of change of the magnetic flux within the rod and the cylindrical air gap between the coil and the rod. The permeability implied by the B-H fit mentioned
above was usually less than the true permeability of the rod enclosed (the permeability of the rod is the quantity of interest). If eddy currents are negligible it is possible to estimate the permeability of the rod via:

\[ \mu_{\text{rod}} \approx \left( \mu_r e^{i\phi_{BH}} \right)_{\text{experiment}} \left( \frac{R_{pui}}{R_{TO}} \right)^2 - \left( \frac{R_{pui}}{R_{TO}} \right)^2 + 1 \mu_0 \]

where: 
- \( \mu_r \) = the relative permeability implied by the experiment,
- \( \phi_{BH} \) = the phase angle between B and H from experiment,
- \( R_{pui} \) = the radius of the pick-up coil, inner (radius), mm
- \( R_{TO} \) = the radius of the "Terfenol" rod, outer (radius), mm, and
- \( \mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m}/(\text{A}\cdot\text{turn}) \).

If eddy currents, or dynamic effects on the magnetic permeability are not negligible, it is impossible (or exceedingly difficult) to separate the effects from the rod's permeability. It is much easier to formulate the electromagnetic-magnetomechanical model to simulate the experiment. It was thought that if the model could predict the experimental B-H results, that would increase confidence in the electromagnetic-magnetomechanical model. Simulations of the transducer's electrical impedance function, are in essence, doing exactly that; only applied to the entire solenoid.

The following specifications apply to the drive/pick-up coil assembly used in this investigation: wire diameter (including insulation) = 0.0167 ±0.0002 inches, coil length = 2.20 inches, turns per layer (approximately) = 130. The inner coil average inside diameter was measured as 0.274 ±0.001 inches. The outer (drive) coil consisted of ten layers, the minimum outside diameter was measured as 0.605 inches. The inner radius for the drive coil was calculated as 3.854 mm. When run as an air-coil, the outer coil behaved as though it had an inductance of 2.75 mH. Figure A.1 is a schematic of the coil assembly showing wire winding orientation for the two coils.

**Theory of Operation**

The voltage over an inductor is given as the negative of the time derivative of the total magnetic flux linkage, aka "Faraday's law of induction," i.e.,

\[ V = -\frac{\partial \Phi_n}{\partial t} = -N_1 \frac{\partial}{\partial t} \int B \cdot d\Delta \]

where: \( V = \) the voltage measured over the leads of the pick-up coil,
\( \phi_m \) = the total magnetic flux threading the pick-up coil,
\( N_i \) = the number of turns in the pick-up coil,
\( \mathcal{B} \) = the magnetic flux density, a vector quantity, and
\( d\mathbf{A} \) = a differential area.

This can also be written as:

\[
\frac{-1}{N_i} \int V(t) \, dt = \int \mathcal{B} \cdot d\mathbf{A}
\]

Note that it is tacitly assumed above that all turns of the coil are threaded by the same flux. Note also that there is an integration over the area within the coil. If eddy effects are present then the flux density is a function of position. For this reason, the electromagnetic-magnetomechanical model was configured to calculate the area integral and that result was compared with the numerical time integration of the experimental measurements of \( V(t) \).

Figure A.1. Sketch of drive/pick-up coil assembly used in dynamic measurements of magnetic flux from applied field. Outer coil is a ten layer drive coil. Inside coil is a single layer pick-up coil.
Realities of Operation

It is important to measure the voltage of the pick-up coil with a very high impedance instrument, that is, it is desirable to keep the current flow in the pick-up coil to a minimum. This is important for two reasons. First, the measurement circuit should not interfere with the phenomena of interest. If the circuit's impedance were too low, power would be extracted from the transducer in the process of taking the measurement. The second reason for keeping the current flow in the pick-up coil to a minimum is that the electromagnetic-magnetomechanical model is formulated assuming that the current density in this area of the transducer is zero. In this investigation, one lead of the pick-up coil was attached to the instrumentation circuit's ground, and the other to a buffer/amplifier (1/2 TL 082 CP "op-amp") with an input impedance of approximately $10^{12}$ Ohms ($\Omega$). Pick-up coil voltages were on the order of a volt, thus currents were "small," i.e., on the order $10^{-12}$ amps. The gain of the op-amp circuit was measured as $10.95 \pm 0.2\%$, from 0 to 20 kHz, with phase lag nearly linear with frequency, phase in degrees $= -\frac{3}{20000} f$, $f$ in Hz.

Two different schemes were used for measuring electric current in the drive coil. The first scheme was simply to measure the voltage drop across a series resistor (with about 0.27 mH of inductance, resulting in about 3.5 degrees of phase shift at 20 kHz). The second scheme was to use the current monitor terminal on the current control module of the Techron amplifier (which amplified the voltage drop across a series precision power resistor within the current control module).

Because different grounds existed for the amplifier and the instrumentation circuit, the circuit needed to have differential inputs. The circuitry for current measurements employed both halves of a TL 082 CP. The first stage was the differential input with 200 k$\Omega$ input impedance. The second was an inverting amplifier. The overall gain of the configuration was measured as $10.03 \pm 0.2\%$ with phase lag of less than two degrees at 20 kHz (the indicated phase was approaching the accuracy of the Tektronix 2630 Fourier Analyzer used in the tests).

See Appendix B for schematics of the instrumentation circuitry designed, built, and calibrated for use in this investigation.
Calculation Details

Parameters $\mu$ and $\phi_{BH}$ were sought for the elliptical B-H model

$$B = \left(\mu e^{i\phi_{BH}}\right)H$$

Parameter estimation was performed in the time domain using single frequency sinusoidal electric current excitation and an integer number of cycles in the window. It was assumed that $H(t)$ and $B(t)$ had the following functional forms:

$$H(t) = H_m \sin(\omega t + \phi_H)$$

$$B(t) = B_m \sin(\omega t + \phi_H + \phi_{BH})$$

where:

- $H_m$ = amplitude of sinusoidal field strength, A-turn/m,
- $\omega$ = the circular frequency of current excitation, rad/s,
- $t$ = time, s,
- $\phi_H$ = the phase in $H(t)$ necessary to assume $t = 0$ for the first sample, radians,
- $B_m$ = the amplitude of the assumed sinusoidal magnetic flux density, T, and
- $\phi_{BH}$ = the phase between $H(t)$ and $B(t)$, radians.

Digital samples of drive coil current and pick-up coil voltage were sampled simultaneously using the instrumentation circuitry and the Tektronix 2630 Fourier Analyzer (AC coupled). A Tektronix utility program was used to transform the data to MATLAB file format for the subsequent processing. Known quantities included the time averaged digital samples, sample time interval, frequency of excitation, circuit gains, and coil parameters (areas, turns, turns per length, etc.).

Numerical routines were written and tested for converting the digitally sampled pick-up coil and current transducer voltages into discrete estimates of $B(t)$ and $H(t)$, respectively. Conversion was performed by the program "BHCONVERSION." Numerical integration was done in the routine called "SIMPSONS" (no relation to Bart). Accuracy of the numerical integration was improved by sampling the original data such that there were an integer number of cycles in the analysis window. Parameters for the elliptical B-H model (detailed above) were estimated in the program "BHPARAMETERS." All of these routines were included in Appendix C.

The entire experimental and parameter identification approach was tested with the solenoid assembly operated as an air solenoid. The relative permeability of the core (air) was estimated by this technique as 1.012 for a 3 Hz, 64 mA drive current, and as 1.009 for an 860 mA
current. The estimates were within approximately one percentage point of the anticipated value, 1.00. This was considered to be acceptable accuracy.

Displacement Modelling

The same procedure, minus the time integration, allows displacement from current, or applied field, to be modelled. In this case, displacement is substituted for $B(t)$, $u_m$ is $B_m$, $\phi_{uH}$ is $\phi_{BH}$, etc. The program "BHCONVERSION" was written with a "switch" to allow either B-H or u-H modelling.
APPENDIX B: INSTRUMENTATION CIRCUITRY SCHEMATICS

Inside coil
Near end

V10 \propto \phi_m

10 K

1 K

+Vcc

IC TL 082 CP Dual Op-Amp

-Vcc
V40 \propto -i(t)

OUTPUT

+Vcc

100 K

100 K

100 K

100 K

2.2 K

IC TL 082 CP Dual Op-Amp

10 K

INPUTS

V31

neg.

current xducer.

V32

pos.

-\text{Vcc}
All @ 100 K except as noted

IC TL 082 CP Dual Op-Amp

V51 neg. voltage INPUT

V52 pos. voltage INPUT

V60 \propto -V(t) OUTPUT

+Vcc

10 K

2.2 k

10 K

-\text{Vcc}
APPENDIX C: COMPUTER PROGRAMS AND FUNCTIONS

Some of the routines written for this investigation are given in this appendix. With the exception of the last program, which is in FORTRAN, the routines were written for use with a down-sized (student) version of the reasonably popular software known as MATLAB, The MathWorks, Inc., Cochituate Place, 24 Prime Parkway, Natick, MA, 01760, Version S3.5, 1-Jan.-92. MATLAB stands for matrix laboratory. The software is not exceedingly expensive, was written by folks with a sense of humor, features editable routines for calculating all sorts of functions including simple Bessel and Hankel functions, curve fit functions, calculates in double precision, understands what a complex number is, plots, writes various types of files, and was fairly easy to learn.

