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Testing the Present Value Model of Farmland Prices

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Abstract
This paper examines the adequacy of the constant expected returns version of the present value model of farm real estate and rent data over a 1921-1985 sample period. The nature of the model's failure to explain these data is remarkably similar to the kind of model failure that Campbell and Shiller [1987] uncovered in their study of U.S. stock market price and dividend time series. More specifically, real farmland prices tend to overreact to movements in real cash rents, falling much too far during periods of declining rents and rising much too far during periods of increasing rents.

Disciplines
Agribusiness | Economic Policy | Public Policy | Taxation-Federal Estate and Gift

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Robert Shiller provided helpful comments. Michael Duffy provided much of the data to me.
This paper examines the adequacy of the constant expected returns version of the present value model of farm real estate and rent data over a 1921-1985 sample period. The nature of the model's failure to explain these data is remarkably similar to the kind of model failure that Campbell and Shiller [1987] uncovered in their study of U.S. stock market price and dividend time series. More specifically, real farmland prices tend to overreact to movements in real cash rents, falling much too far during periods of declining rents and rising much too far during periods of increasing rents.
I. **INTRODUCTION**

Substantial interest has been shown in determining whether the present value model of asset price determination provides a reasonable approximation of the determination of farmland prices. (See, for example, Alston [1986], Burt [1986], Castle and Hoch [1982], and Melichar [1979], among others.) A major problem that has made it difficult to interpret the results of formal statistical tests of the present value model as an explanation of farmland prices is that such tests have typically been joint tests of the present value model and the specific model used to represent the market's forecasts of future net returns. This paper will use Iowa farmland price and cash rent time series data to test the present value model in a framework that largely circumvents this problem. The only assumptions that will be required beyond the present value model itself are i) the presence of a single unit root in the real cash rent series; ii) current and past land prices and rents are contained in the market's current information set; and iii) the market's expectations are rational expectations, in the sense of being optimal linear projections on the market's complete information set, which can be larger than the econometrician's information set.

Iowa farmland seems to be an especially suitable subject for empirically evaluating the present value model as Iowa farmland is relatively homogeneous and is not typically valued for its potential nonagricultural uses. Thus, it has remained in relatively fixed supply over time (and, hence, we can avoid problems that arise in stock market studies due to such features as splits in corporations' shares). Furthermore, the concept of a market price of a typical acre of Iowa farmland and the expected returns to such an acre are relatively unambiguous concepts. Finally, there has long been a very active rental market in Iowa farmland so that a reasonably good
first approximation of the returns to Iowa farmland can be derived from observable, market determined cash rents.

In a recent paper, Falk [1988] used Iowa price and rent data to test whether the present value model could be rejected in favor of a rational bubble model, i.e., in favor of a model in which prices deviate from the predictions of the present value model because of purely speculative, but rational, explosive forces. The present value model could not be rejected against this alternative based upon conventional test criteria. In this paper, we will consider whether the present value model can be rejected for other reasons, such as persistent deviations of returns to land ownership from the "normal" rate of return, and hence, the presence of persistent unexploited profit opportunities in the Iowa land market.

Section II provides the theoretical background and a development of the testing strategy. It is largely a review of Campbell and Shiller [1987, pp. 1064-1070]. The outcome of the tests are described and discussed in Section III. Section IV contains additional analysis of the data to help interpret the economic significance of the test results. The main conclusions of the paper are summarized in Section V.

II. THE THEORETICAL MODEL AND TESTABLE RESTRICTIONS

Let \( Y(t) \) denote the real price per acre of homogeneous farmland at the beginning of year \( t \) and let \( y(t) \) denote the real return (net of capital gains) per acre of farmland during year \( t \). The (constant expected return) present value model of farmland price determination asserts that

\[
Y(t) = \delta \sum_{i=0}^{\infty} \delta^i E[y(t+i)|I(t)]
\]  

(1)
where $\delta$ is a constant discount factor such that $0 < \delta < 1$ and $E[y(t+i)|I(t)]$ denotes the market's forecast of $y(t+i)$ conditional upon the information set $I(t)$. The only restrictions we will need to impose on these forecasts are i) they are optimal linear projections on the information set $I(t)$ and ii) $I(t)$ contains at least $Y(t), Y(t-1), \ldots$ and $y(t), y(t-1), \ldots$.  

