An open loop control scheme to minimize flexible robot response time while minimizing residual vibrations

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An open loop control scheme to minimize flexible robot response time while minimizing residual vibrations

Jang, Wan-Shik, Ph.D.

Iowa State University, 1994
An open loop control scheme to minimize flexible robot response time while minimizing residual vibrations

by

Wan-Shik Jang

A dissertation Submitted to the Graduate Faculty in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY

Department: Aerospace Engineering and Engineering Mechanics
Major: Engineering Mechanics

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1991

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ACKNOWLEDGEMENTS

I would like to thank Dr. K. G. McConnell, my major advisor, for his patient guidance during these years. It was a valuable experience gained by working under his supervision. I am really indebted to him for many valuable suggestions, discussions and encouragement when I was wandering. I could not have accomplished this without him. Also, I wish to thank Dr. W. H. Brockman and Dr. A. B. Flatau for their assistance in this research work.

I wish to thank my wife Kyung-Ok for her support and encouragement, and thank my two daughters, Ah-Mee and Woo-Hyung, for their patience during my study. Also, I must thank my parents for their encouragement and understanding.
CHAPTER 1. INTRODUCTION

Traditionally, robotic manipulator arms are modeled as rigid links that are moved through use of controllers, actuators and sensors. This rigid body concept works quite well in predicting robot behavior provided the robot moves slowly. However, high accelerated motion causes considerable residual vibration to occur after the robot manipulator reaches its defined end point. This residual vibration results from the inherent compliance of the structural elements that make up the robot assemblies; that is, additional potential energy is stored in the robot structure when responding to high inertia loads from fast motion commands. This energy is available to cause residual vibration. Since structural resonances tend to be lightly damped, any residual vibration requires additional settling time before the robot is considered to have completed its task.

Positioning is a fundamental function of robotic manipulators. To realize high speed and accurate positioning, it is necessary to consider the vibratory characteristics of the robotic manipulators in order to reduce residual vibrations so that the robot is cost effective. The faster the motion, the larger the energies that must be removed before the mechanism stops completely. To minimize the manipulator response time, it is necessary to minimize the manipulator’s residual vibration when it arrives at the defined end point. Consequently, in recent years, methods that generate
fast motions with minimum residual vibration are increasingly important.

Several different approaches have been suggested to reduce residual vibration.
In the following sections, past studies are reviewed, and the problem definition for
this thesis is described.

1.1. Review of Previous Studies

In the past two decades, many researchers have examined techniques for reducing
the residual vibration of flexible systems. These techniques can be divided into two
broad categories, dependent upon one's viewpoint. One category is based on closed-
loop feedback control techniques while the other category is based on input command
shaping techniques. Here, the two techniques are reviewed separately.

1.1.1. Closed-loop feedback control techniques

Many researchers have examined closed-loop feedback techniques for reducing
residual vibration at the final position. These techniques have a distinct characteris­
tic where they use continuously measured data of the system's operating state and
modify the feedback according to a prescribed control scheme to achieve vibration
control. The methods from these studies are reviewed in the following paragraphs.

One approach is to explicitly increase the damping of the flexible modes. Active
damping techniques have been proposed that add distributed damping to beam-like
structures.

Burke and Hubbard [3] examined the application of a spatially-shape distributed
actuator to a simply supported beam for vibration control. The actuator consists of
a layer of the piezoelectric polymer polyvinylidene fluoride (PVF$_2$). They generated
a distributed control law by applying a PVF$_2$ bonded to one face of the beam that alters its resistance to bending when given a voltage signal.

Silverberg [26] studied an active damping technique by introducing a uniform damping control to a flexible spacecraft. Because the state of a spacecraft is distributed over its domain, achieving the best dynamic performance will require distributed actuators and sensing devices. Thus they developed the implementation of uniform damping control using discrete actuators and sensing type devices for a spacecraft vibration suppression.

Besides explicit damping augmentation, standard classical and modern feedback methods have been proposed to maneuver flexible structures.

Dougherty et al. [7] used a classical proportional-integral-derivative (PID) controller on the space telescope pointing control system. This approach utilized collocated actuator-sensor pairs, making stability in the presence of controlled system (plant) uncertainties easier to achieve with respect to fine pointing performance. They developed the control method by treating the flexible modes as a separable subsystem. Thus, the PID gains, which are originally chosen for the rigid body alone, need to decrease for lower bandwidth to assure stability when flexible modes are present.

Kosut et al. [15] showed that a well-designed feedback control system exhibits the properties of external disturbance attenuation and performance robustness with respect to plant uncertainty for flexible spacecraft. They evaluated the robustness properties of several Linear Quadratic Geometry (LQG) based control designs using the singular value robustness measures.

Cannon and Schmitz [4] utilized a non-collocated controller in a flexible one link robot by applying torque input at the hub and employed a small light bulb for optical
sensing of the tip position. This approach has been used to actively control both the rigid body angle and the vibration of a flexible system. By using a tip position sensor, they accomplished a more accurate end point positioning.

Kotnik et al. [16] also examined a closed-loop feedback technique that reduced end point vibration of a flexible manipulator arm. The flexible body motion is controlled using the end point acceleration that is fed back through a Butterworth filter with corner frequency of 8 Hz while shaft position is used to control the rigid body motion of the manipulator arm.

Yang et al. [41] developed an adaptive control scheme to control the tip position of a single degree of freedom flexible manipulator for use in high-speed and large amplitude motion situations. Through experimental work, they showed that the control scheme is essentially self-tuning and consists of system identification and pole placement control algorithms. They adjusted controller gains as a function of the estimated parameters corresponding to any changes occurring in the manipulator's dynamic description.

Most of these techniques can only increase damping by a limited amount. If the inherent damping is very low, this increase may be insufficient to adequately improve the response. In addition, these controllers rely on accurate system models, especially if dynamic performance parameters must be estimated. This makes them very sensitive to modeling errors that can degrade performance and, in some cases, produce instabilities.
1.1.2. Input command shaping techniques

Input Command shaping techniques alter the shape of either actuator commands or reference commands by using various control algorithms that reduce residual vibration when the final position is achieved. The earliest work for input command shaping was the use of high speed cam profiles for cam-follower systems to generate a smooth motion.

Sehitoglas and Aristizabal [24] studied the design of a trajectory controller for the industrial robot by using a cylindrical motion profile to generate smooth motions throughout one cycle. This smoothness reduced unwanted dynamics by not introducing high frequency inputs into the system. However, they made no attempt to tune these functions to the dynamics of the system to minimize both move time and residual vibration.

Wiederrich and Roth [37] analyzed the design of high-speed cam profiles using finite trigonometric series to reduce residual vibration. They used a mean squared error minimization performance index to generate high-speed cam profiles for a single degree of freedom cam follower system in order to reduce the residual vibration.

Wiederrich [38] investigated the application of the method [37] to a multiple degree of freedom cam follower system when using the methods of modal analysis.

Optimal control approaches have been used to generate input command shaping for commanding vibratory systems.

Breakwell [2] presented a method for maneuvering a flexible spacecraft from one position to another, while leaving an arbitrary number of bending modes inactive at the end of the maneuver. He developed the optimal theory which allows determination of open-loop control profile to effect desired maneuvers. The generated control
profiles, based on the fixed-time, linear quadratic-loss minimization problem which is common in modern control theory, is converted into a feedback maneuvering law for the closed-loop implementation with time-varying feedback gains.

Chun et al. [5] have used the performance index in combination with Pontryagin's principle to generate optimal control functions for commanding flexible spacecraft. The resulting optimal trajectory of control input profiles is obtained in the form of the solution to the system equations.

Farrenkorf [9] demonstrated that appropriate velocity shaping can be implemented on systems which modally decompose into second order harmonic oscillators. Then, he showed that inputs in the form of the solutions for the decoupled modes can be added so as not to excite vibration. This technique solves for parameters to control a template input function so that the inputs are limited to the form of the template. The parameters, which define the control input, are obtained by minimizing a structural excitation criterion satisfying any end point constraints based on the calculus of variation.

Gupta [10] has used some frequency shaping terms in the optimal formulation with the standard LQG cost functional to generate an input profile. The derivative of the control input is included in a penalty function so that, as with cam profiles, the resulting functions are smooth.

Juang, Turner, and Chun [13] [14] studied the optimal control approach to generate input profiles with time varying a feedback gain. They derived a feedback control law for a class of optimal finite time tracking problems with terminal constraints. Then, they developed analytical solutions for the feedback gain and the closed-loop response trajectory. Such formulations are expressed in recursive forms so that a real
time computer implementation becomes feasible.

Swigert [30] developed an appropriately-shaped torque to optimally move a mechanical element of a structure so the mechanical modes are left unexcited at the desired end position of the movement. This approach not only minimizes residual vibration, but also minimizes the effect of parameter variations which change the modal frequencies.

Turner and Chun [35], and Turner and Junkins [36] used various performance indices in combination with Pontryagin's principle to generate optimal continuous torque functions for performing open-loop maneuvers of flexible spacecraft. The resulting optimal trajectory is obtained in the form of the solution to the system model.

These optimal approaches have two major drawbacks. First, computation is very difficult. Each motion of the system requires recomputation of the control algorithm. Though the papers cited above have shown major advances toward simplifying this step, solving for the input to complex systems is extremely difficult. Second, the value of optimal input strategies depends on move time. Different motions will have different vibration excitation levels. To avoid these difficulties, several researchers developed alternative approaches to generate input command profiles for flexible systems.

Aspinwall [1] studied a pulse-shaping technique based on a short, finite Fourier series expression for the forcing function. This method is used to attenuate residual dynamic response in elastic systems response by several orders of magnitude. Then, the forcing function is based on selecting the Fourier coefficients to depress the envelope of the residual response spectrum in desired regions.
Meckl and Seering [17] developed a feedforward control scheme to achieve fast settling time in robots. They modified the conventional controller with the addition of a suitably frequency-shaped feedforward forcing function which is designed to minimize both move time and residual vibration in robot manipulators.

Another approach to input command shaping was developed by Meckl and Seering [18] [19]. They studied two different types of forcing functions to eliminate the residual vibration of a robot arm at the end position of a movement. One approach they examined is a bang-bang control function for the time optimal response, which is very sensitive to switching accuracy. To overcome these problems, another forcing function uses a series of ramped sinusoids with coefficients chosen to minimize spectral magnitude in this frequency band is constructed to avoid exciting resonance throughout the move.

Wang et al. [39] developed a method that is based on closed-loop simulation to generate the open-loop control input so that a flexible robot manipulator precisely moves along a given trajectory, and the residual vibration of the robot arm is reduced at the end position of the movement. They modeled the actual system as an undamped spring mass system and designed a PID controller for the plant that gave a desired response. Then, they examined the actual input that the controller gave to the plant and used this for the real system. Next, they refined the reference input using an iteration scheme that added the error signal to the reference signal in order to get better tracking of the given trajectory. Additionally, they showed the experimental results [40] for a single degree of freedom flexible manipulator using the developed method [39].

Recently, Meckl and Seering [20] developed another method to generate shaped
force input, which is constructed from a versine series with coefficients of the harmonic terms chosen to maximize kinetic energy for fast motion and minimize excitation energy at the system natural frequencies for the reduction of residual vibration. Next, they incorporated these inputs into a closed-loop system by tuning them to the closed-loop natural frequencies and generating the corresponding position reference profile by integrating shaped force input twice for the implementation of the closed-loop system.

Singer and Seering [27] developed preshaping input commands using the impulse input sequence which significantly reduces or eliminates the end point vibration at the end of a movement. This approach, taken for both open and closed loop systems, expresses the transient residual vibration amplitude of a system as a function of the impulse input sequence. They specified the input so the system's natural tendency to vibrate cancelled residual vibration. Next, they modified the input to include insensitivity to uncertainties.

Most of these techniques have examined the transient vibration of robot manipulators in terms of frequency content of the system inputs and outputs. This approach inherently assumes that the system inputs are not actually transient, but are one cycle of a repeating waveform. In addition, these techniques essentially construct an input function. Thus, this method may become inaccurate when the system characteristics are changed during real time control.

1.2. Problem Definition

The objective of this study is to develop a practical control scheme called three-step input method whereby a flexible robot arm is moved from one position to another
in the least amount of time with a minimum of residual vibration present when the arm reaches its defined end point. The basic premise is to use open-loop self-adjusting input signals that take the system's dynamic behavior into account. There have been relatively few applications of the self-adjusting control scheme in the area of vibration control. In most applications, the approach is to control the force profiles applied to the robot base. These applications require considerable computation time. Consequently, it is common to experience real-time processing difficulties for cases involving high speed motion.

Given the apparent disadvantage of the previous applications of various control algorithms to reduce residual vibration, this thesis will address the issue of using the self-adjusting command input function to eliminate this residual vibration in high speed motion. In particular, the class of problems addressed in this thesis is restricted to the elimination of residual vibration when the fastest response is done in time steps of one-half of the robot's fundamental natural period.

The basic idea of this study comes from the response of an undamped single degree of freedom system to a step input as shown in Figure 1.1. The system wants to ring about its static deflection when subjected to a single step input as shown in Figure 1.1a. However, if a second equal step is applied when the displacement is at a maximum as shown in Figure 1.1b, the system remains at rest at twice the initial static deflection, and there is no residual vibration. Thus, the system has moved from its initial position to its final position in a minimum amount of time of one-half natural period, and there is no residual vibration. This is an ideal situation and defines the physical response limit.

However, real structural systems have small amounts of damping, so that it is
physically impossible to eliminate residual vibrations when the defined end point is achieved when using this dual step input. Thus, a differently shaped command input function is required to eliminate the residual vibration when responding in the least amount of time. This research work is concerned with defining a simple practical method to utilize step inputs to achieve optimum response. The optimum response is achieved by using a self-adjusting command input function that is obtained during a real time process following the initial step input. The shape of this command input function consists of three step inputs, each with different magnitudes and time duration. Three important factors need to be decided. These are: a) the magnitude of the first step input, b) the switching time and magnitude of the second step input, and c) the switching time and magnitude of the last step input. These unknown values are significant parameters obtained according to the parameter estimation model that is described later. The first step input is used to experimentally determine the natural period of the mechanical system. The second step input is used to compensate the difference between the defined end point value and the peak value that would result under the first step input when damping is taken into account. The last step input is used to eliminate the residual vibration when the defined end point is achieved in

Figure 1.1: Basic response of mechanical system to step inputs. a) Single step, ringing, b) dual step, no ringing
the least amount of time.

Among the parameters that define the command input function, only two parameters are computed during real time processing. These two parameters involve the magnitude of the second step input and the switching time of the last step input. The others are defined by the desired end point and the mechanical system's fundamental natural frequency when a flexible robot arm is moved from one position to another. Since the computed parameters are related to the fundamental natural frequency and damping ratio of the mechanical system, it is not necessary to recompute the command input function when a flexible robot arm is moved with various defined end points. Thus, this method uses a small amount of computing time, a critical factor when dealing with high speed motion and real time processing. A further advantage of this method is that it can be applied to an unknown mechanical system, which moves from an initial position to a final position while transporting an unknown mass. In other words, the method is self-adaptive to the robot system being used.

Finally, a multiple level procedure is outlined for large displacement problems. The multiple level procedure is applied to a large motion that cannot be achieved in one step due to either stress or torque limitations. Thus, a large motion is broken into several smaller motions that are within the servo system's ability to handle, where the three-step input method is used a number of times.

The practicality of this control scheme is demonstrated experimentally by using an analog computer to simulate a simple flexible robot and a conventional servo controller. This procedure is shown to use a small amount of digital computing resources.
CHAPTER 2. MATHEMATICAL DEVELOPMENT OF THE
CONTROL PROBLEM

2.1. Development of the Servo and Flexible Manipulator Model

The simplest flexible manipulator is used as a model to study a control method which minimizes both response time and residual vibration. This manipulator is modeled by two lumped masses, a spring and a damper as shown in Figure 2.1. $m_1$, $m_2$, $k$, and $c$ represent the properties of the flexible manipulator, and force $F_d(t)$ is the servo generated forcing function that acts on mass $m_1$ to move mass $m_2$. The dynamic equations for this system are

$$m_1\ddot{x}_1 + c\dot{x}_1 + kx_1 - c\dot{x}_2 - kx_2 = F_d(t) \tag{2.1}$$

$$m_2\ddot{x}_2 + c\dot{x}_2 + kx_2 - c\dot{x}_1 - kx_1 = 0 \tag{2.2}$$

Figure 2.2 shows the schematic diagram for the flexible manipulator control system. As shown in Figure 2.2, a standard proportional controller and a D-C servo motor are used to control the flexible manipulator. The controller base motion $x_1$ is used as a feedback signal, while the D-C servo motor is operated by the constant field current mode.

An analysis of the servo system is required to relate the D-C servo motor force $F_d(t)$ to the the input variable $x_{in}(t)$. The force generated by a perpendicular mag-
Figure 2.1: Damped spring mass system

Figure 2.2: Schematic diagram for the flexible manipulator control system
netic field of strength $B$ and wire current $I$ is given by

$$F_d(t) = BlI = K^*I \quad (2.3)$$

where $l$ is the effective wire length. The back emf $E_m$ generated by wire motion is given by

$$E_m = Bl\dot{x} = K^*\dot{x} \quad (2.4)$$

where $\dot{x}$ is the wire velocity perpendicular to the magnetic field. Thus, the force $F_d(t)$ becomes

$$F_d(t) = \frac{K^*}{R}(E - E_m) = \frac{K^*}{R}(E - K^*\dot{x}) = GK_1E - K_2\dot{x} \quad (2.5)$$

where $R$ is a constant wire resistance, $G$ is servo system gain, $K_1$ is a system physical constant, $K_2$ is a back emf feedback constant, and $E$ is an error signal. Error signal $E$ is defined as

$$E = x_{in}(t) - x_1 \quad (2.6)$$

where $x_{in}(t)$ represents the command input signal that is applied to the servo system, and $x_1$ represents the base’s actual response. Combining Eqs. (2.1), (2.2), (2.5) and (2.6) gives the dynamic equations as

$$m_1\ddot{x}_1 + (c + K_2)\dot{x}_1 + (k + GK_1)x_1 - c\dot{x}_2 - kx_2 = GK_1x_{in} \quad (2.7)$$

$$m_2\ddot{x}_2 + c\dot{x}_2 + kx_2 - c\dot{x}_1 - kx_1 = 0 \quad (2.8)$$
The \(G, K_1,\) and \(K_2\) parameters are used in Fig. 2.2 along with the Laplace operator \(s\). Equations (2.7) and (2.8) describe the block diagram of Fig. 2.2.