Key features of the program (MATLAB) are outlined next. The symbol "%" is used to comment out the rest of the paragraph (everything between it and the next hard return).

Functions, even user defined functions, are invoked by typing: what-you-want = some-function(input-variables), just like y = sin(x).

A semicolon at the end of the statement instructs the program to NOT display that calculated result, for example, "[Ix Kx] = modbessn(order, argument);" returns to vector "Ix" the values of the modified Bessel function of the first type, of order "order," corresponding to each value in the vector "argument," and returns to vector "Kx" the values of the modified Bessel function of the second type, of order
"order," corresponding to each value in the vector "argument," and the semicolon at the end saves the operator the trouble (and time) of values of Ix and Kx scrolling past on the control screen.

Users can write their own programs/routines or functions. Typing the name of a program causes that program to run. Programs can be invoked by other programs. The user can also call-up, edit, and rename most of the existing MATLAB routines. That was handy several times during the course of this research.

The FORTRAN program, "itsolenoid.f", may be of interest to some. It was written to allow iteration to determine the solenoid winding configuration which would result in the lowest axial variation of flux density, B(z). One can manually remove sections of each coil layer and see the effect on the axial distribution of B. Some readers may be familiar with "dumbbell" coils. This program will allow one to estimate B(z) for these types of coils. (It assumes the permeability everywhere is a constant.)

Program: BHCONVERSION

%BHconversion, D. Hall, 14 Jan., 1994
%TimeV = the time vector
%TimeA1 = ch. 1 voltage samples (= dB/dt)
%TimeA2 = ch. 2 voltage samples (= -I)
%TimeA3 = ch. 3 voltage samples (= -V)
%TimeA4 = ch. 4 voltage samples (= ±u)
clear,format compact
no=23264;
rodlength=2*0.0254;
Ni=130;
Rpui=0.00348;
Ai=pi*(Rpui)^2;
Rto=0.00318;
muo=4*pi*1e-7;
%
% ------------------------ change these ------------------------
load f61504e.mat
frequency=200
w=2*pi*frequency
EUnits,pause(1)
%
% Obtain actual values from experimentally measured voltages using
% sensitivities, remove DC components at this point also.
dt=TimeV(2)-TimeV(1);
Vi=TimeA1/EUnits(1); % Vi is the induced voltage in inner coil
% this inserted for use if some OTHER quantity is of interest versus H
switch=1; % switch = 0 is normal BH stuff, switch = 1 is displacement
if switch == 1,Vi=TimeA4/EUnits(4);,muo=1,end
aveVi=mean(Vi)
Vi=Vi-aveVi;
l=TimeA2/EUnits(2); % l is the drive current in amperes
avel=mean(l);
l=l-avel; , maxl=max(abs(l)), clear maxl
V=TimeA3/EUnits(3); % V is the drive voltage in volts
aveV=mean(V);
V=V-aveV;
plot(TimeV,Vi),title('Vi vs TimeV'),grid,pause
plot(TimeV,l),title('l vs TimeV'),grid,pause
plot(TimeV,V),title('V vs TimeV'),grid,pause
% Numerically integrate Vi for B calculations.
if switch == 1,
Boft=Vi;
else
    Boft=simpsons(Vi,dt,1); % assumed periodic for numerical integration
    Boft=Boft/(-Ni*Ai);
end
aveB=mean(Boft)
Boft=Boft-aveB;
Hoft=no*l;
plot(TimeV,Hoft/nnax{Hoft),TimeV,Boft/max(Boft)),title('Normalized H and B of t'),pause
axis('square');
plot(Hoft,Boft),grid,pause
% estimate the relative magnetic permeability as the slope of a pure ellipse.
if switch == 1,
    mur=(max(Boft)-min(Boft))/(max(Hoft)-min(Hoft))
    q=mur/rodlength,% assumes strain x length = displacement
else
    mur=(max(Boft)-min(Boft))/(max(Hoft)-min(Hoft))/muo
end
% compare above with that implied by a linear fit to the BH data.
p=polyfit(Hoft,Boft,1); % this doesn't work so well with only one cycle
p(1)/muo,mur
% if mur > p(1),mur=p(1),q=mur/rodlength,end/%muo;end,% helps with high freq.
noisey data, choose the minimum of the two.
% if p(1)/muo < mur, mur=p(1)/muo,end,% helps with low freq. noise data
BHinfo=sprintf('Average (over area of p/u coil) Relative permeability = %g',mur);
if switch == 1,
    plot(Hoft,Boft*1e6),title(BHinfo),xlabel('Applied Field, H, Amp
turns/meter'),ylabel('displacement, micro-meters'),grid,axis('normal');,pause
else
    plot(Hoft,Boft*1000),title(BHinfo),xlabel('Applied Field, H, Amp
turns/meter'),ylabel('Flux Density, B, milli-Tesla'),grid,axis('normal');,pause
end
BHparameters
Program: BHPARAMETERS

%BHparameters D. Hall 16 Jan., 1994
%Routine for estimating parameters for model:
%H(t) = Hm sin(wt + phiH)
%B(t) = Bm sin(wt + phiH + phiBH)
% = IMAG[ { mu_r mu_0 exp(j phiBH) } Hm exp(j(wt + phiH)) ]
%where: the complex permeability = { mu_r mu_0 exp(j phiBH) }
%from experimental estimates of H(t), B(t), and t.
%MUST run BHconversion BEFORE this routine.

%Below was included during initial verification of program operation.
%dt=7.8125e-3:,TimeV=0:dt:128*dt:,w=2*pi*3:,wt=w*TimeV:
%phase=pi*.3:,magH=1478.3:,Hoft=magH*sin(wt+phase);
%mur=1.01174:,muo=4*pi*1e-7:,magB=mur*muo*magH;
%phasemu=-0.4:,Boft=magB*sin(wt+phase+phasemu);

i = sqrt(-1):
plot(TimeV,Hoft,title('H vs t'),pause
plot(Hoft,Boft,title('B vs H'),pause
%
%find Hoft parameters for Hoft=Hm sin(w*t + phiH) model:
if Hoft(1) ~= 0,
%///////////////////////////////////////////////////////
%find tcross from the FIRST zero crossing of H(t)
temp1 = zc(Hoft);,aa = temp1(1);,clear temp1
tcross = -Hoft(aa)*dt/(Hoft(aa+1)-Hoft(aa)) + TimeV(aa);%from a linear interpolation
between the point before and the point after the sign change
if Hoft(aa) > 0, phiH = pi-w*tcross;
else, phiH = 2*pi-w*tcross;
end
%///////////////////////////////////////////////////////
end %end of if Hoft(1) ~= 0
if Hoft(1) == 0,
phiH = 0;
if Hoft(2) < 0, phiH = pi; end
end

% at this point phiH, 0≤phiH≤2π, is known
% phiH
% errorphiH=(phase-phiH)/phase*100
%
% now estimate Hm
[H1,aaa]=max(abs(Hoft));
Hm=abs(H1/sin(w*TimeV(aaa)+phiH))
% errorHm=(magH-Hm)/magH*100
%
% now estimate B parameters for Boft = Bm sin(w*t + phiH + phiBH) model
Bm=mu*muo*Hm
temp1=zc(Boft); bb=temp1(1);
if bb < aa, aa=temp1(2); else aa=bb; end, clear temp1
tcrossb=Boft(aa)*dt/(Boft(aa+1)-Boft(aa))+TimeV(aa);
phiBH=w*(tcross-tcrossb)
Bmodel=Bm*sin(w*TimeV+phiH+phiBH);
plot(TimeV,Boft,TimeV,Bmodel),title('B, solid, & Bmodel, dashed, vs t'),pause
errormodel=(Bmodel-Boft);
maxerrormodel=max(abs(errormodel)/max(Boft)*100)
aveerrormodel=mean(errormodel)
sderrormodel=std(errormodel)
plot(TimeV,errormodel/max(Boft)*100),title('%error as (Bmodel-
Boft)/max(Boft)*100'),pause
plot(Hoft,Boft,Hoft,Bmodel,'+'),title('B(t), line, and Bmodel, +, vs H(t)'),grid,pause
%
% Adjust permeability to that of the ROD. This is necessary since the p/u coil is NOT wound
on the rod, but is at Rpui. The balance of the coil's volume is full of air-assumed to be at
muo.
disp('This is only an APPROXIMATION. It ASSUMES the absence of eddy currents.\')
murrod=Rpui*Rpui/(Rto*Rto)*(mur*exp(i*phiBH)-1)+1;
magmurrod=abs(murrod)
phmurrod=angle(murrod)