Let $H(t)$ denote the subset of $I(t)$ given by \{Y(t), Y(t-1), \ldots, y(t), y(t-1), \ldots\}. 

Define a new variable, $S(t)$, called "the spread," which is the linear combination of $Y(t)$ and $y(t)$ given by

$$S(t) = Y(t) - \theta y(t), \quad (2)$$

where $\theta$ is determined by the discount factor according to $\theta = \delta/(1-\delta)$. If the present value model is correct, then Campbell and Shiller [1987] show that the spread will be equal to a weighted average of expected future changes in $y(t)$. They also show that if the stochastic processes $Y(t)$ and $y(t)$ are related according to the present value model (1) and if $\Delta y(t)$ is a stationary stochastic process then i) $\Delta Y(t)$ is a stationary stochastic process, and, ii) $Y(t)$ and $y(t)$ are cointegrated of order $(1,1)$. The precondition that $\Delta y(t)$ is stationary and the implication of that condition, that $\Delta Y(t)$ is stationary, can be tested by using Dickey-Fuller type unit root tests. The implication that $Y(t)$ and $y(t)$ are cointegrated of order $(1,1)$ can be tested by using cointegration tests, as described by Engle and Granger [1987, pp. 264-270]. 

Furthermore, consider the following vector autoregressive representation of $\Delta y(t)$ and $S(t)$:
\[
\begin{bmatrix}
\Delta y(t) \\
S(t)
\end{bmatrix} =
\begin{bmatrix}
a(L) & b(L) \\
c(L) & d(L)
\end{bmatrix}
\begin{bmatrix}
\Delta y(t-1) \\
S(t-1)
\end{bmatrix}
+ \begin{bmatrix}
u_1(t) \\
u_2(t)
\end{bmatrix} \tag{3}
\]

where \(a(L), b(L), c(L),\) and \(d(L)\) are \(p\)-th order polynomials in the lag operator \(L\) and \(u_1(t)\) and \(u_2(t)\) are zero-mean and serially independent processes which are mutually uncorrelated at all leads and lags, except that \(E[u_1(t)u_2(t)]\) may be nonzero. The present value model and the stationarity of \(\Delta y(t)\) imply, that the VAR will be characterized by \(S(t)\) linearly Granger-causing the \(\Delta y(t)\) process and by the following cross-equation restrictions:

\[
c_i = \Theta a_i, \quad i = 1, \ldots, p; \quad d_1 = (1/\delta) - \Theta b_1; \quad \text{and} \quad d_i = -\Theta b_i, \quad i = 2, \ldots, p.
\]

The restriction that \(S(t)\) Granger-causes the \(\Delta y(t)\) process, i.e., some of the elements of \(b(L)\) are nonzero, arises because \(S(t)\) is an optimal forecast of a weighted average of future values of \(\Delta y(t)\) based upon the market's complete information set. So long as there is any information in this set beyond lagged values of \(y(t)\) itself that is useful in forecasting future changes in \(y(t)\) this will be reflected in (3) by a nonzero \(b(L)\).

(This is formally proven by Campbell and Shiller [1987]; see their footnote 7.)