The constant values in these equations are determined by the mechanical and servo system parameters such as \(m_1, m_2, k, c, K_1, K_2,\) and \(G\). Inspection of these equations shows that Eq. (2.7) represents the servo system characteristics that include the effects of \(k, c,\) and \(m_1\) on its performance, while Eq. (2.8) represents the mechanical system characteristics. From Eq. (2.7), the servo system natural frequency \(f_1\) and dimensionless damping ratio \(\zeta_1\) are defined by

\[
f_1 = \frac{\omega_1}{2\pi} \quad \text{and} \quad \zeta_1 = \frac{\omega_1}{\omega_1} = \frac{(c + K_2)}{2m_1\omega_1} = \frac{1}{2\sqrt{m_1(k + GK_1)}}
\]

where \(\omega_1\) represents the circular natural frequency of the servo system. From Eq. (2.8), the mechanical system natural frequency \(f_2\) and dimensionless damping ratio \(\zeta_2\) are defined as

\[
f_2 = \frac{\omega_2}{2\pi} \quad \text{and} \quad \zeta_2 = \frac{c}{2m_2\omega_2} = \frac{c}{2\sqrt{m_2k}}
\]

where \(\omega_2\) represents the circular natural frequency of the mechanical system. These natural frequencies and damping ratios apply to each system when they are uncoupled.
from one another. These natural frequency and damping definitions are used in Chapter four.

2.2. Control Strategy

The proposed control method began with trying to satisfy three major objectives when dealing with a flexible robot arm. First, it is desired to move a prescribed distance in the least amount of time. Second, there is to be no residual vibration at the end of the motion. Third, conventional servo system components are to be used along with minimum computational effort to generate the required system input commands. These three objectives are easily met when dealing with an undamped single degree of freedom mechanical system that is subjected to two equal step input forces.

It is well known that a second order system response to a step input force is a "ringing" motion defined by

\[ x(t) = x_{st} [1 - \cos(\omega_n t)] \]  

(2.13)

where \( x_{st} = F_0/k \) is the static deflection and \( \omega_n \) is the undamped natural frequency. Equation (2.13) represents the "fastest possible mechanical system response". However, if a second equal step load is applied at time \( t_p \) so that \( \omega_n t_p = \pi \), then

\[ x(t_p) = 2x_{st} \]  

(2.14)

\[ \dot{x}(t_p) = 0 \]  

(2.15)

and the spring force is exactly equal to the applied force; that is,

\[ k(2x_{st}) = 2F_0 \]  

(2.16)
Equation (2.16) shows that there is no residual force to cause additional motion while Eq. (2.15) shows there is no residual motion in the form of kinetic energy. The system is in equilibrium and a change in position of $2x_{st}$ has occurred. This simple system response indicates that it may be possible to extend such ideas to a servo control system that involves at least one robot arm.

Unfortunately, most structural systems contain some damping so this simple dual-step input causes some residual vibration to occur when the end point is reached. The system requires additional settling time due to this residual vibration before the system can be judged to have completed its task. Thus, a modified control method is required for damped systems that will also employ step inputs but have no residual motion.

The objective of this study is to develop a practical control scheme called "three-step input method", whereby a flexible robot arm is moved from one position to another in the least amount of time with a minimum of residual vibration present when the arm reaches its defined end point. The basic premise is to use the open loop self-adjusting command input function that takes system characteristics into account. For this study, the defined end point is assumed to be as twice the static deflection that occurs when the damped system is excited by a single step input. In this way, the defined end point is independent of the mechanical system damping ratio. The command input function's shape consists of three-step inputs, each with different magnitudes, as shown in Figure 2.3. Here $U_0$ is half of the defined end point value $U_1$, and $\Delta U$ and $t_1$ define the magnitude and switching time of the second step input, and $t_2$ defines the switching time of the last step input. Each input is shown separately as well as added together.
The first step input is used to determine the natural period of the mechanical system. The second step input is used to compensate for the difference between the defined end point value and the peak value under the first step input. This difference depends on the mechanical system damping ratio. The last step input is used to eliminate the residual vibration when the defined end point is achieved in the least amount of time. The parameters of this command input function depend on the characteristics of the mechanical system. The detailed control scheme is described in the following seven steps:

1. Command the first step input with magnitude $U_0$. The response to this step input is used to identify the system’s natural frequency.

2. Time $t_0$ is identified by monitoring either the spring force (servo driver current) or relative motion $(x_1 - x_2)$.

3. Based on time $t_0$, calculate $T_n$, which defines the natural period of the mechanical system, according to the system identification model described in the following subsection.

4. With the calculated $T_n$, select the switching time $t_1$ of the second step input so that $t_1 = 3T_n/8$, as described in section 2.4.

5. Based on $T_n$ and $t_1$, compute the magnitude $\Delta U$ of the second step input and the switching time $t_2$ of the last step input from the parameter estimation model developed in section 2.4.

6. Command the second step input with magnitude $\Delta U$ at time $t_1$. 
Figure 2.3: The shape of command input function
7. Command the last step input with magnitude \((U_0 - \Delta U)\) at time \(t_2\), which represents the minimum response time for the manipulator.

Figure 2.4 shows the block diagram of the three-step input method. This block diagram consists of four parts including the main system, the system identification, parameter estimation, and control law. The main system consists of the mechanical system and the servo control system parts. The mechanical system representing the simplest flexible manipulator is modelled by two lumped masses, a spring and a damper as shown in Figure 2.1. The servo control system has a D-C armature controlled servo motor and a standard proportional controller that uses the base motion \(x_1\) as a feedback signal as shown in Figure 2.2.

2.3. System identification

System identification is concerned with determination of the unknown mechanical system natural period, \(T_n\). For this study, both the manipulator end point response, \(x_2\), and spring force, defined as \(k(x_1 - x_2)\), can be used for monitoring variables. It is evident that both conditions lead to the same result of \(x_1 = x_2\). These monitoring variables are used to identify when the position of the response has reached halfway to the defined end point. There is a question of how to calculate time, \(t_0\), when working with digital feedback signals. A general calibrational procedure is explained in Section 4.3 to solve this question. For this thesis, the manipulator end point response, \(x_2\), is used as a monitoring variable.

When time \(t_0\) is obtained, the natural period of the unknown (or known) mechanical system is computed from a typical step input response of the damped spring
Figure 2.4: Block diagram of overall control system
mass system that is given by

\[ x(t) = \frac{F_0}{k} \left[ 1 - \frac{e^{-\alpha t}}{\sqrt{1 - \zeta^2}} \cos(\omega_d t - \psi) \right] \] (2.17)

with

\[ \tan \psi = \frac{\zeta}{\sqrt{1 - \zeta^2}} \]

\[ \alpha = \zeta \omega_n \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

where \( \frac{F_0}{k} \) is the static deflection under a step input force of \( F_0 \), \( \omega_n \) is the undamped natural frequency of the system, and \( \zeta \) is the system damping ratio. At time \( t_0 \), \( x(t_0) = \frac{F_0}{k} \). Then, Eq. (2.17) becomes

\[ \frac{F_0}{k} = \frac{F_0}{k} \left[ 1 - \frac{e^{-\alpha t_0}}{\sqrt{1 - \zeta^2}} \cos(\omega_d t_0 - \psi) \right] \] (2.18)

which reduces to

\[ \cos(\omega_d t_0 - \psi) = 0 \] (2.19)

Since \( t_0 \) corresponds to the first time Eq. (2.19) is satisfied, the damped natural frequency \( \omega_d \) is given by

\[ \omega_d = \frac{1}{t_0} (\frac{\pi}{2} + \psi) \] (2.20)

and the natural frequency \( \omega_n \) becomes

\[ \omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} = \frac{1}{t_0 \sqrt{1 - \zeta^2}} (\frac{\pi}{2} + \psi) \] (2.21)
When the damping ratio $\zeta$ is very small, Eqs. (2.20) and (2.21) reduce to

\[ \omega_n \approx \omega_d \approx \frac{\pi}{2t_0} \]  

(2.22)

This equation also leads to

\[ t_0 = \frac{T_n}{4} \]  

(2.23)

where $T_n$ represents the natural period of the mechanical system.

### 2.4. Parameter Estimation Model

The three-step input method in this thesis is taken so that the manipulator response time is reduced while minimizing residual vibration at the defined end point. The three-step command input function, shown in Figure 2.3, has five unknown parameters involving the magnitude $U_0$ of the first step input, the magnitude $\Delta U$ and switching time $t_1$ of the second step input, and the magnitude $U_1$ and switching time $t_2$ of the last step input. $U_0$ is defined to be half of the defined end point, while $U_1$ is expressed as $(U_0 - \Delta U)$. Time $t_1$ must be greater than $T_n/4$ and significantly less than $T_n/2$. $t_1$ is arbitrarily selected to be three-eighths of the mechanical system natural period $T_n$ so that there is time for system identification and parameter estimation to be done. Thus, among five unknown parameters, only two parameters, $\Delta U$ and $t_2$, remain as unknowns. Proper choice of these parameters are important factors in making the three-step input method work.

The mechanical system shown in Figure 2.1 is analyzed to determine the proper values of $\Delta U$ and $t_2$ because the manipulator end point motion $x_2$ is used as a monitoring variable. The manipulator end point motion $x_2$ is excited by a horizontal base motion $x_1$, which is excited by the servo generated control force $F_d$. Consider
the free body diagram in Figure 2.5. Using coordinates $x_1$ and $x_2$ measured from inertial reference, the differential equation of motion for mass $m_2$ becomes

$$m_2 \ddot{x}_2 + c \dot{x}_2 + kx_2 = c \dot{x}_1 + kx_1$$  \hspace{1cm} (2.24)

From this equation, the total force available to drive $m_2$ mass system is

$$F_f = c \dot{x}_1 + kx_1$$  \hspace{1cm} (2.25)

where it is seen that $F_f$ is a function of $x_1$ and $\dot{x}_1$. In reality, the base motion $x_1$ is not a step function like the command input, because $x_1$ is delayed due to the finite time it takes the servo system to move mass $m_1$. However, an investigation into frequency response characteristics of the second order servo control system allows us to assume the force, $F_f$, is nearly a step function.

A typical servo frequency response function between base motion $x_1$ and command step input $x_{in}$ is shown in Figure 2.6 where $M$ is the magnitude ratio of $x_1(j\omega)$
Figure 2.6: Servo system frequency response characteristics
divided by \( x_{in}(j\omega) \) and \( \omega_{cf} \) is the corner frequency that is equal to the servo natural frequency \( \omega_1 \). This frequency response function shows the servo characteristics when the servo damping ratio is 70%. From this plot, the passband of the servo frequency response with \( M \approx 1 \) is defined as the frequency range between 0 to \( \omega_p \) where 0 represents the low frequency limit and \( \omega_p \) represents the high frequency limit. In the case of 70% servo damping ratio, \( \omega_p \approx 0.33\omega_1 \) when \( M \approx 1 \) within 0.5%. The mechanical natural frequency \( \omega_2 \) (or \( f_2 \)) is selected to be in this frequency range for this study. This means that the servo system natural frequency \( \omega_1 \) (or \( f_1 \)) is several times larger than the mechanical system natural frequency \( \omega_2 \) (or \( f_2 \)).

When the servo system natural frequency \( \omega_1 \) (or \( f_1 \)) is significantly larger (5 to 10 times) than the mechanical system natural frequency \( \omega_2 \) (or \( f_2 \)), motion \( x_1 \) will not be a true step motion like \( x_{in} \) is assumed to be, but as far as motion \( x_2 \) is concerned, \( F_f \) in Eq. (2.25) is essentially a step force input. Thus, being able to assume that \( F_f \) is a step input force reduces the computational complexity required to determine \( \Delta U \) and \( t_2 \). This means that Eq. (2.24) describes the response of a single degree of freedom system subjected to step input forces as shown in Figure 2.7.

What constraints must this motion satisfy? First, \( x_2 \) must be equal to the desired end point when \( t = t_2 \). Second, there is to be no residual vibratory motion. These conditions are automatically satisfied if \( x_2 \) is equal to the desired end point motion for all \( t \geq t_2 \), for a constant \( x_2 \) value precludes any residual vibration.

Based on the above assumption, the parameter estimation model is developed using a single degree of freedom system under the input command force \( F_f \) as shown in Figure 2.7. In this case, \( m = m_2, \omega_n = \omega_2, \zeta = \zeta_2 \). The subscript is dropped for convenience. The relationship between the steady state value \( x_{ss} \) and a step force
input of $F_f$ is described by

$$x_{ss} = \frac{F_f}{k} \quad (2.26)$$

where $k$ represents the system stiffness.

The response of a single degree of freedom system can be considered to be the sum of independent three step inputs. The first step input has positive magnitude $F_0$, which is the force required to move to one half of the desired final position, the second step input has positive magnitude $\Delta F$ and delayed by the time $t_1$, and the third step input has positive magnitude $(F_0 - \Delta F)$ and delayed by the time $t_2$. The response of the system is obtained using the convolution integral. The convolution integral under the arbitrary excitation $f(t)$ is represented by the integral

$$x(t) = \int_0^t f(\eta)h(t - \eta)d\eta \quad (2.27)$$

where

$$h(t) = \frac{1}{m\omega_d}e^{-at}\sin \omega_d t \quad (2.28)$$

$$\omega_d = \omega_n \sqrt{1.0 - \zeta^2} \quad (2.29)$$

$$a = \zeta \omega_n \quad (2.30)$$

$$b = \frac{\zeta}{\sqrt{1.0 - \zeta^2}} \quad (2.31)$$

For the first step input, the terms of the convolution integral are

$$f(t) = F_0 \quad (2.32)$$

Substituting Eqs. (2.28) and (2.32) into Eq. (2.27), the system response becomes

$$x(t) = \frac{F_0}{k} \left[ 1.0 - e^{-at}(\cos \omega_d t + b \sin \omega_d t) \right] \quad \text{for } t \geq 0 \quad (2.33)$$
Figure 2.7: The expected force command input shape on the damped spring mass system. a). The damped spring mass system b). The input command force
For the second step function starting at \( t_1 \), the excitation force \( f(t) \) is

\[
f(t) = \Delta F
\]  

Similarly, the convolution integral can be solved by the substitution of Eqs. (2.28) and (2.34) into Eq. (2.27) as giving

\[
x(t) = \frac{\Delta F}{k} \left[ 1.0 - e^{-a(t-t_1)}(\cos \omega_d(t - t_1) + b \sin \omega_d(t - t_1)) \right] \text{ for } t \geq t_1
\]  

(2.35)

By superimposing Eqs. (2.33) and (2.35), the system response for \( t \geq t_1 \) becomes

\[
x(t) = \frac{F_0 + \Delta F}{k} - \frac{F_0}{k} \left[ e^{-at_1}(\cos \omega_d t + b \sin \omega_d t) \right]
- \frac{\Delta F}{k} \left[ e^{-a(t-t_1)}(\cos \omega_d(t - t_1) + b \sin \omega_d(t - t_1)) \right]
\]  

(2.36)

For the third step function starting at \( t_2 \), the excitation \( f(t) \) becomes

\[
f(t) = F_0 - \Delta F
\]  

(2.37)

By combining Eqs. (2.27), (2.28), and (2.37), the convolution integral solution for \( t \geq t_2 \) can be written as

\[
x(t) = \frac{F_0 - \Delta F}{k} \left[ 1.0 - e^{-a(t-t_2)}(\cos \omega_d(t - t_2) + b \sin \omega_d(t - t_2)) \right]
\]  

(2.38)

By superimposing Eqs. (2.36) and (2.38), the system response for \( t \geq t_2 \) becomes

\[
x(t) = \frac{2F_0}{k} - \frac{F_0}{k} e^{-at_1} \left[ \cos \omega_d t + b \sin \omega_d t + e^{at_2} \cos \omega_d(t - t_2) + be^{at_2} \sin \omega_d(t - t_2) \right] + \frac{\Delta F}{k} e^{-at_1} \left[ e^{at_2} \cos \omega_d(t - t_2) + be^{at_2} \sin \omega_d(t - t_2) - e^{at_1} \cos \omega_d(t - t_1) - be^{at_1} \sin \omega_d(t - t_1) \right]
\]  

(2.39)
The system response for \( t \geq t_2 \) is a constant, since the three-step input method seeks to eliminate residual vibration at the defined end point. The displacement constraint satisfying the above demand is defined so the displacement is constant with the desired magnitude. Like the control strategy, the desired value that the manipulator must achieve is the defined end point given by

\[
x(t) = \frac{2F_0}{k}
\]  

(2.40)

Equating the displacement constraint Eq. (2.40) to the system response Eq. (2.39) for \( t \geq t_2 \) and using the trigonometric relations,

\[
\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B
\]  

(2.41)

\[
\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B
\]  

(2.42)

and expand this equation for \( t \geq t_2 \) in terms of the arbitrary time \( t \). we obtain

\[
e^{-at} \left[ (1 + e^{at_2} \cos \omega_d t_2 - be^{at_2} \sin \omega_d t_2) - \frac{\Delta F}{F_0} e^{at_2} \cos \omega_d t_2 \right. \\
+ \frac{\Delta F}{F_0} e^{at_2} \sin \omega_d t_2 + \frac{\Delta F}{F_0} e^{at_1} \cos \omega_d t_1 \\
- \frac{\Delta F}{F_0} e^{at_1} \sin \omega_d t_1 \cos \omega_d t \\
+ (b + e^{at_2} \sin \omega_d t_2 + be^{at_2} \cos \omega_d t_2 - \frac{\Delta F}{F_0} e^{at_2} \sin \omega_d t_2 \\
- \frac{\Delta F}{F_0} be^{at_2} \cos \omega_d t_2 + \frac{\Delta F}{F_0} e^{at_1} \sin \omega_d t_1 \\
+ \frac{\Delta F}{F_0} be^{at_1} \cos \omega_d t_1 \sin \omega_d t \right] = 0
\]  

(2.43)

Since \( e^{-at} \neq 0 \), Eq. (2.43) is rewritten as

\[
M^* \cos \omega_d t + N^* \sin \omega_d t = 0
\]  

(2.44)
where

\[ M^* = 1 + e^{at_2} \left( \cos \omega_d t_2 - b \sin \omega_d t_2 \right) - \frac{\Delta F}{F_0} \left( e^{at_2} \cos \omega_d t_2 - b e^{at_2} \sin \omega_d t_2 - e^{at_1} \cos \omega_d t_1 + b e^{at_1} \sin \omega_d t_1 \right) \]  

\[ N^* = b + e^{at_2} \left( \sin \omega_d t_2 + b \cos \omega_d t_2 \right) - \frac{\Delta F}{F_0} \left( e^{at_2} \sin \omega_d t_2 + b e^{at_2} \cos \omega_d t_2 - e^{at_1} \sin \omega_d t_1 - b e^{at_1} \cos \omega_d t_1 \right) \]  

From Eq. (2.44), the appropriate conditions can be obtained for all \( t \) satisfying \( t \geq t_2 \) by setting the constant values \( M^* \) and \( N^* \) equal to zero.

\[ M^* = 0 \]  

\[ N^* = 0 \]  