Function ZC

function [vector] = zc(x)
%[vector] = zc(X), or Zero Crossings, returns a vector of index numbers indicating the
indices BEFORE a sign change within the vector X.
%Here are some examples:

<table>
<thead>
<tr>
<th>GIVEN</th>
<th>RETURNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1 2 -3 -4 5 6 7 -8 -9 10]</td>
<td>[2 4 7 9]</td>
</tr>
<tr>
<td>([0 0 0 1 -1 0])</td>
<td>[3 4]</td>
</tr>
<tr>
<td>([0 0 0 1 -1 0 -1])</td>
<td>[3 4]</td>
</tr>
<tr>
<td>([0 0 0 1 -1 0 1])</td>
<td>[3 4 6]</td>
</tr>
<tr>
<td>([1 0 0 1 -1 0 1])</td>
<td>[4 6]</td>
</tr>
</tbody>
</table>
%It ignores zeros between elements, but if the vector begins with zero it counts the first
element that is not zero as a sign change. D. Hall 4/23/93.

n=length(x);
index=1;
old=sign(x(1));
for i=2:n,
    new=sign(x(i));
    if new ~= 0,
        if new ~= old,vector(index)=i-1;,index=index+1;,old=new;,end
    end
end
Function SIMPSONS

function [a] = simpsons(x,dt,sw)

%[a]=simpsons(x,dt,sw) performs a simple numerical integration of x wrt t using Simpson's Rule for all points but the first two. THIS ROUTINE ASSUMES AN EVEN NUMBER OF POINTS IN [x]. sw = 1 invokes a periodicity assumption to improve the est. of a(1), otherwise, a(1)=0 is returned. a(2) is estimated via a 4th order polyfit, integrated analytically, and evaluated at the end points, dt and zero. [x] = the function to be integrated, dt = the "time" step, and [a] is the "area" under x as a function of t.

l=length(x);
if rem(l,2) > 0,error('SIMPSONS wants an even number of points.'),end
if l < 4,error('SIMPSONS needs at least six points in x.'),end
a(1)=0;

%a(2) is estimated from a 4th order polynomial fit to the first 5 points of x.
p=polyfit([0 1 2 3 4]*dt,[x(1) x(2) x(3) x(4) x(5)],4);
a(2)=(((p(1)/5*dt+p(2)/4)*dt+p(3)/3)*dt+p(4)/2)*dt+p(5))*dt;
a(3)=dt*(x(1)+4*x(2)+x(3))/3;
for k=4:2:l-2
    a(k)=dt*(x(k-2)+4*x(k-1)+x(k))/3+a(k-2);
    a(k+1)=dt*(x(k-1)+4*x(k)+x(k+1))/3+a(k-1);
end
a(l)=dt*(x(l-2)+4*x(l-1)+x(l))/3+a(l-2);
%here is the periodicity for sw == 1:
if sw == 1,a(1)=dt*(x(l-1)+4*x(l)+x(1))/3+a(l-1);,end
FORTRAN Program: ITSOLENOID.F

C itsolenoid.f program. D. Hall, Fall '92.
C This program iterates to find the position of a chosen layer and segment of coil winding
C which results in the least axial deviation of B(z).
INTEGER K,N,I,J,NN,COUNT,LAYER,SEG
PARAMETER (NN=31)
LOGICAL MODIFY, FILEIT,OK,AGAIN,CHANGE
CHARACTER*20 FILE1
REAL RIN,T,L,ROUT,MU,MUO,R(NN),IO,COS30,X2,X1,RTEMP1
REAL RTEMP2,DELTAX,X(NN),B(NN,NN),BNET(NN),BAVE,BSD
REAL XLAY(NN,3,2),BMAX,NEFF,FROM,TO,BY,BESTX,BESTSD
1 COS30=SQRT(3.)/2.
AGAIN=.TRUE.
C RIN = rod dia. in mm
RIN=13.08
C T = wire dia. in mm
T=0.6985
C L = solenoid length in mm
L=45.72
C ROUT = maximum outside radius of solenoid
ROUT=16.89
C K = integer number of layers as dictated above
K=INT((((ROUT-RIN)/T-1.)/COS30)+1
PRINT *,The geometry implies that there is a max.'
PRINT *,of ',K,' layers possible. Would you like'
PRINT *,to decrease that number? T/F'
READ *,OK
IF(OK)THEN
5 PRINT *,Input the number of layers desired.'
READ *,K
END IF
IF(K.LT.1)GOTO 5
IF(K.GT.NN)THEN
PRINT *, 'Increase B matrix dimension to at least', K
PAUSE
STOP
ENDIF
MU=1.
MUO=4.*ACOS(-1.)*1E-7
IO=1.
SCALE=MU*MUO*IO/(2.*T/1000.)
C The 1000 above is due to T being in mm's
C Calculate all of the R's AND square them
R(1)=RIN+T/2.
RTEMP1=R(1)
R(1)=R(1)*R(1)
DO 10 I=2,K
   R(I)=RTEMP1+T*REAL(I-1)*COS30
   R(I)=R(I)*R(I)
10 CONTINUE
X2=L/2.
X1=-X2
C Calculate B(x) for full coil
DELTAX=0.5*L/REAL(NN-1)
C Yields NN-1 values per half cylinder plus center value where x = 0
DO 20 J = 1,NN
   X(J)=REAL(J-1)*DELTAX
20 CONTINUE
DO 30 J=1,NN
   DO 40 I=1,K
      B(J,I)=RTEMP2/SQRT(RTEMP2*RTEMP2+R(I))-
            RTEMP1/SQRT(RTEMP1*RTEMP1+R(I))
   IF(J.EQ.1)THEN
XLAY(I,1,2)=L/2.
XLAY(I,2,2)=0.
XLAY(I,3,2)=0.
XLAY(I,1,1)=0.
XLAY(I,2,1)=0.
XLAY(I,3,1)=0.

ENDIF

40 CONTINUE

30 CONTINUE

CALL SUM(NN,B,BMAX,BNET,K,BAVE,BSD,NEFF,T)

PRINT *,'Here is the normalized field vs x for a stock + solenoid.'
PRINT *,'Wire dia. =','T,' solenoid length =','L,' + No. of layers =','K
PRINT *,' X B/Bmax Bmax =','BMAX
PRINT *,__(mm)__________ Bave/Bmax =','BAVE
DO 60 J =1,NN
   PRINT *,X(J),' •,BNET(J)
60 CONTINUE

PRINT *,_________ Bsd/Bmax =','BSD
PRINT *,Scale by ','SCALE,' for B''s in Webers/m**2 (mu =',' + MU',' lo =','I0,' amp).'
PRINT *,''n_eff =','NEFF,' (turns/meter), H/Amp =',' + NEFF*0.004*ACOS(-1.)' (Oe/Amp), No. of layers =','K
PRINT *,Approx. elec. inductance (mH)=',
   + MU*MU0*NEFF*NEFF*ACOS(-1.)*RIN*RIN*L*1E-6
PRINT *,'Max. outer rad. =','ROUT,' actual =','SQRT(R(K))+T/2.
PRINT *,'Bave =','BMAX*BAVE

IF(AGAIN)THEN
   PRINT *,'Wanna store anything? T/F','CHAR(7)
   READ *,FILEIT
   IF(FILEIT)THEN
      PRINT *,'Input a file name.','CHAR(7)
      READ *,FILE1
   ENDIF
OPEN(UNIT=20,FILE=FILE1)
WRITE(20,"*" 'Stock solenoid file=',FILE1
CALL FILER(NN,T,L,RIN,ROUTE,SQRT(R(K)) + T/2.,K,X,BNET,
+ SCALE,BMAX,BAVE,BSD,MU,MUO,JO,XLAY,NEFF)
WRITE(20,"*" 'End stock solenoid.'
ENDIF
ENDIF

C The stock solenoid values have been calculated by layer, now one is
C offered the opportunity to modify any or all of the layers.
COUNT=0
PRINT *,'Wanna modify windings? T/F',CHAR(7)
80 READ *,MODIFY
IF(MODIFY)THEN
PRINT *,'MANUAL ADJUSTMENT?'
READ *,OK
IF(OK)THEN
CALL RESTRT(NN,XLAY,K,UYER,SEG)
GOTO 100
ENDIF
BESTSD=BSD
82 PRINT *,'Input layer number you wish to modify.'
PRINT *,'1=inside, 2=next, ...K=outside.'
READ *,LAYER
IF((LAYER.LT.1).OR.(LAYER.GT.K))GOTO 82
84 PRINT *,'Input segment number you wish to modify. .LE. 2'
READ *,SEG
IF((SEG.LT.1).OR.(SEG.GT.2))GOTO 84
PRINT *,'Working.'
IF(SEG.EQ.1)THEN
FROM=0.
TO=X(NN)-T
BY=T
ELSE
FROM=XLAY(LAYER,1,1)