To interpret the cross-equation restrictions it will be useful to define a new variable, \(x(t)\), according to

\[
x(t) \equiv Y(t) + ry(t-1) - (1+r)Y(t-1)
\]

where \(r\) is the "normal" real rate of return to holding land and is related to the discount factor \(\delta\) according to \(1/(1+r) = \delta\). Thus, \(x(t)\) measures the
"excess return" to holding land in period t-1. According to the present value model (1):

\[ x(t) = Y(t) - E[Y(t) \mid I(t-1)] \tag{5} \]

i.e., \( x(t) \) can also be interpreted as the innovation in the price of land in period t based upon the market's complete information set in period t-1, I(t-1). Since this information set contains, by assumption, at least \( H(t-1) \), it follows that \( E[x(t) \mid H(t-1)] = 0 \) for all t. In other words, according to the present value model, current excess returns ought to be unpredictable based upon past prices and returns. The cross-equation restrictions of the VAR coefficients reflect this implication of the model.

The Granger causality and cross-equation restrictions are testable, in principle, based upon time series observations of the \( y(t) \) and \( S(t) \) processes. As a practical matter, the issue is complicated by the fact that the order of the VAR lag length is unknown \textit{a priori} and \( S(t) \) is not directly observable, since it depends upon the parameter \( \theta \). Although \( S(t) \) is not observable, the fact that \( S(t) \) is, according to (2), a linear combination of the cointegrated processes \( Y(t) \) and \( y(t) \) means that \( S(t) \) can be constructed based upon the "cointegrating vector" associated with \( Y(t) \) and \( y(t) \), \([1, -\theta]\). This cointegrating vector can be consistently estimated directly from a regression of \( Y(t) \) on \( y(t) \) by using the regression coefficient on \( y(t) \) as an estimate of \( \theta \). (See Engle and Granger, pp. 260-264.) Thus, a simple procedure is available to obtain a consistent estimator of \( S(t) \). Given this measure of \( S(t) \), the problem of the unknown order of the VAR's lag length can be overcome by applying one of a number of procedures that have been used for this purpose.
III. TESTING THE VAR RESTRICTIONS

In this paper, we will measure $Y(t)$ as the estimated average value of an acre of Iowa farmland in year $t$ divided by the Consumer Price Index (all items, 1967=100) for that year. We will measure $y(t)$ as the estimated average annual cash rent per acre of Iowa farmland in year $t$, divided by the CPI. The sample period is 1921-1986. The data are described more thoroughly in an appendix provided at the end of the paper. Figures 1 and 2 provide a graphical description of $Y(t)$ and $y(t)$ and their first differences.

We acknowledge that cash rents provide an imperfect measure of the net returns to holding farmland for a number of well-known reasons. One reason is that there are several types of farmland leasing contracts that are common in Iowa: crop-share leases, cash leases, and combination crop-share and cash leases. It may be the case that there is a systematic relationship between the quality of land and the nature of the leasing arrangement. This could contribute a systematic bias to the cash rent data as a measure of return per acre. Second, in the case of cash leases there are factors other than the amount of cash rent that influence the landowner's net return, such as owner-borne maintenance costs, property taxes, and insurance premiums. Nevertheless, cash rents seem to be the closest widely-available measure of returns that exists. Furthermore, the historically very active cash rent market in Iowa farmland suggests that the use of cash rents to index returns may be less of a problem in this study than it would be in studies of land price determination in other parts of the country.

As part of another study (Falk, [1988]), Dickey-Fuller type unit root tests were applied to these measures of $Y(t)$ and $y(t)$ with the conclusion that they appear to be first-difference stationary processes. That is, both
processes appear to have exactly one unit root. The stationarity of $\Delta y(t)$ is, as was noted above, a precondition for exploiting the Campbell-Shiller tests of the present value model, while the stationarity of $\Delta Y(t)$ is an implication of the model under this precondition.

If $\Delta y(t)$ is stationary then another implication of the present value model is that $Y(t)$ and $y(t)$ are cointegrated of order (1,1) processes. In other words, there exists a unique linear combination of $Y(t)$ and $y(t)$ of the form $Y(t) + \alpha y(t)$ which is stationary. In fact the theory also implies that $\alpha$ is equal to $-\theta$ since $S(t)$ must be stationary. Table 1 shows the results of applying Engle and Granger's "Augmented Dickey-Fuller" test for cointegration of $Y(t)$ and $y(t)$.