These two equations can be solved for \( \Delta F \), respectively. From Eq. (2.47),

\[ \Delta F = \frac{F_0 \left( 1 + e^{at_2} \cos \omega_d t_2 - be^{at_2} \sin \omega_d t_2 \right)}{e^{at_2} \cos \omega_d t_2 - be^{at_2} \sin \omega_d t_2 - e^{at_1} \cos \omega_d t_1 + be^{at_1} \sin \omega_d t_1} \]  

From Eq. (2.48),

\[ \Delta F = \frac{F_0 \left( b + e^{at_2} \sin \omega_d t_2 + be^{at_2} \cos \omega_d t_2 \right)}{e^{at_2} \sin \omega_d t_2 + be^{at_2} \cos \omega_d t_2 - e^{at_1} \sin \omega_d t_1 - be^{at_1} \cos \omega_d t_1} \]  

These two equations are expressed in terms of the command input force function \( (F_f(t)) \) parameters, \( F_0 \) and \( \Delta F \). Using Eq. (2.26), these parameters are expressed in terms of the command input position function \( (x_{in}(t)) \) parameters.

\[ U_0 = \frac{F_0}{k} \]  

\[ \Delta U = \frac{\Delta F}{k} \]
Thus, Eqs. (2.49) and (2.50) are rewritten in terms of the command input function $(x_{in}(t))$ parameters by dividing both terms of each equation by $k$. and using Eqs. (2.51) and (2.52). From Eq. (2.49),

$$U = \frac{L_0(1 + e^{at_2} \cos \omega dt_2 - b e^{at_2} \sin \omega dt_2)}{e^{at_2} \cos \omega dt_2 - b e^{at_2} \sin \omega dt_2}$$

From Eq. (2.50),

$$U = \frac{L_0(b + e^{at_2} \sin \omega dt_2 + b e^{at_2} \cos \omega dt_2)}{e^{at_2} \sin \omega dt_2 + b e^{at_2} \cos \omega dt_2 - e^{at_1} \sin \omega dt_1 - b e^{at_1} \cos \omega dt_1}$$

Recall that $L_0$ is the magnitude of the first step input representing half of the defined end point, and $\Delta U$ is the magnitude of the second step input. These two equations are equated in order to find the unknown parameter, $t_2$.

$$(e^{at_2} + e^{at_1} e^{at_2} \cos \omega dt_1 + b^2 e^{at_1} e^{at_2} \cos \omega dt_1) \sin \omega dt_2$$

$$-(b^2 e^{at_1} e^{at_2} \sin \omega dt_1 + e^{at_1} e^{at_2} \sin \omega dt_1) \cos \omega dt_2$$

$$= e^{at_1} \sin \omega dt_1 + b^2 e^{at_1} \sin \omega dt_1$$

Equation (2.55) can be written as

$$(1 - D_2)e^{at_2} \sin \omega dt_2 - D_1 e^{at_2} \cos \omega dt_2 = D_1$$

where

$$D_1 = e^{at_1} \sin \omega dt_1$$

$$D_2 = e^{at_1} \cos \omega dt_1$$

Equation (2.56) is non-linear with a variable $t_2$. The solution for $t_2$ is obtained by using International Mathematical and Statistical Libraries (IMSL) software developed for the non-linear equation solvers. An additional constraint is that $t_2$ must be a
minimum value subjected to the condition that \( t_2 > t_1 \). The magnitude \( \Delta U \) is acquired from either of two Eqs. (2.53) or (2.54), when the solution \( t_2 \) obtained from the equation (2.56) is substituted.

According to the above developed parameter estimation model, two unknown parameters, \( \Delta U \) and \( t_2 \), are obtained for several cases. These cases are used in both the numerical solution and experimental analog simulation. Tables 2.1 through 2.4 show the values of \( \Delta U \) and \( t_2 \) for damping ratios of 1, 5, 10, and 25 \%. Note that the value of \( \Delta U \) is a constant value for a given value of damping independent of frequency. Similarly, the value of \( t_2 \) is a constant multiplier of system natural period \( T_n \) for a given value of damping. These are convenient results since simple relationship emerges between \( \frac{\Delta U}{L_0} \) and \( \frac{t_2}{T_n} \) vs \( \zeta \). Each functional relationship is obtained in two subregions. The type of functional relationship is obtained by plotting the actual data set with 25 data points in different ways such as linear-linear, linear-log, log-linear, and log-log scales. Figure 2.8 shows \( \frac{\Delta U}{L_0} \) vs \( \zeta \) in two different scales. Figure 2.8a represents \( \frac{\Delta U}{L_0} \) vs \( \zeta \) in linear-log scales while Figure 2.8b represents \( \frac{\Delta U}{L_0} \) vs \( \zeta \) in log-log scales. Figure 2.8a shows a straight line in region 2 \((0.11 < \zeta \leq 0.25)\). This indicates that \( \frac{\Delta U}{L_0} = a + b \ln(\zeta) \) in region 2. Also there is a straight line in \(0.04 < \zeta \leq 0.11\) of region 1 and a nearly linear region in \(0.001 \leq \zeta < 0.005\) of region 1. This means that \( \frac{\Delta U}{L_0} = a + b \ln(\zeta) \) in \(0.04 < \zeta \leq 0.11\) of region 1. Figure 2.8b shows a linear relationship in \(0.001 \leq \zeta < 0.02\) of region 1. This suggests that \( \frac{\Delta U}{L_0} = a(\zeta)^b \) for \(0.001 \leq \zeta < 0.02\) of region 1. It could be that a function like \( \frac{\Delta U}{L_0} = \ln(1 + b\zeta + c\zeta^2) \) might be a possible function that would fit the entire region 1.

Figure 2.9 shows that \( \frac{t_2}{T_n} \) vs \( \zeta \) in linear-linear scales. There are two straight line segments in region 1 \((0 < \zeta \leq 0.11)\) and region 2 \((0.11 < \zeta \leq 0.25)\). This means that
Table 2.1: The values of $\Delta U$ and $t_2$ for a 1% damping ratio

<table>
<thead>
<tr>
<th>Natural frequency</th>
<th>Parameters</th>
<th>Calculated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>$\Delta U$</td>
<td>0.0972536($U'_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5120588($T_n$)</td>
</tr>
<tr>
<td>2 Hz</td>
<td>$\Delta U$</td>
<td>0.0972536($U'_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5120584($T_n$)</td>
</tr>
<tr>
<td>3 Hz</td>
<td>$\Delta U$</td>
<td>0.0972536($U'_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5100093($T_n$)</td>
</tr>
<tr>
<td>5 Hz</td>
<td>$\Delta U$</td>
<td>0.0972536($U'_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5120585($T_n$)</td>
</tr>
</tbody>
</table>

$t_2/T_n = a + b(\zeta)$ or such type of function.

The results of above functional relationship are obtained by using regression analysis method. For $\frac{\Delta U}{U'_0}$ vs $\zeta$, the results are

$$
\frac{\Delta U}{U'_0} = \begin{cases} 
\ln(1 + 10.08\zeta - 33.34\zeta^2) & \text{for } 0 < \zeta \leq 0.11 \\
0.7719 + 0.1991\ln(\zeta) & \text{for } 0.11 < \zeta \leq 0.25 
\end{cases} \tag{2.59}
$$

For $\frac{t_2}{T_n}$ vs $\zeta$, the results are

$$
\frac{t_2}{T_n} = \begin{cases} 
0.5012 + 1.145\zeta & \text{for } 0 < \zeta \leq 0.11 \\
0.5353 + 0.8498\zeta & \text{for } 0.11 < \zeta \leq 0.25 
\end{cases} \tag{2.60}
$$

Figure 2.10 represents the comparison of the actual values (calculated from Eqs. (2.53) or (2.54) and (2.56)) and predicted values (calculated from Eqs. (2.59) and (2.60)) in $\frac{\Delta U}{U'_0}$ and $\frac{t_2}{T_n}$ vs $\zeta$. Also Figure 2.10 shows three important points. First, time $t_2$ is always greater half a period for any damping. Second, time $t_2$ increases with increasing damping. Third, the value of $\Delta U$ also increases with increasing damping.
Table 2.2: The values of $\Delta U$ and $t_2$ for a 5% damping ratio

<table>
<thead>
<tr>
<th>Natural frequency</th>
<th>Parameters</th>
<th>Calculated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>$\Delta U$</td>
<td>0.3505793($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5595300($T_n$)</td>
</tr>
<tr>
<td>2 Hz</td>
<td>$\Delta U$</td>
<td>0.3505793($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5595300($T_n$)</td>
</tr>
<tr>
<td>3 Hz</td>
<td>$\Delta U$</td>
<td>0.5573079($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5595300($T_n$)</td>
</tr>
<tr>
<td>5 Hz</td>
<td>$\Delta U$</td>
<td>0.3505793($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.5595300($T_n$)</td>
</tr>
</tbody>
</table>

Table 2.3: The values of $\Delta U$ and $t_2$ for a 10% damping ratio

<table>
<thead>
<tr>
<th>Natural frequency</th>
<th>Parameters</th>
<th>Calculated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>$\Delta U$</td>
<td>0.5127664($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.6151649($T_n$)</td>
</tr>
<tr>
<td>2 Hz</td>
<td>$\Delta U$</td>
<td>0.5127664($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.6151648($T_n$)</td>
</tr>
<tr>
<td>3 Hz</td>
<td>$\Delta U$</td>
<td>0.6127041($T_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.6151650($T_n$)</td>
</tr>
<tr>
<td>5 Hz</td>
<td>$\Delta U$</td>
<td>0.5127664($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.6151650($T_n$)</td>
</tr>
</tbody>
</table>

Table 2.4: The values of $\Delta U$ and $t_2$ for a 25% damping ratio

<table>
<thead>
<tr>
<th>Natural frequency</th>
<th>Parameters</th>
<th>Calculated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Hz</td>
<td>$\Delta U$</td>
<td>0.6983790($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.7441103($T_n$)</td>
</tr>
<tr>
<td>2 Hz</td>
<td>$\Delta U$</td>
<td>0.6983790($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.7441104($T_n$)</td>
</tr>
<tr>
<td>3 Hz</td>
<td>$\Delta U$</td>
<td>0.6983790($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.7441104($T_n$)</td>
</tr>
<tr>
<td>5 Hz</td>
<td>$\Delta U$</td>
<td>0.6983790($U_0$)</td>
</tr>
<tr>
<td></td>
<td>$t_2$(sec)</td>
<td>0.7441105($T_n$)</td>
</tr>
</tbody>
</table>
Figure 2.8: Plots of $\frac{\Delta U}{U_0}$ vs $\zeta$ in 2 different ways. a) a linear-log plot of $\frac{\Delta U}{U_0}$ vs $\zeta$, b) a log-log plot of $\frac{\Delta U}{U_0}$ vs $\zeta$. 
2.5. Control Law

Control law is concerned with the manipulation of the position input commands through the control hardware. This control law is written in the mathematical form, according to the control strategy expressed previously.

\[ x_{in}(t) = U_0 u(t) + \Delta U_u(t-t_1) + (U_0 - \Delta U)u(t-t_2) \]  

(2.61)

where \( u(t) \) is a unit step function, and \( u(t-t_1) \) and \( u(t-t_2) \) are the delayed unit step functions, and

\[ 0 < t_1 < t_2 \]
Figure 2.10: Comparison of the actual and predicted values in $\frac{\Delta U}{U_0}$ and $\frac{t_2}{T_n}$ vs $\zeta$. 

Legend

- Actual $\frac{\Delta U}{U_0}$
- Predicted $\frac{\Delta U}{U_0}$
- Actual $\frac{t_2}{T_n}$
- Predicted $\frac{t_2}{T_n}$
CHAPTER 3. NUMERICAL SOLUTIONS USING MODAL ANALYSIS

Several numerical methods may be used to calculate the responses of the flexible manipulator and control system as shown in Figure 2.2 when the three-step input method is used to minimize both manipulator response time as well as minimizing residual vibration. One of these methods is called modal analysis. This numerical method is used to test the effectiveness of the three-step input parameter estimation model before the experimental analog simulation is set up.

3.1. Development of the Modal Analysis Method

The differential equations of motion that govern the damped system behavior cannot be uncoupled by the undamped modal matrix except under special circumstances. Thus, different modal analysis techniques are required to uncouple the equation of motion for damped systems. The technique utilized in this study reduces the second order equations of motion to a set of first order differential equations. The analysis of this reduced set of equations is essentially the same as that for the undamped system. For example, the same techniques are used to determine the characteristic frequency equation and to derive the modal orthogonality conditions. The eigenvalues and modal vectors are complex quantities for this general case.
Table 3.1: One example for the characteristics of the control system shown in Figure 2.2

<table>
<thead>
<tr>
<th>Mass $m_1$</th>
<th>Mass $m_2$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Kg</td>
<td>1 Kg</td>
<td>10 Hz</td>
<td>1 Hz</td>
<td>70 %</td>
<td>5 %</td>
</tr>
</tbody>
</table>

Consider that a flexible manipulator control system has the characteristics listed in Table 3.1. Based on these system characteristics, Eqs. (2.7) and (2.8) become

$$2\ddot{x}_1 + 175.924\dot{x}_1 + 7895.68x_1 - 0.6283\ddot{x}_2 - 39.48x_2 = 7855.74x_{in}(t)$$  \hspace{1cm} (3.1)

$$\ddot{x}_2 + 0.6283\dot{x}_2 + 39.48x_2 - 0.6283\dot{x}_1 - 39.48x_1 = 0$$  \hspace{1cm} (3.2)

These equations are rewritten in the matrix form.

$$M\{\ddot{x}\} + C\{\dot{x}\} + K\{x\} = \{F(t)\}$$  \hspace{1cm} (3.3)

where

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 175.924 & -0.6283 \\ -0.6283 & 0.6283 \end{bmatrix}$$

$$K = \begin{bmatrix} 7898.68 & -39.48 \\ -39.48 & 39.48 \end{bmatrix}$$

$$\{F(t)\} = \left\{ \begin{array}{c} 7855.74x_{in}(t) \\ 0 \end{array} \right\}$$

\(^1\)The procedures for determining the constant values are described in detail in Chapter 4.
\{x(t)\} = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}

and \(M, C\) and \(K\) are \(2 \times 2\) symmetric matrices. However, Eq. (3.3) can be expressed as

\[
\begin{bmatrix}
0 & M \\
M & C
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{x}
\end{bmatrix}
+ \begin{bmatrix}
-M & 0 \\
0 & K
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
x
\end{bmatrix}
= \begin{bmatrix}
0 \\
F
\end{bmatrix}
\tag{3.4}
\]

By introducing a \(4 \times 1\) state vector \(\{y(t)\}\)

\[
\{y(t)\} = \begin{pmatrix} \dot{x}(t) \\ x(t) \end{pmatrix}
\tag{3.5}
\]

Eq. (3.4) can be reduced to a set of 4 simultaneous first order equations as

\[
A\{\dot{y}\} + B\{y\} = \{E(t)\}
\tag{3.6}
\]

where

\[
A = \begin{bmatrix}
0 & M \\
M & C
\end{bmatrix}, \quad B = \begin{bmatrix}
-M & 0 \\
0 & K
\end{bmatrix}
\]

\[
\{E(t)\} = \begin{pmatrix} 0 \\ F(t) \end{pmatrix}
\]

and \(A\) and \(B\) are \(4 \times 4\) symmetric matrices, and \(\{E(t)\}\) is \(4 \times 1\) matrix.

The homogeneous form of Eq. (3.6) is given by

\[
A\{\dot{y}\} + B\{y\} = \{0\}
\tag{3.7}
\]
when \( \{E(t)\} = \{0\} \). The solution of Eq. (3.7) gives the system's eigenvalues and eigenvectors. These eigenvalues are obtained by assuming the solution of Eq. (3.7) is of the form

\[
\{y(t)\} = \{\Phi\} e^{\alpha t}
\]

where \( \alpha \) is a complex number, and \( \{\Phi\} \) is a \( 4 \times 1 \) modal vector with complex elements. Substituting Eq. (3.8) into Eq. (3.7) yields

\[
\alpha A \{\Phi\} + B \{\Phi\} = 0 \tag{3.9}
\]

which can be written as

\[
[f(\alpha)]\{\Phi\} = 0 \tag{3.10}
\]

where

\[
[f(\alpha)] = [B + \alpha A]
\]

Note that Eq. (3.9) can be expressed in a different form by premultiplying \( A^{-1} \) and defining \( D = -A^{-1}B \). Thus,

\[
[\alpha I - D] \{\Phi\} = [f(\alpha)]\{\Phi\} = 0 \tag{3.11}
\]

where \( I \) is the identity matrix with order 4.