TO=0.
BY=-T
ENDIF
CHANGE=.FALSE.
IF(SEG.EQ.1)THEN
BESTX=XLAY(LAYER,SEG,1)
ELSE
BESTX=XLAY(LAYER,SEG,2)
ENDIF
DO 90 RTEMP1=FROM,TO,BY
IF(SEG.EQ.1)THEN
XLAY(LAYER,SEG,1)=RTEMP1
ELSE
XLAY(LAYER,SEG,2)=RTEMP1
ENDIF
CALL MODIF(NN,B,K,R,XUY,L,X,LAYER)
CALL SUM(NN,B,BMAX,BNET,K,BAVE,BSD,NEFF,T)
IF(BSD.LT.BESTSD)THEN
BESTSD=BSD
BESTX=RTEMP1
CHANGE=.TRUE.
ENDIF
90 CONTINUE
IF(.NOT.CHANGE)PRINT *,"NO CHANGE",CHAR(7),CHAR(7)
IF(SEG.EQ.1)THEN
XLAY(LAYER,SEG,1)=BESTX
ELSE
XLAY(LAYER,SEG,2)=BESTX
ENDIF
100 CALL MODIF(NN,B,K,R,XLAY,L,X,LAYER)
CALL SUM(NN,B,BMAX,BNET,K,BAVE,BSD,NEFF,T)
COUNT=COUNT+1
PRINT *,"For modification number ",COUNT
CALL DISPLA(NN,BNET,BMAX,K,BAVE,BSD,SCALE,XLAY,
END
STOP
PRINT *",LATTER
IF(AAGAIN)GOTO 1
READ * AGAIN
+ go it again.
PRINT ".Want to change the total number of layers and
IF(FLTR)CLOSE(UNIT=20)
ENDIF
GOTO 80
PRINT ".Modify windings (only) again? T/F:CHAR(?)
ENDIF
ENDIF
WRITE(20,*) End of modification, COUNT
SCALE'BMAX,BAVE,BSD'MNU,IO,XLAY,NEFF)
+ CALL FILER(NN,T,L,RIN,ROUTE,SORT(R(K))+T/2,MNU,IO,XLAY,NEFF)
IF(OK)THEN
READ * OK
PRINT *.Store that one? T/F
IF(FLTR)THEN
+ L,RIN,ROUTE,SORT(R(K))+T/2,MNU,IO,XLAY,NEFF)
DO 20 I=1,K
    BNET(J)=BNET(J)+B(J,I)
20    CONTINUE
    IF(BNET(J).GT.BMAX)BMAX=BNET(J)
10    CONTINUE
DO 30 J=1,NN
    BNET(J)=BNET(J)/BMAX
30    CONTINUE
CALL STATS(NN,BNET,BMAX,BAVE,BSD)
NEFF=BAVE*BMAX*500./T
RETURN
END

C
C Subroutine STATS
C This subroutine calculates the statistics of the net B field wrt
C axial location.
SUBROUTINE STATS(NN,BNET,BMAX,BAVE,BSD)
INTEGER NN,J,N
C PARAMETER (NN=21)
REAL BNET(NN),BMAX,BAVE,BSD,SUM2
N = 2*(NN-1) + 1
BAVE=BNET(1)
DO 10 J=2,NN
    BAVE=BAVE+2*BNET(J)
10    CONTINUE
    BAVE=BAVE/REAL(N)
    SUM2=(BNET(1)-BAVE)**2
    DO 20 J=2,NN
        SUM2=(SUM2+(BNET(J)-BAVE)**2)
20    CONTINUE
RETURN
END
WRITE(20,100)'______________________________'
DO 70 J=1,NN
   WRITE(20,130) X(J),CHAR(9),BNET(J)
130   FORMAT(F10.7,A,F10.7)
70   CONTINUE
WRITE(20,100)'______________________________'
WRITE(20,111)'For B in Webers/m**2 scale by',
   + SCALE/1000.' x 10**-3'
WRITE(20,110)'Also use Bmax =',BMAX
WRITE(20,110)' Bave/Bmax =',BAVE
WRITE(20,110)' Bsd/Bmax =',BSD
WRITE(20,110)' mu (numeric) =',MU
WRITE(20,111)'mu_o (Wb/A m) =',MUO/1E-7,' x 10**-7'
111   FORMAT(1X,A,F10.5,A)
WRITE(20,110)' io (Amps) =',IO
WRITE(20,110)' Bave =',BMAX*BAVE
WRITE(20,110)'Bsd (std.dev) =',BSD*BMAX
WRITE(20,100)'Layer Segment X2 X1'
WRITE(20,100)'______________________________'
DO 20 I=1,K
   DO 30 J=1,3
      IF(XLAY(I,J,2).GT.0.)THEN
         WRITE(20,140)I,' •,J,' •,XLAY(I,J,2),' ',
         + XLAY(I,J,1)
      140   FORMAT(I5,A,I5,A,F8.3,A,F8.3)
      ENDIF
30   CONTINUE
20   CONTINUE
WRITE(20,100)'______________________________'
WRITE(20,100)'x = 0 is the center of the solenoid.'
WRITE(20,100)' '
RETURN
END
C Subroutine MODIF
  SUBROUTINE MODIF(NN,B,K,R,XLAY,L,X,LAYER)
  INTEGER I,J,NN,K,LAYER,SEG,ITEMP
  C PARAMETER(NN=21)
  REAL B(NN,NN),R(NN),XLAY(NN,3,2),L,X2,X1,X(NN)
  REAL X1S1,X1S2,X1S3,X2S1,X2S2,X2S3,RTEMP2,RTEMP1
  LOGICAL OK
  C Determine the number of active segments in the given layer.
  1 5  ITEMP=0
  DO 20 J=1,3
    IF(XLAY(LAYER,J,2).GT.0.)ITEMP=ITEMP+1
  2 0  CONTINUE
  C ITEMP now equals the number of active segments.
  DO 50 I=1,ITEMP
  C Pass through for each active segment and sum the contributions at x.
  DO 60 J=1,NN
    RTEMP2=XLAY(LAYER,I,2)-X(J)
    RTEMP1=XLAY(LAYER,I,1)-X(J)
    IF(I.EQ.1)THEN
      C Replaces previous contents of B(J,LAYER) the first time thru.
      B(J,LAYER)=RTEMP2/SQRT(RTEMP2*RTEMP2+R(LAYER))-
      + RTEMP1/SQRT(RTEMP1*RTEMP1+R(LAYER))
    ELSE
      C Adds to previous contents of B(J,LAYER) for second or third segment's
      C contribution.
      B(J,LAYER)=B(J,LAYER)+
      + RTEMP2/SQRT(RTEMP2*RTEMP2+R(LAYER))-
      + RTEMP1/SQRT(RTEMP1*RTEMP1+R(LAYER))
    ENDIF
  C Catch the other half of the solenoid also.
RTEMP2 = -XLAY(LAYER,L,1) - X(J)
RTEMP1 = -XLAY(LAYER,L,2) - X(J)

C Yes, these should be this way, i.e., x2 rhs = -x1 lhs, and
C x1 rhs = -x2 lhs
B(J,LAYER) = B(J,LAYER) +
  + RTEMP2/SQRT(RTEMP2*RTEMP2 + R(LAYER)) -
  + RTEMP1/SQRT(RTEMP1*RTEMP1 + R(LAYER))

60 CONTINUE
50 CONTINUE

C The matrix B is now modified for the segment changes on this LAYER
RETURN
END

C Subroutine Displa
SUBROUTINE DISPLA(NN,BNET,BMAX,K,BAVE,BSD,SCALE,XLAY,
  + L,RIN,ROUT,T,AROUT,MU,MUO,IO,X,NEFF)
INTEGER NN,K,I,J
C PARAMETER(NN=21)
REAL BNET(NN),BMAX,BAVE,BSD,SCALE,XLAY(NN,3,2),L,RIN,ROUT
REAL T,AROUT,MU,MUO,IO,X(NN),NEFF
LOGICAL OK
C PRINT *
PRINT *,'here are the results.',CHAR(7)
PRINT *, X/Bmax'
PRINT *,'(mm)______________'
DO 10 J =1,NN
  PRINT *,X(J),' ,BNET(J)
10 CONTINUE
PRINT *,'___________________'
PRINT *,'Scale by ',SCALE,' for B\''s in Webers/m\''2 (mu =',
  + MU,' lo =',IO,' amp).'
PRINT *, 'n_eff = ', NEFF, ' turns/meter.',
+  ' Field = ', NEFF * 0.004 * ACOS(-1.) * ' Oe/Amp.
PRINT *, ' Approx. elec. inductance (mH) = ',
+  MU * MU0 * NEFF * NEFF * AC0S(-1.) * RIN * RIN * L * 1E-6
PRINT *, ' Bave = ', BMAX * BAVE, ' Bave/Bmax = ', BAVE,
+  ' Bsd/Bmax = ', BSD, ' Bmax = ', BMAX
PRINT *, ' Display coil dimensions? T/F'
READ *, OK
IF(OK) THEN
  PRINT *, ' Length = ', L, ', inner rad. = ', RIN, ', maximum
  allowed outer rad. = ', ROUT, ',
  PRINT *, ' actual outer rad. for ', K, ' layers = ', AROUT, ',
  PRINT *, ' Wire dia. = ', T, '. All dimensions in millimeters.'
PRINT *
PRINT *, ' Lay. Seg. x2 x1'
  PRINT *, ' ____________________________'
DO 20  I = 1, K
  DO 30  J = 1, 3
    IF(XLAY(I,J,2).GT.0.)
      +      PRINT *, 'I, ', J, ' ', XLAY(I,J,2), ' ', XLAY(I,J,1)
  30 CONTINUE
20 CONTINUE
  PRINT *, ' ____________________________'
  PRINT *, ' x = 0 is the center of the solenoid.'
ENDIF
RETURN
END