If $Y(t)$ and $y(t)$ are cointegrated processes then the conclusion of the test should not depend upon the direction of the cointegrating regression. That is, given the present value relationship between $Y(t)$ and $y(t)$, we would expect to be able to reject the null hypothesis of no cointegration at a chosen test size regardless of which version of the cointegrating regression we choose to consider. However, based upon Engle and Granger's recommended critical value of around 2.75 for a ten-percent test size, we would reject the null hypothesis when the cointegrating regression is run with $y(t)$ as the regressor but not when $Y(t)$ is the regressor. One interpretation of these results is that they are not consistent with the hypothesis that $Y(t)$ and $y(t)$ are cointegrated and, hence, the land price and cash rent data are not consistent with the implications of the present value model. However, the critical values presented by Engle and Granger are based upon very limited Monte Carlo experimentation. Therefore, it would be prudent to interpret this mixed evidence conservatively and proceed without rejecting the cointegration implication of the present value model.
It remains to test the restrictions that the present value model imposes on the VAR representation of $\Delta y(t)$ and $S(t)$. The process $S(t)$ is not directly observable. However, as we have previously pointed out, if $Y(t)$ and $y(t)$ are cointegrated processes, a consistent estimator of $\Theta$ (and, hence, $S(t)$) is the regression coefficient on $y(t)$ in the regression of $Y(t)$ on $y(t)$. Another consistent estimator of $\Theta$ is the reciprocal of the regression coefficient on $Y(t)$ in the regression of $y(t)$ on $Y(t)$. These two estimates of $\Theta$ can be inferred from Table 1 to be 17.75 and 21.74, respectively. These estimates imply annual discount rates of 5.6 percent and 4.6 percent, respectively. Although these estimates of $\Theta$ are close to one another, the remaining steps in the study were executed twice: once on the basis of each of these numbers.

The unrestricted VAR representation of $\Delta y(t)$ and $S(t)$ was assumed to have a third-order lag structure based upon the results of Sims' modified likelihood ratio test. (See Sims [1980].) Table 2 contains a summary of the VAR and the results of tests of the restrictions implied by the present value model. The $R^2$ of the $\Delta y$ equation implies that over fifty percent of change in real cash rents is predictable on the basis of its own past changes and the past history of the spread. The Granger-causality test suggests that the spread Granger-causes changes in returns, which is what we would expect to observe if the present value model is correct and if the market uses information other than past values of $y$ to forecast current and future changes in $y$.

We proceeded to test the cross-equation linear restrictions implied by the present value model. Wald tests were executed in RATS based upon White's Heteroskedasticity-Consistent Covariance Matrix Estimator. The unrestricted model was estimated without restricting the mean of either
\( \Delta y(t) \) or \( S(t) \). However, the present value model implies that \( E[S(t)] = 0 \). Therefore, we tested the cross-equation restrictions without restricting the mean of \( S(t) \) and we tested these restrictions along with the zero restriction on the mean of \( S(t) \). All of these tests clearly reject the cross equation restrictions implied by the present value model at the one-percent level of significance.

IV. FURTHER ANALYSIS

One of the objectives of Campbell and Shiller's 1987 (and their 1988) paper was to propose a strategy that could be used not only to construct a formal statistical test of the present value model, but also to informally evaluate the fit of the model in order to assess the economic significance of a statistical rejection of the model. As they note (p. 1058), "the major advantage of the VAR framework is that it can be used to generate alternative measures of the economic importance, not merely the statistical significance, of deviations from the present value relation." In this section of the paper, we try to evaluate the economic significance of the failure of Iowa farmland price and rent data to satisfy the statistical restrictions implied by the present value model for the VAR representation of \( S(t) \) and \( \Delta y(t) \).