From Eq. (3.10), the eigenvalue problem has a nontrivial solution only if the characteristic determinant is zero,

\[
\Delta(\alpha) = |f(\alpha)| = 0 \tag{3.12}
\]

This determinant leads to the characteristic equation of

\[
\alpha^4 + 88.5903\alpha^3 + 4043.8892\alpha^2 + 5929.3045\alpha + 155140.61 = 0 \tag{3.13}
\]
for the values of $[M]$, $[C]$, and $[K]$ given above. Since this equation is of order 4, there are 4 eigenvalues $\alpha_r$ ($r=1,2,3,4$) that occur in complex conjugate pairs. For this case, they are

\[
\begin{align*}
\alpha_1 &= -43.98195 + 44.88981i \\
\alpha_2 &= -43.98195 - 44.88981i \\
\alpha_3 &= -0.313203 + 6.259625i \\
\alpha_4 &= -0.313203 - 6.259625i
\end{align*}
\] (3.14)

The corresponding modal vectors $\{\Phi\}$ can be determined by substituting eigenvalues, $\alpha_r$, into Eq. (3.10) one at a time. The modal vector can be obtained in a different way by taking any column of adjoint matrix $[F(\alpha)]$ of the matrix $[f(\alpha)]$. The proof of this statement is the same as for the undamped system (see Appendix). By taking the fourth column $[F^{(4)}(\alpha)]$ of the adjoint matrix $[F(\alpha)]$, one obtains

\[
[F^{(4)}(\alpha)] = \begin{bmatrix}
1.2566\alpha^2 + 78.96\alpha \\
4\alpha^3 + 351.848\alpha^2 + 15797.36\alpha \\
1.2566\alpha + 78.96 \\
4\alpha^2 + 351.848\alpha + 15797.36
\end{bmatrix}
\] (3.15)

from which the four modal vectors $\{\phi^{(r)}\}$ corresponding to each $\alpha_r$ are obtained. The results are

\[
\{\phi^{(1)}\} = \begin{bmatrix}
-0.35740 - 0.14170i \\
0.00300 + 0.00000i \\
0.00240 + 0.00560i \\
0.00003 - 0.00003i
\end{bmatrix}
\]
The modal matrix $\Phi$ is a linear combination of the eigenvectors $\{\Phi^{(r)}\}$ and is of order 4. Thus,

$$\Phi = \begin{bmatrix} \{\Phi^{(1)}\} & \{\Phi^{(2)}\} & \{\Phi^{(3)}\} & \{\Phi^{(4)}\} \end{bmatrix}$$  \hspace{1cm} (3.17)

Equation (3.6) can be uncoupled by the means of the modal matrix $\Phi$. Let $\{z(t)\}$ be a new state vector so that

$$\{y(t)\} = \Phi\{z(t)\}$$  \hspace{1cm} (3.18)

and Eq. (3.6) becomes

$$A\Phi\{\dot{z}\} + B\Phi\{z\} = E(t)$$  \hspace{1cm} (3.19)

Premultiplying Eq. (3.19) by the transpose $\Phi^T$ of the modal matrix $\Phi$, we obtain

$$\Phi^T A\Phi\{\dot{z}\} + \Phi^T B\Phi\{z\} = \Phi^T E(t)$$  \hspace{1cm} (3.20)
The product $\Phi^T A \Phi$ becomes

$$
\Phi^T A \Phi = \begin{bmatrix}
\phi(1)^T A \Phi(1) & \phi(1)^T A \Phi(2) & \phi(1)^T A \Phi(3) & \phi(1)^T A \Phi(4) \\
\phi(2)^T A \Phi(1) & \phi(2)^T A \Phi(2) & \phi(2)^T A \Phi(3) & \phi(2)^T A \Phi(4) \\
\phi(3)^T A \Phi(1) & \phi(3)^T A \Phi(2) & \phi(3)^T A \Phi(3) & \phi(3)^T A \Phi(4) \\
\phi(4)^T A \Phi(1) & \phi(4)^T A \Phi(2) & \phi(4)^T A \Phi(3) & \phi(4)^T A \Phi(4)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \phi(2)^T A \Phi(2) & 0 & 0 \\
0 & 0 & \phi(3)^T A \Phi(3) & 0 \\
0 & 0 & 0 & \phi(4)^T A \Phi(4)
\end{bmatrix}
$$

(3.21)

where the off-diagonal terms are zero because of orthogonality. Similarly, performing the product $\Phi^T B \Phi$,

$$
\Phi^T B \Phi = \begin{bmatrix}
\phi(1)^T B \Phi(1) & \phi(1)^T B \Phi(2) & \phi(1)^T B \Phi(3) & \phi(1)^T B \Phi(4) \\
\phi(2)^T B \Phi(1) & \phi(2)^T B \Phi(2) & \phi(2)^T B \Phi(3) & \phi(2)^T B \Phi(4) \\
\phi(3)^T B \Phi(1) & \phi(3)^T B \Phi(2) & \phi(3)^T B \Phi(3) & \phi(3)^T B \Phi(4) \\
\phi(4)^T B \Phi(1) & \phi(4)^T B \Phi(2) & \phi(4)^T B \Phi(3) & \phi(4)^T B \Phi(4)
\end{bmatrix}
$$

$$
= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \phi(2)^T B \Phi(2) & 0 & 0 \\
0 & 0 & \phi(3)^T B \Phi(3) & 0 \\
0 & 0 & 0 & \phi(4)^T B \Phi(4)
\end{bmatrix}
$$

(3.22)
where the off-diagonal terms of matrix $B^*$ are also zero because of orthogonality.

Defining the diagonal matrices $A^*$ and $B^*$,

$$A^* = \Phi^T A \Phi$$  \hspace{1cm} (3.23) \\
$$B^* = \Phi^T B \Phi$$  \hspace{1cm} (3.24)

and a generalized force matrix $N(t)$ is given by

$$N(t) = \Phi^T E(t)$$

\[ = \begin{bmatrix}
18.6120 + 44.3131i \\
18.6120 - 44.3131i \\
61.7197 + 6.1792i \\
61.7197 - 6.1792i
\end{bmatrix}
\begin{bmatrix}
x_{i1}(t) \\
x_{i2}(t) \\
x_{i3}(t) \\
x_{i4}(t)
\end{bmatrix}
= N^* x_{i1}(t) \hspace{1cm} (3.25)

Eq. (3.20) can be written as

$$A^* \{ \ddot{z} \} + B^* \{ z \} = N(t)$$  \hspace{1cm} (3.26)

which represents an uncoupled set of equations of the type

$$A_r^* \ddot{z}_r + B_r^* z_r = N_r(t) \hspace{1cm} \text{for} \hspace{0.5cm} r = 1, 2, 3, 4.$$  \hspace{1cm} (3.27)

Using the following relation,

$$B_r^* = -\alpha_r A_r^*$$  \hspace{1cm} (3.28)
where $\alpha_r$ is the $r$th eigenvalue of the system. Thus, Eq. (3.27) can be further simplified to

$$\dot{z}_r - \alpha_r z_r = \frac{1}{A_r^2} \mathcal{N}_r(t) \quad \text{for } r = 1, 2, 3, 4.$$  \hspace{1cm} (3.29)

The particular solution of Eq. (3.29) is found from the convolution integral.

$$z_r(t) = \frac{1}{A_r^2} \int_0^t e^{\alpha_r(\tau - \tau)} \mathcal{N}_r(\tau) \, d\tau \quad \text{for } r = 1, 2, 3, 4.$$ \hspace{1cm} (3.30)

The initial conditions $\{z(0)\}$ in the $\{z\}$ coordinates are found by the transformation in Eq. (3.18). Assuming zero initial conditions in the $\{x\}$ coordinates, $\{z(0)\}$ are

$$\{z(0)\} = \Phi^{-1} \{y(0)\}$$

$$\Phi^{-1} \begin{bmatrix} \dot{x}(0) \\ x(0) \end{bmatrix}$$

$$= \{0\}$$ \hspace{1cm} (3.31)

Since the initial conditions $\{z(0)\}$ are zeros, the complete solution $z_r$ of Eq. (3.29) is the same as the particular solution shown in Eq. (3.30). Recall that Eq. (3.5) is given by

$$\{y(t)\} = \begin{bmatrix} \dot{x}(t) \\ x(t) \end{bmatrix}$$

so that

$$\{y(t)\} = \Phi \{z(t)\}$$ \hspace{1cm} (3.32)

$$= \begin{bmatrix} \varphi \\ \phi \end{bmatrix} \{z(t)\}$$

where the $2 \times 4$ rectangular matrix $\varphi$ is the upper half of the modal matrix $\Phi$, and the $2 \times 4$ rectangular matrix $\phi$ is the lower half of the modal matrix $\Phi$. Since the
matrix $\phi$ represents the displacement quantities, the displacement vector $\{x(t)\}$ can be written as

$$\{x(t)\} = \phi\{z(t)\}$$

$$= \sum_{r=1}^{4} \phi^r z_r(t)$$

$$= \sum_{r=1}^{4} \frac{1}{A_r^*} \int_{0}^{t} e^{\alpha_r(t-\tau)} N_r(\tau) d\tau$$

(3.33)

Since the system modes appear in complex conjugate pairs, let the $(r+1)$th mode be the complex conjugate of the $r$th mode. Introducing the following complex vector notation

$$A_r^* = |A_r^*| e^{i\theta_r^a} \quad A_{r+1}^* = |A_r^*| e^{-i\theta_r^a}$$

$$\sigma_r^j = |\sigma_r^j| e^{i\theta_r^j} \quad \sigma_{r+1}^j = |\sigma_r^j| e^{-i\theta_r^j}$$

$$N_r(t) = |N_r(t)| e^{i\theta_r^a} \quad N_{r+1}(t) = |N_r(t)| e^{-i\theta_r^a}$$

$$\alpha_r = c_r + d_r i \quad \alpha_{r+1} = c_r - d_r i$$

(3.34)

and grouping pairs of complex conjugates, then $x_j(t)$ are

$$x_j(t) = \sum_{r=1,3} \frac{1}{|A_r^*|} \int_{0}^{t} |N_r(\tau)| e^{i\theta_r^a} \left| e^{\alpha_r(t-\tau)} \right| d\tau$$

$$\times \{e^{i\theta_j^a - \theta_r^a + \theta_r^j} + e^{i\theta_j^a - \theta_r^a - \theta_r^j}\}$$

$$+ \{e^{i\theta_j^a - \theta_r^a - \theta_r^j} - e^{i\theta_j^a - \theta_r^a + \theta_r^j}\}$$

(3.35)

Using the following relation,

$$e^{i\theta} + e^{-i\theta} = 2 \cos \theta$$

(3.36)
Eq. (3.35) can be reduced to

\[ x_j(t) = \sum_{r=1,3} \frac{2|\phi_j^{(r)}|}{|A_r^*|} \int_0^t |N_r(\tau)| \exp[c_r(t - \tau)] \]

\[ \times \cos[d_r(t - \tau) - \theta^r_a + \theta^r_n + \theta^r_j] d\tau \]

(3.37)

where the subscript \( j \) represents the system degrees of freedom. Assuming that the system is excited by the step input function, the magnitude \(|N_r(\tau)|\) in the equation above is constant as shown in Eq. (3.25). Thus, the displacement vector, \( x_j(t) \), can be written as

\[ x_j(t) = \sum_{r=1,3} \frac{2|\phi_j^{(r)}|}{|A_r^*|} \int_0^t \exp[c_r(t - \tau)] \]

\[ \times \cos[d_r(t - \tau) - \theta^r_a + \theta^r_n + \theta^r_j] d\tau \]

(3.38)

Integrating Eq. (3.38) and using the definition for \( N(t) \) from Eq. (3.25), \( x_j(t) \) becomes

\[ x_j(t) = |x_{in}| \sum_{r=1,3} \frac{2|\phi_j^{(r)}|}{|A_r^*|}(c_r^2 + d_r^2)^{1/2} [\cos(\omega_r t + \Omega_r) + d_r \sin(\omega_r t + \Omega_r)] \]

(3.39)

where

\[ \Omega_r = -\theta^r_a + \theta^r_n + \theta^r_j \]

When \( j=1 \), \( x_j(t) \) represents the base response. Similarly, \( x_j(t) \) represents the manipulator end point response when \( j=2 \). Equation (3.39) is the displacement solution of the damped system excited by the step input function \( x_{in}(t) \).
3.2. Numerical Results

The modal analysis solution is used with the "three-step input" function to simulate the system response. The dynamic servo-mechanical system is described by 70% servo damping and 5% mechanical system damping. The servo system has a constant natural frequency of 10 Hz while the mechanical system has either a 1 Hz or a 5 Hz natural frequency. In this way, a frequency ratio \( n \) is used to describe system scaling; that is,

\[
\frac{f_1}{f_2} = \frac{\omega_1}{\omega_2}
\]  

(3.40)

The three-step input method uses the general input form given by Eq. (2.59)

\[
x_{in}(t) = U_0 u(t) + \Delta U u(t - t_1) + (U_0 - \Delta U) u(t - t_2)
\]  

(3.41)

where \( u(t) \) is a unit step function, and \( u(t - t_1) \) and \( u(t - t_2) \) are the delayed unit step functions, and

\[
0 < t_1 < t_2
\]

This function changes to match the mechanical system natural period.

The general modal model solution (3.39) of the damped system excited by the step function is used to get the numerical results for the three-step input method that uses the input function \( x_{in}(t) \) described by Eq. (3.41). For the first step function with a magnitude \( U_0 \), the solution becomes

\[
x_j(t) = U_0 x^*_j(t) \quad \text{for } j = 1, 2.
\]  

(3.42)

For the second step function with a magnitude \( \Delta U \), which is delayed by time \( t_1 \), the solution can be written down by inspection of the above equation as

\[
x_j(t) = \Delta U x^*_j(t - t_1) \quad \text{for } j = 1, 2.
\]  

(3.43)
Similarly, for the last step function with a magnitude \((U_0 - \Delta U)\), which is delayed by time \(t_2\), the solution becomes

\[ x_j(t) = (U_0 - \Delta U)x_j(t - t_2) \quad \text{for } j = 1, 2. \quad (3.44) \]

Thus, based on the above solutions (3.42), (3.43), and (3.44), the response for each time interval is computed. For \(0 < t \leq t_1\), the response is the same as Eq. (3.42).

\[ x_j(t) = U_0x_j^r(t) \quad \text{for } j = 1, 2. \quad (3.45) \]

For \(t_1 \leq t < t_2\), by superimposing Eqs. (3.42) and (3.43), the response becomes

\[ x_j(t) = U_0x_j^r(t) + \Delta Ux_j^s(t - t_1) \quad \text{for } j = 1, 2. \quad (3.46) \]

For \(t \geq t_2\), by superimposing Eqs. (3.42), (3.43), and (3.44) the response becomes

\[ x_j(t) = U_0x_j^r(t) + \Delta Ux_j^s(t - t_1) + (U_0 - \Delta U)x_j^s(t - t_2) \quad \text{for } j = 1, 2. \quad (3.47) \]

Even though the developed modal analysis model is performed in the case of \(n=10\), a similar modal model is developed for the other case with \(n = 2\). For this case, Tables 3.2 and 3.3 show the eigenvalues \(\alpha_r\) and eigenvectors \(\Phi^{(r)}\), respectively. Based on Tables 3.2 and 3.3, the system response \(x_j(t)\) for each time interval is obtained similar to the case of \(n = 10\). In both cases, the servo natural frequency \(f_1\) of 10 Hz is used. On the other hand, the mechanical natural frequency \(f_2\) is changed in order to get the specified frequency ratios of \(n = 2\) and 10. These two values of \(n\) are chosen to investigate the effects of \(n\) on the control algorithm. When \(n\) is larger than 10, the results are nearly ideal, while \(n\) near 2 gives a marginal response.

Figure 3.1 shows both the base input response ratio of \(\frac{X_1}{U_0}\) and the end point response ratio of \(\frac{X_2}{U_0}\) when the frequency ratio is \(n = 10.0\). The corresponding values
Table 3.2: The eigenvalues $\alpha_r$ for $n = 2$

<table>
<thead>
<tr>
<th>$\alpha_r$</th>
<th>eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>$-42.89966 - 45.04960i$</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$-42.89966 - 45.04960i$</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>$-2.652969 + 29.56207i$</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>$-2.652969 - 29.56207i$</td>
</tr>
</tbody>
</table>

Table 3.3: The modal vectors $\Phi^r$ for $n = 2$

<table>
<thead>
<tr>
<th>$\Phi^r$</th>
<th>modal vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi^1$</td>
<td>$-0.8587 + 0.6464i$</td>
</tr>
<tr>
<td></td>
<td>$-0.1498 - 0.1942i$</td>
</tr>
<tr>
<td></td>
<td>$0.0170 + 0.0028i$</td>
</tr>
<tr>
<td></td>
<td>$-0.0006 + 0.0039i$</td>
</tr>
<tr>
<td>$\Phi^2$</td>
<td>$-0.8587 - 0.6464i$</td>
</tr>
<tr>
<td></td>
<td>$-0.1498 + 0.1942i$</td>
</tr>
<tr>
<td></td>
<td>$0.0170 - 0.0028i$</td>
</tr>
<tr>
<td></td>
<td>$-0.0006 - 0.0039i$</td>
</tr>
<tr>
<td>$\Phi^3$</td>
<td>$-0.1068 + 0.5737i$</td>
</tr>
<tr>
<td></td>
<td>$-3.1915 + 3.1077i$</td>
</tr>
<tr>
<td></td>
<td>$0.0196 + 0.0019i$</td>
</tr>
<tr>
<td></td>
<td>$0.1139 + 0.0977i$</td>
</tr>
<tr>
<td>$\Phi^4$</td>
<td>$-0.1068 - 0.5737i$</td>
</tr>
<tr>
<td></td>
<td>$-3.1915 - 3.1077i$</td>
</tr>
<tr>
<td></td>
<td>$0.0196 - 0.0019i$</td>
</tr>
<tr>
<td></td>
<td>$0.1139 - 0.0977i$</td>
</tr>
</tbody>
</table>
of \( \Delta U \) and \( t_2 \) from Table 2.2 are \( \Delta U = 0.3505793U_0 \) and \( t_2 = 0.55953T_2 = 0.55953 \) sec. since \( T_2 = 1.0 \) second. The value of \( t_1 = \frac{2}{3}T_2 = 0.375 \) seconds. The servo is seen to drive the input motion of \( (x_1/U_0) \) to be quite step like in tracking the input motion. \( x_1/U_0 \) becomes equal to 2.0 shortly after \( t = t_2 \). The end point response ratio of \( (x_2/U_0) \) is seen to follow along smoothly and reaches a constant value of unity at \( t = t_2 \). This is precisely the desired motion.

Figure 3.2 shows that both the base input response ratio of \( (x_1/U_0) \) and the end point response ratio of \( (x_2/U_0) \) when the frequency ratio \( n = 2.0 \). The corresponding values from Table 2.2 are \( \Delta U = 0.3505793U_0 \) and \( t_2 = 0.55953T_2 = 0.111906 \) seconds \( (T_2 = 0.2 \) seconds in this case). The value of \( t_1 = \frac{3}{3}T_2 = 0.075 \) seconds for this case. Note that the value of \( \Delta U \) is the same for both values of \( n \) since \( \Delta U \) is dependent only on damping ratio \( \zeta_2 \). It is evident in Figure 3.2a that the input motion of \( (x_1/U_0) \) does not track the three-step input shape since there is insufficient time for the servo system to complete its task; that is, events are happening 5 times faster than in Figure 3.1. This smearing out of the input steps causes motion of \( (x_2/U_0) \) to be completely off of the mark as shown in Figure 3.2b where an 6.9 \% overshoot is seen to occur. Consequently, the fast time of response is nearly achieved but an unacceptable residual vibration is seen to occur. Thus, the poor base response of \( (x_1/U_0) \) due to insufficient time for the servo to react causes the input motion to be non step like and the entire scheme begins to break down.

These results support the assumptions used for developing the parameter estimation model. As mentioned earlier, the base response \( x_1 \) is assumed to be nearly a step input to the mechanical system, by selecting the mechanical system natural frequency in the range between 0 to \( \omega_p \). Thus, end point response \( x_2 \) is very close
to that which occurs for a true step input. From the above reasoning, the parameter estimation model assumes that the damped mechanical system is excited by the true step input function. This assumption means that the servo system frequency needs to be larger than two times the mechanical system frequency as shown in Figures 3.1 and 3.2. This result agrees with the servo frequency response plot shown in Figure 2.6, where the maximum value of the frequency ratio $n$ is approximately 3.3.