C
C Subroutine RESTRT
SUBROUTINE RESTRT(NN,XLAY,K,LAYER,SEG)
INTEGER NN,K,LAYER,SEG
REAL XLAY(NN,3),X2S1,X2S2,X2S3,X1S1,X1S2,X1S3
LOGICAL OK
10 PRINT *, 'Input layer number you wish to modify. 1=inside,
+ 2=next, ..., K=outside.'
READ *, LAYER
IF((LAYER.GT.K).OR.(LAYER.LT.1))GOTO 10
C Assign present segment end locations to interim variables
   X2S1=XLAY(LAYER,1,2)
   X2S2=XLAY(LAYER,2,2)
   X2S3=XLAY(LAYER,3,2)
   X1S1=XLAY(LAYER,1,1)
   X1S2=XLAY(LAYER,2,1)
   X1S3=XLAY(LAYER,3,1)
100 PRINT *, 'Layer Segment X2 X1'
PRINT *, 'LAYER,' 1 ', X2S1,' ',X1S1
PRINT *, 'LAYER,' 2 ', X2S2,' ',X1S2
PRINT *, 'LAYER,' 3 ', X2S3,' ',X1S3
PRINT *, 'Are these okay? T/F', CHAR(7)
READ *, OK
IF(OK)GOTO 110
102 PRINT *, 'Input segment number.'
READ *, SEG
IF(SEG.EQ.1)THEN
   PRINT *, 'Input new X1 for segment 1.'
   READ *, X1S1
   GOTO 100
ENDIF
IF(SEG.EQ.2)THEN
   PRINT *, 'Input X2 and X1 for segment 2.'
   READ *, X2S2,X1S2
   GOTO 100
ENDIF
C IF(SEG.EQ.3)THEN
C PRINT *, 'Input X2 and X1 for segment 3.'
C READ *, X2S3, X1S3
C GOTO 100
C ENDIF
IF((SEG.GT.2).OR.(SEG.LT.1)) GOTO 102
C This is an attempt to verify that the segments are geometrically possible
110 IF((X2S1.GT.L/2.).OR.(X2S1.LT.0.).OR.(X2S1.LT.X1S1).OR.
+ (X1S1.LT.0.).OR.(X1S1.LT.X2S2).OR.
+ (X2S2.GT.L/2.).OR.(X2S2.LT.0.).OR.(X2S2.LT.X1S2).OR.
+ (X1S2.LT.0.).OR.(X1S2.LT.X2S3).OR.
+ (X2S3.GT.L/2.).OR.(X2S3.LT.0.).OR.(X2S3.LT.X1S3).OR.
+ (X1S3.LT.0.)) THEN
    PRINT *, 'Something is amiss!', CHAR(7), CHAR(7), CHAR(7)
    GOTO 100
ENDIF
XI_AY(LAYER, 1, 2) = X2S1
XLAY(LAYER, 2, 2) = X2S2
XLAY(LAYER, 3, 2) = X2S3
XLAY(LAYER, 1, 1) = X1S1
XLAY(LAYER, 2, 1) = X1S2
XLAY(LAYER, 3, 1) = X1S3
C The windings are now modified for the given layer.
C
PRINT *, 'Another SEGMENT change? T/F'
READ *, OK
IF(OK) GOTO 100
PRINT *, 'Display winding dimensions? T/F'
READ *, OK
IF(OK) THEN
    PRINT *, 'Layer Seg. x2 x1'
    PRINT *, '----------------------------------------'
    DO 40 I = 1, K
        DO 30 J = 1, 3
            IF(XLAY(I, J, 2).GT.0.)
30 CONTINUE
40 CONTINUE
   PRINT *,' ',XLAY(I,J,2),',XLAY(I,J,1)
PRINT *,',--------------------------------------'
PRINT *,'More changes? T/F'
READ *,OK
IF(OK)GOTO 102
ENDIF
RETURN
END
APPENDIX D: USER'S GUIDE TO TRANSDUCER MODELS

Pointers will be offered to those who want to apply the modelling procedures developed in the body of this dissertation. This Appendix will begin with an improved method for estimating displacement from current. Then comparisons will be offered assuming eddy current losses were based on the blocked permeability of Terfenol-D, and assuming the losses were based on the dynamic permeability given by Eqn. (3.3.13).

Improved u/l Estimates

In the body of the dissertation (especially Sections 3.5 and 5.6) it was argued that modelling $Z_{ee}$, $Z_{e}$, $z_m$, and $z_L$ allowed calculation of the transduction coefficient, $T_{em}$ via Eqn. (3.5.1). Then that coefficient could be used to estimate $u/l$ from Eqn. (5.6.1). The improved $u/l$ estimate stems from recognizing that $u$ is a consequence of $H$, not $I$. Strain, or equivalently, displacement of Terfenol-D is given by Eqn. (3.2.1a), where it is seen to be a function of the applied stress and applied magnetic field strength, $H$. For these transducers, the applied stress was related to the applied force, which was related to the velocity. In the presence of eddy currents from the external cylindrical housing, the applied field at the outside of the Terfenol-D rod is not simply $H(R_{TO}) = nI$. That was the point behind half of the antics of Chapter 4. Including the effects of the magnitude and phase existing between the applied field strength, at the outside of the magnetostrictive rod, and the electric current in the solenoid, will usually result in improved estimates of displacement from electric current. It is not strictly correct, it is simply an easy method for obtaining better approximations.

For this procedure, one calculates $T_{em}$ as before, since when one does this they are solving the pair of simultaneous equations which describe the linear transduction of the transducer. (One must always consider BOTH equations.) Only this time around, use the damping estimate from the admittance loop, $\zeta_Y$, which was found to be always less than that obtained from the impedance loop. This is empirical. In the previous technique, the average damping was used - that, too, was empirical. $u/l$ can now be estimated as:

$$\frac{u}{l} = \frac{T_{em}}{j\omega(z_m + z_L)} \left\{ \frac{H(R_{TO})}{nI} \right\}$$  \hspace{1cm} (D.1)
where: \( u \) is displacement, 
\( I \) is drive current, 
\( T \) is the transducer's mechanical impedance of the transducer and load, respectively, based on velocity, and 
\( \{H(R_{To}) / nl\} \) is the applied field at the outer radius of the Terfenol-D rod, normalized by the applied field strength of the coil, \( nl \), as given by Eqn. (4.4.20). This is a dimensionless function which varies in magnitude and phase with excitation frequency.

An example simulation will now be detailed. Shown below is the MATLAB screen which resulted from the simulation of \( Z_{ee} \). Explanations have been added in brackets, [ comment ]. The experimental measurement was a swept-sine test at 100 mA drive current amplitude with, \( m_1 = 2.19 \text{ kg} \) (called "basemass" below), and \( m_2 = 0.109 \text{ kg} \) (called "endmass" below). The input parameters for the simulation are shown below, as are the corrected values calculated for the material parameters.

\[
\text{resswitch1} = 1 \quad [1 \Rightarrow Zee, 0 \Rightarrow Ze simulation]
\]
\[
fr = 2780 \quad [\text{input from impedance analysis} = f_0z]
\]
\[
qexp = 2.9648e-09 - 3.8761e-10i \quad [= \text{input} = (u/l) \text{ from experiment} / (n \text{ rodlength})]
\]
\[
twozeta = 6.7120e-02 \quad [= 2 \zeta_Y \text{ input from admittance analysis}]
\]
\[
keff2 = 0.08421 \quad [= \text{input} = 1 - (f_0z/f_0y)^2]
\]
\[
kMH = 2.9903e+07 \quad [\text{calculated}]
\]
\[
EyH = 4.7966e+10 \quad [\text{calculated}]
\]
[the balance are also calculated as explained in the body of the dissertation]
\[
\text{FYI: } q = 3.29484e-09 \text{ m/A at 0 rads., twozeta = 0.06712}
\]
\[
\text{calculating } \muH \quad \muH = 4.42857, \text{ at 0 radians. } k'^2 = 0.0935689
\]
\[
\text{FYI: } mH = 4.01419, \text{ at 0 radians.}
\]

Figure D.1 shows the dimensionless parameter \( \{H(R_{To})/nl\} \) versus frequency, as calculated assuming that the permeability of the magnetostrictive rod was given by Eqn. (3.3.13a) (or (3.3.13b) since \( q, EyH, \) and \( k^2 \) were all assumed to be real valued constants). This is the applied field strength from drive current as would be occurring at the outer radius of the Terfenol-D rod. Note that it is not simply equal to one, i.e., the eddy currents in the external cylindrical permanent
magnet require that it cannot be one. If one were to remove the cylindrical housing, or slit it lengthwise to break-up the path of the large circulating eddy currents, this function would be very nearly one, both in theory and in practice. As mentioned in Chapter 5, eliminating eddy currents in the external housing is a good thing to do. With their elimination, one should anticipate substantial improvement in transducer performance. As shown in Figure D.1, there is a significant amount of magnetic shielding occurring within the transducer, especially near mechanical resonance (2800 Hz). This shielding results in reduced transducer performance.