Let \( S'(t) \) denote the expected present value of all future changes in returns, conditional upon current and past prices and returns, i.e.,

\[
S'(t) = E[\Theta \sum_{i=1}^{\infty} \delta^i \Delta y(t+i) | H(t)]
\]

where as before, we define the information set \( H(t) \) as the set of current
and past values of $Y(t)$ and $y(t)$. Based upon the unrestricted VAR representation of $\Delta y(t)$ and $S(t)$ given by (3), it can be shown (see Campbell and Shiller [1987], p. 1068) that

$$S'(t) = \theta h' \delta A (I - \delta A)^{-1} z(t) \quad (7)$$

where $h'$ is the $1 \times 2p$ row vector $[1 \ 0 \ 0 \ ... \ 0]$; $z(t) = [\Delta y(t), ..., \Delta y(t-p+1), S(t), ..., S(t-p+1)]'$; and $A$ is the $p \times p$ "companion matrix" of the VAR, i.e.,

$$A = \begin{bmatrix}
    a_1 & a_2 & a_3 & ... & a_{p-1} & a_p & b_1 & b_2 & b_3 & ... & b_{p-1} & b_p \\
    1 & 0 & 0 & ... & 0 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\
    0 & 1 & 0 & ... & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    ... \\
    0 & 0 & 0 & ... & 1 & 0 & 0 & 0 & 0 & ... & 0 & 0 \\
    c_1 & c_2 & c_3 & ... & c_{p-1} & c_p & d_1 & d_2 & d_3 & ... & d_{p-1} & d_p \\
    0 & 0 & 0 & ... & 0 & 1 & 0 & 0 & 0 & ... & 0 & 0 \\
    0 & 0 & 0 & ... & 0 & 0 & 1 & 0 & 0 & ... & 0 & 0 \\
    ... \\
    0 & 0 & 0 & ... & 0 & 0 & 0 & 0 & 0 & ... & 1 & 0
\end{bmatrix}$$

Thus, $S'(t)$ can be estimated based upon the estimates of $\theta$ and the VAR coefficients.

The definitions of $x(t)$, $S(t)$ and $S'(t)$ and the VAR representation of $\Delta y(t)$ and $S(t)$ given by (3), imply that $S(t)$, $x(t)$, and $S'(t)$ are related according to
\[ S(t) - S'(t) = \sum_{i=1}^{\infty} \delta_i \mathbb{E}[x(t+i)|H(t)] \] (8)

Thus, the difference between the actual spread, \( S(t) \), and the "theoretical spread," \( S'(t) \), measures the expected present value of predictable future excess returns. Of course, according to the present value model there should not be any predictable excess returns, i.e., the actual and theoretical spreads should be equivalent measures. Therefore, sampling error aside, observed deviations of the actual spread from the theoretical spread index the presence of large predictable and persistent excess positive and/or negative returns in the Iowa farmland market.

Figure 3 contains time series plots of \( S(t) \) and \( S'(t) \), for the case where \( \theta = 17.75 \). (The corresponding plots for \( \theta = 21.74 \) are virtually identical.) The most obvious feature of the relationship between \( S(t) \) and \( S'(t) \) is the strong degree of negative correlation. Indeed, as shown in Table 3, the sample correlation between \( S(t) \) and \( S'(t) \) was estimated to be \(-.8631\) when \( \theta = 17.75 \) and \(-.8523\) when \( \theta = 21.74 \) with numerically estimated standard errors equal to \(.07\) and \(.245\) respectively.\(^7\) The theory predicts that in the absence of sampling error the correlation should be exactly equal to one. The extreme degree of negative correlation seems to be too large to be attributable simply to sampling error or to measurement errors associated with the use of cash rent to measure net returns to land ownership.\(^8\)

On the basis of Figures 3 and 4, it appears as though the statistical rejection of the present value model arises because there are sustained periods of time during which there are (objective) expectations of persistent positive (or negative) excess returns to holding Iowa farmland.
Furthermore, the sign and magnitude of these expected excess returns vary inversely (directly) with the sign and magnitude of the theoretical (actual) spread. Loosely speaking, Iowa farmland prices tend to be excessively high (low) when the present value of expected future changes in returns are unusually low (high).