For a large value of the frequency ratio $n$ such as 10, the three-step input method nearly eliminates residual vibration at the defined end point. On the other hand, for a small value of frequency ratio $n$ such as 2, residual vibration still remains when the manipulator end point response $x_2$ reaches the defined end point. This residual vibration needs an additional time delay to complete the manipulator's task. In the following experimental analog simulation, many cases are performed to show the validity of the three-step input method and the limitation of the value of $n$ for real time control.
Figure 3.1: System responses calculated from modal model with frequency ratio of $n=10$. a) Base response ratio $\frac{x_1}{t_0}$. b) End point response ratio $\frac{x_2}{t_0}$.
Figure 3.2: System responses calculated from modal model with frequency ratio of $n=2$. a) Base response ratio $\frac{x_1}{t_0}$, b) end point response ratio $\frac{x_2}{t_0}$. 
CHAPTER 4. ANALOG SIMULATION

The analog computer performs mathematical operations on a continuous basis while digital computers perform discrete mathematical operations. The Pace TR-10 analog computer is used for this analog simulation. The simulated variables are represented by continuously varying voltages. The electronic analog computer is used to build an electrical model of a physical system in which voltages behave with time in a way similar to the variables of interest in the actual system. It can be said that the actual system and the electrical model are analogous in that the variables which demonstrate their characteristics are described by relations which are mathematically equivalent. Thus, the physical system is "simulated" because of the similarity of the operation of the electrical model and the physical system. These capabilities of analog computers are of great value in performing scientific research or engineering design calculations in that they give an insight into the relationship between the mathematical equations and the response of the physical system. As a simulator, the analog computer performs as an equation solver, since it performs mathematical operations which result in solutions of the equations used to represent the actual system.

Although the analog computer utilizes electronic components and electrical circuit characteristics in its operation, it is not essential that its users have an extensive
knowledge of electrical circuits. The task of preparing an analog computer with the correct electrical model is simple, and the steps necessary for accomplishing this task with the motion equations shown in Eqs. (2.7) and (2.8), which represent the simplest flexible manipulator control system, are described in this chapter.

Once the electrical model is completed, experiments can be performed quickly and with great flexibility to predict the behavior of the physical system under many different conditions. The analog computer is basically a set of building blocks that are easily interconnected where each block is able to perform specific mathematical operations. By constructing an appropriately interconnected group of building blocks, an electrical model is produced in which the voltages at the outputs of the blocks obey the relations given in the mathematical description of a physical system. By applying the appropriate initial conditions and forcing functions to the electrical model, its behavior is determined to be the same as the physical system's behavior. For this simulation, output voltages, which represent a physical system behavior, are acquired using an Analog to Digital (A/D) converter. The A/D converter and the Digital to Analog (D/A) converter are used to control the physical system in a PC type of digital computer.

4.1. Description of the Experimental Apparatus

The following paragraphs describe the experimental apparatus used in the analog simulation.
4.1.1. Analog computer

The Pace TR-10 analog computer shown in Figure 4.1 is used in this study. It produces output voltages of an electrical model used for simulating the dynamic system. This analog computer output is limited to ± 10 volts. The front face of the computer is divided into three five-inch high rows of computing components and their corresponding interconnecting terminations. This will be referred to as the "patch panel." In the top row, there are attenuators for multiplying voltages by positive constants less than unity. Each attenuator consists of a ten-turn 5KΩ carbon potentiometer with an uncalibrated knob. In the bottom row, there are high gain operational amplifiers (Op-Amps) that are used to perform many tasks by connecting appropriate electrical components such as resistances and capacitors. In the middle row, there are capacitive integrator networks for use with the Op-Amps. The built-in integrator network capacitor has a fixed value of 10 μF.

4.1.2. Data acquisition hardware

A Data Translation model DT2801-A analog digital I/O board is used to measure displacement signals corresponding to the base response \( x_1 \) and end point response \( x_2 \). This board contains a 12-bit A/D converter that is used to convert the output voltages of the analog computer into digital values. The board is operated in two different modes that are called single data point mode and block of data mode. In the single data point mode, the board gathers a single voltage when the board is commanded to run. In the block data mode, the board gathers multiple voltages sampled at equal time interval when the board is commanded to run. For this study, the single data point mode is used to acquire the output voltage measurements. The
Figure 4.1: TR-10 analog computer
single data point mode is used a multiple number of times for monitoring when the response reached the half way point of the defined end point.

A 12-bit D/A converter is also available on this board. It is used to output the command input function $x_{in}(t)$ to the analog computer according to the control law.

Since DT2801-A has only a single A/D converter board, another A/D converter is required to obtain the overall system responses. A Digital Equipment Corporation's Professional 300 Series Analog Data Module (ADM) is used to monitor the response voltages. It converts the analog signals representing the dynamic behavior of the system into 16-bit digital values at a user specified sample frequency.

4.1.3. Digital computer

The computer used is the IBM-XT. Its task is to carry out the control algorithm called three-step input method. This involves controlling the A/D conversion, performing real-time computation, and controlling the D/A conversion. To increase control process speed, the program was written in Quick Basic rather than Basic.

The Digital Equipment Corporation's Professional 380 computer (Pro 380) is used to operate ADM, which is used for acquiring the data of the physical system's overall performance.

4.2. Development of the Analog Simulation Model

As discussed earlier, the analog computer is used to acquire the behavior of the dynamic control system involving the one degree of freedom flexible manipulator. This manipulator is modeled as the damped spring-mass system shown in Figure 2.1. The mathematical model on the damped spring-mass system needs to construct
the corresponding electrical model. The following paragraphs explain the general procedures on the analog simulation model.

Rewriting Eqs. (2.7) and (2.8) developed earlier, we have the following form.

\[ \ddot{x}_1 + a_1 \dot{x}_1 + a_2 x_1 - a_3 \dot{x}_2 - a_4 x_2 = a_5 x_{in} \]  

\[ \ddot{x}_2 + b_1 \dot{x}_2 + b_2 x_2 - b_1 \dot{x}_1 - b_2 x_1 = 0 \]

where

\[ a_1 = \frac{(c + K_2)}{m_1} \]

\[ a_2 = \frac{(k + G K_1)}{m_1} \]

\[ a_2 = a_4 + a_5 \]

\[ a_3 = \frac{c}{m_1} \]

\[ a_4 = \frac{k}{m_1} \]

\[ a_5 = \frac{G K_1}{m_1} \]

\[ b_1 = \frac{c}{m_2} \]

\[ b_2 = \frac{k}{m_2} \]

\( c \) represents damping, and \( k \) represents the values of stiffness.

The following paragraphs show the procedures for calculating the above constant values. The constant values in Eqs. (4.1) and (4.2) are obtained in terms of the assumed known values such as \( f_1, f_2, \zeta_1, \zeta_2, m_1, \) and \( m_2 \). First, using the relation

\[ \omega_1 = 2\pi f_1 \]

\[ = \sqrt{a_2} \]  

(4.3)
the constant value \( a_2 \) is obtained in terms of \( f_1 \),

\[
a_2 = (2\pi f_1)^2 \tag{4.4}
\]

where \( \omega_1 \) represents the circular natural frequency of the servo system. Second, from the relation

\[
\zeta_1 = \frac{a_1}{2\omega_1} \tag{4.5}
\]

the constant value \( a_1 \) is obtained as

\[
a_1 = 2\omega_1 \zeta_1 \tag{4.6}
\]

where \( \zeta_1 \) represents the damping ratio of the servo system. Third, using the relation

\[
\omega_2 = 2\pi f_2 = \sqrt{\frac{b_2}{2}} \tag{4.7}
\]

the constant value \( b_2 \) is

\[
b_2 = (2\pi f_2)^2 \tag{4.8}
\]

where \( \omega_2 \) represents the circular natural frequency of the mechanical system. Fourth, from the relation

\[
\zeta_2 = \frac{b_1}{2\omega_2} \tag{4.9}
\]

the constant value \( b_1 \) is obtained as

\[
b_1 = 2\omega_2 \zeta_2 \tag{4.10}
\]

where \( \zeta_2 \) represents the damping ratio of the mechanical system. Fifth, from the definition of \( b_2 \)

\[
b_2 = \frac{k}{m_2} \tag{4.11}
\]
the constant spring stiffness $k$ is calculated as

$$k = m_2 b_2 \quad (4.12)$$

Sixth, using the definition

$$b_1 = \frac{c}{m_2} \quad (4.13)$$

the constant damping value $c$ is

$$c = m_2 b_1 \quad (4.14)$$

Seventh, the constant value $a_3$ is obtained from the definition

$$a_3 = \frac{c}{m_1} = \frac{2m_2 \omega_2 \zeta_2}{m_1} \quad (4.15)$$

Eighth, the constant value $a_4$ is also calculated from the definition

$$a_4 = \frac{k}{m_1} = \frac{m_2 (2\pi f_2)^2}{m_1} \quad (4.16)$$

Finally, the constant value $a_5$ is calculated from the definition

$$a_5 = a_2 - a_4 = (2\pi f_1)^2 - \frac{m_2 (2\pi f_2)^2}{m_1} \quad (4.17)$$

The constant values, $a_i$ and $b_j$, represent the system characteristics. As shown in the above procedures, the constant values, $a_i$ and $b_j$, are varied in order to obtain the desired natural frequencies and damping ratios. The values of $a_i$ and $b_j$ that are used in the experimental analog simulation are shown in Tables 4.1 through 4.4.
Table 4.1: Values $a_i$ and $b_j$ for various frequency ratios $n$ for 1% damping

<table>
<thead>
<tr>
<th>$a_i$ and $b_j$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
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<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$a_2$</td>
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<td>3947.610</td>
<td>3947.610</td>
<td>3947.610</td>
</tr>
<tr>
<td>$a_3$</td>
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<td>0.1885</td>
<td>0.1257</td>
<td>0.0628</td>
</tr>
<tr>
<td>$a_4$</td>
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<td>177.660</td>
<td>79.0</td>
<td>19.740</td>
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<tr>
<td>$a_5$</td>
<td>3454.128</td>
<td>3769.950</td>
<td>3868.60</td>
<td>3927.870</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.6283</td>
<td>0.3770</td>
<td>0.2514</td>
<td>0.1257</td>
</tr>
<tr>
<td>$b_2$</td>
<td>986.965</td>
<td>355.320</td>
<td>158.0</td>
<td>39.480</td>
</tr>
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Table 4.2: Values $a_i$ and $b_j$ for various frequency ratios $n$ for 5% damping

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>$a_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$a_2$</td>
<td>3947.610</td>
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</tr>
<tr>
<td>$a_3$</td>
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<td>0.9425</td>
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<td>$a_4$</td>
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<td>177.660</td>
<td>79.0</td>
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</tr>
<tr>
<td>$a_5$</td>
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<td>3769.950</td>
<td>3868.60</td>
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<td>0.2570</td>
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<tr>
<td>$b_2$</td>
<td>986.965</td>
<td>355.320</td>
<td>158.0</td>
<td>39.480</td>
</tr>
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Table 4.3: Values $a_i$ and $b_j$ for various frequency ratios $n$ for 10% damping

<table>
<thead>
<tr>
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<td>$a_1$</td>
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<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$a_2$</td>
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<td>$a_4$</td>
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<tr>
<td>$a_5$</td>
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</tr>
<tr>
<td>$b_2$</td>
<td>986.965</td>
<td>355.320</td>
<td>158.0</td>
<td>39.480</td>
</tr>
</tbody>
</table>
Table 4.4: Values $a_i$ and $b_j$ for various frequency ratios $n$ for 25% damping

<table>
<thead>
<tr>
<th>$a_i$ and $b_j$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$a_2$</td>
<td>3947.610</td>
<td>3947.610</td>
<td>3947.610</td>
<td>3947.610</td>
</tr>
<tr>
<td>$a_3$</td>
<td>7.8540</td>
<td>4.7125</td>
<td>3.1425</td>
<td>1.5708</td>
</tr>
<tr>
<td>$a_4$</td>
<td>493.483</td>
<td>177.660</td>
<td>79.0</td>
<td>19.740</td>
</tr>
<tr>
<td>$a_5$</td>
<td>3454.128</td>
<td>3769.950</td>
<td>3868.60</td>
<td>3927.870</td>
</tr>
<tr>
<td>$b_1$</td>
<td>15.7080</td>
<td>9.4250</td>
<td>6.2850</td>
<td>3.1415</td>
</tr>
<tr>
<td>$b_2$</td>
<td>986.965</td>
<td>355.320</td>
<td>158.0</td>
<td>39.480</td>
</tr>
</tbody>
</table>

With the substitution of these constant values into Eqs. (4.1) and (4.2), these two equations are rearranged so the highest-order derivatives of the dependent variables appear alone on the left side of the equations. This gives

$$
\ddot{x}_1 = -[a_1 \dot{x}_1 + a_2 x_1 - a_3 \dot{x}_2 - a_4 x_2 - a_5 x_{in}(t)]
$$

(4.18)

$$
\ddot{x}_2 = -(b_1 \dot{x}_2 + b_2 x_2 - b_1 \dot{x}_1 - b_2 x_1)
$$

(4.19)

This arrangement of the equations indicates that the acceleration $\ddot{x}_1$ can be obtained by summing the five quantities in the bracket on the right side of Eq. (4.18). Similarly, $\ddot{x}_2$ can be obtained.

Since it is not necessary to measure the accelerations $\ddot{x}_1$ and $\ddot{x}_2$, Eqs. (4.18) and (4.19) may be alternatively rearranged as

$$
\dot{x}_1 = -\int[a_1 \dot{x}_1 + a_2 x_1 - a_3 \dot{x}_2 - a_4 x_2 - a_5 x_{in}(t)] \, dt
$$

(4.20)

$$
\dot{x}_2 = -\int(b_1 \dot{x}_2 + b_2 x_2 - b_1 \dot{x}_1 - b_2 x_1) \, dt
$$

(4.21)

To avoid the Op-Amp output becoming so large that the Op-Amp overloads or the problem variable becomes so small that the Op-Amp noise is the predominant output signal, magnitude scaling is needed. To find the magnitude scaling factors of the
displacement variables, \( x_1 \) and \( x_2 \), and the velocities, \( \dot{x}_1 \) and \( \dot{x}_2 \), we need to make the rough estimate of the maximum displacements, \( x_{1\text{max}} \) and \( x_{2\text{max}} \), from a knowledge of the physical problem. If the maximum value of \( x_1 \) from Eq. (4.1) is assumed as the symbol \( \rho \), the maximum value of \( x_2 \) can be estimated from Eq. (4.2). Neglecting all terms with derivatives gives

\[
x_{2\text{max}} = x_{1\text{max}} = \rho
\]  

(4.22)

The maximum velocities \( \dot{x}_{1\text{max}} \) and \( \dot{x}_{2\text{max}} \) are calculated by the relation

\[
\dot{x}_{\text{max}} = \omega x_{\text{max}}
\]  

(4.23)

Hence,

\[
\dot{x}_{1\text{max}} = \omega_1 x_{1\text{max}} = \omega_1 \rho
\]  

(4.24)

and

\[
\dot{x}_{2\text{max}} = \omega_2 x_{2\text{max}} = \omega_2 \rho
\]  

(4.25)

Then, the magnitude scale factors are calculated from the following relation

\[
S_d = \frac{\epsilon_{\text{max}}}{x_{\text{max}}}
\]  

(4.26)

\[
S_v = \frac{\epsilon_{\text{max}}}{\dot{x}_{\text{max}}}
\]  

(4.27)

where \( S_d \) represents the displacement scale factor, \( S_v \) represents the velocity scale factor, and \( \epsilon_{\text{max}} \) is the maximum reference voltage of the analog computer, which
is ± 10 volts. From the equation (4.26), the displacement scale factors of $x_1$ and $x_2$ are

$$S_{d1} = \frac{\epsilon_{max}}{x_{1max}} = \frac{10}{\rho}$$  \hspace{1cm} (4.28)

$$S_{d2} = \frac{\epsilon_{max}}{x_{2max}} = \frac{10}{\rho}$$  \hspace{1cm} (4.29)

where $S_{d1}$ represents the displacement scale factor of the base, and $S_{d2}$ represents the displacement scale factor of the manipulator end point. Also, the velocity scale factors of $\dot{x}_1$ and $\dot{x}_2$ are calculated from Eq. (4.27).

$$S_{v1} = \frac{\epsilon_{max}}{\dot{x}_{1max}} = \frac{10}{\omega_1 \rho}$$  \hspace{1cm} (4.30)

$$S_{v2} = \frac{\epsilon_{max}}{\dot{x}_{2max}} = \frac{10}{\omega_2 \rho}$$  \hspace{1cm} (4.31)

where $S_{v1}$ represents the velocity scale factor of the base, and $S_{v2}$ represents the velocity scale factor of the manipulator end point. Let Eqs. (4.20) and (4.21) be multiplied by $S_{v1}$ and $S_{v2}$ given from Eqs. (4.30) and (4.31), respectively.

$$S_{v1} \dot{x}_1 = -\int \left[ S_{v1} a_1 \dot{x}_1 + S_{v1} a_2 x_1 - S_{v1} a_3 \dot{x}_2 - S_{v1} a_4 x_2 \\ - S_{v1} a_5 x_{in}(t) \right] dt$$  \hspace{1cm} (4.32)

$$S_{v2} \dot{x}_2 = -\int (S_{v2} b_1 \dot{x}_2 + S_{v2} b_2 x_2 - S_{v2} b_1 \dot{x}_1 - S_{v2} b_2 x_1) dt$$  \hspace{1cm} (4.33)
Then, modify the lefthand side of Eqs. (4.32) and (4.33), using the appropriate scale factors to produce the new variables that have a voltage unit.

\[
S_{v1} \dot{x}_1 = -\int \left[ a_1(S_{v1} \dot{x}_1) + \frac{S_{v1} a_2}{S_{d1}}(S_{d1} x_1) - \frac{S_{v1} a_3}{S_{v2}}(S_{v2} \dot{x}_2) \right. \\
- \frac{S_{v1} a_4}{S_{d2}}(S_{d2} x_2) - \frac{S_{v1} a_5}{S_{d1}}(S_{d1} x_{in}(t)) \bigg] \, dt 
\]

(4.34)

\[
S_{v2} \dot{x}_2 = -\int \left[ b_1(S_{v2} \dot{x}_2) + \frac{S_{v2} b_2}{S_{d2}}(S_{d2} x_2) - \frac{S_{v2} b_1}{S_{v1}}(S_{v1} \dot{x}_1) \right. \\
- \frac{S_{v2} b_3}{S_{d1}}(S_{d1} x_1) \bigg] \, dt 
\]

(4.35)

where \( S_{v1} \dot{x}_i \), \( S_{d1} x_i \) and \( S_{d1} x_{in}(t) \) are new variables expressed in volt units. With the defined new constant values \( D_i \), rewrite Eqs. (4.34) and (4.35) for simplicity.

\[
S_{v1} \dot{x}_1 = -\int \left[ D_1(S_{v1} \dot{x}_1) + D_2(S_{d1} x_1) - D_3(S_{v2} \dot{x}_2) - D_4(S_{d2} x_2) \right. \\
- D_5(S_{d1} x_{in}(t)) \bigg] \, dt 
\]

(4.36)

\[
S_{v2} \dot{x}_2 = -\int \left[ D_6(S_{v2} \dot{x}_2) + D_7(S_{d2} x_2) - D_8(S_{v1} \dot{x}_1) \right. \\
- D_9(S_{d1} x_1) \bigg] \, dt 
\]

(4.37)

where

\[
D_1 = a_1 \\
= 2\omega_1 c_1 \\
D_2 = \frac{S_{v1} a_2}{S_{d1}} \\
= 2\pi f_1 \\
D_3 = \frac{S_{v1} a_3}{S_{v2}}
\]
\[
D_4 = \frac{S_{v1}a_4}{S_{d2}} = \frac{2\pi f_2}{f_1}
\]
\[
D_5 = \frac{S_{v1}a_5}{S_{d1}} = \frac{1}{\omega_1} \left( \omega_1^2 - \frac{m_2\omega_2^2}{m_1} \right)
\]
\[
D_6 = b_1 = 2\omega_2\zeta_2
\]
\[
D_7 = \frac{S_{v2}b_2}{S_{d2}} = 2\pi f_2
\]
\[
D_8 = \frac{S_{v2}b_1}{S_{v1}} = 2\omega_1\zeta_2
\]
\[
D_9 = \frac{S_{v2}b_2}{S_{d1}} = 2\pi f_2
\]

and note that the constant values \( D_i \) in Eqs. (4.36) and (4.37) are expressed in terms of the known properties of the physical system such as \( m_1, m_2, \zeta_1, \zeta_2, f_1, \) and \( f_2 \).