The large variations in magnitude and phase displayed in Figure D.1 at frequencies near mechanical resonance are due to the displacement

Figure D.1. Magnitude and phase of \{H(R_{TO})/nI\}, the normalized magnetic field strength at the outer radius of the Terfenol-D rod, versus frequency of excitation
enhanced variations in Terfenol-D magnetic permeability, from dynamic effects, and the resulting increase, followed by a decrease in induced eddy currents. (Recall Figure 5.16, the plot of $\mu_T$ vs. frequency.) An increase in the rod's permeability means it carries more flux, thus the permanent magnet experiences more flux linkage, which increases the induced voltage in the magnet, which increases the induced currents and their influence on $H(r)$. A plot of field strength versus radial position, at an excitation frequency of 2800 Hz, is shown in Figure D.2.

Figure D.2 shows the magnitude and phase of the normalized magnetic field strength as a function of a normalized position, $r/R_{To}$, at 2800 Hz. With regard to abscissa values, the center of the Terfenol-D rod is at zero. The outside of the rod is at one. From 1.0 to about 1.2 is the air gap and pick-up coil (recall Figure 4.3 in Section

![Figure D.2. Magnitude and phase of H/|nI versus position, r/R_{To}, at 2800 Hz. R_{To} = 3.175 mm](image)

- $\| H(r/R_{To})/nI \|
- \text{Phase of } H(r/R_{To})/nI, \text{deg.}$

$r/R_{To}, \text{Position}$

$r/R_{To}, \text{Position}$
From 1.2 to 2.4 is the drive coil. From 2.4 to 3 is the air gap around the outside of the drive coil. The cylindrical permanent magnet occupies the values from 3 to 4.

Recall the specifics for the second transducer model. Note that H as a function of r is continuous, that its derivatives are not, that H is constant where the current density is zero (the air gaps), that H is linear through the drive coil, that H goes to zero inside the windings of the drive coil due to destructive interference from the field due to the eddy currents in the magnet, and that H goes to zero at the outside of the permanent magnet. Note also that the normalized field strength at the outside of the rod is not one; it is the 0.72 at nearly zero phase as was displayed at 2800 Hz in Figure D.1. This reduced magnitude is thought to be applicable to the calculation of displacement.

Consider the zone $0 < \frac{r}{R_{T0}} < 1$, the Terfenol-D rod. Note that the field towards the center of the rod is reduced in magnitude, and shifted in phase when compared to the value at the outside. This is the magnetic shielding phenomenon eluded to previously. Eddy currents inside the rod are reducing the "penetration" of the field.

A subtle effect of this magnetic shielding can be envisioned when one considers that the material strains in response to the field strength. Since the values of the field are reduced towards the center of the rod, the magnetostrictive material in that region "wants" to strain less than that towards the outside of the rod. This variation of strain with radial position must, therefore, result in shear stresses within the rod. The inner portions of the rod are acting to reduce the overall strain of the rod because they are excited less, and tend to hold-back the outer portions. Yet another reason to try to eliminate/reduce eddy currents within the magnetostrictive rod.

An even more subtle effect of magnetic shielding exists. Note that the field within the rod possess both a magnitude and a phase. As the rod progresses through a cycle of oscillation, there comes a point in each cycle in time where the outside portions of the rod are "shorter" than the inner portions. Considering the fact that the rod's end is generally butted up against the flat surface of the motion output component of the transducer (e.g. component (a) in Figure 1.5), this means that at different times in each cycle, different portions of the rod are defining the output motion. The result is another deviation from linearity. Even if all portions of the rod produced perfect sinusoidal motion (which they don't), the output motion would be contaminated with harmonic frequencies because the "point of contact"
is assuming different magnitudes and phases at different times in each cycle of rod motion. This was just for fun. This effect is likely a second order effect, completely dwarfed by the huge harmonics produced by other nonlinearities in the magnetostrictive material.

On with the simulation. Figure D.3 shows the simulation and experimental measurement of $Z_{ee}$. As shown in the figure, the simulation was fairly good. As a matter of fact, the percentage of error for each datum is plotted in Figure D.4. The simulation of $Z_{ee}$ was then saved for later use in the calculation of $T = T_{em}$.

Figure D.3. Magnitude and phase of $Z_{ee}$. Solid line is the simulation, dash-dot line is the experimental measurement. See text.
Figure D.4. Percent error versus frequency, where \( \% \text{error} \) is defined as:
\[
\% \text{error} = 100 \frac{\text{simulation-model}}{\text{model at each frequency}}.
\]
Solid line = magnitude errors, dashed = phase errors.
Next, a blocked simulation of the transducer was performed. In this case, the permeability of the magnetostrictive rod is assigned the value of the calculated blocked permeability. Figure D.5 shows the resulting normalized magnetic field at the outside of the Terfenol-D rod, versus frequency. Compare these "smooth" plots with those calculated earlier (Figure D.1). Removing the variations in Terfenol-D permeability eliminated the displacement related eddy current effects.

Figure D.5. Normalized magnetic field versus frequency assuming the magnetic permeability of the Terfenol-D rod was the blocked value, $\mu^\varepsilon = 4.01 \mu_0$. 
The blocked electrical impedance, $Z_e$, was plotted as the solid line in Figure D.6. The dash-dot line is the experimental measurement of $Z_{ee}$ again. Note how the blocked values approach, but theoretically never reach, the measured impedance at frequencies "far removed" from the mechanical resonance. This simulation of the blocked impedance includes the effects of eddy currents occurring in both the rod and the housing. It does not include the increased eddy current effects resulting from the displacement enhanced magnetic permeability. $Z_e$ was saved for the up-coming calculation of $T = T_{em}$.

![Graph](image)

Figure D.6. Magnitude and phase of the simulated blocked impedance, $Z_e$ (solid lines), and the experimental measurement of $Z_{ee}$ (dash-dot lines)
Equation (3.5.1) was used to calculate $T_{em}$. The results of the calculation are displayed in Figure D.7. This function was then used to estimate displacement from current via two different procedures. The first method will be that described in the body of the dissertation, specifically, calculating $u/l$ via Eqn. (5.6.1). The second will employ Eqn. (D1), which includes the effects of the magnitude and phase which exists between the magnetic field at the outer surface of the magnetostrictive rod, and the current in the drive coil.

The model simulation of $u/l$ ignoring the magnitude and phase between $H$ and $I$ is shown in Figure D.8, along with the corresponding experimental measurement (dash-dot lines). The peak value measured experimentally was $30.6 \times 10^{-6}$ m/A at 2800 Hz. The simulation produced a peak value of $37.8 \times 10^{-6}$ m/A at 2780 Hz. Thus, the

![Figure D.7. Magnitude, solid line, and phase, dashed line, of the calculated transduction coefficient, electrical due to mechanical, $T = T_{em}$ from Eqn. (3.5.1)](image-url)
simulation over predicted the peak value by 23%, and missed the frequency by one frequency increment.

This type of simulation has two major problems. First, like most literature concerning Terfenol-D, it is overly optimistic; specifically, the simulated displacements generally exceed the experimental displacements in the frequency range of primary interest, zero to resonance. Second, it is missing a phase component. It was mentioned previously that this could be "fixed" by adding in the "DC phase" of \( q \). This is not entirely satisfactory. To begin with, those who actually

Figure D.8. Magnitude and phase of transducer output displacement from drive current. Solid lines are from calculations using Eqn. (5.6.1). Dash-dot lines are experimental measurements
mention frequency independent phases, e.g., [3 and 12], offer no assurances that they are applicable at frequencies where eddy current effects might be important. It seems that most of the work done in this area is still waiting to be translated from Chinese.

Displacement from current including the effects of \( \{H(R_{T0})/nI\} \), calculated via Eqn. (D.1), is compared with the experimental measurement in Figure D.9. Using this procedure results in more conservative estimates of transducer output displacements. Instead of overestimating displacements by upwards of twenty percent, it underestimates by 2 to 10%. In addition, both the magnitude and phase traces follow the trends displayed by the experimental values better than the previous technique. The magnitude is a much better fit for

![Figure D.9. Magnitude and phase of transducer output displacement from drive current. Solid lines are from calculations using Eqn. (D.1). Dash-dot lines are experimental measurements](image)
frequencies below 3000 Hz. The peak simulation value was $28.8 \times 10^{-6}$ m/A at 2800 Hz, an under prediction of six percent, but at the right frequency. The phase is within 5 to 10% of experimental values from 500 to 6000 Hz. This was thought to be a substantial improvement over the previous technique.