One possible explanation of these features of the data is that traders in the Iowa farmland market act in an extremely myopic manner. Suppose that the changes in returns are positively autocorrelated and there occurs a sequence of decreases in real rents (relative to the mean change in rents). The stationarity of these first differences means that the sequences of decreases will probably be offset by a sequence of increases before stabilizing at their mean value. If, however, traders ignore the tendency of runs of decreases to be offset by run of increases and instead assume that recent changes are permanent or will only be exacerbated over time, then the price of land will be driven downward. In other words, although the theoretical spread may be very large due to a large temporary fall in rents, the actual spread may be very small due to the erroneous perception that the fall is permanent.

Another view is offered by comparing the behavior of the actual price, the ex-ante rational price, and actual rents. Figure 4 contains a plot of the actual real price \( Y(t) \), the ex-ante rational real price \( Y^*(t) \), derived according to \( Y^*(t) = S'(t) + \theta y(t) \), and real rents weighted by \( \theta \) \( (\theta y(t)) \). Notice that \( Y^*(t) \) tends to move less-than-proportionally with respect to changes in \( y(t) \) while \( Y(t) \) tends to move more-than-proportionally with respect to such changes. These tendencies are especially apparent during the most volatile periods of the sample period: the 1930's and early 1940's, and the post-1960 period. They suggest that \( \theta y(t) \) will tend to lie
between \( Y(t) \) and \( Y^*(t) \) and, therefore, \( S(t) \) will be negatively correlated with \( S'(t) \).

V. CONCLUSION

In this paper, we have studied the validity of the constant expected return version of the present value model of land prices by applying a strategy recently proposed by Campbell and Shiller [1987] to annual Iowa farmland price and return data over the 1921-1986 sample period. We found that these data fail to satisfy certain key restrictions which the model imposes on the vector autoregressive representation of the changes in real returns and the (unique) stationary linear combination of real price and real returns. Although measurement error and sampling error offer possible explanations of the failure of the model to explain the data, an informal analysis of the data based upon the unrestricted VAR suggests a more fundamental explanation is needed.

The current decade has seen a large growth spurt in the empirical analysis of stock market price fluctuations, sparked by the volatility tests introduced by Leroy and Porter [1981] and Shiller [1981] and fueled by recent developments in the study of nonstationary time series. This literature, whose current state is nicely summarized by West [1988(b)], appears to be moving toward a consensus that the constant expected return version of the present value model is not an appropriate explanation of stock market price determination. In fact, this is the conclusion drawn by Campbell and Shiller in the study which inspired the present paper. As a result, the development of alternative explanations of stock market pricing and strategies for testing these alternatives have been receiving much attention.
This suggests to me that the most promising directions for future research into the determination of land pricing would be those that are currently being pursued in studies of stock market pricing. These directions include the study of present value models with time varying expected returns (as in, e.g., Campbell and Shiller [1988]) and the development and analysis of "fad" models (as in, e.g., Summers [1986]).
<table>
<thead>
<tr>
<th>Cointegrating Regression</th>
<th>Implied Annual Discount Rate</th>
<th>$\hat{\theta}$</th>
<th>ADF Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t = -0.020 + 17.75y_t$</td>
<td>5.6%</td>
<td>17.75</td>
<td>3.69</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(1.07)</td>
<td></td>
</tr>
<tr>
<td>$y_t = 0.005 + 0.046$</td>
<td>4.6%</td>
<td>21.74</td>
<td>2.21</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Estimated standard errors are in parentheses. The ADF value is the Augmented Dickey-Fuller statistic defined in Engle and Granger [1987, pp. 264-270]. Under the null hypothesis that $Y_t$ and $y_t$ are not cointegrated, Engle and Granger's Monte Carlo experiments suggest using a critical value of about 2.75 for a test at the 10 percent significance level.
TABLE 2

Testing the Restrictions of the VAR Representation of $\Delta y(t)$ and $S(t)$

Sample: 1921-1986, Annual Observations

I. $\delta = 17.75$ (5.6% annual discount rate)

Sims' modified likelihood ratio test suggested a 3-lag VAR.