The constant values \( D_i \) are shown in Tables 4.5 through 4.8.

The analog computer circuit for solving Eqs. (4.36) and (4.37) is shown in Figure 4.2. In this figure, the triangle symbol (\( \vartriangleleft \)) with the number inside represents an operational amplifier, points with the same numbers in rectangular symbols (\( \Box \)) are connected to one another, potentiometers are shown as circles (\( \bigcirc \)) with an \( a_i^* \) next
Table 4.5: Values of $D_i$ for the analog simulation model according to the frequency ratio $n$ for damping ratio 1 %

<table>
<thead>
<tr>
<th>$D_i$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$D_2$</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
</tr>
<tr>
<td>$D_3$</td>
<td>0.152013</td>
<td>0.053349</td>
<td>0.023867</td>
<td>0.005927</td>
</tr>
<tr>
<td>$D_4$</td>
<td>7.4020</td>
<td>2.6649</td>
<td>1.1850</td>
<td>0.2961</td>
</tr>
<tr>
<td>$D_5$</td>
<td>51.812</td>
<td>56.549</td>
<td>58.029</td>
<td>58.918</td>
</tr>
<tr>
<td>$D_6$</td>
<td>0.62832</td>
<td>0.3770</td>
<td>0.2514</td>
<td>0.12566</td>
</tr>
<tr>
<td>$D_7$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
<tr>
<td>$D_8$</td>
<td>1.298528</td>
<td>1.332067</td>
<td>1.32404</td>
<td>1.3320</td>
</tr>
<tr>
<td>$D_9$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
</tbody>
</table>

Table 4.6: Values of $D_i$ for the analog simulation model according to the frequency ratio $n$ for damping ratio 5 %

<table>
<thead>
<tr>
<th>$D_i$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$D_2$</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
</tr>
<tr>
<td>$D_3$</td>
<td>0.760</td>
<td>0.2667</td>
<td>0.1190</td>
<td>0.0296</td>
</tr>
<tr>
<td>$D_4$</td>
<td>7.4020</td>
<td>2.6649</td>
<td>1.1850</td>
<td>0.2961</td>
</tr>
<tr>
<td>$D_5$</td>
<td>51.812</td>
<td>56.549</td>
<td>58.029</td>
<td>58.918</td>
</tr>
<tr>
<td>$D_6$</td>
<td>3.1416</td>
<td>1.8850</td>
<td>1.2570</td>
<td>0.6283</td>
</tr>
<tr>
<td>$D_7$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
<tr>
<td>$D_9$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
</tbody>
</table>
Table 4.7: Values of $D_i$ for the analog simulation model according to the frequency ratio $n$ for damping ratio 10 %

<table>
<thead>
<tr>
<th>$D_i$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$D_2$</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
</tr>
<tr>
<td>$D_3$</td>
<td>1.520129</td>
<td>0.53349</td>
<td>0.23867</td>
<td>0.059274</td>
</tr>
<tr>
<td>$D_4$</td>
<td>7.4020</td>
<td>2.6649</td>
<td>1.1850</td>
<td>0.2961</td>
</tr>
<tr>
<td>$D_5$</td>
<td>51.812</td>
<td>56.549</td>
<td>58.029</td>
<td>58.918</td>
</tr>
<tr>
<td>$D_6$</td>
<td>6.2832</td>
<td>3.770</td>
<td>2.5140</td>
<td>1.2566</td>
</tr>
<tr>
<td>$D_7$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
<tr>
<td>$D_8$</td>
<td>12.98528</td>
<td>13.32067</td>
<td>13.2404</td>
<td>13.320</td>
</tr>
<tr>
<td>$D_9$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
</tbody>
</table>

Table 4.8: Values of $D_i$ for the analog simulation model according to the frequency ratio $n$ for damping ratio 25 %

<table>
<thead>
<tr>
<th>$D_i$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
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</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
<td>87.962</td>
</tr>
<tr>
<td>$D_2$</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
<td>59.214</td>
</tr>
<tr>
<td>$D_3$</td>
<td>3.800323</td>
<td>1.333726</td>
<td>0.596677</td>
<td>0.148184</td>
</tr>
<tr>
<td>$D_4$</td>
<td>7.4020</td>
<td>2.6649</td>
<td>1.1850</td>
<td>0.2961</td>
</tr>
<tr>
<td>$D_5$</td>
<td>51.812</td>
<td>56.549</td>
<td>58.029</td>
<td>58.918</td>
</tr>
<tr>
<td>$D_7$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
<tr>
<td>$D_8$</td>
<td>33.4632</td>
<td>33.30167</td>
<td>33.1010</td>
<td>33.2999</td>
</tr>
<tr>
<td>$D_9$</td>
<td>30.5960</td>
<td>18.832</td>
<td>12.482</td>
<td>6.277</td>
</tr>
</tbody>
</table>
to them, and resistances and capacitors are represented by \( R_i \) and \( C \), respectively.

For the experimental analog simulation, the resistances, \( R_i \), and potentiometer coefficients, \( a_i^* \), need to be calculated. The potentiometer coefficients, \( a_i^* \), represent the portion of the input signal that is used by the remaining portion of the circuit and range from zero to unity. It is recommended that \( a_i^* \) be greater than 0.1 to reduce the error of the potentiometer.

Since the value of the capacitor \( C \) is fixed at 10 \( \mu \text{F} \), using the relation for the integrator of the analog computer.

\[
D_i = \frac{a_i^*}{R_i C} \tag{4.38}
\]

the values of the resistors \( R_i \) are calculated as

\[
R_i = \frac{a_i^*}{D_i C} \tag{4.39}
\]

As shown in Figure 4.2, eight potentiometers are used. Note that in this circuit, four Op-Amps numbered as 3, 4, 7 and 8 are used for the sign inversion with unity gain, and their output voltages are connected to four potentiometers numbered as \( a_2^*, a_3^*, a_5^* \) and \( a_6^* \), respectively. With the appropriate manipulation of this circuit, four potentiometers \( (a_2^*, a_3^*, a_5^*, \text{ and } a_6^*) \) are eliminated as shown in Figure 4.3. In Figure 4.3 those Op-Amps generate the new output voltages \( e_{out}^* \) connected to the integrator directly, these voltages are calculated from

\[
e_{out}^* = a^*e_{out} \tag{4.40}
\]

where \( a^* \) represents the potentiometer coefficients eliminated, and \( e_{out} \) represents the output voltages of the unity gain Op-Amps before the potentiometers are eliminated. Thus, using the general relation for the analog computer Op-Amp gain (\( G \))
Figure 4.2: The original electric circuit for the analog simulation
Figure 4.3: The simplified electric circuit for the analog simulation
\[ G = \frac{e_o}{e_i} \]
\[ = \frac{R_f}{R_{in}} \]  \hspace{1cm} (4.41)

where \( R_{in} \) and \( R_f \) represent the input and feedback resistors of the analog computer Op-Amp, respectively, and \( e_i \) and \( e_o \) represent the input and output voltages of Op-Amp, respectively. Thus, in Figure 4.3, the input and feedback resistors of four Op-Amps connected to the integrators directly are changed as

\[ G^* = \frac{a_i^* S_{V_i} x_i (or a_i^* S_{d_i} x_i)}{S_{V_i} x_i (or S_{d_i} x_i)} \]
\[ = a_i^* \]  \hspace{1cm} (4.42)

In Figure 4.3, the numerical values of the circuit components involving the resistors, \( R_i \), and the coefficients, \( a_i^* \), in several cases are listed in Tables 4.9 through 4.12.

For real time control, the general procedure for the experimental analog simulation is shown in Figure 4.4. Figure 4.4 shows that the analog simulation is performed through the computer program, and this program controls the IBM computer and the DT2801-A I/O board, and the Pro 380 computer and ADM are used to measure the overall system performance signals, and the Tectronix 2201 Digital Storage Oscilloscope is used for visual observation of the overall system performance.

### 4.3. Time Calibration of the Control Computer for Real Time Control

The control scheme in the three-step input method is operated through the computer program written in Quick Basic as shown in Figure 4.4. This program controls the IBM-XT computer and the DT2801-A I/O board through the IBM-XT computer I/O registers. Before analog simulation is performed with the circuit shown
Table 4.9: For 1% clamping ratio, the values of resistors and potentiometer's gain, $S_{d_i}$ and $S_{v_i}$ in the electrical circuit shown in Figure 4.3

<table>
<thead>
<tr>
<th>$R_i$ and $a_j^*$</th>
<th>$n = 2$</th>
<th>$n = 3.3$</th>
<th>$n = 5$</th>
<th>$n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
<td>1.9301 KΩ</td>
<td>1.7684 KΩ</td>
<td>1.723 KΩ</td>
<td>1.697 KΩ</td>
</tr>
<tr>
<td>$R_1$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_2$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_3$</td>
<td>100.0 KΩ</td>
<td>1.0 MΩ</td>
<td>1.0 MΩ</td>
<td>10.0 MΩ</td>
</tr>
<tr>
<td>$R_4$</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_5$</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_6$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
<td>$R_7$</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>1.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
<td>$R_8$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
<td>$R_9$</td>
<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
</tr>
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<td>5.3 KΩ</td>
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<td>15.9 KΩ</td>
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</tr>
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<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
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</tr>
<tr>
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</tr>
<tr>
<td>$R_{14}$</td>
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<td>18.832 KΩ</td>
<td>12.482 KΩ</td>
<td>62.77 KΩ</td>
</tr>
<tr>
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</tr>
<tr>
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<td>100.0 KΩ</td>
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<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{18}$</td>
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<td>53.349 KΩ</td>
<td>23.867 KΩ</td>
<td>59.274 KΩ</td>
</tr>
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<td>0.87962</td>
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<td>0.87962</td>
</tr>
<tr>
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<td>0.1185</td>
<td>0.2961</td>
</tr>
<tr>
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<td>0.3770</td>
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<tr>
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<td>0.12482</td>
<td>0.6277</td>
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Table 4.10: For 5% damping ratio, the values of resistors and potentiometer’s gain, \( S_{d_1} \) and \( S_{v_1} \) in the electrical circuit shown in Figure 4.3

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<tr>
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<th>( n = 2 )</th>
<th>( n = 3.3 )</th>
<th>( n = 5 )</th>
<th>( n = 10 )</th>
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</thead>
<tbody>
<tr>
<td>( R_0 )</td>
<td>1.9301 K( \Omega )</td>
<td>1.7684 K( \Omega )</td>
<td>1.723 K( \Omega )</td>
<td>1.697 K( \Omega )</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>1000.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_5 )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_6 )</td>
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<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_7 )</td>
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<td>10.0 K( \Omega )</td>
<td>10.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_8 )</td>
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<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
<td>1.0 K( \Omega )</td>
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<tr>
<td>( R_9 )</td>
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<td>1.5 K( \Omega )</td>
<td>1.5 K( \Omega )</td>
<td>1.5 K( \Omega )</td>
</tr>
<tr>
<td>( R_{10} )</td>
<td>3.1 K( \Omega )</td>
<td>5.3 K( \Omega )</td>
<td>7.9 K( \Omega )</td>
<td>15.9 K( \Omega )</td>
</tr>
<tr>
<td>( R_{11} )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_{12} )</td>
<td>59.214 K( \Omega )</td>
<td>59.214 K( \Omega )</td>
<td>59.214 K( \Omega )</td>
<td>59.214 K( \Omega )</td>
</tr>
<tr>
<td>( R_{13} )</td>
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<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_{14} )</td>
<td>30.596 K( \Omega )</td>
<td>18.832 K( \Omega )</td>
<td>12.482 K( \Omega )</td>
<td>62.77 K( \Omega )</td>
</tr>
<tr>
<td>( R_{15} )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
</tr>
<tr>
<td>( R_{16} )</td>
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<td>66.2 K( \Omega )</td>
<td>66.1 K( \Omega )</td>
</tr>
<tr>
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<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
<td>100.0 K( \Omega )</td>
</tr>
<tr>
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<td>26.67 K( \Omega )</td>
<td>11.9 K( \Omega )</td>
<td>29.597 K( \Omega )</td>
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<tr>
<td>( a_1^* )</td>
<td>0.87962</td>
<td>0.87962</td>
<td>0.87962</td>
<td>0.87962</td>
</tr>
<tr>
<td>( a_4^* )</td>
<td>0.7402</td>
<td>0.26649</td>
<td>0.1185</td>
<td>0.2961</td>
</tr>
<tr>
<td>( a_5^* )</td>
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<tr>
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<td>0.18832</td>
<td>0.12482</td>
<td>0.6277</td>
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<tr>
<td>( S_{d_1} )</td>
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<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
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<td>100.0</td>
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<tr>
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<tr>
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Table 4.11: For 10 % damping ratio, the values of resistors and potentiometer’s gain, $S_{d_i}$ and $S_{v_i}$ in the electrical circuit shown in Figure 4.3

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<th>$n = 2$</th>
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<th>$n = 5$</th>
<th>$n = 10$</th>
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<tbody>
<tr>
<td>$R_0$</td>
<td>1.9301 KΩ</td>
<td>1.7684 KΩ</td>
<td>1.723 KΩ</td>
<td>1.697 KΩ</td>
</tr>
<tr>
<td>$R_1$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_2$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_3$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>1000.0 KΩ</td>
</tr>
<tr>
<td>$R_4$</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_5$</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
<td>$R_6$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
<td>$R_7$</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_8$</td>
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<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
</tr>
<tr>
<td>$R_9$</td>
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<td>5.3 KΩ</td>
<td>7.9 KΩ</td>
<td>15.9 KΩ</td>
</tr>
<tr>
<td>$R_{10}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{11}$</td>
<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
</tr>
<tr>
<td>$R_{12}$</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{14}$</td>
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<td>18.832 KΩ</td>
<td>12.482 KΩ</td>
<td>62.77 KΩ</td>
</tr>
<tr>
<td>$R_{15}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{17}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{18}$</td>
<td>15.201 KΩ</td>
<td>53.349 KΩ</td>
<td>23.867 KΩ</td>
<td>59.274 KΩ</td>
</tr>
<tr>
<td>$a_{1}^*$</td>
<td>0.87962</td>
<td>0.87962</td>
<td>0.87962</td>
<td>0.87962</td>
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<tr>
<td>$a_{4}^*$</td>
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<td>0.26649</td>
<td>0.1185</td>
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<td>$a_{5}^*$</td>
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<td>0.3770</td>
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<tr>
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<td>0.18832</td>
<td>0.12482</td>
<td>0.6277</td>
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<tr>
<td>$S_{d1}$</td>
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<tr>
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Table 4.12: For 25% damping ratio, the values of resistors and potentiometer's gain, $S_{d_1}$ and $S_{v_1}$ in the electrical circuit shown in Figure 4.3.

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<td>1.723 KΩ</td>
<td>1.697 KΩ</td>
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<tr>
<td>$R_1$</td>
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<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
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<td>1.0 KΩ</td>
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<tr>
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<td>10.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
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<td>10.0 KΩ</td>
<td>10.0 KΩ</td>
</tr>
<tr>
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<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_7$</td>
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<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_8$</td>
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<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
<td>1.0 KΩ</td>
</tr>
<tr>
<td>$R_9$</td>
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<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
<td>1.5 KΩ</td>
</tr>
<tr>
<td>$R_{10}$</td>
<td>3.1 KΩ</td>
<td>5.3 KΩ</td>
<td>7.9 KΩ</td>
<td>15.9 KΩ</td>
</tr>
<tr>
<td>$R_{11}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
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<tr>
<td>$R_{12}$</td>
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<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
<td>59.214 KΩ</td>
</tr>
<tr>
<td>$R_{13}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
<td>$R_{14}$</td>
<td>30.596 KΩ</td>
<td>18.832 KΩ</td>
<td>12.482 KΩ</td>
<td>62.77 KΩ</td>
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<tr>
<td>$R_{15}$</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
<tr>
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<td>33.101 KΩ</td>
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<td>100.0 KΩ</td>
<td>100.0 KΩ</td>
</tr>
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<td>0.87962</td>
<td>0.87962</td>
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<tr>
<td>$a_{14}$</td>
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<td>0.1185</td>
<td>0.2961</td>
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<tr>
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<td>0.18832</td>
<td>0.12482</td>
<td>0.6277</td>
</tr>
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<td>3.1</td>
<td>5.3</td>
<td>7.9</td>
<td>15.9</td>
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</table>
Figure 4.4: For real time control, the general procedure of the experimental analog simulation
in Figure 4.3, two different time calibrations of the IBM-XT control computer and the DT2801-A I/O board are required in order to achieve real time control.

The schematic diagram for these time calibrations is shown in Figure 4.5, where a D/A converter generates output voltages corresponding to the D/A command values, and these output voltages are sent to the Tectronix 2201 Digital Storage Oscilloscope through a TR-10 analog computer that uses the Op-Amp with unity gain. This digital oscilloscope is triggered by output voltages from the analog computer and used to record the time delay between points A and B. Note that a TR-10 analog computer is used to get the time calibrations in the same environment that the experimental analog simulation is performed. In this section, two time calibration procedures are described.