An alternate technique has been advanced for improving model simulations of transducer output displacement from input current. The new technique directly incorporates the effects of the magnitude and phase existing between magnetic field strength and electric current in the drive coil. It was found that the damping estimate provided by electrical admittance analysis generally resulted in the best match between simulation and experiment.

Which Permeability to Use in Eddy Current Calculations?

Section 5.7 of the dissertation discussed the differences in the effects of eddy currents when one used different Terfenol-D magnetic permeabilities in the calculation of $k_T$, (from Eqn. (4.2.9) using Terfenol parameters). The two options were: $\mu_T$ given by Eqn. (3.3.13), or $\mu_T = \mu^e$. Figure D.10 shows yet another simulation of the 100 mA experiment of Figure 5.17, assuming $k_T$ was based on the dynamic permeability given by Eqn. (3.3.13). (The simulation in Figure 5.17 also used that assumption.)

The simulations in this section were calculated assuming $\xi = \xi_y$, and using the improved technique for displacement calculations discussed above. It was not mentioned previously, however, using the lower damping factor from admittance analysis had the tendency to improve the phase agreement at frequencies near resonance. Note how the local minimum in the simulated phase, as shown in Figure D.10, is a better match to experiment than that shown in Figure 5.17 (where the damping was a larger value, i.e., it was the average of that from impedance, and that from admittance analysis). This was a general trend in these simulations. The lower damping translated to a deeper trough in the phase trace. It is important to note, however, that if one were to now perform an old style u/l simulation (using Eqn. (5.6.1)), the lower damping would result in 20 to 50% over estimations of the peak displacement amplitudes.

The simulation in Figure D.10 was different from that in Figure 5.17 in another important way. Recall that the magnitude in the simulation in 5.17 was slightly higher than the experimental measurement, easily
Figure D.10. $Z_{ee}$ magnitude and phase using $k_T$ based on dynamic magnetic permeability of magnetostrictive rod. Solid lines are simulations, dash-dot lines are experimental measurements seen in the figure at 6000 Hz. For Figure D.10, the $q$ value was reduced by 5% to improve the magnitude simulation. Thus, the improved agreement in magnitude at the higher frequencies in Figure D.10, when compared to Figure 5.17, was not due to the reduced damping, but rather it was due to the reduced $q$ (which reduced the calculated value of $\mu_0\sigma$).

So, Figure D.10 assumed $k_T$ was based on the dynamic permeability. Figure D.11 assumes $k_T$ was based on the blocked permeability of Terfenol-D. A comparison of Figures D.10 and D.11 reveals the changes in simulated transducer performance when eddy current losses are based on the classic, constant magnetic permeability. There are at
least two changes to note. First, the simulation in Figure D.11 overestimates the magnitude of $Z_{ee}$ at frequencies around mechanical resonance. As mentioned previously, this is because using the constant permeability ignores the losses due to the displacement enhanced permeability near mechanical resonance. Including these effects resulted in a better match between simulation and experiment, as shown in Figure D.10.

The second trend to note is the under prediction of the phase near mechanical resonance in Figure D.11. This is because the reduction in eddy currents in this simulation translates to better transduction, which reduces the phase near resonance (generally, the deeper the trough, the better the transduction).

Figure D.11. $Z_{ee}$ magnitude and phase using $k_T$ based on blocked magnetic permeability of magnetostrictive rod. Solid lines are simulations, dash-dot lines are experimental measurements.
Other, more subtle trends become apparent when examining the phase predictions of the two different methods. Using the dynamic permeability resulted in better agreement with experiment, numerically, and in trends. Note that using the blocked permeability resulted in the phase being nearly constant from about 3200 to 5200 Hz. In this range, the experimental phase has a local maximum near 3200 Hz, and it is clearly reducing in magnitude from there. When the blocked permeability is used in eddy current calculations, eddy current losses are a smooth function (recall Figure D.1 vs. D.5). There is no mechanism for a local maximum in phase to occur just after resonance. Using the dynamic permeability, the mechanism exists, and the trend is reproduced.

The last trend in phase to be discussed is that displayed between 1000 Hz and mechanical resonance (approximately the local minimum). Basing eddy current losses on the dynamic permeability provides a mechanism, other than simply the increase in frequency, for generating increased losses as frequencies increase towards resonance. Larger eddy currents translate to lower flux linkages and increased ohmic losses, both of which result in reduced electrical impedance phase angles. Comparing the phases displayed in Figures D.10 and D.11 shows that use of the dynamic permeability resulted in improved agreement with experiment.

Transducer output displacement from electric current simulations were also performed. The $Z_{ee}$ simulations above were saved, and a blocked impedance simulation was performed. Transduction coefficients were calculated and u/l was estimated. Figure D.12 is the u/l simulation performed when eddy current losses were based on the dynamic permeability. Figure D.13 was produced using $k_T$ based on the constant, blocked magnetic permeability. Once again, both simulations used $\zeta_Y$ and Eqn. (D.1); the improved technique explained at the beginning of this appendix. Both simulations are more conservative than that presented in Figure 5.20, thus either one is preferred over that shown in Figure 5.20.

Basing $k_T$ on the dynamic permeability resulted in a 3% under prediction of the peak value. For $k_T$ based on $\mu_c$, the peak was over estimated by 5%. Using the dynamic permeability resulted in the magnitude of the simulation doing a slightly better job of following the experimental measurement up the front side of the resonant peak. The most obvious improvement is exhibited by the phase traces. Compare that of Figure D.12 with D.13. The blocked permeability
simulation, Figure D.13, was incapable of following the subtle experimental trends. It missed the boat between 1500 and 2500 Hz; it bulged up. It also resulted in a larger over prediction of the high frequency phase than did the dynamic permeability simulation.

Figure D.12. \( \|u/l\|, m/A \) magnitude and phase using \( k_T \) based on dynamic magnetic permeability of magnetostrictive rod. Solid lines are simulations, dash-dot lines are experimental measurements.
Figure D.13. \( u/l \) magnitude and phase using \( k_T \) based on blocked magnetic permeability of magnetostrictive rod. Solid lines are simulations, dash-dot lines are experimental measurements.
APPENDIX E: SIMULATIONS OF TRANSDUCER MODEL 1 WITH A SLIT CYLINDRICAL PERMANENT MAGNET AND SOLID OR TWO-LAMINA RODS

Transducer Model 2 included the effects of a conducting cylindrical external housing (for the transducer under study, the housing was a cast, Alnico V permanent magnet). Model 2 was investigated in the body of the dissertation. The goal of this Appendix is to investigate Transducer Model 1, the case where there is no cylindrical external conducting housing. In addition, data will be presented on the case where the Terfenol-D rod has been cut lengthwise, then glued back together. This process, called lamination, is supposed to break-up the large conducting paths within the rod resulting in lower eddy current losses.

For the simulations in this Appendix:

1) The average damping will be used. This was also done in the body of the dissertation; however, in Appendix D, the damping from admittance analysis was used.

2) The electrical inductance will be calculated based on the full length of the solenoid. Previous simulations used L based on the Terfenol-D rod length - this is the "calibration" discussed in Chapter 6 as applied to the slit housing transducer.

3) Displacement from current simulations will be adjusted to reflect experimental conditions, i.e., displacement measurements were actually measurements of $x_2$, thus the factor $1/(1+m_2/m_1)$ is required in $u/l$ calculations to compensate. This will not affect the phase of $u/l$.

4) Using the solenoid length in inductance calculations implies that the transducer has that length of active material. That was not the case. Therefore, the factor $(\text{rod length})/(\text{solenoid length})$ is required in $u/l$ calculations to compensate. This will not affect the phase of $u/l$.

Figure E.1 shows a $Z_{ee}$ simulation and the corresponding experimental measurement for the transducer with a longitudinally slit external housing. For the simulation, the electrical conductivity of the Alnico V permanent magnet was changed from $(1/47) \times 10^8$ to $(1/47) \times 10^9$, i.e., the material was modelled as if it had very high resistance to the flow of electric current. This, in essence, removes the eddy currents in the external housing. The simulated magnitude displayed significant (greater than 10%) errors for frequencies below 1000 Hz. Above 1000 Hz, the error ranged between $\pm 5\%$. Considering
the simplicity of the model, axial variations were totally ignored, this was thought to be excellent agreement. As shown in the phase plot of Figure E.1, the simulation provides an over prediction. Transducer output displacement from input current is shown in Figure E.2. (When comparing these simulations with others in this dissertation, keep conditions 3 and 4, stated above, in mind.) Again, the solid lines display the simulation and the dash-dot lines correspond to the experimental measurement. Errors in the simulated magnitude were in the range of ±20%. The peak value was in error by
Figure E.2. Magnitude and phase of transducer displacement from current. Solid lines = simulation, dash-dot lines = experiment. See text only +3%. As shown in the phase plot, the transducer is displaying about 20 to 30 degrees more phase lag than predicted by the simulation procedure.