$\Delta y$ equation $R^2 = .522$; $S$ Granger-causes $\Delta y$ at .00001 level.

$S$ equation $R^2 = .825$; $\Delta y$ Granger-causes $S$ at .018 level.

Test of the Present-Value Model without the Mean Restriction:

$\chi^2(6) = 38.271$, p-value = .9945 E-06

Test of the Present-Value Model with the Mean Restriction:

$\chi^2(7) = 39.376$, p-value = .1657 E-05

II. $\delta = 21.74$ (4.6% annual discount rate)

Sims' modified likelihood ratio test suggested a 3-lag VAR

$\Delta y$ equation $R^2 = .517$; $S$ Granger-causes $\Delta y$ at .00001 level.

$S$ equation $R^2 = .819$; $\Delta y$ Granger-causes $S$ at .041 level.

Test of the Present-Value Model without the Mean Restriction:

$\chi^2(6) = 26.872$, p-value = .00015

Test of the Present-Value Model with the Mean Restriction:

$\chi^2(7) = 27.669$, p-value = .00025
TABLE 3

Summary Statistics

I. \( \theta = 17.75 \)

\[
E(\Delta y) = 0.000093, \quad \sigma(\Delta y) = 0.001715 \\
E(S) = -0.02667, \quad \sigma(S) = 0.07407 \\
E(S') = 0.01179, \quad \sigma(S') = 0.04659
\]

\[
\text{Corr}(S,S') = -0.8631 (.07) \\
\text{Var}(S)/\text{Var}(S') = 2.5276 (1.48)
\]

II. \( \theta = 21.74 \)

\[
E(\Delta y) = 0.000093, \quad \sigma(\Delta y) = 0.001715 \\
E(S) = -0.1131, \quad \sigma(S) = 0.08087 \\
E(S') = 0.0871, \quad \sigma(S') = 0.06365
\]

\[
\text{Corr}(S,S') = -0.8523 (.245) \\
\text{Var}(S)/\text{Var}(S') = 1.613 (2.59)
\]

Note: Numerically estimated standard errors are in parentheses.
FIGURE 1
Real Cash Rents: Levels and Changes
1922-1986

\[ y(t), \text{real rents} + y(t) - y(t-1) \]
FIGURE 2
Real Land Prices: Levels and Changes
1922-1986

\[ Y(t), \text{ real price} + Y(t) - Y(t-1) \]
FIGURE 3
Actual and Theoretical Spreads
1924-1986

\[ S(t), \text{Actual Spread} \quad + \quad S'(t), \text{Theoretical Spread} \]
FIGURE 4

Actual Real Prices, Predicted Real Prices, and Weighted Real Rents
1924-1986
1. The procedures that will be used in this paper can easily be adapted to the case where \( y(t) \) is not known until after \( Y(t) \) is determined. For example, in Campbell and Shiller's 1987 study of the stock market, \( Y(t) \) measured beginning-of-the-period prices and \( y(t) \) measured during-the-period dividends, which are usually not known at the start of the period. The modifications that account for this problem are described by Campbell and Shiller (p. 1074). West [1988(a)] has designed a procedure to test the present value model when \( y(t) \) and \( Y(t) \) are first difference stationary and when \( I(t) \) need not be larger than \( \{y(t-1), y(t-2), \ldots \} \).