The first calibration procedure is performed to assure proper measurement of time \( t_0 \), the time required for the system response to reach the halfway point of the defined end point. In this thesis, the manipulator end point response is used as the variable for estimating \( t_0 \). The question is how to measure time, \( t_0 \), correctly with the equipment available. In the method used, time \( t_0 \) is obtained by using the A/D converter in the single data point mode a multiple number of times until the end point response, \( x_2 \), has reached or passed its halfway point. In this method, the single A/D operation is defined as a process starting with the computer commands for an single A/D conversion to the implementation of a second command that uses a converted voltage value for comparison with the halfway point of the end point response \( x_2 \). Thus, if a total number of \( N \) single A/D operations have been made when the end point response, \( x_2 \), reaches the halfway point at time \( t_0 \), then

\[
t_0 = Nt_{w1}
\]  

(4.43)
where \( t_{w1} \) represents time delay for a single A/D operation. Figure 4.5a shows the experimental diagram for \( t_{w1} \) calibration. Note that in \( t_{w1} \) calibration, the D/A command is used twice. The DT2801-A I/O board manual indicates that a D/A command takes a time delay of 50 \( \mu \)sec. to generate output voltages. A value of 50 \( \mu \)sec. is very small when compared with \( t_{w1} \) and thus is neglected in \( t_{w1} \) calibration.

To calibrate time delay \( t_{w1} \), the following general procedures are used:

1. Set the digital oscilloscope properly for the single sweep display.

2. Command a step input \( u_1 \) through a D/A converter to the digital oscilloscope.

3. Execute a computer program that simulates the iterative single A/D operation process. For example,

   ```
   for \( A = 1 \) to \( NO \)
   gosub 5000
   next \( A \)
   ```

   where subroutine 5000 contains the program for a single A/D operation that includes a single A/D conversion and comparison with an arbitrary constant, and \( NO \) represents the total number of the single A/D operation.

4. Command a step input, \( u_2 \), through the D/A converter.

5. Measure the step input (\( u_1 \)) time delay \( t_{w1}^* \) between points A and B from the digital oscilloscope screen. For convenience of the time measurement, print the step inputs displayed on the digital oscilloscope screen.

6. Calculate the time delay, \( t_{w1} \), using the formula, \( t_{w1} = \frac{t_{w1}^*}{NO} \).
For this time calibration, the above procedures are repeated multiple times to reduce the time measurement error from the digital oscilloscope and the control computer resolution error using different values of \( NO \) with 7 increments of 10, ranging from 40 to 100. Thus, \( t_{w1} \) is obtained by taking an average value on the delay time, \( t_{w1} \), obtained each time. The values of \( t_{w1} \) obtained each time are constant within 5%. Time delay, \( t_{w1} \), for a single A/D conversion is found to be approximately 5.6 msec.

The second calibration procedure is performed for accurate time delay generation, where the second and third input steps need to be executed at the exact switching times of \( t_1 \) and \( t_2 \). The question is how to control the timing of the last two step inputs during real time process. In this study, the idea was to use the computer program to provide the time delay between inputs. A desired time delay \( t_{w2} \) between inputs is achieved by using a “FOR NEXT LOOP” computer program without any statement inside. Since the loop size (\( NNO \)) determines the time delay that the computer takes, a time calibration between \( NNO \) and \( t_{w2} \) is required. This calibration is an important issue to achieve good results because three-step input method’s performance is dependent on the correct execution for step inputs at their switching times, \( t_1 \) and \( t_2 \). Figure 4.5b shows the experimental diagram for the \( t_{w2} \) calibration.

To calibrate \( t_{w2} \), the following general procedures are used:

1. Set the digital oscilloscope properly for the single sweep display.

2. Command a step input, \( u_1 \), through a D/A converter to trigger the digital oscilloscope.
Figure 4.5: Experimental schematic diagram for time calibrations. a). Time delay $t_{w1}$ of a A/D single operation b). Time delay $t_{w2}$ between inputs.
3. Execute a computer program that simulates the time delay. For example,

\[ \text{for } A = 1 \text{ to } NNO \\
\text{next } A \]

4. Command a step input, \( u_2 \), through the D/A converter.

5. Measure the step input \( u_1 \) time delay \( t_{w2} \) between points A and B from the digital oscilloscope screen. For convenience of the time measurement, print the step inputs displayed on the digital oscilloscope screen.

6. Record \( t_{w2} \) and \( NNO \) values to get the calibrational equation.

Since the loop size \( NNO \) determines the time the program waits, the above procedures are repeated using different values of \( NNO \) with 20 increments of 400, ranging from 2000 to 9600 to get enough \( t_{w2} \) and \( NNO \) data. With these \( t_{w2} \) and \( NNO \) data, the calibrational equation is obtained using the interpolation method. This resulted in

\[
NNO = \frac{t_{w2} - 6.5}{0.026}
\]  \hspace{1cm} (4.44)

where \( t_{w2} \) has units of milliseconds. Note that for \( t_{w2} \) calibration, the time delay of 50 \( \mu \)sec. for the D/A command is neglected due to the same reason as in the \( t_{w1} \) calibration.

As described above, these time calibrations should be performed in the three-step input method for real time control. These time calibrations are very hardware and software dependent. Thus, if any control hardware and/or software are changed, the recalibrations for \( t_{w1} \) and \( t_{w2} \) are required to achieve a correctly timed real time control.
CHAPTER 5. EXPERIMENTAL ANALOG SIMULATION RESULTS

This study reveals several interesting facts about manipulator response to high speed motion using the three-step input method, which uses a set of self-adjusting step inputs for real time control, when the flexible manipulator is modeled as a spring-mass system with damping as shown in Figure 2.1. As mentioned earlier, the purpose of this study is to reduce the manipulator response time while minimizing residual vibration. Experimental analog simulation is performed to verify the effectiveness of the three-step input method for real time control. Experimental verification is required to see if the theoretical assumptions for the parameter estimation model are justified when dealing with real physical systems. In order to grasp the meaning of a wide range of experimental results, the following observations should be helpful.

1. Experimental results support the modified parameter estimation model derived from the assumed one degree of freedom system as shown in Figure 2.5. These experimental results are identical with the results of the numerical solutions using the modal analysis method.

2. The minimum time, $t_2$, desired for a manipulator to move from an initial position to a defined end point is found to depend on the mechanical system damping ratio when the servo and mechanical natural frequencies are fixed.
3. The elimination of the manipulator end point residual vibration at the defined end point is found to depend on the natural frequency ratio between the servo and mechanical system when the servo and mechanical system damping ratios are fixed.

4. Another interesting phenomenon is observed only in the heavy mechanical damping ratio of 25 %, which shows that residual vibration is eliminated, irrespective of frequency ratio \( n \) after an additional time delay between the desired (or calculated) minimum time, \( t_2 \), and the time when the defined end point is actually achieved.

5. Finally, the application of the three-step input method is illustrated in the multiple level procedure to achieve a large motion that the servo system cannot handle in a single step. This procedure uses the three-step input method a multiple number of times to achieve a large motion.

Focusing on the above five important factors, the experiments are performed as shown in Figure 4.4, and their results are investigated. First, the experiments focus on point-to-point movement, and then the multiple level procedure experiments are performed.

5.1. Point-to-Point Movement

In this section, the purpose of the study focuses on point-to-point movement of the manipulator. The servo system damping ratio \( \zeta_1 \) is fixed at 70 % for all experiments. Even though the structural damping of most real mechanical systems is in the range of 1 % ~ 6 %, the experiments are performed using the mechanical
damping ratio in the broad range of 1 % ~ 25 % to show the robustness of the three-step input method. In the point-to-point movement study, the experiments are classified as four cases according to the mechanical damping values of \( \zeta_2 = 1, 5, 10, \) and 25 % . The servo natural frequency \( f_1 \) is fixed at 10 Hz while the mechanical natural frequency is varied from 1 to 5 Hz so that \( n \) varies from 2 to 10 . The values of \( f_2 \) and \( n \) used are

\[
f_2 = \begin{cases} 
5 \text{ Hz} & \text{for } n = 2 \\
3 \text{ Hz} & \text{for } n = 3.3 \\
2 \text{ Hz} & \text{for } n = 5 \\
1 \text{ Hz} & \text{for } n = 10 
\end{cases}
\]

Thus, each damping case has four experimental results that correspond frequency ratios of \( n = 2, 3.3, 5, \) and 10 .

The manipulator movement is assumed to be the distance corresponding to the 2 volts, which is calculated to be 2 cm from the analog simulation model. This full scale motion choice gives good signals to noise ratios and convenient scaling.

To observe the effectiveness of the three-step input method for real time control, the following points are investigated through the experimental analog simulation:

First, the experimental results are compared with the modal analysis results for 5 % mechanical damping ratio and frequency ratio of \( n = 2 \) and 10 . Second, the effect of \( n \) is studied, and the effective range of \( n \) for the three-step input method is determined for each case. Third, the effect of the mechanical damping ratios of \( \zeta_2 = 1, 5, 10, \) and 25 % is studied with the same frequency ratio(\( n \)) results obtained from the four cases as to how \( \zeta_2 \) effects the manipulator response. Finally, numerical values, such as a maximum error, are calculated for all experimental results. For this study, a
maximum error is defined as the ratio of the final position value and the difference between the maximum overshoot and the final position value, where the final position represents the point at which the manipulator end point completes its task. Ideally, the final position value is the same as the defined end point value exactly, but actually there is a small amount of error observed through this experimental work, which is measured to be less than 0.3 %, due to the resolution error of the analog computer components such as resistors and capacitors.

5.1.1. Case 1

In Case 1, a lightly damped mechanical system with $\zeta_2 = 1\%$ is used. Also, the experiments are performed with four natural frequency ratios of $n = 10, 5, 3.3,$ and 2. Figures 5.1 through 5.4 show the experimental analog simulation results acquired from Pro 380 ADM A/D converter when using a 250 Hz sampling frequency.

Figure 5.1 shows the results when $n = 10$, where Figure 5.1a is the base response, $x_1$, and Figure 5.1b is the manipulator end point response $x_2$. Figure 5.1b shows that the end point residual vibration has been nearly eliminated. The reason is that even though $x_1$ is not a step input like the command input, the change is nearly a step as far as its end position is concerned, and hence the manipulator end point response, $x_2$, is very close to that which occurs for a true step input (Figure 5.1a). For $n = 10$, the maximum error is about 0.254 %, and time, $t_2$, is 0.512 sec., which is approximately a half period of the mechanical system.

Figure 5.2 shows the results with $n = 5$. In this case, the residual vibration of the end point at the defined end point has also been eliminated as shown in Figure 5.2b because the base response, $x_1$, still appears to be nearly a step input to the
Figure 5.1: For 1% damping ratio, system responses with $n = 10$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.2: For 1% damping ratio, system responses with $n = 5$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.3: For 1% damping ratio, system responses with $n = 3.3$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.4: For 1% damping ratio, system responses with $n = 2$. a) Base response $x_1$, b) End point response $x_2$
mechanical system as shown in Figure 5.2a. These results show the maximum error is 0.325 %, and time, $t_2$, is about 0.256 sec., which is approximately a half period of the mechanical system.

Figure 5.3 shows the results with $n = 3.3$. In this case, there is some residual vibration after the final position as shown in Figure 5.3b because the base response, $x_1$, corresponding to the second step input cannot be followed in time as shown in Figure 5.3a. Even though there still exists a small mount of residual vibration, the maximum error is measured as 1.09 %. On other hand, the response, $x_2$, attains the desired end point several cycles beyond the desired time, $t_2$.

Figure 5.4 shows the results when $n = 2$. As expected, the end point residual vibration has not been eliminated as shown in Figure 5.4b because the base response, $x_1$, cannot follow the command input as required. These results show that the maximum error is about 5.35 %, and response, $x_2$, reaches the defined end point at a time considerably beyond the desired time, $t_2$, of 0.102 sec. It is also seen in Figure 5.4a that $x_1$ oscillates a small amount in response to the residual end point vibration.

5.1.2. Case 2

In Case 2, the mechanical system damping ratio $\zeta_2$ is assumed to be 5 %. Case 2 uses the same values of $n$ as were used in Case 1. Also, the experimental data results are acquired using a Pro 380 ADM A/D converter operating at a sampling frequency of 250 Hz. Figures 5.5 through 5.8 show Case 2 results.

Figure 5.5 shows the results with $n = 10$, where Figure 5.5a is the base response, $x_1$, and Figure 5.5b is the manipulator end point response, $x_2$. As shown in Figure 5.5b, the end point residual vibration has been nearly eliminated because base re-
Figure 5.5: For 5% damping ratio, system responses with $n = 10$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.6: For 5% damping ratio, system responses with $n = 5$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.7: For 5% damping ratio, system responses with $n = 3.3$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.8: For 5% damping ratio, system responses with $n = 2$. a) Base response $x_1$, b) end point response $x_2$. 
response, \( x_1 \), follows the command input closely enough to appear as a step input to the mechanical system as shown in Figure 5.5a. Also, these results are identical with those of the modal analysis method shown earlier, using the same input conditions. For \( n = 10 \), the maximum error is about 0.484 \%, and time, \( t_2 \), is about 0.559 sec., which is about 12 \% longer than a half period of the mechanical system.

Figure 5.6 shows the results with \( n = 5 \). The end point residual vibration at the defined end point has been eliminated as shown in Figure 5.6b because the base response, \( x_1 \), still appears to be nearly a step input to the mechanical system as shown in Figure 5.6a. These results show that the maximum error is 0.538 \%, and time, \( t_2 \), is 0.28 sec., which is about 12 \% longer than the half period of the mechanical system.

Figure 5.7 shows the results with \( n = 3.3 \). In this example, there is some residual vibration at the defined end point as shown in Figure 5.7b because the base response, \( x_1 \), is not fast enough to appear as step inputs to the mechanical system as shown in Figure 5.7a. The maximum error is measured to be approximately 0.87 \% and time, \( t_2 \), is 0.186 sec., which is about 12 \% longer than the mechanical system half period.

Figure 5.8 shows the results when \( n = 2 \) and are similar to those from the modal analysis method. As expected, the end point residual vibration has not been eliminated as shown in Figure 5.8b because the base response, \( x_1 \), cannot follow the command input as shown in Figure 5.8a. These results show that the maximum error is about 4.08 \%, and the end point response, \( x_2 \), initially reaches the defined end point at a time considerably beyond the desired time, \( t_2 \), as shown in Figure 5.8b, where the calculated time, \( t_2 \), is 0.112 sec. It is also seen that the residual vibration damps out much more quickly than the 1 \% damping case.
5.1.3. Case 3

In Case 3, the damping ratio $\zeta_2$ is increased to 10% and the same four frequency ratios of $n$ are used. The experimental data are obtained using a Pro 380 ADM A/D converter operating with a 500 Hz sampling frequency. Figures 5.9 through 5.12 show these output for the four different cases.

Figure 5.9 shows the results when $n = 10$, where Figure 5.9a is the base response, $x_1$, while Figure 5.9b is the end point response, $x_2$. As shown in Figure 5.9b, the end point residual vibration has been nearly eliminated because the base response, $x_1$, can follow the command input closely enough to appear as nearly a step input to the mechanical system as shown in Figure 5.9a. For $n = 10$, the maximum error is about 0.28%, and time, $t_2$, is 0.615 sec, which is about 23% longer than the mechanical system half period of 0.5 sec.

Figure 5.10 shows the results when $n = 5$. The end point residual vibration at the defined end point has also been eliminated as shown in Figure 5.10b, because the base response, $x_1$, still appears to be a step input to the mechanical system as shown in Figure 5.10a. These results show that the maximum error is 0.348%, and time, $t_2$, is 0.307 sec, which is about 23% longer than the mechanical system half period of 0.25 sec.

Figure 5.11 shows the results with $n = 3.3$. In this example, there is a small residual vibration at the defined end point as shown in Figure 5.11b because the base response, $x_1$, is quite slow in responding to the step input as shown in Figure 5.11a. While a small amount of the residual energy still exists in Cases 1 and 2, the residual vibration at the defined end point has been nearly eliminated due to a heavier mechanical damping in this example. From these results, the maximum error
Figure 5.9: For 10% damping ratio, system responses with $n = 10$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.10: For 10% damping ratio, system responses with $n = 5$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.11: For 10% damping ratio, system responses with $n = 3.3$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.12: For 10% damping ratio, system responses with $n = 2$. a) Base response $x_1$, b) end point response $x_2$. 
is measured to be approximately 0.467 %, and time, \( t_2 \), is 0.204 sec., which is about 23 % longer than the mechanical system half period of 0.167 sec.

Figure 5.12 shows the results with \( n = 2 \). As expected, the end point residual vibration has not been eliminated as shown in Figure 5.12b because the base response, \( x_1 \), cannot adequately follow the command input. These results show that the maximum error is about 2.38 %, and response, \( x_2 \), reaches initially the defined end point beyond the calculated time, \( t_2 \), of 0.123 sec. with a considerable time delay.

5.1.4. Case 4

In Case 4, a heavy mechanical system damping ratio of 25 % is used. The experimental data are acquired with a Pro 380 ADM A/D converter that uses a 500 Hz sampling frequency. The results to four different frequency ratios are shown in Figures 5.13 through 5.16.

Figure 5.13 shows the results when \( n = 10 \), where Figure 5.13a is the base response, \( x_1 \), and Figure 5.13b is the end point response, \( x_2 \). As shown in Figure 5.13b, the end point residual vibration has been nearly eliminated because the base response, \( x_1 \), can follow the command input fast enough to appear as a sequence of step inputs to the mechanical system as shown in Figure 5.13a. The maximum error is measured to be approximately 0.31 % when the final position is achieved at time \( t_2 \) of 0.765 sec, which is approximately three-quarters of the mechanical system natural period of 1 sec.

Figure 5.14 shows the results with \( n = 5 \), where the end point residual vibration has also been eliminated as shown in Figure 5.14b, because the base response, \( x_1 \), still appears to be a step input to the mechanical system as shown in Figure 5.14a.
Figure 5.13: For 25 % damping ratio, system responses with $n = 10$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.14: For 25% damping ratio, system responses with $n = 5$. a) Base response $x_1$, b) end point response $x_2$. 
Figure 5.15: For 25% damping ratio, system responses with $n = 3.3$. a) Base response $x_1$, b) end point response $x_2$
Figure 5.16: For 25% damping ratio, system responses with $n = 2$. a) Base response $x_1$, b) end point response $x_2$. 
As shown in Figure 5.14b, the maximum error when the final position is achieved at time, $t_2$, of 0.392 sec is around 0.368 %. Time, $t_2$, is approximately three-quarters of the mechanical system natural period of 0.5 sec.

Figure 5.15 shows the results with $n = 3.3$. As shown in Figure 5.15b, there is little residual vibration after the final position is reached at the actual time, $t_2$, of 0.284 sec measured from the experimental results, while the calculated time, $t_2$, of 0.247 sec from the parameter estimation model. Unlike Cases 1 and 2, residual vibration at $t = 0.284$ sec is nearly eliminated due to a heavy mechanical system damping, even though the base response, $x_1$, is somewhat slow in simulating the command input as shown in Figure 5.15a. The end point response overshoot at the desired time, $t_2$, of 0.247 sec is measured as 2.0 %. Time difference between the calculated time, $t_2$, of 0.247 sec. and the actual time, $t_2$, of 0.284 sec. is due to overshoot of 2 %. As shown in Figure 5.15b, the maximum error when the final position is achieved at the actual time, $t_2$, of 0.284 sec is around 0.12 %.