The results in Figure E.2 suggest that something is amiss. Recall the discussion in Section 5.4 about the "tipped" phase trace. Experimental measurements with a conducting housing displayed about 240° of phase lag at 6000 Hz (see Figure 5.12). Slitting the housing reduced the phase lag to about 215°. Theory, as explained in this dissertation (especially that put forth in Appendix D), suggests that the phase lag, assuming no eddy currents in the cylindrical housing causing a phase between $H(R_{t_0})$ and $I$, should be at worst 185° (= 180° from the second order system + about 5° from the hysteretic, DC phase of $q$, in this measurement). In the simulation, the model (which
ignored the DC phase) produced a phase lag of 190°. The model comes closer to predicting the phase than does the simple explanation of theory. That is a bonus, however, some error still exists. Either there are still some eddy effects from the cylindrical housing remaining (perhaps occurring near the ends), or the neglected effects from other components in the transducer's magnetic circuit are now significant. Considering the simplicity of the modelling procedure, the simulation results shown in Figure E.2 were thought to be acceptable.

The topic of Section 5.7, and part of Appendix D, was the use of a constant valued magnetic permeability for the magnetostrictive rod in eddy current estimates. Figure E.3 is the simulation displayed in Figure E.1, except \( k_T \) of Eqn. (4.2.9) was based on the constant, blocked permeability of the Terfenol-D rod. As shown in the figure, the now familiar over prediction of impedance magnitude at frequencies around the local maximum, is again displayed. The changes between Figures E.1 and E.3 are reminiscent of those between Figures D.10 and D.11 (with eddy currents in the housing). Once again suggesting that the dynamic permeability of Eqn. (3.3.13) should be used when estimating eddy current losses.

Tests were conducted using the slit housing and a laminated Terfenol-D rod (one cut down its center, epoxied back together, ground back to round, resulting in a two-lamina rod). Figure E.4 displays the experimental measurement and the corresponding model simulation of the transducer's electrical impedance. For the simulation, the complete absence of eddy currents was assumed. To simulate this condition, the electrical conductivities of the Terfenol-D rod and the Alnico V permanent magnet were decreased from \( \sigma_T = (1/6) \times 10^7 \) and \( \sigma_{pm} = (1/47) \times 10^8 \) to \( \sigma_T = (1/6) \times 10^0 \) and \( \sigma_{pm} = (1/47) \times 10^0 \) 1/(ohm meter). The simulated electrical impedance is generally greater than the experimentally measured values (for frequencies greater than about 1000 Hz). This trend implies that the assumption of NO eddy currents was optimistic. This is also apparent in the measured phase.
Figure E.3. Magnitude and phase of $Z_{ee}$ as measured experimentally (dash-dot lines) and as simulated (solid lines) basing $k_T$, Eqn. (4.2.9), on the constant, blocked magnetic permeability of the magnetostrictive rod. See text.
Figure E.4. Magnitude and phase of $Z_{ee}$ as measured experimentally (dash-dot lines) and as simulated (solid lines). The transducer contained a two-lamina Terfenol-D rod and the external magnet was slit lengthwise. For the simulation, electrical conductivities of the rod and housing were reduced to eliminate all eddy currents.

Figure E.5 shows the magnitude and phase of the transducer output displacement per ampere, simulation and experiment. As shown in the figure, neglecting all eddy currents resulted in an overly optimistic prediction of the transducer's displacement abilities. The peak value is, however, only about 11% high. Unfortunately, estimates at other frequencies were in error by more than 20%. As to the phase plot, the maximum phase lag from experiment was $204^\circ$; from the simulation, it was $178^\circ$. As mentioned previously, something is (still) amiss.
Figure E.5. Magnitude and phase of transducer displacement from current. Solid lines = simulation, dash-dot lines = experiment. The transducer contained a two-lamina Terfenol-D rod and the external magnet was slit lengthwise. For the simulation, electrical conductivities of the rod and housing were reduced to eliminate all eddy currents.

It is unreasonable to assume that a two-lamina rod would reduce eddy currents within the rod to zero. The laminating process, at best, breaks-up the large cylindrical conducting path within the rod. The result is two, "D" shaped regions, positioned back-to-back. Even assuming that the glue joint acts as a perfect insulator (an unknown), one is still left with the possibility of significant eddy currents within the "D" shaped regions. In an effort to better simulate the
laminated rod/slitr housing actuator, different values of electrical conductivities were tried. Assuming that the laminated rod reduced the eddy currents within the rod by, say 85%, and that the slit housing reduced the effective eddy currents within the housing by, say 80%, resulted in the simulations shown in Figures E.6 and E.7. For these

![Figure E.6. Magnitude and phase of transducer electrical impedance. Solid lines = simulation, dash-dot lines = experiment. The transducer contained a two-lamina Terfenol-D rod and the external magnet was slit lengthwise. For the simulation, electrical conductivities of the rod and housing were reduced by 85 and 80%, respectively.](image-url)
simulations, $\sigma_T = 0.15(1/6) \times 10^7$ and $\sigma_{pm} = 0.20(1/47) \times 10^8$ were used for the Terfenol-D and permanent magnet electrical conductivities. These values were found empirically. However, the model's ability to simulate transducer behavior over the frequency range of interest demonstrates that it may be useful as a design tool, even when the rod geometry defies analytical solutions.

Figure E.7. Magnitude and phase of transducer displacement from current. Solid lines = simulation, dash-dot lines = experiment. The transducer contained a two-lamina Terfenol-D rod and the external magnet was slit lengthwise. For the simulation, electrical conductivities of the rod and housing were reduced by 85 and 80%, respectively.
Additional Experimental Observations

Recall that experiments and simulations were performed using both a solid ("O" shaped) and a slit ("C" shaped) permanent magnet/housing. Material parameters were measured in each case. Experiments using the same rod with the two different housings produced some unexpected, and as yet unexplained changes in the measured material parameters. Average parameters measured in the two cases were:

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SOLID</th>
<th>SLIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_y^H$, GPa</td>
<td>48</td>
<td>40</td>
</tr>
<tr>
<td>$E_y^B$, GPa</td>
<td>53</td>
<td>51</td>
</tr>
<tr>
<td>$2C_{ave}$</td>
<td>7.5</td>
<td>9.5</td>
</tr>
<tr>
<td>$q$, pm/A</td>
<td>3.2</td>
<td>4.1</td>
</tr>
<tr>
<td>$k^2$</td>
<td>0.09</td>
<td>0.21</td>
</tr>
<tr>
<td>$\mu^\sigma/\mu_0$</td>
<td>4.4</td>
<td>2.9</td>
</tr>
<tr>
<td>$\mu^e/\mu_0$</td>
<td>4.0</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The following may be considered editorial in nature.

Consider the relative permeabilities. As mentioned previously, Terfenol-D has a "low" permeability because it is transducing magnetic energy to elastic energy. It seems that slitting the external housing increased (somehow) Terfenol's ability to convert energy. Perhaps more accurately, placing a rod inside of a conducting housing (somehow) limits the rod's abilities to transduce energy. (Blocked relative permeabilities (for bare rods) of from 2 to 3 are common in the literature, whereas a blocked value of 4 is on the high side.) Note the implications of such dramatic decreases in magnetic permeabilities. Eddy current losses generated within the rod itself decrease with the lower permeabilities. Thus, slitting the housing not only reduced the eddy current effects (of particular interest, the heating due to eddy currents) from the housing, it reduced the eddy current effects generated within the rod.

Consider the linear coupling, $q$. Measurements of $q$ were performed at "low" frequencies, that is, at frequencies where dynamic and eddy current effects were thought to be small (100 to 1000 Hz). Why did placing the rod inside of a solid housing yield a 30% decrease in the linear coupling? From an energy viewpoint, $q$ is a measure of how much work a rod can do against a given force (stress). Unintentionally reducing the rod's native ability to transduce energy (placing it in a conducting housing?) would show up as a reduction in $q$. 
Consider the moduli of elasticity. The blocked values, $E^B_y$, appear to be almost constant (for Terfenol-D, these two values are as close to each other as one is liable to measure). That was considered to be a good sign. Reducing the eddy currents within the *transducer* had the effect of making the rod more compliant, i.e., less stiff. It resonated at lower frequencies. Perhaps this was due in part to the reduction in eddy current induced internal shear stresses (recall, reduced eddy currents mean lower field gradients within the rod). Consider also the energy transducing viewpoint. The two moduli were related via energy arguments. The better the transduction, the bigger the difference between the moduli. Reducing the rod's ability to transduce must result in an increase in the lower modulus (the blocked modulus should stay constant).

The next obvious question is, "How does a transducer designer predict the damage they are about to do to the rod's ability to convert energy when they try to use it?" This is a really fine question. How *does* one estimate material parameters for Terfenol-D before the transducer is built? At present, experience seems to be the best guide. This is an area where further research seems very appropriate.