2. Two stochastic processes \( x(t) \) and \( y(t) \) are defined to be cointegrated of order \((1,1)\) if both processes are stationary in their first differences and if there exists a linear combination of the levels of \( x(t) \) and \( y(t) \) which is stationary. Thus, if \( Ay(t), AY(t), \) and \( S(t) \) are stationary processes and \( S(t) \) is equal to \( Y(t) - \Theta y(t) \), then \( y(t) \) and \( Y(t) \) are cointegrated of order \((1,1)\). The vector \([1,-\Theta]\) is called the cointegrating vector and it is unique up to a scale transformation. See Engle and Granger [1987] for a more complete discussion of cointegrated processes.

3. The land price data that were used in this study were spliced together from USDA survey data (1921-1949) and the Iowa Land Value Survey (1950-1986), which are described more fully in the data appendix. The theoretical model assumes that \( Y(t) \) is the beginning of the period price. Although it is not possible to associate a particular part of the year to which the USDA's price measure most closely corresponds, the price reported
for year t by the Iowa Land Value Survey is a fourth quarter of the year (November of year t) price. The results reported in the main body of this paper are based on using the cash price reported in year t to form the measure of Y(t). However, the empirical procedures were also conducted using the price reported for year t-1, divided by the price deflator associated with year t, to form the measure of Y(t). The results obtained in this case, which are available from the author, tended to exaggerate the negative conclusions that are reported in this paper.

4. An interesting property of cointegrated processes is that the unknown parameter of the cointegrating vector \([l, \alpha]\) can be consistently estimated as the regression coefficient in a regression of Y(t) on y(t) and it can be consistently estimated by the reciprocal of the regression coefficient in a regression of y(t) on Y(t). Engle and Granger recommend that their cointegration tests be conducted on the basis of both estimates of the cointegrating vector.

5. The mean annual rate of return in this market over the sample period was 5.7 percent.

6. There is a finite sample problem associated with the Wald test in that alternative algebraically equivalent forms of a set of restrictions can lead to quite different test results. (See, e.g., Phillips and Park [1988].) My application of Wald tests of the nonlinear form of the cross-equation restrictions given by equation (7) in Campbell and Shiller [1987] did not alter my conclusion that the cross-equation restrictions can be rejected at the one-percent level. Furthermore, the discussion in Section IV of this
paper suggests that these rejections are not likely to be explainable in terms of small sample problems with Wald tests.

7. These standard errors were evaluated numerically conditional upon the sample mean of \( S(t) \), its sample variance, and the estimated values of \( \theta \) and \( \delta \).

8. It is interesting to compare these results with the results obtained by Campbell and Shiller [1987] in their study of stock market prices. When they used the sample mean annual rate of return (8.2 percent) to estimate \( \theta \), they computed a correlation between the theoretical and actual spreads of -.46.

9. This explanation and the use of Figure 4 to support it were suggested to me by Bob Shiller who has conjectured that a similar phenomenon accounts for the negative correlation observed in the stock market's actual and theoretical spreads.
DATA APPENDIX

I. Land Prices

The time series of average annual prices per acre of Iowa farmland was constructed by splicing the United States Department of Agriculture's Farm Real Estate Dollar Value, by state, series (1921-1949) with Iowa State University's Extension Service's Iowa Land Value Survey series (1950-1986). These series provide summary measures of the average dollar value per acre of whole farms (i.e., land and buildings) being sold in Iowa. The USDA's data series is described more fully in Barnard and Hexum [1988]. The Iowa Land Value Survey data are described in the Iowa State University Extension Service's FM-1825 publications.

II. Cash Rent

The cash rent time series are average annual dollar rent paid per acre for the rental of whole Iowa farms. These data are estimates produced by the Iowa Crop and Livestock Reporting Service (now known as Iowa Agricultural Statistics Service) based upon their own surveys and are published by the USDA's Economic Research Service.

III. Price Deflator

The raw cash rent and land price series were converted into real rent and value, respectively, by dividing each series by the Consumer Price Index (all items, 1967 = 100) for that year. The CPI data were collected from the U.S. Commerce Department's publication Historical Statistics of the United States: Colonial Times to 1970 (1921-1970) and from various issues of its Business Statistics.

These data are available from the author upon request.
REFERENCES