Figure 5.16 shows the results when $n = 2$, where Figure 5.16a shows the base response, $x_1$, and Figure 5.16b shows the end point response, $x_2$. These results are compared to Cases 1, 2 and 3 when $n = 2$. While the previous three cases have a considerable amount of residual vibration, in this example, residual vibration is nearly eliminated after the manipulator has reached its final position at the actual time, $t_2$, which is measured to be 0.2 sec from experimental results shown in Figure 5.16b. The reason for such good performance is the heavy mechanical system damping, even though the base response, $x_1$, cannot follow the command input as shown in Figure 5.16a. The end point response overshoot at the desired time, $t_2$, of 0.148 sec is measured to be approximately 1.7 %. The time difference between the calculated
time, $t_2$, of 0.148 sec. and the actual time, $t_2$, of 0.2 sec is due to a considerable overshoot of 1.7% at $t = 0.148$ sec. The maximum error when the manipulator has reached its final position at the actual time, $t_2$, of 0.2 sec is measured to be approximately 0.204%.

5.2. The Multiple Level Procedure

In this section, the more general control problem called the multiple level procedure is considered. The manipulator cannot reach its final position in a single three-step input sequence when a large motion is considered due to either structural stress or servo torque limitations. Thus, a large motion is broken into $N$ smaller motions that are within the system's physical limits. There are a number of ways to break a large motion into $N$ smaller motions. However, the minimum response time must be an integer multiples of half of the mechanical system's natural period. Let $S$ be the desired range of motion, let $S_s$ be the servo system's maximum range per three-step input sequence, and let $R_1$ be the ratio given by

$$R_1 = \frac{S}{S_s}$$

Then, the minimum number of three-step input sequences $N$ becomes

$$N \geq R_1$$

where $N$ is the smallest integer value that satisfies this relationship. The most efficient system set-up is to use $N$ equal motion increments so that

$$S_1 = \frac{S}{N}$$
gives each step size, which is generally called the equal-spaced sequence movement. However, it is possible to use \((N - 1)\) equal increments of magnitude \(S_8\) and a residual motion of \(S - (N - 1)S_8\), which is generally called the non-equal-spaced sequence movement. Both approaches are used in this study. Since \(N\) steps are used in both case, this procedure is called here the multiple \((N)\) level procedure.

The initial position velocity and displacement are assumed to be zero. As shown in the point-to-point movement cases with a small mechanical damping, \(\zeta_2\), and frequency ratio of \(n \geq 5\), the residual vibration at the defined end point of the first level procedure is zero so the corresponding zero velocity is used as an initial velocity for the second level procedure. For the first level procedure, if the manipulator response time is \(T\), the second level response is also started at \(t = T\) by using the three-step input method again to create the second level dynamic response during the time of \(T\) to \(2T\). The second level procedure ends at zero velocity, which is used as an initial velocity for the third level procedure. This concept can be applied to the \(N\) level procedure. Each level has an amplitude value corresponding to each smaller motion value and ends at time \(t = NT\).

To investigate the system response for the multiple level procedure, the mechanical system damping ratio, \(\zeta_2\), is selected as 5\% while the servo system damping ratio, \(\zeta_1\), remains at 70\%. The two more realistic natural frequency ratios of 10 and 5 are used for this part of the study. Four different multiple level sequences are listed in Tables 5.1 through 5.4. In the first three sequences, the total movement is divided into equal movement increments. In the fourth sequence (Table 5.4), the total movement is divided into four unequal increments. For the experimental analog simulation of the multiple level procedure, eight different tests are performed to
Table 5.1: A 2-level sequence movement

<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$ (volt)</td>
<td>2.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 5.2: A 3-level sequence movement

<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$ (volt)</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

investigate the effectiveness of the three-step input method.

For each case, a numerical maximum error is calculated. Like point-to-point movement, a maximum error is defined as the ratio of the measured maximum value of $x_2$ that occurs after time $t_2$ and the measured static long term value of $x_2$. Figures 5.17 through 5.24 show the multiple level procedure results.

Figure 5.17 shows the mechanical system response, $x_2$, of the 2-level procedure for $n = 10$. As mentioned earlier, the three-step input method is used twice to achieve a 2-level movement, where each level has a different initial displacement condition, and the initial displacement condition for the second level procedure is the final position of the first level procedure. As shown in Figure 5.17, the end point residual vibration is nearly eliminated at the final position. The maximum error when the manipulator reaches the final position is measured to be approximately 0.225%. The total response time is 1.119 sec. Similarly, the experimental results of 2 points movement using $n = 5$ is shown in Figure 5.18, where the maximum error is around

Table 5.3: A 4-level equal-spaced sequence movement

<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$ (volt)</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Table 5.4: A 4-level non-equal-spaced sequence movement

<table>
<thead>
<tr>
<th>Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_2$ (volt)</td>
<td>0.4</td>
<td>1.2</td>
<td>2.4</td>
<td>4.0</td>
</tr>
</tbody>
</table>

0.45 %, and the total response time is 0.559 sec.

For the 3-level procedure, the three-step input method is applied three times. The experimental results using $n = 10$ and 5 are shown in Figures 5.19 and 5.20, respectively. These results show that the manipulator end point residual vibration at the final position is nearly eliminated. The maximum error and total response time when $n = 10$ are 0.15 % and 1.678 sec, respectively. For $n = 5$, the maximum error is 0.46 % and the total response time is 0.839 sec.

Two different sequences are used for the 4-level procedure, which uses the three-step input method four times. Figures 5.21 and 5.22 show the results of a 4-level equal-spaced sequence using different frequency ratio of $n = 10$ and 5, respectively. These results show that the end point residual vibration is nearly eliminated at the final position of the sequence listed in Table 5.3. The maximum errors when $n = 10$ and 5 are 0.15 % and 0.33 %, respectively, and the total response times using $n = 10$ and 5 are 2.238 sec and 1.119 sec, respectively.

For a unequal-spaced 4 points movement listed in Table 5.4, Figures 5.23 and 5.24 show the end point experimental results of two different frequency ratios of $n = 10$ and 5, respectively. From these results, in both cases the residual vibration is nearly eliminated. The maximum errors when $n = 10$ and 5 are 0.2 % and 0.45 %, respectively, and the total response times using $n = 10$ and 5 are the same as those of a 4-level equal space sequence because the total response depends on the number of the multiple level procedures.
The multiple level procedure is shown to eliminate residual vibration motion to be less than 0.5% when the three-step input sequence is applied three or four times to achieve large motions with a servo system and robot structure that has structural damping of 5% and frequency ratio of 5 and 10.
Figure 5.17: For 5% damping ratio, manipulator end point response, $x_2$, with a 2-level movement when $n = 10$
Figure 5.18: For 5% damping ratio, manipulator end point response, $x_2$, with a 2-level movement when $n = 5$
Figure 5.19: For 5% damping ratio, manipulator end point response, $x_2$, with a 3-level movement when $n = 10$
Figure 5.20: For 5 % damping ratio, manipulator end point response, $x_2$, with a 3-level movement when $n = 5$
Figure 5.21: For 5 % damping ratio, manipulator end point response, $x_2$, with a 4-level equal-spaced movement when $n = 10$
Figure 5.22: For 5% damping ratio, manipulator end point response, $x_2$, with a 4-level equal-spaced movement when $n = 5$
Figure 5.23: For 5% damping ratio, manipulator end point response, $x_2$, with a 4-level unequal-spaced movement when $n = 10$.
Figure 5.24: For 5 % damping ratio, manipulator end point response, $x_2$, with a 4-level equal-spaced movement when $n = 5$
CHAPTER 6. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The objective of this study is to develop a self-adjusting open loop control algorithm called the three-step input method that can be implemented in real time to control residual manipulator vibrations when the manipulator moves from one position to another in the shortest possible time. A three-step command sequence is utilized to achieve the desired manipulator response. The step input size and switching time are decided based on the defined end point and the mechanical properties such as the natural frequency and assumed correct damping. In this study, the flexible manipulator is modeled as two lumped masses connected by a spring with a damper. One mass represents the manipulator end connected to the servo controller and is called the base position, $x_1$. The other mass represents the manipulator end point with motion, $x_2$.

This chapter consists of three sections. The first section summarizes the results obtained through this study, and then conclusions are described in the second section, and finally, recommendations for future research are described in the last section.
6.1. Summary

The three-step input method is tested in two ways; one being the modal analysis method while the other is an experimental analog simulation. Since the parameter estimation model is a key factor in the three-step input method, its effectiveness needs to be tested in achieving a minimized manipulator response time along with a minimum of residual vibration when the manipulator reaches the defined end point. Then, by performing many different experimental analog simulations, the effectiveness and limitations of the three-step input method are found without theoretical approximations.

The combined theoretical and experimental study reveals several important facts. These include:

1. The command input function, \( x_{in}(t) \), used for the three-step input method involves two unknown parameters, \( \Delta U \) and \( t_2 \). These parameters are obtained from the parameter estimation model that is developed by assuming the base motion, \( x_1 \), appears as a step input to the mechanical system. This assumption comes from the fact that \( x_1 \) is not a step input like the command input, but if there is sufficient time for the servo system to move the base, the change is nearly a step input as far as the end position is concerned. This assumption requires a sufficiently large frequency ratio, \( n \), between the servo and mechanical system and it has a big advantage in reducing computation complexity, the most difficult problem encountered for real time control.

2. Two parameters, \( \Delta U \) and \( t_2 \), must be known in order to apply the correct input sizes at the proper times. The \( \Delta U \) values are found to depend on the mechanical damping, \( \zeta_2 \) and the half value of the defined end point, while \( t_2 \) depends on both
the mechanical damping, $\zeta_2$ and the natural frequency, $f_2$. As shown in Tables 2.1, 2.2, 2.3, and 2.4, the ratio of $\Delta U/U_0$ is a constant value for a given value of damping independent of frequency ratio $n$, and the value of $t_2$ is a constant multiplier of the system natural period $T_n$ for a given value of damping. Thus, the parameter estimation model does not require recomputation when the defined end point is changed and the mechanical system damping characteristic is the same.

3. As developed in Chapter 3, the modal analysis method tests the effectiveness of the parameter estimation model. The three-step input method is studied under conditions of $n = 2$ and 10 with $\zeta_2$ fixed at 5%. The modal analysis results show that residual vibration is nearly eliminated with the maximum error of 0.1% when $n = 10$, while for $n = 2$, residual vibration remains with the maximum error of 6.9%. These results support the parameter estimation model that is developed on the assumption of base response, $x_1$, appearing to be as nearly a step input to the mechanical system by selecting a sufficiently large frequency ratio $n$.

4. Before the experimental analog simulation is performed, two time calibrations are required as discussed in detail in Section 4.3. The first calibration procedure is performed to assure proper measurement of time $t_0$; that is, the time required for the system response to reach the halfway point from the defined end point. The second calibration procedure is performed for accurate time delay generation, where the second and third input steps need to be executed at exact switching times of $t_1$ and $t_2$.

5. The results of Chapter 5 demonstrate the effective frequency ratio range and manipulator response time that the three-step input method provides for practical vibration control in real time. The effective frequency ratio range varies with the
mechanical system damping, $\zeta_2$. For small mechanical damping of $\zeta_2 = 1$ and 5 % and frequency ratios of $n = 2$ and 3.3, significant residual vibration still remains when the defined end point is achieved. For the cases of $n = 5$ and 10, residual vibration is nearly eliminated within the maximum error of 0.5 % when the defined end point is achieved. Thus, for light damping, the effective frequency ratio must be $n \geq 5$.

For a mechanical damping, $\zeta_2$, of 10 % and frequency ratios of $n = 5$ and 10, residual vibration is nearly eliminated, giving a maximum error less than 0.5 % like the case when $\zeta_2 = 1$ and 5 %. On the other hand, when $n = 3.3$, residual vibration is nearly eliminated with a maximum error less than 0.5 % due to the heavier mechanical damping of 10 %. Thus, for $\zeta_2$ of 10 %, the effective frequency ratio is $n \geq 3.3$.

Finally, when $\zeta_2 = 25 \%$, residual vibration is nearly eliminated within the maximum error of 0.4 % for $n = 5$ and 10 like in the previous cases. On the other hand, when $n = 2$ and 3.3 along with heavy mechanical damping of $\zeta_2 = 25 \%$, residual vibration is nearly eliminated, giving a maximum error of 0.2 % along with significant additional time delay between the actual response and the desired response.

6. The minimum response time that corresponds to the manipulator reaching the defined end point with minimum residual vibration is found to depend on the mechanical system damping ratio, $\zeta_2$, when the frequency ratio $n$ is fixed. For a small mechanical damping $\zeta_2$ of 1 % with $n \geq 5$, the manipulator response time is very close to one-half of the mechanical natural period. When $\zeta_2 = 5 \%$ with $n \geq 5$, the manipulator response time is about 12 % longer than a half period of the mechanical system, while for $\zeta_2$ of 10 % with $n \geq 3.3$, the manipulator response time is about 23 % longer than half of the mechanical system period. Finally, for a heavy mechanical damping, $\zeta_2$, of 25 %, the manipulator response time is around three-quarters of the
mechanical natural period, except \( n = 3.3 \) and \( 2 \). For \( n = 2 \) and 3.3 with \( \zeta_2 \) of 25 \%, due to some time delay beyond the calculated time \( t_2 \) as shown in Figures 5.15 and 5.16, the manipulator response time is measured to be around one mechanical system period. Thus, the manipulator minimum response time is found to be dependent on both the mechanical damping ratio, \( \zeta_2 \), and frequency ratio \( n \).

7. The multiple level procedure is performed experimentally in four different level sequences for mechanical damping of 5 \% and frequency ratios of \( n = 5 \) and 10. The experimental results show that residual vibration is nearly eliminated within a maximum error of 0.5 \% when the manipulator reaches the defined end point. The manipulator minimum response time is \( NT \), where \( N \) is the total number of multiple levels, and \( T \) is the minimum response time for a single three-step input procedure. For each case, the actual manipulator response time agrees with the desired minimum time of \( t = NT \).

### 6.2. Conclusions

A high performance manipulator must meet two performance requirements: fast response time and minimized residual vibration. For this purpose, the use of three-step input method for commanding the computer controlled simple flexible manipulator showed that significant residual vibration reduction along with the minimized response time can be achieved.

The development of the parameter estimation model used for the three-step input method shows that two parameters, \( \Delta U \) and \( t_2 \), must be known in order to apply the correct input at the proper times. It is known that \( \Delta U \) values are dependent both on the mechanical damping, \( \zeta_2 \) and half-step size \( U_0 \), while \( t_2 \) depends on
both the mechanical damping, $c_2$, and natural frequency, $f_2$. It is also found that this parameter estimation model reduces computation complexity, as discussed in summary statements 1 and 2 of Section 6.1.

The experimental analog simulation study in point-to-point movement shows the suitability of the parameter estimation model when an effective frequency ratio is selected. It is found that the frequency ratio $n$ must be greater than 5 in order to nearly eliminate residual vibration along with the minimum response time when the mechanical damping varies from 1% to 10%. This conclusion is based on summary statements 5 and 6 of Section 6.1. This research work places great emphasis on the importance of the natural frequency ratio $n$ in designing a suitable control system to implement the three-step input method.

The experimental analog simulation study in the multiple level procedure shows that multiple three-step inputs can be used to achieve motions that are too large to accomplish in one step. This step input requires $n > b$. This conclusion is based on summary statement 7 of Section 6.1.

Time calibration of the control computer is required for the experimental implementation of the three-step input method. It is noticed that the recalibration of the control computer is required to achieve a correctly timed real time control if any control hardware and/or software are changed. This conclusion is very hardware software dependent and may be meaningless for a properly designed system that includes a careful handing of time.

A further advantage of the three-step input method is that both system identification and system control occur in real time. This property can be especially desirable in situations where the robot manipulator moves from an initial position
to a final position while transporting either an unknown mass or several significantly different masses.

Finally, this study has demonstrated the potential for the three-step input method in the areas of the robotic manipulator position control in minimum time with minimum residual vibration, while using a small amount of digital computing resources.

6.3. Recommendations

The three-step input method is developed to provide the capability of controlling the flexible robotic manipulator position control in minimum time, while minimizing residual vibration when the manipulator reaches the defined end point. Although the results of this study appear to be encouraging, they also serve to point out some limitations that appeared during the experimental analog simulation and indicate possible areas for future research that are not addressed in this thesis.

These include:

1. It is necessary to pursue the experimental verification of the capability of the three-step input method to deal with a single degree of freedom actual flexible manipulator with either a known or an unknown payload on its end point in conjunction with a real servo system. The reason is that actuators and sensors used in a servo system affect a system's performance.

2. An important aspect is that the suitability of using the three-step input method with multiple degree of freedom systems needs to be investigated.

3. Additional work needs to be done on the role of damping. First, how much can it
vary before it has significant effects on residual vibration? Second, can previous measurements be used to estimate current values? Third, what methods can be used to estimate its value in operating machine, particularly during warm up periods when damping may change significantly.

4. Finally, this experiment is performed by using a conventional servo controller. Actually, several types of control can be implemented for the system control, including proportional control, proportional-integral (PI) control, proportional-derivative (PD) control and proportional-integral-derivative (PID) control. Thus, it is necessary to examine the effects of using different types of controller models and compare the results in order to judge the best controller to use with the three-step or similar input method.
BIBLIOGRAPHY


APPENDIX: Evaluation of Modal Matrix

A method is presented to evaluate the modal matrix. Recall Eq. (3.10).

\[ [f(\alpha)] \{\Phi\} = 0 \]  

(6.1)

From the definition of matrix inverse, the inverse of \([f(\alpha)]\) is

\[ [f(\alpha)]^{-1} = \frac{\text{adj}[f(\alpha)]}{\Delta(\alpha)} \]

(6.2)

where \(\Delta(\alpha)\) is the characteristic determinant of \([f(\alpha)]\), and \([F(\alpha)]\) is the adjoint matrix of \([f(\alpha)]\). Premultiplying Eq. (A.2) by \([f(\alpha)]\) and rearranging,

\[ [f(\alpha)][F(\alpha)] = \Delta(\alpha)I \]  

(6.3)

Since \(\Delta(\alpha_r) = 0\) for \(\alpha = \alpha_r\), Eq. (A.3) yields

\[ [f(\alpha_r)][F(\alpha_r)] = [0] \]  

(6.4)

Note that for each value of \(\alpha_r\), Eq. (A.1) gives

\[ [f(\alpha_r)]\{\Phi^r\} = 0 \]  

(6.5)

Comparing Eqs. (A.4) and (A.5), they show that every column of \([F(\alpha_r)]\) must be proportional to \(\{\Phi^r\}\). Furthermore, each column of \([F(\alpha_r)]\) must be proportional to one another. Hence, any nonzero column of \([F(\alpha_r)]\) can be taken as an eigenvector appropriate to \(\alpha_r\).